Solving open economy models with incomplete markets by perturbation

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Motivation

- A few quotes about state-of-the-art in open economy macro modeling:
  - Maurice Obstfeld (2004): "At the moment we have no integrative general equilibrium monetary model of international portfolio choice, although we need one."
  - Pierre-Olivier Gourinchas (2006): "Looking ahead, the next obvious step is to build general equilibrium models of international portfolio allocation with incomplete markets. I see this as a major task that will close a much needed gap in the literature..."
  - Charles Engel (2023): "I would say that this field is in its infancy. But this infant is getting old very quickly."

Motivation

- The root of the problem? General equilibrium models with portfolio choice turn out to be extremely hard to solve.
- Why? lack of stable dynamics around a well-defined deterministic steady-state prevents the use of the standard perturbation method for DSGE models (Devereux and Sutherland, 2010; Tille and van Wincoop, 2010).
- Intuition: In a deterministic arbitrage-free World, all assets must deliver the same risk-free return. Where to invest your money becomes trivial!

First contribution: two-parameter perturbation

- I propose a small generalization of standard perturbation: the two-parameter perturbation model.
- Key idea: first, approximate the dynamics of a nearby deterministic auxiliary model that you can actually solve.
- Then, apply joint perturbation of parameters σ and ε to reach the model of interest, where
  - σ is the standard parameter scaling innovations (controls risk)
  - ε interacts with other parameters of the auxiliary model (controls auxiliary modifications).

Nice properties of two-parameter perturbation

- It can be implemented with available solution toolboxes (e.g. Dynare)
- It can approximate models around a large subset of the state-space, including the stochastic steady-state
- VERY accurate, even compared to global methods (it builds on theorems!)

Second contribution: a two-country DSGE model with bonds and equities

- I use two-parameter perturbation to study hedging motives behind the Equity Home bias Puzzle (French and Poterba, 1991).
- The application extends the two-period, multi-asset model of Coeurdacier and Gourinchas (2016) to an infinite horizon setting.
- This allows me to perform rigorous quantitative work: put the model to the test!

Main results

- Bonds still matter! A model with bonds and equities can deliver a large equity home bias as in the data, while an equities-only model fails to do so (Coeurdacier and Gourinchas, 2016; Coeurdacier and Rey, 2013).
- Develop a new link between trade and financial integration (Obstfeld and Rogoff, 2000; Heathcote and Perry, 2013).
- Predictions of long-run gross asset positions consistent with observations (Khalil, 2019; Maggiori, Neiman and Schreger, 2020).
- However, excessive international risk-sharing (Backus-Smith puzzle, consumption correlation puzzle), and counterfactual co-movements of gross capital flows (Broner et al., 2013; Davis and van Wincoop,2018).

Standard perturbation model

- Equilibrium conditions of the DSGE model are:

$$\mathbb{E}_t f\left(y_{t+1}, y_t, x_{t+1}, x_t\right) = 0$$

where y_t are the control variables and x_t the state variables, with laws of motion

$$\begin{aligned} y_t &= g(x_t, \sigma) \\ x_{t+1} &= h(x_t, \sigma) + \sigma \eta u_{t+1}, \end{aligned}$$

- The perturbation parameter σ only scales random innovations u_{t+1}.

Pitfall of standard perturbation

- Well-known algorithms approximate g and h with Taylor series around the Deterministic steady-state (DSS)
- But they only work if the model has a unique stable solution at $\sigma = 0$ (deterministic economy).
- In Portfolio-choice DSGE models, it is often the case that

$$f\left(y_{t+1}, y_t, x_{t+1}, x_t\right) = 0$$

admits many trivial solutions. Without risk, all assets are perfect substitutes!

