DOES UNIVERSALIZATION ETHICS JUSTIFY PARTICIPATION IN LARGE ELECTIONS?

TICIPATION IN LARGE ELECTIONS?

Joint work with Ingela Alger (TSE, IAST) and Jean-François Laslier (PSE)

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PEOPLE INCUR COSTS TO VOTE IN LARGE ELECTIONS

Photo: PTI

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RQ: WHY DO PEOPLE INCUR COSTS TO VOTE IN LARGE ELECTIONS?

Hypotheses:

- for instrumental reasons
  [Black 1948; Downs 1957; Ledyard et al. 1981; Palfrey and Rosenthal 1985;
  Austen-Smith and Banks 1996; Myerson 2000; Demichelis and Dhillon 2010]
- for ethical reasons:
  - as a civic duty to support democracy [Downs 1957, Riker and Ordeshook 1968]
  - to conform with ethical standards – rule utilitarianism
    [Harsanyi 1980; Coate and Conlin 2004; Feddersen and Sandroni 2006; Herrera,
    Morelli and Nunnari 2016; Bierbrauer, Tsyvinski and Werquin 2022]
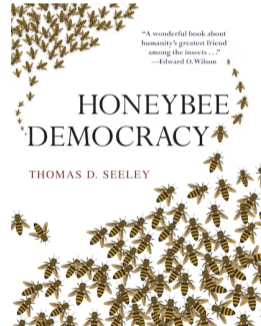- for other reasons

Empirical evidence Blais 2000; Blais and Achen 2019; Gerber, Green and Larimer 2008;
Rogers et al. 2017

RQ: WHY DO PEOPLE INCUR COSTS TO VOTE IN LARGE ELECTIONS?

- Biological roots of political psychology should be considered.
  [Rousseau 1755; Shubert 1982; Masters 1984; Conley, Toossi and Wooders 2006; Sidanius and Kurzban 2013; Petersen 2015; Bergner and Hatemi 2016]

https://www.harvardmagazine.com/2011/04/honeybee-house-hunting

https://press.princeton.edu/books/hardcover/9780691147215/honeybee-democracy

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OUR CONTRIBUTION

- The theory of evolution of preferences by natural selection: among all continuous utility functions, one of them "beats" the others. [Alger and Weibull 2013; Alger and Weibull 2016]
- This function – dubbed *Homo moralis* – implies partial universalization.
- It had not been considered before in economics.
- We show that *Homo moralis* preferences can explain why people are willing to incur costs to vote, even though their impact on the outcome is virtually nil.
- In sum: we propose a novel theory of turnout in large elections.

OUR CONTRIBUTION

- The principle of universalization (introduced in modern economics by Laffont 1975): the decision-maker chooses the strategy that would maximize her own welfare if – hypothetically – this strategy was chosen by all the other players.
  - *Homo moralis* is a less extreme form: *if the strategy was chosen by a fraction $\kappa \in [0, 1]$ of the others* [Alger and Weibull 2013].
- Different from "rule utilitarianism" (Harsanyi 1955; Harsanyi 1980): the decision-maker chooses the strategy that maximizes the total welfare of all involved players.
  - Altruism is a less extreme form: a weight $\alpha \in [0, 1]$ on the other players' welfare [Becker 1974; Lindbeck and Weibull 1988].

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THE POLITICAL SETTING

- A continuum population.
- Two candidates, *A* and *B*.
- *A* is preferred by a mass $\bar{a} = a_0 + a_v$ of voters:
  - $a_0 > 0$: *A*'s base – they always vote
  - $a_v > 0$: *A*'s cost-sensitive supporters
- *B* is preferred by a mass $\bar{b} = b_0 + b_v$ of voters:
  - $b_0 > 0$: *B*'s base – they always vote
  - $b_v > 0$: *B*'s cost-sensitive supporters
- No aggregate uncertainty: *B* is the underdog and *A* is the leader: $\bar{b} < \bar{a}$

