Robust estimation and inference with categorical data

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Who thinks that questionnaire participants always respond accurately and attentively?

I propose a general methodological solution!

…and much more…

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Motivation: Respondent inattention

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A little garbage in, lots of garbage out: Assessing the impact of careless responding in personality survey data

Víctor B. Arias[1] · L. E. Garrido[2] · C. Jenaro[1] · A. Martínez-Molina[3] · B. Arias[4]

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Insufficient Effort Responding: Examining an Insidious Confound in Survey Data

Jason L. Huang and Mengqiao Liu
Wayne State University

Nathan A. Bowling
Wright State University

Motivation: Respondent inattention

Respondent inattention is a big problem in questionnaire studies

- Can lead to biased parameter estimates, invalid inference, deteriorated model fit, errors in hypothesis testing (e.g. Arias et al., 2020; Huang et al., 2015; Meade & Craig, 2012)
- Already prevalence of 5–10% problematic (e.g. Credé, 2010; Woods, 2006)
- Likely present in all questionnaire data (Ward & Meade, 2023)

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Contribution (Welz, 2024)

Develop estimator for categorical data that is robust to inattention/misspecification

- Novel categorical analogue to robust M-estimation theory (Huber, 1964)
- No assumption on the type or magnitude of misspecification
- Generalizes MLE, attractive statistical guarantees
- Statistical test to identify cells/responses that cannot be fitted well

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General setup

A model $\{p(\boldsymbol{\theta}) : \boldsymbol{\theta} \in \boldsymbol{\Theta}\}$ for k-dimensional categorical $\boldsymbol{Z}$, parameter $\boldsymbol{\theta} \in \mathbb{R}^d$

- Categorical outcome takes values in finite sample space $\mathcal{Z} = \{z_1, \ldots, z_m\}$
- Model assigns to each event $z \in \mathcal{Z}$ a probability $p_z(\boldsymbol{\theta}) = \mathbb{P}_{\boldsymbol{\theta}}[Z = z]$
- Examples:
  - Factor models/SEMs on latent variables for survey scales
  - Discrete choice
  - Poisson counting process...

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Model misspecification (Huber, 1964, AoS)

Misspecification: Instead of true $p(\boldsymbol{\theta}_*)$, sample from corrupted mixture

$$f_\varepsilon = (1-\varepsilon)p(\boldsymbol{\theta}_*) + \varepsilon h$$

- Fraction $\varepsilon \in [0,1]$ is degree of misspecification (unspecified)
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No assumptions on misspecification!

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Notation

- Let $\{Z_i\}_{i=1}^N$ denote N samples from f_ε $\implies$ model is possibly misspecified!
- $\widehat{f}_N(z) = \frac{1}{N}\sum_{i=1}^N \mathbb{1}\{Z_i = z\}$ is empirical probability of event $z \in \mathcal{Z}$
- $p_z(\boldsymbol{\theta})$ is theoretical probability of z at $\boldsymbol{\theta}$ (returned by model)

Proposed estimator

The proposed estimator $\widehat{\boldsymbol{\theta}}_N$ minimizes over $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ the loss

$$L\left(\boldsymbol{\theta}, \widehat{f}_N\right) = \sum_{\boldsymbol{z} \in \boldsymbol{\mathcal{Z}}} \rho\left(\frac{\widehat{f}_N(\boldsymbol{z})}{p_{\boldsymbol{z}}\left(\boldsymbol{\theta}\right)}\right) p_{\boldsymbol{z}}\left(\boldsymbol{\theta}\right)$$

The fraction $\widehat{f}_N(z)/p_z\left(\theta\right)$ is called Pearson residual (Lindsay, 1994, AoS)

- Values close to 1 indicate good model fit, far away from 1 poor fit
- Avoid that classes that cannot be fitted well dominate fit
- Idea: Downweight influence of poorly fitted classes via choice of $\rho(\cdot)$

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Choice of discrepancy function $\rho(\cdot)$

Depending on the situation, one may choose from an array of $\rho(\cdot)$ functions

- Theory developed for general $\rho(\cdot)$
- This talk (for simplicity): specific choice of $\rho(\cdot)$

Robust choice of $\rho(\cdot)$(Ruckstuhl & Welsh, 2001, AoS)

For $x = \widehat{f}_N(\boldsymbol{z})/p_{\boldsymbol{z}}(\boldsymbol{\theta})$ a Pearson residual at $\boldsymbol{\theta}$, use function

$$\rho(x) = \begin{cases} x \log(x) & \text{if } x \in [0, c], \\ x(\log(c) + 1) - c & \text{if } x > c, \end{cases}$$

where the constant $c \in [1, \infty]$ is prespecified

- If $x \in [0, c]$: Good fit, loss behaves like MLE $\Longrightarrow$ no need to downweight
- If $x > c$: Bad fit $\Longrightarrow$ downweight influence to be linear

