Auctioning Corporate Bonds: A Uniform-Price under Investment Mandates

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Abstract

This paper examines how budget and risk limits within investment mandates affect the bidding strategy in a uniform-price auction for issuing corporate bonds. I prove the existence of symmetric Bayesian Nash equilibrium and explore how the risk limits imposed on the mandate may mitigate severe underpricing, as the symmetric equilibrium's yield positively relates to the risk limit. Investment mandates with low-risk acceptance inversely affect the equilibrium bid. The equilibrium bid provides insights into the optimal mechanism for pricing corporate bonds conveying information about the bond's valuation, market power, and the number of bidders. These findings contribute to auction theory and have implications for empirical research in the corporate bond market.

Keywords: Bond Market, Auctions, Investment Mandates, Market Design

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1. Introduction

Over the past decade, the use of investment mandates for investment-grade bonds has gained prominence, providing asset managers with strict guidelines for allocations within this asset class[2]. Despite these structured guidelines, the current practice of offering corporate bonds through post-pricing selling[3] often results in significant underpricing and allocation inefficiencies (Benveniste and Spindt, 1989; Cornelli and Goldreich, 2001; Jenkinson and Jones, 2004; Bessembinder et al., 2020).

To address those issues, this study explores the potential of uniform price auctions to enhance the pricing efficiency of corporate bonds in the primary market, with a particular focus on the participation of asset managers operating under specific investment mandates. In this auction format, asset managers submit bids specifying the amount of capital they intend to invest at various interest rates, aiming to resell the bonds in the secondary market. The return on investment for these allocated bonds is derived from the spread between the issuance's yield (purchase price) and the expected secondary market yield (sale price). The bidding strategies employed by asset managers are predominantly influenced by their budget constraints, within the framework of investment-grade bond mandates.

[2]For example, Invesco Global Investment Grade Corporate Bond Fund disseminated a prospectus to potential clients explicitly describing the fund's mandate to invest a portion of its total assets in investment-grade corporate bonds (https://www.invesco.nl/dam/jcr:cb6153ab-0bd2-47fd-80da-8d96921bd6f3/LU1075208725_EN_NL.pdf).

[3]Post-pricing selling, also known as book-building, involves the underwriter marketing the bond issuance to potential investors and gathering non-binding indications of interest before setting the final price. This process can lead to mispricing and inefficient allocation of the bonds.

This research contributes to the literature by examining the integration of a budget constraint and a common knowledge risk limit into the bidding strategy, resulting in a nonunique symmetric Bayesian Nash equilibrium. The equilibrium bid is decomposed into two main components, reflecting both the minimum bid linked to the risk limit and additional strategic factors such as market power and competition. The market power arises endogenously in the equilibrium bid, aligning with existing auction theory (Back and Zender, 1993; Wilson, 1979). The findings suggest that the risk limit, by imposing a boundary on demand, can mitigate underpricing in uniform auctions and book-building processes Kremer and Nyborg (2004a); Cai et al. (2007). This effect holds for downwardly bounded continuous bidding strategies (Kremer and Nyborg, 2004b).

While this study provides important insights, it is based on certain simplifying assumptions. Notably, the model assumes that the only private information of bidders is the budget limit, which they truthfully report through this mechanism, while the risk component for investment-grade bonds is common knowledge. The model does not account for potential correlations among bidders' budget limits or consider asymmetric risk limits. The secondary market yield is treated as an exogenous variable, independent of the bidders' private information.

The existing literature has primarily focused on the pricing of government bonds using two auction formats: uniform or discriminatory[4]. However, neither empirical research (Nyborg and Sundaresan, 1996; Tenorio, 1997; Binmore and Swierzbinski, 2000), nor auction

[4]This is a sealed bid auction for homogeneous items in which each winner pays an amount equal to the sum of the bids placed for the items allocated.

theory (Wilson, 1979; Back and Zender, 1993; Wang and Zender, 2002; Bikhchandani and Huang, 1993; Ausubel et al., 2014) provides a compelling reason to prefer discriminatory auctions over uniform auctions.

Another strand of literature examines fixed-price offerings versus uniform price auctions for initial public offerings (IPOs) (Biais and Faugeron-Crouzet, 2002; Sherman, 2005; Wang, 2021; Benveniste and Spindt, 1989). Experiments have shown that uniform price auctions are more effective than fixed-price offerings in raising revenue and outperform post-pricing selling (Zhang, 2009; Wang, 1998). Fixed-price offerings tend to suffer from underpricing and allocation inefficiencies, similar to the issues observed in post-pricing selling for corporate bonds (Bessembinder et al., 2020; Cornelli and Goldreich, 2001; Jenkinson and Jones, 2004). Despite these findings, the application of auction theory to corporate bond issuance remains relatively underexplored.

