Credit Enforcement and Monetary-Policy Transmission in a Search Economy*

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Abstract

We study how the degree of credit enforcement matters for the transmission of long-run inflation and the welfare effect of private-money creation. We do so in a model with directed and competitive search, where sellers borrow against their search-market income. Intermediaries sell the arising claims as private money to buyers, who use it along with fiat money to transact in the search market. Inflation stimulates borrowing by curbing real interest rates. With sophisticated enforcement, sellers borrow more through ex-ante commitment to more search, as this increases their search-market income. Trade therefore accelerates. With simple enforcement, commitment is infeasible. The inflation-driven increase in borrowing then decelerates trade since debt distorts search incentives ex post. We calibrate the economies with sophisticated and simple enforcement to U.S. data. The extent of private-money creation is close to optimal in the sophisticated-enforcement calibration, whereas in the simple-enforcement calibration, any level of private-money creation reduces welfare.

Keywords: money creation, credit, financial intermediation, directed and competitive search, contract enforcement.

JEL Classification: D83, E40, E50, G21.

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1 Introduction

In today's monetary architecture, financial intermediaries create most of the assets that are used as money. A key feature of this process is that credit extension and money creation are two sides of the same coin—when an intermediary grants a loan to a borrower, it credits the deposit account of the borrower with the exact same amount. Because deposits are accepted as a means of payment, the borrower can use them to buy goods from other agents—the deposits circulate as money. Crucial for this process is the intermediaries' ability to enforce loan repayment.

We show in this paper that the degree of such enforcement matters (i) for whether inflation accelerates or decelerates trade and (ii) for whether financial intermediaries' money-creation privilege improves or impairs GDP and welfare. We use a monetary model in continuous time based on Choi and Rocheteau (2021) and extend their framework in three ways. First, we let bilateral trade take place in directed and competitive search markets as in Wright, Kircher, Julien, and Guerrieri (2021): sellers of search goods compete in the terms of trade they post to attract buyers, who use money to settle search-market transactions. Second, we add financial intermediaries. They extend credit to sellers against sellers' search-market income, which allows sellers to prepone consumption of a non-search good. Intermediaries bundle the arising claims and intermediate them as private money to buyers who use it along with fiat money. Third, we distinguish different enforcement technologies, capturing the degree to which intermediaries can enforce sellers' promises in loan contracts.

A typical loan contract between an intermediary and a seller specifies a claim on the seller's revenue from a bilateral match at a particular date. Loan contracts are therefore contingent claims: the seller receives credit upon writing the contract, but repays only in case of a match. The intermediary and the seller can stipulate a contingent claim up to some upper limit of contract duration. The value of a loan contract is determined by: competitive interest rates; the contingent repayment; and the seller's matching rate at the time of repayment. To differentiate between the enforcement of the repayment and the enforcement of the matching rate, we introduce two technologies: *contract control* and *project control*. Contract control enables intermediaries to observe whether a seller has a match, and to seize the seller's match revenue in case of default on the promised repayment. We assume the availability of contract control throughout. Project control in addition enables intermediaries to enforce the seller's search-market actions. This allows a loan contract to additionally include the terms of trade the seller posts and the search effort s/he devotes at the time of repayment, which jointly determine the seller's matching rate. Project control thus enlarges the set of enforceable contracts. We analyze and compare the economies both with and without project control.

Monetary policy affects borrowing because government-issued fiat money competes with privately-issued money in payment. In particular, inflation reduces the return on fiat money and thus allows intermediaries to pay a lower real interest rate on private money. Since intermediaries need assets on their balance sheets to back the private money they issue, they also reduce the real loan rate to attract borrowers. How this affects search-market activity depends on the intermediaries' command of project control.

Without project control, inflation reduces economic activity beyond its negative effect stemming from higher opportunity costs of holding money. Because intermediaries cannot enforce the sellers' search-market actions, sellers can only borrow more by stipulating a higher contingent repayment. This reduces the ex-post match surplus, since an indebted seller's match revenue accrues to the intermediary until the contingent repayment is met. As a result, search incentives weaken, so that the matching rate is lower than when sellers are less indebted.

With project control, sellers borrow more in response to inflation by contracting search-market actions that increase both the match revenue and the matching rate. They do this to increase the value of the search-market income they borrow against. Yet, in our view, the resulting acceleration of trade cannot be ascribed to the *hot-potato* effect of inflation, which is commonly understood as the buyers' incentive to get rid of money faster due to inflation. The rationale of the hot-potato effect breaks down when sellers, as they do in our model, anticipate inflation and thus become reluctant to accept money. Instead, inflation speeds up trade by curbing real interest rates, which incentivizes sellers to borrow and to commit to more economic activity. This mechanism is in line with the credit channel of monetary policy: loose monetary policy, i.e., high inflation, stimulates borrowing through low real rates.

We quantify how private-money creation, arising from the intermediation of consumptive credit, matters for the effect of long-run inflation on matching rates, welfare, and GDP. To assess the role of credit enforcement for these effects, we calibrate three different economies to U.S. data: a benchmark economy without any intermediation, as well as two intermediation economies, one with project control and one without. We use the method of simulated moments to let our models match empirical moments of money demand and firm-level markups from 1968 to 2019, and we match all moments in each calibration. We consider the Friedman rule as our reference point for monetary policy since it entails first-best allocations by rendering the opportunity cost of holding money zero through slight deflation. We find for the economy with project control that a deviation from the Friedman rule to 3% inflation accelerates matching by 11.03%. The sign and magnitude of this result hinges on the presence of intermediation in general and on the availability of project control in particular: the same deviation decelerates matching by 12.99% and 2.03% in the economy without project control and in the economy without intermediation, respectively. Intermediation without project control thus aggravates the negative effect of inflation on matching rates: inflation imposes not only a cost on monetary activity but it also stimulates distortionary credit extension.

In terms of welfare, a deviation from the Friedman rule to 8% inflation costs 1.08% of first-best consumption in the economy with project control, whereas it costs 1.80% and 1.22% in the economy without project control and in the economy without intermediation. Our models with different frictions in financial intermediation thus imply different welfare costs of inflation while matching salient features of money demand equally well. In that sense, we hit the same note as Lagos and Wright (2005). They stress that the traditional way of looking at the area under the money-demand curve is not enough to understand the welfare-cost of inflation; the microfoundations of money demand matter.

For a normative assessment of financial intermediation, which transforms consumptive credit for sellers into money for buyers, we quantify how the welfare cost of inflation changes when policy shuts down intermediation by imposing 100-percent capital requirements. We find that at all empirically observed levels of inflation, such a policy improves welfare in the economy without project control, whereas it reduces welfare in the economy with project control. At 8% inflation for instance, shutting down intermediation in the no-project-control economy reduces the welfare cost of inflation from 1.80% to 0.48%, whereas in the project-control economy, the same policy increases the welfare cost from 1.08% to 1.34%. Even stronger, the extent of intermediation in the project-control economy is close to optimal in mitigating the welfare cost of inflation, whereas in the no-project-control economy, intermediation unambiguously aggravates the welfare cost. A stance on whether private-money creation is good or bad thus boils down to assessing the degree of commitment in loan contracts.

The response of GDP to inflation, similar to the response of matching rates, qualitatively depends on the availability of project control. In the economy with project control, GDP increases by 0.27% in response to a deviation from the Friedman rule to 3% inflation due to the increase in matching rates; the economy overheats in that GDP increases above its first-best level. The same policy experiment curbs GDP by 4.09% and 3.13% in the economies without project control and without intermediation, respectively, as matching rates decline; the economies cool down. Moreover, at 8% inflation, shutting down intermediation in the project-control economy increases GDP by 6.06%, whereas it increases GDP by 3.59% in the no-project-control economy.

Finally, our model highlights that statistical money velocity, measured by GDP over aggregate money balances, and the actual frequency at which money is spent can move in opposite directions in response to inflation. For instance, in the project-control economy, a deviation from the Friedman rule to 3% inflation increases the spending frequency and statistical velocity by 11.03% and 9.41%. In the no-project-control economy and in the economy without intermediation, the same policy reduces the spending frequency by 12.99% and 2.03%, but increases statistical velocity by 15.39% and 20.07%.

In what follows, we relate our work to the existing literature in Section 2, and we develop the model in Section 3. In Section 4, we describe agents' optimal choices, and we introduce the equilibrium concept as well as the notion of welfare in Section 5. We analyze the transmission of monetary policy at the Friedman rule in Section 6. In Section 7, we discuss equilibrium existence and uniqueness away from the Friedman rule. Finally, in Section 8, the model is calibrated and we quantify the transmission of monetary policy. We conclude the analysis in Section 9. Proofs and derivations are in the appendix.

2 Literature

We combine the work of Choi and Rocheteau (2021) with that of Wright et al. (2021). Choi and Rocheteau (2021) develop a continuous-time version of the money-search model developed by Lagos and Wright (2005) and Rocheteau and Wright (2005), among others. We use continuous time to conveniently model that creditors can only borrow against income arising within a bounded time interval. This facilitates calibrating the length of that time interval, as it is then a continuous variable, and it also allows for a natural notion of matching frequencies because trades arise as Poisson events.

The framework of directed and competitive search was developed by Hosios (1990), Moen (1997), Montgomery (1991), Peters (1984, 1991), and Sattinger (1990), among others. It has been surveyed in depth by Wright et al. (2021) who also applied it to a standard money-search model. Following this approach, sellers, who are also borrowers in our model, post prices and quantities in order to compete for buyers. We model the degree of credit enforcement in terms of whether sellers can ex-ante commit to future terms of trade and also future search—an ability we coin project control.

We find that without project control, sellers' debt generates moral hazard, as the benefits from higher search effort and more attractive terms of trade only partially accrue to the seller. Broadly speaking, our results here confirm those found in static models of moral hazard, e.g., Acharya, Mehran, and Thakor (2016), Biais and Casamatta (1999), Edmans and Liu (2010), Fender and Mitchell (2009), and Hellwig (2009).

Our model with project control also relates to the literature on the hot-potato effect of inflation. As argued by Liu, Wang, and Wright (2011), common wisdom is that inflation makes people spend money faster, as already described by Keynes (1924). Li (1994) confirms this insight by incorporating endogenous search in a money-search model à la Kiyotaki and Wright (1993). It is difficult, though, to examine inflation in their framework, as goods and money are indivisible. Approaches to reconciling money-search models with the hot-potato effect, as developed, e.g., by Ennis (2009), Nosal (2011), and Dong and Jiang (2014), have focused primarily on buyers' incentives to spend money. Liu et al. (2011) focus on the extensive margin, i.e., on buyers' incentives to participate in markets. We take a different perspective and stress that in our model with project control, inflation accelerates trade by curbing the borrowing rate and stimulating credit, rather than by incentivizing buyers to spend money faster.

Closely related to our model, Lagos and Rocheteau (2005) find that inflation accelerates matching in a model of directed and competitive search when buyers' search is endogenous. Our model generalizes their framework by also endogenizing sellers' search. Without credit, we find that while inflation increases buyers' search, it decreases sellers' search. The resulting effects on the matching frequency cancel out if buyers and sellers are symmetric in terms of the search elasticities. We then show that the presence of credit in a symmetric model shifts this result in either one or the other direction: with project control, the matching rate increases; without project control, it decreases.

The way in which we model financial intermediation ties to the idea that private money and credit are two sides of the same coin. In the money dimension, our approach aligns with the existing money-search literature in that money is used by buyers in search markets. In the credit dimension, we contrast existing work, which primarily focuses on credit being used for capital formation or financing firms' operational cost (e.g., Altermatt, 2022; Altermatt & Wang, 2024; Van Buggenum, 2021; Williamson, 2012). We rather let money be backed by sellers' search-market income, who use credit for the sole purpose of preponing consumption. While private-money creation is inessential in that fiat money already facilitates search-market trade, it is not irrelevant since the associated credit extension affects sellers' search-market behaviour. Whether this process stimulates or curbs the economy eventually hinges on the degree of credit enforcement.

3 Model

Time $t \in \mathbb{R}_+$ is continuous and goes on forever. We focus on a setup with perfect foresight—there is no aggregate uncertainty. There are two types of perfectly divisible and non-storable goods: *search goods* and *general goods* (treated as the numéraire). The economy is populated by three unit masses of infinitely-lived agents called *buyers*, *sellers*, and *financial intermediaries* (FIs). All agents can produce and consume general goods. Sellers can produce but cannot consume search goods, whereas buyers wish to consume search goods but cannot produce them. We distinguish two types of markets. Search goods are infrequently traded in decentralized *search markets* where sellers and buyers match bilaterally. General goods are traded in a continuously-open centralized *Walrasian market*, but agents cannot access this market when matched.[1]

Buyers' anonymity and their lack of commitment in the search markets necessitate a medium of exchange. Two perfectly divisible and storable assets, *fiat money* and *private money*, play this role. The government issues intrinsically worthless fiat money M_t at

[1] Our structure emulates the alternating market sequence typical for discrete-time money-search models. We can interpret matches as occurring spatially separated from the general-goods market. Among others, Townsend (1980) provided a theory of money by building upon the idea of spatial separation.

constant growth rate $\gamma = \dot{M}_t/M_t$. Fiat money is injected (withdrawn if $\gamma < 1$) through a cumulative process $\{\Upsilon_t\}_{t=0}^{\infty}$ of real lump-sum transfers (resp. taxes) that go solely to buyers for simplicity. The price of fiat money is ϕ_t and inflation is $\pi_t = -\dot{\phi}_t/\phi_t$.[2] Private money is issued by the FIs, trades at price ψ_t, and pays a flow of real dividend ξ_t, so that its real return is $r_t^p = (\xi_t + \dot{\psi}_t)/\psi_t$. Since the model pins down the real return on private money, we can normalize its price as $\psi_t = \phi_t$.[3]

We allow for several search markets being open concurrently. A particular search market is characterized by the posted terms of trade, the set of active buyers and sellers, and the aggregate search effort they devote. Sellers and buyers can only be active in one search market at a time. Each buyer and each seller can locate himself/herself in one of the open search markets, and sellers can also open new search markets at zero cost.

In an open search market, a measurable subset $A_t^s \subseteq [0,1]$ of sellers and a subset $A_t^b \subseteq [0,1]$ of buyers with respective masses μ_t^s and μ_t^b are active. Seller $i \in A_t^s$ devotes search effort $\varepsilon_t^{s,i} \geq 0$ and buyer $j \in A_t^b$ devotes search effort $\varepsilon_t^{b,j} \geq 0$, so that the average search efforts of sellers and buyers are

$$\overline{\varepsilon}_t^s = \frac{1}{\mu_t^s}\int_{A_t^s} \varepsilon_t^{s,i}\,\mathrm{d}i \qquad \text{and} \qquad \overline{\varepsilon}_t^b = \frac{1}{\mu_t^b}\int_{A_t^b} \varepsilon_t^{b,j}\,\mathrm{d}j. \tag{1}$$

The matching rate is given by matching function $\mathcal{N}(\mu_t^b\overline{\varepsilon}_t^b, \mu_t^s\overline{\varepsilon}_t^s)$, where $\mathcal{N}$ is homogeneous of degree one (constant returns to scale), twice continuously differentiable, strictly increasing, strictly concave, and satisfies $\mathcal{N}(\mu_t^b\overline{\varepsilon}_t^b, 0) = \mathcal{N}(0, \mu_t^s\overline{\varepsilon}_t^s) = 0$. Defining market tightness $\theta_t \equiv \mu_t^b\overline{\varepsilon}_t^b/\mu_t^s\overline{\varepsilon}_t^s$, seller i and buyer j face respective matching rates

$$\frac{\varepsilon_t^{s,i}\mathcal{N}(\mu_t^b\overline{\varepsilon}_t^b, \mu_t^s\overline{\varepsilon}_t^s)}{\mu_t^s\overline{\varepsilon}_t^s} = \varepsilon_t^{s,i}\beta(\theta_t) \qquad \text{and} \qquad \frac{\varepsilon_t^{b,j}\mathcal{N}(\mu_t^b\overline{\varepsilon}_t^b, \mu_t^s\overline{\varepsilon}_t^s)}{\mu_t^b\overline{\varepsilon}_t^b} = \varepsilon_t^{b,j}\alpha(\theta_t), \tag{2}$$

where $\beta(\theta_t) \equiv \mathcal{N}(\theta_t, 1)$ and $\alpha(\theta_t) \equiv \mathcal{N}(1, 1/\theta_t)$. It holds that $\beta(\theta) = \theta\alpha(\theta)$ and $\theta\alpha'(\theta)/\alpha(\theta) = \chi(\theta) - 1$ with matching elasticity $\chi(\theta) \equiv \theta\beta'(\theta)/\beta(\theta)$.[4]

3.1 Preferences

The preferences of a typical seller and a typical buyer are captured by the utility functions

$$\mathcal{U}^s = \mathbb{E}_0\left\{ -\sum_{n=1}^{\infty} e^{-\rho T_{0,n}^s} c\left(q_{T_{0,n}^s}^s\right) - \int_0^{\infty} e^{-\rho\tau}\zeta(\varepsilon_\tau^s)\,\mathrm{d}\tau + \int_0^{\infty} e^{-\rho\tau}\,\mathrm{d}X_\tau^s \right\} \tag{3}$$

[2] Our notation means that $1/\phi_t$ units of fiat money buy one unit of general goods at time t.

[3] We thus replicate the singleness of money: privately created units of money (e.g., JPMorgan USD) trade at par with publicly created units of money (Fed USD).

[4] Constant returns to scale imply that $\chi(\theta) \in [0,1]$.

and

$$\mathcal{U}^b = \mathbb{E}_0\left\{ \sum_{n=1}^{\infty} e^{-\rho T^b_{0,n}} u\left(q^b_{T^b_{0,n}}\right) - \int_0^{\infty} e^{-\rho\tau}\zeta(\varepsilon^b_\tau)\,\mathrm{d}\tau + \int_0^{\infty} e^{-\rho\tau}\,\mathrm{d}X^b_\tau \right\}. \tag{4}$$

Sellers and buyers have the same rate of time preference ρ. The seller derives disutility $c(q)$ from producing search goods, whereas the buyer derives utility $u(q)$ from consuming them. We assume $u'(q) > 0$, $u''(q) < 0$, $u(0) = c(0) = c'(0) = 0$, $u'(0) = +\infty$, $c'(q) > 0$, $c''(q) \geq 0$, and $u'(q^*) = c'(q^*)$ for some $q^* > 0$. Devoting search effort $\varepsilon \geq 0$ entails cost $\zeta(\varepsilon)$, where $\zeta(0) = \zeta'(0) = 0$, $\zeta'(\varepsilon) > 0$, and $\zeta''(\varepsilon) > 0$. A seller's n-th match after time t occurs at arrival time $T^s_{t,n}$, governed by a Poisson process with time-dependent matching rates $\{\varepsilon^s_\tau\beta(\theta^s_\tau)\}^{\infty}_{\tau=t}$.[5] The matching rate at time $\tau \geq t$ is determined by the seller's search effort ε^s_τ and the tightness θ^s_τ of the search market where the seller is active. We define $T^b_{t,n}$ and $\{\varepsilon^b_\tau\alpha(\theta^b_\tau)\}^{\infty}_{\tau=t}$ analogously for the buyer. Finally, X^s_t and X^b_t denote the cumulative net consumption of general goods by the seller and by the buyer.[6] The production and consumption of general goods can take place in flows or discretely.

A typical FI's preferences are captured by the utility function

$$\mathcal{U}^{FI} = \int_0^{\infty} e^{-\rho\tau}\,\mathrm{d}X^{FI}_\tau, \tag{5}$$

where X^{FI}_t denotes the FI's cumulative net consumption of general goods. Since there is a continuum of identical and perfectly competitive FIs, we focus on a representative FI. We next detail the financial side of the economy.

3.2 Loans and Money

Financial intermediaries. The representative FI provides loans, issues private money, and holds its own stake in its operations, i.e., capital.[7] Competing in the markets for private money and loans, the FI takes the processes $\{r^p_t\}^{\infty}_{t=0}$ and $\{r^\ell_t\}^{\infty}_{t=0}$ of real returns on private money and loans, respectively, as given. We let ℓ^{FI}_t be the value of the FI's loan contracts, a^{FI}_t the value of private money issued, and k_t its capital, so that $\ell^{FI}_t = a^{FI}_t + k_t$.[8] We assume the FI can intermediate at most a fraction η of its claims on loan repayment, i.e., its claims from credit extension. Having $\eta < 1$ could be the result of regulation or of the FI's incentive to abscond when it has too much leverage. Thus, $a^{FI}_t \leq \eta\ell^{FI}_t$.

[5] We outline mathematical details about Poisson processes in Appendix B.

[6] X^s_t and X^b_t are restricted to the set of functions of bounded variation on any finite time interval. Hence, the Riemann-Stieltjes integrals of $e^{-\rho t}$ w.r.t. X^s_t and X^b_t, respectively, exist.

[7] In order to examine the FI's dual role of granting loans and creating money simultaneously, we use the term "private money" instead of "deposits".

[8] Private money has maturity zero as it can be withdrawn upon demand, but bank runs are ruled out—the FI can produce general goods to satisfy withdrawals. The loans have a maturity that depends on their underlying contractual specifications, but the FI can trade these contracts in a continuously-open market at the actuarially fair value.

The FI's cumulative consumption reads as

$$X_t^{FI} = a_0^{FI} - \ell_0^{FI} + \int_0^t \left(-\dot{\ell}_\tau^{FI} + r_\tau^\ell \ell_\tau^{FI} + \dot{a}_\tau^{FI} - r_\tau^p a_\tau^{FI} \right) \mathrm{d}\tau. \tag{6}$$

With transversality condition $\lim_{t\to\infty} e^{-\rho t} k_t = 0$, the FI's continuation value thus is

$$V_t^{FI} = \mathbb{1}_{\{t>0\}} k_t + \int_t^\infty e^{-\rho(\tau-t)} \left[(\rho - r_\tau^p) a_\tau^{FI} - (\rho - r_\tau^\ell) \ell_\tau^{FI} \right] \mathrm{d}\tau. \tag{7}$$

We explain the economic meaning of the spreads $\rho - r_t^\ell$ and $\rho - r_t^p$ further below.

Sellers. Sellers can prepone consumption by writing loan contracts with the FI, a process we call borrowing. Loan contracts specify contingent claims on sellers' future revenues from bilateral matches. A seller borrows to exploit the *borrowing discount* $\nu_t \equiv \rho - r_t^\ell$, being the difference between his/her rate of time preference and the interest rate on loans.

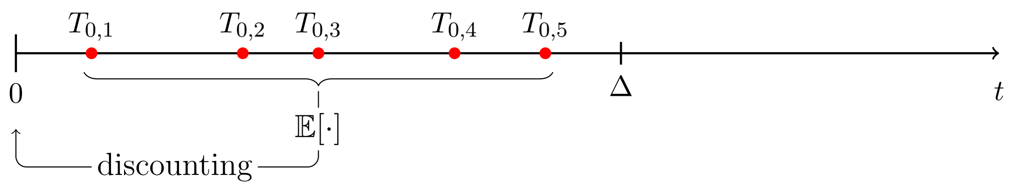
Borrowing works as follows: at time $t = 0$, the seller writes loan contracts specifying the process of contingent repayments $\{d_t^s\}_{t=0}^\Delta$ in time period $(0, \Delta]$. Particularly, payment of d_t^s occurs if and only if the seller obtains a match at time t, and is bounded by the match revenue p_t^s. Parameter $\Delta > 0$ reflects a technological friction in borrowing: loans that are contingently repayable at time t cannot be stipulated earlier than at time $t - \Delta$ since borrowers cannot commit to repayment over longer time spans than Δ. We call Δ the *pledgeability horizon*.[9] In return for the promised contingent repayments, the seller receives a payment of general goods from the FI at $t = 0$ that equals the contract's value

$$\ell_0^s = \int_0^\Delta e^{-\int_0^\tau r_s^\ell \,\mathrm{d}s} \, \varepsilon_\tau^s \beta(\theta_\tau^s) d_\tau^s \,\mathrm{d}\tau. \tag{8}$$

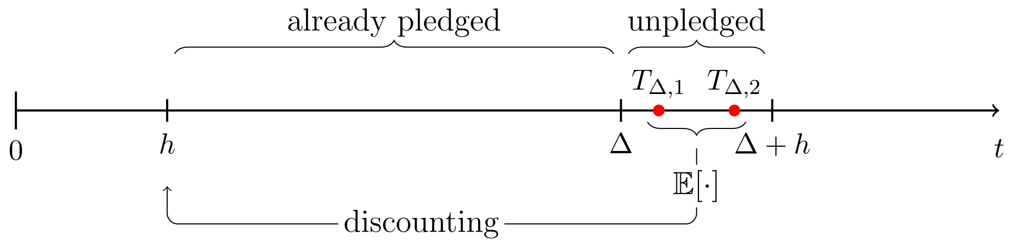
The contract's value equals the expected discounted value of contingent repayments $\{d_t^s\}_{t=0}^\Delta$, which occur at the sellers' matching rates $\{\varepsilon_t^s \beta(\theta_t^s)\}_{t=0}^\Delta$ and are discounted by real borrowing rates $\{r_t^\ell\}_{t=0}^\Delta$. To correctly value the contract at $t = 0$, it is therefore essential that (i) the seller can either ex-ante commit to search effort $\{\varepsilon_t^s\}_{t=0}^\Delta$ and market tightnesses $\{\theta_t^s\}_{t=0}^\Delta$, or that (ii) $\{\varepsilon_t^s\}_{t=0}^\Delta$ and $\{\theta_t^s\}_{t=0}^\Delta$ are in line with the seller's ex-post optimization, i.e., they are incentive feasible once the contract is written. We address these issues in the sellers' optimization problem. The initial borrowing is illustrated in Figure 1a, where $T_{0,1}, \ldots, T_{0,5}$ denote some exemplary arrival times of matches.

We illustrate borrowing at times $t > 0$ by considering what happens when the seller and the FI wait until time $h > 0$ (small) to write another loan contract after the initial borrowing at time $t = 0$. The seller has no reason to change or extend the already existing loan contracts with contingent repayments in time interval $(0, \Delta]$ because of

[9] The pledgeability horizon Δ can be microfounded by the assumption that creditors can deviate from any financial contract, e.g., by running away, after some time span.



(a) Initial borrowing.



(b) Borrowing immediately after time $t = 0$.

Figure 1: Borrowing.

perfect foresight. But similar to the initial borrowing, s/he can now contract contingent repayments $\{d_t^s\}_{t=\Delta}^{\Delta+h}$ within time interval $(\Delta, \Delta+h]$. In return, s/he receives a lump-sum payment at time h that becomes the flow payment

$$\lim_{h\to 0} \frac{1}{h} \int_{\Delta}^{\Delta+h} \exp\left(-\int_h^\tau r_s^\ell \, \mathrm{d}s\right) \varepsilon_\tau^s \beta(\theta_\tau^s) d_\tau^s \, \mathrm{d}\tau = \exp\left(-\int_0^\Delta r_s^\ell \, \mathrm{d}s\right) \varepsilon_\Delta^s \beta(\theta_\Delta^s) d_\Delta^s \tag{9}$$

in transition to borrowing in continuous time. Extending this argument to all $t > 0$, s/he receives the flow

$$\exp\left(-\int_t^{t+\Delta} r_s^\ell \, \mathrm{d}s\right) \varepsilon_{t+\Delta}^s \beta(\theta_{t+\Delta}^s) d_{t+\Delta}^s \tag{10}$$

at time t, and the value of his/her pending loan contracts is

$$\ell_t^s = \int_t^{t+\Delta} \left\{ \exp\left(-\int_t^\tau r_s^\ell \, \mathrm{d}s\right) \varepsilon_\tau^s \beta(\theta_\tau^s) d_\tau^s \right\} \mathrm{d}\tau. \tag{11}$$

We distinguish two monitoring technologies the FI uses to ensure the sellers' loan repayment. With *contract control*, the FI can track which future revenues sellers have already pledged, and can seize sellers' match revenues. Contract control is thus essential for borrowing, so we assume its availability throughout. *Project control* allows to ex-ante contract and ex-post enforce a seller's search-market decisions. The availability of project control thus enlarges the set of enforceable loan contracts since search-market decisions then need not be ex-post optimal for the seller. While Δ captures the degree of contract enforcement in terms of the time horizon over which loan contracts can be written, project control captures the degree of enforcement in terms of contractable actions.