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THE POLITICAL SETTING

- Cost-sensitive voters face uncertainty about the voting cost.
- They follow threshold strategies: vote if the cost falls short of the threshold, abstain otherwise.

$$a(s_A) = \underbrace{a_0}_{\text{base}} + \underbrace{a_v \, F_A(s_A)}_{\text{cost-sensitive voters who vote}}$$

$$b(s_B) = \underbrace{b_0}_{\text{base}} + \underbrace{b_v \, F_B(s_B)}_{\text{cost-sensitive voters who vote}}$$

yielding relative vote margin for A

$$\alpha(s_A, s_B) = \frac{a(s_A) - b(s_B)}{a(s_A) + b(s_B)}.$$

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THE POLITICAL SETTING

- The *A*-supporters obtain instrumental benefits $h(\alpha)$.
- The *B*-supporters obtain material (instrumental) benefits $-\rho \cdot h(\alpha)$.
- *h* captures how power is shared:
  - $h' > 0$
  - $h''(x) < 0$ for all $x > 0$: the marginal benefit is the largest at zero
  - the competition for power is zero-sum
- $\rho \geq 1$ is the stake of the election

EXAMPLE BENEFIT FUNCTION

$$h\left(x\right) = \frac{\arctan\left(mx\right)}{\arctan\left(m\right)}$$
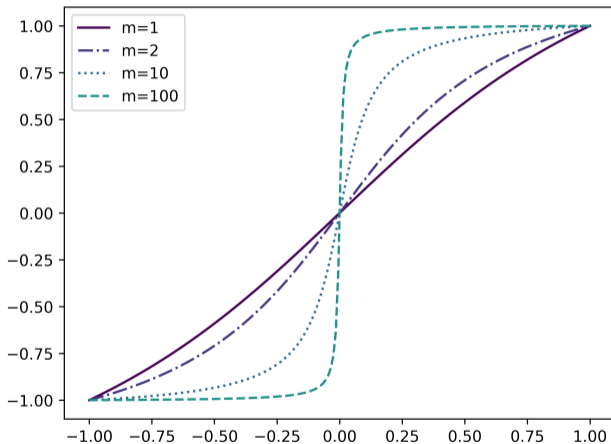
Figure: h for different values of m

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VOTERS' PREFERENCES

- Voters' preferences are not determined solely by the material payoffs.
- Voters have *Homo moralis* preferences (Alger and Weibull 2013):
  - The utility of a *Homo moralis* with degree of universalization $\kappa \in [0, 1]$ is: *the material payoff that the individual would obtain if – hypothetically – a share κ of the others used the same threshold as the individual, rather than the threshold they are actually using*.
- Assume: all the voters have the same degree of universalization κ.
- Two alternative definitions of "the others": partisan vs non-partisan ethics.
- Goal: characterize type-homogenous Nash equilibria.

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KEY TAKE-HOME MESSAGES

- Equilibria do not always exist.
- Multiplicity of equilibria is not rare.
- Equilibrium turnout is strictly positive in most cases.
- A high κ does not guarantee a high turnout.
- For some parameter values the underdog wins (with certainty).

PREFERENCES UNDER PARTISAN ETHICS I

- "The others" are the co-partisans.
- Consider a type-homogenous strategy profile (s_A, s_B).
- An A-supporter i gets expected net material benefits

$$EU_A(s_A, s_B) = h(\alpha(s_A, s_B)) - \int_{c=0}^{s_A} c f_A(c) \, dc$$

but considers the hypothetical number of votes

$$a^\kappa(s_A, s_A^i) = a_0 + a_v F_A(s_A) + \kappa a_v [F_A(s_A^i) - F_A(s_A)]$$

in favor of A

PREFERENCES UNDER PARTISAN ETHICS II

- This yields the corresponding hypothetical relative vote margin

$$\alpha^{\kappa}(s_A, s_B, s_A^i) = \frac{a^{\kappa}(s_A, s_A^i) - b(s_B)}{a^{\kappa}(s_A, s_A^i) + b(s_B)}.$$

