The effect of stock splits on liquidity in a dynamic model

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Introduction

Stock splits and liquidity

- Liquidity is a fundamental property of well-functioning markets $\Rightarrow$ lack of liquidity is at the heart of many episodes of market stress.
- Why do companies split their stocks? This creates "wider" markets.
  - making the stock more accessible to retail investors.
  - allows existing investors to sell part of their holdings more easily.
- As the volume of transactions increases, liquidity conditions should improve.

Counterarguments by Copeland (1979)

Arguments for a decrease in liquidity following a stock split:

- increases in real transaction costs.
- volume increases less than proportionately.
- brokerage revenues increase.
- increases in bid-ask spreads as a percentage of the value of the stock.

Our contribution

- We study the impact of stock splits on liquidity for a recent sample of Dow Jones index component stocks.
- The analysis is based on the daily Amihud illiquidity measure.
- We use the Dynamic Autoregressive Liquidity (DArLiq) model of Hafner, Linton, and Wang (2023).
- We propose tests of permanent and temporary effects in a dynamic framework.
- We find that stock splits cause shifts in the long-term liquidity trend, but no additional effects on short-run liquidity dynamics.

The Model and Estimation

Amihud illiquidity

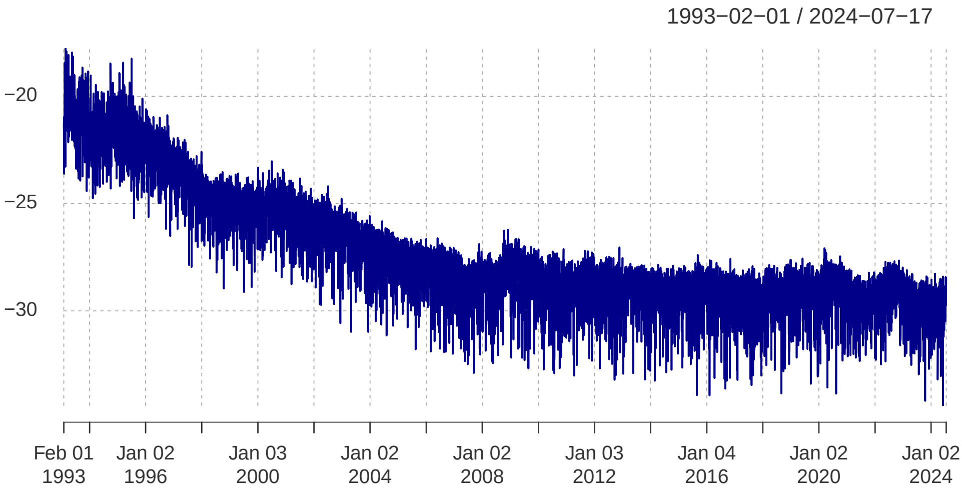
- The Amihud illiquidity measure of a stock at time t, A_t, is

$$A_t = \frac{1}{n_t} \sum_{j=1}^{n_t} \ell_{t_j}, \quad \ell_{t_j} = \frac{|R_{t_j}|}{V_{t_j}},$$

  - R_{t_j} is the intra-period returns.
  - V_{t_j} is the intra-period dollar trading volume.
  - n_t is the number of intra-period returns.

- We focus on the daily Amihud illiquidity ratio $\ell_t = \frac{|R_t|}{V_t}$.

Daily log illiquidity for S&P 500 (SPY)



The DArLiq model for illiquidity

Illiquidity ratio ℓ_t follows a multiplicative process with a nonparametric trend.

$$\ell_t = g(t/T)\lambda_t\zeta_t,$$

$$\lambda_t = \omega + \beta\lambda_{t-1} + \gamma\ell^*_{t-1},$$

$$\ell^*_t = \ell_t/g(t/T),$$

where $\omega > 0, \beta \geqslant 0, \gamma \geqslant 0$ and $\beta + \gamma < 1$.

- $g(.)$ is an unknown function of rescaled time.
- λ_t is stationary with $E[\lambda_t] = 1$ for identification $\Rightarrow$ set $\omega = 1 - \beta - \gamma$.
- ζ_t is a positive random variable with conditional mean one.

