Optimal Public Debt with Redistribution

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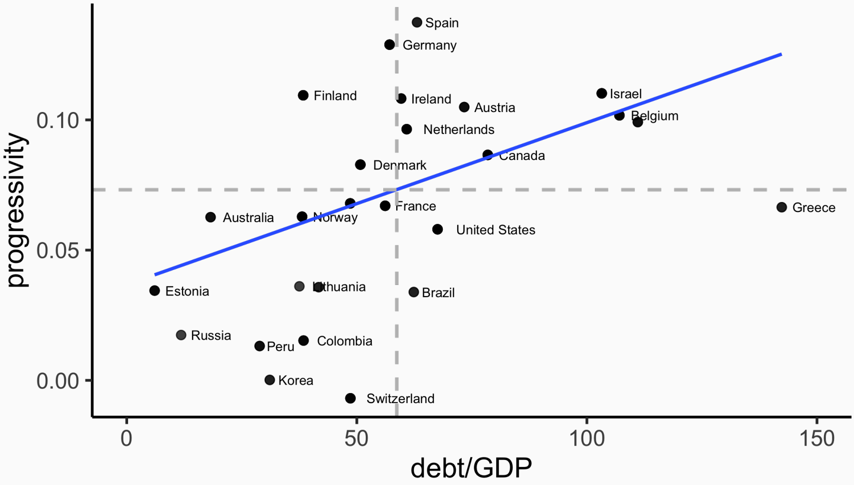
Figure 1: Public debt and progressivity across countries, 1970-2015 [IMF & Qiu and Russo, 2022]



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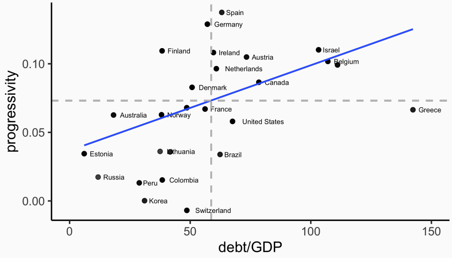
- In *theory:*
  - both can help agents insure against risk



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Blanchard (2019)

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Heathcote et al. (2020)

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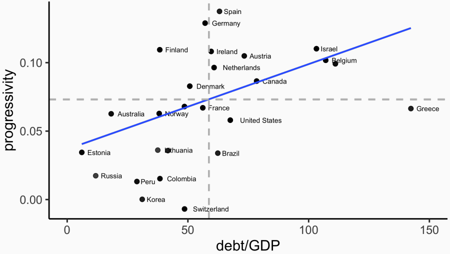

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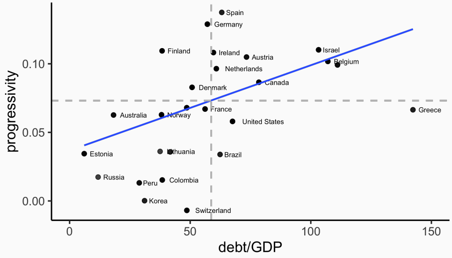

Questions:

1. What is the **optimal mix** of debt and redistributive taxation?
2. How does it depend on social **preferences for redistribution**?

This paper

- Optimal long-run mix of **debt** and **redistributive taxes** in standard het-agent models

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Main results

1. Planners with stronger preferences for **redistribution** favor
   (i) more progressive tax systems
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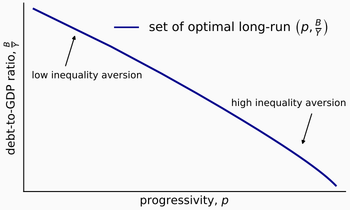

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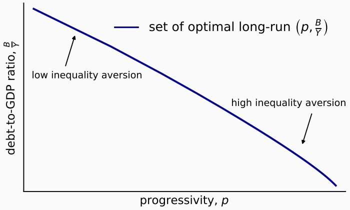

2. …mainly due to novel interest rate channel of progressivity
   – more progressive tax system → more insurance → less precautionary savings

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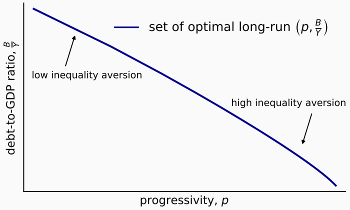

2. ...mainly due to novel interest rate channel of progressivity
   - more progressive tax system $\rightarrow$ more insurance $\rightarrow$ less precautionary savings

3. US social preferences inconsistent with both Utilitarian and Rawlsian criteria
   - SWF that rationalizes status quo features higher weight on well-being of rich

Related literature

1. **Optimal fiscal policy with incomplete markets**: Aiyagari, 1995; Aiyagari and McGrattan, 1998; **Flodén, 2001**; Bakış et al. (2015); Krueger and Ludwig (2016), Boar and Midrigan (2022), Angeletos et al. (2022), Dyrda and Pedroni (2022), Acikgoz et al. (2023), Auclert et al. (2023), LeGrand and Ragot (2023), ...
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2. **Optimal labor income taxation**: Mirrlees (1971), Varian (1980), Saez (2001), Golosov et al. (2006), Farhi and Werning (2013), Heathcote et al. (2017), Chang and Park (2021), Ferriere et al. (2022), ...
   - incorporate public debt into the analysis

Plan for today

1. Model
2. Interest rate channel of progressivity
3. Optimal policy
4. Inverting the optimum

Model

Standard heterogeneous-agent incomplete-markets model

details

- Continuum of households face **uninsurable** idiosyncratic income risk
  - individual productivity θ evolves according to some Markov process

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- Different productivity types are **perfect substitutes** in production
- Government controls supply of safe assets & nonlinear **labor income** tax schedule

Model discussion and calibration

- **CRP** tax schedules [Bénabou, 2002 and Heathcote et al., 2017]

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- Calibrate model to **US** economy, following McKay et al., 2016 Calibration

  (i) β chosen to match a real interest rate of **2%**

  (ii) θ follows an AR(1) process in logs [Floden and Lindé, 2001 and Guvenen et al., 2014]

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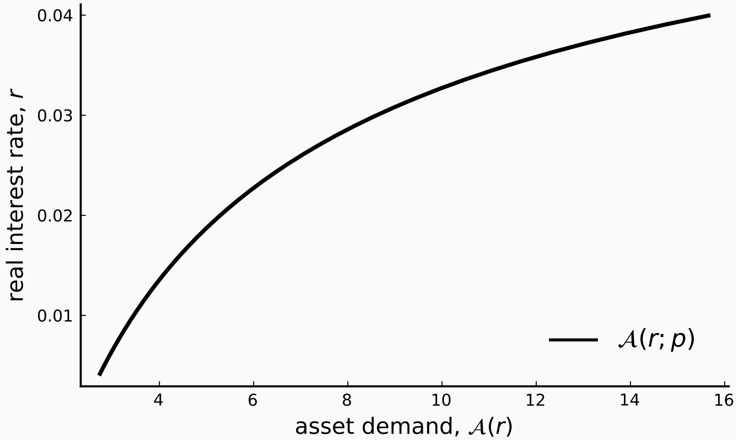

