Debt, Default, and Commitment

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Outline

Motivation I/II

- Quantitative sovereign default literature (*Aguiar et al. 2017*)
  - Rationalise typical emerging market features
  - Evaluate public policy measures (e.g. austerity, third-party loans)
- Workhorse model based on *Eaton and Gersovitz (1981)*
  1. Incomplete financial markets
  2. Ability to default
  3. Complete lack of commitment
- This paper asks:
  - What is the role of (lack of) commitment?

Motivation II/II

- Why?
  - Does commitment matter for model predictions?
  - Degree of commitment matters for many policy measures
  - Study welfare gains of commitment
- How?
  - Introduce loose commitment (see *Debortoli and Nunes, 2010*) into a model à la *Arellano (2008)*
    - Optimal ex-ante plan but re-optimisation ex post with prob. $1 - \lambda$
    - Model nests full commitment ($\lambda = 1$) and no commitment ($\lambda = 0$)
  - Perform quantitative exercises to assess role of commitment

Preview

- Role of commitment for quantitative models of sovereign default?
  - Under commitment
    - Default risk / spread is countercyclical
    - Debt and deficit are countercyclical
    - Consumption is less volatile than income
  - Under loose commitment, new trade-offs arise
  - Welfare gains of commitment mostly due to front-loading motive
- Predictions under commitment provide better fit for European debt crisis

Related literature

- Quantitative sovereign default literature
  - *Aguiar and Gopinath (2006)*; *Arellano (2008)*; *Bocola et al. (2019)*
  - *Cuadra and Sapriza (2008)*; *Hatchondo et al. (2009)*
- Default models with commitment
  - *Adam and Grill (2017)*; *Pouzo and Presno (2022)*; *Mateos-Planas et al. (2023)*
- Loose commitment
  - *Roberds (1987)*; *Schaumburg and Tambalotti (2007)*; *Debortoli and Nunes (2010; 2013)*

Outline

Setting

Economy

- Small open economy is inhabited by household with objective

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \right],$$

with $u(c) = (c^{1-\gamma} - 1)/(1-\gamma)$, $0 < \gamma \neq 1$, and $\beta \in (0, 1)$.

- Economy receives income y_t which follows a first-order Markov process with
  - finite support $\mathbb{Y} = \{y_1, ..., y_Y\}$,
  - conditional transition probabilities $\pi\left(y_{t+1} | y_t\right)$.

- There is a minimum consumption level $\underline{c} \geq 0$.

Setting

Government

- A benevolent government borrows from investors to smooth (and front-load) household consumption
  - Access to non-contingent one-period bond b_t at unit price q_t.
- Can default on debt payments $d_t \in \{0, 1\}$
- Costs of default $d_t = 1$ following *Arellano (2008)*
  - Exclusion from financial markets for $1/\theta$ periods on average
  - Income loss $\phi(y_t) \geq 0$
- Bonds are traded with risk-neutral investors who can borrow or save at real risk-free rate r

$$q_t = \frac{1 - \mathbb{E}_t\left[d_{t+1}\right]}{1 + r}$$

  - Time-inconsistency problem due to default decision ▸ Link

Ramsey problem

Recursive formulation

- As in *Kydland and Prescott (1980)* and *Chang (1998)*
  - Two sub-problems, one for $t = 0$ and one for $t \geq 1$
  - Recursive formulation via additional (co-)state variable
- Conditional on good credit status, problem is recursive in states
  - Debt $b \in \mathbb{B}$
  - Income $y \in \mathbb{Y}$
  - Default promise $d \in \{0, 1\}$
- Promise-keeping constraint,

$$\mathcal{V}^c(b, d, y) = (1 - d)\,\mathcal{V}^r(b, y) + d\mathcal{V}^d(y),$$

enforces state-contingent default default promises made in the past, $\mathbf{d}_{y'} = (d_{y_1}, d_{y_2}, ..., d_{y_Y}) \in \{0, 1\}^Y$.

