

A Theory of Stable Market Segmentations

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joint with Ron Siegel (Penn State)

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- ▶ Homeowners associations
- ▶ Student groups
- ▶ Employer-based prices

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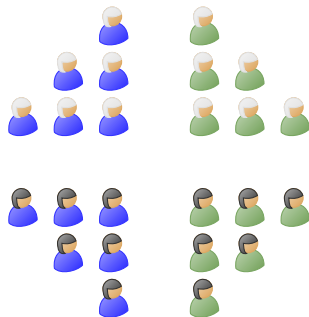
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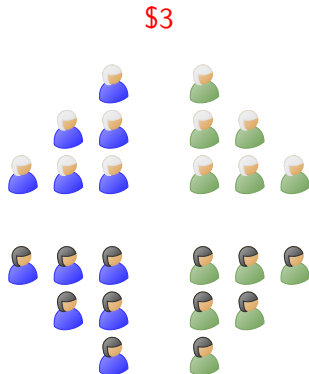


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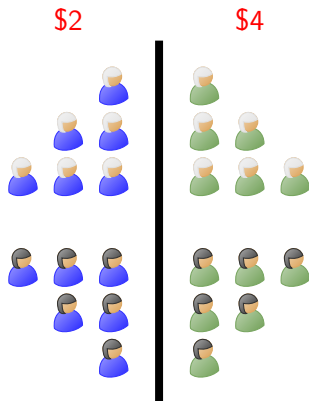


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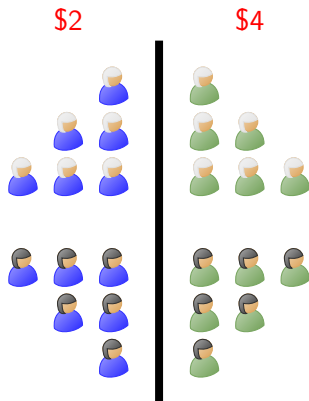
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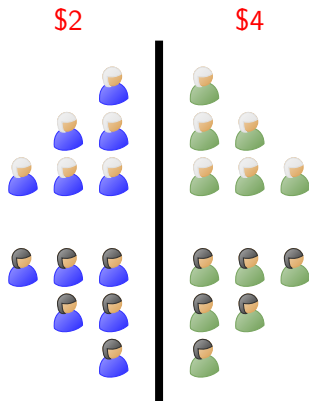
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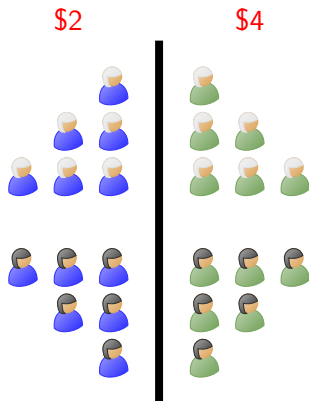
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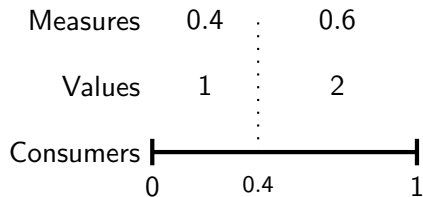
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- ▶ have good welfare properties

