

# What is productive investment?

## Insights from firm–level data for the United Kingdom

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## 1. Introduction

## 2. Data and Empirical Analysis

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- ▶ At the aggregate, UK data suggests a strong positive correlation between corporate debt and investment, whereas the correlation between debt and productivity is more tenuous.
- ▶ At firm level, there is strong evidence in the literature suggesting that high corporate debt leads to lower investment, especially in times of crisis, with negative subsequent effects on productivity. (see e.g. [Duval et al. \(2020\)](#), [Gopinath et al. \(2017\)](#), [Kalemli-Ozcan et al. \(2022\)](#), [Buera and Karmakar \(2021\)](#)).
- ▶ Typically leverage is assumed to be "good" in the boom phase, as it allows firms to invest in their productive capacity. Debt then becomes "bad" in a downturn owing to debt overhang reasons.
- ▶ Rather than studying the *state*-dependency of the debt-investment relationship (i.e., effects of debt and investment depending on the state of the business cycle), we study the *type*-dependency (i.e., effects of debt depending on what type of investments the firm undertakes).

1. What are the characteristics of and inter-dependencies between different types of investment, debt and total factor productivity (TFP) in UK firm-level data?
2. Using empirical firm-level data for the UK and a stylised structural model, can we distinguish between "good" and "bad" leverage in terms of higher debt being associated with investment that is productive (i.e., a positive effect on TFP) vs investment that is not productive (i.e., a zero/negative effect on TFP)?

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- ▶ Worldscope financial account data on UK listed firms.
- ▶ Annual data 1990-2018.
- ▶ Exclude finance and oil sectors (ISIC 2-digit sectors 2, 19, 64–66).
- ▶ 27,712 firm–year observations.
- ▶ Data winzorised at 1st/99th quantiles.

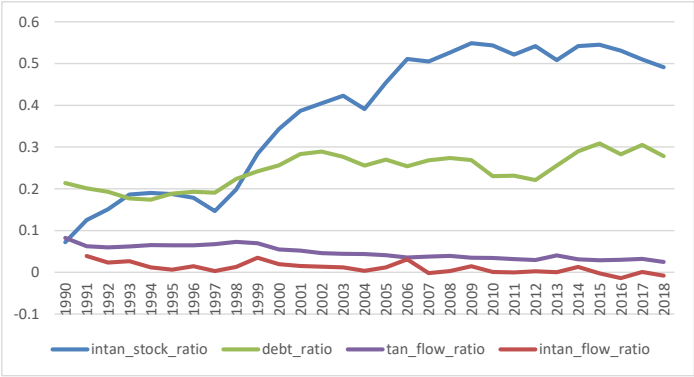


**Table 1: Key variables of interest**

Variable	Definition	Worldscope code
tfp	Akerberg-Caves-Frazer (2015) measure of TFP (log level)	n/a
intan_stock	intan_stock divided by total assets	n/a
intan_flow	y/y change of intan_stock, divided by total assets	n/a
debt	total debt divided by total assets (corrected for industry medians)	totdebt/totass
tan_stock	Fixed tangible assets divided by total assets	ppegross/totass
tan_flow	capital expenditure divided by total assets	tan_stock(t)-tan_stock(t-1)
age	years since incorporation	age
size	real total assets (log level, deflated by aggregate GDP deflator)	totass
profit	profits (EBITDA) divided by total assets	ebitda/totass
cash	cash and short-term investment divided by total assets	csti/totass
equity	total equity divided by total assets	(comstock+capsurpl)/totass
iex	interest expense divided by total debt	intex/totdebt

Measuring Intangibles

Fig 1: Medians over time, as shares of total assets



Sources: Worldscope and own calculations.  
Notes: Data are employment-weighted medians.

Our panel regression specification is the following:

$$z_{it} = \alpha + \beta_1 D_{i,t-1} + \beta_2 R_{i,t-1} + \beta_3 D_{i,t-1} R_{i,t-1} + c_i + f_t + X_{i,t-1} + e_{it} \quad (1)$$

where  $z_{it}$  is (log)level of TFP (estimated with [Ackerberg et al. \(2015\)](#) for consistency) for firm  $i$  at time  $t$ ,  $D_{it}$  is debt\_ratio and  $R_{it}$  is investment variable dummy (=1 for the highest quartile),  $X_{it}$  are firm-level controls (profits, age, size, cash)  $c_i$  and  $f_t$  are firm and year FE, respectively, and  $e_{it}$  is i.i.d. error. Investment variable  $R \in (intans, tans)$ .

- ▶ Interested in effect of investment variable ( $\beta_2$ ), debt ratio ( $\beta_1$ ), the interaction effect ( $\beta_3$ ) and the joint significance of debt ( $\beta_1 + \beta_3$ ).
- ▶ Also estimate with system-GMM to mitigate endogeneity issues, with interest-to-debt ratio as an additional instrument; results similar.

