SHADOW-RATE VARs

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August, 2024

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VARs AND ELB DATA

How to use VARs
w/nominal-interest rates at the ELB?

Bayesian VARs are a great time series tool

But, VARs are ill-equipped to handle bounded data

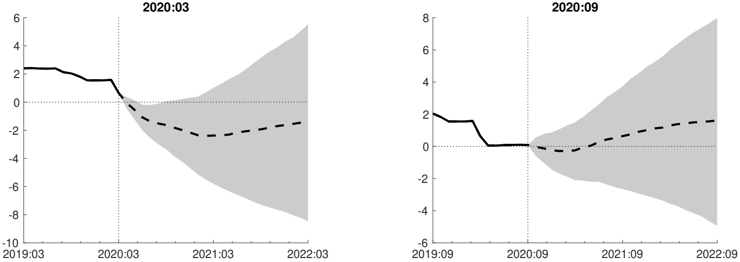
Problem: the standard VAR is linear and unbounded

$$y_t = \Pi_0 + \Pi(L) y_{t-1} + v_t \,, \quad v_t \sim N(0, \Sigma_t)$$

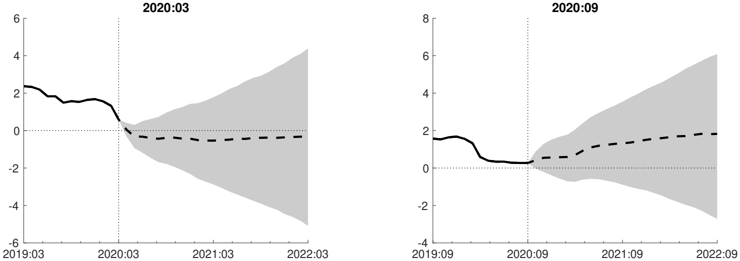
VAR FORECASTS FOR INTEREST RATES IN 2020

Linear forecasts from BVAR-SV with 20 macro and financial variables

Federal funds rate



5-year Treasury yield



Medians and 68% bands

WAYS TO ACCOMODATE ELB

- **Use longer-term yields** and no short-term policy rates (Swanson & Williams, 2014; Debortoli et al., 2019)
- **No-arbitrage models** (Black, 1995): Wu-Xia, Krippner
- **Plug-in VAR**: Use given shadow-rate estimates as data

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- **Plug-in VAR**: Use given shadow-rate estimates as data

Our approach: censored data in a VAR

- **"Shadow rate" as latent state variable in VAR**
- informed by bond yields, financial and macro variables
- Extends works by Mavroeidis (2021), Aruoba et al (2022), Johannsen & Mertens (2021) to medium-sized VAR
- Particular focus: out-of-sample performance

RELATED LITERATURE

No-arbitrage term structure models w/shadow rates

- Black (1995), Krippner (2013/15/20), Wu & Xia (2016/20)
- Bauer & Rudebusch (2014/17), Kim & Singleton (2011), Priebsch (2017), Christensen & Rudebusch (2015, 2016)

Macro models w/negative-rate substitutes

- Sims & Wu (2021), Wu & Zhang (2019)
- Gust et al (2017), Wolf (2021)
- Kulish, Morley, Robinson (2017), Jones, Kulish, Morley (2022)

Time series models w/bounded or censored rates

- Mavroeidis (2021), Ikeda et al (2022), Aruoba, et al (2022)
- Duffy, Mavroeidis & Wycherley (2023a, 2023b)
- Guerron-Quintana, Khazanov & Zhong (2023)
- Johannsen & Mertens (2021), Gonzalez-Astudillo & Laforte (2023)
- Iwata & Wu (2006), Nakajima (2011), Koop & Potter (2011), Chan & Strachan (2014), Bäurle, et al. (2016),
- Chib (1992), Chib & Greenberg (1998)

AGENDA

1. **Shadow-rate VARs**
2. **Empirical results**
3. **Conclusions**

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1. **Shadow-rate VARs**
2. Empirical results
3. Conclusions

Shadow rate s_t

Nominal interest rate that would prevail in the absence of lower bound constraint

Observed Rate i_t

$$i_t = \max\left(s_t, ELB\right)$$

THE SHADOW RATE OF INTEREST

Shadow rate s_t

Nominal interest rate that would prevail in the absence of lower bound constraint

Observed Rate i_t

$$i_t = \max\left(s_t, ELB\right)$$

Our paper

- VAR model for joint dynamics of s_t and other variables
- Estimation treats i_t as censored variable
- Shadow rates identified from historical comovements between interest rates, macro and financial variables
- We study the role of s_t and i_t as predictors in VAR

SIMPLE AND HYBRID SHADOW-RATE VARs

All expressed in companion form omitting intercepts

Setup

- Partition the variable vector:

$$\boldsymbol{y}_t = \begin{bmatrix} \boldsymbol{x}_t \\ \boldsymbol{i}_t \end{bmatrix}$$

SIMPLE AND HYBRID SHADOW-RATE VARs

All expressed in companion form omitting intercepts

Setup

- Partition the variable vector:

$$y_t = \begin{bmatrix} x_t \\ i_t \end{bmatrix}$$

- Define a corresponding shadow-rate VAR vector

$$z_t = \begin{bmatrix} x_t \\ s_t \end{bmatrix}, \qquad i_t = \max(s_t, ELB)$$

Simple shadow-rate VAR

$$z_t = \Pi\, z_{t-1} + v_t$$

SIMPLE AND HYBRID SHADOW-RATE VARs

All expressed in companion form omitting intercepts

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Simple shadow-rate VAR

$$z_t = \Pi \, z_{t-1} + v_t$$

Hybrid shadow-rate VAR

$$\begin{aligned} x_t &= \Pi_{xx} \, x_{t-1} & & + \Pi_{xi} \, i_{t-1} + v_t^x \\ s_t &= \Pi_{sx} \, x_{t-1} + \Pi_{ss} \, s_{t-1} & & + v_t^s \end{aligned}$$

SIMPLE AND HYBRID SHADOW-RATE VARs

All expressed in companion form omitting intercepts

Setup

- Partition the variable vector:

$$y_t = \begin{bmatrix} x_t \\ i_t \end{bmatrix}$$

- Define a corresponding shadow-rate VAR vector

$$z_t = \begin{bmatrix} x_t \\ s_t \end{bmatrix}, \qquad i_t = \max\left(s_t, ELB\right)$$

Simple shadow-rate VAR

$$z_t = \Pi\, z_{t-1} + v_t$$

Fully-Hybrid shadow-rate VAR

$$\begin{aligned} x_t &= \Pi_{xx}\, x_{t-1} + \Pi_{xs}\, s_{t-1} + \Pi_{xi}\, i_{t-1} + v_t^x \\ s_t &= \Pi_{sx}\, x_{t-1} + \Pi_{ss}\, s_{t-1} + \Pi_{si}\, i_{t-1} + v_t^s \end{aligned}$$

RELATIONSHIP TO SVAR OF MAVROEIDIS (2021)

Again, omitting intercepts and higher-order lags

Mavroeidis' "censored and kinked" SVAR

$$i_t = \max(s_t, ELB)$$

$$\begin{bmatrix} A_x & A_s & A_i \end{bmatrix} \begin{bmatrix} x_t \\ s_t \\ i_t \end{bmatrix} = \Pi_x \, x_{t-1} + \Pi_s \, s_{t-1} + \Pi_i \, i_{t-1} + \varepsilon_t$$

Note: $\varepsilon_t \sim \mathcal{N}(0, I)$

RELATIONSHIP TO SVAR OF MAVROEIDIS (2021)

Again, omitting intercepts and higher-order lags

Mavroeidis' "censored and kinked" SVAR

$$i_t = \max\left(s_t, ELB\right)$$

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Need restrictions on A_x, A_s, A_i to ensure **existence and uniqueness** of solutions for s_t and i_t

Note: $\varepsilon_t \sim \mathcal{N}(0, I)$

RELATIONSHIP TO SVAR OF MAVROEIDIS (2021)

Again, omitting intercepts and higher-order lags

Mavroeidis' "censored and kinked" SVAR

$$i_t = \max(s_t, ELB)$$

$$\begin{bmatrix} A_x & A_s & 0 \end{bmatrix} \begin{bmatrix} x_t \\ s_t \\ i_t \end{bmatrix} = \Pi_x \, x_{t-1} + \Pi_s \, s_{t-1} + \Pi_i \, i_{t-1} + \varepsilon_t$$

Need restrictions on A_x, A_s, A_i to ensure existence and uniqueness of solutions for s_t and i_t

Our shadow-rate VARs drops contemporaneous i_t

$A_i = 0$ assures unique solution w/o restrictions on A_x, A_s

Note: $\varepsilon_t \sim \mathcal{N}(0, I)$

RELATIONSHIP TO SVAR OF MAVROEIDIS (2021)

Again, omitting intercepts and higher-order lags

Mavroeidis' "censored and kinked" SVAR

$$\boldsymbol{i_t = \max\left(s_t, ELB\right)}$$

$$\begin{bmatrix} \boldsymbol{A_x} & \boldsymbol{A_s} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{x_t} \\ \boldsymbol{s_t} \\ \boldsymbol{i_t} \end{bmatrix} = \boldsymbol{\Pi_x\, x_{t-1} + \Pi_s\, s_{t-1} + \Pi_i\, i_{t-1} + \varepsilon_t}$$