Figure 1: Equilibrium in the asset market before and after the reform

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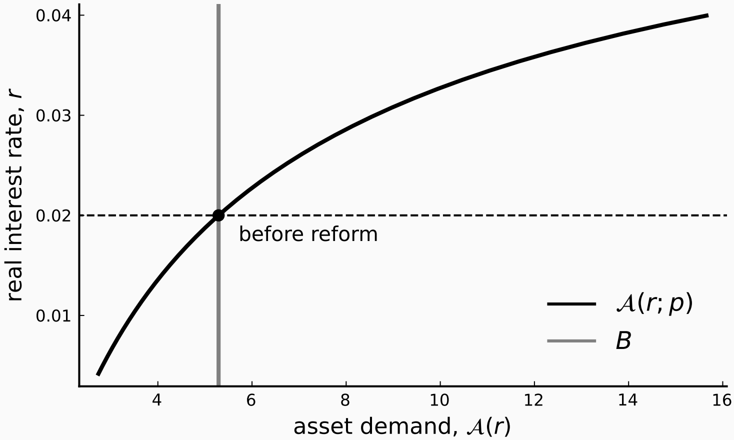

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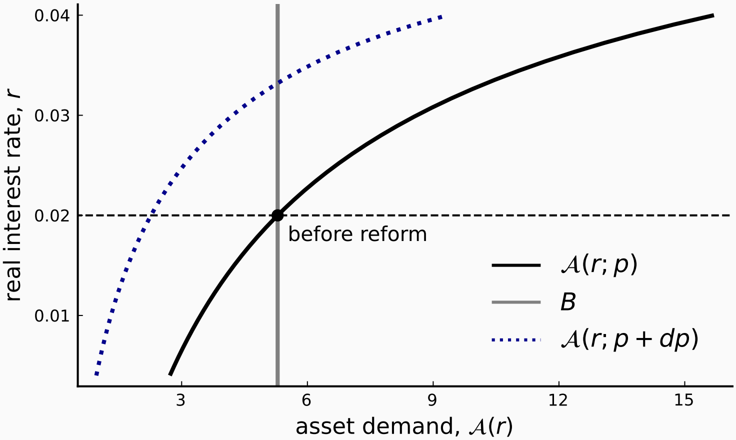

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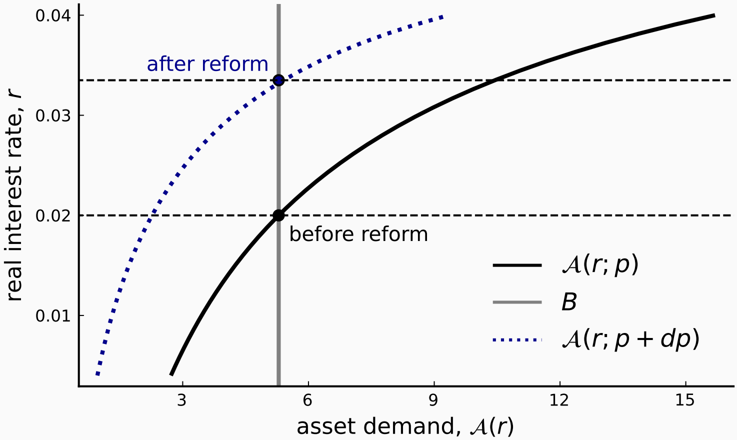

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Optimal policy

Optimal policy problem

- The dynamic **full-commitment** Ramsey problem for this economy is

$$\max_{\{r_t, B_t, p_t, \tau_t\}} \sum_{t=0}^{\infty} \beta^t \mathcal{U}_t\left(\{r_s\}, \{\tau_s\}, \{p_s\}\right) \quad \text{s.t} \quad \begin{cases} \mathcal{A}_t\left(\{r_s\}, \{\tau_s\}, \{p_s\}\right) = B_t, \\ G + (1 + r_{t-1})B_{t-1} = B_t + \mathcal{T}_t\left(\{r_s\}, \{\tau_s\}, \{p_s\}\right) \end{cases}$$

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Note: SWF departs from welfarist approach Phelan, 2006; Farhi and Werning, 2007; Davila and Schaab, 2022

Simple condition for optimal long-run level of debt

For any $u = 0, 1, 2, \ldots$ the following must be true:

$$\sum_{t=0}^{\infty}\sum_{s=0}^{\infty}\beta^{t-u}\frac{\partial \mathcal{U}_t}{\partial r_s}\frac{\partial \boldsymbol{r}_s}{\partial B_u}+\sum_{t=0}^{\infty}\sum_{s=0}^{\infty}\beta^{t-u}\lambda_t\frac{\partial \mathcal{T}_t}{\partial r_s}\frac{\partial \boldsymbol{r}_s}{\partial B_u}+\lambda_u-\beta\lambda_{u+1}(1+\boldsymbol{r}_u)-\sum_{t=0}^{\infty}\beta^{t-u}\lambda_t\frac{\partial \boldsymbol{r}_t}{\partial B_u}B_{t-1}=0$$

Simple condition for optimal long-run level of debt

Proposition

The optimal long-run level of debt B^{RSS}, if it exists, solves

$$\left[\frac{\mathcal{S}_{\mathcal{U},r}}{\lambda^{RSS}} + \mathcal{S}_{\mathcal{T},r}\right]\mathcal{S}_{\mathbf{r},B} + \{1 - \beta(1+\mathbf{r})\} - \mathcal{S}_{\mathbf{r},B}\, B^{RSS} = 0,$$

where $\mathcal{S}_{F,X} \equiv \lim_{u\to\infty} \sum_{t=0}^{\infty} \beta^{t-u} \frac{\partial F_t}{\partial X_u}$ and $\lambda^{RSS} = \lim_{u\to\infty} \lambda_u$.

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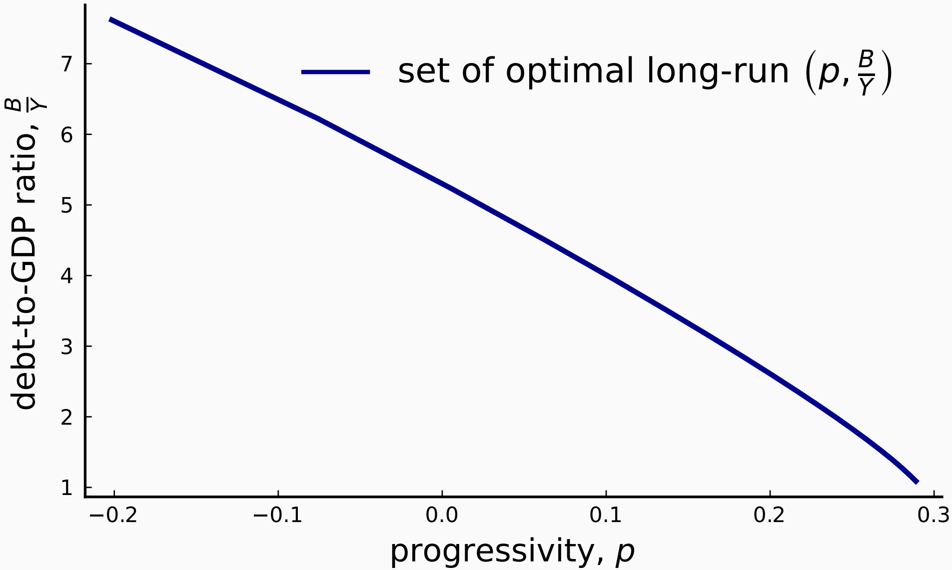
3. sensitivity of interest rates to changes in public debt $\mathcal{S}_{\mathbf{r},B}$