Table 2: Size of effects on TFP

## What is the effect on level of TFP at time t of...

...firm being in the highest quartile of...	
...variable (t-1):	effect :
intan_stock	9.7% ***
intan_flow	2.4% ***
tan_stock	-2.5% **
tan_flow	-2.4% ***

...a 10pp increase in debt ratio and firm being in the highest quartile of...	
...variable (t-1):	effect :
intan_stock	1.1% **
intan_flow	0.9% *
tan_stock	0.6%
tan_flow	0.6%

- ▶ Intangible stocks and flows are associated with higher firm TFP
- ▶ Increases in debt that are used to build up the intangible stock yield more productive firms. No such effect for tangible investment.

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We build a stylised model to illustrate the effects of investment and debt on TFP. Model is based on [Warusawitharana \(2015\)](#) and [Levine and Warusawitharana \(2021\)](#), but we add debt financing stocks and flows as control and state variables.

Value function ( $V$ ) is:

$$V = \max_{i,r} E_0 \int_0^{\infty} e^{\rho t} \pi(z, k, s, b, i, r) dt \quad (2)$$

subject to:  $\dot{z} = \mu_z - \rho_z(z - \mu_z) + \sigma_z dW$ ,  $\dot{k} = \delta_k k + i$ ,  $\dot{s} = \delta_s s + r$ ,  $\dot{b} = \text{if } (\pi < 0)\pi - bx, \text{ else } -bx$

$$\pi(z, k, s, b, i, r) = y(z, k, s) - i - r - \lambda_k \frac{k}{2} \left( \frac{i}{k} - \delta_k \right)^2 - \lambda_s \frac{s}{2} \left( \frac{r}{s} - \delta_s \right)^2 - b(x + B(z, k, s, b, \psi_1, \psi_2)) \quad (3)$$

$V$  is maximised for the four state variables,  $z$  (TFP, O-U process),  $k$  (physical capital)  $s$  (intangible capital), and  $b$  (debt) by choosing tangible investment  $i$  and intangible investment  $r$  optimally.  $\rho$  is the discount factor.  $\delta_k$  and  $\delta_s$  are the depreciation rates of physical and intangible capital.  $x$  is the share of debt a firm pays back in every period,  $f$  is the amount of debt.  $y$  is output, and dividends are affected by convex [Hayashi \(1982\)](#)-type adjustment costs, with size parameters  $\lambda_k$  and  $\lambda_s$ .  $B(k, s, b)$  is a term capturing higher leverage leading to higher financing costs.

Straightforward to turn this stochastic continuous time value function into the HJB with Ito's lemma:

$$\rho V(z, k, s, b) = \pi(z, k, s, b, i, r) + \frac{\delta V}{\delta k}(-\delta_k k + i) + \frac{\delta V}{\delta s}(-\delta_s s + r) + \frac{\delta V}{\delta b}(-xb + f) + \frac{\delta V}{\delta z}(1 - \rho_z(z - 1)) + \frac{\sigma_z^2}{2} \frac{\delta^2 V}{\delta z^2} \quad (4)$$

More important is the output function. We use two:

$y(z, k, s) = zs^{\gamma_1} k^\alpha$  in this case  $\gamma_1 \in [0, 1 - \alpha]$ . We can hence estimate freely whether  $\gamma_1 > \alpha$ .

Once this is established we use:

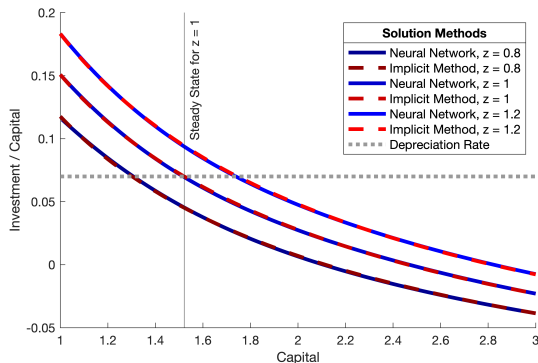
$y(z, k, s) = (z + s^{\gamma_2})(k + s)^\alpha$  which is equivalent to the empirical setup. In this case the estimation requires  $\gamma_2 > 0$ , which would show up as  $\gamma_1 > \alpha$  in the previous estimation. Further we require  $(\delta_s > \delta_k)$ , which is a well - known empirical fact.

