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The geography of energy transitions: a network approach for post-Fukushima Japan

David Castells-Quintana, Alvaro Domínguez and Felipe Santos-Marquez

Universitat Autònoma de Barcelona Asian Growth Research Institute Technische Universität Dresden

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Given the increasing threat of climate change, energy transitions from traditional sources to greener and renewable ones have become a major global need and goal.

However, energy transitions are costly and require time:

- Renewables still represent less than 30 per cent of all global electricity generation and only around 11 per cent of global primary energy.
- Wind electricity generation has shown one of the highest increases among all renewable power technologies but represents only around 7 per cent of electricity generation worldwide.
- At this pace of transition, we are far from meeting the goal of global Net Zero Emissions by 2050 (International Energy Association, 2023)

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Motivation II					

Energy transitions are occurring at different speeds across world regions and sectors:

- Some countries doing really well, while others show no progress, with **important technological and economic obstacles** to be overcome.
- Within countries, the adoption and spread of renewable energy sources is very uneven in space.

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Aim					

O To empirically study the spatial spread of green energy transitions at the local level.

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Aim					

- To empirically study the spatial spread of green energy transitions at the local level.
- Solution Focusing on Japan and exploring the nuclear-to-wind energy transitions triggered by the Fukushima Nuclear Incident (FNI) in 2011.

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- **O** To empirically study the spatial spread of green energy transitions at the local level.
- Focusing on Japan and exploring the nuclear-to-wind energy transitions triggered by the Fukushima Nuclear Incident (FNI) in 2011.
- Building a novel dataset combining detailed gridded data on the location of wind farms and nuclear plants, for 1742 municipalities for the 2001-2020 period.

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- **O** To empirically study the spatial spread of green energy transitions at the local level.
- Focusing on Japan and exploring the nuclear-to-wind energy transitions triggered by the Fukushima Nuclear Incident (FNI) in 2011.
- Building a novel dataset combining detailed gridded data on the location of wind farms and nuclear plants, for 1742 municipalities for the 2001-2020 period.
- Using panel-data econometric techniques, to explore the connection between the proximity to nuclear power plants and the spread of the adoption of Wind Energy Technology (WET).

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Aim					

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- Focusing on Japan and exploring the nuclear-to-wind energy transitions triggered by the Fukushima Nuclear Incident (FNI) in 2011.
- Building a novel dataset combining detailed gridded data on the location of wind farms and nuclear plants, for 1742 municipalities for the 2001-2020 period.
- Using panel-data econometric techniques, to explore the connection between the proximity to nuclear power plants and the spread of the adoption of Wind Energy Technology (WET).
- **and modelling and simulating the diffusion of WET through a network approach**.



1. FNI works as **a natural experiment**, enabling us to identify the causal effects of phasing out nuclear technology on WET diffusion:

· After FNI, more than 90% of nuclear power plants in Japan were shut down.

2. In Japan different energy sources coexist:

· Including fossil fuels, nuclear, and renewables, all having been extensively adopted with leading global technologies.

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3. Availability of **rich, fine-grained geo-located data** of WET adoption along with other factors for Japan for over 20 years.

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Energy Production in Japan



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Note: Figures created with data on power production from IEA

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Related Lite	rature				

Energy transitions, especially to those papers analyzing local adoption of greener technologies:

· Hall and Helmers (2013), Popp et al. (2011), Rode and Weber (2016).

Network diffusion of new technologies:

- · Acemoglu et al., 2011; Beaman et al., 2021
- \cdot especially in the energy sector (Halleck-Vega et al. (2018)).

Energy consequences of exogenous shocks:

• in particular, the Fukushima event in Japan in 2011 (Kawashima and Takeda (2012), Rehdanz et al. (2017) Okubo et al. (2020)).

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Data					

Variable	Available Years	Data Type	Source
Ozone concentration	1995, 2000, 2005, 2010-13.	average	Goodman et al. (2019)
PM2.5 Concentration	1990, 1995, 2000, 2005, 2010-12	average	Goodman et al. (2019)
Population GHSL	1990, 2000, 2015	count	Goodman et al. (2019)
NDVI	1990-2020	average	Goodman et al. (2019)
Lights	1992-2020	count	Li et al. (2020)
Wind farms	1985-2022	count	Wind Power database (2022)

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Note: We construct a balanced panel for 1992-2020. Missing data are replaced with linear interpolation estimates.