The paper is organized as follows. The following section 2 contains a formal analysis and describes the model as a direct revelation mechanism. In the same section, I introduce the concept of risk limit. Section 3, studies bidders' incentives and provides the proof of a Bayes-Nash symmetric equilibrium for independent signals performing the respective comparative statics. The last sections, 4 and 5, discuss the previous sections' outcomes and conclude. Some proofs are expanded on Appendix A.

2. Model

2.1. Preliminaries

I assume a common value auction for the sale of a single unit of perfectly divisible bond with a face value equal to one, and n competitive bidders, defined as a finite set $\mathcal{I}=\{1, 2, 3 \ldots n\}$, with $n \geq 3$. All bidders are risk-neutral and none of them is eligible to bid for the full face value of the bond.

Each bidder i has an upper-bound bidding stipulated by the investment mandate, defined as the budget limit $c_i \in [\underline{c}, \bar{c}]$, as well as a risk limit $\ell_i \in [\underline{r}, \bar{r}]$. The type of i is defined as $\tau_i = (c_i, \ell_i)$, with $\tau \in \mathcal{T}$, which attributes bidders' preferences $\mathcal{T} := [\underline{c}, \bar{c}] \times [\underline{r}, \bar{r}]$ to the eligible real intervals. To simplify, I assume that the risk limit ℓ^* is symmetrical, and common knowledge among all bidders, so only the budget limit c is private information. Therefore, the type of bidder i is $\tau_i = (c_i, \ell^*)$, with a corresponding type space $\mathcal{T} := [\underline{c}, \bar{c}]$. The budget limit c is independently drawn from a continuous distribution $F(c)$, with a density $f(c)$ which is common knowledge among all bidders. Given that ℓ^* is common knowledge, the joint distribution of types simplifies to $f(c, \ell = \ell^*) = f(c)$.

Each type $\tau_i = (c_i, \ell^*)$ has a bidding strategy $b_i(r|c_i)$ that specifies the quantity[5] demanded by bidder i, at the interest rate r, given his type c_i. Since the bond is perfectly divisible, the demand is expressed as a fraction or percentage of the total amount being

[5]Since the bond is perfectly divisible and bidders cannot bid for the entire face value, each bidder will specify a desired quantity of the bond, expressed as a fraction or percentage of the total amount being issued.

issued. This strategy essentially describes a demand function, mapping each interest rate r to a corresponding quantity demanded:

$$b_i(r|c_i) : [\underline{r}, \bar{r}] \to [0, 1] \tag{1}$$

The aggregate demand across all bidders at any given interest rate r is then the sum of all individual demands, expressed as $D(b) = \sum_{i=1}^{n} b_i(r|\tau_i)$.

Assumption 1. All bid strategies are assumed to be continuously differentiable, decreasing to the interest rate r, and an increasing continuous function in the budget limit c.

The issuance yield y functions as an inverse aggregate demand function. It is the highest yield where the total demand $D(b)$ meets or exceeds the supply (i.e., $D(b) \geq 1$).

$$y = \sup\{r \in [\underline{r}, \bar{r}] \mid D(b) \geq 1\}$$

In this context, let $\Theta = \max_{i \in I} \ell_i$ represent the maximum risk limit among the bidders and serves as a benchmark interest rate, often associated with "high-risk" or "junk" bonds, where $b(\Theta|c_i) = 0$. Another parameter θ is an exogenous sensitivity factor that indicates how much the issuance yield decreases per unit increase in aggregate demand $D(b)$. It is assumed that θ is symmetric for all bidders, implying all bidders have an equal impact on the yield's structure.

Given these parameters, the issuance yield y can be specifically expressed as:

$$y = \begin{cases} \Theta - \theta D(b) & \text{, if } D(b) \geq 1, \text{ with } b > 0,\ \theta < \Theta,\ \theta D(b) < \Theta \\ 0 & \text{, otherwise} \end{cases} \tag{2}$$

This linear rule ensures that the issuance yield decreases with an increasing aggregate demand D reflecting the competitive pressure in the auction.

Next, I define the allocation α_i of each bidder i who participates in the auction. This allocation rule establishes how the bonds are distributed among the bidders based on their bid profiles.

Definition 1. Under the absence of ties, the allocation rule is a one-to-one mapping from the set of bid schedules' profiles $b_i(\cdot)_{i=1}^{n}$ to non-negative allocations $\alpha_i \in (0,1)$, $\forall i \in \mathcal{I}$ with $\alpha(0) = 0$, such that $D(b) = 1$. For non-winners $\alpha_j = 0$, $\forall j \in \mathcal{I}$.

All bidders participating in the auction anticipate to sell the bond in the secondary market. Ex-ante, the yield of the secondary market, denoted by s, is an unknown exogenous random variable within the interval $[\underline{r}, \bar{r}]$. The expectation of the secondary market yield is denoted by $\mathbb{E}[s]$.