Permanent shifts

- Allow trend g to be discontinuous at a finite set of points $u_1, \ldots, u_m \in (0,1)$.
- Define the left and right limits of the function and its first two derivatives

$$\lim_{u \uparrow u} g^{(r)}(u) = g_-^{(r)}(u), \quad \lim_{u \downarrow u} g^{(r)}(u) = g_+^{(r)}(u), \quad r = 0, 1, 2,$$

  but we allow that $g_-^{(r)}(u_i) \neq g_+^{(r)}(u_i)$ for $i = 1, \ldots, m$.
- By convention, $g^{(r)}(.)$ is CADLAG, i.e. $g^{(r)}(u_i) = g_+^{(r)}(u_i)$.
- For any $u \notin \{0, u_1, \ldots, u_m, 1\}$, we maintain that $g_-^{(r)}(u) = g_+^{(r)}(u)$, for $r = 0, 1, 2$.

Permanent shifts: size of the jump

- Potential breakpoints $u_1 = t_1/T, \ldots, u_m = t_m/T$ are known in advance.
- Size of jump at u_i is measured in level and in percentage terms respectively by

$$\begin{aligned} \mathcal{J}(u_i) &= g_+(u_i) - g_-(u_i), \\ \mathcal{J}_{\%}(u_i) &= \frac{g_+(u_i) - g_-(u_i)}{\{g_+(u_i) + g_-(u_i)\}/2}. \end{aligned}$$

  This is the effect that remains permanently in the absence of further changes.

- The effects can be aggregated over different breakpoints.

Estimation of the trend function

- We observe a sample of daily illiquidities $\{\ell_t,\ t = 1, \ldots, T\}$.

- Local linear kernel smoother designed to be robust to potential breaks at points $0 < u_1 < u_2 < \cdots < u_m < 1$. Define $\widehat{g}(u) = \widehat{\alpha}(u)$ and for $u \in [u_i, u_{i+1})$

$$(\widehat{\alpha}(u), \widehat{\beta}(u)) = \arg\min_{\alpha,\beta} \sum_{t=1}^{T} K_h(t/T - u) \{\ell_t - \alpha - \beta(t/T - u)\}^2 1(u_i \leqslant t/T < u_{i+1}).$$

- The estimator $\widehat{g}(u)$ is continuous everywhere except at $\{u_1, u_2, \ldots, u_m\}$.

- At point u_i we compute two estimates of $\widehat{g}(u_i)$: a left sider $\widehat{g}_-(u_i)$ and a right sider $\widehat{g}_+(u_i) \Rightarrow$ size of jump $\widehat{\mathcal{J}}(u_i)$ and $\widehat{\mathcal{J}}_{\%}(u_i)$.

Estimation of the parametric component

- We use GMM to estimate $\theta = (\beta, \gamma)^\intercal$ from the conditional moment restriction $E(\ell_t^* | \mathcal{F}_{t-1}) = \lambda_t$, where $\ell_t^* = \ell_t / g(t/T)$, $t = 1, \ldots, T$.

- We work with residuals $\ell_t^* / \lambda_t(\theta) - 1$, which is a martingale difference sequence at the true parameter values. Define $\rho_t(\theta, \widehat{g}) = z_{t-1} \left\{ \widehat{\ell}_t^* / \widehat{\lambda}_t(\theta) - 1 \right\}$

$$\widehat{\theta}_{GMM} = \arg\min_{\theta \in \Theta} \left\| M_T(\theta, \widehat{g}) \right\|_W , \quad M_T(\theta, \widehat{g}) = \frac{1}{T} \sum_{t=1}^{T} \rho_t(\theta, \widehat{g}),$$

  where W is a weighting matrix, while $z_t \in \mathcal{F}_t$ are instruments.

- In our application, we use $z_t = (1, \widehat{\ell}_t^*, \widehat{\ell}_t^* / \widehat{\lambda}_t)'$ and $W = I_3$.

Estimation: improvement

- Given consistent estimates of $\theta, g(.)$, estimation can be improved in terms of efficiency and simplicity of standard errors.
- Note that $E\left(\ell_t/\lambda_t\right) = g(t/T)$, which is an alternative local moment condition that is purged of the short-run variation.
- Use local linear kernel smoother as before but replacing ℓ_t by $\ell_t/\widehat{\lambda}_t$, where $\widehat{\lambda}_t = \widehat{\lambda}_t(\widehat{\theta}_{GMM}, \widehat{g}) \Rightarrow \widetilde{g}(u_i),\ \widetilde{\mathcal{J}}(u_i),\ \widetilde{\mathcal{J}}_{\%}(u_i)$.
- The large sample variance of $\widetilde{g}(u)$ is much simpler to estimate than that of $\widehat{g}(u)$.