Buyers. A buyer's real private (fiat) money demand is a_t^b (resp. m_t^b) and his/her transversality condition reads as $\lim_{t\to\infty} e^{-\rho t}[a_t^b + m_t^b] = 0$. When the economy starts, the buyer holds zero money and buys $a_0^b, m_0^b \geq 0$. In contrast to sellers, buyers cannot borrow since they lack any future revenues they could pledge to the FI. The continuation value the buyer derived from holding $\{a_\tau^b\}_{\tau=t}^\infty$ and $\{m_\tau^b\}_{\tau=t}^\infty$ reads as

$$- \mathbb{1}_{\{t=0\}}[a_0^b + m_0^b] + \int_t^\infty e^{-\rho(\tau-t)}[-\dot{a}_\tau^b - \dot{m}_\tau^b + r_\tau^p a_\tau^b - \pi_\tau m_\tau^b]\,\mathrm{d}\tau$$
$$= \mathbb{1}_{\{t>0\}}[a_t^b + m_t^b] - \int_t^\infty e^{-\rho(\tau-t)}\left[\iota_\tau^p a_\tau^b + \iota_\tau^m m_\tau^b\right]\mathrm{d}\tau, \quad (12)$$

where $\iota_t^p \equiv \rho - r_t^p$ and $\iota_t^m \equiv \rho + \pi_t$ are the *opportunity costs* of holding private and fiat money, respectively.

4 Optimal Choices

We now consider the agents' optimal choices, which we describe as dependent on the borrowing discounts $\{\nu_t\}_{t=0}^\infty$ and opportunity costs $\{\iota_t^p\}_{t=0}^\infty$ and $\{\iota_t^m\}_{t=0}^\infty$. From an individual agent's perspective, a search market is sufficiently characterized by (q, p, θ)—posted terms of trade (q, p), i.e., the quantity of traded search goods and the associated payment, and market tightness θ. For convenience and following the literature, p refers to the total payment a seller receives for producing q. We let Ω_t be the set of open search markets $(q, p, \theta) \in \mathbb{R}_+^3$ and denote the option not to participate in any search market, i.e., to drop out of the search markets, as being in *pseudo market* $(0, 0, 0)$.

4.1 Buyers

A buyer's flow value when locating himself/herself in search market $(q_t^b, p_t^b, \theta_t^b)$ reads as

$$v_t^b = \varepsilon_t^b \alpha(\theta_t^b)[u(q_t^b) - p_t^b]\mathbb{1}_{\{a_t^b + m_t^b \geq p_t^b\}} - \zeta(\varepsilon_t^b) - [\iota_t^p a_t^b + \iota_t^m m_t^b]. \quad (13)$$

S/he receives the match surplus $u(q_t^b) - p_t^b$ at matching rate $\varepsilon_t^b \alpha(\theta_t^b)$ if s/he carries enough money to make payment p_t^b, and s/he faces opportunity cost $\iota_t^p a_t^b + \iota_t^m m_t^b$ of holding money. It must hold that $\iota_t^p, \iota_t^m \geq 0$ for money demand to be bounded, which we assume to hold true in what follows. The buyer's continuation value thus reads as

$$V_t^b = \int_t^\infty e^{-\rho(\tau-t)} v_\tau^b\,\mathrm{d}\tau + \mathbb{1}_{\{t>0\}}[a_t^b + m_t^b] + \int_t^\infty e^{-\rho(\tau-t)}\dot{\Upsilon}_\tau\,\mathrm{d}\tau + \mathbb{1}_{\{t=0\}}\Upsilon_0, \quad (14)$$

where s/he receives cumulative lump-sum transfers $\{\Upsilon_t\}_{t=0}^{\infty}$ regardless of his/her participation in any search market. For every t, s/he solves the static optimization problem

$$\max_{q_t^b, p_t^b, \theta_t^b, \varepsilon_t^b, a_t^b, m_t^b} \left\{ \varepsilon_t^b \alpha(\theta_t^b)[u(q_t^b) - p_t^b] - \zeta(\varepsilon_t^b) - [\iota_t^p a_t^b + \iota_t^m m_t^b] \right\}, \tag{15}$$

$$\text{s.t.} \qquad p_t^b \leq a_t^b + m_t^b \quad \text{and} \quad (q_t^b, p_t^b, \theta_t^b) \in \Omega_t \cup \{(0,0,0)\}. \tag{16}$$

S/he can always attain flow value $v_t^b = 0$ by participating in pseudo market $(0,0,0)$. Throughout, we focus on equilibria in which private and fiat money are at positive supply; without coexistence, there is no transmission of monetary policy. Since both assets are perfect substitutes in payment, this requires they earn the same real return; otherwise, the buyers strictly prefer holding one over the other. We therefore use without loss of generality

Assumption 1. $r_t^p = -\pi_t$ *for all* $t \in [0, \infty)$.

This directly implies that $\iota_t^p = \iota_t^m \equiv \iota_t$, and the buyer's money demand fulfills

$$a_t^b + m_t^b = (\geq)\ p_t^b \qquad \text{if} \qquad \iota_t > (=)\ 0. \tag{17}$$

4.2 Financial Intermediaries

The FI writes loan contracts and creates private money. It is clear from the FI's continuation value V_t^{FI} in Equation (7) that for every t, it solves the static maximization problem

$$\max_{\ell_t^{FI}, a_t^{FI}} \left\{ \iota_t a_t^{FI} - \nu_t \ell_t^{FI} \right\}, \tag{18}$$

$$\text{s.t.} \qquad a_t^{FI} \leq \eta \ell_t^{FI}, \quad \ell_t^{FI} \geq 0, \quad \text{and} \quad a_t^{FI} \geq 0. \tag{19}$$

Lemma 1. *The FI's loan issuance* ℓ_t^{FI} *and private-money creation* a_t^{FI} *are positive and bounded only if both* $\iota_t \geq 0$ *and* $\nu_t = \eta \iota_t$. *If* $\iota_t > 0$, *it holds that* $a_t^{FI} = \eta \ell_t^{FI}$.

If $\iota_t < 0$, the FI does not create any private money since it is costly to do so. If $\iota_t > 0$, private-money creation is profitable, so that the FI creates as much private money $a_t^{FI} = \eta \ell_t^{FI}$ as it can back given the loans on its balance sheet. The identity $\nu_t = \eta \iota_t$ must then hold for private-money creation to be positive and bounded; for $\eta \iota_t > \nu_t$, money creation would be excessive, and for $\eta \iota_t < \nu_t$, money creation would not be profitable enough to compensate for the costs of loan issuance. Rewriting the FI's profit, where r_t^k denotes the real return on capital $k_t = \ell_t^{FI} - a_t^{FI}$, yields

$$\iota_t a_t^{FI} - \nu_t \ell_t^{FI} = r_t^\ell \ell_t^{FI} - r_t^p a_t^{FI} - r_t^k k_t \qquad \Rightarrow \qquad r_t^k = \rho. \tag{20}$$

Hence, capital earns the rate of time preference, reflecting its illiquidity to the FI. If $\iota_t = 0$, money creation is neither profitable nor costly, so that the FI is indifferent between all feasible levels $a_t^{FI} \leq \eta \ell_t^{FI}$. However, $\nu_t = 0$ is necessary to have positive and bounded loan issuance.

4.3 Sellers

A seller's flow value when locating himself/herself in search market $(q_t^s, p_t^s, \theta_t^s)$ and writing a loan contract with repayment d_t^s contingent on being matched at time $t + \Delta$ reads as

$$v_t^s = \varepsilon_t^s \beta(\theta_t^s)[p_t^s - d_t^s - c(q_t^s)] - \zeta(\varepsilon_t^s) + \exp\left(-\int_t^{t+\Delta} (\rho - \nu_s)\, \mathrm{d}s\right) \varepsilon_{t+\Delta}^s \beta(\theta_{t+\Delta}^s) d_{t+\Delta}^s. \tag{21}$$

S/he expects a match with surplus $p_t^s - c(q_t^s)$ net of repayment d_t^s to occur at matching rate $\varepsilon_t^s \beta(\theta_t^s)$. S/he incurs search cost $\zeta(\varepsilon_t^s)$ and benefits from writing a new loan contract. Taking into account initial borrowing, his/her continuation value reads as

$$V_t^s = \int_t^\infty e^{-\rho(\tau - t)} v_\tau^s \, \mathrm{d}\tau + \mathbb{1}_{\{t=0\}} \int_0^\Delta \exp\left(-\int_0^\tau (\rho - \nu_s)\, \mathrm{d}s\right) \varepsilon_\tau^s \beta(\theta_\tau^s) d_\tau^s \, \mathrm{d}\tau. \tag{22}$$

The nature of the seller's optimization problem depends on whether project control is present.

Project control. The seller's optimization problem reduces to a sequence of static problems. S/he solves

$$\max_{q_t^s, p_t^s, \theta_t^s, \varepsilon_t^s, d_t^s} \left\{ \varepsilon_t^s \beta(\theta_t^s) \left[\left(\exp\left(\int_{\max\{t-\Delta,0\}}^t \nu_s \, \mathrm{d}s \right) - 1 \right) d_t^s + p_t^s - c(q_t^s) \right] - \zeta(\varepsilon_t^s) \right\}, \tag{23}$$

$$\text{s.t.} \quad \max_{\varepsilon_t^{b\prime}} \left\{ \varepsilon_t^{b\prime} \alpha(\theta_t^s)[u(q_t^s) - p_t^s] - \zeta(\varepsilon_t^{b\prime}) - \iota_t p_t^s \right\} \geq v_t^{b\prime}, \tag{24}$$

$$d_t^s \leq p_t^s, \tag{25}$$

at time $\max\{t - \Delta, 0\}$. Particularly, at time $\max\{t - \Delta, 0\}$, s/he contracts search effort ε_t^s, market choice $(q_t^s, p_t^s, \theta_t^s)$, and contingent repayment d_t^s, which affects $v_{\max\{t-\Delta,0\}}^s$ through borrowing and v_t^s through net match surplus and search cost. Equation (24) reflects that a buyer must be able to realize flow value $v_t^{b\prime}$ in search market $(q_t^s, p_t^s, \theta_t^s)$, where $v_t^{b\prime}$ is the highest flow value a buyer can realize given the set of open search markets Ω_t.

The seller borrows up to the limit, i.e., $d_t^s = p_t^s$, when the accumulative borrowing discount $\int_{\max\{t-\Delta,0\}}^t \nu_\tau \, \mathrm{d}\tau$ is positive. When $\int_{\max\{t-\Delta,0\}}^t \nu_\tau \, \mathrm{d}\tau = 0$, the seller is indifferent between all levels of contingent repayment $d_t^s \leq p_t^s$, since s/he does not benefit from preponing consumption. The terms of trade s/he posts are characterized in

Lemma 2. *If the FI exerts project control, the seller optimally demands payment*

$$p_t^s = [1 - \omega_t^s]u(q_t^s) + \omega_t^s \exp\left(-\int_{\max\{t-\Delta,0\}}^t \nu_\tau \, \mathrm{d}\tau\right) c(q_t^s), \tag{26}$$

where

$$\omega_t^s \equiv \frac{\chi(\theta_t^s)\left[1 + \frac{\iota_t}{\varepsilon_t^{b\prime}\alpha(\theta_t^s)}\right]}{\chi(\theta_t^s)\left[1 + \frac{\iota_t}{\varepsilon_t^{b\prime}\alpha(\theta_t^s)}\right] + 1 - \chi(\theta_t^s)}, \tag{27}$$

and s/he offers quantity q_t^s, implicitly determined by

$$\frac{u'(q_t^s)}{c'(q_t^s)} = \exp\left(-\int_{\max\{t-\Delta,0\}}^t \nu_\tau \, \mathrm{d}\tau\right)\left[1 + \frac{\iota_t}{\varepsilon_t^{b\prime}\alpha(\theta_t^s)}\right]. \tag{28}$$

Everything else constant, p_t^s decreases with $\int_{\max\{t-\Delta,0\}}^t \nu_\tau \, \mathrm{d}\tau$. This allows the seller to exploit the borrowing discount to a larger extent: by posting a lower p_t^s, s/he attracts more buyers and incentivizes them to devote more search effort $\varepsilon_t^{b\prime}$. This increases the matching rate and thus the present value of the future income s/he can borrow against. The buyers' *effective opportunity cost* $\iota_t / \varepsilon_t^{b\prime}\alpha(\theta_t^s)$ of carrying money—loosely speaking, the product of the opportunity cost and the buyers' expected arrival time of the next match—drives down p_t^s for standard reasons: buyers require compensation for the time they hold costly money balances until they are able to spend them. Equation (28) implies that q_t^s increases with $\int_{\max\{t-\Delta,0\}}^t \nu_\tau \, \mathrm{d}\tau$ for the same reason as p_t^s decreases—the seller exploits the borrowing discount by posting more attractive terms of trade. However, q_t^s decreases with $\iota_t / \varepsilon_t^{b\prime}\alpha(\theta_t^s)$, as, ceteris paribus, the payment the seller can demand for each quantity decreases according to Constraint (24).

No project control. The seller now cannot commit to search effort ε_t^s and market choice $(q_t^s, p_t^s, \theta_t^s)$ at time $\max\{t - \Delta, 0\}$. Instead, given the contingent repayment d_t^s contracted at time $\max\{t - \Delta, 0\}$, s/he chooses ε_t^s and $(q_t^s, p_t^s, \theta_t^s)$ at time t to solve the ex-post static maximization problem[10]

$$\max_{q_t^s, p_t^s, \theta_t^s, \varepsilon_t^s} \left\{\varepsilon_t^s \beta(\theta_t^s)[-c(q_t^s) + p_t^s - d_t^s] - \zeta(\varepsilon_t^s)\right\}, \tag{29}$$

$$\text{s.t.} \qquad \max_{\varepsilon_t^{b\prime}} \left\{\varepsilon_t^{b\prime}\alpha(\theta_t^s)[u(q_t^s) - p_t^s] - \zeta(\varepsilon_t^{b\prime}) - \iota_t p_t^s\right\} \geq v_t^{b\prime}. \tag{30}$$

[10] To be precise, the seller's match surplus is $-c(q_t^s) + \max\{p_t^s - d_t^s, 0\}$, as the seller cannot be forced to repay more than p_t^s. We can, however, write $\max\{p_t^s - d_t^s, 0\} = p_t^s - d_t^s$ w.l.o.g., as $p_t^s \leq d_t^s$ would immediately imply that $\varepsilon_t^s = 0$. The matching rate of would become zero and the precise value of the promised repayment would be irrelevant.

Let $(\tilde{q}^s_t, \tilde{p}^s_t, \tilde{\theta}^s_t, \tilde{\varepsilon}^s_t)$ denote the associated choices as functions of d^s_t and $v^{b\prime}_t$. Anticipating these ex-post choices, the seller contracts d^s_t at time $\max\{t-\Delta, 0\}$ to solve

$$\max_{d^s_t} \left\{ \tilde{\varepsilon}^s_t \beta(\tilde{\theta}^s_t) \left[\left(\exp\left(\int_{\max\{t-\Delta,0\}}^t \nu_s \,\mathrm{d}s \right) - 1 \right) d^s_t + \tilde{p}^s_t - c(\tilde{q}^s_t) \right] - \zeta(\tilde{\varepsilon}^s_t) \right\}. \tag{31}$$

Comparing the problems in Equations (23) and (31) clarifies that the absence of project control reduces the vector of contractable objects from $(q^s_t, p^s_t, \theta^s_t, \varepsilon^s_t, d^s_t)$ to d^s_t.

We write $(q^s_t, p^s_t, \theta^s_t, \varepsilon^s_t) = (\tilde{q}^s_t, \tilde{p}^s_t, \tilde{\theta}^s_t, \tilde{\varepsilon}^s_t)$ in what follows because of incentive compatibility. Applying the envelope theorem to the maximization problem in (29), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}d^s_t} \Big[\varepsilon^s_t \beta(\theta^s_t)[-c(q^s_t) + p^s_t - d^s_t] - \zeta(\varepsilon^s_t) \Big] = -\, \varepsilon^s_t \beta(\theta^s_t). \tag{32}$$

Hence, the first-order condition w.r.t. d^s_t from the optimization in (31) reads as

$$0 = \varepsilon^s_t \beta(\theta^s_t) \left[\exp\left(\int_{\max\{t-\Delta,0\}}^t \nu_\tau \,\mathrm{d}\tau \right) - 1 \right] + \frac{\mathrm{d}[\varepsilon^s_t \beta(\theta^s_t)]}{\mathrm{d}d^s_t} \exp\left(\int_{\max\{t-\Delta,0\}}^t \nu_\tau \,\mathrm{d}\tau \right) d^s_t. \tag{33}$$

An increase of d^s_t has two effects on the seller's utility. The first term in Equation (33) captures the direct effect, namely, the seller scales up his/her gains from the borrowing discount. The second term reflects the indirect effect of d^s_t on the seller's ex-post behavior at time t: because d^s_t acts as a fixed cost in the maximization problem in (29), it lowers $\varepsilon^s_t \beta(\theta^s_t)$ and thus the actuarially fair value of the seller's loan contract. The relation between $\int_{\max\{t-\Delta,0\}}^t \nu_\tau \,\mathrm{d}\tau$ and d^s_t is characterized in

Lemma 3. *If $\int_{\max\{t-\Delta,0\}}^t \nu_\tau \,\mathrm{d}\tau = 0$, then $d^s_t = 0$. Moreover,*

$$\frac{\mathrm{d}d^s_t}{\mathrm{d}\left[\int_{\max\{t-\Delta,0\}}^t \nu_\tau \,\mathrm{d}\tau\right]} > 0. \tag{34}$$

In contrast to the case with project control, the seller is not indifferent between all $d^s_t \leq p^s_t$ when $\int_{\max\{t-\Delta,0\}}^t \nu_\tau \,\mathrm{d}\tau = 0$, but s/he chooses $d^s_t = 0$. If s/he chose $d^s_t > 0$ instead, s/he could not realize positive gains from borrowing, but s/he would only suffer from a lower debt-induced matching rate.

The seller's optimal terms of trade are characterized in

Lemma 4. *If the FI does not exert project control, the seller optimally demands payment*

$$p^s_t = [1 - \omega^s_t] u(q^s_t) + \omega^s_t [c(q^s_t) + d^s_t], \tag{35}$$

where ω^s_t is defined as in Equation (27), and s/he offers quantity q^s_t, implicitly determined *by*

$$\frac{u'(q_t^s)}{c'(q_t^s)} = 1 + \frac{\iota_t}{\varepsilon_t^{b\prime}\alpha(\theta_t^s)}. \tag{36}$$

The effective opportunity cost $\iota_t / \varepsilon_t^{b\prime}\alpha(\theta_t^s)$ affects (q_t^s, p_t^s) in the same direct way as in the case with project control. However, the accumulative borrowing discount $\int_{\max\{t-\Delta,0\}}^{t} \nu_\tau \, \mathrm{d}\tau$ has only an indirect effect. Since the seller's gains from borrowing have already materialized, $\int_{\max\{t-\Delta,0\}}^{t} \nu_\tau \, \mathrm{d}\tau$ is irrelevant for the seller's ex-post problem—only d_t^s matters by acting as a fixed cost in search-good production. Higher $\int_{\max\{t-\Delta,0\}}^{t} \nu_\tau \, \mathrm{d}\tau$ increases d_t^s though since it stimulates borrowing, and thus reduces the ex-post match surplus. In particular, d_t^s increases the payment p_t^s since the seller requires partial compensation for his/her contingent repayment.

5 Equilibrium

We concentrate on equilibria in which private and fiat money coexist—this justifies Assumption 1—and in which besides the pseudo market only one search market is open at a time, where all active sellers and buyers locate themselves. Regardless of the enforcement technology, the sellers can attain a positive flow value $v_t^s > 0$ in any equilibrium, so that all sellers are active ($\mu_t^s = 1$). However, this is not true for buyers; their flow value v_t^b hits zero if the search market implies a large effective opportunity cost $\iota_t / \varepsilon_t^b\alpha(\theta_t)$ of holding money. Buyers then are indifferent between being active in the search market and staying passive, i.e., locating themselves in the pseudo market $(q_t, p_t, \theta_t) = (0, 0, 0)$. This can result in partial participation by buyers ($\mu_t^b < 1$). We thus distinguish two types of equilibria: we call equilibria in which $\mu_t^b = 1$ *full-participation equilibria* (FPE), and we call equilibria in which $\mu_t^b < 1$ *partial-participation equilibria* (PPE). Our notion of general equilibrium is formalized in

Definition 1. *Given the process of fiat-money supply $\{M_t\}_{t=0}^\infty$, an equilibrium is a process of cumulative transfers $\{\Upsilon_t\}_{t=0}^\infty$, search markets $\{\Omega_t\}_{t=0}^\infty = \{q_t, p_t, \theta_t\}_{t=0}^\infty$, masses of active buyers $\{\mu_t^b\}_{t=0}^\infty$, search efforts $\{\varepsilon_t^s, \varepsilon_t^b\}_{t=0}^\infty$, contingent repayments $\{d_t\}_{t=0}^\infty$, values of loan contracts $\{\ell_t\}_{t=0}^\infty$, private- and fiat-money holdings $\{a_t, m_t\}_{t=0}^\infty$, money prices $\{\phi_t\}_{t=0}^\infty$, and opportunity costs and borrowing discounts $\{\iota_t, \nu_t\}_{t=0}^\infty$, so that*

(i) $(\ell_t, \mu_t^b a_t)$ *solves the FI's maximization problem in* (18)*, s.t.* (19)*;*

(ii) $(q_t, p_t, \theta_t, \varepsilon_t^b, a_t, m_t)$ *solves the buyers' maximization problem in* (15)*, s.t.* (16)*, opportunity costs* $\iota_t^p = \iota_t^m = \iota_t$*, and transversality condition* $\lim_{t\to\infty} e^{-\rho t}[a_t + m_t] = 0$*;*

(iii) the sellers take the buyers' flow value $v_t^{b\prime} = \varepsilon_t^b\alpha(\theta_t)[u(q_t) - p_t] - \zeta(\varepsilon_t^b) - \iota_t[a_t + m_t]$ *as given, and*

(a) with project control, $(q_t, p_t, \theta_t, \varepsilon_t^s, d_t)$ *solves the maximization problem in* (23)*, s.t.* (24) *and* (25)*, and,*

(b) without project control, $(q_t, p_t, \theta_t, \varepsilon_t^s)$ *solves the maximization problem in* (29)*, s.t.* (30)*, and* d_t *solves the maximization problem in* (31)*;*

(iv) the search-market tightness, the mass of active buyers, and optimal search efforts correspond: $\theta_t = \mu_t^b \, \varepsilon_t^b / \, \varepsilon_t^s$*;*

(v) the government's budget constraint holds: $\Upsilon_t = \phi_0 M_0 + \int_0^t \phi_\tau \dot{M}_\tau \, \mathrm{d}\tau$*;*

(vi) the value of loan contracts is actuarially fair: ℓ_t *fulfills Equation* (11)*;*

(vii) the money market clears: $\phi_t M_t = \mu_t^b m_t$ *and* $\iota_t = \rho - \dot{\phi}_t / \phi_t$*.*

Welfare. Utilitarian welfare $\mathcal{W} = \mathcal{U}^s + \mathcal{U}^b + \mathcal{U}^{FI}$ is the aggregate utility of sellers, buyers, and the FI. We call the allocation $\{\varepsilon_t^{s\star}, \varepsilon_t^{b\star}, \mu_t^{b\star}, q_t^\star\}_{t=0}^\infty$ maximizing $\mathcal{W}$ *first best.*[11]

Proposition 1. *In equilibrium, welfare reads as*

$$\mathcal{W} = \int_0^\infty e^{-\rho t} \left[\mathcal{N}(\mu_t^b \, \varepsilon_t^b, \varepsilon_t^s)[u(q_t) - c(q_t)] - \zeta(\varepsilon_t^s) - \mu_t^b \zeta(\varepsilon_t^b) \right] \mathrm{d}t. \tag{37}$$

The first-best allocation is unique and time-invariant, it features $\mu^{b\star} = 1$*, and it satisfies the necessary and sufficient first-order conditions*

$$0 = u'(q^\star) - c'(q^\star), \tag{38}$$

$$\zeta'(\varepsilon^{s\star}) = \beta(\theta^\star)[1 - \chi(\theta^\star)][u(q^\star) - c(q^\star)], \qquad \textit{and} \tag{39}$$

$$\zeta'(\varepsilon^{b\star}) = \alpha(\theta^\star)\chi(\theta^\star)[u(q^\star) - c(q^\star)] \tag{40}$$

with $\theta^\star \equiv \varepsilon^{b\star} / \, \varepsilon^{s\star}$*.*

Only search-market activity matters for welfare since agents' utility is affine in general-goods consumption. The first-best quantity maximizes the match surplus, and the first-best search efforts equalize the agents' marginal search costs to the respective marginal matching rate, multiplied by the match surplus. The first-best allocation features full participation by buyers ($\mu^{b\star} = 1$) due to the convexity of ζ.

Monetary policy. We focus on stationary equilibria to study the transmission of long-run monetary policy. We drop time indices in what follows as long as clarity is maintained. Real fiat-money supply is constant in stationary equilibrium, so that

$$0 = \frac{(\phi \dot{M})}{\phi M} \qquad \Leftrightarrow \qquad -\frac{\dot{\phi}}{\phi} = \frac{\dot{M}}{M} \qquad \Leftrightarrow \qquad \pi = \gamma. \tag{41}$$

[11] We can easily aggregate welfare within each group of agents because of quasi-linear preferences.

We thus think of monetary policy determining inflation π as well as the opportunity cost of holding money $\iota = \pi + \rho$ through the growth rate of nominal money supply. Since ι rather than π matters for the cost of monetary activity, we consider ι as the primary monetary-policy variable. Only $\iota \geq 0$ are implementable since money demand would be unbounded otherwise. The policy $\iota = 0$ corresponds to the *Friedman rule* (FR), rendering the opportunity cost of holding money zero through a slight deflation.

In stationary equilibrium, ι is the *Fisher rate* since it solves the Fisher equation: $\iota = \pi + \rho$ is the sum of long-run inflation π and the natural rate, being identically equal to the rate of time preference ρ in a model without any notion of growth. Although we consider the Fisher rate as the relevant monetary-policy variable, we stress here that the Fisher rate should not be confused with the policy rate. To see this, recall that the Fisher equation is the long-run counterpart of the canonical Euler equation

$$\frac{1}{1+i_t} = \mathbb{E}_t \left[\frac{SDF_{t+1}\phi_{t+1}}{\phi_t} \right] \tag{42}$$

in a standard discrete-time asset-pricing model, where SDF_{t+1} is the stochastic discount factor, ϕ_t/ϕ_{t+1} is inflation, and i_t is the nominal interest rate on an illiquid short-term bond. Herrenbrueck and Wang (2019) argue that this Euler equation prices an asset we do not observe in reality. In particular, i_t should not be interpreted as the policy rate, e.g., the Federal funds rate in the U.S., since the policy rate prices liquid reserve balances. Moreover, the Fisher rate captures a positive long-run relationship of nominal interest rates with inflation that need not hold in the short run. In fact, the policy rate and the Fisher rate are arguably negatively correlated in the short run, e.g., when rate hikes are used to curb inflation.