Need restrictions on $\boldsymbol{A_x}, \boldsymbol{A_s}, \boldsymbol{A_i}$ to ensure existence and uniqueness of solutions for $\boldsymbol{s_t}$ and $\boldsymbol{i_t}$

Our shadow-rate VARs drops contemporaneous i_t

$A_i = 0$ assures unique solution w/o restrictions on A_x, A_s

Ongoing work: Estimation of general reduced-form w/ $\boldsymbol{A_i \neq 0}$

Note: $\boldsymbol{\varepsilon_t \sim \mathcal{N}(0, I)}$

AGENDA

1. Shadow-rate VARs
2. **Empirical results**
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DATA AND ESTIMATION SETUP

Data

- 20 variables, including FFR and 5 other yields (6m-10y maturities)
- All data from same FRED-MD vintage
- Monthly observations from 1959:03 – 2022:08

Full-sample (smoothed) estimates

reflect data through 2022:08

Quasi-real time estimates

- MCMC over growing estimation windows
- Evaluation window 2009:01 – 2017:12 (similar results through 2022:08)
- Forecasts up to two years out ($h = 24$)

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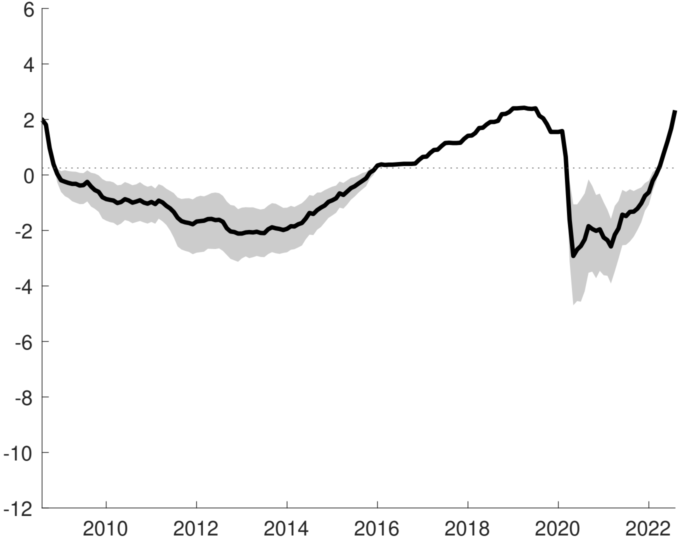
SHADOW RATE ESTIMATES

simple shadow-rate VAR (w/yields), full-sample median and 90% bands

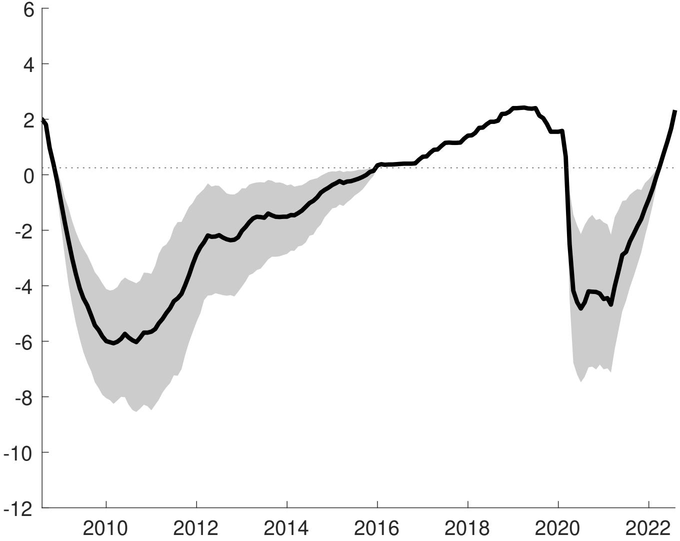


Reflects historical comovements of FFR w/other variables

SHADOW RATE ESTIMATED W/O YIELDS

FFR

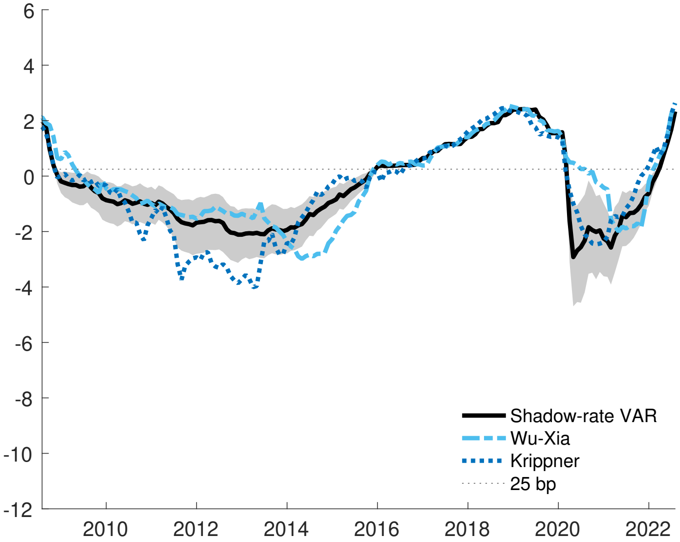
simple shadow-rate VAR, full-sample median and 90% bands



Deeper path when not informed by yields

SHADOW RATE ESTIMATES

simple shadow-rate VAR (w/yields), full-sample median and 90% bands



Between finance-based estimates of Krippner and Wu-Xia

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SUMMARY OF FORECAST RESULTS

Compared to the standard VAR, the hybrid . . .

- . . . delivers **superior interest-rate forecasts**
- . . . with **some gains for macro** variables

Also: Hybrid VAR . . .

- . . . **does better than simple shadow-rate VAR** (which generates somewhat poorer macro forecasts than the standard VAR)
- . . . **beats plug-in VAR** (with Wu-Xia or Krippner rates)
- . . . **expects reasonably delayed departure from ELB** compared to censored predictions from linear VAR

ROLE OF ACTUAL VS SHADOW RATES

Which interest rates serve as predictors in macro equations of VAR?

Standard VAR:

unconstrained actual-rate projections

Simple shadow-rate VAR:

shadow rates

(Block-)Hybrid shadow-rate VAR:

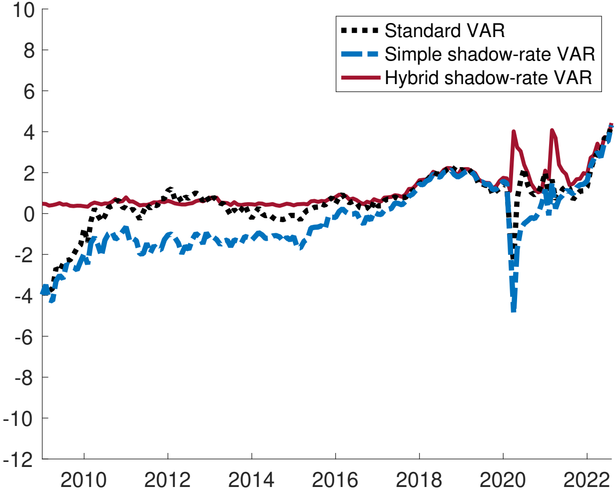
actual rates

(as implied by censored shadow-rate projections)

SHORT-RATE EXPECTATIONS

FEDFUNDS

Average two-year-ahead (shadow-)interest rates at different forecast origins



Stronger stimulus predicted by simple shadow-rate VAR, and (initially) also the linear VAR

HYBRID VS STANDARD VAR

	RMSE			**MAE**			**CRPS**		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income									
Consumption									
IP									
Cap. Util.									
Unemp.									
Nfm Pyrlls									
Hours									
H. Earnings									
PPI (Fin.)									
PPI (Metals)									
PCE Prices									
Hsng Strts									
S&P 500									
USD / GBP									
FFR									
6m Tbill									
1y Trsy									
5y Trsy									
10y Trsy									
BAA Yld									

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

HYBRID VS STANDARD VAR

Values below one indicate improvement of hybrid over standard VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income									
Consumption									
IP									
Cap. Util.									
Unemp.									
Nfm Pyrlls									
Hours									
H. Earnings									
PPI (Fin.)									
PPI (Metals)									
PCE Prices									
Hsng Strts									
S&P 500									
USD / GBP									
FFR	0.22**	0.29	0.50	0.13***	0.28*	0.38**	0.15***	0.25**	0.39***
6m Tbill	0.36*	0.51	0.68	0.24***	0.36**	0.48***	0.26***	0.40**	0.51***
1y Trsy	0.63*	0.66	0.78	0.51***	0.46**	0.56***	0.52***	0.53***	0.56***
5y Trsy	0.95	0.82***	0.75**	0.95	0.90	0.89	0.98	0.88**	0.83***
10y Trsy	0.96	0.88*	0.78***	0.96	0.88	0.97	0.98	0.93	0.95
BAA Yld	0.97	1.00	0.99	1.00	1.01	1.07	0.99	1.02	1.07