Ramsey problem

Recursive formulation (cont'd)

- In the repayment case, the government solves

$$\mathcal{V}^r(b, y) = \max_{b' \in \mathbb{B},\, \mathbf{d}_{y'} \in \{0,1\}^Y} \left\{ \begin{array}{c} u\left(y + q\left(\mathbf{d}_{y'}, y\right) b' - b - \underline{c}\right) \\ +\beta \mathbb{E}_{y'|y}\left[\mathcal{V}^c(b', d_{y'}, y')\right] \end{array} \right\},$$

with bond price schedule

$$q\left(\mathbf{d}_{y'}, y\right) = \frac{1 - \mathbb{E}_{y'|y}\left[d_{y'}\right]}{1 + r}.$$

- In the default case, the government solves

$$\mathcal{V}^d(y) = \max_{\mathbf{d}_{y'} \in \{0,1\}^Y} \left\{ \begin{array}{c} u\left(y - \phi(y) - \underline{c}\right) \\ +\left(1 - \theta\right) \beta \mathbb{E}_{y'|y}\left[\mathcal{V}^d(y')\right] \\ +\theta \beta \mathbb{E}_{y'|y}\left[\mathcal{V}^c(0, d_{y'}, y')\right] \end{array} \right\}.$$

Ramsey problem

First-period problem

- In the first period, there is no time-inconsistency problem.
- The government therefore solves

$$\mathcal{V}^o(b, y) = \max_{d \in \{0,1\}} \mathcal{V}^c(b, d, y),$$

  with option value of default $\mathcal{V}^o(b, y)$.
- Let $\mathcal{D}(b, y)$ be the policy function that solves this problem.

Loose commitment

- Now government re-optimises ex-ante plan with probability $1 - \lambda$.
- Conditional on good credit status:
  - Promise *d* determines repayment with probability λ
  - Function $\mathcal{D}(b, y)$ determines repayment with probability $1 - \lambda$
- Ramsey ($\lambda = 1$) and Markov ($\lambda = 0$) policies as special cases
- Under loose commitment ($0 < \lambda < 1$), government knows its promises might not be kept.

Loose commitment (cont'd)

- In the repayment case, the government now solves

$$\mathcal{V}^r(b,y) = \max_{b' \in \mathbb{B},\, \mathbf{d}_{y'} \in \{0,1\}^Y} \left\{ \begin{array}{c} u\left(y + q\left(b', \mathbf{d}_{y'}, y\right) b' - b - \underline{c}\right) \\ +\lambda\beta\mathbb{E}_{y'|y}\left[\mathcal{V}^c(b', d_{y'}, y')\right] \\ +(1-\lambda)\beta\mathbb{E}_{y'|y}\left[\mathcal{V}^o(b', y')\right] \end{array} \right\},$$

  with bond price schedule

$$q\left(b', \mathbf{d}_{y'}, y\right) = \frac{1 - \mathbb{E}_{y'|y}\left[\lambda d_{y'} + (1-\lambda)\mathcal{D}(b', y')\right]}{1+r}.$$

- In the default case, the government now solves

$$\mathcal{V}^d(y) = \max_{\mathbf{d}_{y'} \in \{0,1\}^Y} \left\{ \begin{array}{c} u\left(y - \phi(y) - \underline{c}\right) + (1-\theta)\,\beta\mathbb{E}_{y'|y}\left[\mathcal{V}^d(y')\right] \\ +\theta\lambda\beta\mathbb{E}_{y'|y}\left[\mathcal{V}^c(0, d_{y'}, y')\right] \\ +\theta(1-\lambda)\beta\mathbb{E}_{y'|y}\left[\mathcal{V}^o(0, y')\right] \end{array} \right\}.$$

▸ Link

Outline

Outline

Two exercises

- First exercise
  - Standard calibration under no-commitment assumption $(\lambda = 0)$
    - Match short- and long-run properties for Argentina
  - What are the implications of different degrees of commitment?
- Second exercise
  - Application to European debt crisis
    - The role of commitment for debt and spread dynamics
  - Horse race between no- and full-commitment model

Functional forms

- Recursive preferences (*Epstein and Zin, 1991; Weil, 1990*):

$$\mathcal{V}_t = u(c_t) + \beta \frac{\left(\mathbb{E}_t\left[\left(1+(1-\beta)(1-\gamma)\mathcal{V}_{t+1}\right)^{\frac{1-\alpha}{1-\gamma}}\right]\right)^{\frac{1-\gamma}{1-\alpha}} - 1}{(1-\beta)(1-\gamma)}$$

- Default costs as in *Chatterjee and Eyigungor (2012)*

$$\phi(y) = \max\left\{0, \phi_1 y + \phi_2 y^2\right\}$$

- Support $\mathbb{Y}$ and transition probabilities $\pi\left(y'|y\right)$ are obtained by discretising the log-normal AR(1)-process

$$\ln y_t = \rho \ln y_{t-1} + \sigma\varepsilon_t,\ \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,1),$$

via the method proposed by *Tauchen (1986)*.