Optimal long-run mix of debt and progressivity



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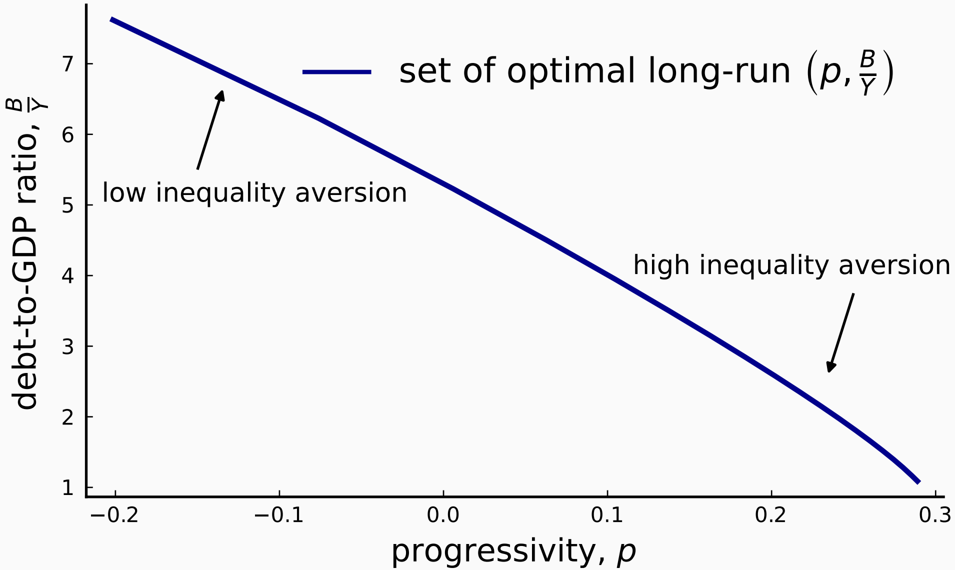

Extensions

1. Optimal policy without transitions figure
   - maximize **steady-state welfare** à la Aiyagari and McGrattan (1998) OSS problem
   - can use more standard SWFs figure
2. Multiple safe assets & taxes on savings figure
   - production technology $F(K, L)$ and allow firms to issue claims to capital
   - qualitative properties of optimal mix unchanged but quantitative differences
3. Alternative labor income tax schedules figure
   - introduce lumpsum transfers
   - jointly tax capital and labor income

Inverting the optimum

Basic idea behind the exercise

Q: What preferences for redistribution can rationalize **observed mix** of B and p?

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- Recall SWF

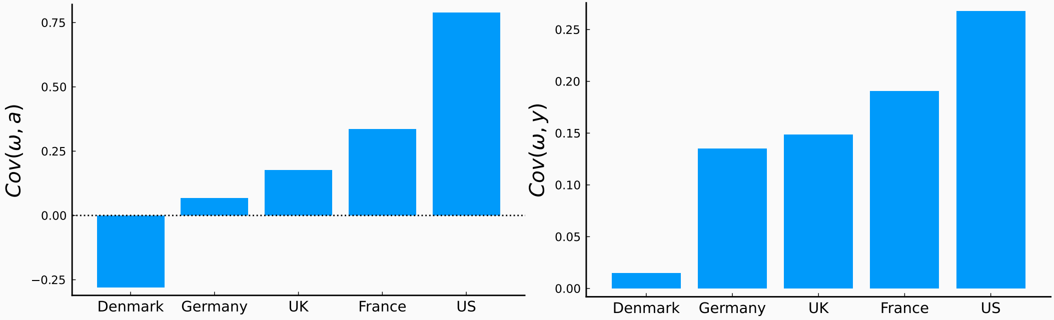
$$\sum_{t=0}^{\infty} \beta^t \int_i \omega_t(\theta_t^i, a_t^i) U(c_t^i, l_t^i) \, di$$

  with social welfare weights $\omega(\theta, a) \propto \exp\left(-\alpha_\theta \theta - \alpha_a a\right)$

- Find α_a and α_θ so that long-run solution gives $p^{RSS} = p^{US}$ and $\frac{B^{RSS}}{Y^{RSS}} = \frac{B^{US}}{Y^{US}}$

- Look at implied $Cov(\omega, a)$ and $Cov(\omega, y)$

Inverting the optimum in selected advanced economies



(a) Implied $Cov(\omega, a)$ **(b)** Implied $Cov(\omega, y)$

Figure 2: Inferred covariances of welfare weights and assets/income in advanced economies

Conclusion

Takeaways:

- inequality-averse planners prefer lower levels of B due to GE effects of p, even if
  1. transitional dynamics are taken into account
  2. multiple safe assets
  3. relax restrictions on the tax system

- *BONUS:* aversion to inequality can help find an interior RSS

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Future work:

1. What happens along transition to Ramsey steady state?
2. Political economy considerations?

Thank You!

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Household block

- Given $\{r_t\}$ and $\{T_t(\cdot)\}$, agent entering period t in state $x = (a, \theta)$ solves

$$V_t(a,\theta) = \max_{\ell,c,a'} u(c) - v(\ell) + \beta \mathbb{E}_{\theta'|\theta}\left[V_{t+1}(a',\theta')\right] \quad \text{s.t} \quad \begin{cases} c + a' = (1+r_t)a + \theta\ell - T_t(\theta\ell) \\ a' \geq -\phi. \end{cases}$$

Household block

- Given $\{r_t\}$ and $\{T_t(\cdot)\}$, agent entering period t in state $x = (a, \theta)$ solves

$$V_t(a,\theta) = \max_{\ell,c,a'} u(c) - v(\ell) + \beta \mathbb{E}_{\theta'|\theta}\left[V_{t+1}(a',\theta')\right] \quad \text{s.t} \quad \begin{cases} c + a' = (1+r_t)a + \theta\ell - T_t(\theta\ell) \\ a' \geq -\phi. \end{cases}$$

- Policy functions: $\boldsymbol{c}_t(x)$, $\boldsymbol{a}_t(x)$, $\boldsymbol{y}_t(x)$ and $\boldsymbol{z}_t(x) = \boldsymbol{y}_t(x) - T_t(\boldsymbol{y}_t(x))$

- Measure of households with productivity θ that have assets in set A at t

$$D_t(\theta, A) = Pr\{\theta_t = \theta, a_t \in A\}$$

back

Government budget constraint and market clearing

- Given **exogenous spending** G, government's budget constraint:

$$G + (1 + r_{t-1})B_{t-1} = B_t + \underbrace{\int T_t(\mathbf{y}_t(x))dD_t(x)}_{=\mathcal{T}_t(\{r_s\},\{T_s(\cdot)\})}$$

Government budget constraint and market clearing

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- Asset market clearing:

$$\int \underbrace{\mathbf{a}_t(x)dD_t(x)}_{=\mathcal{A}_t(\{r_s\},\{T_s(\cdot)\})} = B_t$$