Two-parameter perturbation

- New model includes auxiliary modifications (e.g. portfolio adjustment costs), and two perturbation parameters, ε and σ, where
  - σ only scales futures innovations
  - ε enters directly in the equilibrium conditions:

$$\mathbb{E}_t \widetilde{\mathsf{f}}\left(\mathsf{y}_{t+1}, \mathsf{y}_t, \mathsf{x}_{t+1}, \mathsf{x}_t, \varepsilon\right) = 0$$

- When $\varepsilon = 1$ we have the same equilibrium conditions as before: $\widetilde{\mathsf{f}} = \mathsf{f}$.
- When $\sigma = \varepsilon = 0$ modifications are active and the model has a unique and stable solution. No more indeterminacy problems!

Implementation

- We can implement Two-parameter perturbation with current DSGE software (e.g. Dynare) by treating ε as an exogenous state variable constant over time:

$$\varepsilon_{t+1} = \varepsilon_t$$

- The DSGE algorithm computes Taylor series to approximate

$$\begin{aligned} y_t &= g(x_t, \varepsilon_t, \sigma) \\ x_{t+1} &= h(x_t, \varepsilon_t, \sigma) + \sigma \eta u_{t+1} \end{aligned}$$

- Evaluated at $\sigma = \varepsilon_t = 1$, these functions are the solution to the model of interest.

Stochastic steady-state

- By introducing enough auxiliary parameters (inactive when $\varepsilon = 1$) we control the DSS that satisfies

$$\widetilde{f}\left(y_d, y_d, x_d, x_d, 0\right) = 0.$$

- The paper develops a simple algorithm that chooses as x_d the stochastic steady-state (SSS) of the model of interest. This is the point where agents choose to stay if they expect future risk (Coeurdacier, Rey and Winant, 2011)
- We can compute SSS portofolios and the dynamic solution around them. Similar concept to the Zero-order portfolios of Devereux and Sutherland (2011) and Tille and van Wincoop (2010).

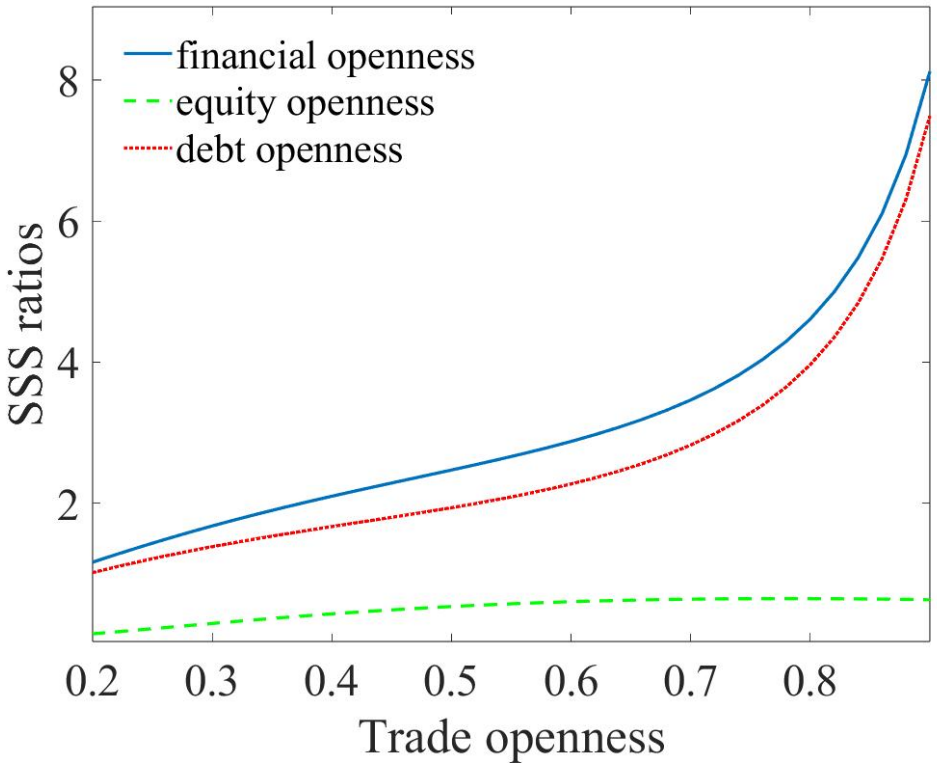
Two-country two-good DSGE Model: key ingredients