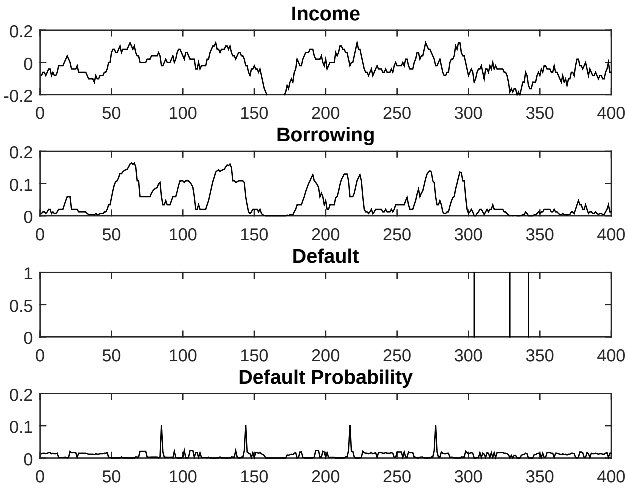
Model parameters

Baseline calibration for Argentina

Parameter	Description	Value
α	Coefficient of CRRA	2
β	Discount factor	0.966
γ	Inverse of IES	2
ρ	Persistence of income	0.945
σ	Std. dev. of income shock	0.025
ϕ_1	Default cost parameter	-1.187
ϕ_2	Default cost parameter	1.228
λ	Degree of commitment	0
θ	Probability of exiting autarky	0.250
$\overline{b}$	Debt limit	0.250
$\underline{b}$	Saving limit	0
$\underline{c}$	Minimum consumption level	0
r	Risk-free rate	0.010

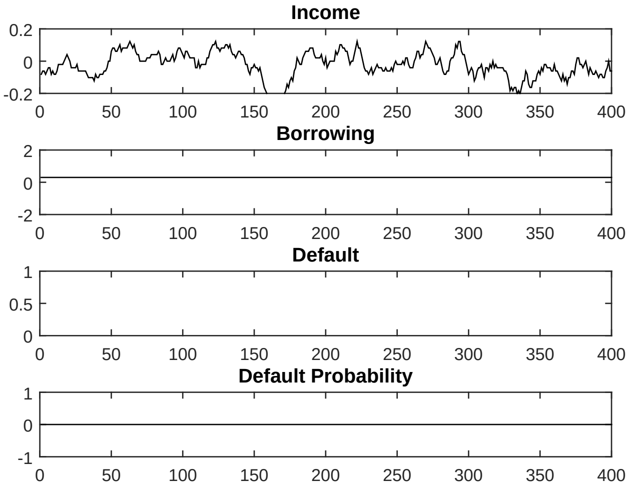
Results

Sample path without commitment ($\lambda = 0$)



Results

Sample path with commitment ($\lambda = 1$)



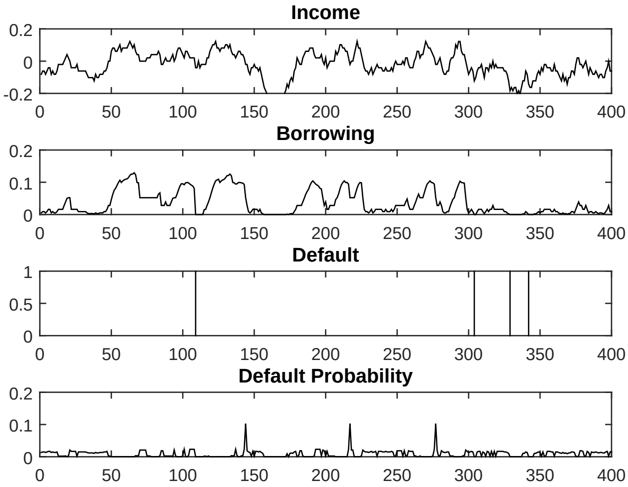
Results

Model statistics

	$\lambda = 0$	$\lambda = 0.4$	$\lambda = 0.7$	$\lambda = 0.9$	$\lambda = 1$
Mean					
Def. prob. overall (annual)	0.030	0.026	0.049	0.113	0.000
Def. prob. comm. (annual)	-	0.000	0.001	0.000	0.000
Def. prob. no comm. (annual)	0.030	0.042	0.152	0.754	-
Debt-service-to-output	0.055	0.061	0.075	0.157	0.251
Interest rate spread (annual)	0.033	0.029	0.059	0.156	0.000
Standard deviation					
Output	0.079	0.079	0.079	0.079	0.079
Consumption	0.081	0.082	0.082	0.082	0.079
Surplus-to-output	0.012	0.013	0.014	0.018	0.000
Correlation with output					
Consumption	0.989	0.988	0.985	0.976	1.000
Interest rate spread (annual)	-0.506	-0.052	-0.320	-0.820	-
Debt-issuance-to-output	0.890	0.888	0.819	0.217	-0.998
Surplus-to-output	-0.167	-0.173	-0.161	-0.057	-0.602