- Hence, voter i's expected utility is

$$EU_A^{\kappa}(s_A, s_B, s_A^i) = h(\alpha^{\kappa}(s_A, s_B, s_A^i)) - \int_{c=0}^{s_A^i} cf_A(c)\, dc.$$

PREFERENCES UNDER PARTISAN ETHICS III

- A B-supporter j gets expected net material benefits

$$EU_B(s_A, s_B) = h(\beta(s_A, s_B)) - \int_{c=0}^{s_B} cf_B(c)\, dc$$

  but considers the hypothetical number of votes

$$b^\kappa(s_B, s_B^j) = b_0 + b_v F_B(s_B) + \kappa b_v[F_B(s_B^i) - b_v F_B(s_B)]$$

  in favor of B, with the corresponding hypothetical relative vote margin

$$\beta^\kappa(s_A, s_B, s_B^j) = \frac{b^\kappa(s_B, s_B^j) - b(s_B)}{b^\kappa(s_B, s_B^j) + a(s_A)}.$$

- Hence, voter j's expected utility is

$$EU_B^\kappa(s_A, s_B, s_B^j) = h(\beta^\kappa(s_A, s_B, s_B^j)) - \int_{c=0}^{s_B^j} cf_B(c)\, dc.$$

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A "NEVER-A-BEST-RESPONSE" RESULT

Proposition

Under partisan ethics,

- if $\kappa = 0$, there exists a unique equilibrium, $(a^*, b^*) = (a_0, b_0)$;
- if $\kappa \in (0, 1]$, $s_A^i = 0$ is never a best response for an A-supporter; hence, any equilibrium (a^*, b^*) is such that at least some cost-sensitive A-supporters vote, i.e., $a^* > a_0$.
- Idem for B-supporters.

THE UNDERDOG SOMETIMES WINS THE ELECTION

Proposition

Suppose that the leader supporters enjoy a cost advantage over the underdog supporters:

- their base is at least as large ($a_0 \geq b_0$),
- their mass of cost-sensitive voters is at least as large ($a_v \geq b_v$), and
- the cost distributions favor them ($F_A(c) \geq F_B(c)$ for all $c \in \mathbb{R}_+$),

with at least one of the inequalities holding strictly.

Then:

1. if the stake is neutral or almost neutral (i.e., $\rho \geq 1$ is close enough to 1), the leader wins ($\alpha(a^*, b^*) > 0$) at any equilibrium (a^*, b^*);
2. for a large enough stake ρ, there may exist equilibria (a^*, b^*) in which the underdog wins ($\alpha(a^*, b^*) < 0$).

THE UNDERDOG SOMETIMES WINS THE ELECTION

$a_0 = 0.85$

$a_v = 0.9$

$b_0 = 0.6$

$b_v = 0.64$

$\rho = 5$
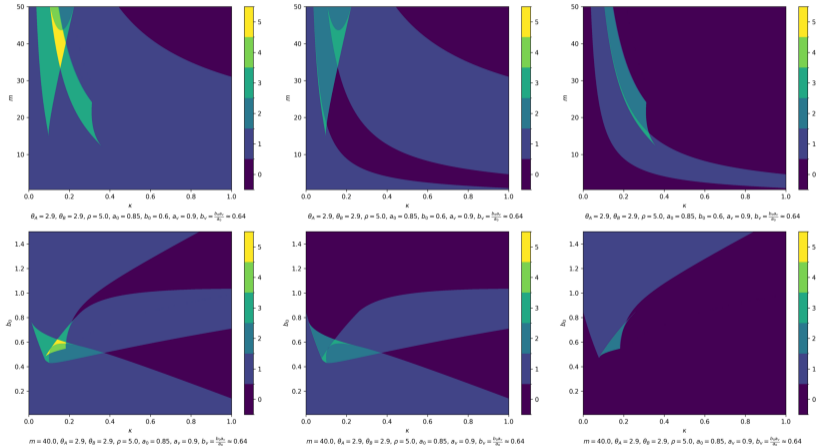
(a) Number of equilibria   (b) Number of equilibria with   (c) Number of equilibria with
                            $a > b$ (A wins) $b > a$ (B wins)