The payoff function $\mathbb{E}_{b_{-i}}[\pi_i]$ represents the expected profit of a risk-neutral bidder i given a common knowledge risk limit ℓ^* and their private budget limit c_i. This expectation is taken over the possible strategies of the other bidders $b_{-i}(\cdot|c_{-i})$, where c_{-i} represents the private budget limits of the other bidders:

$$\mathbb{E}_{b_{-i}}[\pi_i(b_i)] = \mathbb{E}_{b_{-i}(\cdot|c_{-i})}\Big[\Big(y(b_i, b_{-i}) - \mathbb{E}[s]\Big)\alpha_i(b_i, b_{-i})\Big] \tag{3}$$

Each bidder anticipates gaining a positive spread[6] over the issuance i.e. the difference between the issuance yield $y(b_i, b_{-i})$, and the expected secondary market yield $\mathbb{E}[s]$. This spread represents the profit margin of each bidder and neutralizes when there is a high demand for the bond. If $y \geq \mathbb{E}[s]$ the bidder expects to sell the bond in the secondary market with a nonnegative payoff.

2.2. Market Mechanism

The auction is designed to satisfy the revelation principle. Given the common knowledge and symmetry of the risk limit ℓ^*, the only information that is truthfully reported is the budget limit c.

Lemma 1. For each bidder, i, truthfully revealing the budget limit c_i is a dominant strategy incentive compatible.

Proof. Suppose bidder i misreports the budget limit $\tilde{c}_i$ such that $\tilde{c}_i > c_i$. By Assumption 1, the bid corresponding to the misreported budget limit $\tilde{c}_i$ is $\tilde{b}(\tilde{c}_i) \geq b(c_i)$ and bidder i is allocated a higher share over the issuance, $\tilde{\alpha} \geq \alpha$. Given that $\theta < \Theta$ from equation (2), the yield decreases with an increase in the aggregate demand, resulting from $\tilde{b}(\tilde{c}_i) \geq b(c_i)$, i.e. $\tilde{y} \leq y$. Thus, each extra allocated unit of the bond is less profitable, $\tilde{y} - \mathbb{E}[s] \leq y - \mathbb{E}[s]$.

[6]The inverse relationship between bond prices and yields is fundamental in bond pricing. When a bond is issued at a higher yield, its price is lower, making it cheaper for the bidder to purchase. Conversely, if the secondary market yield decreases after issuance, the bond's price increases because the present value of its future cash flows is discounted at a lower rate. This allows the bidder to sell the bond at a higher price than the purchase price, capturing the positive spread.

The reduction in profit per bond due to the decreased margin results in a lower overall nonnegative expected payoff, $(\tilde{y} - \mathbb{E}[s])\tilde{\alpha} \leq (y - \mathbb{E}[s])\alpha$. If the budget limit $\tilde{c}_i < c_i$, then bidder i will not be included among the winners. This results in a lower nonnegative payoff for the bidder compared to truthful reporting. So, regardless of the other bidders' strategies, bidder i cannot strictly benefit from misreporting the budget limit.

Example 1. Suppose 3 bidders $I = \{1, 2, 3\}$, with a bidding function $b_i(r|\tau_i) = c_i(1 - \frac{r}{\ell^*})$. All bidders invest their budget c_i and have the same mandate to invest in AA rating bond, i.e., a symmetric risk limit $\ell^* = 500$. For symmetrical $\theta = 0.03$ and $\Theta = 0.05$ all information concerning the bidding strategies is listed in Table 1.

Bidder	1 (AA)	2 (AA)	3 (AA)
c_i	0.5	0.4	0.6
ℓ^*	500	500	500
r	160	200	150
$b(r\|\tau)$	0.34	0.24	0.42

Table 1: Competitive bids

The auctioneer starts with bidder 3 and allocates the demanded bonds, $b_3 = \alpha_3 = 42\%$ of the issuance at $r_3 = 150bps$. This leaves 58% of the issuance yet to be allocated. The next bidder is 1 with $b_1 = \alpha_1 = 34\%$. The final is bidder 2 with $b_2 = \alpha_2 = 24\%$. After bidder 2 the issuance is fully covered because $D(b) = 1$ and $y = 200bps$. Now, suppose that bidder 1 misreports a budget limit $\tilde{c_1} = 0.8$ at $r_1 = 160bps$. Then $\tilde{b_1} = \tilde{\alpha_1} = 54\%$ leaving 46% of the issuance for allocation. The bids of bidder 3 and 2 are the same as before however now bidder 2 will be allocated the remaining 4%. At $r = 200bps$ the aggregate demand is

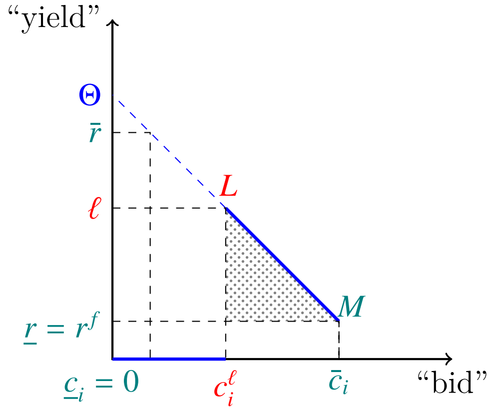

Figure 1: Aggregate Inverse demand curve with risk and budget constraints.

