Parenting styles with externalities

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Introduction: research question

Education combines elements that require no interaction, but in other parts collaboration is important. Cultures vary about which elements of the technology to emphasize.

We want to understand when a society, or groups within it, may decide to pursue one model or the other.

That allows us to understand:

- The externalities that some parents impose on others when different educational cultures coexist.
- In turn, this can lead to self-segregation efforts.
- We also point to a kind of overlooked peer effect, that derived from parental educational cultures.

Introduction: model

- Children's utility depends on individual and collective effort. Individual effort has a fixed return per unit of time. The return to collective effort depends on the average dedication of other members of the students' peer group.
- Parents can influence the relative rate of return of collective vs. individualistic effort, with some limits.
- The utility of parents is the same as the child's, except that the value placed on collective effort is smaller.
- The action of the parents is the influence on the relative rate of return of the two activities, and is taken first.

Introduction: model

- We characterize the equilibria of this game.
  - First, in an environment with homogeneous random matching in the population.
  - Then, society is separated into groups, and students mainly interact with own group.
  - We then study local interaction.
- In all models parents can be dissatisfied with their environment in equilibrium.
- Then, we study the possibility for parents to self-segregate.
- Finally, we explore what happens when different parents have different ability to control the utility of their children.
- Crucial for equilibria: productivity of individual effort, and of standalone and synergy components of collaborative effort, uncertainty of parents.

Introduction: results

- In separated societies and with local interaction, the degree of separation, and the network structure are crucial.
- Parents with higher belief in value collaboration have more incentives to segregate.
- Equilibrium multiplicity can explain why one observes different local educational cultures within homogeneous countries.
- It can dynamically generate segregation, or social pressure to conform. It is also a good reason for intervention.
- With local interaction we study whether local homogeneous clusters can coexist. This requires some relative isolation.

Introduction: main policy takeaways

- With high parental skepticism the policy just needs to shift expectations (multiple equilibria). With low skepticism, the main issue can be to change the parental abilities to control children.
- With differences in socioeconomic status that lead to some parents having a high control ability. In most cases, except for one specific range of parameters, and only if parents coordinate their expectations, it is a good idea to increase the control ability of all parents
- If parents differ in their returns to individual effort, there are incentives for secession but also to level the returns for all.
- With heterogenity in beliefs, secession is always to be encouraged with optimistic expectations. These expectations are reasonable with secession costs and forward induction arguments.

The model

Children choose how to split their time between collaborative and individual learning.

Let x fraction of time in individual learning. Child's utility is:

$$a\left(K_1 + K_2\left(1 - \overline{x}\right)\right)\left(1 - x\right) + Rx \tag{1}$$

which can be rewritten as

$$a\left(K_1 + K_2\left(1 - \overline{x}\right)\right) + \left(R - a\left(K_1 + K_2\left(1 - \overline{x}\right)\right)\right)x$$

Children choose x to maximize (1) yielding the best reply

$$x_1 = \begin{cases} 1 \text{ if } R > a\left(K_1 + K_2\left(1 - \overline{x}\right)\right) \\ 0 \text{ if } R \leq a\left(K_1 + K_2\left(1 - \overline{x}\right)\right) \end{cases} \tag{2}$$

The model

The general parental utility with $a \in [\underline{a}, \overline{a}]$ is given by

$$U^P(a) = \mu a \left(K_1 + (1 - \overline{x}) K_2\right)(1 - x_1) + R x_1 \tag{3}$$

where $\mu \in (0, 1)$ represents the weight of the parent on the collaborative activities of the child. Using the child's best response (2) parental utility becomes

$$U^P(a) = \begin{cases} R \text{ if } \frac{R}{K_1 + (1 - \overline{x}) K_2} \geq a \\ \mu a \left(K_1 + (1 - \overline{x}) K_2\right) \text{ if } \frac{R}{K_1 + (1 - \overline{x}) K_2} < a \end{cases} \tag{4}$$

The complete mixing society I

Children are born with $\overline{a}$ and the investment of parents consists in reducing a. With a tiny cost $\varepsilon \to 0$ per unit of reduction in a

Lemma 1

If $\mu \leq \frac{\underline{a}}{\overline{a}}$ we have the following symmetric pure strategy equilibria,

$$a = \overline{a} \text{ and } x = 0 \text{ for } R \leq K_1 \underline{a}$$

$$\left.\begin{array}{c} a = \overline{a} \text{ and } x = 0 \\ a = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ and } x = 1 \end{array}\right\} \text{ for } K_1 \underline{a} \leq R \leq \underline{a}\left(K_1 + K_2\right)$$

$$a = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ and } x = 1 \text{ for } R > \underline{a}\left(K_1 + K_2\right)$$