- Goods market clearing:

$$G + \int \underbrace{\mathbf{c}_t(x)dD_t(x)}_{=\mathcal{C}_t(\{r_s\},\{T_s(\cdot)\})} = \int \underbrace{\mathbf{y}_t(x)dD_t(x)}_{=\mathcal{Y}_t(\{r_s\},\{T_s(\cdot)\})}$$

back

Optimal mix of debt and progressivity in the RSS

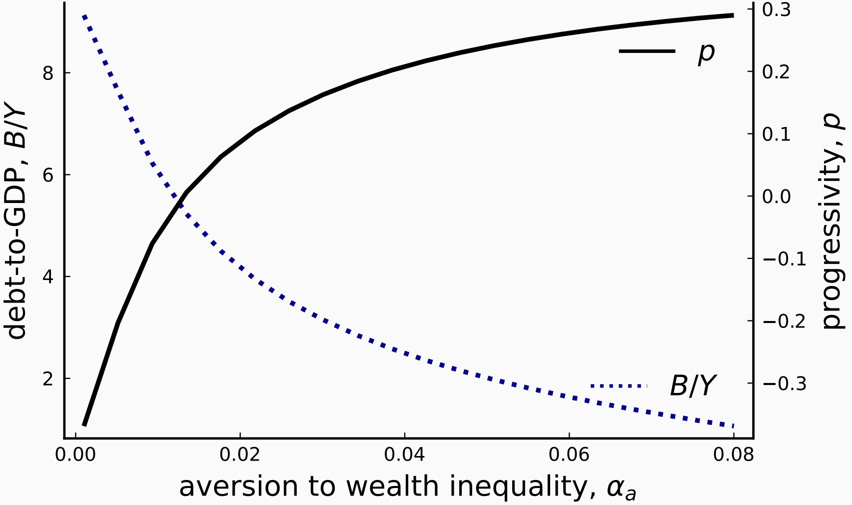


Figure 3: Optimal mix of debt and progressivity in the RSS

Calibration

Parameter	*Description*	*Value*
β	discounting	0.9879
ρ	persistence of AR (1)	0.966
σ	variance of AR(1)	0.703
EIS	curvature in *u*	1
Frisch	curvature in *v*	1/2
G/Y	spending-to-GDP	0.088
B/Y	debt-to-GDP	1.4
p	progressivity of taxes	0.181
τ	level of taxes	0.6740

Table 1: Parameters back

Optimal mix of debt and progressivity - comparative static wrt ϕ

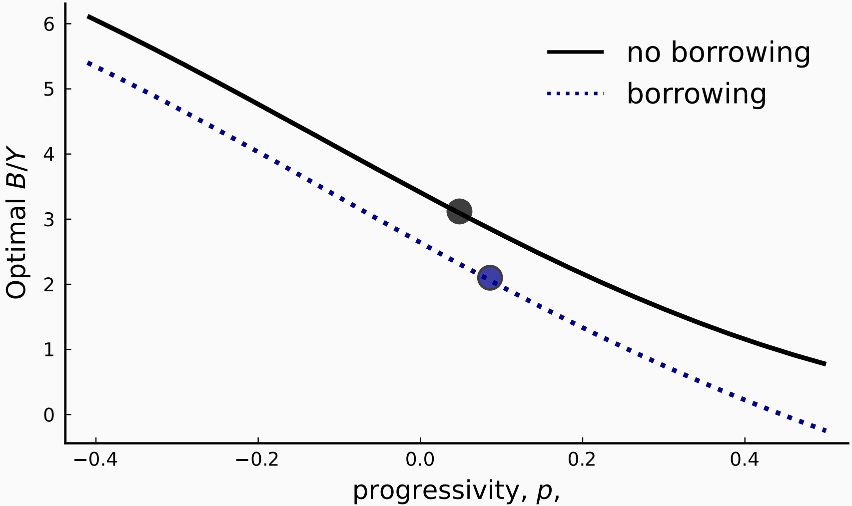


Figure 4: Optimal mix of debt and progressivity with and without borrowing

back

Optimal mix of debt and progressivity with lumpsum transfers

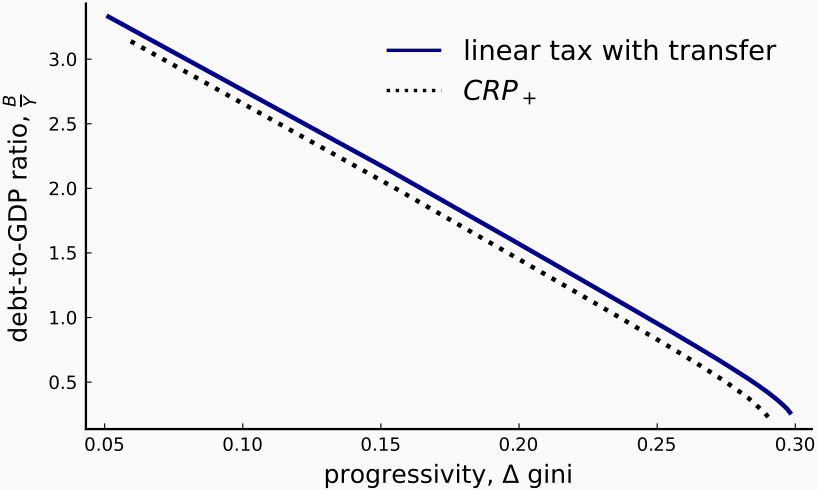


Figure 5: Optimal mix of debt and progressivity with lump-sum transfers

back

Optimal mix of debt and lumpsum transfers

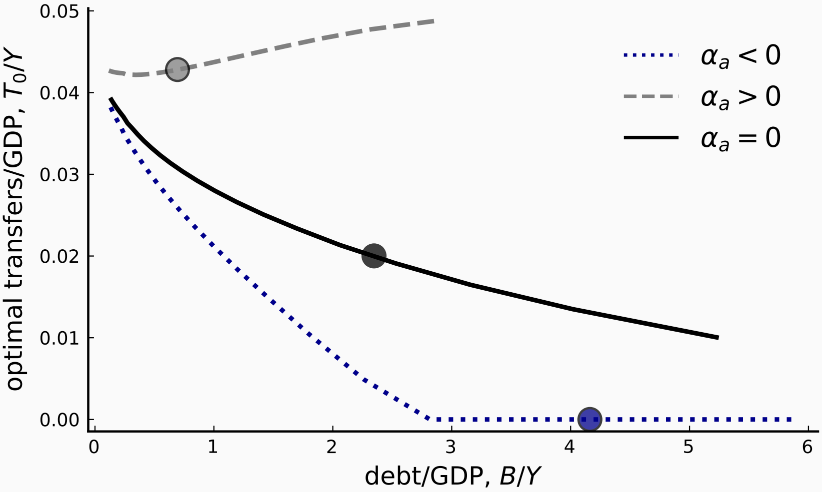


Figure 6: Optimal mix of debt and lump-sum transfers

back

Optimal long-run mix of debt and progressivity ignoring transitions

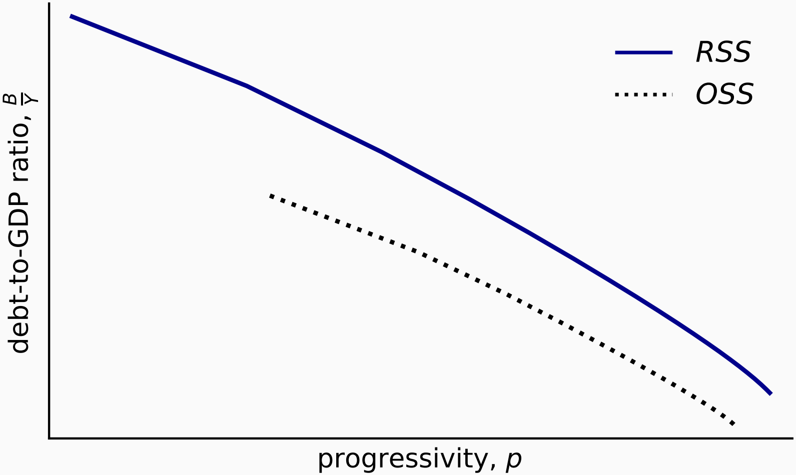


Figure 7: Optimal long-run mix of debt and progressivity across solution concepts