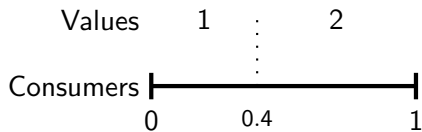


Coalitions, segments, and segmentations

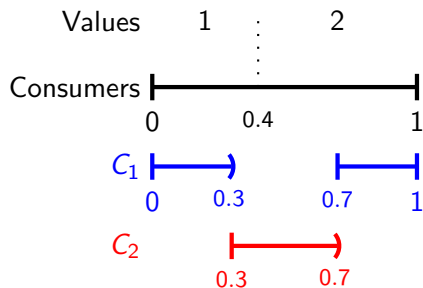
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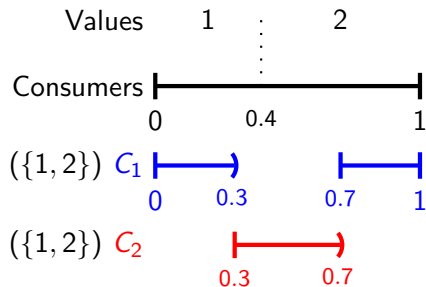
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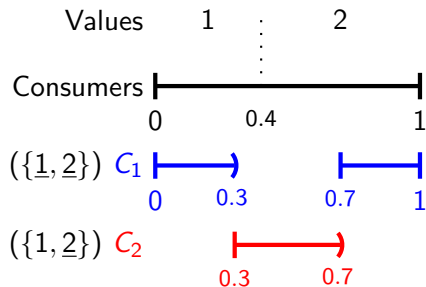
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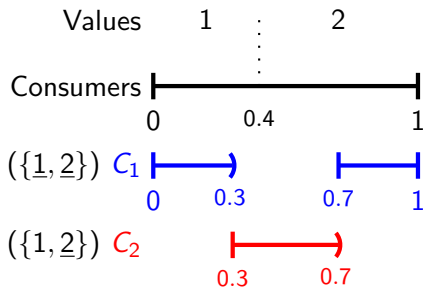


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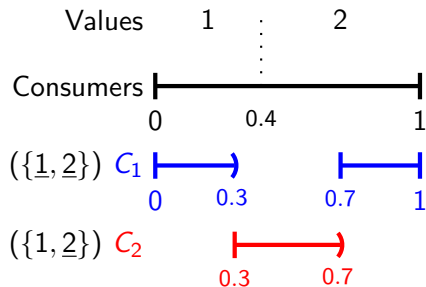
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Segmentation $S = \{(C_1, 1), (C_2, 2)\}$ s.t. coalitions partition $[0, 1]$

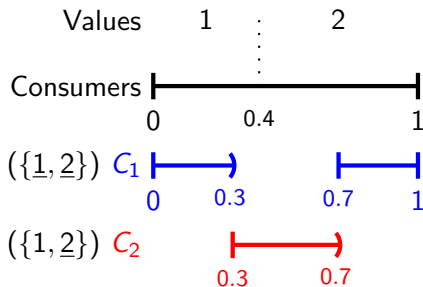


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$$CS(c, S) = \begin{cases} \max\{v(c) - 1, 0\} & \text{if } c \in C_1, \\ \max\{v(c) - 2, 0\} & \text{if } c \in C_2. \end{cases}$$



Outline

- 1 Core
- 2 Stability

The core

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A segment (C, p) objects to segmentation S if

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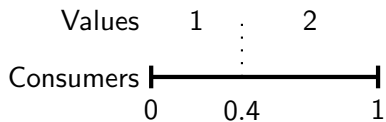
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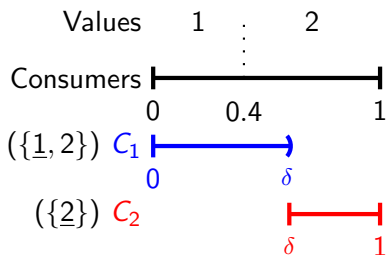
S is in the core if \nexists segment (C, p) that objects to S

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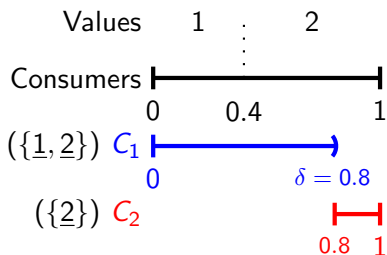
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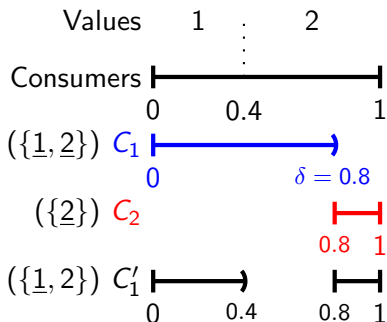


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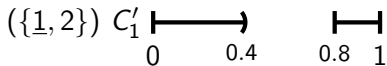
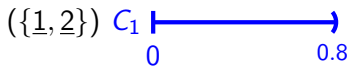
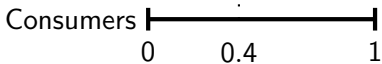
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- ▶ Segment $(C'_1, 1)$ objects

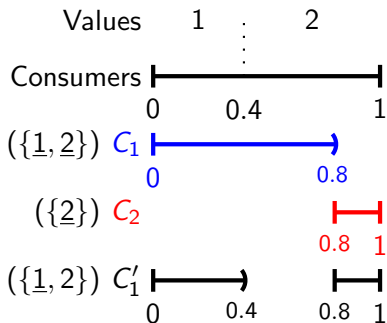


Values 1 ... 2



Definition (Weakened Core)