6 Policy Transmission at the Friedman Rule

We consider monetary policy in the neighbourhood of the FR to provide analytical results—the non-linear nature of the model renders global analytical solutions intractable without imposing functional forms. We distinguish three different economies: a baseline economy with only fiat money (F), an economy with financial intermediation under project control (P), and an economy with financial intermediation with no project control (NP). Economy F is a special case of both economies P and NP when private-money creation is shut down through 100-percent capital requirements: $\eta = 0$. We consider two equilibria as equivalent if they feature the same allocation $(\varepsilon^s, \varepsilon^b, \mu^b, q)$. As a benchmark result, we derive

Lemma 5. *In each of the economies F, P, and NP, there is a unique equilibrium at the*

FR. This equilibrium yields the first-best allocation, and the payment

$$p^\star = [1 - \chi(\theta^\star)]u(q^\star) + \chi(\theta^\star)c(q^\star) \tag{43}$$

satisfies Hosios's (1990) efficiency condition.

Lemma 5 says that all three economies feature the same allocation at the FR. In particular, the equilibrium features full participation by buyers, so that buyers and sellers face the same matching rate $\mathcal{N}$. The FR renders the opportunity cost of holding money and the borrowing discount zero, such that there is no distortion either through private-money creation or through credit extension. Directed and competitive search then implies the payment $p^\star$ is such that all agents are remunerated according to the importance of their search in the matching function—the Hosios condition is satisfied. We next characterize the effects of a marginal deviation from the FR on matching rates in Proposition 2 and on search-market terms of trade in Proposition 3. We explain these propositions jointly below.

Proposition 2. *The local effects of a deviation from the FR on matching rates in economies F, P, and NP relate as*

$$\begin{aligned}&\left[\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\iota}\right]_P - \left[\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\iota}\right]_F \\ &= \Xi \left[\frac{1-\chi}{\sigma^s}\left[1 + \left[\frac{1-\chi-\varphi}{1-\chi}\right]\frac{1}{\sigma^b}\right]u(q^\star) + \frac{\chi}{\sigma^b}\left[1 + \left[\frac{1-\chi-\varphi}{1-\chi}\right]\frac{1}{\sigma^s}\right]c(q^\star)\right]\Delta\eta > 0\end{aligned} \tag{44}$$

and

$$\left[\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\iota}\right]_{NP} - \left[\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\iota}\right]_F = \Xi\left[\chi\left[\frac{1}{\sigma^b} + \frac{1-\chi}{\chi}\frac{1}{\sigma^s}\right] + \left[\frac{1-\chi-\varphi}{1-\chi}\right]\frac{1}{\sigma^b\sigma^s}\right]\left[\frac{\mathrm{d}d}{\mathrm{d}\iota}\right] < 0 \tag{45}$$

for

$$\Xi = \left[\frac{\mathcal{N}}{1 + [1-\chi-\varphi]\left[\frac{1}{\sigma^b} + \frac{\chi}{1-\chi}\frac{1}{\sigma^s}\right]}\right]\frac{1}{u(q^\star) - c(q^\star)} > 0 \tag{46}$$

and $\sigma(\varepsilon) \equiv \varepsilon\zeta''(\varepsilon)/\zeta'(\varepsilon)$ *and* $\varphi(\theta) \equiv \theta\chi'(\theta)/\chi(\theta)$ *as well as* $\chi \equiv \chi(\theta^\star)$, $\varphi \equiv \varphi(\theta^\star)$, $\sigma^s \equiv \sigma(\varepsilon^{s\star})$, *and* $\sigma^b \equiv \sigma(\varepsilon^{b\star})$.

Proposition 3. *The local effects of a deviation from the FR on search-market quantities in economies F, P, and NP relate as*

$$\left[\frac{\mathrm{d}q}{\mathrm{d}\iota}\right]_{NP} - \left[\frac{\mathrm{d}q}{\mathrm{d}\iota}\right]_F = 0 \qquad \text{and} \qquad \left[\frac{\mathrm{d}q}{\mathrm{d}\iota}\right]_P - \left[\frac{\mathrm{d}q}{\mathrm{d}\iota}\right]_F = \Delta\eta\left[\frac{c'(q^\star)}{c''(q^\star) - u''(q^\star)}\right] > 0. \tag{47}$$

The local effects of a deviation from the FR on search-market payments relate as

$$\left[\frac{\mathrm{d}p}{\mathrm{d}\iota}\right]_{NP} - \left[\frac{\mathrm{d}p}{\mathrm{d}\iota}\right]_{F} = \chi\Psi\left[\frac{\mathrm{d}d}{\mathrm{d}\iota}\right] \tag{48}$$

and

$$\left[\frac{\mathrm{d}p}{\mathrm{d}\iota}\right]_{P} - \left[\frac{\mathrm{d}p}{\mathrm{d}\iota}\right]_{F} = \Delta\eta\left[\frac{u'(q^\star)^2}{c''(q^\star)-u''(q^\star)} + \left[\frac{\varphi\chi[u(q^\star)-c(q^\star)]}{1+[1-\chi-\varphi]A}\right]\frac{1}{\sigma^s} - \Psi\chi c(q^\star)\right] \tag{49}$$

for

$$\Psi \equiv 1 + \frac{\varphi}{1+[1-\chi-\varphi]A}\left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]. \tag{50}$$

Baseline economy F. We elaborate on baseline economy F in relation to the existing literature before studying the role of financial intermediation. The economics behind how agents adjust their search to changes in ι become clear from sellers' and buyers' first-order conditions with respect to ε^s and ε^b,

$$\zeta'(\varepsilon^s) = \beta(\theta)[1-\omega][u(q)-c(q)] \quad \text{and} \quad \zeta'(\varepsilon^b) = \alpha(\theta)\omega[u(q)-c(q)], \tag{51}$$

where ω, as defined in Equation (27), denotes the buyers' share of the match surplus $u(q)-c(q)$. The agents devote search effort that equalizes their marginal cost and benefits from search. From the proof of Proposition 2, we immediately obtain

Corollary 1. *Consider economy F. It holds at the FR that*

$$\frac{\mathrm{d}\theta}{\mathrm{d}\iota} = \frac{[1-\chi]A}{1+[1-\chi-\varphi]A}\left[\frac{\theta}{\mathcal{N}}\right] > 0 \qquad \textit{and} \qquad \frac{\mathrm{d}\omega}{\mathrm{d}\iota} = \frac{1+[1-\chi]A}{1+[1-\chi-\varphi]A}\left[\frac{\chi[1-\chi]}{\mathcal{N}}\right] > 0, \tag{52}$$

and that

$$\frac{\mathrm{d}\,\varepsilon^s}{\mathrm{d}\iota} = -\frac{\frac{\varepsilon^s}{\sigma^s}\left[\frac{\chi}{\mathcal{N}}\right]}{1+[1-\chi-\varphi]A} < 0 \qquad \textit{and} \qquad \frac{\mathrm{d}\,\varepsilon^b}{\mathrm{d}\iota} = \frac{\frac{\varepsilon^b}{\sigma^b}\left[\frac{1-\chi}{\mathcal{N}}\right]}{1+[1-\chi-\varphi]A} > 0, \tag{53}$$

with $A \equiv 1/\sigma(\varepsilon^{b\star}) + [1/\sigma(\varepsilon^{s\star})]\chi/[1-\chi]$. *Moreover,*

$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\iota} = \left[\frac{1}{\sigma(\varepsilon^{b\star})} - \frac{1}{\sigma(\varepsilon^{s\star})}\right]\left[\frac{\chi[1-\chi]}{1+[1-\chi-\varphi]A}\right] \tag{54}$$

and

$$\frac{\mathrm{d}q}{\mathrm{d}\iota} = -\left[\frac{c'(q^\star)}{c''(q^\star)-u''(q^\star)}\right]\frac{1}{\mathcal{N}}. \tag{55}$$

In response to a deviation from the FR, buyers face a higher opportunity cost of holding money and reduce their real balances, so that sellers post a smaller quantity q and $u(q)-c(q)$ decreases. Sellers compensate buyers for this cost by posting a payment that assigns a larger surplus share ω to buyers. For a small deviation from the FR, the total surplus suffers only a second-order reduction since q is at its efficient level $q^\star$ at the FR. However, the increase in ω is of first-order importance because it shifts surplus from sellers to buyers. Buyers' marginal benefit from search thus increases, stimulating their search effort ε^b, whereas sellers' marginal benefit reduces, curbing their search effort ε^s. Market tightness θ consequently increases.

The net effect of agents changing search on the matching rate $\mathcal{N}$ is positive if and only if buyers increase their search by more than sellers decrease it. This is the case if the buyers' elasticity of search effort $1/\sigma(\varepsilon^{b\star})$ with respect to their marginal benefit from search is larger than the sellers' elasticity $1/\sigma(\varepsilon^{s\star})$ since the Hosios condition $\omega = \chi(\theta)$ holds at the FR: all agents are already remunerated according to the importance of their search in the matching function. If $\sigma(\varepsilon^{b\star}) = \sigma(\varepsilon^{s\star})$, i.e., if sellers and buyers adjust their search equally strongly in response to changes in their marginal benefits from search, a deviation from the FR has a zero first-order effect on $\mathcal{N}$.[12]

Corollary 1 confirms and generalizes the results of Lagos and Rocheteau (2005). They consider a fiat-money economy with directed and competitive search that differs from our baseline economy F in that only buyers' search is endogenous, whereas sellers' search is exogenous.[13] They find that the matching rate $\mathcal{N}$ increases in response to a deviation from the FR. Our model nests their setup in the limit $\sigma(\varepsilon^{s\star}) \to \infty$, i.e., when sellers' search is inelastic. In that case, Equation (54) implies $\mathrm{d}\mathcal{N}/\,\mathrm{d}\iota > 0$. Intuitively, sellers do not respond to the decline of their surplus share, so that the increase in buyers' search unambiguously results in a higher matching rate $\mathcal{N}$. Endogenizing also sellers' search, the reduction in sellers' search mitigates or even overturns the increase in $\mathcal{N}$ when $\sigma(\varepsilon^{s\star})$ is sufficiently small.

Economies P and NP. Economies P and NP differ from economy F in that they feature financial intermediation. We compare economies P and NP to understand how the presence of project control matters for how a deviation from the FR affects search-market trade. Relative to baseline economy F, the effects of the Fisher rate ι on the matching rate $\mathcal{N}$ and terms of trade (q, p), are characterized in Propositions 2 and 3.

In economy NP, a deviation from the FR reduces the matching rate: the Fisher rate ι generates a borrowing discount $\nu = \eta\iota$ that incentivizes sellers to increase their contingent repayment d as shown in Lemma 3. Because d is a fixed cost in the sellers' ex-post maximization problem (29), the sellers devote less search effort while reducing the posted quantity in the same way as in economy F. As a result, sellers and buyers match less frequently and $\mathcal{N}$ is lower than in economy F. When search elasticities of sellers and

[12]This, e.g., holds true if $\zeta(\varepsilon) = \varepsilon^{1+x}/(1+x)$ with $x > 0$, or if $\mathcal{N}(\varepsilon^b, \varepsilon^s) = (\varepsilon^b\,\varepsilon^s)^{1/2}$.

[13]Lagos and Rocheteau (2005) let buyers instead of sellers post terms of trade. However, it does not matter for equilibrium outcomes who posts terms of trade.

buyers are equal at the FR ($1/\sigma(\varepsilon^{s\star}) = 1/\sigma(\varepsilon^{b\star})$), sellers also unambiguously increase the posted payments.

In economy P, project control enables the sellers to affect the future income they borrow against through ex-ante commitment. The borrowing discount thus incentivizes sellers to commit to posting more attractive terms of trade and to devoting more search effort. In particular, sellers post larger q relative to economy F, whereas the adjustment of the posted payment is ambiguous. The matching rate $\mathcal{N}$ increases in contrast to economy F: credit accelerates matching when project control is present.

This acceleration is distinct from the hot-potato effect of inflation: long-run inflation $\pi = \iota - \rho$ accelerates trade but this is not because of the buyers' incentive to spend money faster when inflation increases, since sellers anticipate inflation and, ceteris paribus, become more reluctant to accept money. It is rather the transmission of inflation to a lower real interest rate on loans, which positively affects economic activity in the presence of project control.

Credit channel in economy P. Our results point towards a *credit channel* of long-run monetary policy in the presence of project control. An increase of long-run inflation lowers the real return on fiat money, and consequently, also the real return on private money decreases because of the perfect substitutability of fiat and private money in payment. The reduced real return on private money lowers the FI's funding costs, translating into a lower real return on loans. The borrowing discount increases, the volume of credit expands, and economic activity is stimulated, as described in Proposition 2.

7 Equilibrium away from the Friedman Rule

We now narrow down equilibrium existence and uniqueness away from the FR to obtain additional insights useful for calibrating the model and for understanding global equilibrium behaviour. We impose functional forms common in the literature:

Assumption 2. *Let $\sigma \in (0,1)$, $\kappa \geq 0$, $\tau > 0$, as well as $\chi \in (0,1)$. We define*

$$u(q) = \frac{q^{1-\sigma}}{1-\sigma}, \quad c(q) = \frac{q^{1+\kappa}}{1+\kappa}, \quad \zeta(\varepsilon) = \frac{\varepsilon^{1+\tau}}{1+\tau}, \quad \text{and} \quad \mathcal{N}(\varepsilon^b, \varepsilon^s) = (\varepsilon^b)^{\chi}(\varepsilon^s)^{1-\chi}. \quad (56)$$

Economy P. The effective opportunity cost $B \equiv \iota / \varepsilon^b \alpha(\theta)$ is the key endogenous variable determining equilibrium outcomes; B captures the cost of monetary activity in general and drives the terms of trade, as seen in Lemma 2. It turns out convenient to characterize the equilibrium relationship between ι, determined by policy, and B since this relationship yields equilibrium uniqueness and determines whether the equilibrium is an FPE or a PPE. We obtain

Proposition 4. *Consider economy P.*

1. *If*

$$1-\chi\left[\frac{\tau}{1+\tau}+\frac{\sigma+\kappa}{1+\kappa}\right]\leq 0, \tag{57}$$

any $\iota \geq 0$ *induces a unique FPE through*

$$\underbrace{\iota^{\tau}\exp\left(-\Delta\eta\left[\frac{1+\kappa}{\sigma+\kappa}-\chi\right]\iota\right)}_{\equiv f(\iota)}=\underbrace{B^{\tau}\omega^{\chi}(1-\omega)^{1-\chi}\mathcal{S}_B}_{\equiv g(B)}, \tag{58}$$

where

$$\omega\equiv\frac{\chi[1+B]}{1-\chi+\chi[1+B]},\qquad \mathcal{S}_B\equiv u(q_B)-c(q_B),\qquad \text{and}\qquad \frac{u'(q_B)}{c'(q_B)}\equiv 1+B. \tag{59}$$

2. *Suppose that Inequality* (57) *does not hold true. Let*

$$\overline{B}\equiv\frac{\left[\frac{\sigma+\kappa}{1+\kappa}\right]\frac{\chi\tau}{1+\tau}}{1-\chi\left[\frac{\tau}{1+\tau}+\frac{\sigma+\kappa}{1+\kappa}\right]}. \tag{60}$$

Every

$$\iota\in\left\{\iota\geq 0\ :\ f(\iota)>g(\overline{B})\right\}\equiv I \tag{61}$$

induces a unique PPE with $B=\overline{B}$ *and*

$$\mu^b=\left[\frac{g(\overline{B})}{f(\iota)}\right]^{\frac{1}{\tau(1-\chi)}}. \tag{62}$$

Every $\iota\in[0,\infty)\setminus I$ *induces a unique FPE with* $f(\iota)=g(B)$.

There is exactly one equilibrium for all $\iota \geq 0$. This equilibrium is either an FPE or a PPE. If Inequality (57) holds true, the buyers' flow match surplus $\varepsilon^b\alpha(\theta)[u(q)-p]$ is sufficiently large as compared to the cost of carrying money ιp and devoting search effort $\zeta(\varepsilon^b)$ to facilitate full participation by buyers ($\mu^b = 1$) for all ι. Hence, for all ι, there cannot be a PPE. The primitives σ, κ, χ, and τ must be jointly large enough for Inequality (57) to hold: if σ and κ are large, the total match surplus $u(q)-c(q)$ is large; if χ is large, buyers receive a large share of this surplus; if τ is large, search is cheap. If Inequality (57) does not hold true, there is a maximum $\overline{B}$ of all effective opportunity costs B that can occur in equilibrium; any $B > \overline{B}$ would drive the buyers' flow utility v^b below zero and would thus violate the buyers' participation constraint.

If Inequality (57) holds true, ι pins down B through Equation (58). If moreover $\Delta = 0$, Equation (58) describes a monotonically increasing map $\iota \mapsto B$ since ι curbs economic activity through making money balances expensive, so that ι spurs the effective

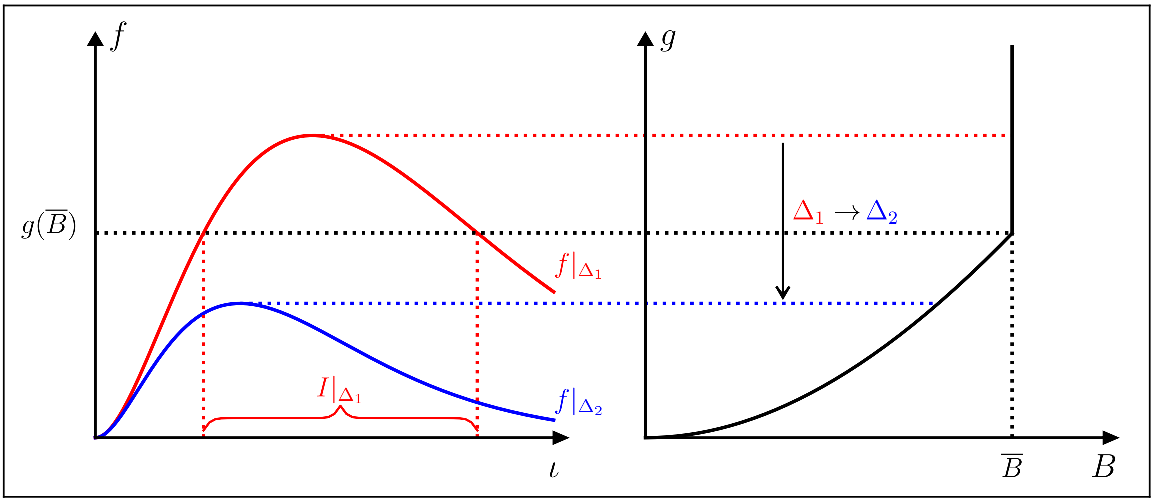


Figure 2: The existence of FPEs and PPEs in economy P dependent on $\Delta \in \{\Delta_1, \Delta_2\}$ with $0 < \Delta_1 < \Delta_2$.

opportunity cost B. If $\Delta > 0$, this choking effect of ι is outweighed by the stimulating effect of the borrowing discount $\nu = \eta\iota$ for large ι. This feature is captured by the unimodal shape of f together with the monotonicity of g, as shown in Figure 2.

If Inequality (57) does not hold true, PPEs can only emerge if Δ is small; sellers then have little incentives to increase their economic activity to exploit the borrowing discount, so that B reaches high levels. If Δ is sufficiently small to feature $\max_{\iota \geq 0} f|_\Delta(\iota) > g(\overline{B})$, any $\iota \in I|_\Delta$ induces a unique PPE with $B = \overline{B}$. The mass of participating buyers μ^b then adjusts endogenously to guarantee $v^b = 0$. If, however, Δ is large enough to have $\max_{\iota \geq 0} f|_\Delta(\iota) \leq g(\overline{B})$, it holds that $I|_\Delta = \emptyset$, so that all ι induce a unique FPE. Figure 2 illustrates these two cases for $\Delta \in \{\Delta_1, \Delta_2\}$ with $0 < \Delta_1 < \Delta_2$. Taking stock, we find that a long pledgeability horizon crowds in buyers in the project-control economy.

Economy NP. We examine how Fisher rate ι and the sellers' contingent debt d jointly determine search-market outcomes in Proposition 5.

Proposition 5. *Consider economy NP. Let all sellers enter their ex-post problem in* (29)*, subject to Constraint* (30)*, with contingent repayment* $d \geq 0$*, and let* $\overline{B}(d)$ *be the unique solution of*

$$B = \frac{\left[\frac{\sigma+\kappa}{1+\kappa} - \left[\frac{1-\sigma}{1+\kappa}\right] \frac{d}{c(q_B)}\right] \chi F}{1 - \chi \left[F + \frac{\sigma+\kappa}{1+\kappa} - \left[\frac{1-\sigma}{1+\kappa}\right] \frac{d}{c(q_B)}\right]} \tag{63}$$

in B*. It holds that* $\overline{B}'(d) < 0$*. Let* $\iota \geq 0$*.*

1. *If*

$$f(\iota) \leq g(\overline{B}(d), d), \tag{64}$$

ι and d pin down a unique search-market outcome with $\mu^b = 1$ through

$$\underbrace{\iota^\tau}_{\equiv f(\iota)} = \underbrace{B^\tau \omega^\chi (1-\omega)^{1-\chi}[S_B - d]}_{\equiv g(B,d)}. \tag{65}$$

2. *If Inequality* (64) *does not hold true, ι and d pin down a unique search-market outcome with partial participation by buyers, such that $B = \overline{B}(d)$ and*

$$\mu^b = \left[\frac{g(\overline{B}(d), d)}{f(\iota)}\right]^{\frac{1}{\tau(1-\chi)}}. \tag{66}$$

Proposition 5 says that there is a unique search-market outcome given ι and d. Moreover, it implies that for increasing ι, the set of d that induce full participation by buyers shrinks. This becomes clear from Figure 3. We note that g bends clockwise and $\overline{B}(d)$ falls for increasing d since for buyers to derive flow utility $v^b = 0$, the effective opportunity cost B must fall if the match surplus decreases due to an increase in d. Hence, $\bar{d}_\iota$, which is implicitly defined through $f(\iota) = g(\overline{B}(\bar{d}_\iota), \bar{d}_\iota)$, decreases in ι. Since for a given ι, any d features a search-market outcome with full participation by buyers if and only if $d \leq \bar{d}_\iota$, the set of such d decreases in ι. The intuition is that both the opportunity cost of holding money ι and sellers' contingent repayment d reduce surplus and thus make buyers more likely to drop out of the market ($\mu^b < 1$).

In general equilibrium, sellers determine d in their ex-ante optimization problem in (31). Lemma 3 says that there is a unique solution to this problem at the FR. It is challenging and beyond the scope of this paper to analytically generalize equilibrium uniqueness on a global level due to the dynamic non-linear nature of the sellers' ex-ante problem. However, this problem has a unique solution in d in all model parametrizations we have checked, in particular in those we use in the calibration procedure below.

8 Quantitative Results

We calibrate the models of the three economies P, NP, and F separately to U.S. data to quantify how long-run inflation and financial intermediation matter for equilibrium outcomes. To be precise, we conduct three different calibrations; one for each of the three economies. Apart from the fact that $\Delta = 0$ in economy F by construction, so that Δ need not be calibrated, the procedure is identical across the three calibrations.

The unit of time is normalized to one year, and we use quarterly data series from January 1968 to December 2019. While we infer some primitives directly from the data and impose functional forms, we calibrate the remaining parameters jointly using the method of simulated moments.

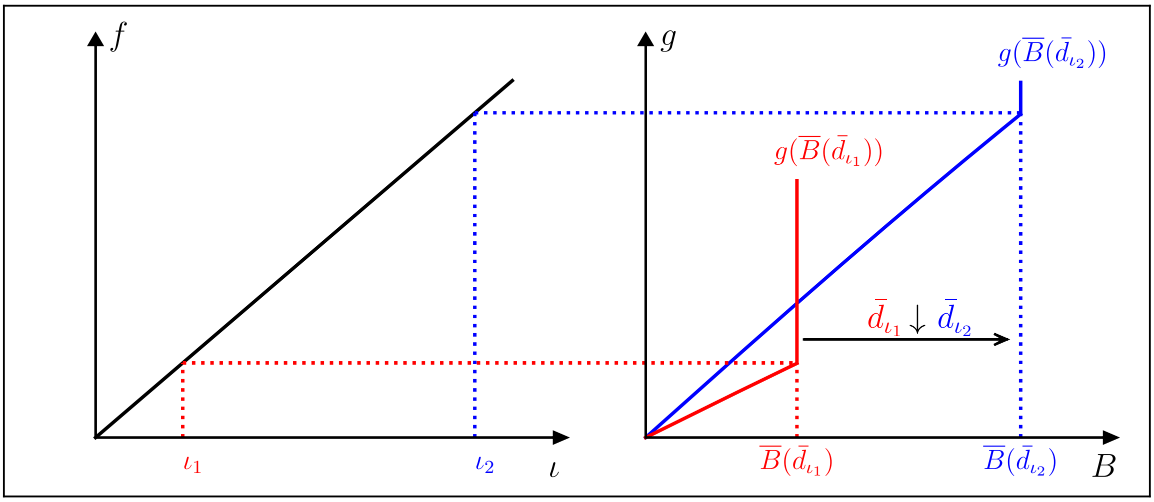


Figure 3: Search-market outcomes in economy NP for $\iota_1 < \iota_2$, inducing $\bar{d}_{\iota_1} > \bar{d}_{\iota_2}$.

8.1 Calibration

External calibration. We let the yearly rate of time preference be $\rho = 0.02$ (see, e.g., Ait Lahcen, Baughman, Rabinovich, & Van Buggenum, 2022). We derive the capital share $(1-\eta)$ in the model from the average of the ratios of common equity to tangible assets of global systemically important U.S. banks and obtain $\eta = 0.9434$.[14] We impose the functional forms $c(q) = q^{1+\kappa}/(1+\kappa)$, $\zeta(\varepsilon) = \varepsilon^{1+\tau}/(1+\tau)$, and $\mathcal{N}(\varepsilon^b, \varepsilon^s) = (\varepsilon^b)^\chi(\varepsilon^s)^{1-\chi}$ as in Assumption 2, and we set $\kappa = 0$ and $\tau = 1$. To separate the effect of the level of u on the size of the match surplus from the effect of the curvature of u, we characterize the sellers' search-good utility function as

$$u(q) = \left(\frac{1-\sigma}{\sigma}\right)^\sigma \frac{q^{1-\sigma}}{1-\sigma}, \tag{67}$$

so that $u(q^\star) - c(q^\star) = 1$ for any $\sigma \in (0,1)$.

Internal calibration. We calibrate the remaining parameters $\sigma \in (0,1)$, $\chi \in (0,1)$, and $\Delta > 0$—we set $\Delta = 0$ in economy F—by using empirically observed money demand, firm-level markups, and the share of the monetary base M0 in an augmented version of M1, called "NewM1". NewM1 includes money-market deposit accounts in addition to the monetary aggregate M1, as suggested by Lucas and Nicolini (2015).[15] We understand money demand as the relationship between the money-to-GDP ratio and the Fisher rate.

[14] The Board of Governors of the Federal Reserve System (2024b) provides data on bank equity from 1985 up until now.

[15] Since the competition for deposits significantly increased during the Great Inflation in the 1970s, money market funds were established to circumvent the interest-rate cap of Regulation Q. Money market funds are however economically equivalent to deposits (see Gorton & Zhang, 2023) and can help explain the relation of money demand to the T-Bill rate.