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

HYBRID VS STANDARD VAR

Values below one indicate improvement of hybrid over standard VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	1.00	1.00	0.65	0.99	1.01	0.97	1.00	1.01	0.99
Consumption	1.00	0.99	0.88***	0.99*	1.01	0.90*	1.00	0.99	0.97**
IP	1.00	1.02	0.96	1.01	1.01	0.98	1.01	1.01	1.00
Cap. Util.	1.00	1.02	0.96	1.00	1.00	0.95	1.00	1.01	0.97
Unemp.	1.00	0.99	0.91***	1.00	0.98	0.91***	1.00	0.99	0.94***
Nfm Pyrlls	0.98**	1.04	0.99	0.99**	1.01	0.91**	0.99**	1.01	0.97
Hours	1.00	1.02	1.03	1.00	1.02	1.03	1.00	1.01	1.01
H. Earnings	1.00	1.01	1.03**	1.00	1.01**	1.01	1.00	1.01**	1.01**
PPI (Fin.)	1.00	0.97	1.00	1.00	0.99	1.01	1.00	0.99	1.00
PPI (Metals)	1.00	0.99	1.02	1.00	0.99	1.00	1.00	1.00	1.01
PCE Prices	1.00	0.97	1.07*	0.99	0.98	1.08***	0.99	0.99	1.05***
Hsng Strts	1.00	0.91	0.90	1.01	0.91	0.90	1.00	0.94	0.93
S&P 500	1.00	1.01**	1.08*	0.99**	1.01	1.02**	0.99	1.01*	1.01***
USD / GBP	1.00	1.00	1.02	1.00	1.02	1.05	1.00	1.00	1.02
FFR	0.22**	0.29	0.50	0.13***	0.28*	0.38**	0.15***	0.25**	0.39***
6m Tbill	0.36*	0.51	0.68	0.24***	0.36**	0.48***	0.26***	0.40**	0.51***
1y Trsy	0.63*	0.66	0.78	0.51***	0.46**	0.56***	0.52***	0.53***	0.56***
5y Trsy	0.95	0.82***	0.75**	0.95	0.90	0.89	0.98	0.88**	0.83***
10y Trsy	0.96	0.88*	0.78***	0.96	0.88	0.97	0.98	0.93	0.95
BAA Yld	0.97	1.00	0.99	1.00	1.01	1.07	0.99	1.02	1.07

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

STANDARD VS FULLY-HYBRID VAR

Values below one indicate improvement over standard VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	1.01*	1.04	4.27	1.03	1.08	1.19	1.02	1.10**	1.47**
Consumption	1.01	1.07**	6.69*	1.00	1.04	1.11	1.01	1.10**	1.47**
IP	1.01	1.12*	6.41**	1.01	1.06	1.13	1.00	1.09	1.55**
Cap. Util.	0.97	1.05	4.52*	0.96	1.03	1.16	0.98	1.10*	1.46*
Unemp.	1.02	1.07*	4.07	1.00	1.01	1.03	1.01	1.10**	1.31*
Nfm Pyrlls	0.97	1.08	13.66	0.96	1.08*	1.22*	0.99	1.12***	1.68**
Hours	1.05***	1.21	7.86	1.04**	1.13**	1.18***	1.05***	1.16**	1.61**
H. Earnings	1.00	1.09**	14.43	1.00	1.02	1.07*	1.01	1.11**	1.54**
PPI (Fin.)	1.01	1.01	3.97	1.00	1.01	1.06	1.01	1.04*	1.38**
PPI (Metals)	1.00	1.01	4.72	1.00	1.00	1.05***	1.00	1.04**	1.43**
PCE Prices	0.99	0.95	3.86**	0.99	0.95	0.94	1.00	1.02	1.33**
Hsng Strts	1.00	0.98	2.09**	1.00	0.98	0.97	1.01	1.03	1.27*
S&P 500	0.99	1.08	19.81	0.98	1.00	1.04	1.00	1.07**	1.48**
USD / GBP	0.99	1.04	6.22	0.99	0.97*	1.01	1.00	1.04*	1.37*
FFR	0.23*	0.31	1.49	0.15***	0.29*	0.39**	0.16***	0.27*	0.44***
6m Tbill	0.41*	0.60	2.86	0.26***	0.40*	0.51***	0.30***	0.46**	0.60***
1y Trsy	0.71	0.76	2.45	0.56***	0.55**	0.59***	0.58***	0.61**	0.66***
5y Trsy	0.98	0.96	2.89*	0.97	0.98	1.01	1.00	0.99	1.03
10y Trsy	1.00	0.99	3.71	0.99	1.01	1.10	1.01	1.05	1.22*
BAA Yld	0.99	1.14***	4.64	1.01	1.14***	1.06	1.01	1.15***	1.33**

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

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ADDITIONAL RESULTS

Time-invariant VAR parameters

- Linear VAR exhibits significant drift in VAR parameters (in particular for interest rate equations)
- Shadow-rate VAR does not (except for COVID-19)
- Poor performance when shock impact matrix $\boldsymbol{A}$ allowed to switch at ELB

Estimates w/o yields (other than FFR)

- on balance, poorer macro forecasts
- deeper, and more uncertain shadow rate path
- IRF qualitatively similar

Alternative ELB values

- 12.5bp: Similar (albeit limited) differences as w/25bp
- 50bp: problematic for hybrid (b/o assumed 50bp path)

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CONCLUSIONS

Our solution to handle ELB: Shadow-rate VARs

- **Internally consistent** inference
- **Scalable** to multiple interest-rate maturities at ELB
- **Hybrid VAR** conditions macro variables on actual rates

Structural Analysis at/away from ELB:

- strong differences in nominal rate responses
- some differences in macro responses
- consistent w/some effectiveness of unconventional policies

Forecasts for interest rates and macro variables

- **Interest-rate forecasts superior** to standard VAR
- **Hybrid** typically better than simple shadow-rate VAR
- ... and with **some gains in macro** over linear VAR

APPENDIX

- **Estimation method**
- **Data set**
- **Shadow-rate sampling**
- **Additional shadow rate estimates**
- **Interest rate predictions**
- **Additional forecast comparisons**
- **SVAR responses to financial conditions shock**

VAR parameters

$$\begin{aligned} z_t &= \Pi_0 + \Pi(L) z_{t-1} + \Pi_i(L) i_{t-1} + v_t \\ v_t &\sim \mathcal{N}(0, \Sigma_t) \end{aligned}$$

- transition coefficients Π
- stochastic volatility in shock vector $\Sigma_t = A^{-1} \Lambda_t A^{-1\prime}$

For given values of z_t, we know how to draw Π and Σ_t

MCMC SAMPLER FOR SHADOW-RATE VAR
Overview

VAR parameters

$$z_t = \Pi_0 + \Pi(L) z_{t-1} + \Pi_i(L) i_{t-1} + v_t$$
$$v_t \sim \mathcal{N}(0, \Sigma_t)$$

- transition coefficients Π
- stochastic volatility in shock vector $\Sigma_t = A^{-1} \Lambda_t A^{-1\prime}$

For given values of z_t, we know how to draw Π and Σ_t

Additional step: draw shadow rates consistent with ELB

Sample entire path for s_t from
"missing data" distribution truncated at ELB

Throughout, we treat ELB as known value of 25bp

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20-VARIABLE DATA SET

Monthly obs from 1959:03 to 2022:08; FRED-MD vintage 2022:09

Variable	FRED-MD code	transformation
Real Income	RPI	$\Delta \log(x_t) \cdot 1200$
Real Consumption	DPCERA3M086SBEA	$\Delta \log(x_t) \cdot 1200$
IP	INDPRO	$\Delta \log(x_t) \cdot 1200$
Capacity Utilization	CUMFNS	
Unemployment	UNRATE	
Nonfarm Payrolls	PAYEMS	$\Delta \log(x_t) \cdot 1200$
Hours	CES0600000007	
Hourly Earnings	CES0600000008	$\Delta \log(x_t) \cdot 1200$
PPI (Fin. Goods)	WPSFD49207	$\Delta \log(x_t) \cdot 1200$
PPI (Metals)	PPICMM	$\Delta \log(x_t) \cdot 1200$
PCE Prices	PCEPI	$\Delta \log(x_t) \cdot 1200$
Housing Starts	HOUST	$\log(x_t)$
S&P 500	SP500	$\Delta \log(x_t) \cdot 1200$
USD / GBP FX Rate	EXUSUKx	$\Delta \log(x_t) \cdot 1200$
Federal Funds Rate	FEDFUNDS	
6m Tbill	TB6MS	
1-Year Yield	GS1	
5-Year Yield	GS5	
10-Year Yield	GS10	
Corporate Bond Yield	BAA	

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SHADOW-RATE SAMPLING

Data augmentation step (Chib, 1992) within MCMC sampler for VAR

Shadow-rate setup in static form

- $\boldsymbol{Y}$, vector of all $\boldsymbol{y_t} = \begin{bmatrix} \boldsymbol{x_t} \\ \boldsymbol{i_t} \end{bmatrix}$
- $\bar{\boldsymbol{Y}}$, all of $\boldsymbol{Y}$ except for $\boldsymbol{i_t}$ when ELB binds
- $\boldsymbol{S}$, all shadow rates $\boldsymbol{s_t}$ when ELB binds