$\tilde{D}(b) = 1.20$. This corresponds to $\tilde{y} = 140bps$, based on Equation 2. If $\mathbb{E}[s]$=100 bps, then the margin from misreporting for bidder 1 is $\tilde{y} - \mathbb{E}[s] = 40$bps, while with truthfully bidding is $y - \mathbb{E}[s] = 100$ bps, and the respective payoffs are $(\tilde{y} - \mathbb{E}[s])\tilde{\alpha}$=0.0022 and $(y - \mathbb{E}[s])\alpha$=0.0034.

2.3. The impact of risk limit

Now, let's explain how the risk limit affects the infimum bidding amount and the payoff of each asset manager. Suppose bidder i has to comply with an investment mandate with a supremum risk $\ell \in [r^f, \bar{r}]$, where r^f is the risk-free rate, and the budget limit of bidder i is $c_i \in [0, \bar{c}]$, with c_i^ℓ to be the infimum bid associated with the risk limit ℓ. Figure 1 plots the inverse aggregate demand function from equation 2. The vertical axis denotes the different "yield" levels associated with the bid. The horizontal axis depicts the amount that each asset manager i is willing to "bid" given his budget limit c_i.

Bidders who participate in the auction will always demand a high stake at a low yield to maximize payoffs subject to the expectation for the secondary market. If all auction participants demand a high stake over the issuance, i.e., $c_i = \bar{c}$ the yield will be further reduced up to $\underline{r} = r^f$. However, at point M, bidders are indifferent to investing in issuance with a yield equal to the risk-free rate, so $b(\bar{c}) = 0$. In reverse, if all auction participants demand the infimum stake over the issuance, i.e., $c_i = c^\ell$, the yield will be set at the supremum risk limit ℓ.

From the individual demand perspective, the infimum demand of bidder i is c_i^ℓ, including an interest rate ℓ in his bidding. His incentive is to exhaust budget c_i in an investment-grade bond, requesting a relatively low yield that will ensure not only his winning but also a positive spread given the secondary market. Thus, the bidder will bid between c_i^ℓ and $\bar{c}$ for an interest rate r that would result in a nonnegative spread and simultaneously ensure winning. The shaded area in Figure 1 expresses bidders' willingness to participate in the auction and defines the boundaries for the asset manager's payoff.

Example 2. Consider bidder 1 from Example 1 with $\ell = 500$ and $\bar{c} = 0.5$.

c_i	0.5	0.4	0.3	0.24
r	100	200	300	499
$b(r\|c_i)$	33%	20%	10%	0.06%

In Example 1, the issuance yield at D=1 is y = 200 bps. Bidder 1 invests all his budget $c_i = 0.5$ for an interest rate $r = 100$ bps. This bid ensures winning, however, he becomes indifferent between investing in the bond as the spread with the secondary market becomes zero, while for an interest rate $r = \ell = 500$ bps the bid will be zero. For $r \approx 500$ bps, the infimum budget is $c^\ell = 0.3$ associated with a bid that approaches zero.

3. Existence of symmetric equilibrium

In this section, we assume that all bidders adopt the symmetric bidding strategy $b^*(\cdot)$, which maximizes their expected payoff and we prove the existence of symmetric equilibrium. The minimum bid is defined by $\lambda \in (0, 1)$, where $b(c^{\ell^*}) = \lambda$ with an allocation equal to $\alpha(c^{\ell^*})$. The strategy profile of the remaining $n - 1$ bidders, except for bidder i, is captured in the random variable $x = x^{n-1}$.

The expected profit of bidder i from equation (3) is given by:

$$\mathbb{E}(\pi_i) = \int_{\mathbf{c}^{\ell}}^{\bar{\mathbf{c}}} \alpha_i(b_{c_i}, b_{c_{-i}}) \Big[y(b_{c_i}, b_{c_{-i}})) - \mathbb{E}[s] \Big] f_{x|c_i} dx \tag{4}$$

where $\bar{\mathbf{c}} = \max_{j \in N/\{i\}} \bar{c}_j, \mathbf{c}^{\ell^*} = \max_{j \in N/\{i\}} c_j^{\ell^*}$ respectively.