For the former, we use the ratio of NewM1 over GDP; for the latter, we use the 3 Month Treasury Bill Rate (T-bill rate).[16] We apply the HP filter (Hodrick & Prescott, 1997) with smoothing parameter $\epsilon = 1600$ on the series of T-bill rates and logarithmic money demand to extract the trend component of the respective series. We do that for two reasons. First, we essentially use steady-state comparative statics in the calibration procedure, which is common practice in the money-search literature since it focuses on the long run. It is therefore important that cyclical components are filtered out of the data before using them for calibration. Second, the T-bill rate prices a comparatively liquid asset and might thus behave uncorrelated with the Fisher rate in the short run. To make our calibration comparable with existent work, which commonly uses the T-bill rate as a proxy for the Fisher rate, we capture the long-run behavior of the Fisher rate with the trend component of the T-bill rate.[17] Empirical money demand reads as

$$\mathrm{MD}_t^{\mathrm{em}} \equiv \frac{\mathrm{NewM1}_t}{\mathrm{GDP}_t}. \tag{68}$$

As theoretical counterpart, we use

$$\mathrm{MD}_t^{\mathrm{th}} \equiv \left.\frac{\mu^b p}{\mathcal{N}(\mu^b \varepsilon^b, \varepsilon^s)p + G}\right|_{\iota=\iota_t}, \tag{69}$$

where the subscript t refers to the time-series dimension of the data, instead of indicating dynamics within the model—for each observation of ι_t, we compute the respective steady state. The real value of aggregate money holdings $\mu^b p$ is the payment p weighted with the mass of active buyers μ^b. Theoretical real GDP $= \mathcal{N}(\mu^b \varepsilon^b, \varepsilon^s)p + G$ is equal to the flow value $\mathcal{N}(\mu^b \varepsilon^b, \varepsilon^s)p$ of search-good production plus an additional parameter $G > 0$. G captures the value of goods and services outside of the search sector, including general goods produced to extend and redeem credit, as well as goods and services produced in sectors our model does not explicitly account for. We target the estimated parameters (a_0^{em}, a_1^{em}) of the empirical regression

$$\log \mathrm{MD}_t^{\mathrm{em}} = a_0^{em} + a_1^{em}\iota_t + v_t \tag{70}$$

[16] The U.S. Bureau of Economic Analysis (2024) provides data on GDP. We retrieve the data for the 3-month T-bill rate, money-market funds and the monetary aggregates M0 and M1 from the Board of Governors of the Federal Reserve System (2024a, 2024c, 2024d, 2024e).

[17] It is common to identify the Fisher rate, i.e., the opportunity cost of holding money, with the difference between the 3-month T-bill rate and the interest rate on demand deposits. However, before the House of Representatives of the United States of America in Congress (2010) passed the Dodd-Frank Wall Street Reform and Consumer Protection Act, Regulation Q was in place, prohibiting the payment of interest on demand deposits. Moreover, from 2011 to 2019, deposit rates were negligibly low, so that we can identify the opportunity cost with the T-bill rate from 1968 to 2019 without loss.

with their theoretical counterparts (a_0^{th}, a_1^{th}). Loosely speaking, G drives the intercept a_0^{em} of the estimated theoretical money-demand curve, and σ drives its slope a_1^{em}, which is the semi-elasticity of money demand.

We let the model match firm-level markups to determine χ since χ captures the importance of buyers' search in the matching function, which, in a model of directed and competitive search, determines the split of match surplus. De Loecker, Eeckhout, and Unger (2020) estimate an average markup $\overline{\varrho}^{em} = 36\%$ for U.S. firms across all sectors from 1955 to 2016.[18] The theoretical firm-level markup is

$$\varrho_t^{th} \equiv \left. \frac{p/q - c'(q)}{c'(q)} \right|_{\iota=\iota_t}, \tag{71}$$

which relates the per-unit price of search goods to the marginal cost of production. We define $\overline{\varrho}^{th} \equiv \sum_t \varrho_t^{th}/T$ as the theoretical counterpart of $\overline{\varrho}^{em}$, where T denotes the number of observations.

For economies P and NP, we also need to calibrate the pledgeability horizon Δ, which drives the extent of private-money creation. We therefore target the share

$$\lambda_t^{em} \equiv \frac{\text{M0}_t}{\text{NewM1}_t} \tag{72}$$

of the monetary base M0 in the monetary aggregate NewM1. The theoretical share of public money in the monetary aggregate is

$$\lambda_t^{th} \equiv \frac{m_t}{a_t + m_t}. \tag{73}$$

We define $\overline{\lambda}^{th} \equiv \sum_t \lambda_t^{th}/T$ as the theoretical counterpart of $\overline{\lambda}^{em} \equiv \sum_t \lambda_t^{em}/T$.

For economies P and NP, we calibrate $(\sigma, \chi, G, \Delta)$ by solving

$$\min_{\sigma,\chi,G,\Delta} \left\{ \left(\frac{a_0^{em} - a_0^{th}}{a_0^{th}} \right)^2 + \left(\frac{a_1^{em} - a_1^{th}}{a_1^{th}} \right)^2 + \left(\frac{\overline{\varrho}^{em} - \overline{\varrho}^{th}}{\overline{\varrho}^{th}} \right)^2 + \left(\frac{\overline{\lambda}^{em} - \overline{\lambda}^{th}}{\overline{\lambda}^{th}} \right)^2 \right\}^{\frac{1}{2}}, \tag{74}$$

and for economy F, we calibrate (σ, χ, G) by solving

$$\min_{\sigma,\chi,G} \left\{ \left(\frac{a_0^{em} - a_0^{th}}{a_0^{th}} \right)^2 + \left(\frac{a_1^{em} - a_1^{th}}{a_1^{th}} \right)^2 + \left(\frac{\overline{\varrho}^{em} - \overline{\varrho}^{th}}{\overline{\varrho}^{th}} \right)^2 \right\}^{\frac{1}{2}}. \tag{75}$$

The calibrated parameters thus minimize the Euclidean distance of the theoretical moments from their empirical counterparts.

[18]The estimates also suggest that markups have been rising recently, and our calibration can be repeated with higher average markups for the most recent period.

Parameter	Description	Economy P	NP	F
σ	curvature of search-good utility function u	0.3636	0.3954	0.3679
χ	buyers' bargaining power	0.3547	0.5997	0.4797
G	production not accounted for by the model	6.9900	5.6785	6.3875
Δ	pledgeability horizon	1.0642	17.6024	-

(a) Parameters.

Moment	Description	Data	Economy P	NP	F
a_0	level of money demand	-1.0934	-1.0936	-1.0935	-1.0934
a_1	interest-rate semi-elasticity	-7.5169	-7.5131	-7.5171	-7.5166
$\overline{\varrho}$	average price markup	0.3600	0.3600	0.3601	0.3601
$\overline{\lambda}$	average share of M0 in NewM1	0.3278	0.3279	0.3278	-
	Euclidean distance $\times 10^4$	-	1.4324	0.5688	0.1940

(b) Moments and errors.

Table 1: Calibration.

Calibration results. Table 1 provides the results of applying the calibration procedure to each of the three economies P, NP, and F. All targets are hit for all three calibrations. Figure 4 shows the resulting theoretical money-demand curves $\mathrm{MD}_t^{\mathrm{th}}$ and empirical money demand $\mathrm{MD}_t^{\mathrm{em}}$ for each of the economies P, NP, and F. The kinks in the theoretical money-demand curves arise due to the drop out of buyers for large Fisher rates.

Two differences in the calibrated parameters across the three economies are salient. First, economy NP has a considerably larger pledgeability horizon Δ than economy P. This is because for a given Δ, economy P features more private-money creation than economy NP. In particular, project control facilitates borrowing without distorting sellers' ex-post search incentives and thus makes the FI willing to extend more credit ex ante. In economy P, a smaller Δ thus suffices to hit the empirical share $\overline{\lambda}^{em}$ of public money in the monetary aggregate.

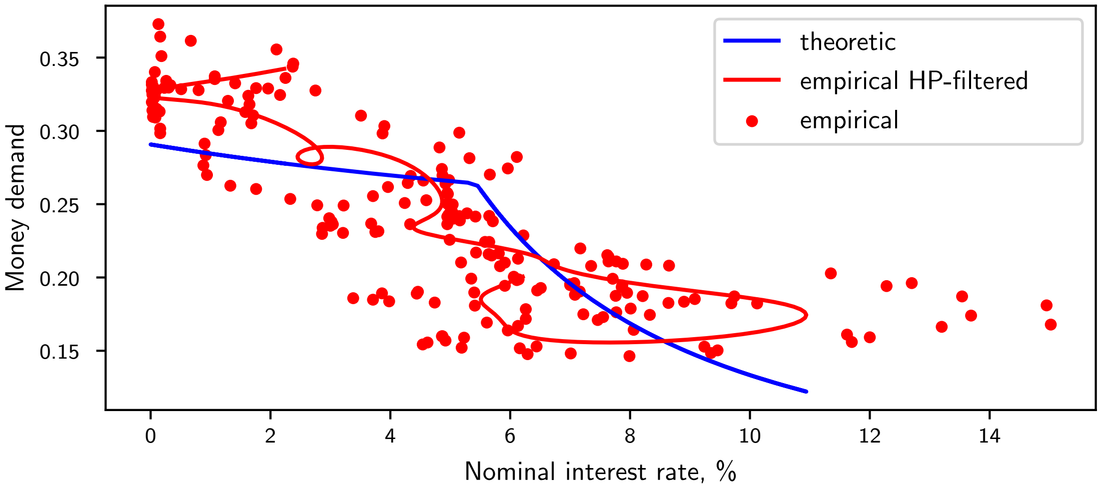
Moreover, the calibrations of economies P, F, and NP feature values for χ in ascending order, respectively. In economy P, χ proxies the buyers' share of the ex-ante surplus $u(q)-\exp(-\Delta\nu)c(q)$, which includes the sellers' gains from exploiting borrowing discount ν, whereas in economy NP, χ proxies the buyers' share of the ex-post surplus $u(q)-c(q)-d$, where the sellers' gains of exploiting ν are not accounted for. Hence, χ must be larger in economy NP than in economy P to match the empirical markup $\overline{\mu}^{em}$.

8.2 Monetary Policy

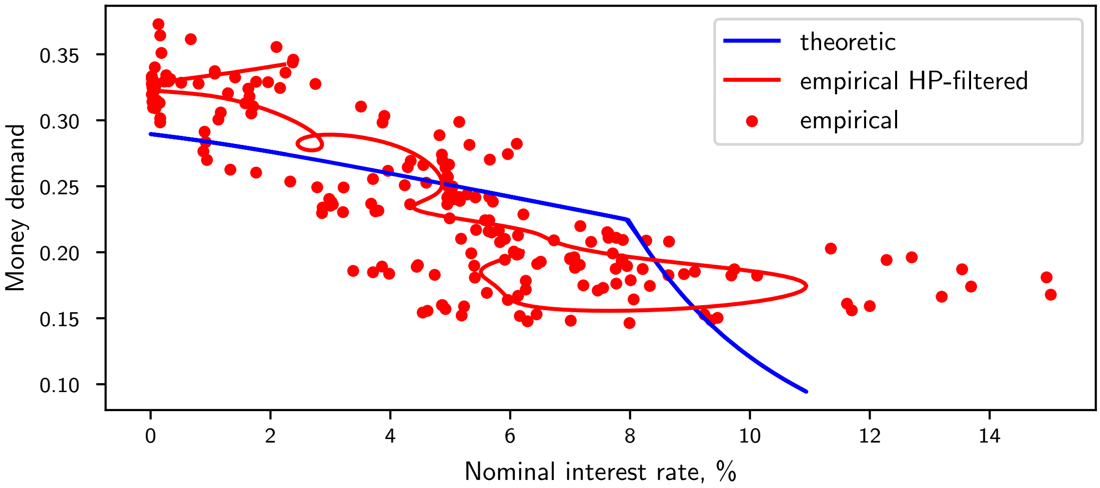
We study how equilibrium variables change in the Fisher rate ι in each of the calibrated economies P, NP, and F. Figure 5 illustrates the results and Table 2a quantifies them.

Matching rates. Figures 5a and 5b show sellers' and buyers' matching rates $\mathcal{N}^s$ and $\mathcal{N}^b$, where $\mathcal{N}^s$ is also the aggregate matching rate. The paths of $\mathcal{N}^s$ and $\mathcal{N}^b$ generalize Proposition 2 for all Fisher rates ι that feature full participation by buyers: ι stimulates credit and thus accelerates matching in economy P and decelerates matching in economy NP. For ι large, $\mathcal{N}^s$ however decreases in all economies since the market tightness changes in the sellers' disadvantage as buyers drop out of the search market. $\mathcal{N}^b$ then linearly increases in economy P rather mechanically: the effective opportunity cost $\iota/\mathcal{N}^b = \overline{B}$ is constant for all ι that feature partial participation by buyers, so that any increase in ι is one-to-one offset by an increase in $\mathcal{N}^b$. In economy NP, $\mathcal{N}^b$ increases at a rate even larger than one—though this is hard to see in the Figure 5b—since ι reduces $\overline{B}(d)$ by increasing d (see the discussion at Proposition 5).

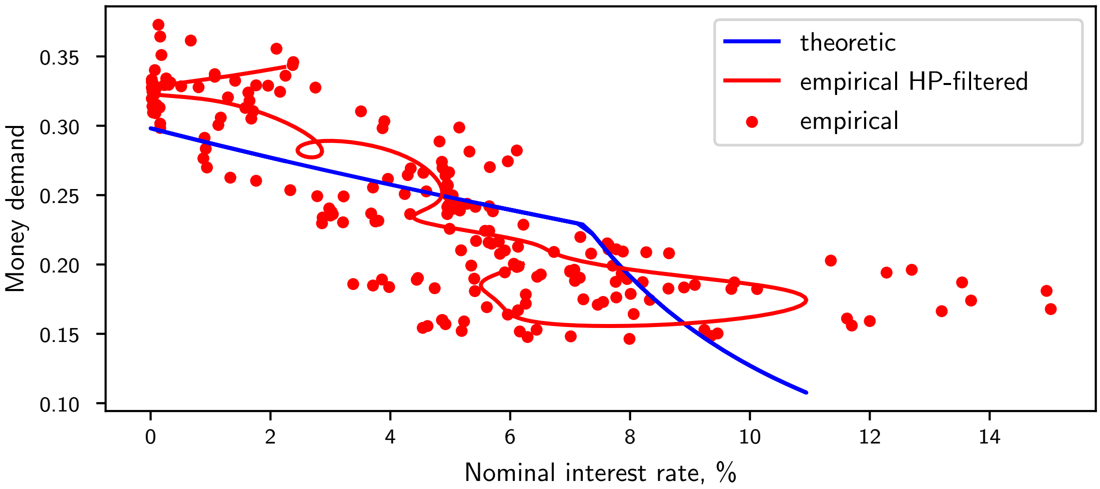
Table 2a shows that a deviation from the FR to $\iota = 5\%$ accelerates aggregate matching in economy P by 11.03%. This is overturned for deviations to $\iota = 10\%$ and $\iota = 15\%$, which decelerate matching by 5.15% and 8.57%, respectively. In economies NP and F, any deviation from the FR unambiguously decelerates matching. The effect in economy NP



(a) Economy P.

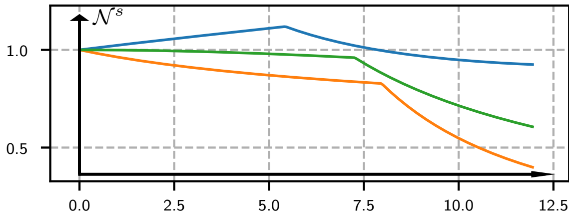


(b) Economy NP.

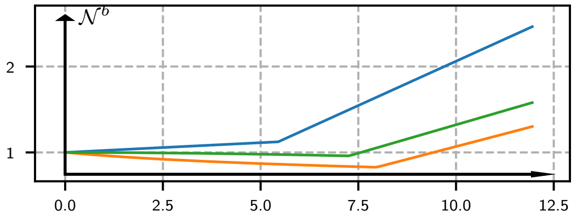


(c) Economy F.

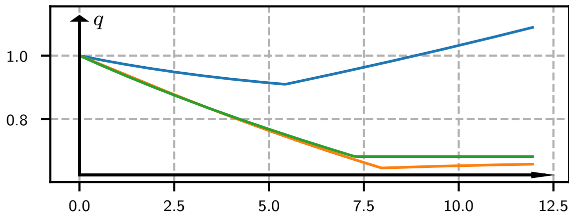
Figure 4: Money demand in the calibrated models.



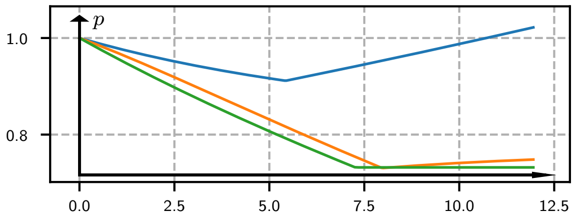
(a) Sellers' matching rate $\mathcal{N}^s = \mathcal{N}(\mu^b \varepsilon^b, \varepsilon^s)$.



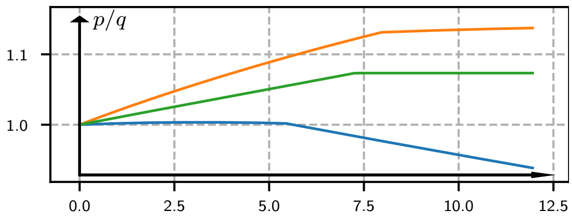
(b) Buyers' matching rate $\mathcal{N}^b = \mathcal{N}(\varepsilon^b, \varepsilon^s/\mu^b)$.



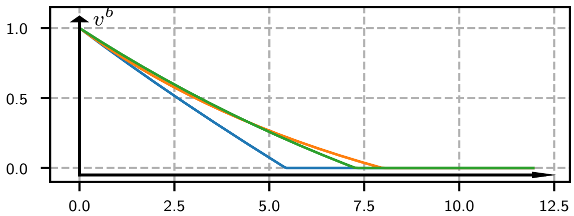
(c) Search-good quantity q.



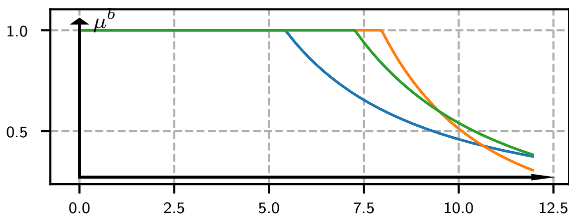
(d) Payment p.



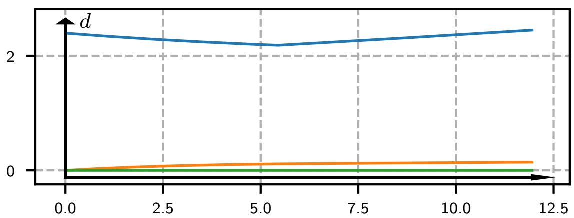
(e) Payment over search-good quantity p/q.



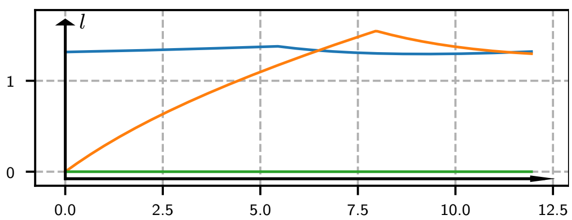
(f) Buyers' flow value v^b.



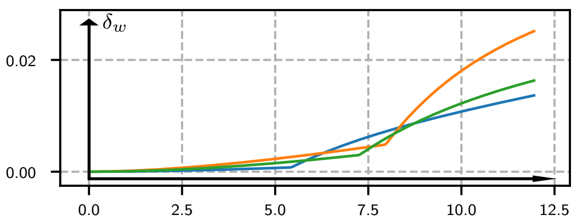
(g) Mass of active buyers μ^b.



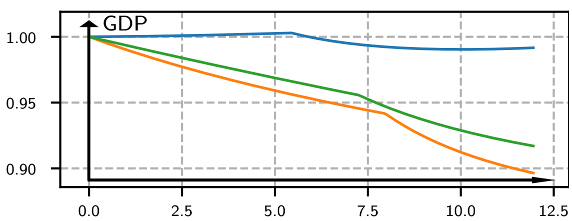
(h) Contingent repayment d.



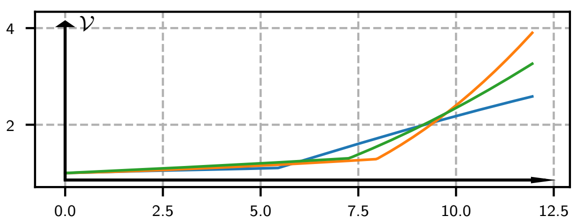
(i) Present value of pending loan contracts l.



(j) Welfare cost δ_w.



(k) GDP.



(l) Statistical velocity $\mathcal{V}$.

Figure 5: Equilibrium variables for economies P (●), NP (●), and F (●). The horizontal axes show the Fisher rate in percent. The vertical axes show equilibrium variables. All variables but d, l, and δ_w are normalized at their levels at the Friedman rule.

	Economy P			Economy NP			Economy F		
	$\iota = 5\%$	$\iota = 10\%$	$\iota = 15\%$	$\iota = 5\%$	$\iota = 10\%$	$\iota = 15\%$	$\iota = 5\%$	$\iota = 10\%$	$\iota = 15\%$
$\mathcal{N}^s$	11.03	-5.15	-8.57	-12.99	-45.20	-72.99	-2.03	-28.53	-50.82
$\mathcal{N}^b$	11.03	106.16	209.23	-12.99	6.85	66.55	-2.03	32.19	98.28
q	-8.59	3.18	18.43	-23.66	-34.69	-33.71	-23.2	-31.80	-31.80
p	-8.36	-1.24	7.81	-16.88	-25.88	-24.44	-19.32	-26.80	-26.80
p/q	0.25	-4.29	-8.97	8.88	13.49	13.98	5.06	7.33	7.33
v^b	-92.71	-100.00	-100.00	-73.4	-100.00	-100.00	-74.28	-100.00	-100.00
μ^b	0.00	-53.99	-70.43	0.00	-48.72	-83.78	0.00	-45.93	-75.20
δ_w	0.07	1.08	1.79	0.23	1.80	3.14	0.15	1.22	2.06
GDP	0.27	-0.96	-0.22	-4.09	-8.77	-11.76	-3.13	-7.11	-9.55
$\mathcal{V}$	9.41	117.97	213.03	15.39	139.99	620.22	20.07	134.68	398.19

(a) Changes in response to a deviation from the Friedman rule.

	Economy P			Economy NP		
	$\iota = 5\%$	$\iota = 10\%$	$\iota = 15\%$	$\iota = 5\%$	$\iota = 10\%$	$\iota = 15\%$
$\mathcal{N}^s$	-13.82	-31.08	-42.78	13.04	71.96	144.42
$\mathcal{N}^b$	-7.16	0.00	0.00	13.04	-11.81	-31.07
q	-14.33	-24.10	-33.87	3.02	-5.06	-15.25
p	-9.58	-16.09	-23.14	-1.82	-10.16	-19.04
p/q	5.54	10.55	16.23	-4.70	-5.37	-4.47
v^b	-100.00	0.00	0.00	84.34	0.00	0.00
μ^b	-7.17	-31.08	-42.78	0.00	94.99	254.57
δ_w	0.15	0.26	0.03	-0.09	-1.32	-1.40
GDP	-3.40	-6.06	-8.41	1.22	3.59	3.34
$\mathcal{V}$	15.09	62.46	108.3	3.10	-40.87	-64.00

(b) Changes in response to shutting down financial intermediation.

Table 2: Changes of equilibrium variables in response to deviations from the Friedman rule and to shutting down financial intermediation. All changes are in percentage terms.

is moreover considerably stronger than in economy F: deviations to $\iota = 5\%$ and $\iota = 15\%$ decelerate matching by 12.99% and 72.99% in economy NP but only by 2.03% and 50.82% in economy F. In the presence of intermediation without project control, inflation thus distorts search not only through increasing the opportunity cost of holding money, as it is standard in money-search models, but also through stimulating distortionary credit extension.

Contingent repayment. Figure 5h shows that in economy NP, the contingent repayment d is monotonically increasing in ι, generalizing the local result in Lemma 3 at the FR, whereas in economy P, d decreases for ι small and increases for ι large. This difference in the behaviour of d across economies traces back to the ways in which sellers exploit the borrowing discount. In economy NP, the only way in which sellers can increase borrowing is contracting a larger repayment d, so that any increase in the borrowing discount $\nu = \eta\iota$ results in an increase in d. In economy P, sellers instead borrow against their full future revenues by setting $d = p$. The path of d is thus determined by the price-posting behavior of sellers, as described below.

Terms of trade. Figures 5c and 5d plot terms of trade (q, p) against ι. The paths of q and p generalize Proposition 3 for all ι that feature full participation by buyers. We identify two effects of ι on q, and analogously on p. There is a (i) demand-side effect in all economies that operates through the effective opportunity cost $B = \iota/\mathcal{N}^b$ (see Equations (28) and (36)): B drives down q since buyers require compensation for carrying money, which reduces the sellers' incentive to produce large quantities.[19] The weaker decrease of q in ι close to the FR in economy P as compared to economies NP and F thus traces back to the steeper increase of $\mathcal{N}^b$ in economy P. Beyond that, there is a (ii) supply-side effect only present in economy P: the increasing borrowing discount $\nu = \eta\iota$ stimulates q since sellers increase their borrowing by committing to post more favorable terms of trade and thus to attract more buyers.

In economy P, the demand-side effect is the driving force for low ι. However, this effect is shut down when buyers start dropping out at high ι since $B = \overline{B}$ in all PPEs. The supply-side effect thus begins to dominate, so that q increases. In economy NP, q begins to increase as well when buyers drop out but again due to the demand-side effect: the increase in d, as shown in Figure 5h and discussed at Proposition 5, makes $B = \overline{B}(d)$ fall, so that buyers are willing to carry more money and sellers are willing to produce more.

Finally, Figure 5e shows that the unit price p/q is small in economy P as compared to economies NP and F, confirming our earlier intuition that terms of trade are relatively favorable for buyers in economy P. This is due to sellers' ability to increase borrowing

[19]Recall the discussion at Lemma 2, which elaborates on this demand-side effect.

through commitment. In economy NP, terms of trade are relatively less favorable since d imposes an additional cost for sellers ex post.

Welfare. We add calibrated non-search production G to our notion of welfare to account for non-search sectors not modelled explicitly.[20] Flow welfare is thus

$$w = \mathcal{N}(\mu^b \varepsilon^b, \varepsilon^s)[u(q) - c(q)] - \zeta(\varepsilon^s) - \mu^b \zeta(\varepsilon^b) + G. \tag{76}$$

We measure the welfare cost of inflation in consumption-equivalent terms. In particular, we ask what fraction δ_w of total consumption a planner would give up to prevent a deviation from the FR to Fisher rate ι. If ι induces flow welfare w, δ_w thus solves

$$w = \mathcal{N}(\varepsilon^{b\star}, \varepsilon^{s\star})[u((1-\delta_w)q^\star) - c((1-\delta_w)q^\star)] - \zeta(\varepsilon^{s\star}) - \zeta(\varepsilon^\star) + (1-\delta_w)G. \tag{77}$$

Figure 5j shows how δ_w unambiguously increases in ι in all three economies. However, the size of δ_w considerably varies across economies. Table 2a quantifies δ_w for deviations from the FR to $\iota \in \{5\%, 10\%, 15\%\}$ in all three economies. For instance, a deviation to $\iota = 10\%$ costs 1.08% of first-best consumption in economy P, whereas it costs 1.22% and even 1.80% of first-best consumption in economies F and NP.

This finding is analogous to that of Lagos and Wright (2005). Although they can match a standard money-search model to empirical money demand equally well for different levels of bargaining power, they find very different values for the welfare cost of inflation. We likewise find that economies P, NP, and F can fit money demand equally well while inducing very different welfare costs of inflation. This echoes Lagos and Wright's (2005) point that "[k]nowing the empirical 'money demand' curve is not enough: one really needs to understand the micro foundations [...] in order to correctly estimate the welfare cost of inflation" (p. 480).