SHADOW-RATE SAMPLING

Data augmentation step (Chib, 1992) within MCMC sampler for VAR

Shadow-rate setup in static form

- $\boldsymbol{Y}$, vector of all $\boldsymbol{y_t} = \begin{bmatrix} \boldsymbol{x_t} \\ \boldsymbol{i_t} \end{bmatrix}$
- $\bar{\boldsymbol{Y}}$, all of $\boldsymbol{Y}$ except for $\boldsymbol{i_t}$ when ELB binds
- $\boldsymbol{S}$, all shadow rates $\boldsymbol{s_t}$ when ELB binds

Missing-value problem (given Π, Σ_t, and $\forall\, t$)

$$\boldsymbol{S}|\bar{\boldsymbol{Y}} \sim \boldsymbol{N}(\boldsymbol{\mu}, \boldsymbol{\Omega})$$

can be obtained from standard Kalman smoothing

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Shadow-rate sampling problem

$$S|Y \sim \text{truncN}\left(\mu, \Omega, S \leq ELB\right)$$

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Data augmentation step (Chib, 1992) within MCMC sampler for VAR

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can be obtained from standard Kalman smoothing

Shadow-rate sampling problem

$$S|Y \sim \text{truncN}\,(\mu, \Omega, S \leq ELB)$$

- Johannsen & Mertens (2021): Rejection sampling
- Here: Direct sampling from truncated multivariate normal

APPENDIX

- Estimation method
- Data set
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- **Additional shadow rate estimates**
  - Shadow rate estimates for other short rates
  - Shadow rate estimates from hybrid VAR
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- Additional forecast comparisons

AGENDA

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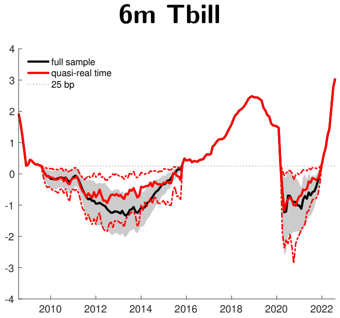
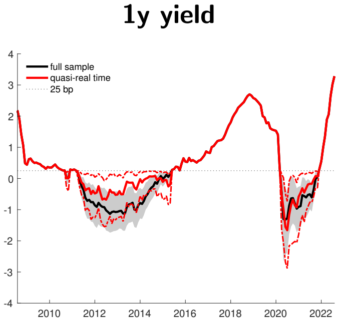
SHADOW RATE ESTIMATES FOR 6M and 1Y RATES

medians and 90% bands, simple shadow-rate VAR

in addition to FFR,

6M Tbill and 1y yield have data at ELB





(similar results with hybrid shadow-rate VAR)

AGENDA

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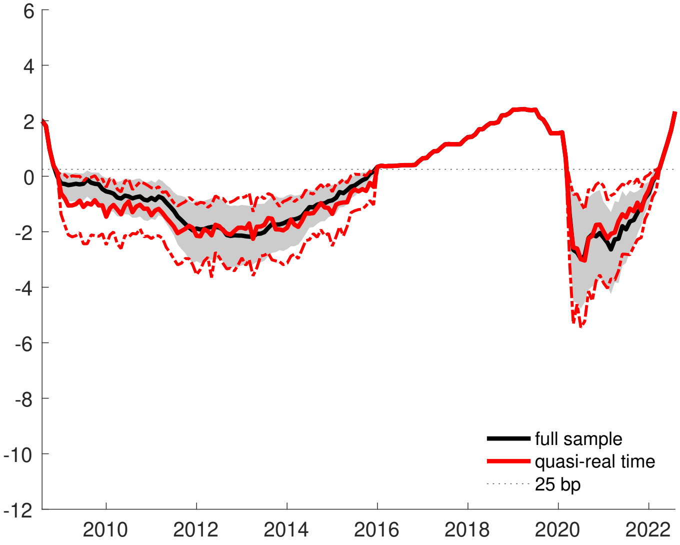
SHADOW RATE ESTIMATES

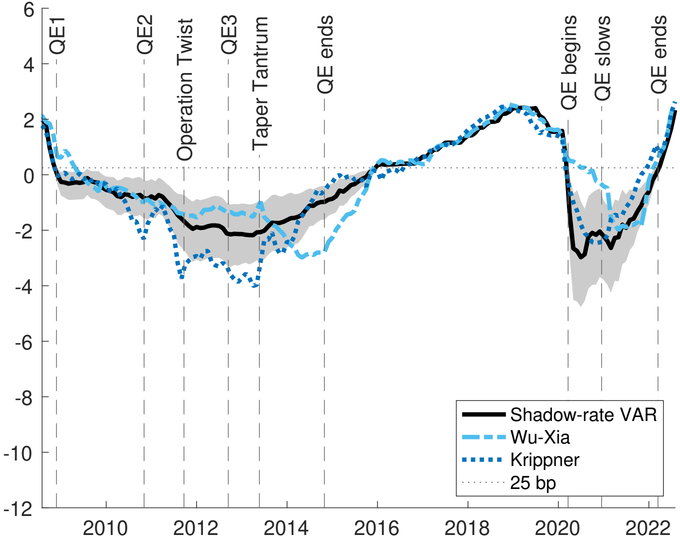
hybrid shadow-rate VAR, full-sample median and 90% bands



hybrid VAR: somewhat more uncertainty in quasi-real time

SHADOW RATE ESTIMATES

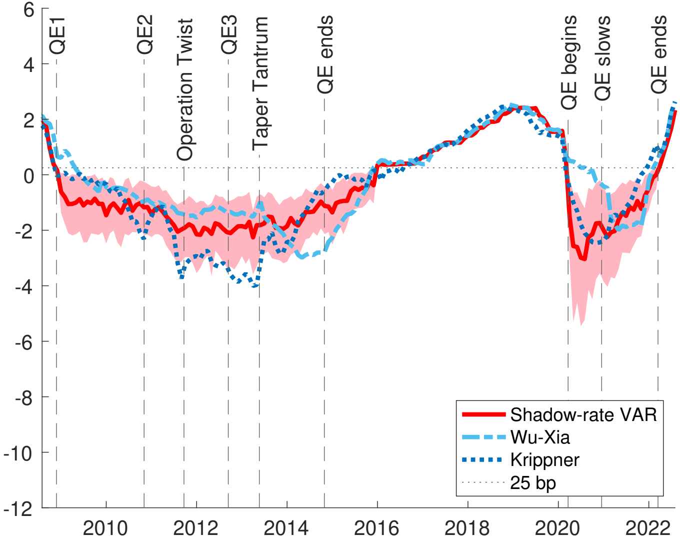
hybrid shadow-rate VAR, full-sample median and 90% bands



Some reactions to major balance sheet events

SHADOW RATE ESTIMATES

hybrid shadow-rate VAR in quasi-real time, median and 90% bands



Some reactions to major balance sheet events

AGENDA

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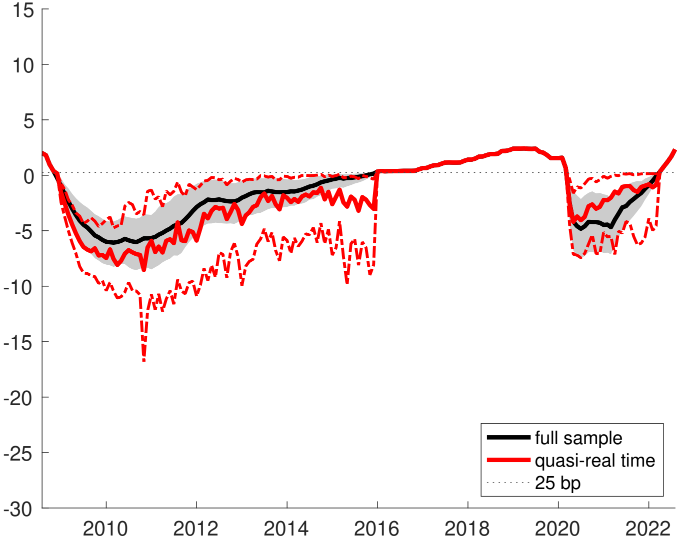
SHADOW RATE ESTIMATED W/O YIELDS