- A World with two countries (Home and Foreign).
- Time is discrete and infinite.
- A tree in each country delivers one type of good as income.
- Goods are imperfect substitutes, and can be traded (with home-bias in preferences).
- Agents can trade Home and Foreign equities S_t (pay dividends from the tree).

key ingredients (continued)

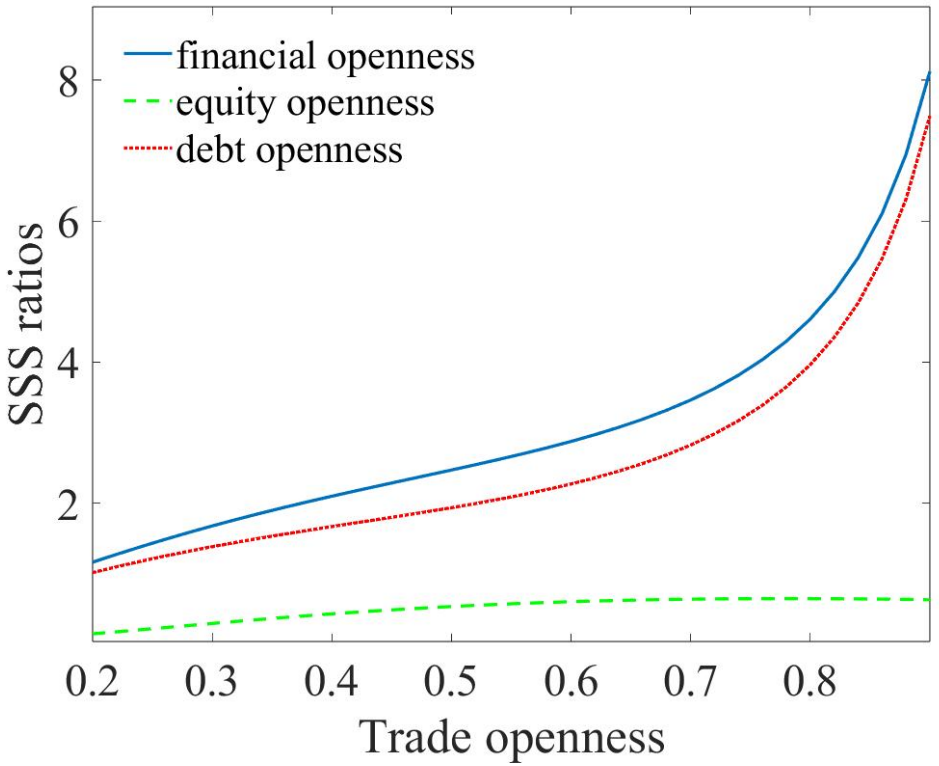
- Agents can also trade Home and Foreign real bonds B_t (pay in the price index of each country).
- Most of the income from the tree goes to country residents as non-financial income.
- Different sources of risk makes financial markets incomplete: income, redistributive, and preference risk.
- Goods and financial markets are competitive, and clear every period (sequential trading).
- Agents (identical within each country) maximize expected utility subject to budget constraints.