We next turn to the welfare cost of financial intermediation in economies P and NP. We ask how the welfare cost of inflation δ_w at a given Fisher rate ι changes when policy shuts down intermediation through 100-percent capital requirements: $\eta = 0$. Table 2b quantifies these changes.[21] For instance, at $\iota = 10\%$, δ_w increases by 0.26 percentage points in economy P, whereas δ_w falls by 1.32 percentage points in economy NP. Hence, the sign and magnitude of the effect of financial intermediation on welfare depend on the availability of project control. A normative stance on the welfare effect of financial intermediation thus requires assessing the degree of commitment in loan contracts.

We note that financial intermediation affects search-market allocations only through the accumulative borrowing discount $\Delta\eta\iota$, as can be seen in the sellers' optimization

[20] This reflects the idea that G is measured in general goods.

[21] Table 2b likewise quantifies the effect of 100-percent capital requirements on the other equilibrium variables.

problems, so that the product $\Delta\eta$ matters rather than the individual levels of Δ and η. To generalize the results in Table 2b, we thus plot in Figure 6 how δ_w behaves in $\Delta\eta$ and ι while keeping all the other calibrated parameters fixed.[22] Figure 6a suggests the calibrated $\Delta = 1.06$ and $\eta = 0.94$ are close to their welfare-maximizing levels in economy P; the welfare-improving potential of financial intermediation is almost fully utilized. In contrast, Figure 6b suggests financial intermediation in general impairs welfare in economy NP; it is best to set $\eta = 0$.

GDP. Figure 5k plots GDP ($= \mathcal{N}^s p + G$) against ι. Any increase of ι drives the equilibrium allocations away from their first-best levels. These inefficiencies make GDP decrease in economies NP and F; the economies cool down. In economy P, however, GDP increases in ι for ι small due to the increase in the value of search-good production $\mathcal{N}^s p$ as trade accelerates in ι; the economy overheats. Table 2a quantifies that a deviation from the FR to $\iota = 5\%$ makes GDP increase by 0.27% in economy P. For larger values of ι, GDP decreases relative to its values at the FR in all three economies. Table 2b draws the overall picture that for a given Fisher rate, financial intermediation under project control stimulates the economy, whereas financial intermediation without project control curbs it. For instance, when shutting down intermediation at $\iota = 10\%$, GDP drops by 6.05% in economy P, whereas GDP increases by 3.58% in economy NP when intermediation is shut down.

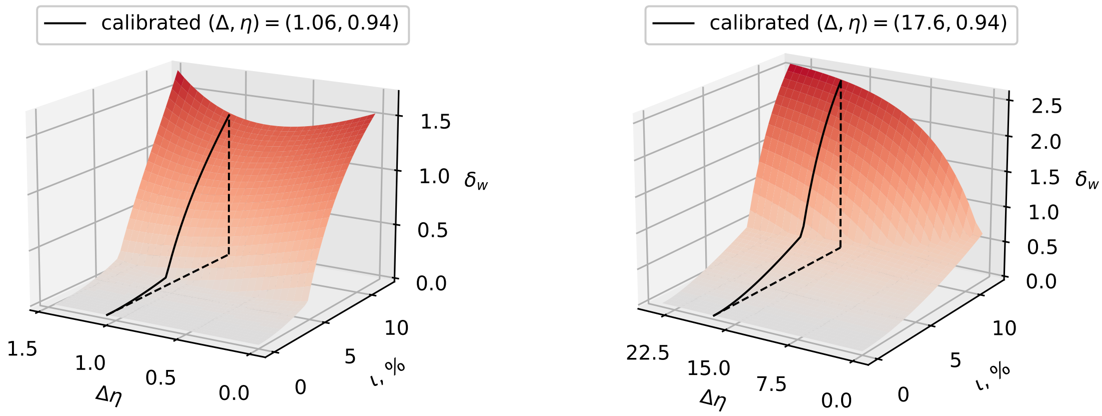
Velocity. Finally, we investigate how matching frequencies and the *statistical* velocity of money

$$\mathcal{V} \equiv \frac{1}{\mathrm{MD}^{\mathrm{th}}} = \frac{\mathcal{N}(\mu^b \varepsilon^b, \varepsilon^s)p + G}{\mu^b p} = \mathcal{N}^b + \frac{G}{\mu^b p}. \tag{78}$$

With $\mathcal{V}$ being defined as GDP over aggregate money holdings, $\mathcal{V}$ is additive in that it is the sum of sectoral productions over aggregate money holdings. Since the search sector is the only monetary sector in our model, search-sector production over aggregate money holdings is equal to the frequency $\mathcal{N}^b$ at which active buyers match and thus spend money. Non-search production over money holdings $G/\mu^b p$ is however unrelated to the frequency at which money is spent. In that sense, statistical velocity is contaminated by the presence of non-monetary activity in GDP.

Figure 5l shows that $\mathcal{V}$ increases in ι in both economies P and NP, and Table 2a quantifies these effects. For instance, in economy P, a deviation from the FR to $\iota = 5\%$ increases $\mathcal{N}^b$ and $\mathcal{V}$ by 11.03% and 9.41%. In economies NP and F, the same policy however reduces $\mathcal{N}^b$ by 12.99% and 2.03%, but increases $\mathcal{V}$ by 15.39% and 20.07%. Hence, the responses of $\mathcal{N}^b$ and $\mathcal{V}$ to deviations from the FR differ in sign and magnitude, which highlights the contamination of $\mathcal{V}$.

[22]We do the same exercise for all other variables than w depicted in Figure 5 in Appendix A.



(a) Project control.

(b) No project control.

Figure 6: Welfare cost δ_w in percentage terms.

9 Conclusion

We provide a monetary model relating the degree of credit enforcement to monetary-policy transmission. In a framework of directed and competitive search, sellers take out consumptive credit by borrowing against their future search-market income. Financial intermediaries write loan contracts with sellers and intermediate the arising claims as private money to buyers who use it to settle transactions in search-market trade. We find that the intermediaries' ability to enforce contractual promises—a technology we call project control—qualitatively and quantitatively matters for the long-run transmission of monetary policy. Particularly, inflation speeds up trade when project control is present, while the economy cools down when it is absent. We calibrate both the economies with and without project control as well as an economy without intermediation to U.S. data to quantify these effects.

The calibration results are particularly interesting in light of recurrent sovereign-money initiatives seeking to eliminate intermediaries' money-creation privilege. Imposing 100-percent capital requirements on intermediaries—an effective ban of private-money creation—decelerates trade and impairs welfare when intermediaries exert project control, whereas such a policy has the the opposite effect in the absence of project control. Even stronger, the calibrated extent of intermediation in the project-control economy is close to optimal in terms of what intermediation can do about mitigating the welfare cost of inflation. When enforcement is advanced, the credit extension that goes along with private-money creation can thus help improve welfare although private money is not essential as a payment instrument when the government issues fiat currency.

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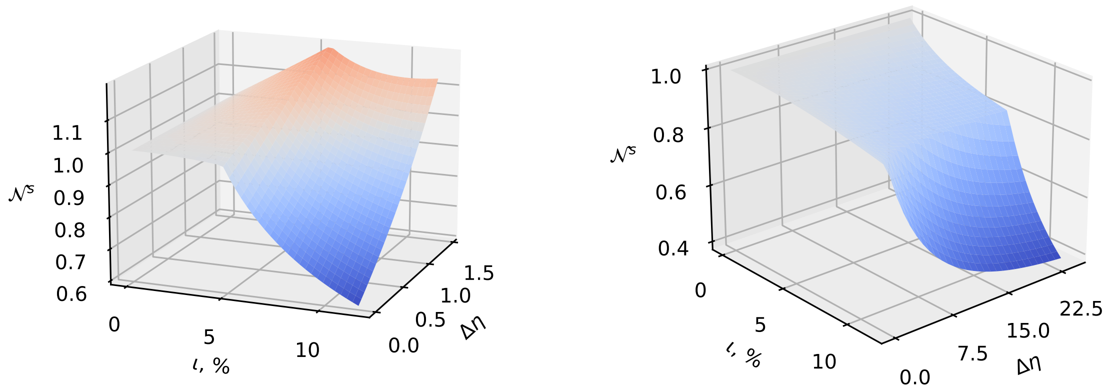
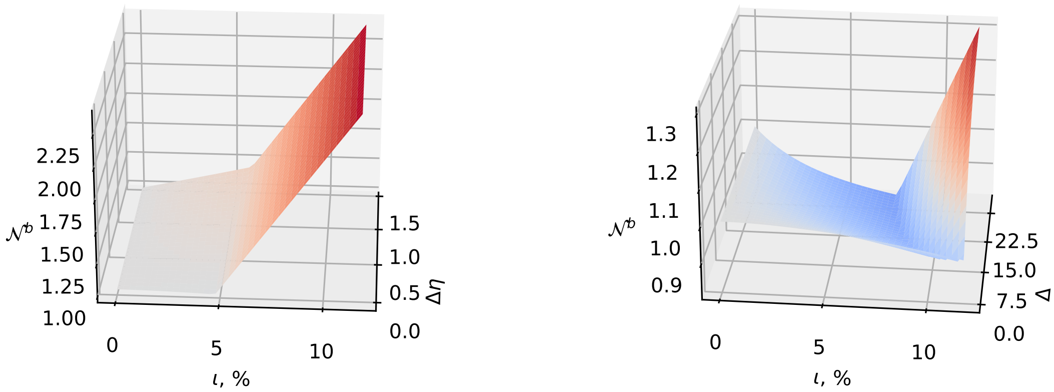
A Complementary Figures



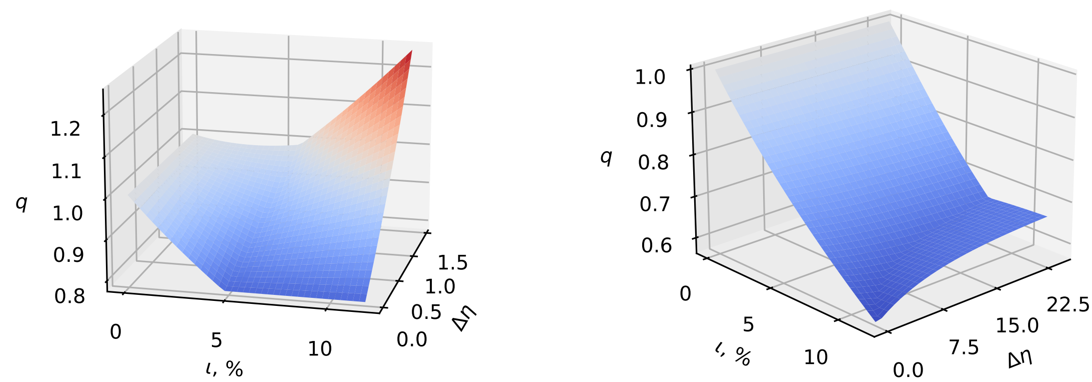
(a) Project control. (b) No project control.

Figure 7: Sellers' matching rate $\mathcal{N}^s$, normalized to one at the Friedman rule.



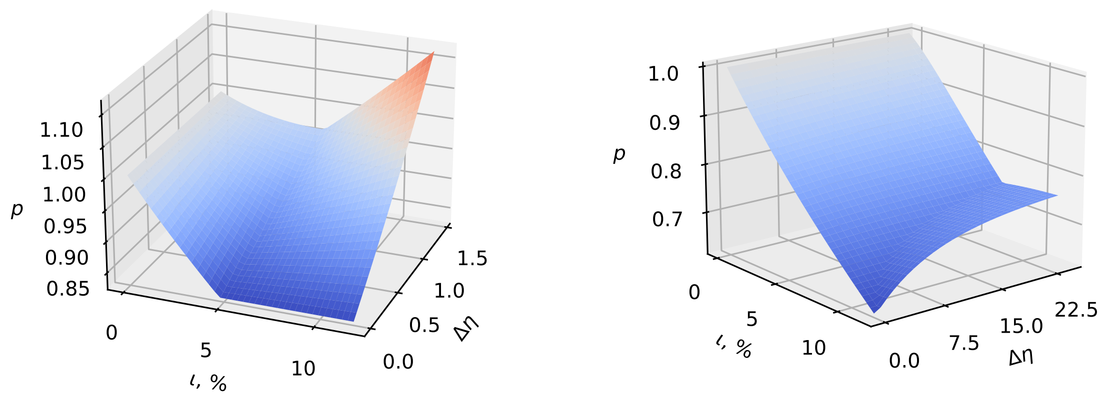
(a) Project control. (b) No project control.

Figure 8: Buyers' matching rate $\mathcal{N}^b$, normalized to one at the Friedman rule.



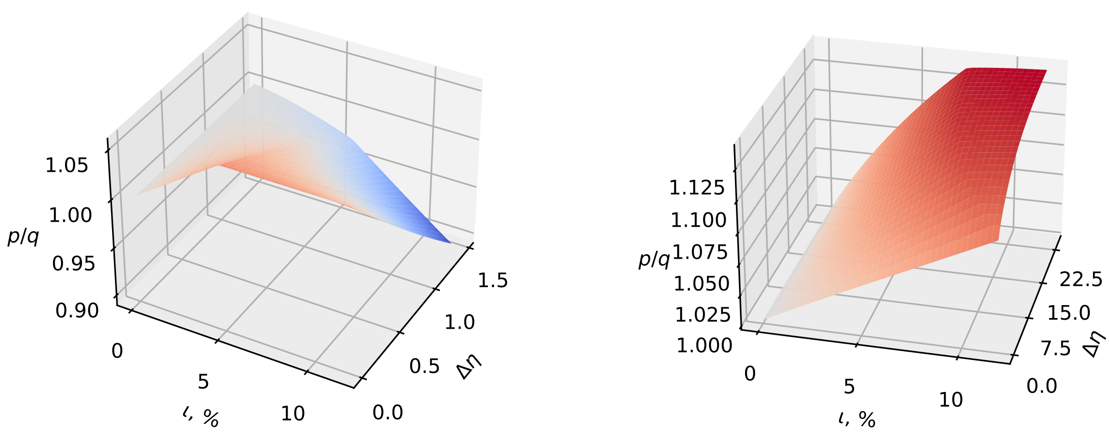
(a) Project control. (b) No project control.

Figure 9: Search-good quantity q, normalized to one at the Friedman rule.



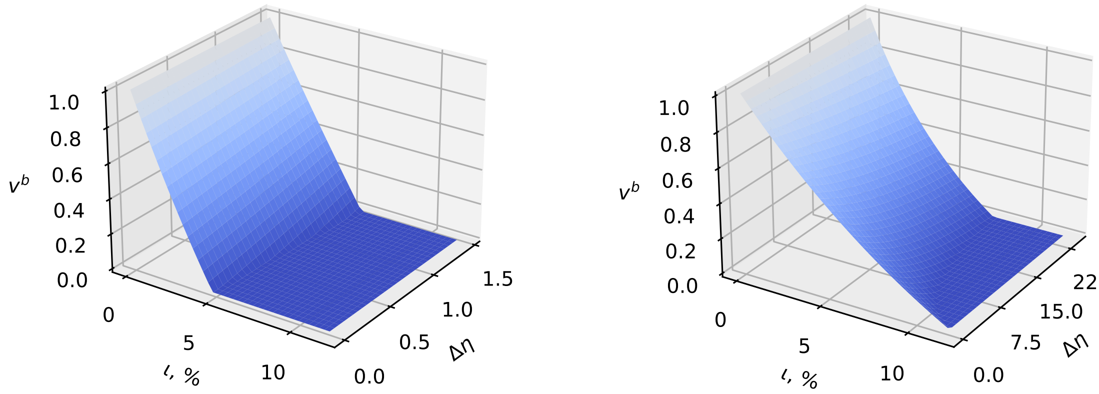
(a) Payment p. (b) No project control.

Figure 10: Payment p, normalized to one at the Friedman rule.



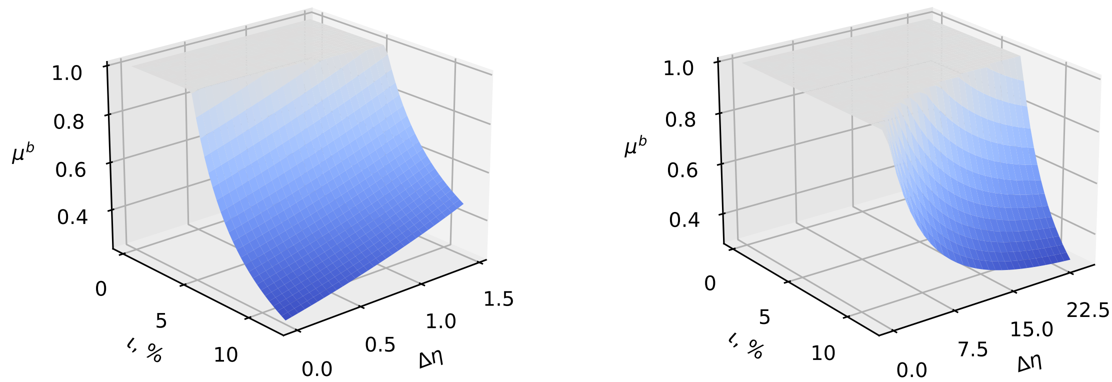
(a) Project control. (b) No project control.

Figure 11: Per-unit price p/q, normalized to one at the Friedman rule.



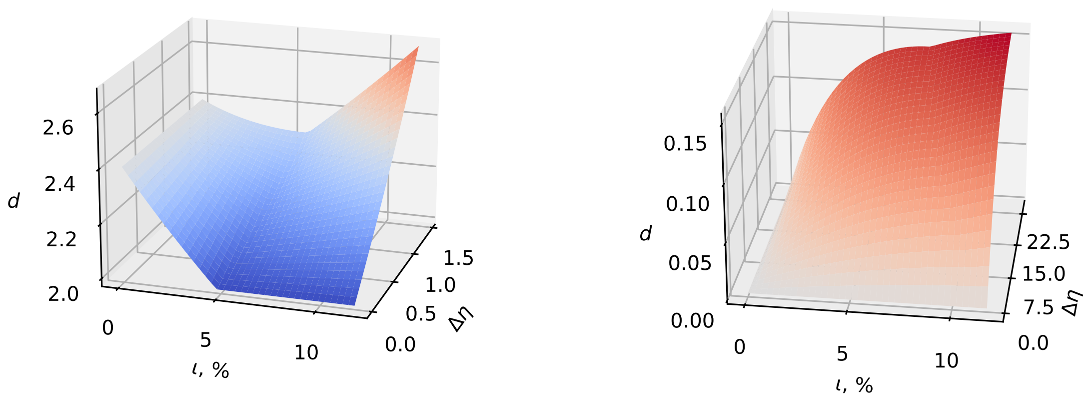
(a) Project control. (b) No project control.

Figure 12: Buyers' flow value v^b, normalized to one at the Friedman rule.



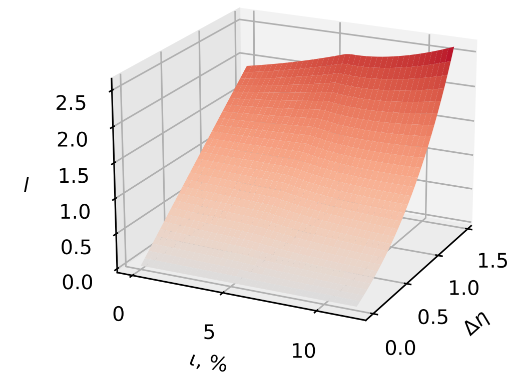
(a) Project control. (b) No project control.

Figure 13: Mass of active buyers μ^b, normalized to one at the Friedman rule.

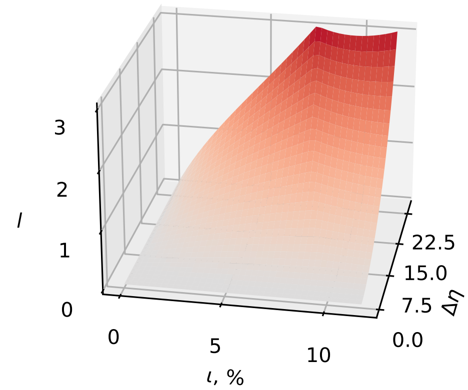


(a) Project control. (b) No project control.

Figure 14: Contingent repayment d.

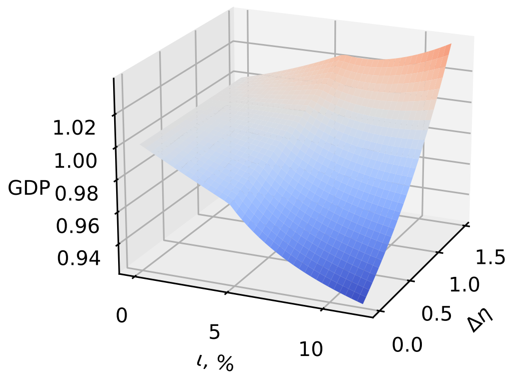


(a) Project control.

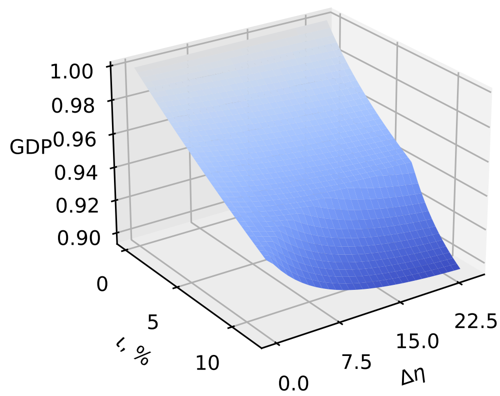


(b) No project control.

Figure 15: Present value of pending loan contracts l.

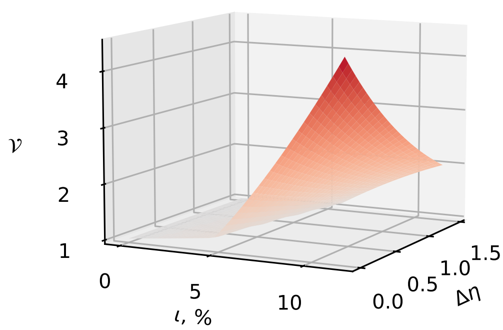


(a) Project control.

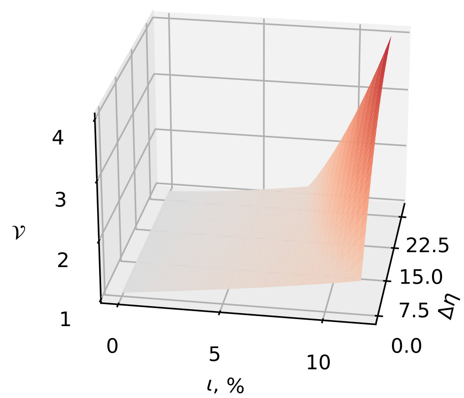


(b) No project control.

Figure 16: GDP, normalized to one at the Friedman rule.



(a) Project control.



(b) No project control.

Figure 17: Statistical velocity $\mathcal{V}$, normalized to one at the Friedman rule.

B Poisson Processes of Matches

This appendix explains how we can express the expectation of the sum over utility streams, whose occurrence is modelled by a Poisson process, as an integral over the respective Lebesgue-measurable matching rates. We do so by considering a typical buyer. In particular, we prove that

$$\mathbb{E}_t\left[\sum_{n=1}^{\infty} e^{-\rho(T^b_{t,n}-t)} u\left(q^b_{T^b_{t,n}}\right)\right] = \int_t^{\infty} e^{-\rho(\tau-t)}\, \varepsilon^b_\tau \alpha(\theta^b_\tau) u(q^b_\tau)\, \mathrm{d}\tau. \tag{79}$$

The number of a buyer's matches follows an inhomogeneous Poisson process with Lebesgue-measurable matching-rate process $\{\varepsilon^b_t \alpha(\theta^b_t)\}_{t=0}^{\infty}$. The matching rate $\varepsilon^b_t \alpha(\theta^b_t)$ at time t depends on both the market tightness θ^b_t and the individual search effort ε^b_t the buyer chooses. The number of matches $N^b(s,t] \in \mathbb{N}$ within time interval $(s,t]$ that the buyer faces, is distributed according to

$$\mathbb{P}\left[N^b(s,t] = n\right] = \frac{\Lambda^b(s,t)^n}{n!} \exp(-\Lambda^b(s,t)), \qquad \forall n \in \mathbb{N}, \tag{80}$$

where $\Lambda^b(s,t) = \int_s^t \varepsilon^b_\tau \alpha(\theta^b_\tau)\, \mathrm{d}\tau$. For every $t \in [0,\infty)$, we derive from basic statistics that the n-th arrival time $T^b_{t,n}$ on the interval (t,∞) associated with the above Poisson process has probability density function

$$f^b_{t,n}(\tau) = \frac{\Lambda^b(t,\tau)^{n-1}}{n!}\, \varepsilon^b_\tau \alpha(\theta^b_\tau) \exp\left(-\Lambda^b(t,\tau)\right), \qquad \forall \tau \in [t,\infty). \tag{81}$$

This particularly implies that $\mathbb{P}[T^b_{t,1} \geq t] = 1$. We can write with *Fubini's* theorem

$$\begin{aligned}
&\mathbb{E}_t\left[\sum_{n=1}^{\infty} e^{-\rho(T^b_{t,n}-t)} u\left(q^b_{T^b_{t,n}}\right)\right] \\
&\quad= \sum_{n=1}^{\infty} \int_t^{\infty} e^{-\rho(\tau-t)} u(q^b_\tau) f^b_{t,n}(\tau)\, \mathrm{d}\tau \\
&\quad= \sum_{n=1}^{\infty} \int_t^{\infty} e^{-\rho(\tau-t)} u(q^b_\tau) \frac{\Lambda^b(t,\tau)^{n-1}}{n!}\, \varepsilon^b_\tau \alpha(\theta^b_\tau) \exp(-\Lambda^b(t,\tau))\, \mathrm{d}\tau \\
&\quad= \int_t^{\infty} \underbrace{\left[\sum_{n=1}^{\infty} \frac{\Lambda^b(t,\tau)^{n-1}}{n!}\right]}_{=\exp\left(\Lambda^b(t,\tau)\right)} \exp\left(-\Lambda^b(t,\tau)\right) e^{-\rho(\tau-t)} u(q^b_\tau)\, \varepsilon^b_\tau \alpha(\theta^b_\tau)\, \mathrm{d}\tau \\
&\quad= \int_t^{\infty} e^{-\rho(\tau-t)}\, \varepsilon^b_\tau \alpha(\theta^b_\tau) u(q^b_\tau)\, \mathrm{d}\tau.
\end{aligned} \tag{82}$$

The reasoning for a seller's Poisson process of matches is analogous.

C Proofs

C.1 Proof of Lemma 1

The proof is explained in the main body.