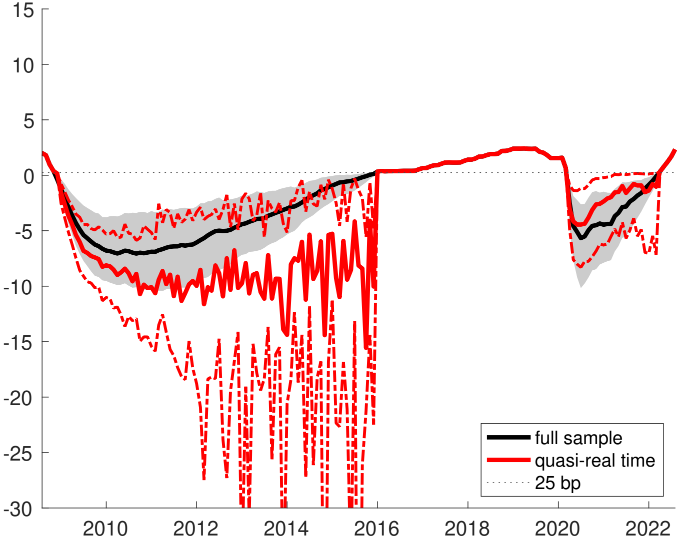
simple shadow-rate VAR, median and 90% bands



more quasi-real-time uncertainty w/o yields

SHADOW RATE ESTIMATED W/O YIELDS

hybrid shadow-rate VAR, median and 90% bands

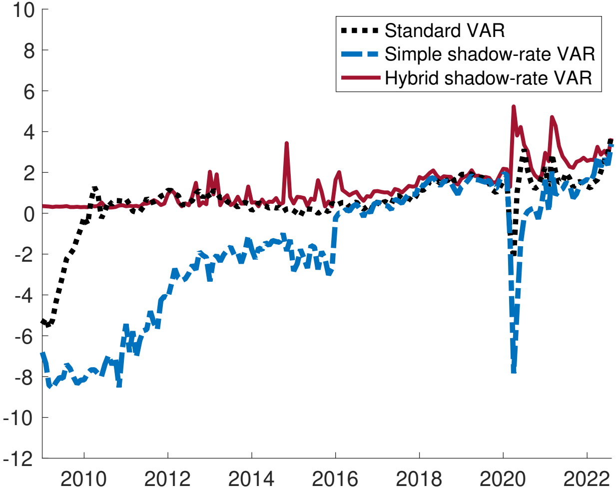


much more quasi-real-time uncertainty for hybrid w/o yields

SHORT-RATE EXPECTATIONS

MODELS W/O YIELDS

Average two-year-ahead (shadow-)interest rates at different forecast origins



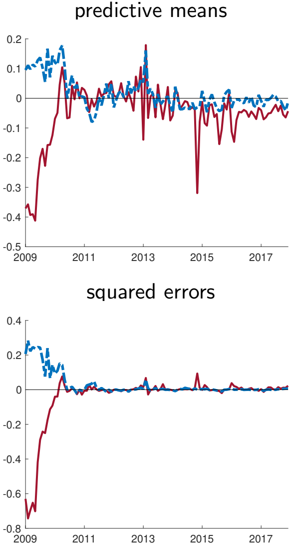
Absent yield data, even stronger stimulus predicted by simple shadow-rate VAR (but not by hybrid VAR)

DIFFERENCES IN FORECASTS

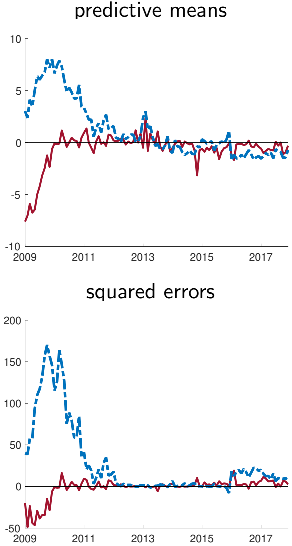
MODELS W/O YIELDS

Simple (blue) and hybrid (red) shadow-rate VARs relative to linear, $h = 24$

Housing starts



Capacity utilization



Even starker differences between simple and hybrid VAR when yields (other than FFR) are excluded from model

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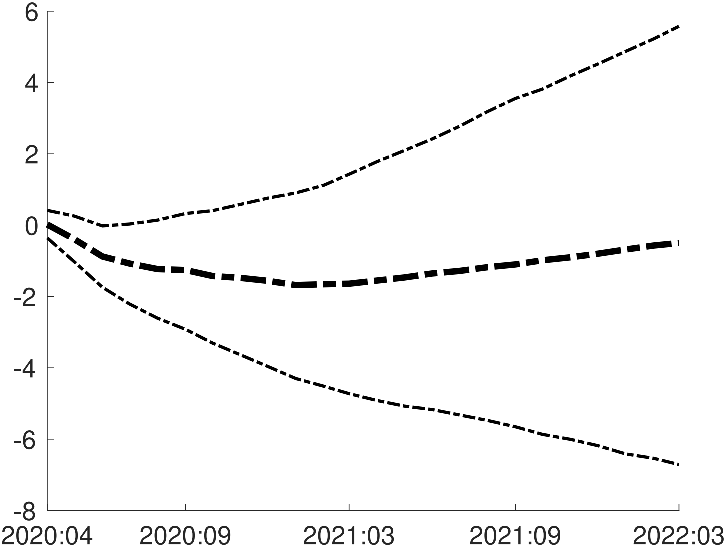
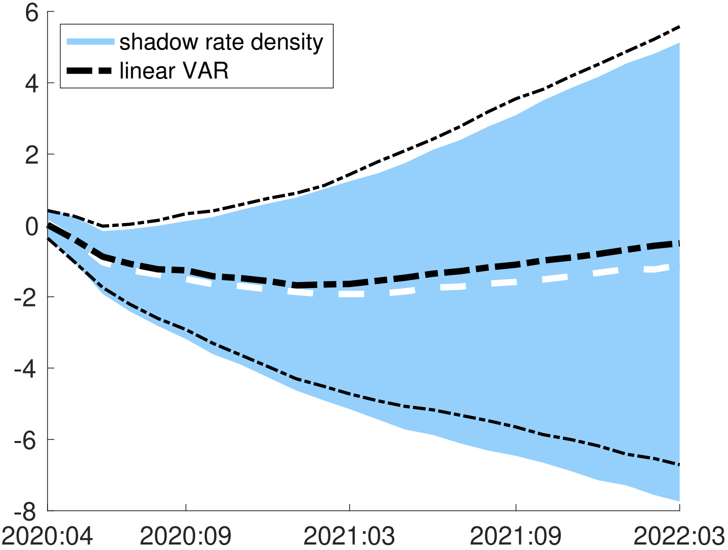
STANDARD VAR: FUNDS RATE PREDICTIONS



Median and 68% bands of predictive densities.

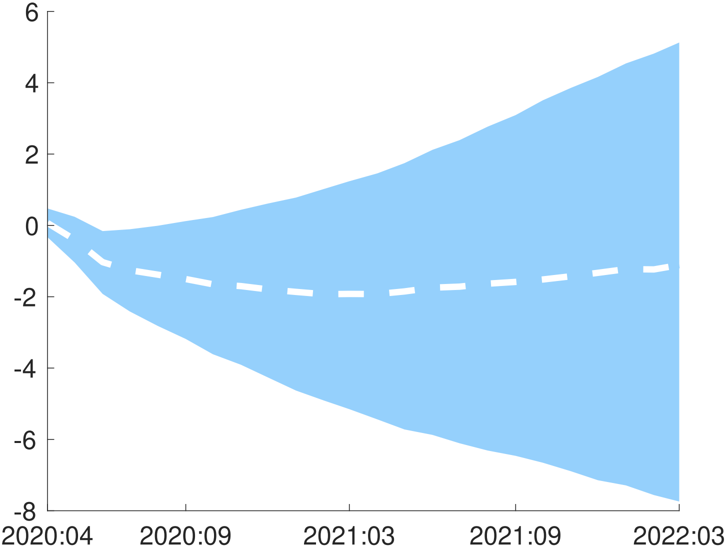
STANDARD VS SHADOW-RATE PREDICTIONS



Median and 68% bands of predictive densities.

SHADOW-RATE VAR PREDICTIONS

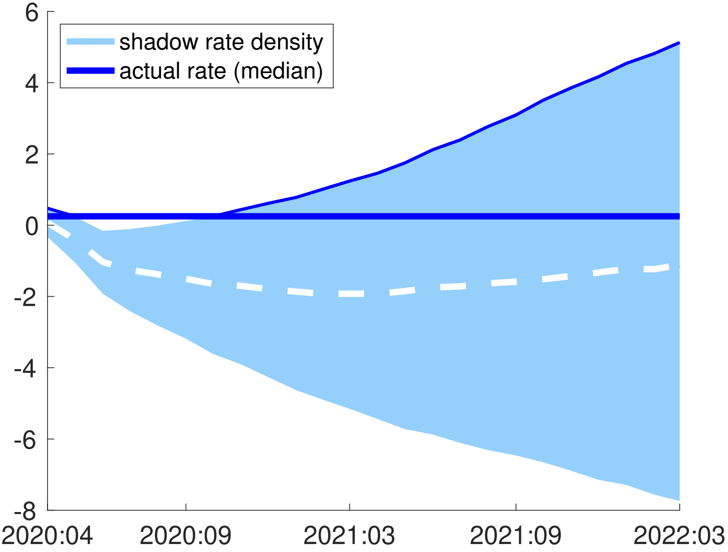
Shadow-rate density (light blue)



Medians and 68% bands of predictive densities.

SHADOW-RATE VAR PREDICTIONS

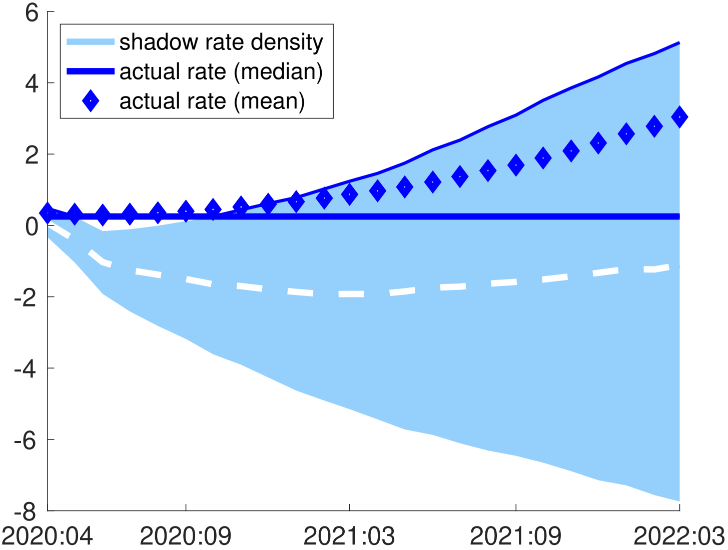
Shadow-rate (light blue) and actual-rate (dark blue) densities



Medians and 68% bands of predictive densities.