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THE UNDERDOG SOMETIMES WINS THE ELECTION

Proposition

Suppose that the underdog's base exceeds that of the leader ($b_0 > a_0$). Then, for some parameter values, there exist equilibria (a^*, b^*) in which the underdog wins ($\alpha(a^*, b^*) < 0$), even if the stake is neutral ($\rho = 1$).
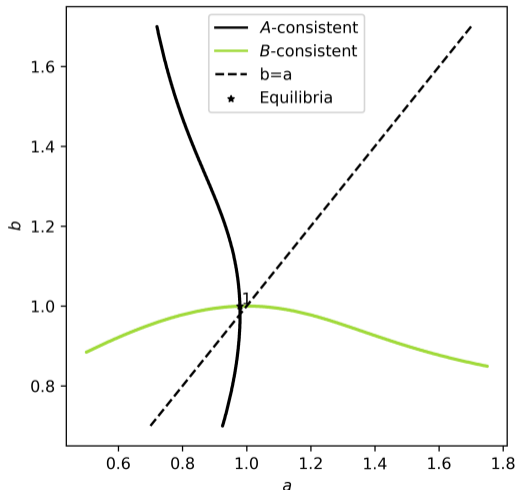
$m = 3.0, \kappa = 0.5, \rho = 1, \theta_A = 2.0, \theta_B = 2.0,$
$a_0 = 0.5, a_v = 1.25, b_0 = 0.7, b_v = 1.0$

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THE WINNER-TAKE-ALL LIMIT CASE

The pure winner-take-all case corresponds to a discontinuous step-function $\text{sign}(\cdot)$.

We proceed by approximation, by studying a sequence of benefit functions h_t that converges to the winner-take-all benefit function $\text{sign}(\cdot)$.

Proposition

Let (a, b) be sustained as a limit equilibrium of the winner-take-all case. Then:

1. $(a, b) = (a_0, b_0)$ if $a_0 \neq b_0$;
2. no such pair (a, b) exists if $a_0 = b_0$.

CONTRAST WITH GROUP-BASED VOTER PARTICIPATION MODELS

- Coate and Conlin 2004; Feddersen and Sandroni 2006; Herrera, Morelli and Nunnari 2016; Bierbrauer, Tsyvinski and Werquin 2022
- A group's equilibrium cost threshold optimally trades off the probability of winning against the expected voting cost, given the other group's threshold: *"this model involves an equilibrium between two party planners, or representative agents, on each side, A and B. In the solution, each planner looks at the total electoral benefit net of the total cost of voting incurred by his supporters, taking the other planner's turnout strategy as given"* (Herrera, Morelli and Nunnari 2016, p. 612).
- By applying its group's optimal threshold, an ethical voter receives a constant "duty" payoff $D > 0$.

CONTRAST WITH GROUP-BASED VOTER PARTICIPATION MODELS

- At the individual level, each ethical voter would be happy to incur any positive voting cost, since *D* is taken to exceed the largest possible cost realization.

- The duty that such a voter feels obliged to fulfill thus consists in reducing the aggregate cost of voting, by abstaining from voting when the realized cost is above the equilibrium threshold: some voters *"receive a [duty] payoff for not voting"* (Feddersen and Sandroni 2006, p.1272).

- By contrast, in our model costly voting is individually rational.

- The desire to deviate from abstaining is driven by a utility kick from envisaging the loss that would arise if others also abstained.

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OTHER INNOVATIONS (OR RE-CONNECTIONS WITH OLDER LITERATURE)

- We have two kinds of voters: the base voters and the cost-sensitive voters.
  - By reducing the cost of achieving a certain benefit, a large enough base can motivate underdog supporters enough to make the underdog win.
- We do not adopt assumptions which guarantee existence and uniqueness.
  - When there are multiple equilibria: some can yield low turnouts, even if $\kappa \approx 1$.
  - Under non-partisan ethics, such multiplicity represents a pure coordination problem among co-partisans (since in the other group only the base votes).
  - In the examples, the voters prefer the equilibrium with the highest turnout.