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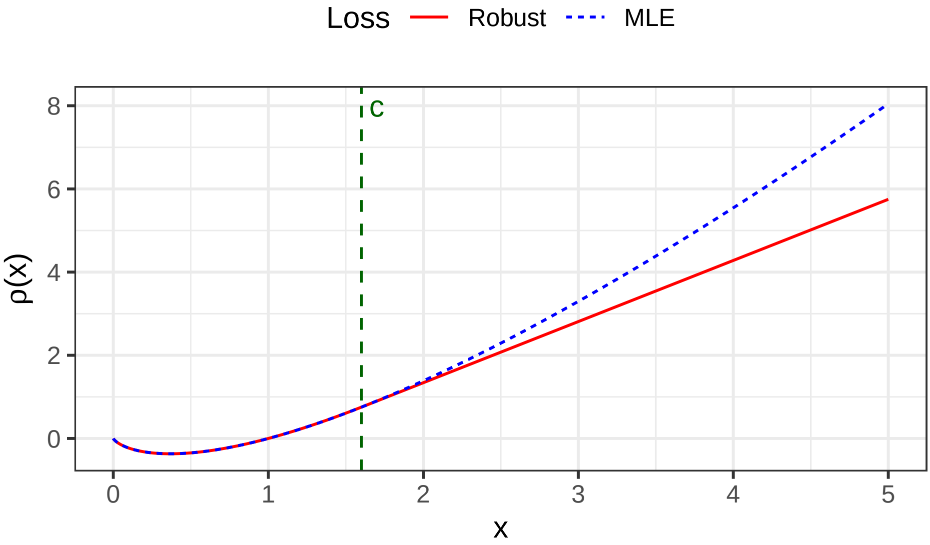
Robust choice of $\rho(\cdot)$, for $c = 1.6$



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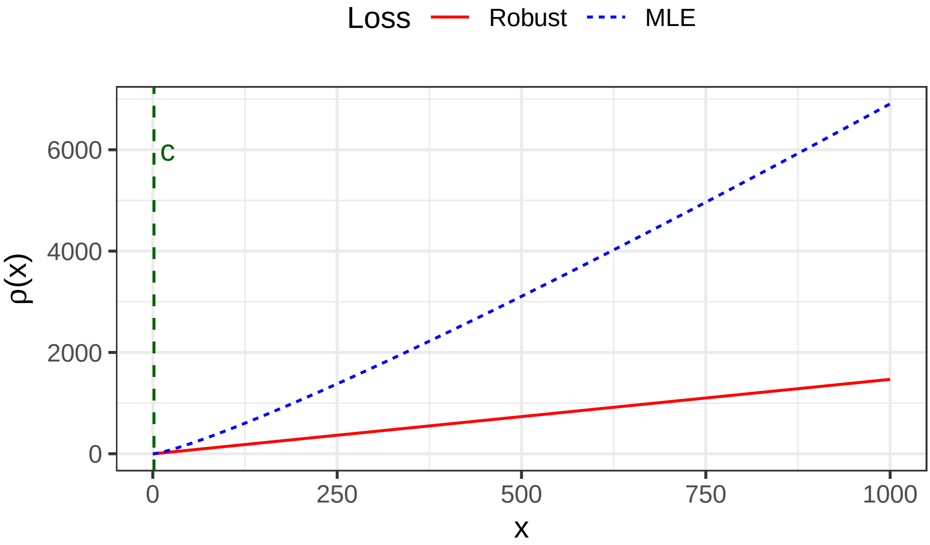

Asymptotics

$$\widehat{\boldsymbol{\theta}}_N = \arg\min_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} \sum_{\boldsymbol{z}\in\boldsymbol{\mathcal{Z}}} \rho\left(\frac{\widehat{f}_N(\boldsymbol{z})}{p_{\boldsymbol{z}}(\boldsymbol{\theta})}\right) p_{\boldsymbol{z}}(\boldsymbol{\theta}) \qquad \rho(x) = \begin{cases} x\log(x) & \text{if } x \in [0, c], \\ x(\log(c)+1) - c & \text{if } x > c. \end{cases}$$

Estimand: $\boldsymbol{\theta}_0 = \arg\min_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} L\left(\boldsymbol{\theta}, f_\varepsilon\right)$, equals $\boldsymbol{\theta}_*$ if $\varepsilon = 0$ (Fisher consistent)

Theorem (Consistency & asymptotic normality)

Under standard mild regularity conditions assumptions, it holds that

$$\widehat{\theta}_N \xrightarrow{\text{a.s.}} \theta_0$$

as $N \to \infty$, and

$$\sqrt{N}\left(\widehat{\theta}_N - \theta_0\right) \xrightarrow{\text{d}} \text{N}_d\left(0, \Sigma\left(\theta_0\right)\right),$$

where $\Sigma(\theta) = \boldsymbol{M}(\theta)^{-1}\boldsymbol{U}(\theta)\boldsymbol{M}(\theta)^{-1}$ is MLE variance at true model def more.