Comparative statics: trade and financial openness



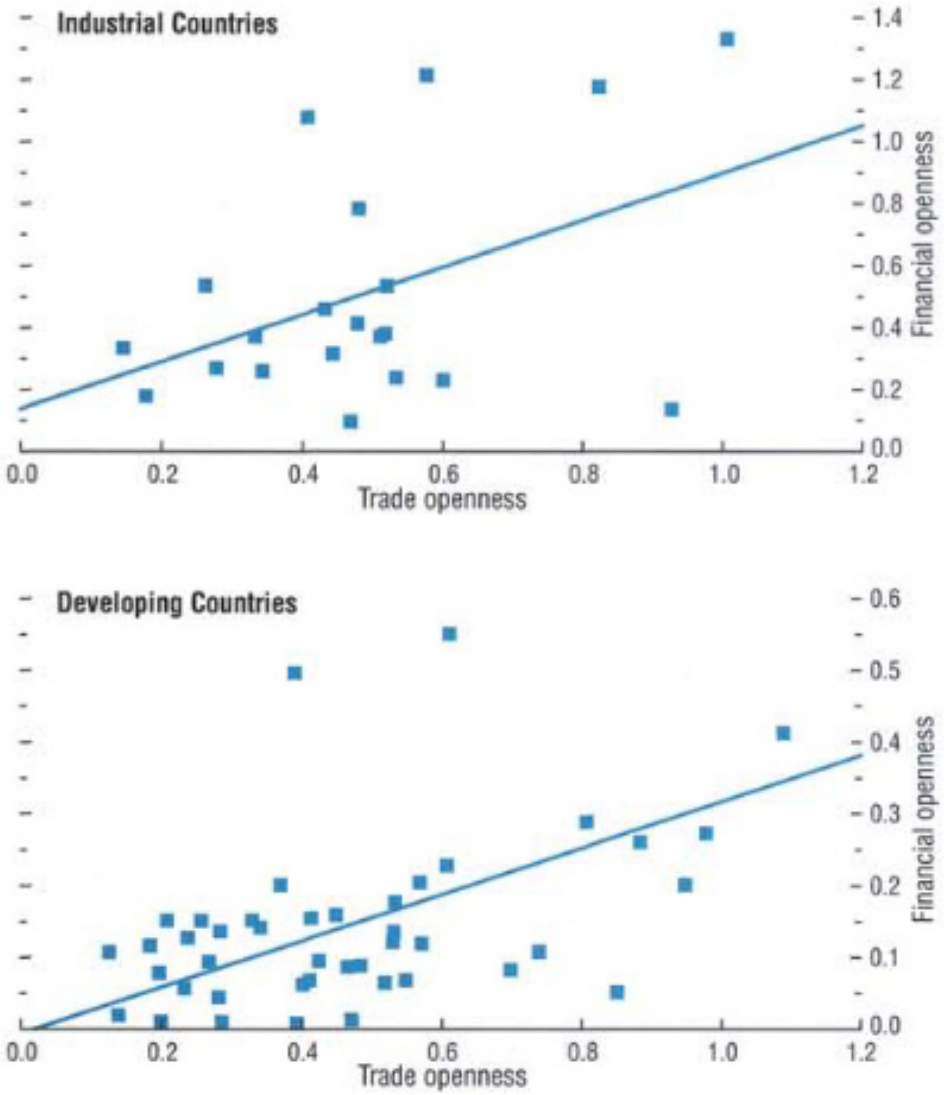
- Trade openness (exports+imports over GDP) is exogenous (controlled by Home bias in goods).
- Financial openness (external assets+liabilities over GDP) is endogenous (SSS portfolios).
- Decomposition shows that the positive relationship is driven by debt openness.

The intuition



- The gross debt position generates risk-sharing transfers because real debt returns are imperfectly correlated
- But more trade openness synchronizes returns and reduces transfer size
- Solution: enlarge the gross debt position

A testable implication!



- The model's goal is to account for the Equity Home bias. So the positive relationship is a byproduct.
- In the data (IMF), cross-country plots exhibit sizable correlation
- The model is consistent with the evidence!

However.. counterfactual correlations

Table: Second moments from simulations

	Data	Model
Cross-Correlations:		
RER and relative consumption	0.71	-0.37
Home and Foreign consumption	0.60	0.89
Capital inflows and outflows	0.78	-0.99

In conclusion

- Now researchers can use two-parameter perturbation to solve DSGE models with incomplete markets just like any other standard model
- This should help to provide quantitative answers to questions such as:
  - What are the determinants of the size and composition of gross external assets?
  - How does portfolio composition affects business cycles and the transmission of shocks?
  - What are the implications for monetary policy?

That is all

Thanks!