Coarse Memory and Plausible Narratives

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⇒ Strategic supply of (false) **narratives** ≡ stochastic maps from policy to outcome space.

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Plausible narratives ≡ compatible with coarse memory.

- Optimism for novelty ⇒ incumbency disadvantage (Paldam, 1986) and polictical cycles.
- Limited property rights on outcomes ⇒ polarized worldviews.

↪ Case study of U.S. congress members' tweets on ACA.

Theoretical literature on **narratives in political economics**.

- Narratives as **DAGs**: Eliaz and Spiegler (2020); Eliaz, Galperti and Spiegler (2022).
- Narratives as **information structures**: Schwartzstein and Sunderam (2021); Izzo, Martin and Callander (2023); Aina (2024).

Models of **dynamic political competition**

- Policy and polarization **cycles**: Levy, Razin and Young (2022); Levy and Razin (2023).
- Bias in **retrospective voting**: Esponda and Pouzo (2017, 2018).

Partial identification in econometrics and **optimal transport**

- Identification with corrupted and **contaminated** data: Horowitz and Manski (1995).

Imperfect memory in game theory

- **Statistical memory**: Battigalli and Generoso (2023).

Optimal Narrative Design

- Two policies $\{s, t\} \ni a$ and two outcomes $\{g, b\} \ni y$.

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- Interventional distributions describe the stochastic impact of policies on outcome:

$$\text{For} \quad a \in \{s, t\} \quad a \mapsto (\mathbb{P}(y = g|a), \mathbb{P}(y = b|a)) \equiv (\mu_a^*, 1 - \mu_a^*)$$

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  - $\mu_a^* \in [0, 1] \equiv$ objective probability that g occurs when policy a is implemented: effectiveness.
- Call $\mu^* \equiv (\mu_s^*, \mu_t^*) \in [0, 1]^2$ the true model of the economy.

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  - Policy implementation $(\alpha_s, \alpha_t = 1 - \alpha_s) \in [0, 1]^2$.
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$\hookrightarrow$ Any history of policies and outcomes is perceived coarsely through (α_s, ν_g).

Correctness: The true model μ^* relates α_s and ν_g, via the law of total probabilities

$$\nu_g(\alpha_s, \mu^*) = \alpha_s \mu_s^* + (1 - \alpha_s)\mu_t^* \qquad \text{(LoE)}$$

$\hookrightarrow$ Marginals convey *some* information on true model $\rightsquigarrow$ plausibility.

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Coarse Falsification

Given $(\alpha_s, \nu_g) \in [0,1]^2$, and $\mu = (\mu_s, \mu_t) \in [0,1]^2$ the voter

1. Computes the outcome frequency $\alpha_s \mu_s + (1 - \alpha_s)\mu_t$ implied by α_s and μ.
2. Retrieves ν_g from memory.
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- Plausibility is a history-dependent predicate

$$\mathcal{M}(\alpha_s, \nu_g) = \left\{ (\mu_s, \mu_t) \in [0,1]^2 \;\middle|\; \alpha_s \mu_s + (1 - \alpha_s)\mu_t = \nu_g \right\}. \tag{PN}$$

Politician's Problem

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Attribution problem:

$$\max_{(\mu_s^S, \mu_t^S) \in [0,1]^2} \quad \mu_s^S \tag{P$_{\text{simple}}$}$$

$$\text{subj. to:} \quad \alpha_s \mu_s^S + (1-\alpha_s)\mu_t^S = \nu_g .$$

Solution:

$$\implies \hat{\mu}_s^S = \min\left\{1, \frac{\nu_g}{\alpha_s}\right\} \qquad \hat{\mu}_t^S = \max\left\{\frac{\nu_g - \alpha_s}{1-\alpha_s}, 0\right\} \tag{SolP$_{\text{simple}}$}$$

$\hookrightarrow$ $\hat{\mu}_s^S$ (resp. $\hat{\mu}_t^S$) is upper (resp. lower) identification bound for s (resp. for t).