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Example: Analysis of survey scales

Standard model to analyze responses to survey scales (e.g. Muthén, 1984)

- Suppose $Z_j \in \{1, \dots, K_j\}$ is a response to j-th survey question, $j \in [k]$
- Discrete Z_j is governed by unobserved discretization of latent ξ_j (e.g., utility)
- Identification: Assume $\boldsymbol{\xi} = (\xi_1, \dots, \xi_k)^\top$ is multivariate standard normal
- $\boldsymbol{\theta}$ holds the correlation parameters and discretization thresholds

Goal: Estimate correlation structure of $\boldsymbol{\xi}$ from discrete $\boldsymbol{Z} = (Z_1, \dots, Z_k)^\top$

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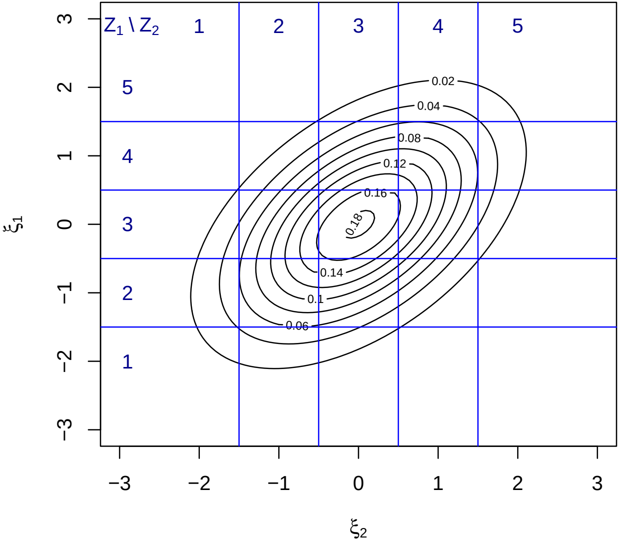
Example: Analysis of survey scales

The τ-thresholds discretize $\boldsymbol{\xi}$ to $\boldsymbol{Z}$ via

$$Z_j = \begin{cases} 1 & \text{if } \xi_j < \tau_{j,1}, \\ 2 & \text{if } \tau_{j,1} \leq \xi_j < \tau_{j,2}, \\ 3 & \text{if } \tau_{j,2} \leq \xi_j < \tau_{j,3}, \\ \vdots & \\ K_j & \text{if } \tau_{j,K_j-1} \leq \xi_j, \end{cases}$$



Simulation: Correlation in bivariate 5-point-scale more

- Sample $N = 1,000$ responses to $k = 2$ questions with 5 answer categories
- True $\rho_* = \text{Cor}[\xi_1, \xi_2] = 0.5$, thresholds $\boldsymbol{\tau}_* = (-1.5, -0.5, 0.5, 1.5)^\top$
- Estimate $\rho_* = 0.5$ with robust estimator, MLE, and sample correlation
- What happens to estimates if a fraction ε is inattentive? Repeat 1,000 times details

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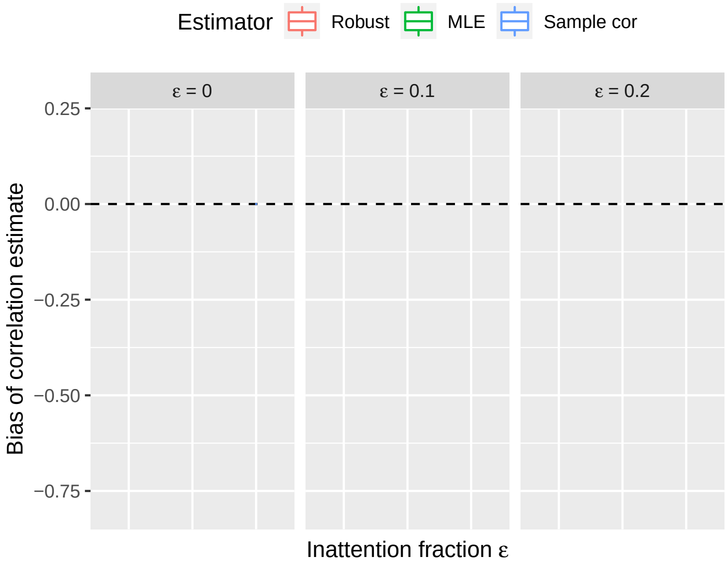

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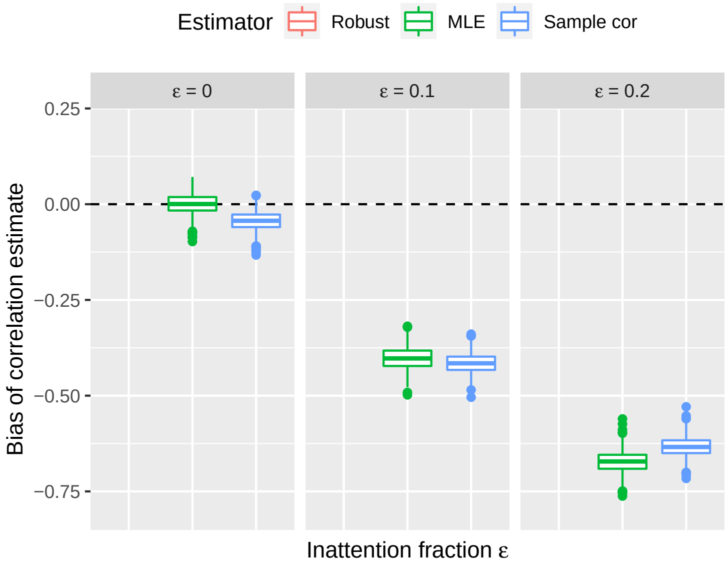