SHADOW-RATE VAR PREDICTIONS

Shadow-rate (light blue) and actual-rate (dark blue) densities



Medians and 68% bands of predictive densities.

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SIMPLE SHADOW-RATE VS LINEAR VAR

Values below one indicate improvement over standard VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	1.00	1.00	0.63	0.98*	0.99	0.99	0.99**	1.01	1.00
Consumption	1.00	1.02	0.96	1.00	1.02	0.99	1.00	1.02	1.00
IP	1.03***	1.06***	0.98	1.04***	1.06***	1.03**	1.03***	1.05***	1.02***
Cap. Util.	1.04**	1.11**	1.13**	1.05**	1.16***	1.12*	1.03***	1.11***	1.10***
Unemp.	1.00	1.02	1.03	1.00	0.97	0.98	1.00	1.02	1.01
Nfm Pyrlls	0.98	1.04	1.05	0.99	1.11*	1.03	1.00	1.06	1.03
Hours	1.02*	1.01	1.04	1.02	1.03	1.06	1.02**	1.02	1.02
H. Earnings	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00
PPI (Fin.)	1.00	1.00	0.98**	1.00	1.00	0.99	1.00	1.00	1.00
PPI (Metals)	0.99*	1.00	1.01	0.99	1.00	1.00	0.99	1.00	1.00
PCE Prices	1.00	1.00	0.99	1.00	1.00	1.02**	1.00	1.00	1.01**
Hsng Strts	1.01	0.97	0.92	1.00	0.99	0.93	1.01	0.97	0.93
S&P 500	0.99	1.01	1.08	0.97	1.00	1.01**	0.99	1.01	1.01***
USD / GBP	0.99**	0.99	0.98**	0.98**	0.98*	0.98**	0.98***	0.99*	0.99
FFR	0.22**	0.30	0.53	0.13***	0.29*	0.39**	0.15***	0.26**	0.43**
6m Tbill	0.37*	0.54	0.76	0.24***	0.39*	0.55**	0.26***	0.44**	0.58***
1y Trsy	0.64	0.70	0.85	0.51***	0.52**	0.65**	0.53***	0.58**	0.64***
5y Trsy	0.94	0.84***	0.83	0.94	0.89**	0.89***	0.97	0.89***	0.85***
10y Trsy	0.95	0.88***	0.80***	0.95	0.89**	0.89*	0.97	0.93	0.93*
BAA Yld	0.97	1.02	0.93	0.99	1.03	1.01	0.99	1.03	1.04

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

HYBRID VS SIMPLE SHADOW-RATE VAR

Values below one indicate improvement over simple shadow-rate VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	1.00	1.01	1.04	1.01	1.02	0.98	1.01	1.00	0.99
Consumption	1.00	0.97	0.92	0.99	0.99	0.90	1.00	0.98**	0.96**
IP	0.97***	0.96	0.98	0.98*	0.95*	0.95	0.98**	0.96**	0.98
Cap. Util.	0.96**	0.92	0.85**	0.95***	0.86**	0.85**	0.97***	0.91**	0.88***
Unemp.	1.00	0.96	0.88	1.00	1.01	0.92	1.00	0.97	0.93
Nfm Pyrlls	1.00	1.00	0.94	0.99	0.91	0.88**	0.98	0.95	0.94***
Hours	0.99	1.00	0.99	0.99	0.99	0.97	0.99	0.99	0.99
H. Earnings	1.00	1.01	1.03*	0.99	1.02	1.01	1.00	1.01**	1.01
PPI (Fin.)	0.99	0.97	1.02	1.00	0.99	1.02	1.00	0.99	1.01
PPI (Metals)	1.00	0.99	1.01	1.00	0.99	1.00	1.00	0.99**	1.00
PCE Prices	1.00	0.97	1.08**	0.99	0.98	1.06***	0.99	0.99	1.04***
Hsng Strts	0.99	0.93	0.98	1.00	0.92	0.97	0.99	0.97	1.00
S&P 500	1.01	1.00	1.00	1.02	1.01**	1.01	1.00	1.00	1.00
USD / GBP	1.02	1.01	1.03	1.02*	1.04*	1.07	1.01	1.01	1.03
FFR	0.99	0.97	0.93***	0.98	0.97*	0.99	0.98	0.94	0.91***
6m Tbill	0.99	0.94***	0.90***	0.98	0.92***	0.87***	0.98	0.91***	0.88***
1y Trsy	0.99	0.94***	0.91***	1.01	0.89***	0.86***	0.99	0.92***	0.89***
5y Trsy	1.01	0.97	0.91***	1.01	1.02	1.01	1.01	0.99	0.97
10y Trsy	1.01	1.00	0.97	1.01	1.00	1.09	1.00	1.00	1.02
BAA Yld	1.00	0.98	1.06	1.01	0.98	1.06	1.00	0.99	1.03

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

HYBRID VS PLUG-IN VAR W/WU-XIA RATE

Values below one indicate improvement over plug-in VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	0.99	1.00	0.86	0.98	0.96	0.89**	0.99	0.98	0.96**
Consumption	0.99	0.95**	0.82**	0.98	0.97	0.82***	0.99	0.96***	0.92***
IP	1.03*	1.01	0.78	1.03*	1.01	0.95	1.02*	1.01	0.98
Cap. Util.	1.00	0.98	0.82**	0.99	0.95	0.81**	0.99	0.97	0.86***
Unemp.	0.99	0.99	0.82***	1.01	1.01	0.81***	0.99	0.98	0.86***
Nfm Pyrlls	1.00	1.02	0.82**	1.01	0.97	0.82**	1.00	0.99	0.90***
Hours	0.99	0.99	0.92*	0.97	0.99	0.91	0.98	0.98	0.93*
H. Earnings	0.99	0.98	1.03	0.98	0.99	1.02	0.99	0.98	1.00
PPI (Fin.)	1.00	0.94**	0.90*	1.00	0.96**	0.94	1.00	0.96***	0.94
PPI (Metals)	0.98	0.96***	0.93	0.98	0.95***	0.95	0.97**	0.96***	0.98
PCE Prices	0.98	0.94**	1.01	0.98	0.96	0.99	0.97	0.95*	0.99
Hsng Strts	0.99	0.88	0.93	1.00	0.85	0.83	0.99	0.91	0.92
S&P 500	0.98**	1.01	1.11	0.98	1.00	1.00	0.99	1.01	1.01**
USD / GBP	1.00	1.00	1.00	1.01	1.03	1.06	1.00	1.00	1.01
Policy Rate	0.57*	0.96	1.13*	0.55*	1.06	1.10	0.61**	1.03	1.10
6m Tbill	0.86	0.98	0.99	0.93	0.93	0.92	0.84	0.97	1.00
1y Trsy	1.00	1.07	1.04	1.06	0.98	0.99	0.97	1.05	1.02
5y Trsy	0.94	1.05	1.08	0.88	0.97	1.08	0.90	0.94	1.02
10y Trsy	0.96	0.96	0.99	0.91	0.90	1.00	0.93	0.90	0.97
BAA Yld	0.90*	0.75*	0.78*	0.92*	0.80**	0.78*	0.91**	0.81**	0.84***

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

HYBRID VS PLUG-IN VAR W/KRIPPNER RATE

Values below one indicate improvement over plug-in VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	0.99*	1.00	0.90	0.97	0.98	0.92**	0.98	0.99	0.95***
Consumption	0.99	0.96	0.88*	0.99	0.97	0.85*	0.99	0.97**	0.93***
IP	1.00	0.98	0.95	1.02	0.98	0.93	1.00	0.97*	0.95
Cap. Util.	1.02	0.98	0.81**	1.01	0.98	0.81*	0.99	0.97	0.84***
Unemp.	0.96	1.02	0.92	0.98	1.03	0.97	0.97	1.00	0.93
Nfm Pyrlls	1.01	1.05	0.87	1.01	0.99	0.84	1.00	0.99	0.89**
Hours	0.98	1.03	0.94	0.97	1.03	0.94	0.97	1.00	0.92
H. Earnings	1.01	1.01	1.05***	0.99	1.01	1.05***	1.00	1.00	1.01
PPI (Fin.)	0.99	0.96	0.96	1.00	0.98	0.97	0.99	0.97*	0.96
PPI (Metals)	0.97	0.94**	0.94	0.98	0.92**	0.92*	0.96*	0.95***	0.95*
PCE Prices	0.98	0.95	1.04	0.99	0.96	1.03	0.99	0.96*	0.99
Hsng Strts	1.00	0.98	1.04	1.00	1.00	1.02	1.00	0.99	1.01
S&P 500	0.98	1.00	1.07	1.01	1.00	0.98	0.98	1.00	0.99
USD / GBP	1.01	1.01	1.02	1.02	1.04	1.06	1.01	1.01	1.01
Policy Rate	0.89	0.89*	0.98	1.05	0.99	1.03	1.01	0.96	1.07
6m Tbill	0.56***	0.77*	0.78*	1.13	0.94	0.90	0.61***	0.84	0.88
1y Trsy	0.69***	0.88	0.85	1.16	1.02	1.02	0.78**	0.97	0.95
5y Trsy	1.01	1.09	1.02	0.95	1.00	0.87	0.94	0.96	0.95
10y Trsy	0.98	1.01	1.03	0.97	0.99	0.89	0.96	0.93	0.92
BAA Yld	0.95	0.89	0.90**	0.97	0.95	0.85**	0.96	0.90**	0.86***