# Solve with a Neural Network

- ▶ Could solve with the implicit method described in [Achdou et al. \(2020\)](#).
- ▶ Quick to solve but extensive coding for four states.
- ▶ Short coding and longer solve with a neural network (similar to [Fernández-Villaverde et al. \(2023\)](#)).
- ▶ Easy to scale in states and performs similarly – shown for a simplified model with two states:

$$V = \max_{i,r} E_0 \int_0^{\infty} e^{\rho t} \pi(z, k) dt \quad (5)$$

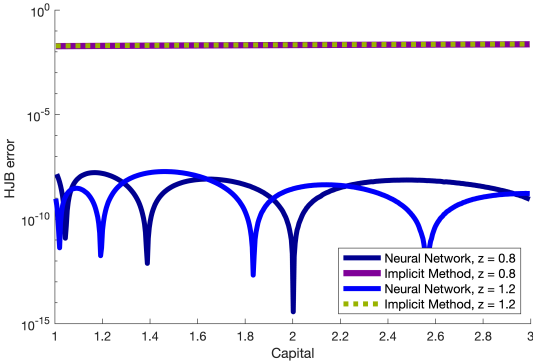
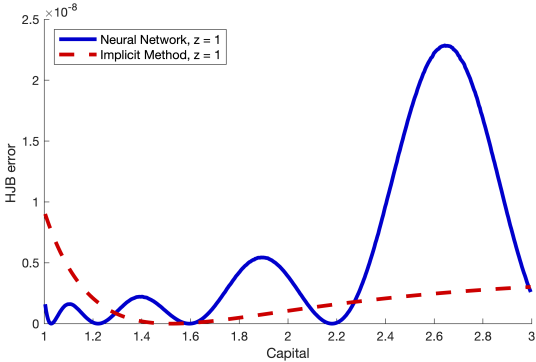
$$\text{subject to: } \dot{z} = \mu_z - \rho_z(z - \mu_z) + \sigma_z dW, \\ \dot{k} = \delta_k k + i$$





# Neural Network performs solves the problem over the whole state space

## HJB Errors for different productivity states.



- ▶ **We methodologically contribute to the literature by proposing a different estimate value function parameters based on the observed states and investment choices in the data.**
  - ▶ We propose to take the states  $X_1 = [z, k, s, b]$  and add a selection of the parameters  $\Psi_X = [\gamma, \psi_1, \psi_2, \dots]$  as additional states to solve the value function for. We then estimate the parameters  $\Psi_X$  governing the value function given data  $Y$ .
  - ▶ Our approach is matching moments; we are matching the investment and debt choices and productivity to quartiles of the state space found in our empirical data.
  - ▶ The advantage of this approach is that we are able to estimate parameters, which would be more difficult to calibrate based on empirics.
- ▶ We then use the model to see if we can replicate the relative relationships between key variables we found in the empirical section, and also draw some aggregate level TFP conclusions.

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**Algorithm 1** Estimating Value function standard

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- 1: Solve  $V(X|\Psi_1)$  producing output  $\hat{Y}$  ( $\hat{Y}$  can be conditional on  $X$ )
  - 2: **for**  $N$  less than Maxit **do**
  - 3:   Check weighted distance  $\|\hat{Y} - Y\|$
  - 4:   Update  $\Psi$  given distance e.g. Newton
  - 5:   Solve  $V(X_1|\Psi_{N+1})$  producing output  $\hat{Y}$
  - 6:   If:  $|\Psi_{N+1} - \Psi_N| < \epsilon$  Break the Loop
  - 7: **end for**
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**Algorithm 2** Estimating Value function with by solving for to be estimated parameters

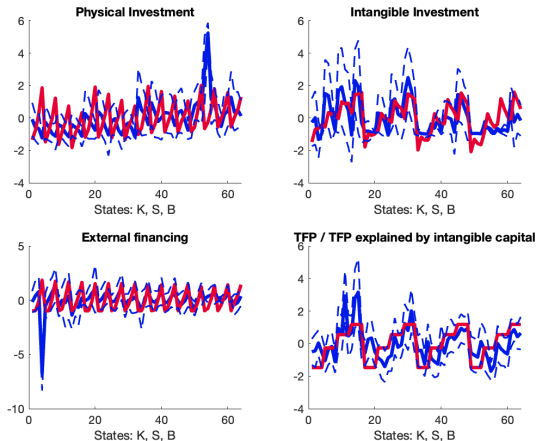
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- 1: Solve  $V([X\Psi_X]|\bar{\Psi})$  producing output  $\hat{Y}(\Psi_X)$
  - 2: Alter  $\Psi_X$  such that  $\min\|\hat{Y}(\Psi_X) - Y\|$  e.g.  $\min_{\Psi_X}(\hat{Y}(\Psi_X) - Y)W^{-1}(\hat{Y}(\Psi_X) - Y)'$
  - 3: When  $\|\hat{Y}(\hat{\Psi}_X) - Y\|$  is minimised  $\hat{\Psi}_X$  is the estimate
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Table 3: Model parameter calibration and estimation results