back

Optimal long-run mix of debt and progressivity across SWFs

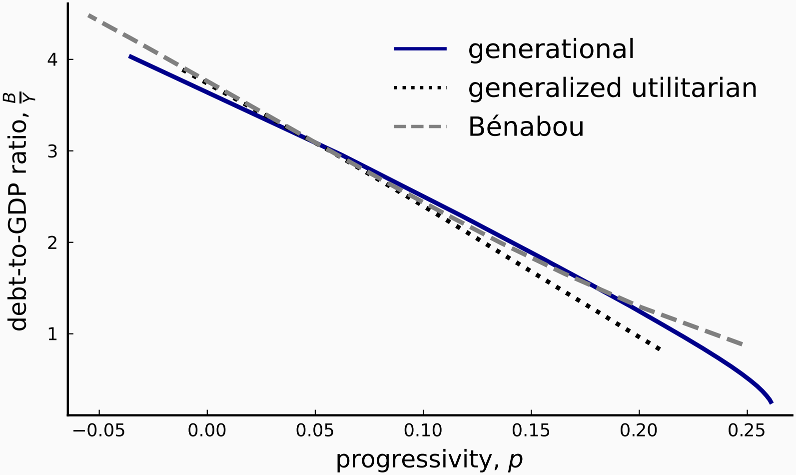


Figure 8: Optimal long-run mix of debt and progressivity across SWFs

back

Optimal mix of debt and progressivity in the OSS with capital and τ_k

In OSS, **golden rule** holds $\implies$ planner chooses τ_k in order to implement $F_K = \delta$

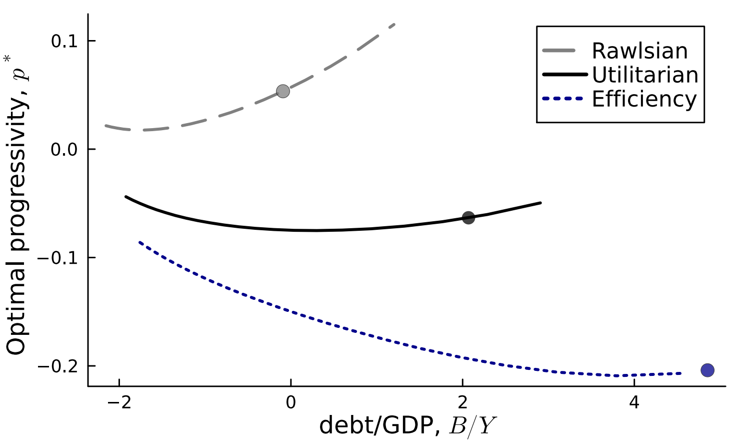


Figure 9: Optimal progressivity vs debt/GDP in the model with capital and τ_k

back

Optimal mix of debt and progressivity in the RSS with capital and τ_k

Modified golden rule holds $\implies$ planner chooses τ_k to implement $F_K = \rho + \delta$

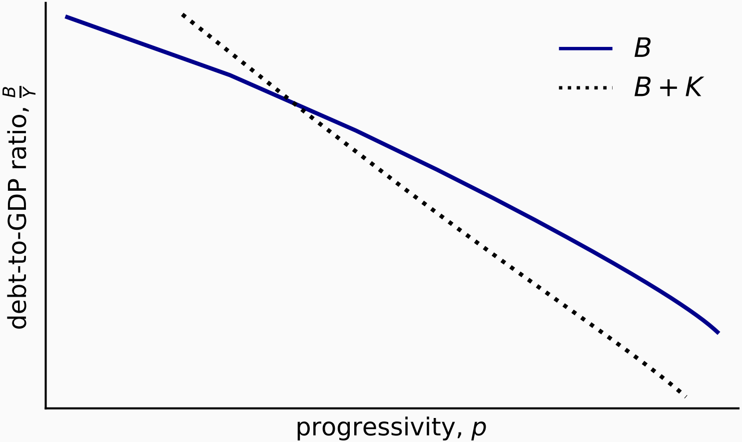
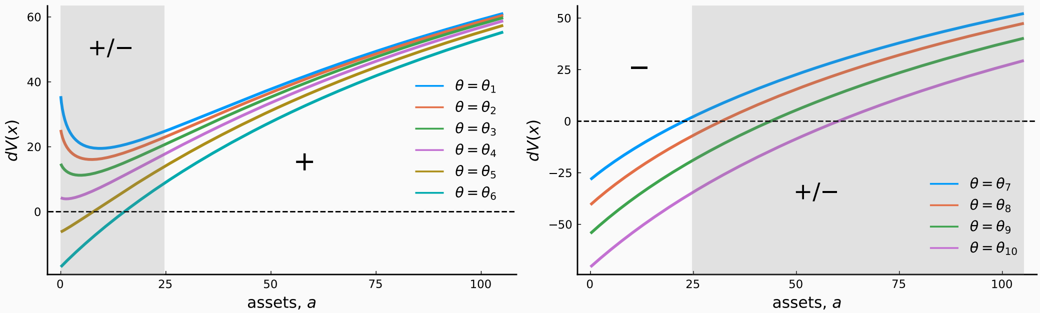


Figure 10: Optimal mix of debt and progressivity in the model with capital and τ_k

back

First-order effects of progressivity: total effect



(a) Total effect at the bottom of the θ distribution

(b) Total effect at the top of the θ distribution

Figure 11: Individual responses across the state space back

Takeaway: GE effect can dominate PE effect due to interest rate channel of progressivity