CONCLUDING REMARKS

- Our analysis was prompted by the discovery of a novel preference class, dubbed *Homo moralis*
  [Alger and Weibull 2013; surveys: Alger and Weibull 2019; Alger 2023].
- Other theoretical work on the implications of such semi-Kantian concerns (and of Kantian equilibrium):
  Laffont 1975; Bergstrom 1995; Alger and Weibull 2017; Alger and Weibull 2019; Sarkisian 2017; Sarkisian 2021; Roemer 2019; Norman 2020; De Donder, Llavador and Roemer 2021; Eichner and Pethig 2021; Eichner and Pethig 2022; Ayoubi and Thurm 2023; Muñoz Sobrado 2022; Alger and Laslier 2022; Salonia 2023; Dizarlar and Karagözoğlu 2023; Juan-Bartroli 2024; Juan-Bartroli and Karagözoğlu 2024
- Related experimental work:
  Capraro and Rand 2018; Miettinen et al. 2020; Levine et al. 2020; Bénabou, Falk and Henkel 2024; van Leeuwen and Alger 2024; Juan-Bartroli 2024; Alger and Rivero-Wildemauwe 2024

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Q & A

Thank you for your attention!

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NON-PARTISAN ETHICS I

- "The others" are all the cost-sensitive voters.
- A cost-sensitive voter *i* chooses a pair of thresholds $s^i = (s_A^i, s_B^i) \in [0, \infty]^2$: the voter abstains when her cost is larger than s_A^i if she prefers candidate *A*, and when her cost is larger than s_B^i if she prefers candidate *B*.
- Two possible interpretations:
  - there is *ex ante* uncertainty regarding the candidate that *i* prefers, and she selects the strategy behind the veil of ignorance;
  - there is no such uncertainty, but due to her ethical concern the individual adopts the viewpoint of Harsanyi's impartial observer, by inserting a veil of ignorance in her reasoning.

NON-PARTISAN ETHICS II

> *"In any social situation, each participant will tend to look at the various issues from his own, self-centered, partisan point of view. In contrast, if anybody wants to assert the situation from a moral point of view in terms of some standard of justice and equity, this will essentially amount to looking at it from the the standpoint of an impartial but humane and sympathetic observer."* *(Harsanyi 1977, p. 623)*

PREFERENCES UNDER NON-PARTISAN ETHICS

- Consider the strategy profile $s = (s_A, s_B)$.
- A cost-sensitive voter gets expected net material benefits

$$\frac{1}{a_v + b_v} \left[a_v h(\alpha(s)) + b_v \rho h(\beta(s)) - a_v \int_{c=0}^{s_A} cf_A(c)\,dc - b_v \int_{c=0}^{s_B} cf_B(c)\,dc \right]$$

$$(a_v - \rho b_v)\, h(\alpha(s)) - a_v \int_{c=0}^{s_A} cf_A(c)\,dc - b_v \int_{c=0}^{s_B} cf_B(c)\,dc.$$

but considers the hypothetical margin

$$\alpha^\kappa(s, s^i) = \frac{(1-\kappa)[a(s_A) - b(s_B)] + \kappa[a(s_A^i) - b(s_B^i)]}{(1-\kappa)[a(s_A) + b(s_B)] + \kappa[a(s_A^i) + b(s_B^i)]}.$$

and gets expected utility

$$EU^\kappa(s, s^i) = (a_v - \rho b_v)\, h(\alpha^\kappa(s, s^i)) - a_v \int_{c=0}^{s_A^i} cf_A(c)\,dc - b_v \int_{c=0}^{s_B^i} cf_B(c)\,dc.$$

EQUILIBRIUM EXISTENCE AND PROPERTIES

Proposition

An equilibrium always exists.