The complete mixing society II

Lemma 1

If $\mu > \frac{\underline{a}}{\overline{a}}$ we have the following symmetric pure strategy equilibria

$$a = \overline{a} \text{ and } x = 0 \text{ for } R < \overline{a}\mu K_1$$

$$\left.\begin{array}{c} a = \overline{a} \text{ and } x = 0 \\ a = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ and } x = 1 \end{array}\right\} \text{for } \overline{a}\mu K_1 \leq R \leq \overline{a}\mu\left(K_1 + K_2\right)$$

$$a = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ and } x = 1 \text{ for } R > \overline{a}\mu\left(K_1 + K_2\right)$$

Welfare and policy

If a social planner wants to maximize parental utility, she wants children to choose $x = 1$ when $R > \overline{a}\mu\left(K_1 + K_2\right)$ and $x = 0$ when $R < \overline{a}\mu\left(K_1 + K_2\right)$.

1. For $\mu > \frac{\underline{a}}{\overline{a}}$ parents can always induce their preferred options. Unique eq. always efficient. When multiple eq. possible, high individual effort eq. inefficient.
2. For $\frac{\underline{a}}{\overline{a}}\frac{K_1}{K_1+K_2} < \mu < \frac{\underline{a}}{\overline{a}}$ Unique eq. efficient. When multiple eq. possible, low individual effort eq. efficient for $K_1\underline{a} < R < \overline{a}\mu\left(K_1 + K_2\right)$ while high individual effort eq. efficient for $\overline{a}\mu\left(K_1 + K_2\right) < R < \underline{a}\left(K_1 + K_2\right)$. When R high, collab. externalities fall short, but parents cannot induce individual effort.
3. For $\mu < \frac{\underline{a}}{\overline{a}}\frac{K_1}{K_1+K_2}$ the low individual effort eq. inefficient even when unique if $\overline{a}\mu\left(K_1 + K_2\right) < R \leq K_1\underline{a}$. Incentives of parents and children lie very far apart. When multiple eq. exist, the high individual effort eq. is efficient and also when it is unique eq..

Welfare and policy

From these results, we can generate some policy insights.

1. If μ is high (case 2 of the Lemma), then the only possible inefficiency arises when there are multiple equilibria. "All" the policy needs to do is to shift expectations to coordinate their behaviors on $x = 0$. This could be done with temporary measures.
2. If μ is low (case 1 of the Lemma), then there are circumstances (like 4 above) where the issue is not just about expectations, but about the ability of parents to control the activities of the children. That is, $\underline{a}$ is not low enough. This could be more costly and difficult. An alternative could be to raise K_2 which may require (re)training the parents and teachers.

The imperfect mixing society

Assume now that there are two groups in the population. Is it possible that these two different groups choose different equilibria?

Lemma 2

An equilibrium with two separate subgroups where parents of the high individual effort children choose $a = \min\left[\overline{a}, \frac{R}{K_1+\lambda K_2}\right]$ *and parents of the low individual effort children choose* $a = \overline{a}$ *exists when*

$$\underline{a}\left(K_1 + (1-\lambda) K_2\right) < R < \underline{a}(K_1 + \lambda K_2) \text{ for } \mu < \frac{\underline{a}}{\overline{a}} \tag{5}$$

or when

$$\overline{a}\left(K_1 + (1-\lambda) K_2\right) < R < \mu\overline{a}(K_1 + \lambda K_2) \text{ for } \mu > \frac{\underline{a}}{\overline{a}} \tag{6}$$

Welfare and policy

1. The conditions for coexistence of two separate subgroups lie strictly inside the bounds for the existence of multiple equilibria in the complete mixing society.
2. This implies when multiplicity of behaviors exist, one is inefficient.
3. One policy tool would be to bus children from the inefficient school to the efficient school, but this policy might backfire.
4. For $\mu\bar{a}(K_1 + K_2) > R > \max\left[\mu\bar{a}(K_1 + \lambda K_2), \underline{a}(K_1 + \lambda K_2)\right]$ the school bus policy would move both groups to high individual effort eq. when inefficient.
5. When $\mu > \frac{\underline{a}}{\bar{a}}$ and $R < \mu\bar{a}(K_1 + (1-\lambda)K_2)$ this policy will always achieve the efficient low individual effort equilibrium.
6. For $\mu < \frac{\underline{a}}{\bar{a}}$ the low individual effort eq. might be inefficient for $\mu\bar{a}(K_1 + K_2) < R < \underline{a}(K_1 + (1-\lambda)K_2)$. Hence busing children does not necessarily improve welfare.