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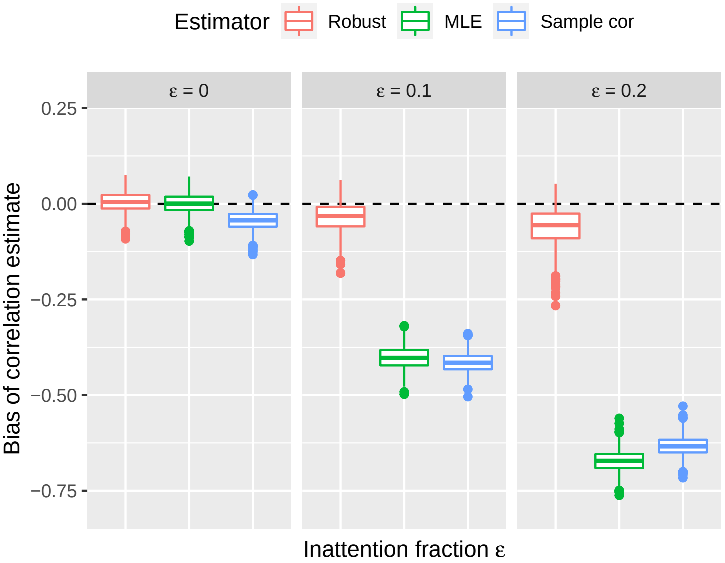

Empirical application: Inattention in Big Five

Use data in Arias et al. (2020, BRM) of 100 unipolar markers of Big 5

- Traits measured by pairs of opposite adjectives (e.g. “talkative" vs. “silent")
- 5-point Likert scale on agreement with each adjective item
- Theory expects strong negative correlation between opposite adjectives
- $N = 725$, but some are probably inattentive (Arias et al., 2020)

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Correlation btw. *Neuroticism* adjective pair "envious" vs. "not envious":

	Sample cor		MLE		Robust	
Parameter	Estimate	SE	Estimate	SE	Estimate	SE
ρ	−0.562	0.031	−0.618	0.025	−0.925	0.062
$\tau_{1,1}$			−1.370	0.061	−1.570	0.276
$\tau_{1,2}$			−0.476	0.043	−0.560	0.203
$\tau_{1,3}$			0.121	0.042	0.109	0.187
$\tau_{1,4}$			1.060	0.054	1.080	0.105
$\tau_{2,1}$			−0.857	0.049	−0.905	0.073
$\tau_{2,2}$			−0.004	0.041	−0.040	0.091
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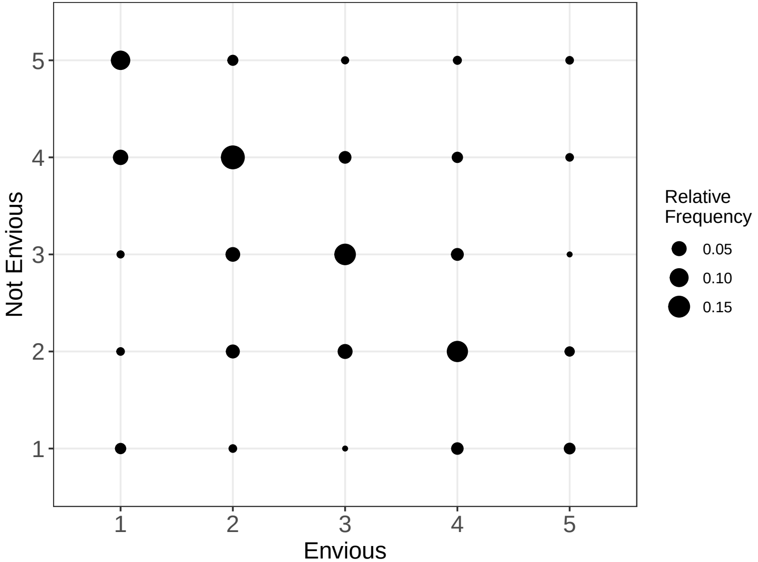
$\tau_{2,4}$			1.580	0.071	1.171	0.811

Frequency of “envious” vs. “not envious” in Arias et al. (2020)