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

HYBRID VS CENSORING OF LINEAR VAR

Values below one indicate improvement over censoring of linear VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	1.00	1.01	0.70	0.99	1.00	0.98	1.00	1.01**	1.00
Consumption	0.99	0.99	0.96	0.99	1.00	0.99	0.99	1.00	0.99
IP	1.01	1.02	1.01	1.01	1.02	1.05**	1.00	1.02*	1.03**
Cap. Util.	1.01	0.98	0.92*	1.01	0.97	0.92	1.01	0.99	0.96
Unemp.	1.00	0.98	0.84***	1.00	0.97**	0.83***	1.00	0.98	0.89***
Nfm Pyrlls	1.00	0.94**	0.82***	1.00	0.94**	0.80***	1.00	0.96**	0.91***
Hours	1.02**	1.00	0.98	1.01	1.01	0.97	1.01***	1.00	0.99
H. Earnings	1.00	1.01	1.02	1.00	1.00	1.02	1.00	1.00	1.01
PPI (Fin.)	1.00	0.99	0.99	1.00	1.00	1.00	1.00	0.99	1.00
PPI (Metals)	0.99*	0.99*	1.00	1.00	1.00	1.00	1.00	1.00	1.00
PCE Prices	1.01**	1.01	1.05*	1.01	1.03***	1.07***	1.01*	1.02***	1.05***
Hsng Strts	1.00	0.92*	0.88**	1.00	0.93*	0.85***	1.00	0.94*	0.89**
S&P 500	0.99	1.01*	1.09*	0.98***	1.00	1.01	1.00	1.01	1.01***
USD / GBP	0.99**	1.00	1.00	0.99	1.00	1.01	0.99	1.00	1.01*
FFR	0.60***	0.60**	0.67**	0.38***	0.49*	0.64	0.42***	0.48**	0.67
6m Tbill	0.76***	0.79**	0.81*	0.53***	0.55**	0.68	0.57***	0.64**	0.78
1y Trsy	0.79***	0.82**	0.83*	0.60***	0.58**	0.72	0.68***	0.69**	0.80
5y Trsy	0.94	0.80**	0.68	0.95	0.89	0.91	0.97	0.89	0.90
10y Trsy	0.96	0.83*	0.66	0.96	0.84	0.88	0.97	0.90	0.95
BAA Yld	0.96	0.90	0.90	0.97	0.93	1.13	0.98	0.97	1.10

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

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- Interest rate predictions
- Additional forecast comparisons
- **SVAR responses to financial conditions shock**

Responses to Gilchrist-Zakrajsek EBP shock

- EBP added at top of VAR
- At MCMC draw m:

$$\Psi_{t,h}^{m} \equiv E_t^m \left(y_{t+h} \mid \tilde{\varepsilon}_{t+1}^{EBP} = \varepsilon_{t+1}^{EBP} + \sigma \right) - E_t^m \left(y_{t+h} \right)$$

while simulating all other shocks from baseline density

- Full-sample parameters, SV simulated at jump off

SVAR SETUP

IRF of Goncalves et al (2021/23)

Responses to Gilchrist-Zakrajsek EBP shock

- EBP added at top of VAR
- At MCMC draw m:

$$\Psi^m_{t,h} \equiv E^m_t \left(y_{t+h} \mid \tilde{\varepsilon}^{EBP}_{t+1} = \varepsilon^{EBP}_{t+1} + \sigma \right) - E^m_t \left(y_{t+h} \right)$$

while simulating all other shocks from baseline density

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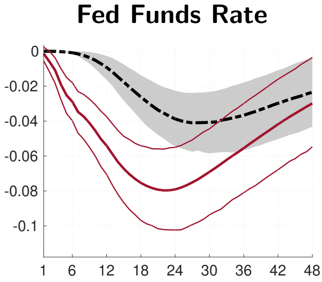
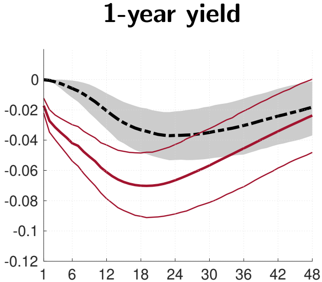
Hybrid VAR generates differences at/away from ELB

- Directly on interest rates
- Indirectly on all variables via lagged $\boldsymbol{i}_t$

IRF: 2006 vs 2012

Away from ELB (red, 2006 Dec) vs at ELB (black, 2012 Dec)





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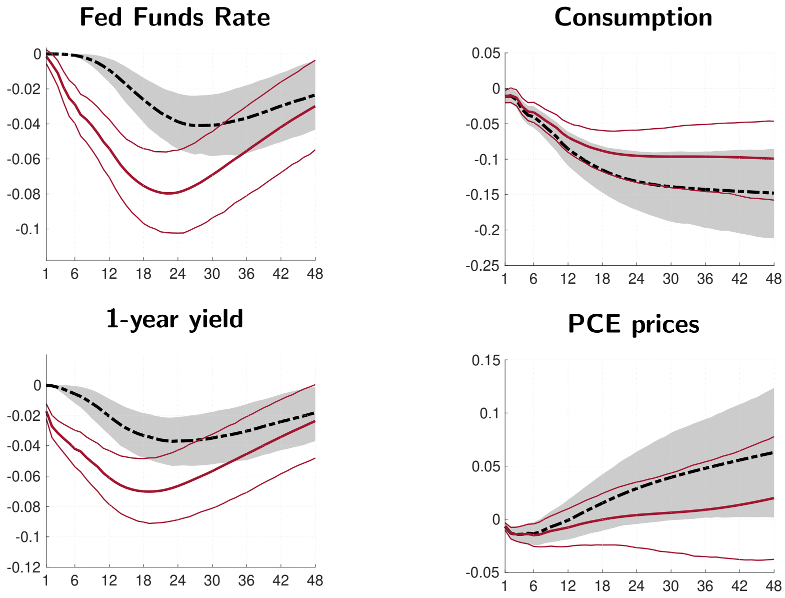

SVAR SUMMARY

In response to adverse shock to financial conditions . . .

- Similar to Gilchrist et al (2019), Arouba et al (2021)
- **Pre 2008:** (G)IRF of hybrid **similar to linear** VAR
- **Persistence** of shadow rates **prolongs stay at ELB**
- **Negative nominal-rate responses** obey the ELB, and **reflect path running below baseline** forecast
- **Macro responses affected by prolonged expectation** of actual rates at ELB
  - Differ between origins at or away from ELB
  - For given origin t: (almost) linear and symmetric

APPENDIX

- Estimation method
- Data set
- Shadow-rate sampling
- Additional shadow rate estimates
- Interest rate predictions
- Additional forecast comparisons
- SVAR responses to financial conditions shock

DETAILS ABOUT SHADOW-RATE SAMPLING

Problem: $S|Y \sim \text{truncN}\left(\mu, \Omega, S \leq ELB\right)$

- $\boldsymbol{\mu}$ and Ω implied by VAR and $\bar{Y}$
- With $\boldsymbol{T^*}$ obs at the ELB, $\boldsymbol{S}$ is large $(\boldsymbol{T^*} \cdot \boldsymbol{N_s})$

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- **. . . while exploiting Markov structure of VAR**

INFORMATION CONTENT OF BINDING ELB FOR VAR

Let's consider the following thought experiments

Let's record . . .