Local interaction: the circle

- We study the incentives of parents living on the boundary of low and high individual effort regions to switch to the other equilibrium.
- A necessary condition for the survival of two clusters in a circle is that two different parental subgroups can exist with $\lambda = \frac{1}{2}$. But then the conditions (5) and (6) can only happen non generically. Therefore, local interaction on a circle will always lead to a unique equilibrium.
- However, this unique equilibrium is not always efficient.
- When this happens, one could have a policy to avoid local interaction, so everyone connects to everyone else, to allow for convergence to an efficient equilibrium.

Local interaction: other structures

Example 3

n-max distance interaction in m dimensions. Each player interacts with all players who are less than n steps away in each of the m dimensions. The threshold at which you get contagion is $\frac{n(2n+1)^{m-1}}{(2n+1)^m-1}$

$$\underline{a}\left(K_1+\frac{n(2n+1)^{m-1}}{(2n+1)^m-1}K_2\right) < R < \underline{a}\left(K_1+\left(1-\frac{n(2n+1)^{m-1}}{(2n+1)^m-1}\right)K_2\right)$$

$$\text{if } \mu < \frac{\underline{a}}{\overline{a}} \text{ or}$$

$$\mu\overline{a}\left(K_1+\frac{n(2n+1)^{m-1}}{(2n+1)^m-1}K_2\right) < R < \mu\overline{a}\left(K_1+\left(1-\frac{n(2n+1)^{m-1}}{(2n+1)^m-1}\right)K_2\right)$$

$$\text{if } \mu > \frac{\underline{a}}{\overline{a}}$$

Incentives for secession

- Suppose we have two population groups with different μ parameters $\mu_M > \mu_m$ where the M group has size $\gamma \in (0,1)$ and the m group $1-\gamma$.
- We first characterize equilibria for different values of μ_M and μ_m.
- We then start our analysis with what we defines as pessimistic expectations (the least favorable equilibrium for the seceding parents will occur). Observe that only parents with μ_M might want to secede.
- The μ_M parents want to secede if after secession the unique equilibrium is choosing $\overline{a}$ and inducing low individual effort. This requires a sufficiently high μ_M.
- Optimistic expectations have less stringent conditions for secession.

Forward induction and equilibrium selection

- We argue that optimistic expectations is the reasonable case. Secession is likely to entail some costs, and those costs can focus expectations.
- Note that this implies that a very high subsidy for secession is not always a good idea.
- Suppose the following game of secession. μ_M parents decide whether to sign a contract. If X of sign, each pays d for a new school. If X sign, the signatories secede.

Forward induction and equilibrium selection

Let $U^*_{B\mu_M}$ be the equilibrium payoffs for μ_M parents in the original schooling game.

Proposition 4

Assume $U^*_{B\mu_M} \leq \overline{a}\mu_M (K_1 + K_2) - d$ *and* $\max\{\overline{a}\mu_M (K_1), \underline{a}(K_1)\} < R < \max\{\overline{a}\mu_M (K_1 + K_2), \underline{a}(K_1 + K_2)\}$, *then in any equilibrium that survives forward induction, the* μ_M *parents decides to sign the contract to secede, and then their children choose* $x = 0$ *in the new school.*

Under the assumptions, it is weakly dominated to sign and take actions that lead children to choose $x = 1$ after secession. Then, after secession, if parents expect others in new school take actions that lead to $x = 0$, choosing actions that lead to $x = 0$ after secession optimal.

Secession by μ_m parents

In our model only μ_M want to secede, because they benefit from positive externality.

If μ_m had one, they may also want to secede.

But collaborative effort may have a negative externality on individualistic effort. Then

$$a\left(K_1 + K_2\left(1-\overline{x}\right)\right)\left(1-x\right) + Rx\left(1-\delta\left(1-\overline{x}\right)\right) \tag{7}$$

The parental utility with $a \in [\underline{a}, \overline{a}]$ is now

$$U^P\left(a\right) = \mu a\left(K_1 + \left(1-\overline{x}\right)K_2\right)\left(1-x_1\right) + Rx_1\left(1-\delta\left(1-\overline{x}\right)\right) \tag{8}$$

The main change is different ranges for equilibria. This creates an incentive for μ_m parents to segregate.

Authoritarian parents

If parents were authoritarian, they could force the children to have a specific x.

Specifically, both parents want to manipulate x if

$$\underline{a}\left(K_1 + (1-\overline{x})\,K_2\right) > R > \mu_M \underline{a}\left(K_1 + (1-\overline{x})\,K_2\right),$$

and only the μ_m parents if

$$\mu_M \underline{a}\left(K_1 + (1-\overline{x})\,K_2\right) > R > \mu_m \underline{a}\left(K_1 + (1-\overline{x})\,K_2\right),$$

One way this this authoritarian control can be achieved is to send the child to a school where teaching emphasizing only individual work, and a very competitive environment.