details

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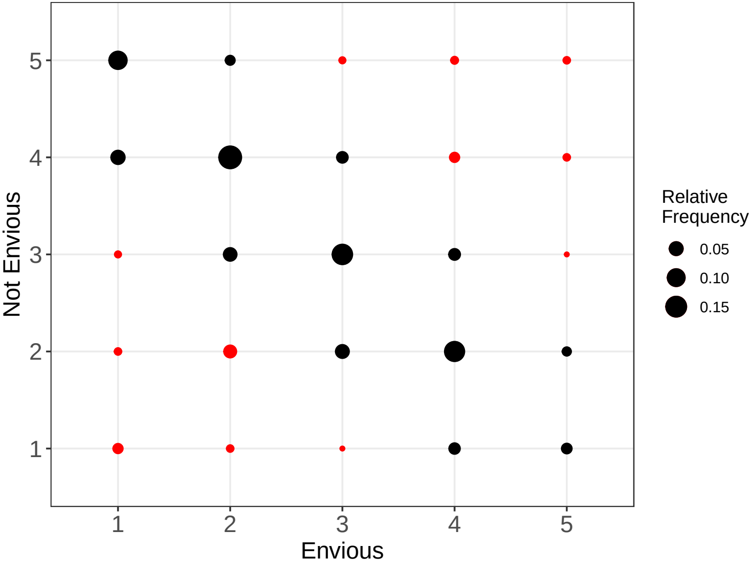

details

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- Developed robust estimator for categorical data
- Generalizes MLE, categorical analogue to *M*-estimation
- Proposed diagnostic test to identify "outlying" cells (omitted) details
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Thank you! QR code to the paper, Welz (2024):

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Robust choice of $\rho(\cdot)$ (Ruckstuhl & Welsh, 2001, AoS)

For $x = \widehat{f}_N(\boldsymbol{z})/p_{\boldsymbol{z}}(\boldsymbol{\theta})$ a Pearson residual at $\boldsymbol{\theta}$, use function

$$\rho(x) = \begin{cases} x \log(x) & \text{if } x \in [0, c], \\ x(\log(c) + 1) - c & \text{if } x > c, \end{cases}$$

where the constant $c \in [1, \infty]$ is prespecified

- If $x \in [0, c]$: Good fit, loss behaves like MLE $\Longrightarrow$ no need to downweight
- If $x > c$: Bad fit $\Longrightarrow$ downweight influence to be linear
- Similar idea as Huber loss: linear loss in tails, quadratic in center

Assumptions asymptotics

Assumption Set 1

A1. $c \in [1, +\infty]$,

A2. $\boldsymbol{\Theta} \subset \mathbb{R}^d$ *is compact,*

A3. $\boldsymbol{\theta}_0 = \arg\min_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} L(\boldsymbol{\theta}, f_\varepsilon)$ *is a unique global minimum, and* $\boldsymbol{\theta}_0 \in \operatorname{int} \boldsymbol{\Theta}$*, where* $f_\varepsilon(z) = (1 - \varepsilon)p_{\mathbf{z}}(\boldsymbol{\theta}_*) + \varepsilon h(z)$ *is the sampling distribution,*

A4. $p_{\mathbf{z}}(\boldsymbol{\theta})$ *is continuously differentiable with respect to* $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ *and twice differentiable at* $\boldsymbol{\theta}_0$*, for all cells* $\mathbf{z} \in \boldsymbol{\mathcal{Z}}$*,*

A5. $\left\| \frac{\partial p_{\mathbf{z}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right\| < \infty$ *for all* $\boldsymbol{\theta} \in \boldsymbol{\Theta}, \mathbf{z} \in \boldsymbol{\mathcal{Z}}$*,*

A6. $p_{\mathbf{z}}(\boldsymbol{\theta}) > 0$ *for all* $\boldsymbol{\theta} \in \boldsymbol{\Theta}$*,* $\mathbf{z} \in \boldsymbol{\mathcal{Z}}$*.*

Assumptions (cont'd) asymptotics

Assumption Set 1

A7. $\#\{\boldsymbol{z} \in \boldsymbol{\mathcal{Z}} : f_\varepsilon(\boldsymbol{z}) > 0\} > d$,

A8. $L(\boldsymbol{\theta}, f_\varepsilon)$ *is convex in a neighborhood of* $\boldsymbol{\theta}_0$,

A9. $\frac{f_\varepsilon(\boldsymbol{z})}{p_{\boldsymbol{z}}(\boldsymbol{\theta}_0)} \neq c$ *for any* $\boldsymbol{z} \in \boldsymbol{\mathcal{Z}}$.

Consistency

$$\widehat{\boldsymbol{\theta}}_N = \arg\min_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} \sum_{\boldsymbol{z}\in\boldsymbol{\mathcal{Z}}} \rho\left(\frac{\widehat{f}_N(\boldsymbol{z})}{p_{\boldsymbol{z}}(\boldsymbol{\theta})}\right) p_{\boldsymbol{z}}(\boldsymbol{\theta}) \quad \rho(x) = \begin{cases} x\log(x) & \text{if } x \in [0, c], \\ x(\log(c)+1) - c & \text{if } x > c. \end{cases}$$

Theorem (Consistency)

Under Assumptions A1–A6, it holds true that

$$\widehat{\boldsymbol{\theta}}_N \xrightarrow{\text{a.s.}} \boldsymbol{\theta}_0,$$

as $N \to \infty$.