Tests for permanent and temporary shifts

Permanent effects: single split

- We consider $H_0 : g_-(u_i) = g_+(u_i)$. Test for discontinuity at u_i is based on

$$\widetilde{\tau}(u_i) = \sqrt{Th}\frac{\widetilde{g}_+(u_i) - \widetilde{g}_-(u_i)}{\sqrt{\|K^+\|^2 \left\{\widetilde{g}_+^2(u_i) + \widetilde{g}_-^2(u_i)\right\} \widehat{\sigma}_\zeta^2}}, \quad \widehat{\sigma}_\zeta^2 = \sum_{t=1}^{T}(\widehat{\zeta}_t - \overline{\widehat{\zeta}})^2/T.$$

- Under H_0 and the condition that $Th^5 \to \gamma$, we have $\widetilde{\tau}(u_i) \to_d N(\rho_i, 1)$, where ρ_i is an asymptotic bias/standard error term, i.e. $\rho_i = \lim_{T\to\infty} \frac{b(u_i)}{SE(u_i)}$.

- We consider three approaches for inference
  - undersmoothing.
  - bias correction.
  - "honest" confidence intervals, Armstrong and Kolesár (2020).

Temporary effects

- We allow for short-term adjustments that eventually die out.
- Include dummy variables in the dynamic equation

$$\lambda_t = \omega + \beta\lambda_{t-1} + \sum_{j=1}^{J} \alpha_j D_{jt} + \gamma \ell^*_{t-1},$$

- If $u_j = t_j / T$ is a stock split day, set $D_{jt} = 1$ if $t \in \{t_j - E, \ldots, t_j + E\}$ for some event window $\mathcal{E}$ of length $J = 2E + 1$.
- With multiple splits, we include dummy variables around all the key dates.

Temporary effects: test statistic

- Consider the single event setting with event window $\{t_1 - E, \ldots, t_1 + E\}$.
- We consider $H_0 : \alpha_1 = \ldots = \alpha_J = 0$.
- Assume $\zeta_t - 1$ is a stationary mixing MDS with unknown distribution F.
- Obtain the residuals $\widehat{\zeta}_t = \ell_t / \widetilde{g}(t/T)\widehat{\lambda}_t$, $t = 1, \ldots, T$. Define abnormal illiquidity and cumulative abnormal illiquidity as

$$AIL_\tau = \widehat{\zeta}_{t_1 - E + \tau} - 1, \quad CAIL(\tau) = \sum_{s=0}^{\tau} AIL_s, \quad \tau = 0, \ldots, 2E.$$

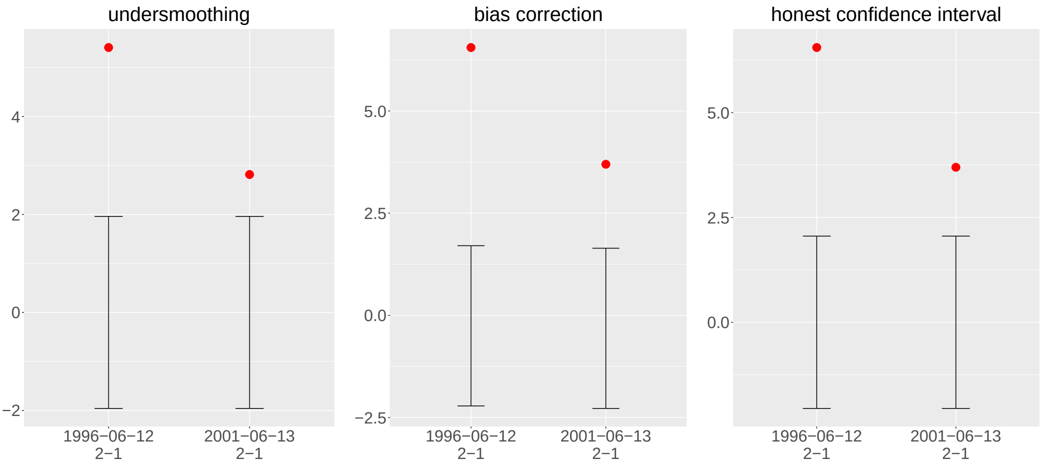
- Critical values are estimated based on data outside the event window.