Theorem 1. The $n - tuple$ $(b^*, \ldots, b^*)$ is a symmetric Bayes-Nash equilibrium under uniform-price auctions when bidders follow the same bidding strategy b^* given a symmetric budget limit c^* and a risk limit ℓ^*. When $\xi = \dfrac{\theta}{\Theta - \mathbb{E}[s]}$ and $\xi < \frac{1}{\lambda n}$ the equilibrium bidding strategy is given by:

$$b^*(c^*) = \lambda \frac{\alpha(c^{\ell^*})}{\alpha(c^*)} + \frac{1}{\xi n} \left[1 - \frac{\alpha(c^{\ell^*})}{\alpha(c^*)} \right], \tag{5}$$

with $c^* \in [c^{\ell^*}, \bar{c}]$.

Proof. See the Appendix A. □

Similar to a symmetric Cournot oligopoly, increasing the number of bidders lowers the equilibrium bidding strategy. The benchmark rate captured in parameter Θ is used to calculate a spread from the asset manager's expectation for the secondary market. The parameter θ in the ξ attributes the oligopolistic effect of bidders upon the issuance yield. As expected, the instructions on the investment mandate directly impact the bidding strategy through the minimum bid λ and allocation $\alpha(c^{\ell^*})$. Additionally, the ratio of the minimum allocation $\alpha(c^{\ell^*})$ to symmetric allocation $\alpha(c^*)$ affects the equilibrium bid.

Next, I provide some basic comparative statics.

Corollary 1. In symmetric equilibrium, the higher the oligopolistic power of bidders upon the market clearance yield, the lower the equilibrium bid.

Proof. The result follows trivially since the equilibrium bid depends inversely on θ. □

Recall that the market power is the slope of inverse demand function (2) and depends on the risk limit ℓ and the infimum budget that an investment manager is willing to invest in the issuance (Figure 1). An asset manager with strong market power has low-risk acceptance (ℓ) and may induce equilibrium to a lower market clearance yield.

Corollary 2. In a Bayesian Nash symmetric equilibrium, the issuance yield is given by,

$$y = \mathbb{E}[s] + (\ell^* - \mathbb{E}[s])\frac{\alpha(c^{\ell})}{\alpha(c^*)},$$

where ℓ^* is the symmetric risk limit for a symmetric minimum bid $b^*(c^{\ell}) = \lambda^*$.

Proof. See the Appendix Appendix B. □

Since bidding strategies respond to investors' budgets, the issuance yield in the symmetric equilibrium reflects information about the bond's value concerning the risk limit imposed on the mandate. The maximum spread an asset manager can earn from the resale in the secondary market seems to have a positive relationship. The value of the bond from equation (4) and Corollary 2 in the symmetry, equals to $(\ell^* - \mathbb{E}[s])\frac{\alpha(c^\ell)}{\alpha(c^*)}$. It is easy to conclude that the risk limit is a boundary for the issuance yield. An investment mandate with low-risk acceptance, ceteris paribus, moves ℓ downwardly, and the ratio $\frac{\alpha(c^\ell)}{\alpha(c^*)}$ declines (see Figure 1). This can further decrease the issuance yield in the symmetric equilibrium.

From corollary 2 and Figure 1, the symmetric budget limit on the mandate, ceteris paribus, has a negative relation to the symmetric issuance yield. Bidders exhaust the available budget set by the investment mandate, i.e., up to $\bar{c}$. From the inverse demand function in Figure 1 the issuance yield reaches its lower bound $\underline{\mathrm{r}}$.

Proposition 1 shows that a high demand guided by narrow investment mandates would allocate the bond to more bidders since any bidder who participates in the issuance in the limit would bid the minimum.

Proposition 1. In symmetric equilibrium, ceteris paribus, as the risk limit of bidders become strict (lower), i.e., ℓ goes to r^f, in the limit, the equilibrium bid equals λ.

Proof. By the yield function, lower yields are associated with higher bids. I.e., for a decreasing sequence $(\ell_k)_{k\in N}$ corresponds to an increasing sequence $(c_k^\ell)_{k\in N}$. By letting c^ℓ to increase and given that $c^\ell < c^*$ then the ratio $\frac{\alpha(c^\ell)}{\alpha(c^*)}$ approaches to one. By equation (5) the result follows immediately. □

4. Discussion

This analysis reveals that the equilibrium is not unique, a characteristic commonly observed in uniform auctions (Ausubel et al., 2014). The risk limit sets a boundary in demand, protecting the issuance from underpricing, which is a common issue in uniform auctions Kremer and Nyborg (2004a) and in the book-building process of corporate bonds (Cai et al., 2007).

The equilibrium bid can be decomposed into two main components, each reflecting distinct strategic considerations. The first component, $\lambda\frac{\alpha(c^{\ell^*})}{\alpha(c^*)}$, captures how bidders adjust their bids based on their budget limits and the auction's competitive structure. The parameter λ represents a baseline level of participation, ensuring that all bidders contribute a minimum bid linked to the risk limit imposed by the investment mandate. The ratio $\frac{\alpha(c^{\ell^*})}{\alpha(c^*)}$ measures the sensitivity of the bidding strategy to variations in budget limits. Ceteris paribus, a higher ratio suggests that even bidders with lower budgets can secure a reasonable allocation, leading to more conservative bidding behavior. Conversely, a lower ratio indicates a more competitive environment, where bidders must bid more aggressively to secure their desired allocation.

