The Endogenous Growth and Asset Prices Nexus Revisited with Closed-form Solution

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Outline

Motivation

- Long-run risk is one example to solve the equity premium puzzle (others: rare disasters, habits, behavioural, etc.)
- Previous literature proposed long-run risk (e.g. Bansal and Yaron (2004) exogenous growth endowment model)
- Endogenous growth generates long-run risk (Kung and Schmid (2015) in a production model)
- 1. Without closed-form illustration it is unclear how endogenous growth produces procyclical price-consumption ratio (see also Bansal et al. 2010)
- 2. Unlike previous literature we match R&D spending-to-GDP ratio, the driver of endogenous growth
- 3. Patent obsolescence is the key to calibrate low growth rate and low risk-free rate (precautionary savings effect)
- 4. Numerical solution (third-order perturbation)

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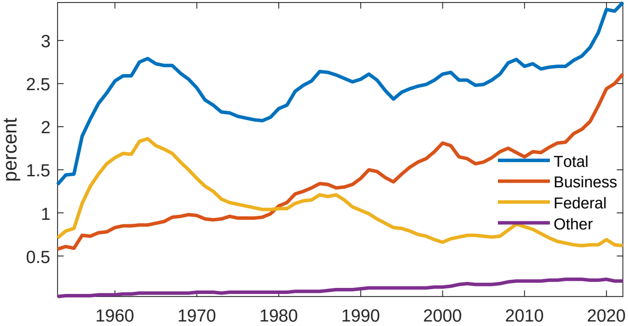
R&D spending-to-GDP in US data 1953-2023



We target US data 1929-2017: i) $E[\Delta c] = 1.82$, ii) $\sigma[\Delta c]^{bc} = 1.34$, iii) $E[R\&D/GDP] = 2.46$ (1953-2017), iv) and risk-free rate, $E[r_f] = 2.40$ (inconsistent with US data).

The simplified endogenous growth model of Fornaro and Wolf (2023)

- 3 main equations + shock process:
- 1. Risk-free Euler: $1 = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\psi}} G_{t+1}^{-\frac{1}{\psi}} r_t,$
- 2. Investment Euler:
  $\boldsymbol{s_t^{1-\zeta} = \left(\frac{1}{1+r_t+\eta}\right)\left(\varsigma\chi\bar{\omega}Z_{t+1}L + (1-\phi)s_{t+1}^{1-\zeta}\right)},$
- 3. Market clearing: $\Psi Z_t L = c_t + s_t,$
- Shock process: $log(Z_{t+1}/Z) \equiv \widehat{z}_{t+1} = \rho\widehat{z}_t + \sigma_z\varepsilon_{t+1}.$

Decomposition

We decompose expected consumption growth into:

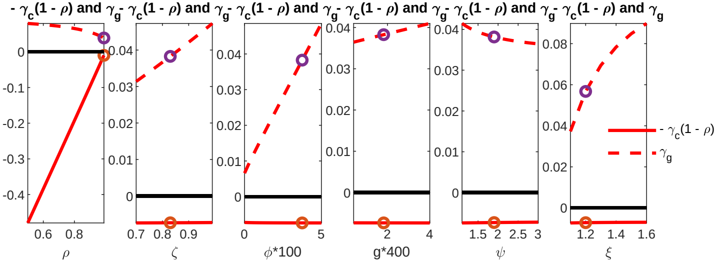
$$
\begin{aligned}
E_t \Delta \hat{c}^{total}_{t+1} &= E_t[\underbrace{\Delta \hat{c}_{t+1}}_{cyclical} + \underbrace{\hat{g}_{t+1}}_{trend}] \\
&= \gamma_c E_t[\Delta \hat{z}_{t+1}] + \gamma_g \hat{z}_t \\
&= \underbrace{-\gamma_c(1-\rho)}_{sign:-} z_t + \underbrace{\gamma_g}_{sign:+} \hat{z}_t \qquad (1)
\end{aligned}
$$

where

$$\hat{c}_t = \gamma_c z_t \text{ and } \hat{g}_{t+1} = \gamma_g z_t \qquad (2)$$

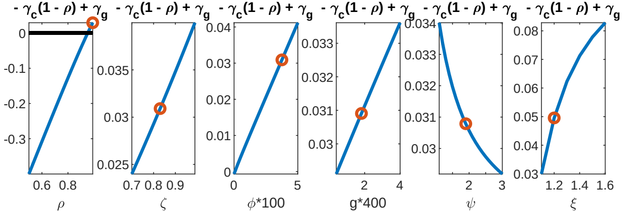
where variables are written in log-deviation from steady-state: $\hat{c}_t \equiv log(c_t/c)$ and $\hat{g}_{t+1} = log(G_{t+1}/G)$

Sensitivity of the cyclical, $-(1-\rho)\gamma_c$ and trend, γ_g components of expected consumption growth to parameters and growth rate, g



Circles: benchmark calibration. ρ is the AR(1) of the shock, ζ is the curvature of the investment function, ϕ is patent obsolescence, g is growth rate, ψ is elasticity of intertemporal substitution, and ξ is the gross markup.

Sensitivity of total expected consumption growth, $-(1-\rho)\gamma_c + \gamma_g$ to params. and growth rate, g



Circles denote benchmark calibration. Note: expected consumption growth rises to positive shocks if persistence of the shock is high, $\rho > 0.95$.