Parameter	Value	Description
$\rho$	0.05	Annual discount factor of 0.9512.
$\delta_k$	0.07	Depreciation rate of physical capital (from the data)
$\delta_s$	0.2	Depreciation rate of intangible capital (from the data)
$\alpha$	0.17	Exponent on combined (Baseline) or physical (C-D) capital (from the data)
$x$	0.06	Principal pay back rate of loans
<i>Estimated Parameters</i>		
$\gamma$	Estimated	Exponent on intangible capital
$\psi_1$	Estimated	Cost of leverage
$\psi_2$	Estimated	Additional cost of intangible heavy balance sheet ( <a href="#">Falato et al. (2022)</a> )
$\lambda_k$	Estimated	Adjustment cost of physical capital
$\lambda_s$	Estimated	Adjustment cost of intangible capital

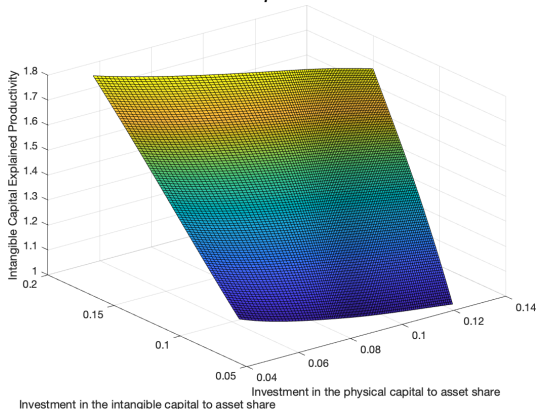
# Estimated model results I



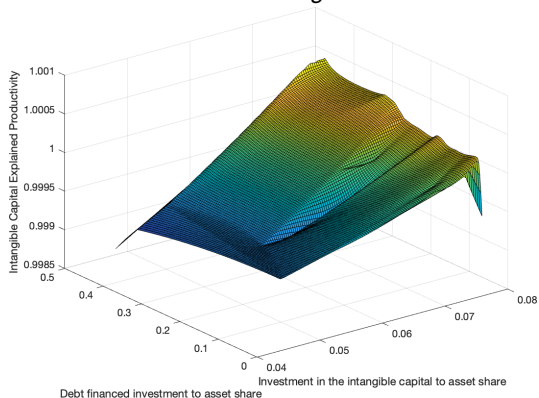
Estimated Parameters	$\psi_1$	$\psi_2$	$\gamma$
Weighted fit	0.0309	0.0154	0.0103
Weighted fit C-D	0.00701	0.0317	0.1712

Fit shows positive contribution of intangible capital to productivity. Apply this results to the change in the growth rate in intangibles post GFC => **explain around 11% of the productivity slowdown post GFC in the UK.**

**Fig 2a:** *Investment in Physical & Intangible Capital*



**Fig 2b:** *Investment in Intangible Capital & External Financing*



**Note:** The model is parameterised according to the calibration and estimation. The results are then interpolated based on the model predictions, collapsing the model predictions for investment activity given the three model state variables and the predicted investment outcomes.

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- ▶ We study the effects of different types of investment and levels of debt on productivity, using firm-level data on UK listed firms.
- ▶ We combine these empirical regression results with a stylised structural model, featuring a dynamic profit–maximisation problem of the firm, in a novel way.
- ▶ We find intangibles investments to be a good proxy for "productive" investment, as they have a positive effect on TFP. Our results also suggest that a combination of high debt and high intangibles investment has a positive effect on TFP.
- ▶ Applying our structural model to aggregate TFP dynamics in the UK suggests that around 11% of TFP slowdown in the UK since the Global Financial Crisis can be attributed to weaker intangibles investments in our sample of firms.



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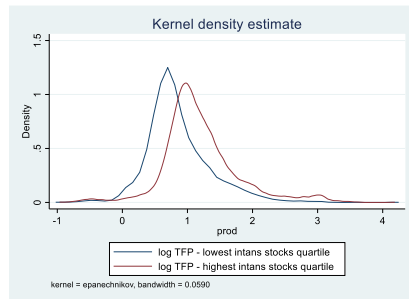
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Peters and Taylor (2017) introduces a methodology for measuring intangible stocks and flows, including a method for accumulating expensed intangibles items into stocks. In our definitions, we follow their methodology. Intangible capital stock ( $K_{it}^{int}$ ) is defined as:

$$K_{it}^{int} = INT_{it} + A_{it} + B_{it} \quad (6)$$

where  $INT_{it}$  is intangible assets in the balance sheet for firm  $i$  at time  $t$ ,  $A_{it}$  is accumulated R&D (RD) spending (defined below) and  $B_{it}$  is accumulated sales, general and administrative (SGA) spending (defined below).

Figure: TFP intan densities



Sources: Worldscope.

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