Relationship between debt and progressivity in the OSS

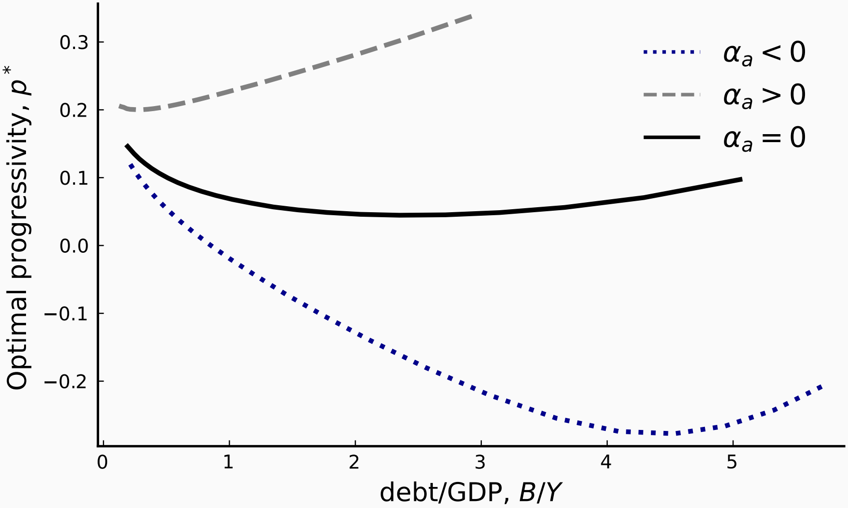
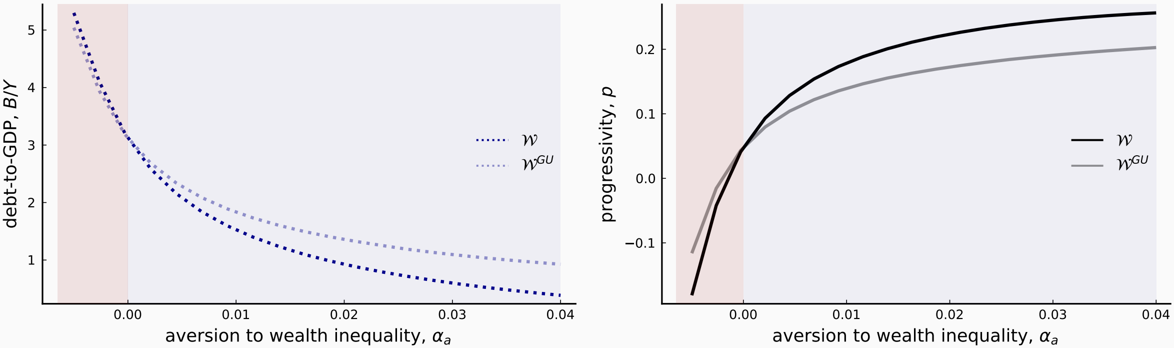


Figure 12: Optimal progressivity vs debt/GDP in the OSS back

Optimal mix of debt and progressivity across SWFs



(a) Optimal debt-to-GDP, $\mathcal{W}$ vs $\mathcal{W}^{GU}$

(b) Optimal progressivity, $\mathcal{W}$ vs $\mathcal{W}^{GU}$

Figure 13: Optimal mix with generational and generalized utilitarian planners

back

Alternative welfare criteria

1. **Benchmark planners** [Davila and Schaab, 2022 or Phelan, 2006 & Farhi and Werning, 2007]

$$\mathcal{W}(r, \tau, p) = \sum_{t=0}^{\infty} \beta^t \int \omega(x) u(\boldsymbol{c}(x)) dD(x)$$

Alternative welfare criteria

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2. **Generalized utilitarian planners**

$$\mathcal{W}^{GU}(r, \tau, p) = \int \omega(x) V(x) dD(x)$$

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2. **Generalized utilitarian planners**

$$\mathcal{W}^{GU}(r,\tau,p) = \int \omega(x) V(x) dD(x)$$

3. **Bénabou planners** [Bénabou, 2002 & Boar and Midrigan, 2022]

$$\mathcal{W}^{\alpha}(r,\tau,p) = \left(\int \bar{c}(x)^{1-\frac{1}{\alpha}} dD(x) \right)^{\frac{1}{1-\frac{1}{\alpha}}},$$

with $\bar{c}(x)$ equal to consumption CE

back

Optimal steady state problem (OSS)

OSS Problem:

back

- Choose **time-invariant** tax code $\{\tau, p\}$ and steady state level of public debt B to

$$\max_{\{r,B,p,\tau\}} \mathcal{W}(r,\tau,p) \quad \text{s.t} \quad \begin{cases} \mathcal{A}(r,\tau,p) = B, \\ G + rB = \mathcal{T}(r,\tau,p) \end{cases}$$

- Alternative welfare criteria:

1. **Generational** planners

$$\mathcal{W}(r,\tau,p) = \sum_{t=0}^{\infty} \beta^t \int_i \omega(\theta_t^i, a_t^i) U(c_t^i, l_t^i)\, di$$

2. **Generalized utilitarian** planners

$$\mathcal{W}^{GU}(r,\tau,p) = \int_i \omega(\theta_0^i, a_0^i) V(\theta_0^i, a_0^i) di$$

3. **Bénabou** planners

[Bénabou, 2002 & Boar and Midrigan, 2022]

$$\mathcal{W}^{\alpha}(r,\tau,p) = \left(\int \bar{c}(\theta_0^i, a_0^i)^{1-\frac{1}{\alpha}} di \right)^{\frac{1}{1-\frac{1}{\alpha}}}, \quad \text{with } \bar{c}(\theta, a) = \text{“consumption CE”}$$

First-order effects of progressivity: direct and indirect effects

dV

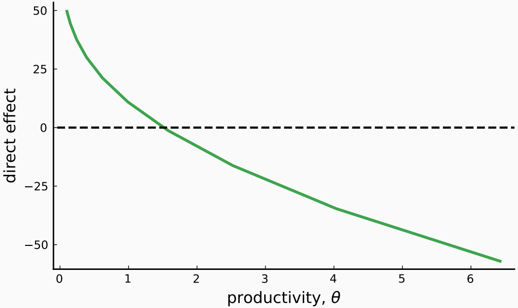
Q: How does a small **permanent** change in p affect equilibrium outcomes?

First-order effects of progressivity: direct and indirect effects

$$dV(x) = \sum_{s=0}^{\infty} \beta^s \mathbb{E}\left[u'(c_s) \left(\underbrace{y_s^{1-p} d\tau + a_s\, dr}_{\text{indirect effect in } s} - \underbrace{z_s \log y_s}_{\text{direct effect in } s} \right) \Bigg| x_0 = x \right].$$

First-order effects of progressivity: direct and indirect effects

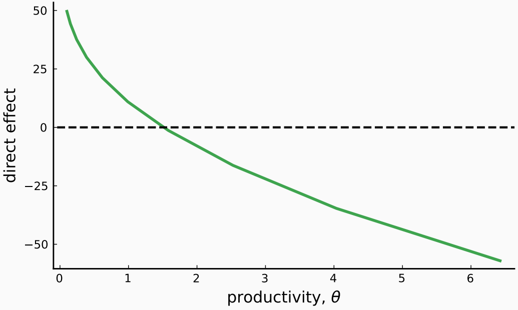
$$dV(x) = \sum_{s=0}^{\infty} \beta^s \mathbb{E}\left[u'(c_s) \left(\underbrace{y_s^{1-p} d\tau + a_s\, dr}_{\text{indirect effect in } s} - \underbrace{z_s \log y_s}_{\text{direct effect in } s} \right) \Bigg| x_0 = x \right].$$



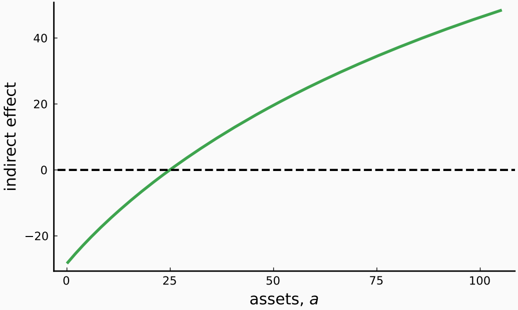
(a) Direct effect along the productivity dimenstion