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The expansio	n of WET c	over time			



Source: created with data from the Wind Power database and GeoQuery Goodman et al. $\left(2019 \right)$

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Nuclear powe	r plants in .	lapan			



Source: created with data from the Global Power Plant Database v1.3.0 and Goodman et al. (2019)

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Summary	Statistics 2001	, 2011 & 2020			

	2001		:	2011		2020	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	
wind_farms	0.061	0.36	0.11	0.54	0.14	0.6	
ozone	58	3.1	59	3.7	60	3.9	
pm2.5	15	4.8	14	3.7	14	4	
рор	72401	178056	72731	182222	72864	183986	
lights pc	0.21	0.36	0.18	0.29	0.18	0.29	
ndvi mean	5351	1425	5232	1410	4407	1256	
log_lights	8	1.1	7.9	1.1	7.7	1.1	

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Summarv	Statistics 20	01-2020			

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	Unique (#)	Missing (%)	Mean	SD	Min	Median	Max
unit	1711	0	868	503	0	869	1741
year	20	0	2010	5.8	2001	2010	2020
wind_farms	8	0	0.1	0.51	0	0	7
ozone	21862	0	59	3.6	44	60	65
pm2.5	21948	0	14	4.1	5.2	14	37
рор	25665	0	72 690	181684	22	25 641	3 665 297
lights_pc	33961	0	0.2	0.33	0	0.1	8.2
ndvi_mean	34200	0	5237	1406	122	5605	7661
log_lights	10955	0	7.9	1.1	0	7.9	11

The post-Fukushima adoption of wind energy: a Difference-in-Differences approach

To assess to what extent the 2011 Fukushima incident translated into an increase in the adoption WET in areas surrounding nuclear power plants, we estimate the following difference-in-differences specification:

 $\log(WF)_{rt} = \alpha_r + \alpha_t + \beta T_{rt} + \delta X_{rt} + \varepsilon_{rt},$

- \cdot WF is the number of wind farms in municipality r at time t
- \cdot α_r are municipality fixed effects and α_t year fixed effects
- T is a treatment dummy which takes the value of 1 if municipality r is at a distance lower than 120 kilometers from any nuclear reactor for all years after 2011
- $\cdot X_{rt}$ is a vector of controls
- $\cdot \ \varepsilon_{rt}$ is an idiosyncratic error term

And the following event-study design:

$$\log(WF)_{rt} = \alpha_r + \alpha_t + \sum_{\tau=-1}^{-2} \gamma_{\tau} D_{rt}^{\tau} + \sum_{\tau=0}^{m} \delta_{\tau} D_{rt}^{\tau} + \delta X_{rt} + \varepsilon_{rt}$$

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DiD regressio	n estimates				

Table: Regression estimates of theDifference-in-Differences model

Dependent Variable:	$\log(\text{wind}_{\text{farms}+1})$				
woder:	(1)	(2)	(3)		
<i>Variables</i> treatment	0.0214*** (0.0031)	0.0104** (0.0049)	0.0119^{**} (0.0050)		
Fixed-effects municipality year Controls	Yes	Yes Yes	Yes Yes Yes		
<i>Fit statistics</i> R ²	0.90594	0.90673	0.90719		

Notes: Clustered (municipality) standard-errors in parentheses. Signif. Codes: ***: 0.01, **: 0.05, *: 0.1. This table reports the regressions estimates based on Equation (1). When specified in the model, we control for ozone concentration, $PM_{2.5}$ concentration, population, the normalized difference vegetation index, and night-time lights.





wind farms 120 km

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Heterogeneity	, analysis - d	distance			

Dependent Variables: Model:	log(total (1)	_power+1) (2)	log(wind (3)	_farms+1) (4)
Variables treatment	0.1089** (0.0543)	0.4702^{***} (0.1293)	0.0119** (0.0050)	0.0513^{***} (0.0138) 0.0005***
		(0.0014)		(0.0002)
<i>Fixed-effects</i> unit year <i>Controls</i>	Yes Yes Yes	Yes Yes Yes	Yes Yes Yes	Yes Yes Yes
<i>Fit statistics</i> Observations R ²	34,220 0.84844	34,220 0.84946	34,220 0.90719	34,220 0.90805

Clustered (unit) standard-errors in parentheses Signif. Codes: ***: 0.01, **: 0.05, *: 0.1

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Robustness c	hecks				

Our main results are robust to:

- Excluding Tokyo's 23 wards
- Excluding municipaities close to Fukushima Daiichi nuclear power plant
- Permutation tests
- Using alternative wind outcomes (extensive and intensive margines and total power)

We also find higher point estimates as we use smaller distance thresholds:

- Different treatment assignment for distance <=120
- ES estimation for treatment assignment for distances <= 150

But smaller (and mostly insignificant) as we use larger distance thresholds:

 $\bullet\,$ Different treatment assignment for distances >=120

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WET adoptio	on: the role	e of neighbors			

0.0125** (0.0050) 0.0278** (0.0116)	$\begin{array}{c} 0.0107^{**} \\ (0.0051) \\ 0.0245^{**} \\ (0.0110) \\ 0.0079 \\ (0.0072) \end{array}$	0.0124** (0.0051) 0.0225** (0.0093)	0.0072 (0.0052) 0.0181* (0.0093)	0.0119** (0.0050)	$\begin{pmatrix} 0.0015\\ (0.0051) \end{pmatrix}$	
(0.0050) 0.0278** (0.0116)	(0.0051) 0.0245^{**} (0.0110) 0.0079 (0.0072)	(0.0051) 0.0225** (0.0093)	(0.0052) 0.0181* (0.0093)	(0.0050)	(0.0051)	
(0.0116)	(0.0110) 0.0079 (0.0072)	0.0225** (0.0093)	0.0181^{*} (0.0093)			
	(0.0072)	0.0225** (0.0093)	0.0181^{*} (0.0093)			
		(0.0093)	(0.0093)			
			0.0123^{**}			
			(0.0062)	-0.0014	-0.0064	
				(0.0000)	(0.0003) 0.0174^{***} (0.0058)	
Yes	Yes	Yes	Yes	Yes	Yes	
Yes	Yes	Yes	Yes	Yes	Yes	
Yes	Yes	Yes	Yes	Yes	Yes	
34,220	34,220	34,220	34,220	34,220	34,220	
	Yes Yes Yes 34,220 0.90742	Yes Yes Yes Yes Yes Yes 34,220 34,220 0.90742 0.90745	Yes Yes Yes Yes Yes Yes Yes Yes Yes 34,220 34,220 34,220 0.90742 0.90745 0.90742	Yes Yes <thyes< th=""> <thyes< th=""> <thyes< th=""></thyes<></thyes<></thyes<>	Yes Yes <th td="" th<="" yes<=""></th>	

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From the definition of the network we identify 3 subnetworks:

• The first with 798 municipalities (mostly central Japan), the second with 109 (north), and the third one with 259 (south).

To simulate diffusion, we set thresholds as follows:

- · Find a q_i (i.e., proportion of neighbours that adopted for a given municipality to consider adoption) such that if we add ϵ to it, there is no diffusion.
- · Multiply q_i by 0.25, 0.5, and 0.75 and obtain its Q1, Q2, Q3 & Q4.
- \cdot We use these four threshold values in simulating the diffusion process though different targeting strategies.



Initial seeds vs Optimal seeding:



Seeds in the first network (Central Japan - 798 municipalities)







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Network anal	ysis: takea	way			

Policymakers could accelerate WET adoption by targeting central connectors that link densely populated areas.

- \cdot By focusing resources and generating incentives at these key nodes, they can enhance the geographical diffusion of WET.
- In this particular case, the "Betweenness" approach (i.e., targeting municiplaities that are in the shortests paths in the network between other municipalities) shows faster capacity for geographical diffusion.

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Conclusions					

Using novel panel data, we have shown how the exogenous shock of Fukushima incident of 2011 led to an increase in the adoption of wind farms.

This happened unevenly in space, with municipalities close to i) nuclear plants, and ii) adopting neighbours, being more likely to transit to WET.

Using a network diffusion model, we have analysed this geographical spread of WET, identifying policy interventions with the potential to foster the pace of adoption.

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Thank You! Link or QR code to the working paper https://tud.link/z8wypx



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Sample without Tokyo's wards

Table: Sample without Tokyo's wards

Dependent Variable: Model:	log(wind_ (1)	farms+1) (2)	
<i>Variables</i> treatment	0.0106** (0.0050)	0.0115^{**} (0.0050)	
Fixed-effects municipality year Controls	Yes Yes	Yes Yes Yes	
<i>Fit statistics</i> Observations R ²	33,760 0.90689	33,760 0.90739	

Notes: Clustered (municipality) standarderrors in parentheses. Signif. Codes: ***: 0.01, **: 0.05, *: 0.1. Estimates for the sample of all municipalities except the Tokyo's 23 special wards.