Empirical Application

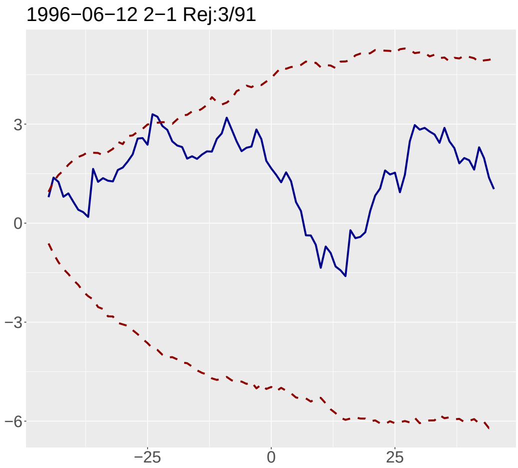
Data

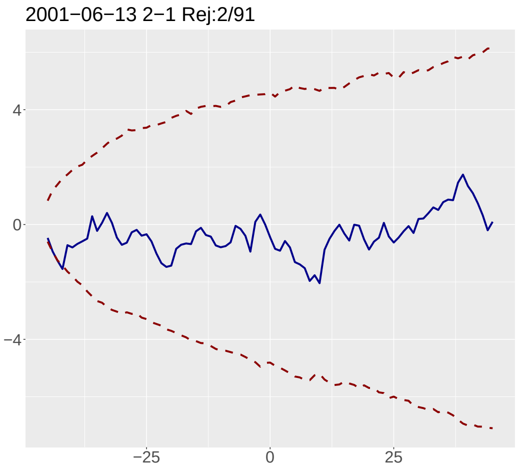
- We use historical daily price and volume data for the Dow Jones index component stocks.
- The sample period starts from each asset's first available data point until December 31, 2023.
- There are in total 76 splits and 62 of them are two-to-one splits.

Individual stocks (permanent effects): Johnson & Johnson



Individual stocks (temporary effects): Johnson & Johnson





Permanent effects: pooled tests

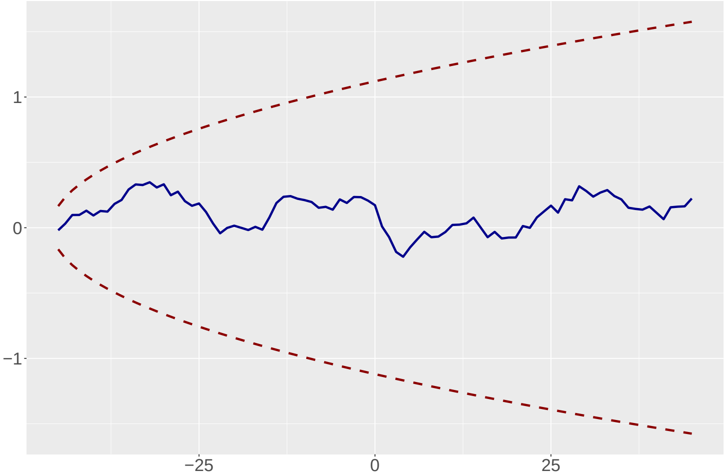
Note: τ_w is the aggregated directional statistic and is asymptotically $N(0,1)$ under the null hypothesis.