First-order effects of progressivity: direct and indirect effects

$$dV(x) = \sum_{s=0}^{\infty} \beta^s \mathbb{E}\left[u'(c_s) \left(\underbrace{y_s^{1-p} d\tau + a_s\, dr}_{\textit{indirect effect in } s} - \underbrace{z_s \log y_s}_{\textit{direct effect in } s} \right) \middle| \, x_0 = x \right].$$



(a) Direct effect along the productivity dimenstion



(b) Indirect effect along the asset dimension

Relationship between debt and progressivity in the OSS

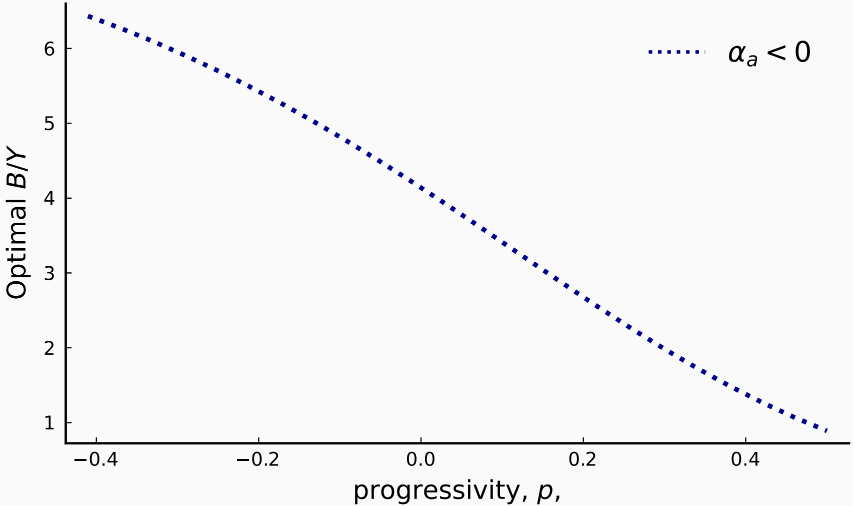


Figure 15: Optimal debt/GDP vs progressivity in the OSS

$\phi > 0$

Relationship between debt and progressivity in the OSS

p vs B

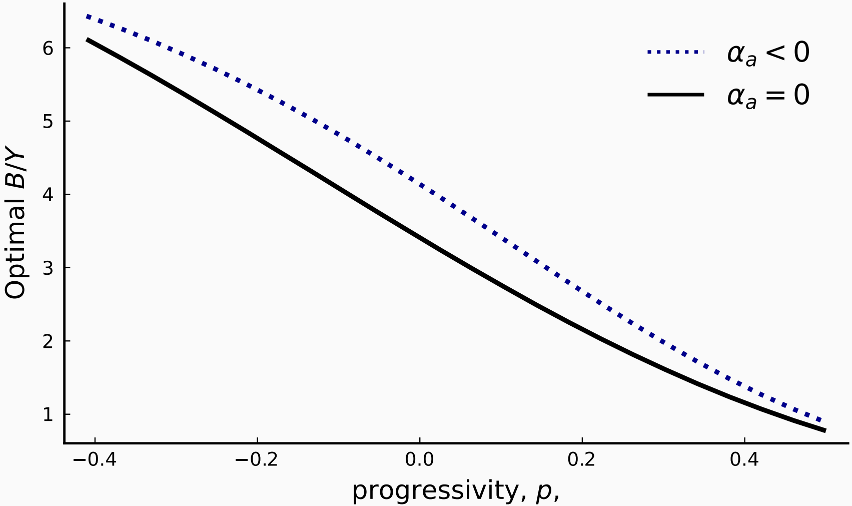


Figure 15: Optimal debt/GDP vs progressivity in the OSS

$\phi > 0$

Relationship between debt and progressivity in the OSS

p vs B

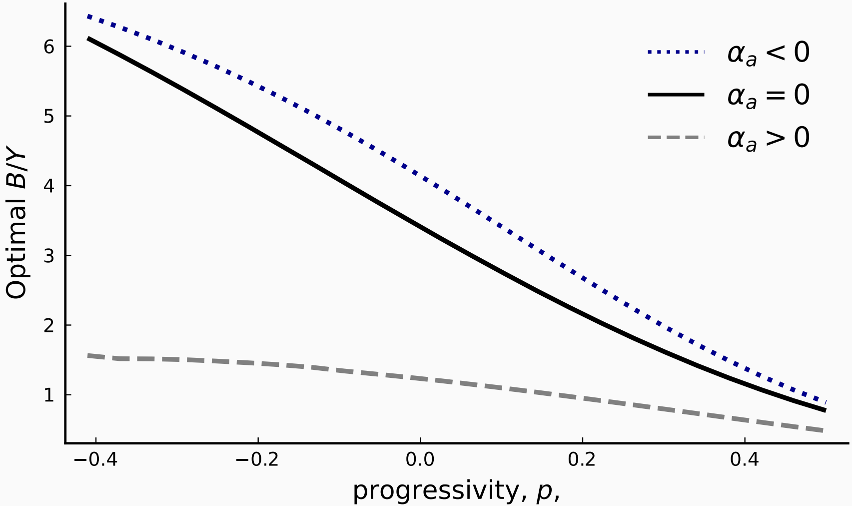


Figure 15: Optimal debt/GDP vs progressivity in the OSS $\phi > 0$

Optimal mix of debt and progressivity in the OSS

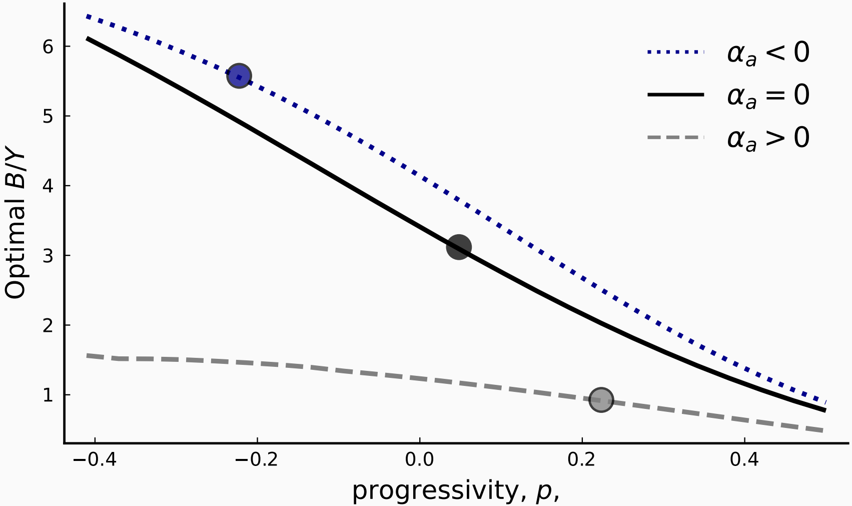


Figure 16: Optimal progressivity vs debt/GDP in the OSS

Optimal mix of debt and progressivity and aversion to inequality

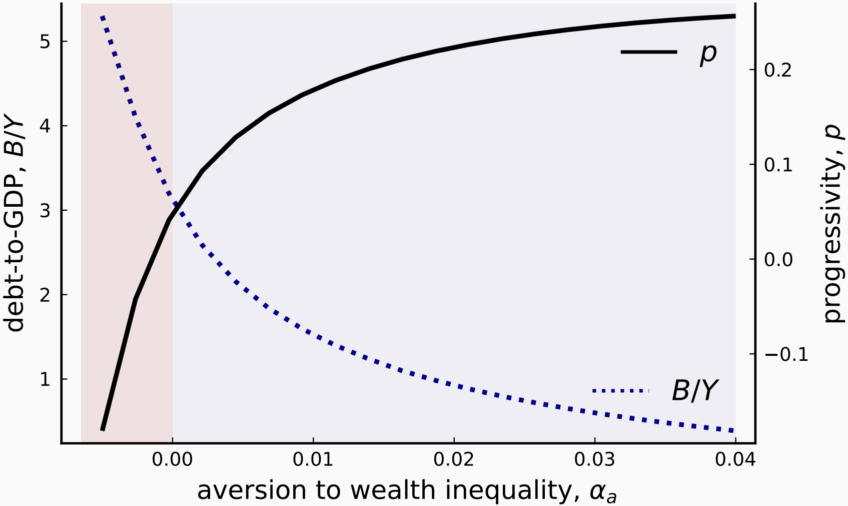


Figure 17: Optimal mix of debt and progressivity in the OSS

Generalized utilitarian

Two concepts of long-run optimality with heterogeneous agents

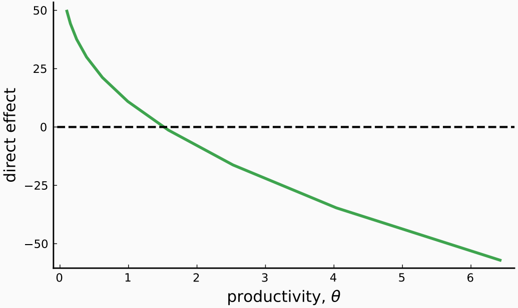
1. **Optimal steady state** $\max \mathcal{W}$
   - used by Aiyagari and McGrattan (1998)
   - maximize welfare in **steady state**
   - ignores transitions $\implies$ EASY

Two concepts of long-run optimality with heterogeneous agents