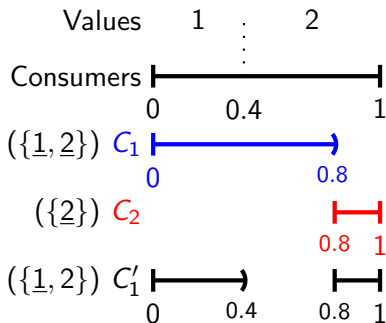
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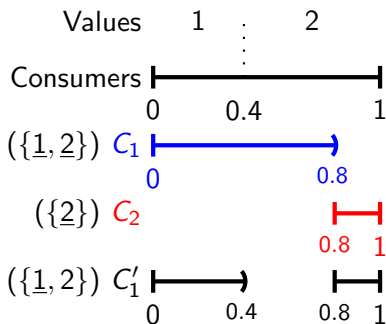
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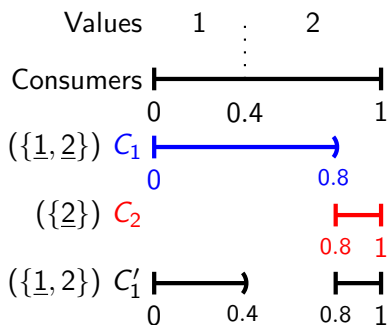
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S is in **weakened core** if \nexists segment (C', p') that **strictly objects** to S :

- 1 (C', p') objects to S
- 2 $\forall (C, p) \in S$ that is "broken apart" by C' ($C \cap C'$ and $C \setminus C'$ have positive measures), \exists consumers in $C \cap C'$ who strictly benefit ($CS(c, p') > CS(c, S)$)

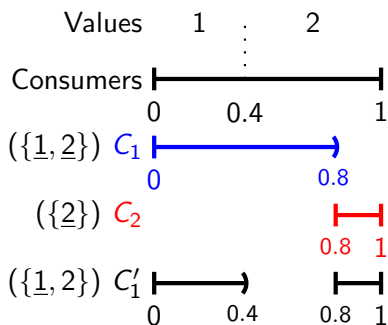


Cost-based justification



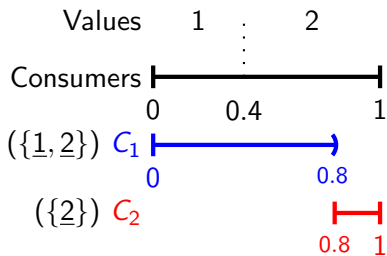
Cost-based justification

- ▶ Small fee to **break up existing coalitions**
- ▶ Must be paid by **members who want to deviate** (in $C_1 \cap C'_1$)
- ▶ If not paid, objection fails



Two type illustration and weakened core

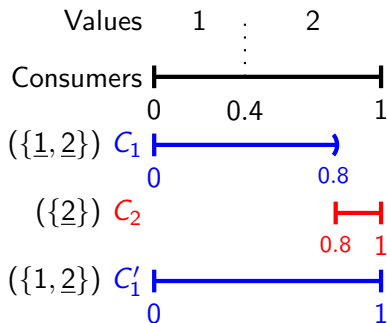
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$S = \{(C_1, 1), (C_2, 2)\}$ is in the weakened core

$S' = \{(C'_1, 2)\}$ is not in the weakened core



Characterization

Proposition

For any segmentation S , the following are equivalent

- ① *S is in the weakened core*
- ② *the induced canonical segmentation of S efficient and saturated*