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$$(\alpha_s, \nu_g) \begin{cases} \alpha_s \leq \nu_g & \Rightarrow \text{claim full effectiveness for oneself, concede residual to opponent.} \\ \alpha_s > \nu_g & \Rightarrow \text{take some blame for oneself, claim full ineffectiveness for opponent.} \end{cases}$$

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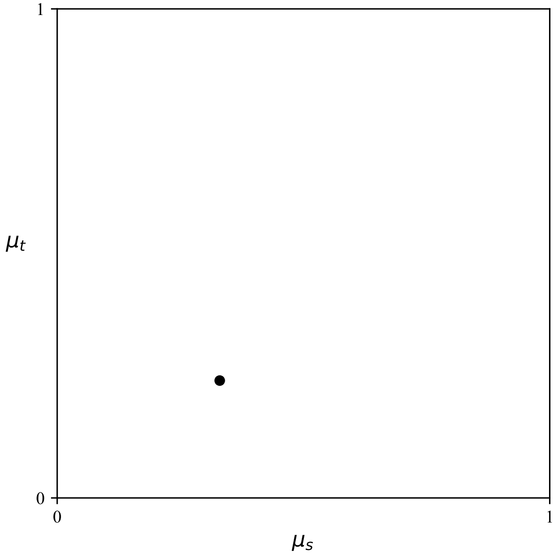
coarse memory ≈ limited "property rights"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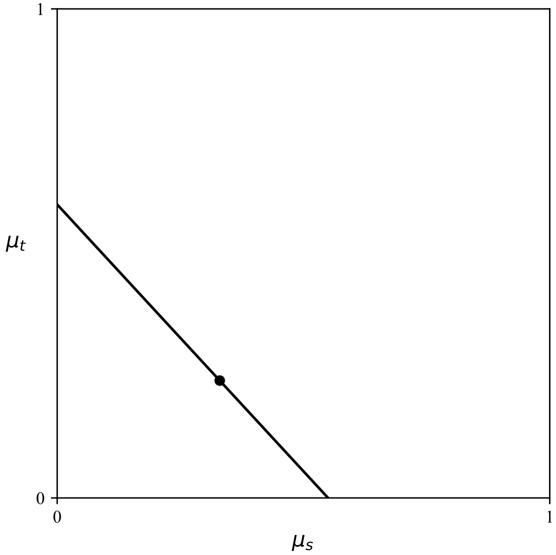
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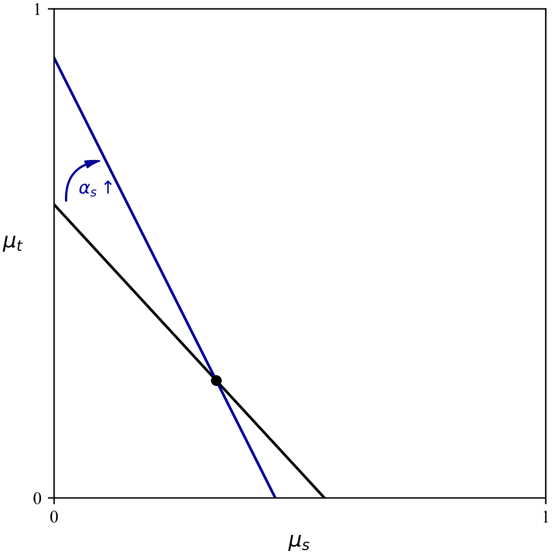
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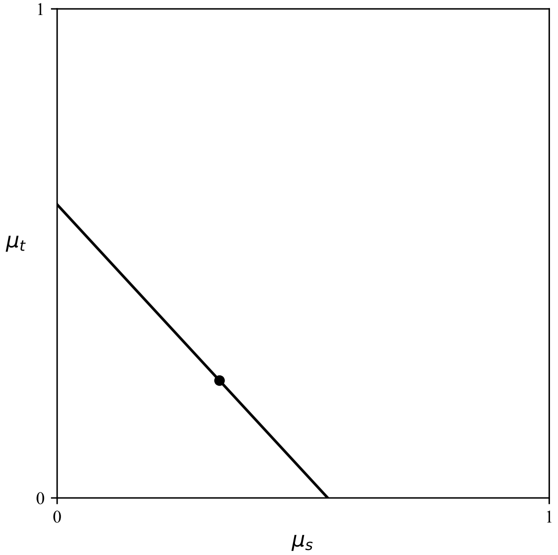
$\alpha_s \uparrow$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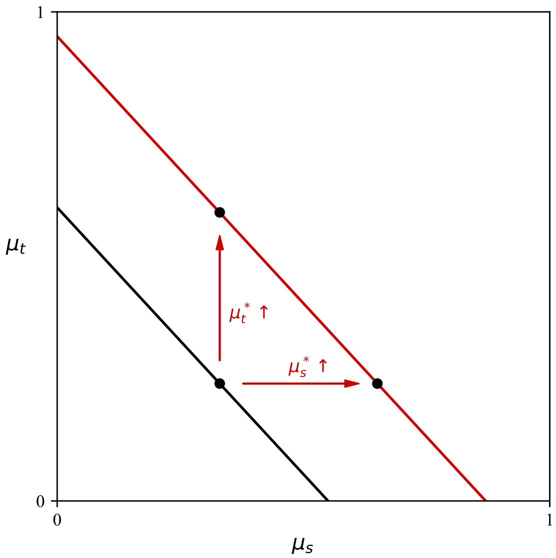
1
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$\mu_t^* \uparrow$
$\mu_s^* \uparrow$

- Policies $\mathcal{A} = \{a_1, \ldots, a_n\}$, measurable space of outcomes $\mathcal{Y} \subseteq \mathbb{R}^d$, measurable $u : \mathcal{Y} \to \mathbb{R}$.
- Models of the economy $\mu, \mu^* : \mathcal{A} \to \Delta(\mathcal{Y})$.
- Coarse memory $(\alpha, \nu) \in \Delta(\mathcal{A}) \times \Delta(\mathcal{Y})$ such that $\int_{\mathcal{A}} \mu^* d\alpha = \nu$.
- Plausible narratives $\mathcal{M}(\alpha, \nu) = \{\mu : \mathcal{A} \to \Delta(\mathcal{Y}) \mid \int_{\mathcal{A}} \mu d\alpha = \nu\}$.

$$V_a(\alpha, \nu) = \max_{\mu:\mathcal{A}\to\Delta(\mathbb{R})} \quad E_{\mu(a)}[u(Y)] \qquad (\mathrm{P}_{\mathrm{general}})$$

$$\text{subj. to:} \quad \mu \in \mathcal{M}(\alpha, \nu)$$

- An Optimal Transport problem for (supermodular) surplus $\Phi(a', y) = \frac{u(y)}{\alpha(a)} \mathbf{1}_{a'=a}$.
- If u strictly increasing, Partial Identification problem (Manski and Horowitz, 1995).

Concentrate $\mu(a)$ on the "best" **superset** of u allowed by plausibility.

$\hookrightarrow$ Application of the **Bathtub Principle** from measure theory.

Theorem 1

In any optimal narrative $\hat{\mu} \in \mathcal{M}(\alpha, \nu)$, $\hat{\mu}(a)$ has the following density with respect to ν

$$\frac{d\hat{\mu}(a)}{d\nu} = \frac{1}{\alpha(a)}[\mathbb{1}_{u(y)>\hat{u}} + c\mathbb{1}_{u(y)=\hat{u}}] \quad \text{where} \quad \begin{cases} \hat{u} = \inf\{r \mid \nu(\{y \mid u(y) > r\}) \leq \alpha(a)\} \\ c\nu(\{y|u(y) = \hat{u}\}) = \alpha(a) - \nu(\{y|u(y) > \hat{u}\}) \end{cases}$$

The value of the problem is

$$V_a(\alpha, \nu) = \frac{1}{\alpha(a)} \int_{\{y|\ u(y)\geq\hat{u}\}} u(y)d\nu(y) = \mathbb{E}_\nu[u|u \geq \hat{u}]$$

- Proof Intuition
- **Sufficient representation**, pin down $\mu(\neg a) = \frac{1}{1-\alpha(a)} \sum_{a'} \alpha(a')\mu(a')$ Corollary 1
- Comparative statics generalizes via **majorization** orders.

Narratives and Electoral Competition

Competition Game

- Two politicians S and T committed respectively to policies s and t.
- True model of the economy $\mu^* : \{s, t\} \to \Delta(\mathcal{Y})$.

[1] If none plausible, breaks tie at random. If only one plausible, votes for proponent.

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- Voter V with memory $(\alpha \equiv \alpha_S, \nu)$ and utility u, receives μ^S, μ^T and tests for their plausibility.
- If both plausible[1] elects S if and only if

$$E_{\mu^S(s)}[u(Y)] \geq E_{\mu^T(t)}[u(Y)] + \phi \qquad \phi \sim \mathcal{U}\left(\left[-\frac{1}{2\zeta}, \frac{1}{2\zeta}\right]\right), \ \zeta > 0,$$

and for T otherwise.

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Candidate A announces the solution to $V_a(\alpha, \nu)$ in equilibrium.

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- Kolmogorov-Smirnov metric on $\Delta(\mathbb{R})$, $d^{KS}(\lambda, \lambda') = \sup_y |F_\lambda(y) - F_{\lambda'}(y)|$.
- Define distance between narratives

$$d^{\mathcal{M}}(\mu, \mu') = \frac{1}{2}[d^{KS}(\mu(s), \mu'(s)) + d^{KS}(\mu(t), \mu'(t))].$$

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Proposition 5

Let u be strictly monotone. Fix any $\alpha \in \Delta(\mathcal{A})$.

The equilibrium narratives $(\hat{\mu}^S, \hat{\mu}^T)$ maximise $d^{\mathcal{M}}(\mu, \mu')$ over $\mathcal{M}(\alpha, \nu(\alpha, \mu^*))$.

Moreover, for any continuous μ^*, $d^{\mathcal{M}}(\hat{\mu}^S, \hat{\mu}^T)(\alpha_s)$ is maximised at $\alpha_s = \frac{1}{2}$.

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Intuition:

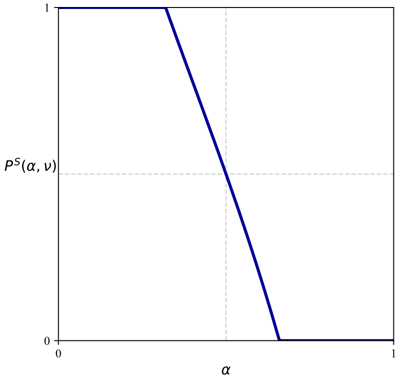
- Optimal competing narratives move mass in opposite directions.
- When memory is balanced, politicians can disagree on both policies.
  - $\hookrightarrow$ As α_s departs from $\frac{1}{2}$ they are forced to increasingly agree on the most implemented one.

Define the narrative advantage

$$\delta(\alpha,\nu) = V_s(\alpha,\nu) - V_t(\alpha,\nu) = \mathbb{E}_\nu[u|u \geq \hat{u}^S] - \mathbb{E}_\nu[u|u \geq \hat{u}^T].$$

$\Rightarrow$ Probability that S wins at (α,ν) is $P^S(\alpha,\nu) = F_\phi(\delta(\alpha,\nu))$.

- $P^S(\alpha_s)$ is **decreasing**.
  Implementation reduces success.
- $P^S(\alpha_s)$ has a **fixed point** at $\alpha_s = 1/2$.
  Narrative advantage independent from quality.
- $P^S(\alpha_s)$ is **determinstic** for extreme α_s.
  (If ϕ sufficiently concentrated).



Myopic Dynamics

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5. The voter's memory tracks time-averages

$$\begin{cases} \alpha_s^{\tau+1} & = \frac{\tau+1}{\tau+2}\alpha_s^\tau + \frac{1}{\tau+2}w^\tau \\ \nu^{\tau+1} & = \alpha_s^{\tau+1}\mu^*(s) + (1-\alpha_s^{\tau+1})\mu^*(t) \end{cases}$$

Dynamics is the realized path an SDS. Fixed initial condition $\alpha_0 \in [0,1]$

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$\Rightarrow$ Both S and T win infinitely often, same asymptotic frequency, recurrence times increase.

$\Rightarrow$ System trapped in state maximizing polarization and minimizing grip of plausibility.

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$$\begin{cases} W^\tau \sim Bern(P(\alpha_S^\tau)). \\ \alpha_S^{\tau+1} = \frac{\tau+1}{\tau+2}\alpha_S^\tau + \frac{1}{\tau+2}W^\tau \end{cases}$$

Theorem 2

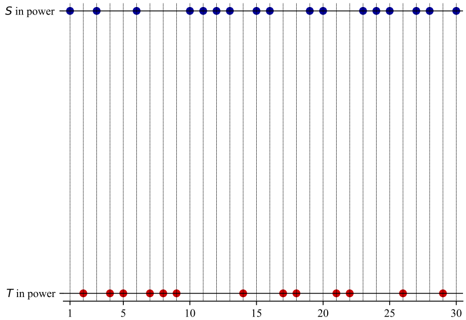
For any μ^*, it holds that $\alpha_S^\tau \xrightarrow{p} \frac{1}{2}$.

⇒ Both S and T win infinitely often, same asymptotic frequency, recurrence times increase.

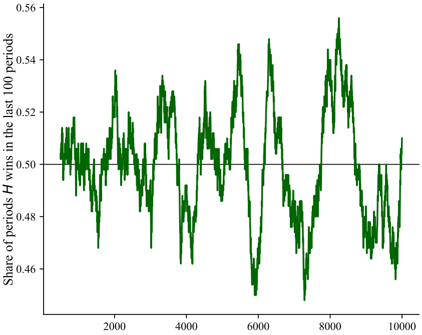
⇒ System trapped in state maximizing polarization and minimizing grip of plausibility.

- Proof based on **Doob's Optional Stopping Theorem**.
- Intuition: incumbency disadvantage.
  - When A in power: $\alpha(a)\uparrow$ and $\alpha(\neg a)\downarrow$
  - Set of plausible $\mu(a)$ shrinks around $\mu^*(a)$ and set of plausible $\mu(\neg a)$ inflates away from $\mu^*(\neg a)$.

Qualitative result is robust to alternative laws of motion Propositions A6-A7



S in power
T in power
1
5
10
15
20
25
30

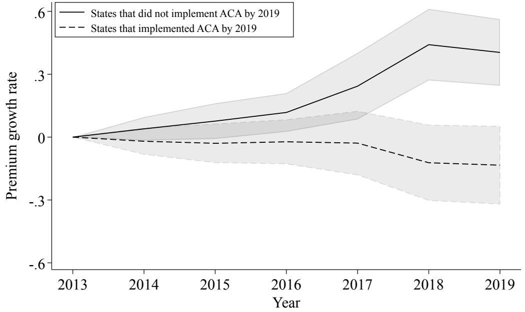


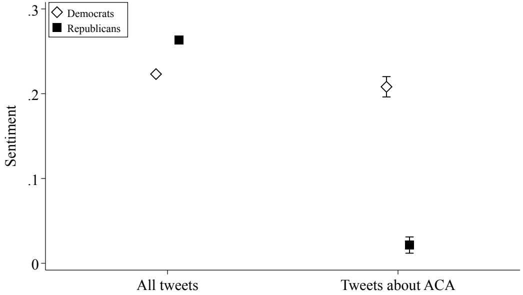
Share of periods H wins in the last 100 periods
0.56
0.54
0.52
0.50
0.48
0.46
2000
4000
6000
8000
10000

Supporting Evidence

Data ≈ 1.6M tweets by congress memebers during 2012-2019 + ACA diffusion data.

Methods Event study based on dictionary methods & VADER classification.



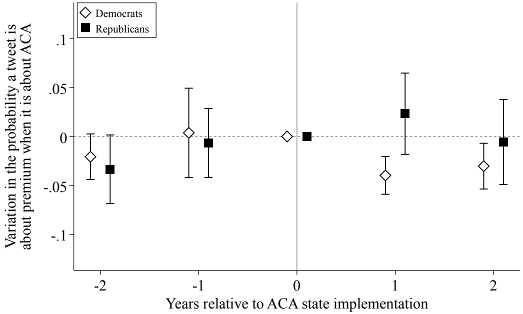


Why ACA ?

- Salient in our period of interest: heated debate, polarized sentiment. Sample of Tweets
- Staggered implementation across U.S. states. Figures
- Desirable but delayed effect: insurance premia increase less in ACA states.

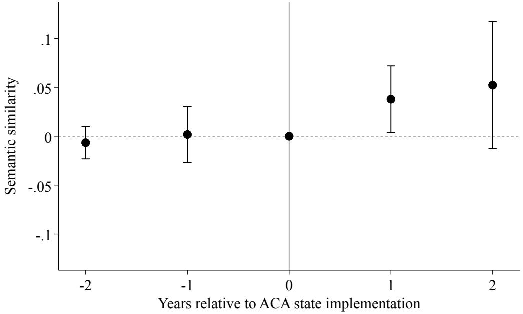
(i) Dems cannot claim credit for success, while Reps keep blaming ACA.

- Focus on pbty of "*premium*" given "*ACA*".
- Driven by states where ACA is *more* effective.
- D's and R's keep tweeting ACA/premium.
  - ↪ Effect is not mechanical.



(ii) Dems and Reps forced to reduce their disagreement throughout staggered implementation.

- SBERT for tweets embedding vectors
- Quantify distance through cosine similarity.
  - Avg similarity bw politician and other group.



Thanks for your attention!

Link to the Paper

Appendix

Intuition for Strictly Monotone Utility

$$\mathcal{M}(\alpha,\mu^*) = \left\{ \mu : \mathcal{A} \to \Delta(\mathcal{Y}) \;\middle|\; \forall y \in \mathcal{Y}\; \underbrace{F_\mu(y|\neg a) - F_{\mu^*}(y|\neg a)}_{\Delta_{\neg a}} = -\overbrace{\left(\frac{\alpha(a)}{1-\alpha(a)}\right)}^{\text{price}(a)} \left[\underbrace{F_\mu(y|a) - F_{\mu^*}(y|a)}_{\Delta_a}\right] \right\}$$

(a) u strictly increasing $\Rightarrow$ outcome ranking is isomorphic to $\mathbb{R}$

(b) Feasibility $\mathcal{M}(\alpha,\mu^*)$ $\Rightarrow$ cost of improving on $F_{\mu^*}(a)$, $\frac{\alpha(a)}{1-\alpha(a)}$, is constant across y

$\Rightarrow$ Try to **concentrate** μ on outcomes as high as possible.

$\hookrightarrow$ Set $F_\mu(y|a) = 0$ until plausible, while $F_\mu(y|a') = 1$ for every $a' \neq a$ as early as plausible.

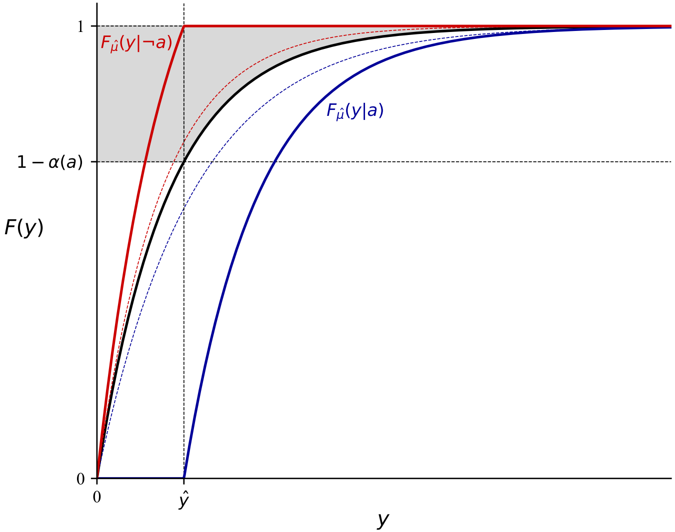
$\Rightarrow$ **Outcome threshold** $\hat{y} = F_\nu^{-1}(1-\alpha(a))$

$\hookrightarrow$ Top-$\alpha(a)$ quant. attributed to a, bottom-$1-\alpha(a)$ quant. to $\neg a$.

$\Rightarrow$ Check that the construction work: $F_\mu(y|a)$ **FOSD** any other plausible $F_{\mu'}(y|a)$

- **Value** $\equiv \mathbb{E}[u(Y)|\underbrace{Y \geq F_\nu^{-1}(1-\alpha(a))}_{Y\text{gets only top}-\alpha(a)\text{ quantiles}}]$

The FOSD approach yields self contained result in the monotone case Proposition 1


1
$F_{\hat{\mu}}(y|\neg a)$
$F_{\hat{\mu}}(y|a)$
$1-\alpha(a)$
$F(y)$
0
0
$\hat{y}$
y

Corollary 1

In any optimal narrative $\hat{\mu}$, $\hat{\mu}(\neg a)$ has density

$$\frac{d\hat{\mu}(\neg a)}{d\nu} = \frac{1}{1-\alpha(a)}[\mathbb{1}_{u(y)<\hat{u}} - c\mathbb{1}_{u(y)=\hat{u}}]$$

Hence, any optimal narrative $\hat{\mu}$ induces the same $(\hat{\mu}(a), \hat{\mu}(\neg a))$, which we call a sufficient representation.

Back

Strictly increasing $u : \mathcal{Y} \subseteq \mathbb{R} \to \mathbb{R}$

$$V_a(\alpha, \nu) = \max_{\mu:\mathcal{A}\to\Delta(\mathbb{R})} \quad E_{\mu(a)}[u(Y)] \qquad \text{subj. to:} \quad \mu \in \mathcal{M}(\alpha, \nu) \tag{P_{general}}$$

Proposition 1

In the case where u is increasing, the optimal narrative is determined by the following CDFs:

$$\begin{cases} F_{\hat{\mu}(a)} & = \max\left\{\frac{F_\nu - (1-\alpha(a))}{\alpha(a)}, 0\right\} = \frac{F_\nu(y)-(1-\alpha(a))}{\alpha(a)} \mathbf{1}_{y \geq \hat{y}} \\ F_{\hat{\mu}(\neg a)} & = \min\left\{\frac{F_\nu}{1-\alpha(a)}, 1\right\} = \frac{F_\nu(y)}{1-\alpha(a)} \mathbf{1}_{y \leq \hat{y}} + \mathbf{1}_{y > \hat{y}} \end{cases}$$

where $\hat{y} = F_\nu^{-1}(1 - \alpha(a))$.

Back

- Given $\alpha, \alpha' \in \Delta(\mathcal{A})$ say that α' a-majorizes α if

$$\alpha'(a) \geq \alpha(a) \text{ and } \forall a' \neq a \; \alpha'(a') \leq \alpha(a')$$

- Given $\mu^*, \mu^{*\prime} : \mathcal{A} \to \Delta(\mathcal{Y})$ say that $\mu^{*\prime}$ is weakly more productive than μ^* if

$$\forall a \in \mathcal{A}, \; r \in \mathbb{R} \; \mu^{*\prime}(a)(S(r)) \geq \mu^*(a)(S(r))$$

where $S(r) = \{y | u(y) \geq r\}$ is the superset of u of height r.

Proposition 2

Fix any $a \in \mathcal{A}$. The following comparative statics holds:

1. Fix μ^*. If α' a-majorizes α then $V_a(\alpha', \nu(\alpha, \mu^*)) \leq V_a(\alpha, \nu(\alpha, \mu^*))$
2. Fix α. If $\mu^{*'}$ is weakly more productive than μ^* then $V_a(\alpha, \nu(\alpha, \mu^{*'})) \geq V_a(\alpha, \nu(\alpha, \mu^*))$

Consider $(\kappa^\tau)_{\tau \in \mathbb{N}} \subseteq (0,1)$ such that $\kappa^\tau \uparrow \kappa^\infty \in [0,1]$. Let:

$$\begin{cases} \alpha_h^{\tau+1} & = \kappa^\tau \alpha_s^\tau + (1-\kappa^\tau) w^\tau \\ \nu^{\tau+1} & = \alpha_h^{\tau+1} \mu^*(s) + (1-\alpha_s^{\tau+1}) \mu^*(t) \end{cases}$$

Proposition A6

If $\zeta > \max\left\{\frac{1}{2|\underline{\delta}|}, \frac{1}{2||\overline{\delta}|}\right\}$, α_h^τ is asymptotically bound in

$$\left[\kappa^\infty \delta^{-1}\left(\frac{1}{2\zeta}\right), \kappa^\infty \delta^{-1}\left(-\frac{1}{2\zeta}\right) + (1-\kappa^\infty)\right] \subset [0,1]$$

Hence both candidates win infinitely often.

If, for all τ, $\kappa^{\tau} = \kappa$ we have a Markov chain.

Proposition A7

The Markov chain for voter's memory has a unique ergodic stationary distribution $\tilde{\pi} \in \Delta([0,1])$.

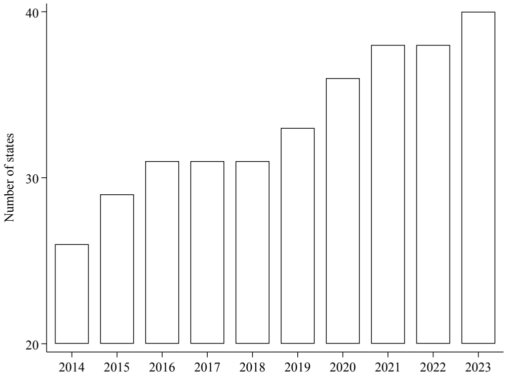
Moreover, it holds

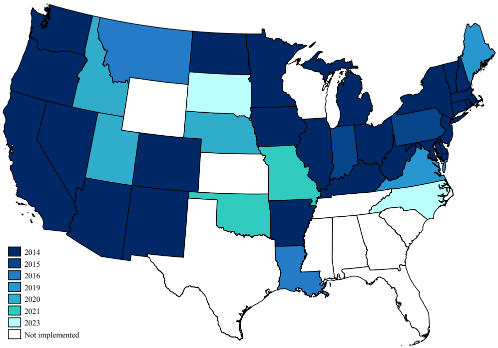
$$\mathbb{P}\left(\tilde{W} = \mathbb{E}_{\tilde{\pi}}[P^H(\alpha)]\right) = 1 \tag{1}$$

In simulations $\mathbb{E}_{\tilde{\pi}}[P^H(\alpha)] = \frac{1}{2}$.

Back

ACA's Implementation across U.S. States





Back

Democrats

- Great news about 7 of 9 health insurers who participate in the Obamacare market in Michigan reducing their premiums for next year. We'll keep working to make health care and prescription drugs universally affordable. #ForThePeople
- Without the ACA's protections for pre-existing conditions, insurance companies will again be able to deny coverage or charge higher premiums for things like high blood pressure, mental illness, or being a woman.
- The ACA prevented insurers from raising premiums of Americans with pre-existing conditions. #GrahamCassidy would end that protection.

Republicans

- Statement on today's news of massive health insurance premium hikes in Indiana under Obamacare.#INSen https://t.co/QiHwFbHccu
- ObamaCare is causing more premium increases – perhaps as much as 20%. This is not reasonable: http://t.co/lqyYQxoDHK #LASEN
- Obamacare = higher premiums for plans Americans don't want or need. #ObamacareRepeal efforts must continue. https://t.co/YfdIclfTG8

Back