Inequality and mixed equilibria: Differences in SES

- Parents are split into two groups that differ in $\overline{a}_i$ where $i \epsilon \{L, H\}$ with $\overline{a}_H > \overline{a}_L$. This reflects socio-economic status (SES). γ is proportion of parents with $\overline{a}_H$.
- There can be a mixed equilibrium where $\overline{a}_H$ parents do not influence their children and choose $\overline{a}_H$ leading to no individualistic effort by the child $x_H = 0$ and $\overline{a}_L$ parents influence their children leading to full individualistic effort $x_L = 1$
- Obviously, when the mixed equilibrium occurs this would create reasons for segregation as in the section with parental secession.
- Would we always reach a better equilibrium if a policy intervention allowed H type parents to also have $\underline{a}_L$? In many cases, yes, but in some cases there is the danger of miscoordination when multiple equilibria exist.

Inequality and mixed equilibria: Differences in returns

- Society is split into two groups with $R \in \{L, H\}$ with $R_L < R_H$ where an upper class child has R_H and the proportion of upper class children is given by γ.
- Existence of a mixed equilibrium where different types of parents induce their children to behave differently exists if the following condition holds.

$$R_L < \max\left\{\underline{a}\left(K_1 + \gamma K_2\right), \overline{a}\mu\left(K_1 + \gamma K_2\right)\right\} < R_H \tag{9}$$

- Again, there are incentives for segregation for the parents with lower returns to individualistic education. They would rather have their children being surrounded by only collaborative learners.

Total rather than average effort

- Up to now only the average collaborative effort mattered. We can examine our setup when the total collaborative effort affects the externality. The child's best response function in this case becomes

$$x_i = \begin{cases} 1 \text{ if } R > a\left(K_1 + NK_2\left(1 - \overline{x}\right)\right) \\ 0 \text{ if } R \leq a\left(K_1 + NK_2\left(1 - \overline{x}\right)\right) \end{cases}$$

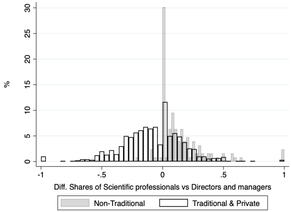
  where N is the size of the population.
- Most of our results would not be affected qualitatively.
- Note, though, that for secession the size of the seceding group matters.
- If a fraction γ of parents prefers collaborative effort, they have to take into account that under secession their children will only interact with γN other children. This lowers the interest of secession.

Empirical illustration

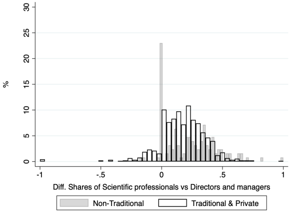
- “Alternative” pedagogies (Montesssori, Waldorf,..) are more supportive of collaborative learning.
- People in scientific or technical professions more likely to understand collaboration.
- Thus we hypothesize more children of scientific and technical parents in alternative schools, than children of managers (similar SES). S
- Data from occupation in parent questionnaire on standardized exams to all Grade 3, 6 and 10 students in Madrid region from 2015 to 2019.
- “alternative“ schools, from Ludus https://ludus.org.es/ plus manual check.

Figure: Share Parents with Tech. and Science vs Directors and Managers.

(a) Father Occupation



(b) Mother Occupation



(c) At least one parent

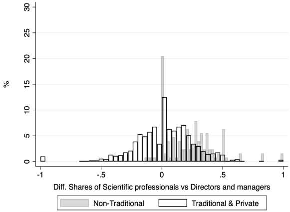
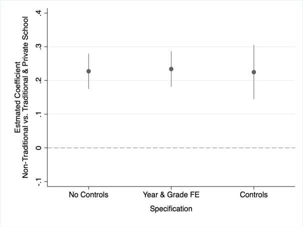
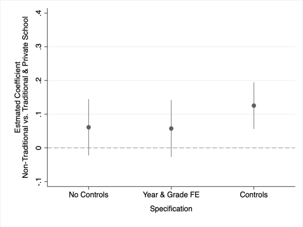


Figure: Est. Coefficient of Non-Traditional vs. Traditional & Private school

(a) Father Occupation



(b) Mother Occupation



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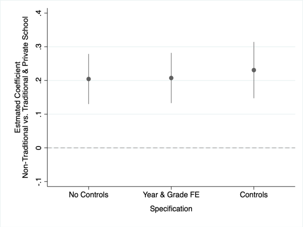

Conclusion

- Parents influence K_2 when they want children to avoid a certain subgroup of the population.
- Intergenerational transmission, probably of μ.Then, without investment child gets μ of each group with a probability proportional to population weights. Investment gets it closer to parental μ.
- Empirics.
- More research is needed. Especially by us.
- Thanks for your comments and patience.