	UNH	MSFT	HD	AMGN	MCD	CAT	BA	HON
τ_w	**5.73**	**8.78**	**1.99**	**4.00**	1.64	**3.61**	-0.28	**6.96**

	TRV	AAPL	JPM	JNJ	WMT	IBM	PG	CVX
τ_w	**7.43**	**8.40**	**29.08**	**5.81**	**2.37**	**3.94**	**7.48**	**4.08**

	MRK	MMM	NKE	KO	CSCO	INTC	VZ	WBA
τ_w	**7.53**	**6.91**	**2.22**	**2.64**	**5.51**	**7.91**	**3.58**	**2.01**

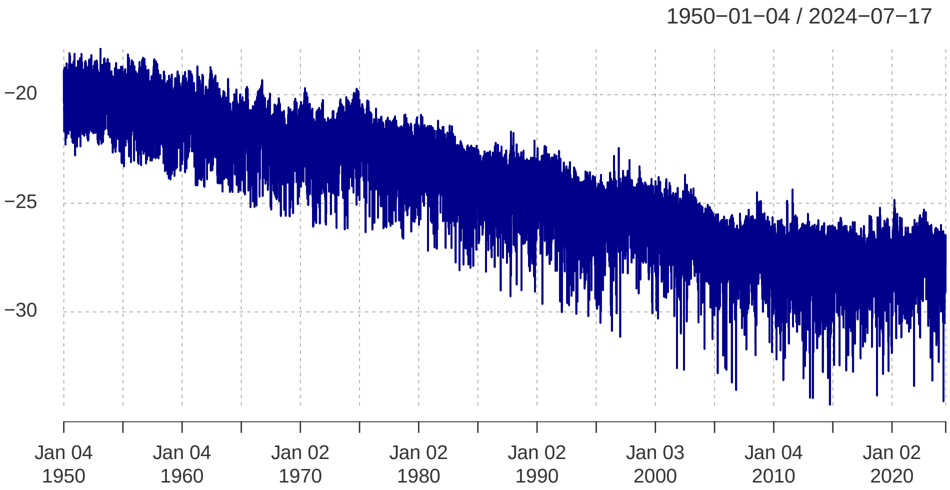
Temporary effects: aggregated



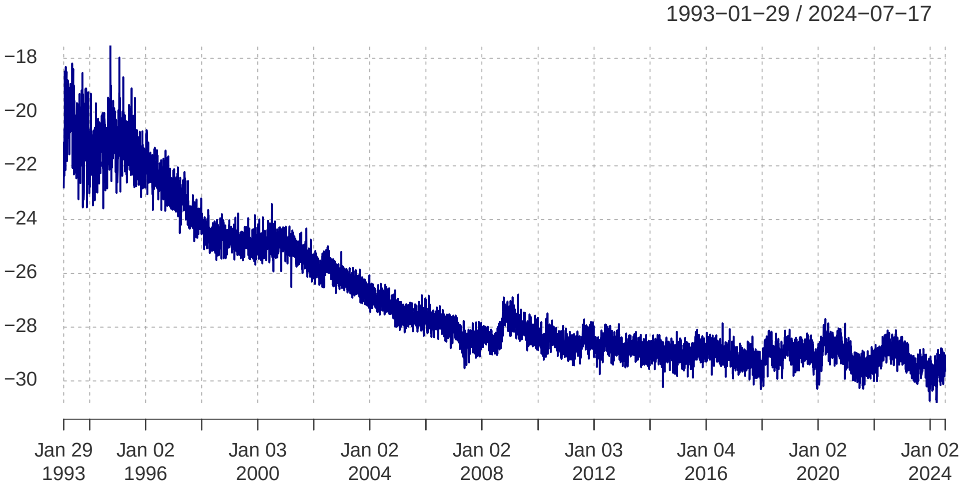
Conclusions

- We propose tests to detect both permanent and temporary breaks in illiquidity in a dynamic framework.
- We find strong empirical evidence for an increase in the long-run illiquidity component after stock splits.
- We do not find significant effects on the short-run illiquidity dynamics.

Daily log illiquidity for S&P 500 index



Daily log illiquidity for S&P 500 (SPY)



Empirical evidence

- Evidence for worsened liquidity following stock splits:
  - Lamoureux and Poon (1987).
  - Lakonishok and Lev (1987) and Huang, Liano, and Pan (2015): only temporary improvements on split announcement, then decline.
  - Han (1995): liquidity improves after reverse splits.
- Evidence for improved liquidity following stock splits:
  - Chern et al. (2008); Guo, Liu, and Song (2008); Yu and Webb (2009): reduce bid-ask spreads, increase number of small traders.
  - Mohanty and Moon (2007): improvement in the average trading volume.

Permanent effects: multiple splits

- Joint test for the null hypothesis of no breaks at any u_i. Consider the statistic

$$W = \sum_{i=1}^{m} \widetilde{\tau}(u_i)^2,$$

Under H_0, $W \to_d \sum_{i=1}^{m}(Z_i + \rho_i)^2$, where Z_i are i.i.d. $N(0,1)$ random variables.

- A directional test where we pool the jumps across splits. For some weighting scheme w_i, we have

$$\widetilde{\tau}_w = \frac{\sum_{i=1}^{m} w_i \widetilde{\tau}(u_i)}{\sqrt{\sum_{i=1}^{m} w_i^2}},$$

Under H_0, $\widetilde{\tau}_w \to_d N(\rho_w, 1)$ where $\rho_w = \sum_{i=1}^{m} w_i \rho_i / \sqrt{\sum_{i=1}^{m} w_i^2}$.