C.2 Proof of Lemma 2

Let ϑ_t and ς_t denote the Lagrange multipliers for Constraints (24) and (25) associated with (23). The first-order conditions (FOCs) of this maximization problem are

$$\varepsilon_t^s \ : \quad 0 = \beta(\theta_t^s)[-c(q_t^s) + p_t^s - d_t^s] - \zeta'(\varepsilon_t^s) + \exp\left(\int_{\max\{t-\Delta,0\}}^t \nu_s \, \mathrm{d}s\right) \beta(\theta_t^s) d_t^s, \tag{83}$$

$$\theta_t^s \ : \quad 0 = \varepsilon_t^s \beta'(\theta_t^s) \left(\left[\exp\left(\int_{\max\{t-\Delta,0\}}^t \nu_s \, \mathrm{d}s \right) - 1 \right] d_t^s + p_t^s - c(q_t^s) \right) + \vartheta_t \, \varepsilon_t^{b\prime} \alpha'(\theta_t^s)[u(q_t^s) - p_t^s], \tag{84}$$

$$p_t^s \ : \quad 0 = \varepsilon_t^s \beta(\theta_t^s) - \vartheta_t[\varepsilon_t^{b\prime} \alpha(\theta_t^s) + \iota_t] + \varsigma_t, \tag{85}$$

$$q_t^s \ : \quad 0 = -\, \varepsilon_t^s \beta(\theta_t^s) c'(q_t^s) + \vartheta_t \, \varepsilon_t^{b\prime} \alpha(\theta_t^s) u'(q_t^s), \tag{86}$$

$$d_t^s \ : \quad 0 = \varepsilon_t^s \beta(\theta_t^s) \left[\exp\left(\int_{\max\{t-\Delta,0\}}^t \nu_s \, \mathrm{d}s \right) - 1 \right] - \varsigma_t, , \tag{87}$$

Equations (85) and (87) yield

$$\vartheta_t = \frac{\varepsilon_t^s \beta(\theta_t^s) \exp\left(\int_{\max\{t-\Delta,0\}}^t \nu_s \, \mathrm{d}s \right)}{\varepsilon_t^{b\prime} \alpha(\theta_t^s) + \iota_t}. \tag{88}$$

We infer from Equation (87) that $d_t^s = p_t^s$ if $\int_{\max\{t-\Delta,0\}}^t \nu_s \, \mathrm{d}s > 0$, so that we can replace d_t^s with p_t^s in Equation (84). Plugging ϑ_t into Equation (84), we obtain

$$0 = \varepsilon_t^s \beta'(\theta_t^s) \left[\exp\left(\int_{\max\{t-\Delta,0\}}^t \nu_s \, \mathrm{d}s \right) p_t^s - c(q_t^s) \right] + \frac{\varepsilon_t^s \beta(\theta_t^s) \exp\left(\int_{\max\{t-\Delta,0\}}^t \nu_s \, \mathrm{d}s \right)}{\varepsilon_t^{b\prime} \alpha(\theta_t^s) + \iota_t} \, \varepsilon_t^{b\prime} \alpha'(\theta_t^s)[u(q_t^s) - p_t^s]. \tag{89}$$

Solving Equation (89) for p_t^s yields

$$\begin{aligned}
p_t^s &= \frac{\varepsilon_t^s \beta'(\theta_t^s)[\varepsilon_t^{b\prime}\alpha(\theta_t^s)+\iota_t]\exp\left(-\int_{\max\{t-\Delta,0\}}^t \nu_s\,\mathrm{d}s\right)c(q_t^s)-\varepsilon_t^s\beta(\theta_t^s)\,\varepsilon_t^{b\prime}\alpha'(\theta_t^s)u(q_t^s)}{\varepsilon_t^s\beta'(\theta_t^s)[\varepsilon_t^{b\prime}\alpha(\theta_t^s)+\iota]-\varepsilon_t^s\beta(\theta_t^s)\,\varepsilon_t^{b\prime}\alpha'(\theta_t^s)} \\
&= \frac{\chi(\theta_t^s)[\varepsilon_t^{b\prime}\alpha(\theta_t^s)+\iota_t]\exp\left(-\int_{\max\{t-\Delta,0\}}^t \nu_s\,\mathrm{d}s\right)c(q_t^s)+[1-\chi(\theta_t^s)]\,\varepsilon_t^{b\prime}\alpha(\theta_t^s)u(q_t^s)}{\chi(\theta_t^s)[\varepsilon_t^{b\prime}\alpha(\theta_t^s)+\iota_t]+[1-\chi(\theta_t^s)]\,\varepsilon_t^{b\prime}\alpha(\theta_t^s)} \qquad (90)\\
&= \frac{\chi(\theta_t^s)[\varepsilon_t^{b\prime}\alpha(\theta_t^s)+\iota_t]\exp\left(-\int_{\max\{t-\Delta,0\}}^t \nu_s\,\mathrm{d}s\right)c(q_t^s)+[1-\chi(\theta_t^s)]\,\varepsilon_t^{b\prime}\alpha(\theta_t^s)u(q_t^s)}{\chi(\theta_t^s)\iota_t+\varepsilon_t^{b\prime}\alpha(\theta_t^s)},
\end{aligned}$$

where we have used that $\theta\alpha'(\theta)/\alpha(\theta)=\chi(\theta)-1$.

The condition on q_t^s in Equation (28) is an immediate consequence from Equation (86) and the expression of ϑ_t in Equation (88).

C.3 Proof of Lemma 3

Optimality of $d_t^s = 0$. Suppose that $\int_{\max\{t-\Delta,0\}}^t \nu_\tau\,\mathrm{d}\tau = 0$. Equation (33) then reads as

$$0 = \frac{\mathrm{d}[\varepsilon_t^s\beta(\theta_t^s)]}{\mathrm{d}d_t^s}d_t^s, \qquad (91)$$

so that $d_t^s = 0$ satisfies the seller's first-order condition w.r.t. d_t^s.

We next pin down the uniqueness of $d_t^s = 0$ as a solution of the seller's ex-ante problem in (31), which reads as

$$\max_{d_t^s}\left\{\tilde{\varepsilon}_t^s\beta(\tilde{\theta}_t^s)[-c(\tilde{q}_t^s)+\tilde{p}_t^s]-\zeta(\tilde{\varepsilon}_t^s)\right\}, \qquad (92)$$

where $(\tilde{q}_t^s,\tilde{p}_t^s,\tilde{\theta}_t^s,\tilde{\varepsilon}_t^s)$ solves the problem in (29), subject to Constraint (30), given d_t^s. Note that any solution $(\hat{d}_t^s,\hat{q}_t^s,\hat{p}_t^s,\hat{\theta}_t^s,\hat{\varepsilon}_t^s)$ of

$$\max_{d_t^s,q_t^s,p_t^s,\theta_t^s,\varepsilon_t^s}\left\{\varepsilon_t^s\beta(\theta_t^s)[-c(q_t^s)+p_t^s-d_t^s]-\zeta(\varepsilon_t^s)\right\}, \qquad (93)$$

subject to Constraint (30), features $\hat{d}_t^s = 0$, since the objective function decreases in d_t^s, while d_t^s does not directly enter in Constraint (30). Hence, by choosing $d_t^s = 0$ in the problem in (31), the seller can induce any solution to the problem in (93), subject to Constraint (30), and the value of the problem in (31) is equal to the value of the problem in (93), subject to Constraint (30).

Suppose that $d_t^s > 0$ solves the problem in (31). It then holds that the seller's FOC w.r.t. ε_t^s, corresponding to the problem in (29), subject to Constraint (30), reads as

$$0 = \beta(\tilde{\theta}_t^s)[-c(\tilde{q}_t^s)+\tilde{p}_t^s-d_t^s]-\zeta'(\tilde{\varepsilon}_t^s), \qquad (94)$$

where $(\tilde{q}_t^s, \tilde{p}_t^s, \tilde{\theta}_t^s, \tilde{\varepsilon}_t^s)$ is induced by the choice of d_t^s. Note that $(0, \tilde{q}_t^s, \tilde{p}_t^s, \tilde{\theta}_t^s, \tilde{\varepsilon}_t^s)$ is also a solution of the problem in (93), subject to Constraint (30), as otherwise

$$\tilde{\varepsilon}_t^s \beta(\tilde{\theta}_t^s)[-c(\tilde{q}_t^s) + \tilde{p}_t^s] - \zeta(\tilde{\varepsilon}_t^s) < \hat{\varepsilon}_t^s \beta(\hat{\theta}_t^s)[-c(\hat{q}_t^s) + \hat{p}_t^s] - \zeta(\hat{\varepsilon}_t^s), \tag{95}$$

which would contradict the optimal choice of d_t^s in the problem in (31). Hence, the seller's FOC w.r.t. ε_t^s, corresponding to the problem in (93), subject to Constraint (30), reads as

$$0 = \beta(\tilde{\theta}_t^s)[-c(\tilde{q}_t^s) + \tilde{p}_t^s] - \zeta'(\tilde{\varepsilon}_t^s). \tag{96}$$

Equations (94) and (96) contradict each other.

Response of d_t^s to the borrowing discount. To determine $\mathrm{d}d_t^s / \mathrm{d}\left[\int_{\max\{t-\Delta,0\}}^t \nu_\tau \, \mathrm{d}\tau\right]$, we simplify notation by letting

$$x_t \equiv \exp\left(\int_{\max\{t-\Delta,0\}}^t \nu_\tau \, \mathrm{d}\tau\right). \tag{97}$$

The second-order condition of optimality w.r.t. d_t^s, which we obtain from differentiating the right-hand side of Equation (33) with respect to d_t^s, reads as

$$\begin{aligned} 0 &> \frac{\mathrm{d}[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}d_t^s}[x_t - 1] + \frac{\mathrm{d}^2[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}(d_t^s)^2} x_t d_t^s + \frac{\mathrm{d}[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}d_t^s} x_t \\ &= \frac{\mathrm{d}[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}d_t^s}[2x_t - 1] + \frac{\mathrm{d}^2[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}(d_t^s)^2} x_t d_t^s. \end{aligned} \tag{98}$$

Since Equation (33) holds for all levels of x_t and associated choice variables of the seller, differentiating Equation (33) w.r.t. x_t yields

$$\begin{aligned} 0 &= \left[\frac{\mathrm{d}[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}d_t^s}[x_t - 1] + \frac{\mathrm{d}^2[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}(d_t^s)^2} x_t d_t^s + \frac{\mathrm{d}[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}d_t^s} x_t\right] \frac{\mathrm{d}d_t^s}{\mathrm{d}x_t} + \varepsilon_t^s \beta(\theta_t^s) + \frac{\mathrm{d}[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}d_t^s} d_t^s \\ &= \left[\frac{\mathrm{d}[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}d_t^s}[2x_t - 1] + \frac{\mathrm{d}^2[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}(d_t^s)^2} x_t d_t^s\right] \frac{\mathrm{d}d_t^s}{\mathrm{d}x_t} \\ &\quad + \frac{1}{x_t}\left[\varepsilon_t^s \beta(\theta_t^s)(x_t - 1) + \frac{\mathrm{d}[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}d_t^s} x_t d_t^s + \varepsilon_t^s \beta(\theta_t^s)\right]. \end{aligned} \tag{99}$$

With Equation (33) and Inequality (98), we obtain

$$\frac{\mathrm{d}d_t^s}{\mathrm{d}x_t} = -\frac{\varepsilon_t^s \beta(\theta_t^s)}{x_t}\left[\frac{\mathrm{d}[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}d_t^s}[2x_t - 1] + \frac{\mathrm{d}^2[\varepsilon_t^s \beta(\theta_t^s)]}{\mathrm{d}(d_t^s)^2} x_t d_t^s\right]^{-1} > 0, \tag{100}$$

which immediately implies that $\mathrm{d}d_t^s / \mathrm{d}\left[\int_{\max\{t-\Delta,0\}}^t \nu_\tau \, \mathrm{d}\tau\right] > 0$.

C.4 Proof of Lemma 4

Let ϑ_t denote the Lagrange multiplier for Constraint (30) associated with (29). The FOCs of this ex-post maximization problem are

$$\varepsilon_t^s \ : \quad 0 = \beta(\theta_t^s)[-c(q_t^s) + p_t^s - d_t^s] - \zeta'(\varepsilon_t^s), \tag{101}$$

$$\theta_t^s \ : \quad 0 = \varepsilon_t^s \beta'(\theta_t^s)[-c(q_t^s) + p_t^s - d_t^s] + \vartheta_t\, \varepsilon_t^{b\prime} \alpha'(\theta_t^s)[u(q_t^s) - p_t^s], \tag{102}$$

$$p_t^s \ : \quad 0 = \varepsilon_t^s \beta(\theta_t^s) - \vartheta_t[\varepsilon_t^{b\prime}\alpha(\theta_t^s) + \iota_t], \tag{103}$$

$$q_t^s \ : \quad 0 = -\,\varepsilon_t^s \beta(\theta_t^s) c'(q_t^s) + \vartheta_t\, \varepsilon_t^{b\prime}\alpha(\theta_t^s) u'(q_t^s). \tag{104}$$

Equation (103) yields

$$\vartheta_t = \frac{\varepsilon_t^s \beta(\theta_t^s)}{\varepsilon_t^{b\prime}\alpha(\theta_t^s) + \iota_t}. \tag{105}$$

Plugging ϑ_t into Equation (102), we obtain

$$0 = \varepsilon_t^s \beta'(\theta_t^s)\,[p_t^s - c(q_t^s) - d_t^s] + \frac{\varepsilon_t^s \beta(\theta_t^s)}{\varepsilon_t^{b\prime}\alpha(\theta_t^s) + \iota_t}\, \varepsilon_t^{b\prime}\alpha'(\theta_t^s)[u(q_t^s) - p_t^s], \tag{106}$$

from which we obtain

$$\begin{aligned} p_t^s &= \frac{\varepsilon_t^s \beta'(\theta_t^s)[\varepsilon_t^{b\prime}\alpha(\theta_t^s) + \iota_t][c(q_t^s) + d_t^s] - \varepsilon_t^s \beta(\theta_t^s)\, \varepsilon_t^{b\prime}\alpha'(\theta_t^s) u(q_t^s)}{\varepsilon_t^s \beta'(\theta_t^s)[\varepsilon_t^{b\prime}\alpha(\theta_t^s) + \iota] - \varepsilon_t^s \beta(\theta_t^s)\, \varepsilon_t^{b\prime}\alpha'(\theta_t^s)} \\ &= \frac{\chi(\theta_t^s)[\varepsilon_t^{b\prime}\alpha(\theta_t^s) + \iota_t][c(q_t^s) + d_t^s] + [1 - \chi(\theta_t^s)]\, \varepsilon_t^{b\prime}\alpha(\theta_t^s) u(q_t^s)}{\chi(\theta_t^s)[\varepsilon_t^{b\prime}\alpha(\theta_t^s) + \iota_t] + [1 - \chi(\theta_t^s)]\, \varepsilon_t^{b\prime}\alpha(\theta_t^s)} \\ &= \frac{\chi(\theta_t^s)[\varepsilon_t^{b\prime}\alpha(\theta_t^s) + \iota_t][c(q_t^s) + d_t^s] + [1 - \chi(\theta_t^s)]\, \varepsilon_t^{b\prime}\alpha(\theta_t^s) u(q_t^s)}{\chi(\theta_t^s)\iota_t + \varepsilon_t^{b\prime}\alpha(\theta_t^s)}, \end{aligned} \tag{107}$$

where we have used that $\theta\alpha'(\theta)/\alpha(\theta) = \chi(\theta) - 1$.

The condition on q_t^s in Equation (36) is an immediate consequence from Equation (104) and the expression of ϑ_t.

C.5 Proof of Proposition 1

Welfare. The equilibrium features one search market with tightness $\theta_t = \mu_t^b\, \varepsilon_t^b / \, \varepsilon_t^s$ and posted terms of trade (q_t, p_t) at time t. It holds that $\varepsilon_t^s \beta(\theta_t^s) = \mathcal{N}(\mu_t^b\, \varepsilon_t^b, \varepsilon_t^s) = \mu_t^b\, \varepsilon_t^b \alpha(\theta_t^s)$. Using the agents' continuation values V_t^{FI}, V_t^b, and V_t^s in Equations (7), (14), and (22), welfare reads as

$$\begin{aligned}\mathcal{W} = &\int_0^\infty e^{-\rho t}\left[\mathcal{N}(\mu_t^b\,\varepsilon_t^b,\varepsilon_t^s)\left[p_t - c(q_t) + \left(\exp\left(\int_{\max\{t-\Delta,0\}}^t \nu_\tau\,\mathrm{d}\tau\right)-1\right)d_t^s\right] - \zeta(\varepsilon_t^s)\right]\mathrm{d}t\\ &+\int_0^\infty e^{-\rho t}\mu_t^b\underbrace{\left[\frac{\mathcal{N}(\mu_t^b\,\varepsilon_t^b,\varepsilon_t^s)}{\mu_t^b}[u(q_t)-p_t] - \zeta(\varepsilon_t^b) - \iota_t[a_t^b + m_t^b]\right]}_{\equiv v_t^b}\mathrm{d}t\\ &+\int_0^\infty e^{-\rho t}\dot{\Upsilon}_t\,\mathrm{d}t + \Upsilon_0 + \int_0^\infty e^{-\rho t}\left[\iota_t a_t^{FI} - \nu_t \ell_t^{FI}\right]\mathrm{d}t.\end{aligned} \tag{108}$$

We exploited that $\mu_t^b < 1$ only if $v_t^b = 0$; otherwise, all buyers are active, such that $\mu_t^b = 1$. Using the expression in Equation (11) for the present value of loan contracts written by sellers, the sellers' lifetime utility from writing loan contracts reads as

$$\begin{aligned}&\int_0^\infty e^{-\rho t}\mathcal{N}(\mu_t^b\,\varepsilon_t^b,\varepsilon_t^s)\left(\exp\left(\int_{\max\{t-\Delta,0\}}^t \nu_\tau\,\mathrm{d}\tau\right)-1\right)d_t^s\,\mathrm{d}t\\ &=\int_0^\Delta e^{-\rho t}\mathcal{N}(\mu_t^b\,\varepsilon_t^b,\varepsilon_t^s)\exp\left(\int_0^t \nu_\tau\,\mathrm{d}\tau\right)d_t^s\,\mathrm{d}t - \int_0^\infty e^{-\rho t}\mathcal{N}(\mu_t^b\,\varepsilon_t^b,\varepsilon_t^s)d_t^s\,\mathrm{d}t\\ &\quad+\int_0^\infty e^{-(t+\Delta)\rho}\mathcal{N}(\mu_{t+\Delta}^b\,\varepsilon_{t+\Delta}^b,\varepsilon_{t+\Delta}^s)\exp\left(\int_t^{t+\Delta}\nu_\tau\,\mathrm{d}\tau\right)d_{t+\Delta}^s\,\mathrm{d}t\\ &=\int_0^\Delta \mathcal{N}(\mu_t^b\,\varepsilon_t^b,\varepsilon_t^s)\exp\left(-\int_0^t(\rho-\nu_\tau)\,\mathrm{d}\tau\right)d_t^s\,\mathrm{d}t\\ &\quad+\int_0^\infty e^{-\rho t}\left[\mathcal{N}(\mu_{t+\Delta}^b\,\varepsilon_{t+\Delta}^b,\varepsilon_{t+\Delta}^s)\exp\left(-\int_t^{t+\Delta}(\rho-\nu_\tau)\,\mathrm{d}\tau\right)d_{t+\Delta}^s - \mathcal{N}(\mu_t^b\,\varepsilon_t^b,\varepsilon_t^s)d_t^s\right]\mathrm{d}t\\ &=\ell_0^s + \int_0^\infty e^{-\rho t}\left[\dot{\ell}_t^s - r_t^\ell \ell_t^s\right]\mathrm{d}t.\end{aligned} \tag{109}$$

Loan-market clearing $\ell_t^s = \ell_t^{FI}$ yields

$$\int_0^\infty e^{-\rho t}\nu_t\ell_t^{FI}\,\mathrm{d}t = \ell_0^{FI} + \int_0^\infty e^{-\rho t}\left[\dot{\ell}_t^{FI} - r_t^\ell\ell_t^{FI}\right]\mathrm{d}t = \ell_0^s + \int_0^\infty e^{-\rho t}\left[\dot{\ell}_t^s - r_t^\ell\ell_t^s\right]\mathrm{d}t. \tag{110}$$

The binding government-budget constraint $\Upsilon_t = \phi_0 M_0 + \int_0^t \phi_\tau \dot{M}_\tau\,\mathrm{d}\tau$ and money-market clearing $\phi_t M_t = \mu_t^b m_t$ with $\iota_t = \rho - \dot{\phi}_t/\phi_t$ yield

$$\begin{aligned}\int_0^\infty e^{-\rho t}\iota_t\mu_t^b m_t^b\,\mathrm{d}t = \int_0^\infty e^{-\rho t}\iota_t\phi_t M_t\,\mathrm{d}t &= \int_0^\infty e^{-\rho t}[\rho\phi_t M_t - \dot{\phi}_t M_t]\,\mathrm{d}t\\ &= \phi_0 M_0 + \int_0^\infty e^{-\rho t}\phi_t\dot{M}_t\,\mathrm{d}t = \Upsilon_0 + \int_0^\infty e^{-\rho t}\dot{\Upsilon}_t\,\mathrm{d}t.\end{aligned} \tag{111}$$

We used integration by parts. Finally, private-money market clearing $a_t^{FI} = \mu_t^b a_t^b$ proves that

$$\mathcal{W} = \int_0^\infty e^{-\rho t}\left[\mathcal{N}(\mu_t^b\,\varepsilon_t^b,\varepsilon_t^s)[u(q_t)-c(q_t)] - \zeta(\varepsilon_t^s) - \mu_t^b\zeta(\varepsilon_t^b)\right]\mathrm{d}t. \tag{112}$$

First-best allocation. The necessary conditions that characterize a first-best allocation $(\varepsilon_t^{s\star}, \varepsilon_t^{b\star}, \mu_t^{b\star}, q_t^\star)$ at time t are given by the FOCs

$$\begin{aligned}
\varepsilon_t^s &: \quad 0 = \frac{\partial \mathcal{N}(\mu_t^b\, \varepsilon_t^b, \varepsilon_t^s)}{\partial\, \varepsilon^s}[u(q_t) - c(q_t)] - \zeta'(\varepsilon_t^s), \\
\varepsilon_t^b &: \quad 0 = \frac{\partial \mathcal{N}(\mu_t^b\, \varepsilon_t^b, \varepsilon_t^s)}{\partial\, \varepsilon^b}[u(q_t) - c(q_t)] - \zeta'(\varepsilon_t^b), \\
\mu_t^b &: \quad 0 \leq \frac{\partial \mathcal{N}(\mu_t^b\, \varepsilon_t^b, \varepsilon_t^s)}{\partial\, \varepsilon^b}\, \varepsilon_t^b[u(q_t) - c(q_t)] - \zeta(\varepsilon_t^b) \qquad \text{with “}<\text{” only if } \mu_t^b = 1, \\
q_t &: \quad 0 = u'(q_t) - c'(q_t).
\end{aligned} \tag{113}$$

It is clear that $q_t^\star = q^\star = q^*$ is unique and stationary. The convexity of ζ with $\zeta(0) = 0$ and the FOC w.r.t. ε_t^b yield that $\mu_t^{b\star} = \mu^{b\star} = 1$ ist stationary as well.

To prove the uniqueness and stationarity of $\varepsilon_t^{s\star}$ and $\varepsilon_t^{b\star}$, we note that

$$\begin{aligned}
\frac{\partial \mathcal{N}(\varepsilon^b, \varepsilon^s)}{\partial\, \varepsilon^s} &= \frac{\partial\left[\varepsilon^s \beta\left(\frac{\varepsilon^b}{\varepsilon^s}\right)\right]}{\partial\, \varepsilon^s} = \beta\left(\frac{\varepsilon^b}{\varepsilon^s}\right)\left[1 - \chi\left(\frac{\varepsilon^b}{\varepsilon^s}\right)\right] \qquad \text{and} \\
\frac{\partial \mathcal{N}(\varepsilon^b, \varepsilon^s)}{\partial\, \varepsilon^b} &= \frac{\partial\left[\varepsilon^b \alpha\left(\frac{\varepsilon^b}{\varepsilon^s}\right)\right]}{\partial\, \varepsilon^b} = \alpha\left(\frac{\varepsilon^b}{\varepsilon^s}\right)\chi\left(\frac{\varepsilon^b}{\varepsilon^s}\right).
\end{aligned} \tag{114}$$

Using that $\theta = \varepsilon^{b\star}/\,\varepsilon^{s\star}$, the FOCs w.r.t. ε^s and ε^b read as

$$\zeta'(\varepsilon^s) = \beta(\theta)[1 - \chi(\theta)][u(q^\star) - c(q^\star)] \quad \text{and} \quad \zeta'(\varepsilon^b) = \alpha(\theta)\chi(\theta)[u(q^\star) - c(q^\star)]. \tag{115}$$

To prove that these equations uniquely pin down $\varepsilon^{s\star}$ and $\varepsilon^{b\star}$, recall that $\chi(\theta) = \theta\beta'(\theta)/\beta(\theta)$, that $\alpha(\theta) = \beta(\theta)/\theta$, and that $\beta''(\theta) < 0$, so that

$$\begin{aligned}
\frac{\partial}{\partial\theta}\left[\beta(\theta)(1 - \chi(\theta))\right] &= \frac{\partial}{\partial\theta}\left[\beta(\theta) - \theta\beta'(\theta)\right] = -\theta\beta''(\theta) > 0 \qquad \text{and} \\
\frac{\partial}{\partial\theta}\left[\alpha(\theta)\chi(\theta)\right] &= \frac{\partial}{\partial\theta}\left[\left(\frac{\beta(\theta)}{\theta}\right)\left(\frac{\theta\beta'(\theta)}{\beta(\theta)}\right)\right] = \beta''(\theta) < 0.
\end{aligned} \tag{116}$$

The inverse ζ^{-1} of ζ is an increasing function on $\mathbb{R}_+$, so that the functions

$$\begin{aligned}
\hat{e}^s(\theta) &= \zeta^{-1}\left(\beta(\theta)(1 - \chi(\theta))[u(q^\star) - c(q^\star)]\right) \qquad \text{and} \\
\hat{e}^b(\theta) &= \zeta^{-1}\left(\alpha(\theta)\chi(\theta)[u(q^\star) - c(q^\star)]\right)
\end{aligned} \tag{117}$$

are increasing and decreasing, respectively, in θ. We infer that the function $\hat{\theta}(\theta) = \hat{e}^b(\theta)/\hat{e}^s(\theta)$ is decreasing in θ and can intersect at most once with the identity function $\mathrm{id}(\theta) = \theta$. Since $\hat{\theta}(\theta^\star) = \mathrm{id}(\theta^\star)$ is a necessary condition for $\theta^\star$ to be featured in equilibrium, the uniqueness of $(\varepsilon^{s\star}, \varepsilon^{b\star})$ is proven.

C.6 Proof of Lemma 5

At the FR, Lemma 1 implies that $\nu = \eta\iota = 0$. For economy NP, Lemma 3 implies that $d = 0$. Hence, the FOCs that characterize the seller's search effort ε^s and market choice (q, p, θ) are the same in economies P and NP, as can be seen in Equations (83) to (86) and in Equations (101) to (104), respectively. Since economy F is a special case of both economies P and NP when $\Delta\eta = 0$, these FOCs also apply to economy F. Lemmas 2 and 4 yield $q = q^\star$ and

$$p = [1 - \chi(\theta)]u(q^\star) + \chi(\theta)c(q^\star). \tag{118}$$

Hence, the sellers' FOC w.r.t. ε^s in Equation (83) (and in Equation (101)) reads as Equation (39), so that $\varepsilon^s = \varepsilon^{s\star}$. It is also clear from Constraint (24) (and Constraint (30)) that the buyers choose $\varepsilon^b = \varepsilon^{b\star}$, as characterized in Equation (40).