Asymptotic normality

Theorem (Asymptotic normality)

Under Assumption Set 1, it holds that

$$\sqrt{N}\left(\widehat{\boldsymbol{\theta}}_N - \boldsymbol{\theta}_0\right) \xrightarrow{\mathrm{d}} \mathrm{N}_d\Big(0, \boldsymbol{\Sigma}\left(\boldsymbol{\theta}_0\right)\Big),$$

as $N \to \infty$, where

$$\boldsymbol{\Sigma}\left(\boldsymbol{\theta}\right) = \boldsymbol{M}(\boldsymbol{\theta})^{-1}\boldsymbol{U}(\boldsymbol{\theta})\boldsymbol{M}(\boldsymbol{\theta})^{-1}.$$

$\boldsymbol{\Sigma}\left(\boldsymbol{\theta}_0\right)$ can be consistently estimated by plug-in principle.

Diagnostic test for identifying outlying cells

Is the model misspecified for an *individual* event $\boldsymbol{z} \in \mathcal{Z}$? Test formulation:

$$\mathrm{H}_0 : p_{\boldsymbol{z}}\left(\boldsymbol{\theta}_0\right) = f_{\varepsilon}(\boldsymbol{z}) \qquad \text{vs.} \qquad \mathrm{H}_1 : p_{\boldsymbol{z}}\left(\boldsymbol{\theta}_0\right) < f_{\varepsilon}(\boldsymbol{z})$$

Corollary (Limit distribution of test statistic)

Under $\mathrm{H}_0 : p_{\boldsymbol{z}}\left(\boldsymbol{\theta}_0\right) = f_{\varepsilon}(\boldsymbol{z})$ and the assumptions of Theorem 2, the test statistic

$$T_N(\boldsymbol{z}) = \frac{p_{\boldsymbol{z}}\left(\widehat{\boldsymbol{\theta}}_N\right) - f_{\varepsilon}(\boldsymbol{z})}{\sqrt{\sigma^2_{\boldsymbol{z}}\left(\boldsymbol{\theta}_0\right) / N}}$$

converges to $\mathrm{N}(0,1)$ as $N \to \infty$, where $\sigma^2_{\boldsymbol{z}}\left(\boldsymbol{\theta}\right) = \left(\frac{\partial p_{\boldsymbol{z}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)^\top \boldsymbol{\Sigma}\left(\boldsymbol{\theta}\right) \left(\frac{\partial p_{\boldsymbol{z}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)$.

That is, the test rejects if the model is misspecified for $\boldsymbol{z}$, and otherwise does not

Definitions used in theorems asymptotics

$$\boldsymbol{\Omega} = \mathrm{diag}(\boldsymbol{f}_\varepsilon) - \boldsymbol{f}_\varepsilon \boldsymbol{f}_\varepsilon^\top \qquad \text{[estimable via } \widehat{f}_N\text{]}$$

$$\boldsymbol{U}(\boldsymbol{\theta}) = \boldsymbol{W}(\boldsymbol{\theta}) \boldsymbol{\Omega} \boldsymbol{W}(\boldsymbol{\theta})^\top,$$

$$\boldsymbol{M}(\boldsymbol{\theta}) = \sum_{z \in \mathcal{Z}} f_\varepsilon(z) \left(w' \left(\frac{f_\varepsilon(z)}{p_z(\boldsymbol{\theta}_0)} \right) \frac{f_\varepsilon(k)}{p_z(\boldsymbol{\theta})} \boldsymbol{s}_z(\boldsymbol{\theta}) \boldsymbol{s}_z(\boldsymbol{\theta})^\top - w \left(\frac{f_\varepsilon(z)}{p_z(\boldsymbol{\theta})} \right) \boldsymbol{Q}_z(\boldsymbol{\theta}) \right),$$

$$\boldsymbol{s}_z(\boldsymbol{\theta}) = \frac{\partial}{\partial \boldsymbol{\theta}} \log p_z(\boldsymbol{\theta}) = \frac{1}{p_z(\boldsymbol{\theta})} \left(\frac{\partial}{\partial \boldsymbol{\theta}} p_z(\boldsymbol{\theta}) \right),$$

$$\boldsymbol{W}(\boldsymbol{\theta}) = \left(\boldsymbol{s}_{z_1}(\boldsymbol{\theta}) \mathbb{1} \left\{ \frac{f_\varepsilon(z_1)}{p_{z_1}(\boldsymbol{\theta})} \in [0, c] \right\}, \cdots, \boldsymbol{s}_{z_m}(\boldsymbol{\theta}) \mathbb{1} \left\{ \frac{f_\varepsilon(z_m)}{p_{z_m}(\boldsymbol{\theta})} \in [0, c] \right\} \right),$$

$$w(x) = \mathbb{1}\{x \in [0, c]\} + c \mathbb{1}\{x > c\} / x,$$

$$\boldsymbol{Q}_z(\boldsymbol{\theta}) = \frac{1}{p_z(\boldsymbol{\theta})} \left(\frac{\partial^2}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^\top} p_z(\boldsymbol{\theta}) \right) - \boldsymbol{s}_z(\boldsymbol{\theta}) \boldsymbol{s}_z(\boldsymbol{\theta})^\top.$$