Proposition

If $\kappa = 0$ or $a_v = \rho b_v$, then $(a^*, b^*) = (a_0, b_0)$ is the unique equilibrium, while if $\kappa \in (0, 1]$:
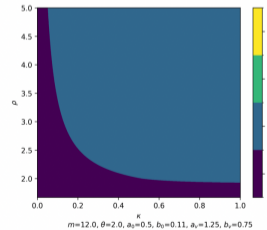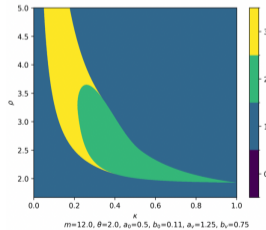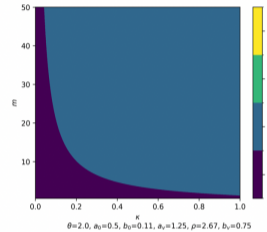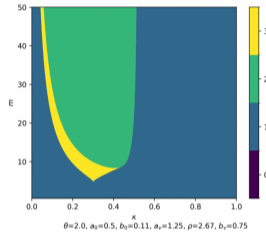
- if $a_v > \rho b_v$, any equilibrium is such that $a^* > a_0$ and $b^* = b_0$;
- if $\rho b_v > a_v$ any equilibrium is such that $a^* = a_0$ and $b^* > b_0$.

THE UNDERDOG SOMETIMES WINS THE ELECTION

$$a_0 + a_v = 1.75$$
$$b_0 + b_v = 0.86$$
$$\rho = 2.67$$

(a) Number of equilibria

(b) Number of equilibria with $b > a_0$ (B wins)

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THE UNDERDOG SOMETIMES WINS THE ELECTION

Proposition

Suppose that $\kappa \in (0, 1]$. Then:

- if $a_v > \rho b_v$ and $a_0 \geq b_0$, there is a unique equilibrium (a^*, b_0). At this equilibrium, the leader wins: $\alpha(a^*, b_0) > 0$;
- if $\rho b_v > a_v$ and $b_0 \geq a_0$, there is a unique equilibrium (a_0, b^*). At this equilibrium, the underdog wins: $\beta(a_0, b^*) > 0$.

## VOTERS SOMETIMES FACE A COORDINATION PROBLEM

Proposition

Suppose that $\kappa = 1$. If there exist multiple equilibria, they all generate the same expected utility to the cost-sensitive voters.

By contrast, if $\kappa < 1$, if there are multiple equilibria, they do not necessarily yield the same expected utility.

In other words, in the non-partisan case the voters who vote may face a coordination problem.

VOTERS SOMETIMES FACE A COORDINATION PROBLEM

$$m = 14.0, \kappa = 0.4, \rho = 3.0, \theta = 2.0,$$
$$a_0 = 0.45, a_v = 0.6, b_0 = 0.1, b_v = 0.4$$
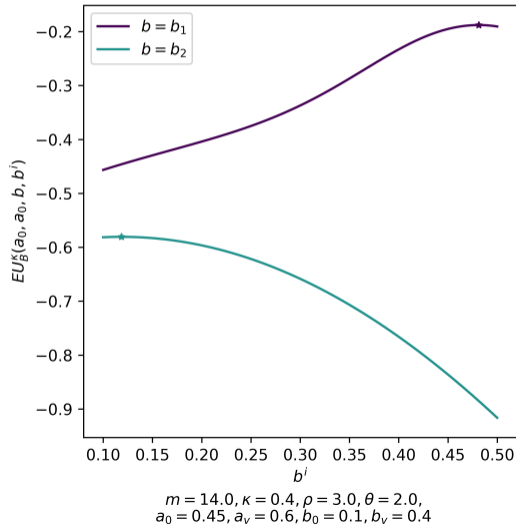
THE WINNER-TAKE-ALL LIMIT CASE

Proposition

Let (a, b) be sustained as a limit equilibrium of the winner-take-all case. Then, any equilibrium is of one of the two following types:

1. $(a, b) = (a_0, b_0)$;
2. $a = b = \max\{a_0, b_0\}$.

The two equilibrium types can co-exist.