Temporary effects: distribution under H_0

- Let F_{w_τ} denote the distribution of $\{w_{r,\tau}\}$, where $w_{r,\tau} = \sum_{s=0}^{\tau}(\zeta_{r+s} - 1)$.
- Estimate the distributions F and F_{w_τ} based on the data not including the event window, $S = \{1, \dots, T\}\backslash\{t_1 - E, \dots, t_1 + E\}$.
- Let $\widehat{w}_{r,\tau} = \sum_{s=0}^{\tau}(\widehat{\zeta}_{r+s} - 1)$, we define $\widehat{F}_{\widehat{w}_\tau}(x) = \frac{1}{T_S}\sum_{t\in S} 1\left(\widehat{w}_{t,\tau} \leqslant x\right)$, where T_S is the cardinality of the set S, and $\widehat{F}(x) = \widehat{F}_{\widehat{w}_0}(x)$.
- Reject H_0 if $CAIL(\tau) \notin [\widehat{F}^{-1}_{\widehat{w}_\tau}(\alpha/2),\ \widehat{F}^{-1}_{\widehat{w}_\tau}(1-\alpha/2)]$ for $\tau = 0, \dots, 2E$.

Permanent effects: joint tests

Note: $\tilde{\mathcal{J}}_{\%w}$ is the average jump in percentage. p_W is the p-value of the aggregated statistic $W = \sum_{i=1}^{m} \tau(u_i)^2$.

	UNH	MSFT	HD	AMGN	MCD	CAT	BA	HON
# of splits	5	5	1	3	2	3	1	2
$\tilde{\mathcal{J}}_{\%w}$	43%	37%	35%	49%	11%	31%	-4%	43%
p_W	0.00	0.00	0.05	0.00	0.26	0.00	0.78	0.00
	TRV	AAPL	JPM	JNJ	WMT	IBM	PG	CVX
# of splits	2	2	1	2	2	2	2	2
$\tilde{\mathcal{J}}_{\%w}$	58%	107%	239%	42%	32%	31%	42%	31%
p_W	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00
	MRK	MMM	NKE	KO	CSCO	INTC	VZ	WBA
# of splits	1	2	5	2	5	5	1	3
$\tilde{\mathcal{J}}_{\%w}$	33%	57%	9%	20%	63%	64%	64%	23%
p_W	0.00	0.00	0.04	0.03	0.00	0.00	0.00	0.15

Reverse splits

- We consider constituents of S&P 400, S&P 500 and S&P 600 with reverse splits and a pre-event price level below \$5.
- We focus on stocks with at most one reverse split in the sample $\Rightarrow$ 53 stocks.
- Individual statistics τ are significantly negative for 32 stocks, which indicates a decrease in stock illiquidity.

Reverse splits

	AAON	ACLS	AIG	ARWR	ASRT	BANR	BCEI	BCOR	BKNG	C	CAR
Split size	1-4	1-4	1-20	1-10	1-4	1-7	1-111.6	1-10	1-6	1-10	1-10
τ	-0.09	**-2.71**	**-21.37**	**-10.53**	**4.42**	**15.40**	**-8.41**	**-7.87**	**-11.77**	**8.64**	**-17.14**

	CBB	CCOI	CIEN	CIVI	COO	CPE	CPF	CSII	CYTK	EPAC	EXPR
Split size	1-5	1-20	1-7	1-111.6	1-3	1-10	1-20	1-10	1-6	1-5	1-20
τ	-0.01	**-2.20**	**3.83**	**-8.41**	0.49	**-2.71**	**-9.85**	**-6.97**	**-24.56**	**-14.86**	-0.46

	FBP	FTR	HAFC	HPR	HSKA	IART	KEM	KLXE	LCI	LPI	MSTR
Split size	1-15	1-15	1-8	1-50	1-10	1-2	1-3	1-5	1-4	1-20	1-10
τ	**-4.26**	-1.90	**-7.65**	**-4.10**	**-7.48**	**4.55**	**-3.93**	1.02	**-2.19**	**-7.99**	**-19.93**

	MTH	NEU	ODP	OPCH	PFBC	PPBI	RRC	SANM	SBCF	SNV	SPPI
Split size	1-3	1-5	1-10	1-4	1-5	1-5	1-15	1-6	1-5	1-7	1-25
τ	-0.63	**-4.48**	**-4.76**	1.27	**-4.15**	0.47	-1.33	**-13.75**	**-8.22**	-1.69	**6.36**

	SSP	THRM	TISI	UCBI	UFI	UIS	VIAV	XPO	ZD
Split size	1-3	1-5	1-10	1-5	1-3	1-10	1-8	1-4	1-4
τ	**12.78**	1.05	**-2.22**	0.66	**-6.77**	**-18.41**	**-2.24**	-0.09	**-3.33**