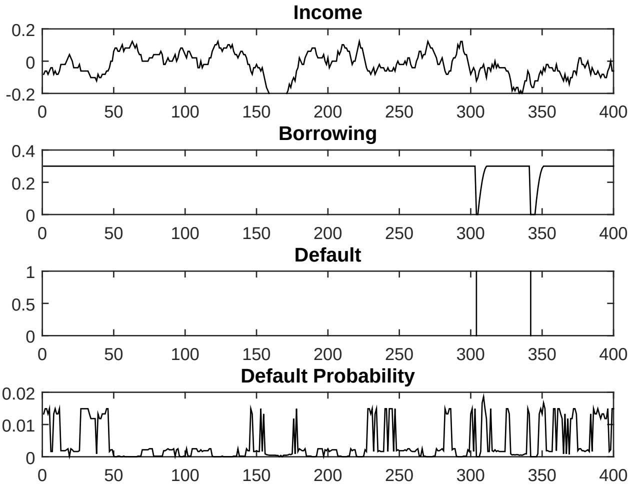
Results

Sample path without commitment ($\lambda = 0$) and with high risk aversion ($\alpha = 10$)



Results

Sample path with commitment ($\lambda = 1$) and high risk aversion ($\alpha = 10$)



Results

Model statistics ($\alpha = 10$)

	$\lambda = 0$	$\lambda = 0.4$	$\lambda = 0.7$	$\lambda = 0.9$	$\lambda = 1$
Mean					
Def. prob. overall (annual)	0.030	0.036	0.067	0.141	0.020
Def. prob. comm. (annual)	-	0.007	0.024	0.042	0.020
Def. prob. no comm. (annual)	0.030	0.052	0.144	0.726	-
Debt-service-to-output	0.042	0.048	0.060	0.139	0.243
Interest rate spread (annual)	0.032	0.049	0.092	0.212	0.022
Standard deviation					
Output	0.079	0.079	0.079	0.079	0.079
Consumption	0.081	0.081	0.081	0.080	0.079
Surplus-to-output	0.009	0.010	0.012	0.018	0.010
Correlation with output					
Consumption	0.994	0.992	0.988	0.975	0.993
Interest rate spread (annual)	-0.385	-0.150	-0.234	-0.747	-0.674
Debt-issuance-to-output	0.878	0.861	0.763	0.246	-0.238
Surplus-to-output	-0.163	-0.163	-0.126	0.019	0.063

Results

Summary of main findings

- Under commitment ($\lambda = 1$), no default with standard calibration.
    - With higher risk aversion, default occurs under (loose) commitment.
- Model economy under full commitment
    - Countercyclical default risk
    - Countercyclical debt and deficit
    - Consumption less volatile than income
- Role of degree of commitment λ
    - Hump-shaped effect of λ on average interest rate spread
    - Average debt increases with λ

Results

Welfare gains of commitment

- Welfare-equivalent consumption variation Δ for different degrees of commitment λ

$$\Delta = \frac{\sum_y \left(1 + (1-\beta)(1-\gamma)\mathcal{V}^o_\lambda(0,y)\right)^{1/(1-\gamma)} \Pi(y)}{\sum_y \left(1 + (1-\beta)(1-\gamma)\mathcal{V}^o_0(0,y)\right)^{1/(1-\gamma)} \Pi(y)} - 1.$$

- Δ-values (in %) for baseline calibration

	$\lambda = 0.4$	$\lambda = 0.7$	$\lambda = 0.9$	$\lambda = 0.97$	$\lambda = 1$
$\alpha = 2$	0.006	0.018	0.114	0.318	0.484
$\alpha = 10$	0.004	0.018	0.129	0.341	0.508

European debt crisis

The role of commitment

- European debt crisis governments (Italy, Portugal, Spain, ...)
  - Government borrowing is countercyclical – even at sizable default risk
  - Behaviour at odds with standard no-commitment model
- *Bocola et al. (2019)* propose recalibration with $\underline{c} > 0$
  - Make government more averse to low income states.
  - $\underline{c}$ as implicit (fixed) commitment device.
- No-commitment government is forced to behave like government naturally does under commitment.
  - How does model performance with $\lambda = 1$ compare to $\lambda = 0$?