The second component, $\frac{1}{\xi n}\left(1-\frac{\alpha(c^{\ell^*})}{\alpha(c^*)}\right)$, reflects additional strategic factors influencing bidding behavior, including the bidder's market power, competition level, and the spread between expected yields in the secondary market and benchmark bond yields. A smaller ξ or a larger n encourages more aggressive bidding due to increased competitive pressure, while the factor $\left(1-\frac{\alpha(c^{\ell^*})}{\alpha(c^*)}\right)$ further adjusts the strategy based on the relative position of a bidder's budget limit within the auction. Moreover, asset managers with low-risk acceptance provide stronger market power, leading to a downward adjustment in bids, similar to dynamics observed in a Cournot oligopoly. While this market power arises endogenously within the equilibrium bid, the risk limitations imposed by the mandate can mitigate underpricing, especially under downwardly bounded continuous bidding strategies (Kremer and Nyborg, 2004b). If bidders anticipate lower yields (higher prices) in the secondary market, they increase their demand for the issuance. Conversely, if they expect yields close to those of high-risk bonds, the equilibrium strategy leans more heavily on the infimum bid component.

This study is based on certain assumptions which simplify the analysis but also introduce limitations. Notably, the model does not account for potential correlations among bidders' types or asymmetric risk limits. Additionally, the secondary market yield is treated as an exogenous variable, independent of the bidders' private information. Future research could explore the impact of relaxing these assumptions, particularly by introducing correlations between bidders' types and examining the effects of asymmetric risk limits on the equilibrium strategy.

5. Conclusion

This study develops a symmetric Bayesian Nash equilibrium model for pricing corporate bonds, incorporating key elements of the primary market and constraints imposed by investment mandates. The analysis demonstrates that the equilibrium, while not unique—an expected outcome in uniform auctions—reveals how risk limits can effectively reduce the risk of underpricing, a common issue in both uniform auctions and the book-building process for corporate bonds. While the model is based on certain simplifying assumptions, such as the use of symmetric risk limits and independent budget constraints, these aspects provide a foundation for future research. Further exploration of these assumptions could lead to a deeper understanding of their impact on the equilibrium strategy and corporate bond pricing practices.

Appendix A. Proof of symmetric equilibrium

By Lemma 1, bidders directly reveal their types by announcing their budget limits. With a slight abuse of notation $b(\cdot|c, \ell^*) = b(c)$.

Thus, the expected profit from (4) can be rewritten as:

$$\mathbb{E}(\pi_i) = \int_{c^\ell}^{\bar{c}} \alpha_i(c_i, x) y(b_i(c_i), b_{-i}(x)) f(x|c_i) dx - \mathbb{E}[s] \int_{c^\ell}^{\bar{c}} \alpha_i(c_i, x) f(x|c_i) dx$$

By the continuity property of the distribution F, the probability of CDF for each bidder i to bid a budget, $c_i \leq c_i^\ell$, equals zero. Because none of the bidders will place a bid above their risk limit ℓ (Figure 1). This means that when $b_{-i}(c^\ell, \ell) = a_i(c^\ell)0$, the bond will not be issued, as none of the bidders can buy the whole issuance. In other words, the yield $y(b_i(c_i), b_{-i}(c^\ell)) = y(b_i(c_i), 0)) = 0$.

Applying integration by parts:

$$\begin{aligned}\int_{c^\ell}^{\bar{c}} \alpha_i(c_i, x) y(b_i(c_i), b_{-i}(x)) f(x|c_i)\, dx = \alpha_i(c_i, \bar{c}) y(b_i(c_i), b_{-i}(\bar{c})) F(\bar{c}|c_i) \\ - \int_{c^\ell}^{\bar{c}} F(x|c_i) \left(\frac{\partial \alpha_i(c_i, x)}{\partial x} y(b_i(c_i), b_{-i}(x)) \right. \\ \left. + \alpha_i(c_i, x) \frac{\partial y(b_i(c_i), b_{-i}(x))}{\partial x} \right) dx\end{aligned}$$

Combining parts, the expected payoff can be rewritten:

$$\begin{aligned}\mathbb{E}(\pi_i) = {} & \alpha_i(c_i, \bar{c}) y(b_i(c_i), b_{-i}(\bar{c})) F(\bar{c}|c_i) \\ & - \int_{c^\ell}^{\bar{c}} F(x|c_i) \left(\frac{\partial \alpha_i(c_i, x)}{\partial x} y(b_i(c_i), b_{-i}(x)) + \alpha_i(c_i, x) \frac{\partial y(b_i(c_i), b_{-i}(x))}{\partial x} \right) dx \\ & - \mathbb{E}[s] \alpha_i(c_i, \bar{c}) F(\bar{c}|c_i)\end{aligned}$$