Loglinear-lognormal asset pricing

- 1. macroeconomic model is loglinearised.
- 2. asset are priced with i) Epstein-Zin preferences with pricing kernel

$$M_{t+1} = \beta^{\theta} \left(\frac{c_{t+1}}{c_t} G_{t+1} \right)^{-\frac{\theta}{\psi}} r_{c,t+1}^{\theta-1}$$

  and ii) lognormality assumption:
  $0 = E_t[m_{t+1} + r_{c,t+1}] + \frac{1}{2} Var_t[m_{t+1} + r_{c,t+1}]$
- where the return, $r_{c,t+1}$ follows Campbell-Shiller (1988):

$$r_{c,t+1} = \kappa_0 + \kappa_1 p_{c,t+1} - p_{c,t} + \Delta c_{t+1}^{total}.$$

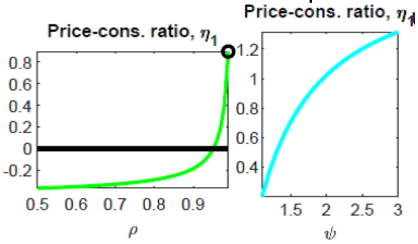
- Guess that price-consumption ratio: $p_{c,t} = \eta_0 + \eta_1 z_t$, where η_0 and η_1 coefficients to be determined.

When asset prices rise to positive technology shocks?

Price-consumption ratio as a function of technology shocks:

$$\eta_1 = \frac{(1-1/\psi)[-\gamma_c(1-\rho)+\gamma_g]}{1-\kappa_1\rho},$$

Takeaway: the price-consumption ratio is procyclical, $\eta_1 > 0$ only when the shock is rather persistent, $\rho > 0.95$ and $\psi = EIS > 1$.



Closed-forms for equity premium and risk-free rate

- Levered equity premium:

$$\phi_{lev} log E_t[exp(r_{c,t+1} - r_{f,t})] = \underbrace{\phi_{lev}\gamma\gamma_c\kappa_1\eta_1\sigma_z^2}_{CRRA} + \underbrace{\phi_{lev}(1-\theta)\kappa_1^2\eta_1^2\sigma_z^2}_{Epstein-Zin, \theta\neq 1}$$

- Risk-free rate:

$$r_{f,t} = \underbrace{-log(\beta) + \frac{g}{\psi}}_{\text{without uncertainty}} \underbrace{-\frac{1}{\psi}(-\gamma_c(1-\rho)+\gamma_g)z_t}_{\text{conditional on the shock, } z_t} \underbrace{-\frac{1}{2}\gamma^2\gamma_c^2\sigma_z^2}_{\text{CRRA, prec. savings}} (3)$$

$$\underbrace{+\frac{1}{2}(1/\psi-\gamma)(1-\gamma)\gamma_c^2\sigma_z^2 + \frac{1}{2}(\theta-1)\kappa_1^2\eta_1^2\sigma_z^2}_{\text{Epstein-Zin, precautionary savings}}$$

Importance of the growth rate, g and R&D spending-to-GDP ratio, $1 - s_c$

Circles denote benchmark calibration.

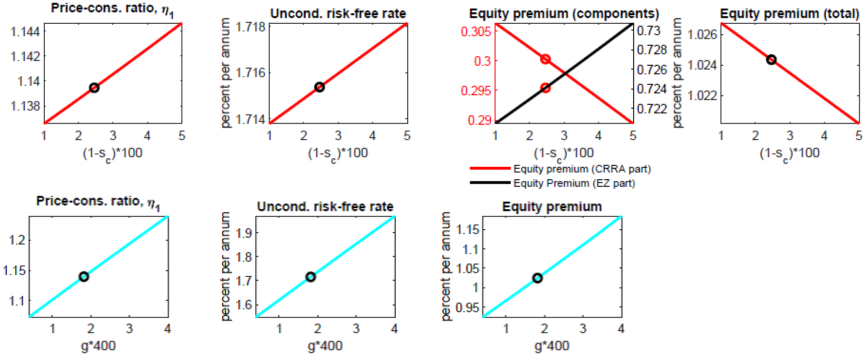


Table 2: Empirical and Simulated Moments

	US data	LL (1)	LL+SV (2)	NL (3)	NL+SV (4)	NL1 (5)
$E[s/y]$	2.46	2.46	2.46	2.59	2.66	2.46
$E[\Delta c^{total}]$	1.82	1.82	1.82	2.48	3.05	1.77
$AC1(\Delta c^{total})$	0.48			0.08	0.37	0.05
$\sigma(\Delta c^{total})$	2.10	0.65	0.65	2.65	4.09	2.63
$\sigma(\Delta c^{total})(bc)$	1.34			1.34	1.67	1.35
$\sigma(\Delta c^{total})(gc)$	0.81			0.32	0.66	0.30
$E[r_f]$	0.32	1.72	1.38	2.45	2.57	2.40
$\sigma(r_f)$	2.78	0.25	1.97	1.02	1.52	0.98
$\sigma(r_f)(bc)$	1.54			0.13	0.19	0.12
$\sigma(r_f)(gc)$	1.30			0.23	0.32	0.23
$\phi_{lev}E[r_c - r_f]$	5.81	1.02	1.49	1.30	1.43	0.31
$\sigma(r_c)$	19.49	5.37	6.54	15.16	35.19	5.54
$\sigma(r_c)(bc)$	17.03			5.19	11.26	2.75
$\sigma(r_c)(gc)$	4.74			4.18	5.16	4.10

Summary

- The trend component of consumption growth generate rising asset prices to positive shocks if the shock is persistent enough.
- Patent obsolescence keeps the risk-free rate low.
- Small rises in the R&D spending-to-GDP ratio has large effects on growth and risk-premia at the stochastic stead-state.
- When R&D spending-to-GDP is calibrated to US data at the stochastic steady-state the fit of the model to financial moments is poor.