2nd order equivalence with MLE at true model back

Lemma

Under Assumption Set 1, it holds true that

$$\lim_{\varepsilon \downarrow 0} \boldsymbol{U}\left(\boldsymbol{\theta}_0(\varepsilon)\right) = \boldsymbol{J}\left(\boldsymbol{\theta}_*\right) \qquad \textit{and}$$

$$\lim_{\varepsilon \downarrow 0} \boldsymbol{M}\left(\boldsymbol{\theta}_0(\varepsilon)\right) = \begin{cases} \boldsymbol{J}\left(\boldsymbol{\theta}_*\right) & \textit{if } c > 1 \\ \boldsymbol{J}\left(\boldsymbol{\theta}_*\right) - \sum_{\boldsymbol{z} \in \mathcal{Z}} \mathbb{1}\left\{h(\boldsymbol{z}) > p_{\boldsymbol{z}}\left(\boldsymbol{\theta}_*\right)\right\} p_{\boldsymbol{z}}\left(\boldsymbol{\theta}_*\right) \boldsymbol{s}_{\boldsymbol{z}}\left(\boldsymbol{\theta}_*\right) \boldsymbol{s}_{\boldsymbol{z}}\left(\boldsymbol{\theta}_*\right)^\top & \textit{if } c = 1 \end{cases}$$

This lemma implies that at the true model, the asymptotic covariances of $\widehat{\boldsymbol{\theta}}_N$ and $\widehat{\boldsymbol{\theta}}_N^{\mathrm{MLE}}$ coincide

Influence function (1/3) back

Definition (Influence function)

Let $\widehat{\boldsymbol{\theta}}_N$ be an estimator that estimates a model $\{\boldsymbol{p}(\boldsymbol{\theta}) : \boldsymbol{\theta} \in \boldsymbol{\Theta}\}$ with finite support $\boldsymbol{\mathcal{Z}}$. Evaluated at model density $p_{\boldsymbol{z}}(\boldsymbol{\theta}), \theta \in \boldsymbol{\theta}$, the estimator's *influence function* at a data point $\boldsymbol{z} \in \boldsymbol{\mathcal{Z}}$ is given by

$$\mathrm{IF}\left(\boldsymbol{z}, \widehat{\boldsymbol{\theta}}_N, \boldsymbol{p}(\boldsymbol{\theta})\right) = \lim_{\varepsilon \downarrow 0} \frac{\widehat{\boldsymbol{\theta}}_N\big((1-\varepsilon)p_{\boldsymbol{z}}(\boldsymbol{\theta}) + \varepsilon \Delta_{\boldsymbol{z}}\big)}{\varepsilon} = \frac{\partial}{\partial \varepsilon} \widehat{\boldsymbol{\theta}}_N\big((1-\varepsilon)p_{\boldsymbol{z}}(\boldsymbol{\theta}) + \varepsilon \Delta_{\boldsymbol{z}}\big)\Big|_{\varepsilon=0},$$

where $\boldsymbol{\mathcal{Z}} \ni \boldsymbol{y} \mapsto \Delta_{\boldsymbol{z}}(\boldsymbol{y}) = \mathbb{1}\{\boldsymbol{z} = \boldsymbol{y}\}$ is the point mass density at point $\boldsymbol{z}$.

Influence function (2/3) back

Theorem (Influence function)

Grant Assumption Set 1. Then, the influence function of estimator $\widehat{\boldsymbol{\theta}}_N$ at cell $\boldsymbol{z} \in \boldsymbol{\mathcal{Z}}$ and true density $\boldsymbol{p}(\boldsymbol{\theta}_*)$ is given by

$$\mathrm{IF}\left(\boldsymbol{z}, \widehat{\boldsymbol{\theta}}_N, \boldsymbol{p}(\boldsymbol{\theta}_*)\right) = \begin{cases} \mathrm{IF}\left(\boldsymbol{z}, \widehat{\boldsymbol{\theta}}_N^{\mathrm{MLE}}, \boldsymbol{p}(\boldsymbol{\theta}_*)\right) & \text{if } c > 1, \\ \left[\boldsymbol{J}(\boldsymbol{\theta}_*) - p_{\boldsymbol{z}}(\boldsymbol{\theta}_*)\, \boldsymbol{s}_{\boldsymbol{z}}(\boldsymbol{\theta}_*)\, \boldsymbol{s}_{\boldsymbol{z}}(\boldsymbol{\theta}_*)^\top\right]^{-1} \boldsymbol{s}_{\boldsymbol{z}}(\boldsymbol{\theta}_*)\, p_{\boldsymbol{z}}(\boldsymbol{\theta}_*) & \text{if } c = 1, \end{cases}$$

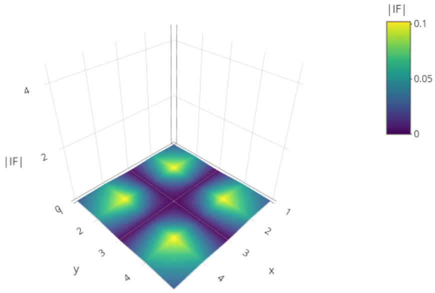
where

$$\mathrm{IF}\left(\boldsymbol{z}, \widehat{\boldsymbol{\theta}}_N^{\mathrm{MLE}}, \boldsymbol{p}(\boldsymbol{\theta})\right) = \boldsymbol{J}(\boldsymbol{\theta})^{-1} \boldsymbol{s}_{\boldsymbol{z}}(\boldsymbol{\theta}),$$

$$\boldsymbol{J}(\boldsymbol{\theta}) = -\sum_{\boldsymbol{z} \in \boldsymbol{\mathcal{Z}}} \boldsymbol{Q}_{\boldsymbol{z}}(\boldsymbol{\theta}) p_{\boldsymbol{z}}(\boldsymbol{\theta})$$