The optimization problem is:

$$\begin{aligned}\max_{c_i} \mathbb{E}(\pi_i) = {} & \alpha_i(c_i, \bar{c}) y(b_i(c_i), b_{-i}(\bar{c})) F(\bar{c}|c_i) \\ & - \int_{c^\ell}^{\bar{c}} F(x|c_i) \left(\frac{\partial \alpha_i(c_i, x)}{\partial x} y(b_i(c_i), b_{-i}(x)) + \alpha_i(c_i, x) \frac{\partial y(b_i(c_i), b_{-i}(x))}{\partial x} \right) dx \\ & - \mathbb{E}[s] \alpha_i(c_i, \bar{c}) F(\bar{c}|c_i)\end{aligned}$$

$$\text{s.t.} \int_{c^\ell}^{\bar{c}} F(x|c_i) \left(\frac{\partial \alpha_i(c_i, x)}{\partial x} y(b_i(c_i), b_{-i}(x)) + \alpha_i(c_i, x) \frac{\partial y(b_i(c_i), b_{-i}(x))}{\partial x} \right) dx \leq 0$$

Ex ante, at the optimum, the expected yield cannot be further diminished, hence the aforementioned constraint is satisfied with equality. For the symmetric common type c^*, the cumulative distribution function satisfies $F(\bar{c} = c^*|c^*) = 1$. To be a symmetric Bayesian Nash equilibrium, the first-order conditions must be zero.

$$
\begin{aligned}
0 &= \frac{\partial \mathbb{E}(\pi_i)}{\partial c_i}\bigg|_{(c=c^*)} \\
&= \left(\Big[y(b_i(c_i), b_{-i}(\bar{c}))F(\bar{c}|c_i) - \mathbb{E}[s]F(\bar{c}|c_i)\Big]\alpha_i(c_i, \bar{c})\right)' \\
&= \alpha_i'(c^*)y(b_i(c^*), b_{-i}(\bar{c} = c^*))F(\bar{c} = c^*|c^*) \\
&\quad - \alpha_i'(c^*)\mathbb{E}[r^s]F(\bar{c} = c^*|c^*) \\
&\quad + \alpha_i(c^*)y'(b_i(c^*), b_{-i}(\bar{c} = c^*))F(\bar{c} = c^*|c^*) \qquad \text{(A.1)}
\end{aligned}
$$

I substitute (2) to (A.1) and for simplicity reasons, I denote $\rho(c^*) = \frac{\alpha_i'(c^*)}{\alpha_i(c^*)}$, which is the relative rate of change for the symmetric allocation $\alpha_i = \alpha^*$, and with b^* the symmetric bidding strategy. Thus,

$$
\rho(c^*)\,[\Theta - n\,\theta\,b^*(c^*)] + [-\theta\,n\,b^{*\prime}(c^*)]\; - \rho(c^*)\,\mathbb{E}[s] = 0
$$

Denominating with $(-\theta n)$ and by substitution of $\xi = \dfrac{\theta}{\Theta - \mathbb{E}[s]}$, where $\xi < \frac{1}{\lambda n}$, I result to a first-order non-homogeneous differential equation:

$$
b^{*\prime}(c^*) + \rho(c^*)\,b^*(c^*) = \frac{\rho(c^*)}{\xi\,n}
$$

The solution to the first-order differential equation is given by

$$
\begin{aligned}
b^*(c^*) &= \mathrm{e}^{-\int \rho(c^*)\,dc^*}\Big(\int \mathrm{e}^{\int \rho(c^*)\,dc^*}\,\frac{\rho(c^*)}{\xi\, n}\; dc^* \;+\Gamma\Big)\\
&= \mathrm{e}^{\ln \alpha^{-1}(c^*)}\Big(\int \mathrm{e}^{\ln \alpha(c^*)}\frac{\rho(c^*)}{\xi\, n}\; dc^* \;+\Gamma\Big)\\
&= \frac{1}{\alpha(c^*)}\Big(\int \frac{\alpha'(c^*)}{\xi\, n}\; dc^* \;+\Gamma\Big)\\
&= \frac{1}{\alpha(c^*)}\Big(\frac{\alpha(c^*)}{\xi\, n} \;+\Gamma\Big)
\end{aligned}
$$

where Γ is an arbitrary constant. Thus, I conclude

$$
b^*(c^*) = \frac{1}{\xi n} + \frac{\Gamma}{a(c^*)},\ \text{with}\, c^* \in [c^\ell, \bar{c}] \tag{A.2}
$$