European debt crisis

Recalibration

- $\underline{c}$ matters for natural debt limit (*Adam and Grill, 2017*)
  - $\overline{b} \equiv (1 + r)(y_1 - \underline{c})/r$
  - Set $\underline{c}$ to match debt service under commitment
- Calibrate model to Spain as in *Bocola et al. (2019)* for $\lambda = 1$
  - $(r, \gamma, \theta, \rho, \sigma) = (0.0045, 2, 0.282, 0.97, 0.01)$ as *Bocola et al. (2019)*
  - $(\beta, \underline{c}, \phi_1, \phi_2)$ chosen to match targets from *Bocola et al. (2019)*

European debt crisis

Model statistics

	Data	*Bocola et al. (2019)*	$\lambda = 1$
Average interest rate spread	0.32	0.09	0.19
Average debt-service-to-output	8.43	8.52	8.52
Interest rate spread volatility	0.88	0.83	0.88
Debt service cyclicality	-0.87	-0.29	-0.75

Outline

1. Introduction

2. Model

3. Quantitative Analysis

4. Conclusion

Conclusion

- Role of commitment for quantitative models of sovereign default?
  - Under commitment
    - Default risk / spread is countercyclical
    - Debt and deficit are countercyclical
    - Consumption is less volatile than income
  - Under loose commitment, new trade-offs arise.
  - Welfare gains of commitment mostly due to front-loading motive.
- Predictions under commitment provide a better fit for European debt crisis countries
  - Alternative to no-commitment model version
  - Different welfare and policy implications!

Ramsey problem

Sequential formulation

$$\max_{\{b_t, c_t, d_t, h_t, q_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

subject to

$$c_t = y_t - \underline{c} - h_t \phi(y_t) + (1 - h_t)\left(q_t b_t - b_{t-1}\right), \tag{1}$$

$$h_t = (d_t \xi_t + 1 - \xi_t) h_{t-1} + d_t (1 - h_{t-1}), \tag{2}$$

$$q_t = \frac{1 - \mathbb{E}_t\left[d_{t+1}\right]}{1 + r}, \tag{3}$$

$$b_t \in \mathbb{B}, \quad c_t \geq \underline{c}, \quad d_t \in \{0, 1\} \tag{4}$$

$$0 = b_t h_t, \quad 0 = d_t (1 - \xi_t) h_{t-1}, \quad 0 = q_t h_t, \tag{5}$$

given initial values $b_{-1} \in \mathbb{B}$ and $h_{-1} \in \{0, 1\}$.

Back

Policy trade-offs

Debt

- The Euler equation for debt is

$$
\begin{aligned}
&\underbrace{\lambda \mathbb{E}_{y'|y}\left[(1-d_{y'})\left(u_c(c)-\tilde{\beta}u_c(c')\right)\right]}_{\text{consumption smoothing (commitment part)}} \\
&+\underbrace{(1-\lambda)\mathbb{E}_{y'|y}\left[(1-d')\left(u_c(c)-\tilde{\beta}u_c(c')\right)\right]}_{\text{consumption smoothing (no-commitment part)}} \\
=\quad &\underbrace{(1-\lambda)u_c(c)\frac{\partial \mathbb{E}_{y'|y}\left[\mathcal{D}(b',y')\right]}{\partial b'}b'}_{\text{time inconsistency}}\ \underbrace{-\mu_{\underline{b}}+\mu_{\overline{b}}}_{\text{debt/savings constraints}},
\end{aligned}
$$

where $\tilde{\beta} \equiv \beta(1+r)$.

Policy trade-offs

Default

- Optimal default under discretion satisfies

$$\mathcal{D}(b', y') = \begin{cases} 1, & \text{if} \quad \mathcal{V}^d(y') - \mathcal{V}^r(b', y') > 0, \\ 0, & \text{if} \quad \mathcal{V}^d(y') - \mathcal{V}^r(b', y') \leq 0. \end{cases}$$

- Optimal default under commitment satisfies

$$\underbrace{u_c(c)b'}_{\text{marginal cost}} = \underbrace{\tilde{\beta}\left[\mathcal{V}^d(\hat{y}) - \mathcal{V}^r(b', \hat{y})\right]}_{\text{marginal benefit}} \underbrace{-\mu_{\underline{y}} + \mu_{\overline{y}}}_{\text{inequality constraints}},$$

where

$$d_{y'} = \begin{cases} 1, & \text{if} \quad y' < \hat{y}, \\ 0, & \text{if} \quad y' \geq \hat{y}, \end{cases}$$

for all $y' \in \mathbb{Y}$.

Back