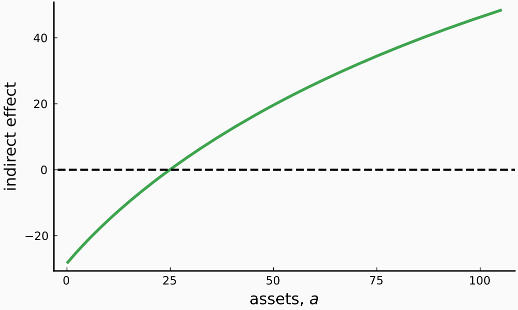
1. **Optimal steady state** max $\mathcal{W}$
   - used by Aiyagari and McGrattan (1998)
   - maximize welfare in **steady state**
   - ignores transitions $\implies$ EASY
2. **Ramsey steady state** max $\sum_t \beta^t \mathcal{W}_t$
   - formulated by Aiyagari (1995)
   - limiting steady state of **dynamic** Ramsey problem w/ full commitment
   - transition dynamics matter $\implies$ HARD

Unintended effects of progressive tax reforms

$$dV(x) = \sum_{s=0}^{\infty} \beta^s \mathbb{E}\left[u'(c_s)\left(\underbrace{y_s^{1-p} d\tau + a_s\, dr}_{\text{indirect effect in } s} - \underbrace{z_s \log y_s}_{\text{direct effect in } s}\right) \Big| x_0 = x\right].$$



(a) Direct effect along the productivity dimension



(b) Indirect effect along the asset dimension

back

Existence of interior steady state with inequality-averse planners

Ramsey problem w/ **utilitarian SWF** does not converge to an interior steady state ...

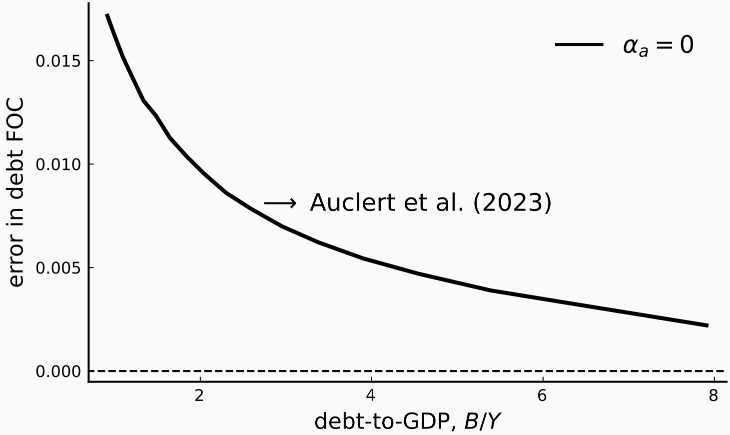


Figure 19: Verifying existence of interior steady state

back

Existence of interior steady state with inequality-averse planners

…but interior steady state exists with **inequality-averse** planners

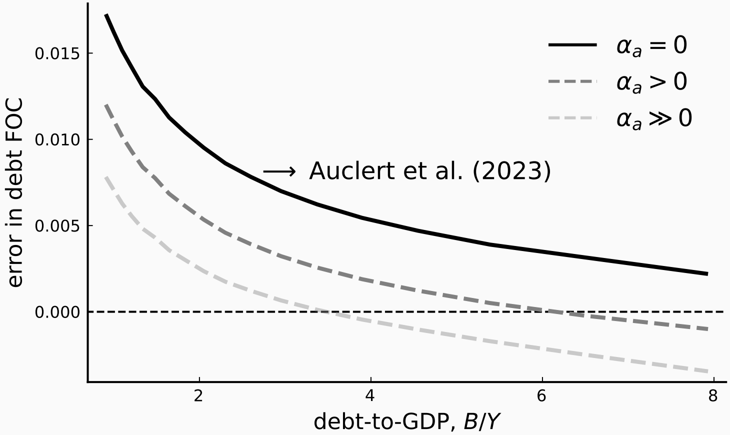
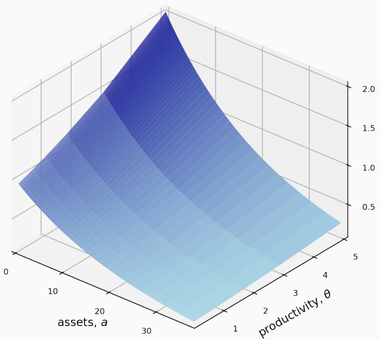


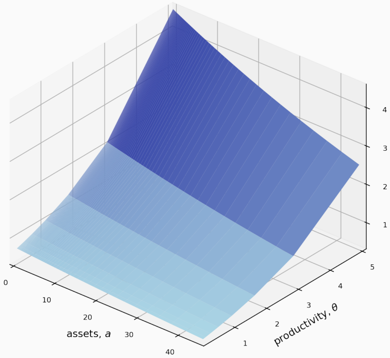
Figure 19: Verifying existence of interior steady state

back

Inverting the optimum: Denmark vs the United States



(a) Implied welfare weights for Denmark



(b) Implied welfare weights for the United States

Figure 20: Inferred welfare weights for Denmark and the United States

back