Now since $b(c^\ell) = \lambda$ is the initial condition of the differential equation, then the value of the constant $\Gamma = \alpha(c^\ell)[\lambda - \frac{1}{\xi n}]$, where $\alpha(c^\ell) \in (0, 1)$ and corresponds to the minimum allocation of the winning bidder. Thus, the solution of equation (A.2) is unique and can be re-written:

$$
\begin{aligned}
b^*(c^*) &= \frac{1}{\xi\, n} + \frac{\alpha(c^\ell)[\lambda - \frac{1}{\xi\, n}]}{a(c^*)}\\
&= \frac{1}{\xi\, n} + \frac{\alpha(c^\ell)\,\lambda}{\alpha(c^*)} - \frac{\alpha(c^\ell)}{\alpha(c^*)\,\xi\, n}\\
&= \lambda\frac{\alpha(c^\ell)}{\alpha(c^*)} + \frac{1}{\xi\, n}\Big[1 - \frac{\alpha(c^\ell)}{\alpha(c^*)}\Big].
\end{aligned}
$$

Bidders maximize a linear expected payoff function under a linear constraint. Hence, it is anticipated that the second derivative is zero with respect to the strategic variable, that by direct revelation mechanism is defined to be the budget limit c_i. Below, it is illustrated that the second-order derivative becomes zero when I substitute the equilibrium bid:

$$\frac{\partial^2 \mathbb{E}(\pi_i)}{\partial c_i^2}\Big|_{(c_i=c^*)} = \alpha_i''(c^*)y(b_i(c^*), b_{-i}(\bar{c} = c^*))F(\bar{c} = c^*|c^*)$$

$$+ 2\alpha_i'(c^*)y'(b_i(c^*), b_{-i}(\bar{c} = c^*))F(\bar{c} = c^*|c^*)$$

$$+ \alpha_i(c^*)y''(b_i(c^*), b_{-i}(\bar{c} = c^*))F(\bar{c} = c^*|c^*)$$

$$- \alpha_i''(c^*)\mathbb{E}[r^s]F(\bar{c} = c^*|c^*)$$

Because the allocation rule is a linear increasing and differential function on the budget limit c, I have $\alpha_i''(c^*) = 0$ and by substituting in (4)

$$= -2\alpha_i'(c^*)\, n\, \theta\, b^{*\prime}(c^*)\, F(\bar{c} = c^*|c^*) - \alpha_i(c^*)\, n\, \theta\, b^{*\prime\prime}(c^*)\, F(\bar{c} = c^*|c^*)$$

Replacing the first and the second derivative of equation (6)

$$= 2\rho^2(c^*)\theta\, n\, \lambda\, \alpha(c^\ell)F(c^*|c^*) - 2\rho^2(c^*)\left(\Theta - \mathbb{E}[s]\right)\alpha(c^\ell)F(c^*|c^*)$$

$$- 2\rho^2(c^*)\theta\, n\, \lambda\, \alpha(c^\ell)F(c^*|c^*) + 2\rho^2(c^*)\left(\Theta - \mathbb{E}[s]\right)\alpha(c^\ell)F(c^*|c^*) = 0$$

It is a standard result in uniform auctions that there is no unique equilibrium (Ausubel et al., 2014). Though a recent paper shows that under certain conditions it exists uniqueness (Burkett and Woodward, 2020). □

Appendix B. Proof of Corollary 2.2

In the symmetric case, the symmetric equilibrium yield from (2) is:

$$y = \Theta - \theta\, n\, b^*(c^*)$$

Substituting equation (5) in (2) I can rewrite equivalently:

$$
\begin{aligned}
y &= \Theta - \theta n\Big(\lambda \frac{a(c^{\ell})}{a(c^{*})} + \frac{1}{\xi n}\Big[1 - \frac{a(c^{\ell})}{a(c^{*})}\Big]\Big) \\
&= \Theta - \frac{\theta\, n\, \lambda\, \alpha(c^{\ell})}{\alpha(c^{*})} - \Theta + \mathbb{E}[s] + (\Theta - \mathbb{E}[s])\frac{\alpha(c^{\ell})}{\alpha(c^{*})} \\
&= \mathbb{E}[s] + (\Theta - \theta\, n\, \lambda - \mathbb{E}[s])\frac{\alpha(c^{\ell})}{\alpha(c^{*})} \\
&= \mathbb{E}[s] + (\Theta - \theta\, n\, b^{*}(c^{\ell}) - \mathbb{E}[s])\frac{\alpha(c^{\ell})}{\alpha(c^{*})}
\end{aligned}
$$

By substituting with the symmetric minimum bid $b^{*}(c^{\ell}) = \lambda$, I result in an equilibrium yield equal to the symmetric risk limit that is $\ell^{*} = \Theta - \theta\, n\, b^{*}(c^{\ell})$. Thus, I conclude that

$$
y = \mathbb{E}[s] + (\ell^{*} - \mathbb{E}[s])\frac{\alpha(c^{\ell})}{\alpha(c^{*})}.
$$

□

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