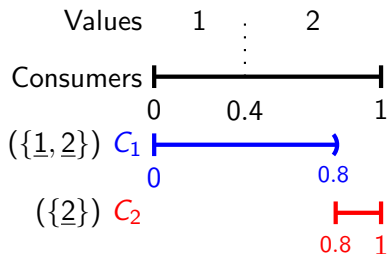
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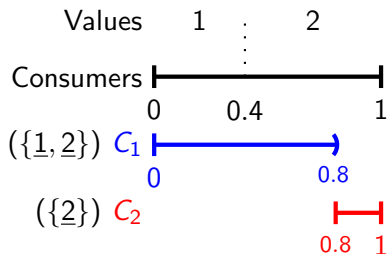
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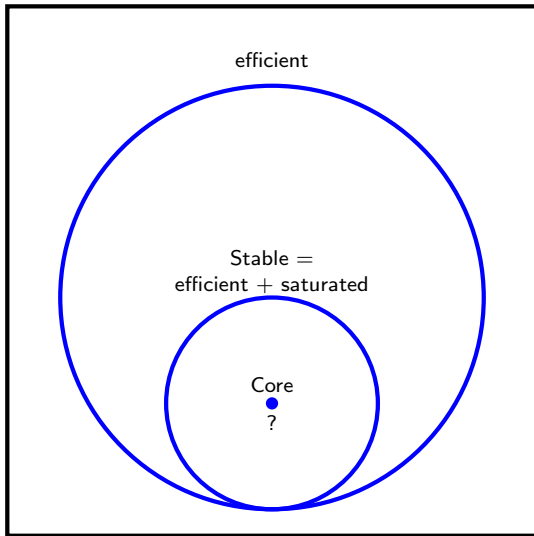
For any segmentation S , the following are equivalent

- 1 S is in the weakened core
- 2 the induced canonical segmentation of S efficient and saturated
- 3 S is "stable": $\forall S' \neq S, \exists (C, p) \in S$ that objects to S'

$$S = \{(C_1, 1), (C_2, 2)\}$$

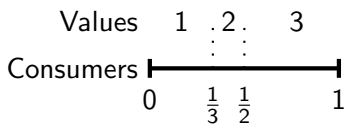


Segmentations



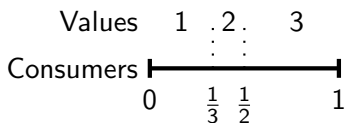
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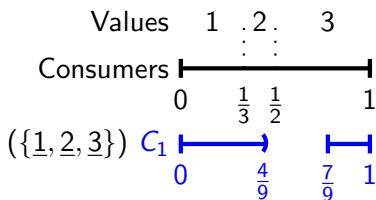
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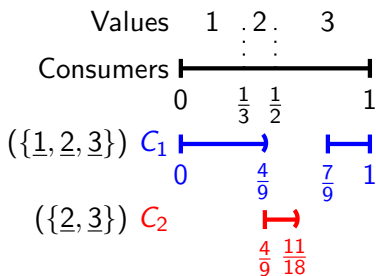
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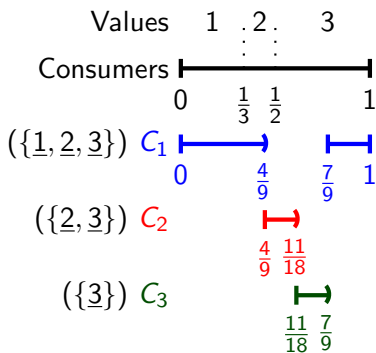
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Is defined recursively. Let $\bar{C} = [0, 1]$, $S = \emptyset$

- 1 $C :=$ largest coalition where all prices (among remaining values in \bar{C}) are revenue-maximizing
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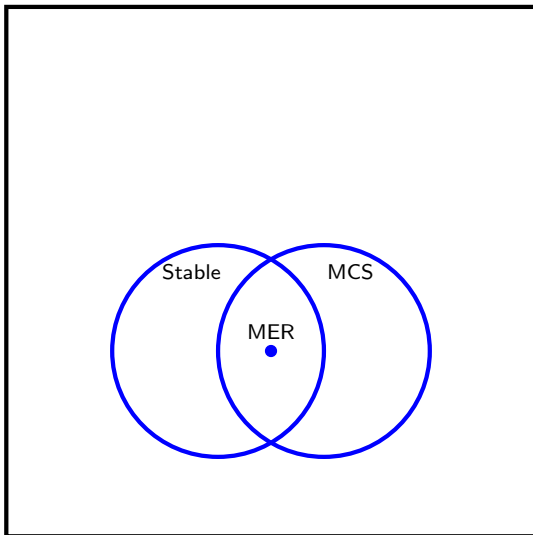
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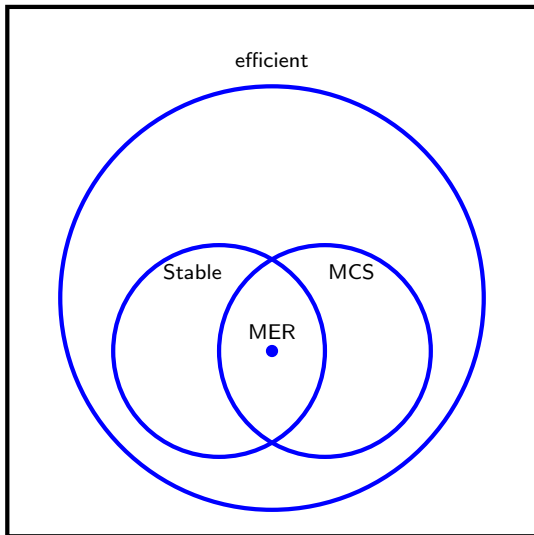
Bergemann, Brooks, Morris (2015):