1. Shadow-rate draws $S|Y$, from shadow-rate VAR

Purpose:

1. This is our baseline

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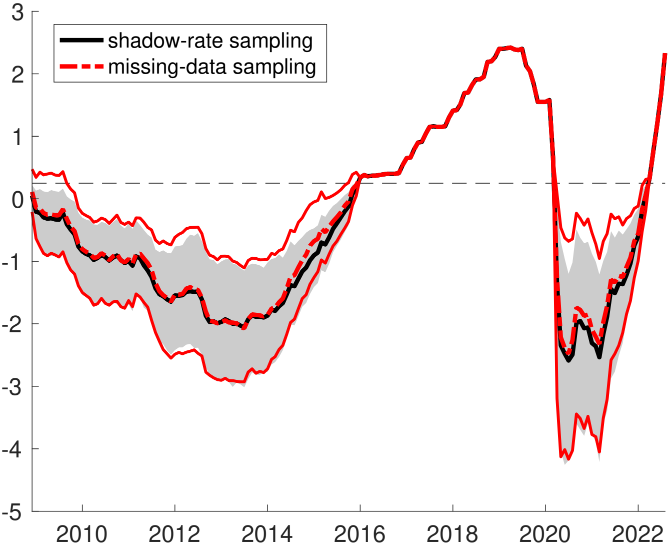
2. Missing-data draws, $\boldsymbol{S}|\bar{\boldsymbol{Y}}$, from shadow-rate VAR
3. Missing-data draws, $\boldsymbol{S}|\bar{\boldsymbol{Y}}$, from a missing-data VAR estimated on $\bar{\boldsymbol{Y}}$ rather than $\boldsymbol{Y}$ (e.g. Del Negro, et al., BPEA, 2017)

Purpose:

1. This is our baseline
2. Tells us to what extent ELB is binding for the sampler *at the parameters estimated with shadow-rate VAR*
3. Shows us if ELB sampling shifted VAR parameters

EFFECT OF CONDITIONING ON ELB

Cases 1&2: $S|Y$ (black) vs $S|\bar{Y}$ (red) with $\Pi, \Sigma_t|Y$, median/90% bnds.



Truncation appears negligible with shadow-rate parameters and SV

EFFECT OF CONDITIONING ON ELB

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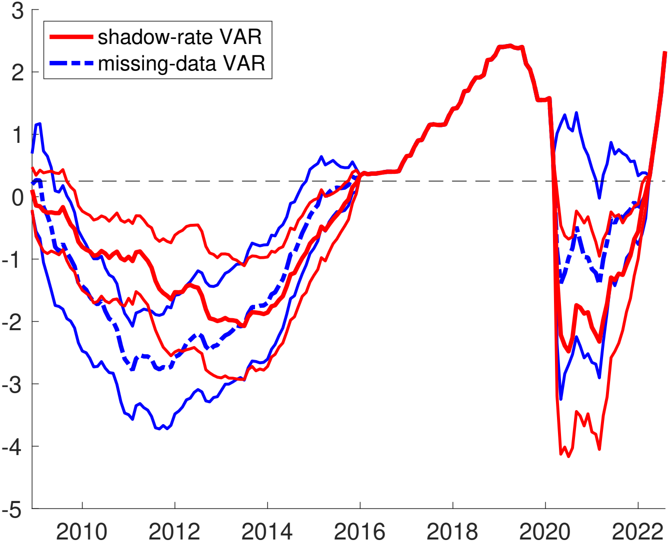
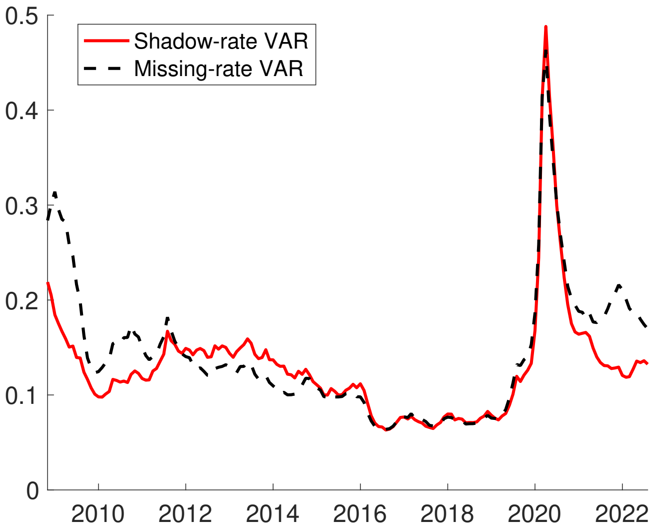


$S|\bar{Y} > ELB$ more often w/missing-data SV & parameters

EFFECTS ON SV FROM CONDITIONING ON ELB

SV of Choleski residual in shadow-rate equation of FFR



Different shadow-rate SV in 2009/2010

(with other SV paths essentially unchanged)

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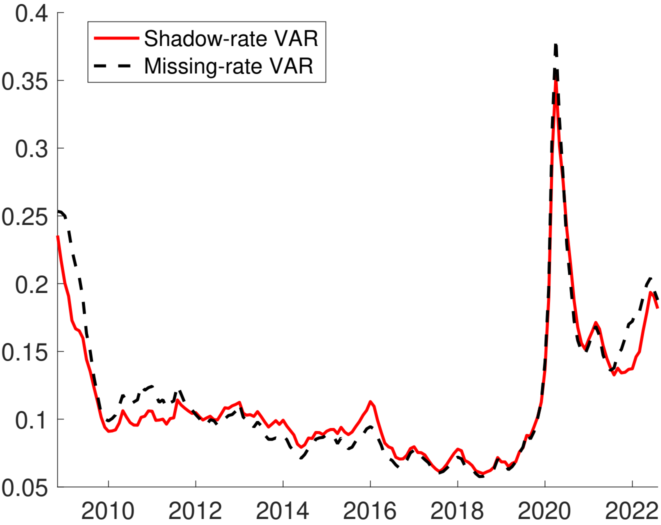

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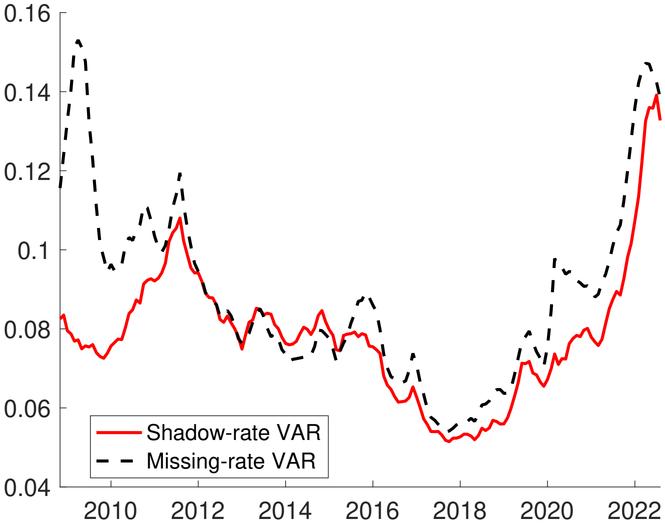

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EX YIELDS

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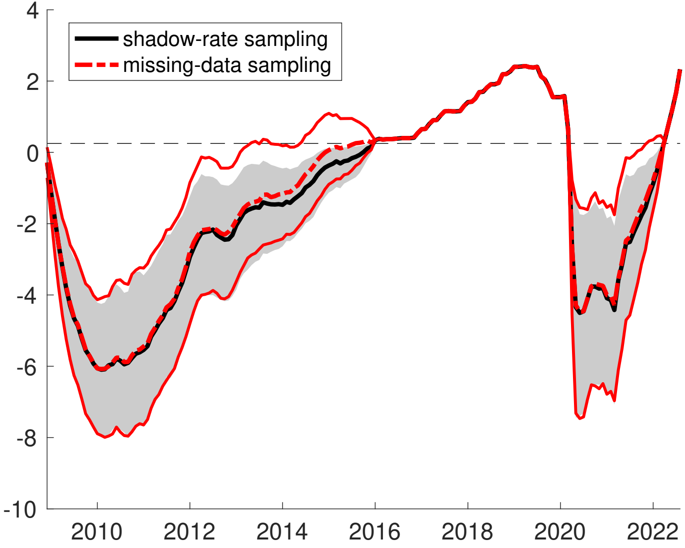

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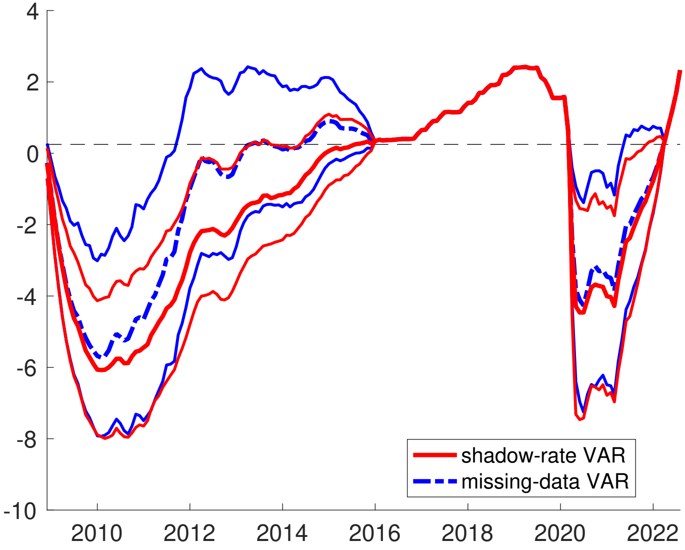

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CONCLUSIONS

Our solution to handle ELB: Shadow-rate VARs

- **Internally consistent** inference
- **Scalable** to multiple interest-rate maturities at ELB
- **Hybrid VAR** conditions macro variables on actual rates

Structural Analysis at/away from ELB:

- strong differences in nominal rate responses
- some differences in macro responses
- consistent w/some effectiveness of unconventional policies

Forecasts for interest rates and macro variables

- **Interest-rate forecasts superior** to standard VAR
- **Hybrid** typically better than simple shadow-rate VAR
- . . . and with **some gains in macro** over linear VAR