Subsample: distance to Fukushima Daiichi nuclear power plant

Dependent Variable: Model:	baseline	log(wind 50 km	_farms+1) 150 km	229 km
<i>Variables</i> treatment	0.0119^{**} (0.0050)	0.0107^{**} (0.0049)	0.0120^{**} (0.0051)	0.0146*** (0.0056)
<i>Fixed-effects</i> municipality year <i>Controls</i>	Yes Yes Yes	Yes Yes Yes	Yes Yes Yes	Yes Yes Yes
<i>Fit statistics</i> Observations R ²	34,220 0.90719	33,740 0.91292	30,900 0.91445	25,640 0.91029

Notes: Clustered (municipality) standard-errors in parentheses. Signif. Codes: ***: 0.01, **: 0.05, *: 0.1. Regression estimates for the sample in which municipalities are at a distance larger than a given threshold from the Fukushima Daiichi nuclear power plant. 229 km represents the distance at which 25% of municipalities are removed from the sample.

Treatment assignment for distances $\leq = 120$

Dependent Variable: Model:	30 km	log(wind_farms+1) 30 km 60 km 90 km 1			
<i>Variables</i> treatment	0,0653*** (0.0202)	0.0267*** (0.0082)	0.0189^{***} (0.0051)	0.0119^{**} (0.0050)	
<i>Fixed-effects</i> municipality year <i>Controls</i>	Yes Yes Yes	Yes Yes Yes	Yes Yes Yes	Yes Yes Yes	
<i>Fit statistics</i> Observations R ²	34,220 0.90841	34,220 0.90762	34,220 0.90746	34,220 0.90719	

Notes: Clustered (municipality) standard-errors in parentheses. Signif. Codes: ***: 0.01, **: 0.05, *: 0.1. Regression estimates for samples in which the treatment varies according to the distance from any nuclear power plant.

Treatment assignment for distances >= 120

Dependent Variable:	log	s+1)	210 km	
Model:	120 km (baseline)	180 km		
<i>Variables</i>	0.0119^{**}	0.0097	0.0152*	0.0144
treatment	(0.0050)	(0.0074)	(0.0078)	(0.0093)
Fixed-effects municipality year Controls	Yes Yes Yes	Yes Yes Yes	Yes Yes Yes	Yes Yes Yes
<i>Fit statistics</i> Observations R ²	34,220 0.90719	34,220 0.90711	34,220 0.90714	34,220 0.90712

Notes: Clustered (municipality) standard-errors in parentheses. Signif. Codes: ***: 0.01, **: 0.05, *: 0.1. Regression estimates for samples in which the treatment varies according to the distance from any nuclear power plant. We control for ozone concentration, $PM_{2.5}$ concentration, population, the normalized difference vegetation index, and night-time lights.

Permutation test



Note: this figure shows the distribution of the point estimates for the DiD treatment effect based on equation (1). The treatment status is assigned randomly to 1263 municipalities based on 999 draws. The black line represents our baseline estimate. Once the 1000 estimates, 999 from the draws and our baseline, are ranked, the baseline estimate ranks 982 which may be interpreted as a p-value with a significance level below 5%.

Event study plot - with controls



Back to robustness checks

Event study plot - with controls



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Alternative wind farm outcomes



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Alternative wind farm outcomes



wind turbines 120 km

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Alternative wind farm outcomes



Back to robustness checks

The Model I

Set of agents $N = \{1, ..., n\}$ in a network, represented through a graph G = (N, E).

E is the set of edges connecting different agents, with no self-loops.

We say that an edge $ij \in E$ connects *i* to *j*, and the network is undirected, thus ij = ji.

The set of neighbors of agent $i \in N$ is defined as $i \in N$ as $N_i(G) = \{j \mid ij \in E\}$.

We assume that at iteration k = 0, a subset of individuals $\Psi(0) \subseteq N$ is selected as seeds.

At the next iteration, an individual $i \in N \setminus \Psi(0)$ will **consider** adopting the innovation if at least $q_i \in (0, 1]$ fraction of neighbors are in seed set:

$$rac{|\Psi(0)\cap N_i(G)|}{|N_i(G)|}\geq q_i\Rightarrow i\in \Psi(1).$$

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The Model II

For $k \ge 0$ we generalize this as:

A node $i \in N \setminus \bigcup_{t=0}^{k-1} \Psi(t)$ will consider adoption of a new technology at k if

$$\frac{|\{\bigcup_{t=0}^{k-1}\Psi(t)\}\cap N_i(G)|}{|N_i(G)|} \ge q_i \Rightarrow i \in \Psi(k).$$
(2)

The Model II

For $k \ge 0$ we generalize this as:

A node $i \in N \setminus \bigcup_{t=0}^{k-1} \Psi(t)$ will consider adoption of a new technology at k if

$$\frac{|\{\bigcup_{t=0}^{k-1}\Psi(t)\}\cap N_i(G)|}{|N_i(G)|} \ge q_i \Rightarrow i \in \Psi(k).$$
(2)

We define a subset $\mathcal{H} \subseteq N$ to be cohesive if

$$\frac{|\mathcal{H} \cap N_i(G)|}{|N_i(G)|} > 1 - q_i, \ \forall i \in \mathcal{H}.$$
(3)

A set of agents make a cohesive set if, for each member of the set, the proportion of neighbors in the set is strictly greater than the individual specific threshold.

the rest, with $0 < \epsilon \ll 1$.



An example



- Assume q_i is 0.5ϵ for nodes 1, 2, and $0.5 + \epsilon$ for the rest, with $0 < \epsilon \ll 1$.
- $\cdot \ \mathcal{H} = \{\{3,4,5,6\},\{4,5,6\},\{1,2,3,4,5\}\}$ and more!

- · Assume q_i is 0.5ϵ for all nodes, with $0 < \epsilon \ll 1$.
- · If we strategically target nodes 3, 5, or 6, we can obtain a complete diffusion.
- This shows the importance of targeting specific nodes in the network.

We say that for a given graph and threshold values, a nonempty set Ψ^* will be a fixed point of the threshold model if

$$\Psi(0) = \Psi^* \Rightarrow \Psi(k) = \emptyset, \ \forall k > 0.$$
 (4)

Equation 4 says that a nonempty set is a fixed point if an innovation initiated at that particular set can not propagate through the rest of the network.

We say that for a graph G with thresholds $\{q_i\}_{i \in N}$ an adopter set Ψ^* is a fixed point $\Leftrightarrow (\Psi^*)^c = N \setminus \Psi^*$ is a cohesive set.

We say that for a given graph and threshold values, a nonempty set Ψ^* will be a fixed point of the threshold model if

$$\Psi(0) = \Psi^* \Rightarrow \Psi(k) = \emptyset, \ \forall k > 0. \tag{4}$$

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We say that for a graph G with thresholds $\{q_i\}_{i \in N}$ an adopter set Ψ^* is a fixed point $\Leftrightarrow (\Psi^*)^c = N \setminus \Psi^*$ is a cohesive set.

Lemma 1 (Acemoglu et al. 2011):

For a given graph G and threshold values $\{q_i\}_{i \in N}$, an adopter set Ψ^* is a fixed point \Leftrightarrow $(\Psi^*)^c = N \setminus \Psi^*$ is a cohesive set.

This proof is based on Morris [Proposition 1]: Members of a cohesive set \mathcal{H} cannot satisfy Eq. (2) unless there exists an individual inside \mathcal{H} who has previously adopted the innovation.

Therefore, if one initializes an innovation from a set Ψ^* whose complement is a cohesive set, the innovation will not be adopted by the members of the complement set.

In the previous examples, we observe several fixed points:

$$\begin{array}{l} \cdot \ \Psi^* = \{\{1,2\},\{1,2,3\},\{6\}\} \ \text{for case 1.} \\ \cdot \ \Psi^* = \{\{1,2\},\{4\},\{1,2,4\}\} \ \text{for case 2.} \end{array} \end{array}$$

Additionally, the universal set $\{1, 2, 3, 4, 5, 6\}$ is also a fixed point.

Lemma 2 (Acemoglu et al. 2011):

For a given graph G, threshold values $\{q_i\}_{i \in N}$ and seed set $\Psi(0)$, denote $\{\Psi_t^*\}_{t=1}^K$, $K \ge 1$ as the set of fixed points for which $\Psi(0) \subseteq \Psi_t^*$ holds. Then,

$$\Psi^* = \bigcap_{t=1}^{K} \Psi_t^* \tag{5}$$

is the set of final adopters.

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Corollary 1:

Given a graph G, with threshold values $\{q_i\}_{i\in N}$ and seed set $\Psi(0)$, denote $\{\mathcal{H}_t\}_{t=1}^{K}$, $K \geq 1$ as the set of cohesive subsets of N for which $\Psi(0) \cap \mathcal{H}_t = \emptyset$ holds. Then,

$$\Psi^* = \left(\bigcup_{t=1}^{\kappa} \mathcal{H}_t\right)^c \tag{6}$$

is the set of final adopters.

Number of adoptions for the first subnetwork