- ▶ The MER segmentation maximizes consumer surplus
- ▶ But is not the only one

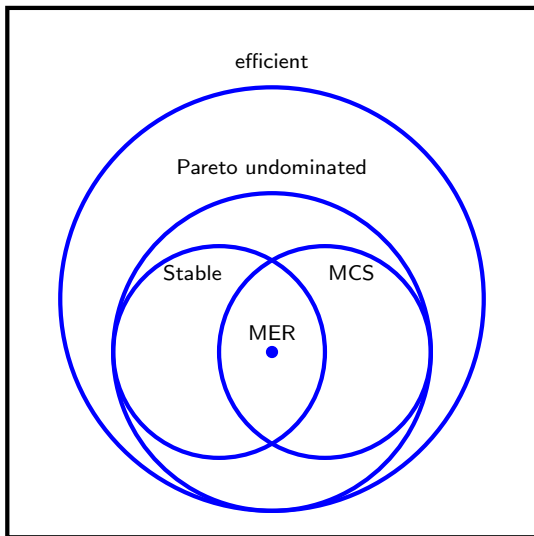
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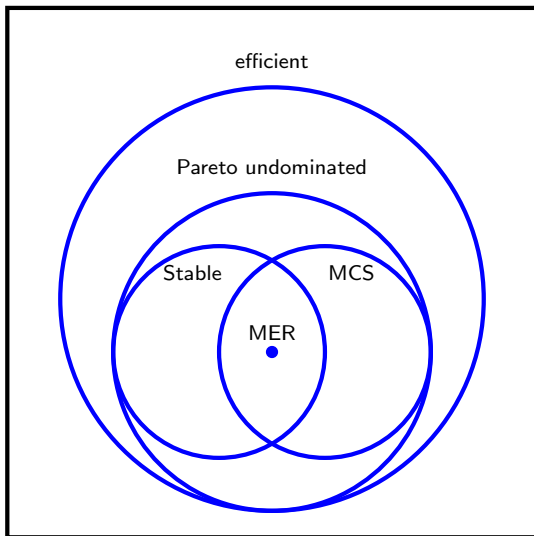
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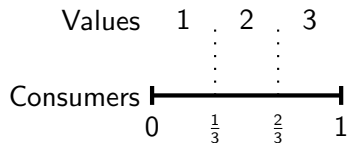
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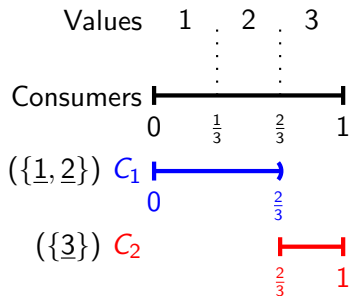
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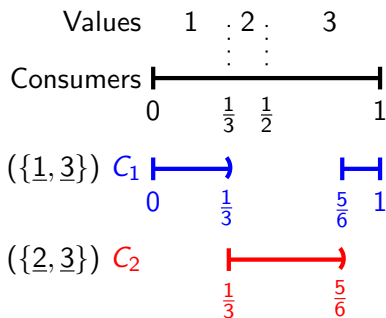
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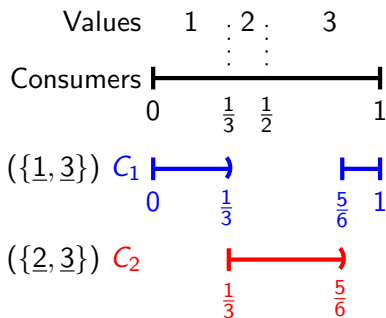
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S is not saturated and so not stable:

