Mechanism Design with Costly Inspection

Amirreza Ahmadzadeh

TSE

Stephan Waizmann

Yale University

Tuesday 27th August, 2024

Motivation

US Defense Contract Audit Agency conducts inspections which

are generally completed before contract award where DCAA evaluates [...] how much it will cost the contractor to provide goods or services to the government.

Research Question

1. What is the optimal combination of screening menus (**quantities** and **transfers**) and **inspection**?
2. How does ability to inspect affect procured quantity?

Contribution

Combine literature on CSV and monopolistic screening.

Study trade-off between quantity distortions and inspection costs.

Contribution

Combine literature on CSV and monopolistic screening.

Study trade-off between quantity distortions and inspection costs.

Methodological Contribution:

- Incentive constraints do not bind locally in any optimal mechanism.
- Analytically characterize *which* incentive constraints bind.

Overview of Results

1. Incentives to the producer are provided only through inspection and bonus payments when the producer has reported his cost truthfully.
2. The firm produces the efficient quantity if his cost is low enough (even if not inspected).
3. Quantity procured from produces with higher costs is inefficiently low.

Model: Players and Mechanism

- Two players: principal ("she"), agent ("he").
- Agent's cost $\theta \in [\underline{\theta}, \overline{\theta}], 0 < \underline{\theta} < \overline{\theta}$, is his private information.
- Principal's belief over cost given by cdf F with density $f > 0$.
- In case of inspection: principal perfectly observes θ.
- Principal commits to mechanism based on report $\hat{\theta}$ and cost θ,

$$(x(\hat{\theta}), q^N(\hat{\theta}), t^N(\hat{\theta}), q^I(\hat{\theta}, \theta), t^I(\hat{\theta}, \theta))$$

Model: Players and Mechanism

- Two players: principal ("she"), agent ("he").
- Agent's cost $\theta \in [\underline{\theta}, \overline{\theta}], 0 < \underline{\theta} < \overline{\theta}$, is his private information.
- Principal's belief over cost given by cdf F with density $f > 0$.
- In case of inspection: principal perfectly observes θ.
- Principal commits to mechanism based on report $\hat{\theta}$ and cost θ,

$$(x(\underset{\text{reported cost}}{\hat{\theta}}), q^N(\hat{\theta}), t^N(\hat{\theta}), q^I(\hat{\theta}, \underset{\text{true cost}}{\theta}), t^I(\hat{\theta}, \theta))$$

Model: Players and Mechanism

- Two players: principal ("she"), agent ("he").
- Agent's cost $\theta \in [\underline{\theta}, \overline{\theta}], 0 < \underline{\theta} < \overline{\theta}$, is his private information.
- Principal's belief over cost given by cdf F with density $f > 0$.
- In case of inspection: principal perfectly observes θ.
- Principal commits to mechanism based on report $\hat{\theta}$ and cost θ,

$$(\underset{\substack{\downarrow \\ \text{proba inspection}}}{x}(\hat{\theta}), \overbrace{q^N(\hat{\theta}), t^N(\hat{\theta})}^{\substack{\text{quantity/transfer} \\ \text{w/o inspection}}}, \underbrace{q^I(\hat{\theta}, \theta), t^I(\hat{\theta}, \theta)}_{\substack{\text{quantity/transfer} \\ \text{w/ inspection}}})$$

Model: Payoffs

Utility of cost θ of the agent from quantity q and transfer t is

$$-q\theta + t.$$

Model: Payoffs

Utility of cost θ of the agent from quantity q and transfer t is

$$-q\theta + t.$$

Utility of the principal is

$$V(q) - t - \kappa \mathbb{1}_{\text{inspection}}$$

$\kappa > 0$, V is twice continuously differentiable, strictly increasing and concave, Inada conditions: $V'(q) \to_{q \searrow 0} \infty$ and $V'(q) \to_{q \to \infty} 0$.

Model: Payoffs

Agent can reject mechanism ex-post:

Optimal mechanism must satisfy, for all reports $\hat{\theta}$ and costs θ,

$$-q^N(\hat{\theta})\hat{\theta} + t^N(\hat{\theta}) \geq 0,$$
$$-q^I(\hat{\theta}, \theta)\theta + t^I(\hat{\theta}, \theta) \geq 0.$$

Model: Payoffs

Agent can reject mechanism ex-post:

Optimal mechanism must satisfy, for all reports $\hat{\theta}$ and costs θ,

$$-q^N(\hat{\theta})\hat{\theta} + t^N(\hat{\theta}) \geq 0,$$
$$-q^I(\hat{\theta}, \theta)\theta + t^I(\hat{\theta}, \theta) \geq 0.$$

Minimal inspection: for some $\underline{x} > 0$,

$$x(\theta) \geq \underline{x}.$$

Principal's problem

Maximize

$$\left\{ \text{Expected value of quantity - cost of transfer - cost of inspection} \right\}.$$

Principal's problem

Maximize

$$\left\{ \text{Expected value of quantity - cost of transfer - cost of inspection} \right\}.$$

Over

- **inspection probability**.
- **quantity w/o** inspection.
- **transfer w/o** inspection.

Principal's problem

Maximize

$$\Big\{ \text{Expected value of quantity - cost of transfer - cost of inspection} \Big\}.$$

Over

- **inspection probability**.
- **quantity w/o** inspection.
- **transfer w/o** inspection.

Subject to:

- truth-telling,
- agent does not reject mechanism ex-post.

Principal's problem

Maximize over $\left(x(\cdot), q^I(\cdot,\cdot), q^N(\cdot), t^I(\cdot,\cdot), t^N(\cdot)\right)$

$$\int_{\underline{\theta}}^{\overline{\theta}} x(\theta)\left(V(q^I(\theta,\theta)) - t^I(\theta,\theta) - \kappa\right) + (1 - x(\theta))\left(V(q^N(\theta)) - t^N(\theta)\right) \mathrm{d}F(\theta).$$

Subject to, for all $\theta, \hat{\theta}$

$$x(\theta)\left(-q^I(\theta,\theta)\theta + t^I(\theta,\theta)\right) + (1 - x(\theta))\left(-q^N(\theta)\theta + t^N(\theta)\right) \tag{IC}$$

$$\geq x(\hat{\theta})\left(-q^I(\hat{\theta},\theta)\theta + t^I(\hat{\theta},\theta)\right) + (1 - x(\hat{\theta}))\left(-q^N(\hat{\theta})\theta + t^N(\hat{\theta})\right),$$

$$-q^N(\theta)\theta + t^N(\theta) \geq 0,$$

$$-q^I(\hat{\theta},\theta)\theta + t^I(\hat{\theta},\theta) \geq 0,$$

$$\underline{x} \leq x(\theta) \leq 1.$$

Related Literature

Classic: Townsend (1979), Diamond (1984), Gale & Hellwig (1985).

Deterministic inspection, and no quantity.

Monopoly Regulation: Baron & Myerson (1982), Baron & Besanko (1984), Palonen & Pekkarinen (2022).

Payoff after inspection is zero in case of truthful report.

Taxation: Border & Sobel (1987), Mookherjee & Png (1989), Chander & Wilde (1998).

Restriction on transfers and no quantity.

CSV without transfers, multiple agents, probabilistic verification: Ben-Porath et al. (2014), Erlanson & Kleiner (2019), Halac & Yared (2020), Ball & Kattwinkel (2022), Kattwinkel & Knoepfle (2023), Ahmadzadeh (2024),...

Results: Providing Incentives

Lemma (informal)

1 Punishment is maximal after inspection and misreport:

$$-q^I(\hat{\theta},\theta)\theta + t^I(\hat{\theta},\theta) = 0 \ \forall \hat{\theta} \neq \theta.$$

2 Agent is reimbursed cost of production when not inspected:

$$-q^N(\theta)\theta + t^N(\theta) = 0.$$

3 Quantity after inspection and truth-telling is first-best,

$$q^I(\theta,\theta) = q^{FB}(\theta).$$

Results: IC

$$x(\theta)\left(-q^I(\theta,\theta)\theta+t^I(\theta,\theta)\right)+(1-x(\theta))\left(-q^N(\theta)\theta+t^N(\theta)\right) \tag{IC}$$
$$\geq x(\hat{\theta})\left(-q^I(\hat{\theta},\theta)\theta+t^I(\hat{\theta},\theta)\right)+(1-x(\hat{\theta}))\left(-q^N(\hat{\theta})\theta+t^N(\hat{\theta})\right)$$

Results: IC

$$x(\theta)\left(-q^{FB}(\theta)\theta + t^{I}(\theta,\theta)\right) + (1-x(\theta))\overbrace{\left(-q^{N}(\theta)\theta + t^{N}(\theta)\right)}^{0} \tag{IC}$$
$$\geq x(\hat{\theta})\underbrace{\left(-q^{I}(\hat{\theta},\theta)\theta + t^{I}(\hat{\theta},\theta)\right)}_{0} + (1-x(\hat{\theta}))\left(-q^{N}(\hat{\theta})\theta + t^{N}(\hat{\theta})\right)$$

Results: IC

$$x(\theta)\left(-q^{FB}(\theta)\theta + t^{I}(\theta,\theta)\right) \geq (1 - x(\hat{\theta}))\left(-q^{N}(\hat{\theta})\theta + t^{N}(\hat{\theta})\right) \tag{IC}$$

Incentives to the agent are provided only through

- inspection,
- bonus payments after truthful report.

Results: IC

Transfer after inspection satisfies

$$x(\theta)(-q^{FB}(\theta)\theta + t^{I}(\theta,\theta)) = \sup_{\hat{\theta}}(1 - x(\hat{\theta}))q^{N}(\hat{\theta})(\hat{\theta} - \theta)$$

Results: IC

Transfer after inspection satisfies

$$x(\theta)(-q^{FB}(\theta)\theta + t^I(\theta,\theta)) = \sup_{\hat{\theta}}(1 - x(\hat{\theta}))q^N(\hat{\theta})(\hat{\theta} - \theta)$$

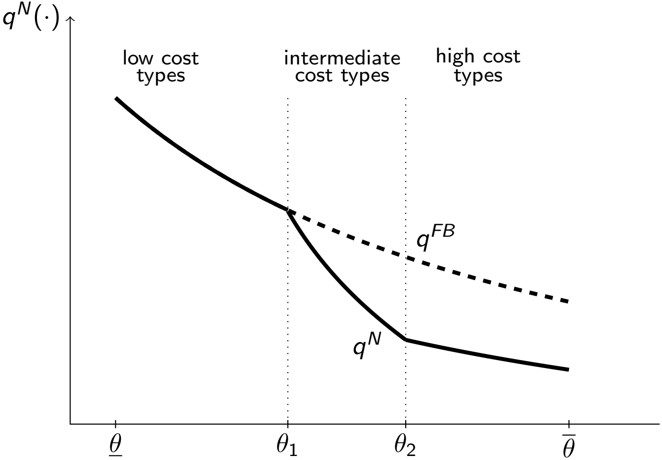
Implies that IC constraints

bind only upwards

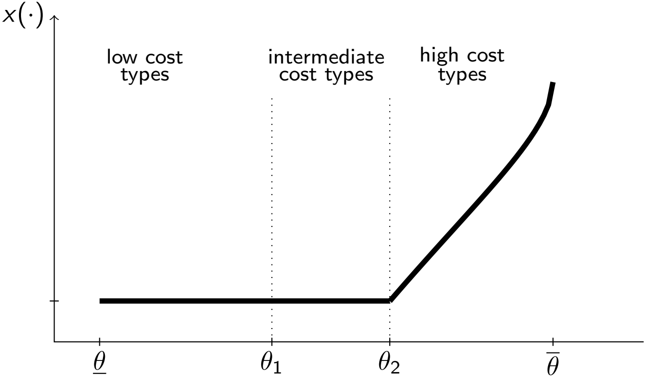
do not bind locally

Results: Quantity

Quantity after inspection is equal first-best.



Results: Inspection



Methodological Contribution

Information rent of cost θ is

$$\sup_{\hat{\theta}}(1 - x(\hat{\theta}))q^N(\hat{\theta})(\hat{\theta} - \theta)$$

Methodological Contribution

Information rent of cost θ is

$$\sup_{\hat{\theta}} (1 - x(\hat{\theta})) q^N(\hat{\theta})(\hat{\theta} - \theta)$$

Challenge: which incentive constraints bind?

$$\hat{\theta}(\theta) = \arg\max_{\hat{\theta}} \, (1 - x(\hat{\theta})) q^N(\hat{\theta})(\hat{\theta} - \theta)$$

Methodological Contribution

Information rent of cost θ is

$$\sup_{\hat{\theta}} (1 - x(\hat{\theta}))q^N(\hat{\theta})(\hat{\theta} - \theta)$$

Challenge: which incentive constraints bind?

$$\hat{\theta}(\theta) = \arg\max_{\hat{\theta}} \, (1 - x(\hat{\theta}))q^N(\hat{\theta})(\hat{\theta} - \theta)$$

Contribution: characterize $\hat{\theta}(\cdot)$ explicitly in optimal mechanism

Conclusion

How does ability to inspect affect incentives and the procured quantity?

1. Incentives are provided only through inspection and bonus payments when the producer reports truthfully.
2. The producer produces the first–best quantity even when not inspected if his cost is low enough.
3. The quantity procured from costs with higher costs is distorted downwards from the first–best benchmark.

Thank you!

Appendix – Minimal Inspection Probability $\underline{x}$

Lemma: The quantities in an optimal mechanism do not depend on the minimal inspection probability. Formally, let $\underline{x}, \underline{x}' \in (0,1)$. Then there is a solution $\mathbb{M}_{\underline{x}} = \left(x_{\underline{x}}(\cdot), q^I_{\underline{x}}(\cdot,\cdot), t^I_{\underline{x}}(\cdot,\cdot), q^N_{\underline{x}}(\cdot), t^N_{\underline{x}}(\cdot)\right)$ to $\mathcal{P}_{\underline{x}}$ and a solution to $\mathcal{P}_{\underline{x}'}$, $\mathbb{M}_{\underline{x}'} = \left(x_{\underline{x}'}(\cdot), q^I_{\underline{x}'}(\cdot,\cdot), t^I_{\underline{x}'}(\cdot,\cdot), q^N_{\underline{x}'}(\cdot), t^N_{\underline{x}'}(\cdot)\right)$, such that

$$\left(q^I_{\underline{x}'}(\cdot,\cdot), q^N_{\underline{x}'}(\cdot), t^N_{\underline{x}'}(\cdot)\right) = \left(q^I_{\underline{x}}(\cdot,\cdot), q^N_{\underline{x}}(\cdot), t^N_{\underline{x}}(\cdot)\right).$$

Moreover, $x_{\underline{x}}$ and $x_{\underline{x}'}$ are related by

$$\frac{1 - x_{\underline{x}}(\theta)}{1 - \underline{x}} = \frac{1 - x_{\underline{x}'}(\theta)}{1 - \underline{x}'}$$

and

$$t^I_{\underline{x}}(\theta) - \theta q^{FB}(\theta) = \frac{1}{x_{\underline{x}}(\theta)} \left(\frac{1 - \underline{x}}{1 - \underline{x}'} - (1 - x_{\underline{x}}(\theta))\right) (t^I_{\underline{x}'} - \theta q^{FB}(\theta)).$$

Appendix – Properties of Optimal Mechanisms

Proposition:

1. For every incentive compatible mechanism that satisfies IC there exists such a mechanism such that the transfer without inspection equals the cost of production, i.e.,

$$t^N(\theta) = \theta q^N(\theta),$$

   and both mechanisms have the same quantity allocation and inspection probability. Moreover, both mechanisms yield the same payoff to the Principal.

2. In any optimal mechanism,

$$q^N(\theta) = q^{FB}(\theta) \text{ or } t^N(\theta) = \theta q^N(\theta).$$

Appendix – Properties of Optimal Mechanisms

Proposition (continue):

3 For $\delta > 0$ let

$$B_\delta = \{\hat{\theta} | t^N(\hat{\theta}) \geq q^N(\hat{\theta})\hat{\theta} + \delta\}$$

and

$$\hat{\theta}_\delta(\theta) = \{\hat{\theta} | (1 - x(\hat{\theta}))(-q^N(\hat{\theta})\theta + t^N(\hat{\theta})) \geq \pi(\theta) - \delta > 0\}.$$

If, for a positive measure of types θ,

$$\hat{\theta}_\delta(\theta) \subset B_\delta,$$

and B_δ has positive measure, then the mechanism $\mathbb{M}$ is not optimal.