C.7 Proof of Proposition 4

The relation of ι and B in equilibrium. The buyers' and the sellers' FOCs w.r.t. search effort read as

$$(\varepsilon^b)^\tau = \theta^{\chi-1}\omega\frac{\mathcal{S}}{D} \qquad \text{and} \qquad (\varepsilon^s)^\tau = \theta^\chi(1-\omega)\mathcal{S}, \tag{119}$$

where

$$\omega \equiv \frac{\chi[1+B]}{1-\chi+\chi[1+B]}, \tag{120}$$

and where $\mathcal{S} \equiv Du(q) - c(q)$ with $D \equiv \exp(\Delta\eta\iota)$. Moreover, Equation (26) reads as

$$\frac{Du'(q)}{c'(q)} = 1 + B \qquad \Leftrightarrow \qquad q = D^{\frac{1}{\sigma+\kappa}} q_B \qquad \text{with} \qquad q_B \equiv \left[\frac{1}{1+B}\right]^{\frac{1}{\sigma+\kappa}}, \tag{121}$$

so that

$$\mathcal{S} = Du(q) - c(q) = D^{\frac{1+\kappa}{\sigma+\kappa}}\mathcal{S}_B \qquad \text{with} \qquad \mathcal{S}_B \equiv u(q_B) - c(q_B). \tag{122}$$

Note that $\theta = \mu^b\,\varepsilon^b/\,\varepsilon^s$ and that $\alpha(\theta) = \theta^{\chi-1}$. Hence,

$$\begin{aligned}[\varepsilon^b\alpha(\theta)]^\tau &= (\mu^b)^{\tau(\chi-1)}[(\varepsilon^b)^\chi(\varepsilon^s)^{1-\chi}]^\tau = (\mu^b)^{\tau(\chi-1)}\omega^\chi(1-\omega)^{1-\chi}\mathcal{S}D^{-\chi} \\ &= (\mu^b)^{\tau(\chi-1)}\omega^\chi(1-\omega)^{1-\chi}\mathcal{S}_B D^{\frac{1+\kappa}{\sigma+\kappa}-\chi}.\end{aligned} \tag{123}$$

Recall that $\varepsilon^b\alpha(\theta) = \iota/B$. It thus holds that

$$f(\iota) = \left[\frac{1}{\mu^b}\right]^{\tau(1-\chi)} g(B) \tag{124}$$

for

$$f(\iota) \equiv \iota^{\tau} \exp\left(-\Delta\eta \left[\frac{1+\kappa}{\sigma+\kappa} - \chi\right] \iota\right) \qquad \text{and} \qquad g(B) \equiv B^{\tau}\omega^{\chi}(1-\omega)^{1-\chi}\mathcal{S}_B. \quad (125)$$

Equation (124) relates the opportunity cost of holding money ι with the effective opportunity cost B in equilibrium.

The buyers' participation constraint dependent on B. The buyers' participation constraint reads as

$$v^b \geq 0 \qquad \Leftrightarrow \qquad \varepsilon^b \alpha(\theta)[u(q) - p] - \iota p - \zeta(\varepsilon^b) \geq 0, \quad (126)$$

where p is given in Equation (26). The functional form of ζ and the buyers' FOC w.r.t. search effort allow us to write

$$\zeta(\varepsilon^b) = \left[\frac{1}{1+\tau}\right] \varepsilon^b \zeta'(\varepsilon^b) = \left[\frac{1}{1+\tau}\right] \varepsilon^b \alpha(\theta)\omega\frac{\mathcal{S}}{D}. \quad (127)$$

Hence, Inequality (126) can be rewritten as

$$\left[\frac{\tau}{1+\tau}\right] \varepsilon^b \alpha(\theta)\omega\frac{\mathcal{S}}{D} - \iota p \geq 0 \qquad \Leftrightarrow \qquad \left[\frac{\tau}{1+\tau}\right] \omega\frac{\mathcal{S}}{D} - Bp \geq 0, \quad (128)$$

which is equivalent to

$$F\chi[1+B]\left[u(q) - \frac{c(q)}{D}\right] - B\left[(1-\chi)u(q) + \chi[1+B]\frac{c(q)}{D}\right] \geq 0 \quad (129)$$

$$\Leftrightarrow \qquad [F\chi[1+B] - (1-\chi)B]\, u(q) - \chi\left[F[1+B] + B[1+B]\right]\frac{c(q)}{D} \geq 0 \quad (130)$$

$$\Leftrightarrow \qquad F\chi[1+B] - (1-\chi)B - \chi\left[\frac{1-\sigma}{1+\kappa}\right][F+B] \geq 0 \quad (131)$$

$$\Leftrightarrow \qquad F\chi - (1-\chi)\left[\frac{B}{1+B}\right] - \chi\left[\frac{1-\sigma}{1+\kappa}\right]\frac{F+B}{1+B} \geq 0, \quad (132)$$

where $F \equiv \tau/(1+\tau)$. The left-hand side of Inequality (132) is strictly decreasing in B. Hence, Inequality (132) holds for all $B \geq 0$ if and only if

$$\left[\frac{\tau}{1+\tau}\right]\chi - (1-\chi) - \chi\left[\frac{1-\sigma}{1+\kappa}\right] \geq 0 \qquad \Leftrightarrow \qquad 1 - \chi\left[\frac{\tau}{1+\tau} + \frac{\sigma+\kappa}{1+\kappa}\right] \leq 0. \quad (133)$$

In a PPE, it must hold that $v^b = 0$. Hence, if Inequality (133) holds true, there cannot arise a PPE for any $\iota \geq 0$.

If Inequality (133) does not hold true, it holds that

$$
\begin{aligned}
& [1+B]F\chi-(1-\chi)B-\chi\left[\frac{1-\sigma}{1+\kappa}\right][F+B]=0 \\
\Leftrightarrow \quad & \left[F\chi-(1-\chi)-\chi\left[\frac{1-\sigma}{1+\kappa}\right]\right]B+F\chi-\chi\left[\frac{1-\sigma}{1+\kappa}\right]F=0 \\
\Leftrightarrow \quad & B=\frac{\left[\frac{\sigma+\kappa}{1+\kappa}\right]\chi F}{1-\chi\left[F+\frac{\sigma+\kappa}{1+\kappa}\right]}\equiv\overline{B}.
\end{aligned}
\tag{134}
$$

$\overline{B}$ thus is the largest effective opportunity cost that can be featured in equilibrium.

Behavior of g. To prove equilibrium existence and uniqueness, we narrow down the behavior of $g(B)$ in $B\in[0,\infty)$. From the definition of g in Equation (125), we obtain

$$
\begin{aligned}
g(B) &= B^{\tau}\left[\frac{\chi^{\chi}(1-\chi)^{1-\chi}[1+B]^{\chi}}{1-\chi+\chi[1+B]}\right]\left[\frac{1}{1-\sigma}-\frac{1}{1+\kappa}\frac{1}{1+B}\right]\left[\frac{1}{1+B}\right]^{\frac{1-\sigma}{\sigma+\kappa}} \\
&= B^{\tau+\chi-\frac{1+\kappa}{\sigma+\kappa}}\underbrace{\left[\frac{\chi^{\chi}(1-\chi)^{1-\chi}[1+B]^{\chi}B^{1-\chi}}{1-\chi+\chi[1+B]}\right]\left[\frac{1}{1-\sigma}-\frac{1}{1+\kappa}\frac{1}{1+B}\right]\left[\frac{B}{1+B}\right]^{\frac{1-\sigma}{\sigma+\kappa}}}_{\equiv\Phi(B)}.
\end{aligned}
\tag{135}
$$

Note that $\Phi'(B)>0$ and that

$$
\lim_{B\to\infty}\Phi(B)=\frac{1}{1-\sigma}\left[\frac{1-\chi}{\chi}\right]^{1-\chi}.
\tag{136}
$$

Hence,

$$
\lim_{B\to\infty}g(B)=\begin{cases}\infty & \text{if } \tau+\chi-\frac{1+\kappa}{\sigma+\kappa}>0,\\ \frac{1}{1-\sigma}\left[\frac{1-\chi}{\chi}\right]^{1-\chi} & \text{if } \tau+\chi-\frac{1+\kappa}{\sigma+\kappa}=0,\\ 0 & \text{otherwise.}\end{cases}
\tag{137}
$$

To pin down $g'(B)$, we determine $\mathrm{d}g(B)/g(B)$ by using the specification of g in Equation (125). Note that

$$
\begin{aligned}
\frac{\mathrm{d}\mathcal{S}_B}{\mathcal{S}_B} &= \left[\frac{\frac{1}{1+\kappa}\frac{1}{1+B}}{\frac{1}{1-\sigma}-\frac{1}{1+\kappa}\frac{1}{1+B}}-\frac{1-\sigma}{\sigma+\kappa}\right]\frac{B}{1+B}\frac{\mathrm{d}B}{B} \\
&= \frac{1-\sigma}{\sigma+\kappa}\left[\frac{\frac{\sigma+\kappa}{1+\kappa}\frac{1}{1+B}}{1-\frac{1-\sigma}{1+\kappa}\frac{1}{1+B}}-1\right]\frac{B}{1+B}\frac{\mathrm{d}B}{B} \\
&= \frac{1-\sigma}{\sigma+\kappa}\left[\frac{\frac{\sigma+\kappa}{1+\kappa}}{\frac{\sigma+\kappa}{1+\kappa}+B}-1\right]\frac{B}{1+B}\frac{\mathrm{d}B}{B}=-\frac{1-\sigma}{\sigma+\kappa}\left[\frac{B}{\frac{\sigma+\kappa}{1+\kappa}+B}\right]\frac{B}{1+B}\frac{\mathrm{d}B}{B},
\end{aligned}
\tag{138}
$$

and that

$$\frac{\mathrm{d}\omega}{\omega} = \left[1 - \frac{\chi[1+B]}{1-\chi+\chi[1+B]}\right]\frac{B}{1+B}\frac{\mathrm{d}B}{B} = [1-\omega]\frac{B}{1+B}\frac{\mathrm{d}B}{B}. \tag{139}$$

We obtain

$$\begin{aligned}\frac{\mathrm{d}g(B)}{g(B)} &= \tau\frac{\mathrm{d}B}{\mathrm{d}B} + \left[\chi - \frac{(1-\chi)\omega}{1-\omega}\right]\frac{\mathrm{d}\omega}{\omega} + \frac{\mathrm{d}\mathcal{S}}{\mathcal{S}} \\ &= \underbrace{\left[\tau + \left[\chi(1-\omega) - (1-\chi)\omega - \frac{1-\sigma}{\sigma+\kappa}\left[\frac{B}{B+\frac{\sigma+\kappa}{1+\kappa}}\right]\right]\frac{B}{1+B}\right]}_{\equiv\Psi(B)}\frac{\mathrm{d}B}{B}.\end{aligned} \tag{140}$$

Note that $\Psi'(B) < 0$ and that $\lim_{B\to\infty}\Psi(B) = \tau + \chi - (1+\kappa)/(\sigma+\kappa)$. Hence, g is either strictly increasing, or g has unique maximum. We distinguish now between the case in which Inequality (133) holds and the case in which it does not.

Suppose that Inequality (133) **holds true.** Then there cannot arise any PPE for any $\iota \geq 0$. Hence, we infer $\mu^b = 1$. Since $\Psi'(B) < 0$, it suffices to show that

$$\lim_{B\to\infty}\Psi(B) = \tau + \chi - \frac{1+\kappa}{\sigma+\kappa} > 0 \tag{141}$$

to prove that g is strictly increasing in $B \in [0,\infty)$ and that $\lim_{B\to\infty} g(B) = \infty$. Note that Inequality (133) is equivalent to

$$\frac{\sigma+\kappa}{1+\kappa} \geq \frac{1}{\chi} - \frac{\tau}{1+\tau} \qquad \Leftrightarrow \qquad \frac{1+\kappa}{\sigma+\kappa} \leq \frac{\chi(1+\tau)}{1+(1-\chi)\tau}, \tag{142}$$

so that

$$\tau + \chi - \frac{1+\kappa}{\sigma+\kappa} \geq \tau + \chi - \frac{\chi(1+\tau)}{1+\tau(1-\chi)} \geq \tau + \chi - \chi(1+\tau) > 0. \tag{143}$$

Hence, $f(\iota) = g(B)$ induces a map $\iota \mapsto B|_\iota$ for any $\iota \geq 0$, so that any $\iota \geq 0$ features a unique FPE.

Suppose that Inequality (133) **does not hold.** Then $\overline{B}$ is the largest effective opportunity cost B that can be featured in equilibrium. Since $\Psi'(B) < 0$, it suffices to show that $\Psi(\overline{B}) > 0$ to prove that g is strictly increasing in $B \in [0, \overline{B}]$. Note that

$$1 + \overline{B} = \frac{1 - \chi\left[F + \frac{\sigma+\kappa}{1+\kappa}\right] + \left[\frac{\sigma+\kappa}{1+\kappa}\right]\chi F}{1 - \chi\left[F + \frac{\sigma+\kappa}{1+\kappa}\right]}, \tag{144}$$

and that

$$\begin{aligned}\tau\left[\frac{1+\overline{B}}{\overline{B}}\right] &= \tau\left[\frac{1-\chi\left[F+\frac{\sigma+\kappa}{1+\kappa}\right]+\left[\frac{\sigma+\kappa}{1+\kappa}\right]\chi F}{\left[\frac{\sigma+\kappa}{1+\kappa}\right]\chi F}\right] = \left[\frac{1+\kappa}{\sigma+\kappa}\right]\left[\frac{\tau}{\chi F}-\tau\right]-\frac{\tau}{F}+\tau \\ &= \left[\frac{1+\kappa}{\sigma+\kappa}\right]\left[\frac{1+\tau}{\chi}-\tau\right]-(1+\tau)+\tau = \left[\frac{1+\kappa}{\sigma+\kappa}\right]\left[\frac{1}{\chi}+\frac{(1-\chi)\tau}{\chi}\right]-1 \\ &= \frac{1}{\chi}+\left[\frac{1-\sigma}{\sigma+\kappa}\right]\frac{1}{\chi}+\left[\frac{1+\kappa}{\sigma+\kappa}\right]\frac{(1-\chi)\tau}{\chi}-1 \\ &= \frac{1-\chi}{\chi}+\left[\frac{1-\sigma}{\sigma+\kappa}\right]\frac{1}{\chi}+\left[\frac{1+\kappa}{\sigma+\kappa}\right]\frac{(1-\chi)\tau}{\chi}.\end{aligned} \tag{145}$$

Furthermore, it holds that

$$\chi(1-\omega)-(1-\chi)\omega = \frac{\chi(1-\chi)-(1-\chi)\chi[1+B]}{1+\chi+\chi[1+B]} > -(1-\chi). \tag{146}$$

Hence,

$$\begin{aligned}\left[\frac{1+\overline{B}}{\overline{B}}\right]\Psi(\overline{B}) &= \tau\left[\frac{1+\overline{B}}{\overline{B}}\right]+\underbrace{\chi(1-\omega)-(1-\chi)\omega}_{>-(1-\chi)}-\left[\frac{1-\sigma}{\sigma+\kappa}\right]\underbrace{\frac{\overline{B}}{\overline{B}+\frac{\sigma+\kappa}{1+\kappa}}}_{<1} \\ &> \frac{1-\chi}{\chi}+\left[\frac{1-\sigma}{\sigma+\kappa}\right]\frac{1}{\chi}+\left[\frac{1+\kappa}{\sigma+\kappa}\right]\frac{(1-\chi)\tau}{\chi}-(1-\chi)-\frac{1-\sigma}{\sigma+\kappa} \\ &= \frac{(1-\chi)^2}{\chi}+\left[\frac{1-\sigma}{\sigma+\kappa}\right]\frac{1-\chi}{\chi}+\left[\frac{1+\kappa}{\sigma+\kappa}\right]\frac{(1-\chi)\tau}{\chi} > 0.\end{aligned} \tag{147}$$

This inequality establishes that g is strictly increasing in $B \in [0, \overline{B}]$. Hence, every

$$\iota \in \left\{\iota \geq 0 \ : \ f(\iota) > g(\overline{B})\right\} \equiv I \tag{148}$$

features a unique PPE with

$$\mu^b = \left[\frac{g(\overline{B})}{f(\iota)}\right]^{\frac{1}{\tau(1-\chi)}}, \tag{149}$$

but no FPE. Moreover, the equality $f(\iota) = g(B)$ induces a map $\iota \mapsto B|_\iota$ on the domain $[0, \infty) \setminus I$. Hence, every $\iota \in [0, \infty) \setminus I$ features a unique FPE, but no PPE.

C.8 Proof of Proposition 5

Without loss, we focus on $d > 0$. For d, the ex-post problem is equivalent to the sellers' problem in economy P with $\Delta = 0$, which we discuss in Proposition 4.

The relation of ι and B in the ex-post problem. We analyze how ι together with d pins down B in the ex-post problem. To keep notation simple, we define

$$\mathcal{S}_{B,d} \equiv \mathcal{S}_B - d \qquad \text{and} \qquad \tilde{d}_B = \frac{d}{c(q_B)}. \tag{150}$$

$\mathcal{S}_{B,d}$ is decreasing in B, whereas $\tilde{d}_B$ is increasing in B with $\lim_{B\to\infty} \tilde{d}_B = \infty$. In analogy to the proof of Proposition 4, Equation (124) holds in equilibrium with

$$f(\iota) \equiv \iota^\tau \qquad \text{and} \qquad g(B,d) \equiv B^\tau \omega^\chi (1-\omega)^{1-\chi} \mathcal{S}_{B,d}. \tag{151}$$

g depends through $\mathcal{S}_{B,d}$ on both B and the exogenous d. Clearly, $\partial g(B,d)/\partial d < 0$.

The buyers' participation constraint dependent on B. The buyers' participation constraint reads as in Inequality (126). The payment p is given by Equation (35). Inequality (126) thus is equivalent to

$$\begin{aligned} F\chi[1+B]\left[u(q_B) - [1+\tilde{d}_B]c(q_B)\right] - B\left[(1-\chi)u(q_B) + \chi[1+B][1+\tilde{d}_B]c(q_B)\right] \geq 0 \\ \Leftrightarrow \qquad F\chi - (1-\chi)\left[\frac{B}{1+B}\right] - \chi\left[\frac{1-\sigma}{1+\kappa}\right]\left[\frac{F+B}{1+B}\right][1+\tilde{d}_B] \geq 0. \end{aligned} \tag{152}$$

Since $\tilde{d}_B$ is increasing in B with $\lim_{B\to\infty} \tilde{d}_B = \infty$, the left-hand side of Inequality (152) decreases in B. There is a unique B, such that[23]

$$\begin{aligned} & [1+B]F\chi - (1-\chi)B - \chi\left[\frac{1-\sigma}{1+\kappa}\right][F+B][1+\tilde{d}_B] = 0 \\ \Leftrightarrow \qquad & \left[F\chi - (1-\chi) - \chi\left[\frac{1-\sigma}{1+\kappa}\right][1+\tilde{d}_B]\right]B + F\chi - \chi\left[\frac{1-\sigma}{1+\kappa}\right][1+\tilde{d}_B]F = 0 \\ \Leftrightarrow \qquad & B = \frac{\left[\frac{\sigma+\kappa}{1+\kappa} - \left[\frac{1-\sigma}{1+\kappa}\right]\tilde{d}_B\right]\chi F}{1 - \chi\left[F + \frac{\sigma+\kappa}{1+\kappa} - \left[\frac{1-\sigma}{1+\kappa}\right]\tilde{d}_B\right]}. \end{aligned} \tag{154}$$

We call the B that is implicitly defined through this equation $\overline{B}(d)$. $\overline{B}(d)$ thus is the largest effective opportunity cost that can prevail in equilibrium, given d, and $\overline{B}(d)$ features $v^b = 0$. Moreover, $\partial\overline{B}(d)/\partial d < 0$.

Behavior of g. To prove the uniqueness of the outcome of the ex-post problem, we narrow down the behavior of g in $B \in [0, \overline{B}(d))$. In analogy to the proof of Proposition

[23] Since any equilibrium features $v^b \geq 0$, d cannot arise in equilibrium if, corresponding to $B = 0$,

$$0 > F\chi - \chi\left[\frac{1-\sigma}{1+\kappa}\right]F[1+\tilde{d}_0] = \chi F\left[\frac{\sigma+\kappa}{1+\kappa} - \left[\frac{1-\sigma}{1+\kappa}\right]\frac{d}{c(q^\star)}\right]. \tag{153}$$

4, we pin down $\partial g/\partial B$ by determining $\mathrm{d}g(B,d)/g(B,d)$. With $\mathrm{d}\mathcal{S}_B/\mathcal{S}_B$ and $\mathrm{d}\omega/\omega$ from the proof of Proposition 4, we obtain

$$\begin{aligned}\frac{\mathrm{d}g(B,d)}{g(B,d)} &= \tau\frac{\mathrm{d}B}{B} + \left[\chi - \frac{(1-\chi)\omega}{1-\omega}\right]\frac{\mathrm{d}\omega}{\omega} + \frac{\mathcal{S}_B}{\mathcal{S}_B - d}\frac{\mathrm{d}\mathcal{S}_B}{\mathcal{S}_B} \\ &= \underbrace{\left[\tau + \left[\chi(1-\omega) - (1-\chi)\omega - \frac{1-\sigma}{\sigma+\kappa}\left[\frac{B}{B+\frac{\sigma+\kappa}{1+\kappa}}\right]\frac{\mathcal{S}_B}{\mathcal{S}_B - d}\right]\frac{B}{1+B}\right]}_{\equiv\Psi(B,d)}\frac{\mathrm{d}B}{B},\end{aligned} \tag{155}$$

where $\partial\Psi(B,d)/\partial B < 0$ as well as $\partial\Psi(B,d)/\partial d < 0$. Note that

$$\begin{aligned}\tau\left[\frac{1+\overline{B}(d)}{\overline{B}(d)}\right] &= \tau\left[\frac{1-\chi\left[F + \frac{\sigma+\kappa}{1+\kappa} - \left[\frac{1-\sigma}{1+\kappa}\right]\tilde{d}_B\right]}{\chi F\left[\frac{\sigma+\kappa}{1+\kappa} - \left[\frac{1-\sigma}{1+\kappa}\right]\tilde{d}_B\right]} + 1\right] \\ &= \frac{\frac{1+\tau}{\chi} - \tau}{\frac{\sigma+\kappa}{1+\kappa} - \left[\frac{1-\sigma}{1+\kappa}\right]\tilde{d}_B} - (1+\tau) + \tau \\ &\geq \left[\frac{1}{1-\left[\frac{1-\sigma}{\sigma+\kappa}\right]\tilde{d}_B}\right]\left[\frac{1+\kappa}{\sigma+\kappa}\left[\frac{1}{\chi} + \frac{(1-\chi)\tau}{\chi}\right] - 1\right] \\ &= \left[\frac{1}{1-\left[\frac{1-\sigma}{\sigma+\kappa}\right]\tilde{d}_B}\right]\left[\frac{1-\chi}{\chi} + \left[\frac{1-\sigma}{\sigma+\kappa}\right]\frac{1}{\chi} + \left[\frac{1+\kappa}{\sigma+\kappa}\right]\frac{(1-\chi)\tau}{\chi}\right],\end{aligned} \tag{156}$$

and that

$$\begin{aligned}\frac{\mathcal{S}_B}{\mathcal{S}_B - d} &= \frac{\left[\frac{1}{1-\sigma} - \frac{1}{1+\kappa}\frac{1}{1+B}\right]\left[\frac{1}{1+B}\right]^{\frac{1-\sigma}{\sigma+\kappa}}}{\left[\frac{1}{1-\sigma} - \frac{1}{1+\kappa}\frac{1+\tilde{d}_B}{1+B}\right]\left[\frac{1}{1+B}\right]^{\frac{1-\sigma}{\sigma+\kappa}}} = \frac{1+B-\frac{1-\sigma}{1+\kappa}}{1+B-\frac{1-\sigma}{1+\kappa}[1+\tilde{d}_B]} \\ &= \frac{\frac{\sigma+\kappa}{1+\kappa} + B}{\frac{\sigma+\kappa}{1+\kappa} + B - \left[\frac{1-\sigma}{1+\kappa}\right]\tilde{d}_B} \leq \frac{1}{1-\left[\frac{1-\sigma}{\sigma+\kappa}\right]\tilde{d}_B}.\end{aligned} \tag{157}$$

Hence,

$$\begin{aligned}\left[\frac{1+\overline{B}(d)}{\overline{B}(d)}\right]\Psi(\overline{B}(d),d) > &\left[\frac{1}{1-\left[\frac{1-\sigma}{\sigma+\kappa}\right]\tilde{d}_B}\right]\left[\frac{1-\chi}{\chi} + \left[\frac{1-\sigma}{\sigma+\kappa}\right]\frac{1}{\chi} + \left[\frac{1+\kappa}{\sigma+\kappa}\right]\frac{(1-\chi)\tau}{\chi}\right] \\ &- (1-\chi) - \frac{1-\sigma}{\sigma+\kappa}\left[\frac{1}{1-\left[\frac{1-\sigma}{\sigma+\kappa}\right]\tilde{d}_B}\right] > 0.\end{aligned} \tag{158}$$

This inequality establishes that $g(B,d)$ is strictly increasing in $B \in [0, \overline{B}(d))$. Hence, every

$$\iota \in \left\{\iota \geq 0 \ : \ f(\iota) > g(\overline{B}(d),d)\right\} \equiv I(d) \tag{159}$$

features a unique PPE with

$$\mu^b = \left[\frac{g(\overline{B}(d), d)}{f(\iota)}\right]^{\frac{1}{\tau(1-\chi)}}, \tag{160}$$

but no FPE. Moreover, the equality $f(\iota) = g(B, d)$ induces a map $\iota \mapsto B|_\iota$ on the domain $[0, \infty) \setminus I(d)$. Hence, every $\iota \in [0, \infty) \setminus I(d)$ features a unique FPE, but no PPE.

C.9 Proof of Proposition 2

To simplify notation, we define $\sigma(\varepsilon) \equiv \varepsilon\zeta''(\varepsilon)/\zeta'(\varepsilon)$ and $\varphi(\theta) \equiv \theta\chi'(\theta)/\chi(\theta)$ as well as $\mathcal{N} \equiv \mathcal{N}(\varepsilon^b, \varepsilon^s)$, $\chi \equiv \chi(\theta)$, $\sigma^s \equiv \sigma(\varepsilon^s)$, and $\sigma^b \equiv \sigma(\varepsilon^b)$. Note that

$$\frac{\mathrm{d}\mathcal{N}}{\mathcal{N}} = \frac{\mathrm{d}\left[\varepsilon^s \beta\right]}{\varepsilon^s \beta} = \frac{\mathrm{d}\,\varepsilon^s}{\varepsilon^s} + \chi\frac{\mathrm{d}\theta}{\theta} = \frac{\mathrm{d}\,\varepsilon^s}{\varepsilon^s} + \chi\left[\frac{\mathrm{d}\,\varepsilon^b}{\varepsilon^b} - \frac{\mathrm{d}\,\varepsilon^s}{\varepsilon^s}\right] = (1-\chi)\frac{\mathrm{d}\,\varepsilon^s}{\varepsilon^s} + \chi\frac{\mathrm{d}\,\varepsilon^b}{\varepsilon^b}. \tag{161}$$

We consider economies P and NP separately.