Appendix – Optimal Mechanism

Proposition: The following holds in the optimal mechanism.

1. Low-cost types produce their first-best quantity and are inspected with the minimal probability: there exists a $\theta_1 > \underline{\theta}$ such that

$$\text{for all } \theta < \theta_1, \quad x(\theta) = \underline{x} \text{ and } q^N(\theta) = q^{FB}(\theta).$$

2. Intermediate cost types are inspected with the minimal probability $\underline{x}$ and produce a quantity strictly less than first-best: there exists a $\theta_2, \theta_1 < \theta_2 \leq \overline{\theta}$, such that $x(\theta) = \underline{x}$ and $q^N(\theta) < q^{FB}(\theta)$ for all types $\theta \in [\theta_1, \theta_2)$.
3. for all types θ such that $\underline{x} < x(\theta) < 1$ the quantity without inspection is strictly less than first-best, strictly decreasing in θ and independent of $x(\theta)$. It is given as the unique solution $q = q^N(\theta)$ to

$$V(q^{FB}(\theta)) - \theta q^{FB}(\theta) - \kappa = V(q) - qV^{'}(q). \quad \text{(quantity-interior-inspection)}$$

Appendix – Steps of Proof

Reformulated problem

$$\max_{x(\cdot),q^N(\cdot)} \int x(\theta)\left(V(q^{FB}(\theta) - q^{FB}(\theta)\theta - \kappa\right)$$

$$+ (1 - x(\theta))\left(V(q^N(\theta)) - q^N(\theta)\theta\right)$$

$$- \sup_{\hat{\theta}} (1 - x(\hat{\theta}))q^N(\hat{\theta})(\hat{\theta} - \theta)\,\mathrm{d}F(\theta)$$

$$\text{subject to}$$

$$\underline{x} \leq x(\theta) \leq 1 \text{ for all } \theta.$$

Appendix – Steps of Proof

Quantity for $x(\theta) > \underline{x}$.

Lemma

For each θ such that $\underline{x} < x(\theta) < 1$, the quantity without inspection is the unique solution to

$$V(q^{FB}(\theta)) - q^{FB}(\theta)\theta - \kappa = V(q) - V'(q)q.$$

Appendix – Steps of Proof

Lemma

There exists a solution such that

1. $(1 - x(\cdot))q^N(\cdot)$ *is a differentiable function that is strictly decreasing when positive;*
2. $\hat{\theta}(\cdot)$ *is single-valued and, viewed as a function, increasing.*

Appendix – Steps of Proof

Auxiliary variable – which constraint binds?

Lemma

At all points of differentiability of $\hat{\theta}(\cdot)$,

$$(\hat{\theta}(\theta) - \theta)f(\theta) = \hat{\theta}'(\theta)\left(V'(q^N(\hat{\theta}(\theta)) - \hat{\theta}(\theta)\right)f(\hat{\theta}(\theta)).$$

Boundary condition: $\hat{\theta}(\overline{\theta}) = \overline{\theta}$.

Appendix – Steps of Proof

Quantity without inspection when inspection is minimal.

Lemma

For a fixed type $\theta_1 \in (\underline{\theta}, \overline{\theta})$ let $(q_1, \hat{\theta}_1)$ be the solution to

$$-q^{'}(\hat{\theta})(\hat{\theta} - \theta) = q(\hat{\theta}),$$
$$\hat{\theta} = \hat{\theta}(\theta) \text{ solves } \;(\text{ode}\,\hat{\theta}),$$

with the boundary conditions $q_1(\theta_1) = q^{FB}(\theta_1), \hat{\theta}_1(\underline{\theta}) = \theta_1$.

Then $q^N(\theta) = q_1(\theta)$, for all $x(\theta) = \underline{x}$.