Economy NP. First, we consider economy NP. Using the expression for p in Lemma 4, the sellers' and the buyers' FOCs w.r.t. ε^s and ε^b, derived from Equations (29) and (30), read as

$$\zeta'(\varepsilon^s) = \beta(\theta)[1 - \omega(\theta, \mathcal{N}, \iota)]\mathcal{S} \qquad \text{and} \qquad \zeta'(\varepsilon^b) = \alpha(\theta)\omega(\theta, \mathcal{N}, \iota)\mathcal{S}, \tag{162}$$

where

$$\omega(\theta, \mathcal{N}, \iota) \equiv \frac{\chi(\theta)\left[1 + \frac{\iota}{\mathcal{N}}\right]}{1 - \chi(\theta) + \chi(\theta)\left[1 + \frac{\iota}{\mathcal{N}}\right]} \tag{163}$$

denotes the buyers' share of the ex-post match surplus $\mathcal{S} \equiv u(q) - c(q) - d$. The differentials of the above equations thus read as

$$\sigma^s\frac{\mathrm{d}\,\varepsilon^s}{\varepsilon^s} = \chi\frac{\mathrm{d}\theta}{\theta} - \left[\frac{\omega}{1-\omega}\right]\frac{\mathrm{d}\omega}{\omega} + \frac{\mathrm{d}\mathcal{S}}{\mathcal{S}} \tag{164}$$

and

$$\sigma^b\frac{\mathrm{d}\,\varepsilon^b}{\varepsilon^b} = -(1-\chi)\frac{\mathrm{d}\theta}{\theta} + \frac{\mathrm{d}\omega}{\omega} + \frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}. \tag{165}$$

We consider the FR $\iota = 0$. Note that $\omega = \chi$ and that

$$\frac{\mathrm{d}\omega}{\omega} = \frac{\partial\omega}{\partial\theta}\frac{\theta}{\omega}\frac{\mathrm{d}\theta}{\theta} + \frac{\partial\omega}{\partial\iota}\frac{\mathrm{d}\iota}{\omega} + \frac{\partial\omega}{\partial\mathcal{N}}\frac{\mathcal{N}}{\omega}\frac{\mathrm{d}\mathcal{N}}{\mathcal{N}} = \varphi\frac{\mathrm{d}\theta}{\theta} + \frac{1-\chi}{\mathcal{N}}\,\mathrm{d}\iota. \tag{166}$$

Since $\mathcal{N}$ is a concave function, we have $\beta''(\theta) < 0$, and with $\chi = \theta\beta'(\theta)/\beta$, it follows that

$$\varphi = \frac{\theta\chi'(\theta)}{\chi} = 1 - \chi + \frac{\theta\beta''(\theta)}{\beta'(\theta)} < 1 - \chi. \tag{167}$$

We obtain with $\theta = e^b/e^s$ that

$$
\begin{aligned}
&\frac{\mathrm{d}\theta}{\theta} = -\left[\frac{1-\chi}{\sigma^b} + \frac{\chi}{\sigma^s}\right]\frac{\mathrm{d}\theta}{\theta} + \left[\frac{1}{\sigma^b} + \frac{\chi}{1-\chi}\frac{1}{\sigma^s}\right]\frac{\mathrm{d}\omega}{\omega} + \left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}} \\
\Leftrightarrow\quad &\frac{\mathrm{d}\theta}{\theta} = \left[\frac{1}{\sigma^b} + \frac{\chi}{1-\chi}\frac{1}{\sigma^s}\right]\left[-[1-\chi-\varphi]\frac{\mathrm{d}\theta}{\theta} + \frac{1-\chi}{\mathcal{N}}\,\mathrm{d}\iota\right] + \left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}} \\
\Leftrightarrow\quad &\frac{\mathrm{d}\theta}{\theta} = \frac{A}{1+[1-\chi-\varphi]A}\left[\frac{1-\chi}{\mathcal{N}}\right]\mathrm{d}\iota + \frac{1}{1+[1-\chi-\varphi]A}\left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}
\end{aligned}
\tag{168}
$$

for $A \equiv 1/\sigma^b + [1/\sigma^s]\chi/[1-\chi]$. It follows from Equation (161) and with the decomposition of $\mathrm{d}\omega/\omega$ in Equation (166) that

$$
\begin{aligned}
\frac{\mathrm{d}\mathcal{N}}{\mathcal{N}} &= \chi\frac{\mathrm{d}\,\varepsilon^b}{\varepsilon^b} + (1-\chi)\frac{\mathrm{d}\,\varepsilon^s}{\varepsilon^s} \\
&= -\chi(1-\chi)\left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\frac{\mathrm{d}\theta}{\theta} + \chi\left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\frac{\mathrm{d}\omega}{\omega} + \left[\frac{\chi}{\sigma^b} + \frac{1-\chi}{\sigma^s}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}} \\
&= -\chi[1-\chi-\varphi]\left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\frac{\mathrm{d}\theta}{\theta} + \chi\left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\frac{1-\chi}{\mathcal{N}}\,\mathrm{d}\iota + \left[\frac{\chi}{\sigma^b} + \frac{1-\chi}{\sigma^s}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}} \\
&= \left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\left[\chi - \frac{\chi[1-\chi-\varphi]A}{1+[1-\chi-\varphi]A}\right]\left[\frac{1-\chi}{\mathcal{N}}\right]\mathrm{d}\iota \\
&\quad + \left[\frac{\chi}{\sigma^b} + \frac{1-\chi}{\sigma^s} - \frac{\chi[1-\chi-\varphi]}{1+[1-\chi-\varphi]A}\left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]^2\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}} \\
&= \left[\chi\left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right] - \Phi A\right]\left[\frac{1-\chi}{\mathcal{N}}\right]\mathrm{d}\iota + \left[\frac{\chi}{\sigma^b} + \frac{1-\chi}{\sigma^s} - \left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\Phi\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}
\end{aligned}
\tag{169}
$$

for

$$
\Phi \equiv \left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\left[\frac{\chi[1-\chi-\varphi]}{1+[1-\chi-\varphi]A}\right]. \tag{170}
$$

Because agents trade $q^\star$, and because $d = 0$, it holds that

$$
\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}} = \frac{\mathrm{d}[u(q) - c(q) - d]}{u(q) - c(q) - d} = -\frac{1}{u(q^\star) - c(q^\star)}\left[\frac{\mathrm{d}d}{\mathrm{d}\iota}\right]\mathrm{d}\iota. \tag{171}
$$

Recall that, according to Lemma 3, $\mathrm{d}d/\,\mathrm{d}(\Delta\nu) > (=) 0$ if $\Delta\eta > (=) 0$. It follows from $\iota = \nu/\eta$ that $\mathrm{d}d/\,\mathrm{d}\iota > 0$.

Economy P. We now consider economy P. Using the expression for p in Lemma 2, the sellers' and the buyers' FOCs w.r.t. ε^s and ε^b, derived from the problem in (23), read as

$$
\zeta'(\varepsilon^s) = \beta(\theta)[1-\omega(\theta,\mathcal{N},\iota)]\mathcal{S} \qquad \text{and} \qquad \zeta'(\varepsilon^b) = \alpha(\theta)\omega(\theta,\mathcal{N},\iota)\exp(-\Delta\eta\iota)\mathcal{S}, \tag{172}
$$

where $\omega(\theta,\mathcal{N},\iota)$ reads as in Equation (163) and $\mathcal{S}\equiv\exp(\Delta\eta\iota)u(q)-c(q)$. The differentials of the above equations thus read as

$$\sigma^s\frac{\mathrm{d}\,\varepsilon^s}{\varepsilon^s}=\chi\frac{\mathrm{d}\theta}{\theta}-\left[\frac{\omega}{1-\omega}\right]\frac{\mathrm{d}\omega}{\omega}+\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}} \tag{173}$$

and

$$\sigma^b\frac{\mathrm{d}\,\varepsilon^b}{\varepsilon^b}=-(1-\chi)\frac{\mathrm{d}\theta}{\theta}+\frac{\mathrm{d}\omega}{\omega}-\Delta\eta\,\mathrm{d}\iota+\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}. \tag{174}$$

With the decomposition of $\mathrm{d}\omega/\omega$ in Equation (166), we obtain

$$\begin{aligned}
&\frac{\mathrm{d}\theta}{\theta}=\left[\frac{1}{\sigma^b}+\frac{\chi}{1-\chi}\frac{1}{\sigma^s}\right]\left[-[1-\chi-\varphi]\frac{\mathrm{d}\theta}{\theta}+\frac{1-\chi}{\mathcal{N}}\,\mathrm{d}\iota\right]-\left[\frac{\Delta\eta}{\sigma^b}\right]\mathrm{d}\iota+\left[\frac{1}{\sigma^b}-\frac{1}{\sigma^s}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}\\
\Leftrightarrow\quad&\frac{\mathrm{d}\theta}{\theta}=\frac{A}{1+[1-\chi-\varphi]A}\left[\frac{1-\chi}{\mathcal{N}}-\frac{\Delta\eta}{A\sigma^b}\right]\mathrm{d}\iota+\frac{1}{1+[1-\chi-\varphi]A}\left[\frac{1}{\sigma^b}-\frac{1}{\sigma^s}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}.
\end{aligned} \tag{175}$$

It follows from Equations (161) and (166) that

$$\begin{aligned}
\frac{\mathrm{d}\mathcal{N}}{\mathcal{N}}&=\chi\frac{\mathrm{d}\,\varepsilon^b}{\varepsilon^b}+(1-\chi)\frac{\mathrm{d}\,\varepsilon^s}{\varepsilon^s}\\
&=-\chi(1-\chi)\left[\frac{1}{\sigma^b}-\frac{1}{\sigma^s}\right]\frac{\mathrm{d}\theta}{\theta}+\chi\left[\frac{1}{\sigma^b}-\frac{1}{\sigma^s}\right]\frac{\mathrm{d}\omega}{\omega}-\left[\frac{\chi\Delta\eta}{\sigma^b}\right]\mathrm{d}\iota+\left[\frac{\chi}{\sigma^b}+\frac{1-\chi}{\sigma^s}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}\\
&=-\chi[1-\chi-\varphi]\left[\frac{1}{\sigma^b}-\frac{1}{\sigma^s}\right]\frac{\mathrm{d}\theta}{\theta}+\chi\left[\left[\frac{1}{\sigma^b}-\frac{1}{\sigma^s}\right]\frac{1-\chi}{\mathcal{N}}-\frac{\Delta\eta}{\sigma^b}\right]\mathrm{d}\iota+\left[\frac{\chi}{\sigma^b}+\frac{1-\chi}{\sigma^s}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}\\
&=\left[\frac{1}{\sigma^b}-\frac{1}{\sigma^s}\right]\left[\chi-\frac{\chi[1-\chi-\varphi]A}{1+[1-\chi-\varphi]A}\right]\left[\frac{1-\chi}{\mathcal{N}}\right]\mathrm{d}\iota\\
&\quad-\frac{\Delta\eta}{\sigma^b}\left[\chi-\left[\frac{1}{\sigma^b}-\frac{1}{\sigma^s}\right]\frac{\chi[1-\chi-\varphi]}{1+[1-\chi-\varphi]A}\right]\mathrm{d}\iota\\
&\quad+\left[\frac{\chi}{\sigma^b}+\frac{1-\chi}{\sigma^s}-\frac{\chi[1-\chi-\varphi]}{1+[1-\chi-\varphi]A}\left[\frac{1}{\sigma^b}-\frac{1}{\sigma^s}\right]^2\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}\\
&=\left[\left[\chi\left[\frac{1}{\sigma^b}-\frac{1}{\sigma^s}\right]-\Phi A\right]\frac{1-\chi}{\mathcal{N}}-\frac{[\chi-\Phi]\Delta\eta}{\sigma^b}\right]\mathrm{d}\iota+\left[\frac{\chi}{\sigma^b}+\frac{1-\chi}{\sigma^s}-\left[\frac{1}{\sigma^b}-\frac{1}{\sigma^s}\right]\Phi\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}.
\end{aligned} \tag{176}$$

Note that

$$\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}=\Delta\eta\left[\frac{u(q)}{u(q)-c(q)}\right]\mathrm{d}\iota. \tag{177}$$

for $q=q^\star$

Comparison of changes in matching rates. It holds that

$$\left[\frac{\mathrm{d}\mathcal{N}}{\mathcal{N}}\right]_P-\left[\frac{\mathrm{d}\mathcal{N}}{\mathcal{N}}\right]_F=\Delta\eta\left[\left[\frac{\chi}{\sigma^b}+\frac{1-\chi}{\sigma^s}-\left[\frac{1}{\sigma^b}-\frac{1}{\sigma^s}\right]\Phi\right]\frac{u(q)}{u(q)-c(q)}-\frac{\chi-\Phi}{\sigma^b}\right]\mathrm{d}\iota. \tag{178}$$

Hence, we obtain

$$\left(\left[\frac{\mathrm{d}\mathcal{N}}{\mathcal{N}}\right]_P - \left[\frac{\mathrm{d}\mathcal{N}}{\mathcal{N}}\right]_F\right)[1+[1-\chi-\varphi]A]\frac{1}{\Delta\eta}\frac{1}{\mathrm{d}\iota}$$
$$= \left[\left[\frac{1}{\sigma^b} + \frac{1-\chi}{\chi}\frac{1}{\sigma^s}\right]\left[\chi + \chi[1-\chi-\varphi]\left[\frac{1}{\sigma^b} + \frac{\chi}{1-\chi}\frac{1}{\sigma^s}\right]\right]\right.$$
$$\left. - \left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]^2 \chi[1-\chi-\varphi]\right]\frac{u(q)}{u(q)-c(q)}$$
$$- \frac{1}{\sigma^b}\left[\chi\left[1+[1-\chi-\varphi]\left[\frac{1}{\sigma^b} + \frac{\chi}{1-\chi}\frac{1}{\sigma^s}\right]\right] - \left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\chi[1-\chi-\varphi]\right]$$
$$= \left[\chi\left[\frac{1}{\sigma^b} + \frac{1-\chi}{\chi}\frac{1}{\sigma^s}\right] + \chi[1-\chi-\varphi]\underbrace{\left[\frac{\chi}{1-\chi} + \frac{1-\chi}{\chi} + 2\right]}_{=\frac{1}{\chi(1-\chi)}}\frac{1}{\sigma^s\sigma^b}\right]\frac{u(q)}{u(q)-c(q)}$$
$$- \frac{1}{\sigma^b}\left[\chi + \chi[1-\chi-\varphi]\left[\frac{1}{1-\chi}\right]\frac{1}{\sigma^s}\right]\frac{u(q)-c(q)}{u(q)-c(q)}$$
$$= \frac{1-\chi}{\sigma^s}\left[1+\left[\frac{1-\chi-\varphi}{1-\chi}\right]\frac{1}{\sigma^b}\right]\frac{u(q)}{u(q)-c(q)} + \frac{\chi}{\sigma^b}\left[1+\left[\frac{1-\chi-\varphi}{1-\chi}\right]\frac{1}{\sigma^s}\right]\frac{c(q)}{u(q)-c(q)}. \tag{179}$$

Moreover, it holds that

$$\left[\frac{\mathrm{d}\mathcal{N}}{\mathcal{N}}\right]_{NP} - \left[\frac{\mathrm{d}\mathcal{N}}{\mathcal{N}}\right]_F = -\left[\frac{1}{u(q)-c(q)}\right]\left[\frac{\chi}{\sigma^b} + \frac{1-\chi}{\sigma^s} - \left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\Phi\right]\left[\frac{\mathrm{d}d}{\mathrm{d}\iota}\right]\mathrm{d}\iota. \tag{180}$$

Hence, we obtain

$$-\left(\left[\frac{\mathrm{d}\mathcal{N}}{\mathcal{N}}\right]_{NP} - \left[\frac{\mathrm{d}\mathcal{N}}{\mathcal{N}}\right]_F\right)[1+[1-\chi-\varphi]A]\left[\frac{u(q)-c(q)}{\left[\frac{\mathrm{d}d}{\mathrm{d}\iota}\right]}\right]\frac{1}{\mathrm{d}\iota}$$
$$= \left[\frac{1}{\sigma^b} + \frac{1-\chi}{\chi}\frac{1}{\sigma^s}\right]\left[\chi + \chi[1-\chi-\varphi]\left[\frac{1}{\sigma^b} + \frac{\chi}{1-\chi}\frac{1}{\sigma^s}\right]\right] - \left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]^2\chi[1-\chi-\varphi]$$
$$= \chi\left[\frac{1}{\sigma^b} + \frac{1-\chi}{\chi}\frac{1}{\sigma^s}\right] + \left[\frac{1-\chi-\varphi}{1-\chi}\right]\frac{1}{\sigma^b\sigma^s}. \tag{181}$$

C.10 Proof of Proposition 3

As in the proof of Proposition 2, we consider economies NP and P separately.

Economy NP. To derive $\mathrm{d}q/q$, we use Equation (36) in Lemma 4, i.e.,

$$\frac{u'(q)}{c'(q)} = 1 + \frac{\iota}{\mathcal{N}}, \tag{182}$$

so that

$$\frac{\mathrm{d}q}{q} = \left[\frac{c'(q^\star)^2}{u''(q^\star)c'(q^\star) - u'(q^\star)c''(q^\star)}\right]\left[\frac{1}{q^\star\mathcal{N}}\right]\mathrm{d}\iota = -\left[\frac{c'(q^\star)}{c''(q^\star) - u''(q^\star)}\right]\left[\frac{1}{q^\star\mathcal{N}}\right]\mathrm{d}\iota, \quad (183)$$

using $u'(q^\star) = c'(q^\star)$.

To derive $\mathrm{d}p/p$, define $\mathcal{S} \equiv u(q) - c(q) - d$ as in the proof of Proposition 2, and note that $p = u(q) - \omega(\theta, \mathcal{N}, \iota)\mathcal{S}$. The decomposition of $\mathrm{d}\omega/\omega$ in Equation (166) and the expression for $\mathrm{d}\theta/\theta$ in Equation (168) yield

$$\frac{\mathrm{d}\omega}{\omega} = \frac{1 + [1-\chi]A}{1 + [1-\chi-\varphi]A}\left[\frac{1-\chi}{\mathcal{N}}\right]\mathrm{d}\iota + \frac{\varphi}{1 + [1-\chi-\varphi]A}\left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}. \quad (184)$$

With $\omega = \chi$ at the FR, we thus obtain

$$\begin{aligned}
\frac{\mathrm{d}p}{p} &= \left[\frac{u'(q^\star)q^\star}{p^\star}\right]\frac{\mathrm{d}q}{q} - \left[\frac{\chi\mathcal{S}}{p^\star}\right]\left[\frac{\mathrm{d}\omega}{\omega} + \frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}\right] \\
&= \left[\frac{u'(q^\star)q^\star}{p^\star}\right]\frac{\mathrm{d}q}{q} - \left[\frac{\chi\mathcal{S}}{p^\star}\right]\left[\frac{1 + [1-\chi]A}{1 + [1-\chi-\varphi]A}\right]\left[\frac{1-\chi}{\mathcal{N}}\right]\mathrm{d}\iota - \Psi\left[\frac{\chi\mathcal{S}}{p^\star}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}} \\
&= -\left(\left[\frac{u'(q^\star)q^\star}{p^\star}\right]\left[\frac{c'(q^\star)}{c''(q^\star) - u''(q^\star)}\right]\left[\frac{1}{q^\star\mathcal{N}}\right] + \left[\frac{\chi\mathcal{S}}{p^\star}\right]\left[\frac{1 + [1-\chi]A}{1 + [1-\chi-\varphi]A}\right]\left[\frac{1-\chi}{\mathcal{N}}\right]\right)\mathrm{d}\iota \\
&\quad - \Psi\left[\frac{\chi\mathcal{S}}{p^\star}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}},
\end{aligned} \quad (185)$$

where we used

$$\Psi \equiv 1 + \frac{\varphi}{1 + [1-\chi-\varphi]A}\left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]. \quad (186)$$

Economy P. To derive $\mathrm{d}q/q$, we use Equation (28) in Lemma 2, i.e.,

$$\frac{u'(q)}{c'(q)} = \exp(-\Delta\eta\iota)\left[1 + \frac{\iota}{\mathcal{N}}\right], \quad (187)$$

so that

$$\frac{\mathrm{d}q}{q} = -\left[\frac{c'(q^\star)}{c''(q^\star) - u''(q^\star)}\right]\left[\frac{1}{q^\star\mathcal{N}} - \frac{\Delta\eta}{q^\star}\right]\mathrm{d}\iota. \quad (188)$$

To derive $\mathrm{d}p/p$, define $\mathcal{S} \equiv \exp(\Delta\eta\iota)u(q) - c(q)$ as in the proof of Proposition 2, and note that $p = u(q) - \omega(\theta, \mathcal{N}, \iota)\exp(-\Delta\eta\iota)\mathcal{S}$. The decomposition of $\mathrm{d}\omega/\omega$ in Equation (166) and the expression for $\mathrm{d}\theta/\theta$ in Equation (175) yield

$$\begin{aligned}
\frac{\mathrm{d}\omega}{\omega} = \frac{1 + [1-\chi]A}{1 + [1-\chi-\varphi]A}\left[\frac{1-\chi}{\mathcal{N}}\right]\mathrm{d}\iota - \frac{\varphi}{1 + [1-\chi-\varphi]A}\left[\frac{\Delta\eta}{\sigma^b}\right]\mathrm{d}\iota \\
+ \frac{\varphi}{1 + [1-\chi-\varphi]A}\left[\frac{1}{\sigma^b} - \frac{1}{\sigma^s}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}.
\end{aligned} \quad (189)$$

We obtain

$$
\begin{aligned}
\frac{\mathrm{d}p}{p} &= \left[\frac{u'(q^\star)q^\star}{p^\star}\right]\frac{\mathrm{d}q}{q} - \left[\frac{\chi\mathcal{S}}{p^\star}\right]\left[\frac{\mathrm{d}\omega}{\omega} + \frac{\mathrm{d}\mathcal{S}}{\mathcal{S}} - \Delta\eta\,\mathrm{d}\iota\right] \\
&= \left[\frac{u'(q^\star)q^\star}{p^\star}\right]\frac{\mathrm{d}q}{q} - \left[\frac{\chi\mathcal{S}}{p}\right]\left[\frac{1+[1-\chi]A}{1+[1-\chi-\varphi]A}\right]\left[\frac{1-\chi}{\mathcal{N}}\right]\mathrm{d}\iota \\
&\quad + \Delta\eta\left[\frac{\chi\mathcal{S}}{p^\star}\right]\left[1 + \frac{\varphi}{1+[1-\chi-\varphi]A}\left[\frac{1}{\sigma^b}\right]\right]\mathrm{d}\iota - \Psi\left[\frac{\chi\mathcal{S}}{p^\star}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}} \\
&= -\left(\left[\frac{u'(q^\star)q^\star}{p^\star}\right]\left[\frac{c'(q^\star)}{c''(q^\star)-u''(q^\star)}\right]\left[\frac{1}{q^\star\mathcal{N}}\right] + \left[\frac{\chi\mathcal{S}}{p^\star}\right]\left[\frac{1+[1-\chi]A}{1+[1-\chi-\varphi]A}\right]\left[\frac{1-\chi}{\mathcal{N}}\right]\right)\mathrm{d}\iota \\
&\quad + \Delta\eta\left(\left[\frac{u'(q^\star)q^\star}{p^\star}\right]\left[\frac{c'(q^\star)}{c''(q^\star)-u''(q^\star)}\right]\frac{1}{q^\star} + \frac{\chi\mathcal{S}}{p^\star}\left[1 + \frac{\varphi}{1+[1-\chi-\varphi]A}\left[\frac{1}{\sigma^b}\right]\right]\right)\mathrm{d}\iota \\
&\quad - \Psi\left[\frac{\chi\mathcal{S}}{p^\star}\right]\frac{\mathrm{d}\mathcal{S}}{\mathcal{S}}.
\end{aligned}
\tag{190}
$$

Comparison of changes in terms of trade. Recall that economy F is a particular instance of economy NP if $\Delta\eta = 0$ with $[\mathrm{d}\mathcal{S}/\mathcal{S}]_F = 0$. Hence,

$$
\left[\frac{\mathrm{d}q}{q}\right]_{NP} - \left[\frac{\mathrm{d}q}{q}\right]_F = 0 \qquad \text{and} \qquad \left[\frac{\mathrm{d}q}{q}\right]_P - \left[\frac{\mathrm{d}q}{q}\right]_F = \frac{\Delta\eta}{q^\star}\left[\frac{c'(q^\star)}{c''(q^\star)-u''(q^\star)}\right]\mathrm{d}\iota \tag{191}
$$

With the expressions for $\mathrm{d}\mathcal{S}/\mathcal{S}$ in economies NP and P in Equations (171) and (177), respectively, we obtain

$$
\left[\frac{\mathrm{d}p}{p}\right]_{NP} - \left[\frac{\mathrm{d}p}{p}\right]_F = \frac{\chi\Psi}{p^\star}\left[\frac{\mathrm{d}d}{\mathrm{d}\iota}\right]\mathrm{d}\iota \tag{192}
$$

and

$$
\begin{aligned}
&\left[\frac{\mathrm{d}p}{p}\right]_P - \left[\frac{\mathrm{d}p}{p}\right]_F \\
&= \Delta\eta\left(\frac{u'(q^\star)}{p^\star}\left[\frac{c'(q^\star)}{c''(q^\star)-u''(q^\star)}\right] + \frac{\chi\mathcal{S}}{p^\star}\left[1 + \frac{\varphi}{1+[1-\chi-\varphi]A}\left[\frac{1}{\sigma^b}\right]\right]\right)\mathrm{d}\iota \\
&\quad - \Delta\eta\Psi\left[\frac{\chi u(q^\star)}{p^\star}\right]\mathrm{d}\iota \\
&= \Delta\eta\left(\frac{u'(q^\star)}{p^\star}\left[\frac{c'(q^\star)}{c''(q^\star)-u''(q^\star)}\right] + \frac{\chi\mathcal{S}}{p^\star}\left[\Psi + \frac{\varphi}{1+[1-\chi-\varphi]A}\left[\frac{1}{\sigma^s}\right]\right]\right)\mathrm{d}\iota \\
&\quad - \Delta\eta\Psi\left[\frac{\chi\mathcal{S}}{p^\star} + \frac{\chi c(q^\star)}{p^\star}\right]\mathrm{d}\iota \\
&= \Delta\eta\left(\frac{u'(q^\star)}{p^\star}\left[\frac{c'(q^\star)}{c''(q^\star)-u''(q^\star)}\right] + \frac{\chi\mathcal{S}}{p^\star}\left[\frac{\varphi}{1+[1-\chi-\varphi]A}\right]\frac{1}{\sigma^s} - \Psi\left[\frac{\chi c(q^\star)}{p^\star}\right]\right)\mathrm{d}\iota.
\end{aligned}
\tag{193}
$$

C.11 Proof of Corollary 1

Economy F differs from economy NP only in that $[d]_F = 0$ and $[\mathrm{d}d/\,\mathrm{d}\iota]_F = 0$, so that $[\mathrm{d}\mathcal{S}/\mathcal{S}]_F = 0$. We obtain from Equations (168) and (184) that

$$\frac{\mathrm{d}\theta}{\theta} = \frac{[1-\chi]A}{1+[1-\chi-\varphi]A}\left[\frac{1}{\mathcal{N}}\right]\mathrm{d}\iota \qquad \text{and} \qquad \frac{\mathrm{d}\omega}{\omega} = \frac{1+[1-\chi]A}{1+[1-\chi-\varphi]A}\left[\frac{1-\chi}{\mathcal{N}}\right]\mathrm{d}\iota, \quad (194)$$

and from Equations (164) and (165), we obtain that

$$\frac{\mathrm{d}\,\varepsilon^s}{\varepsilon^s} = \frac{1}{\sigma^s}\left[\chi\frac{\mathrm{d}\theta}{\theta} - \frac{\chi}{1-\chi}\frac{\mathrm{d}\omega}{\omega}\right] = -\frac{1}{\sigma^s}\left[\frac{\chi}{\mathcal{N}}\right]\left[\frac{1}{1+[1-\chi-\varphi]A}\right]\mathrm{d}\iota \quad (195)$$

and

$$\frac{\mathrm{d}\,\varepsilon^b}{\varepsilon^b} = \frac{1}{\sigma^b}\left[-(1-\chi)\frac{\mathrm{d}\theta}{\theta} + \frac{\mathrm{d}\omega}{\omega}\right] = \frac{1}{\sigma^b}\left[\frac{1-\chi}{\mathcal{N}}\right]\left[\frac{1}{1+[1-\chi-\varphi]A}\right]\mathrm{d}\iota. \quad (196)$$