Influence function (3/3) back

(a) $c > 1$



(b) $c = 1$

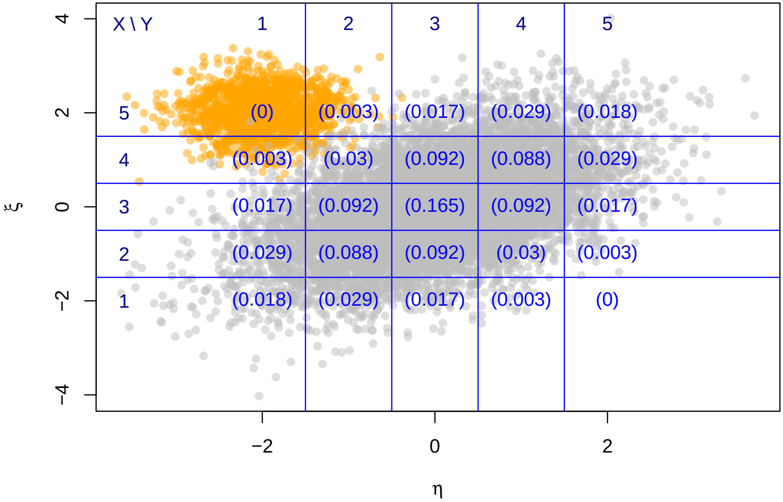
Simulation design (1/2) back

For misspecification fraction ε,

- Sample fraction $1 - \varepsilon$ from bivariate standard normal with $\rho_* = 0.5$
- Sample fraction ε from $\mathrm{N}_2\left((2, -2)^\top, \boldsymbol{I}\right)$
- Use discretization process to obtain (Z_1, Z_2)

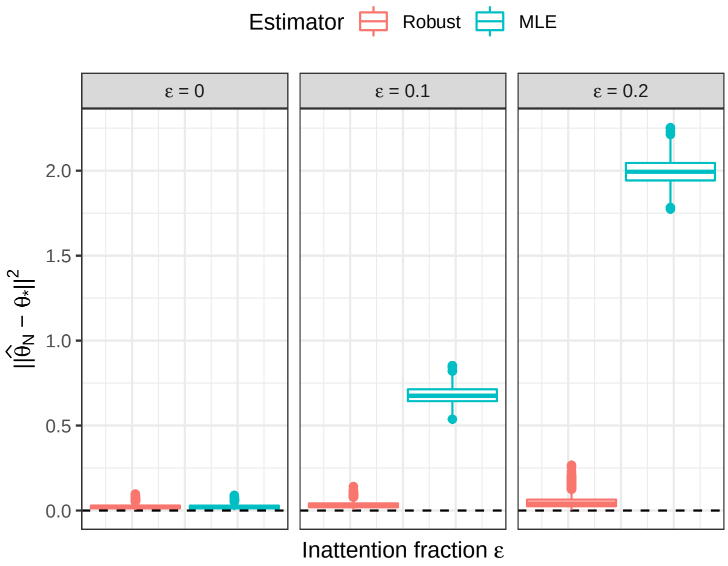
$\Longrightarrow$ Cells $(Z_1, Z_2) = (5, 1), (5, 2), (4, 1)$ are inflated, but some overlap with model distribution

Simulation design (2/2)

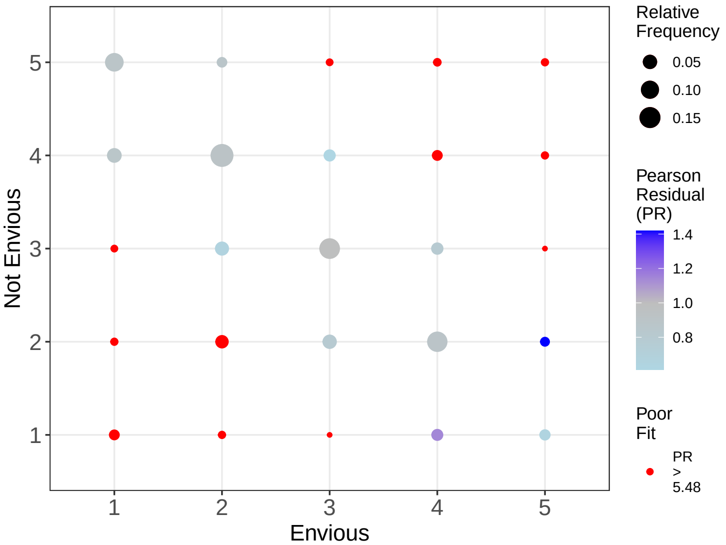


Simulation: Correlation of bivariate 5-point scale

back

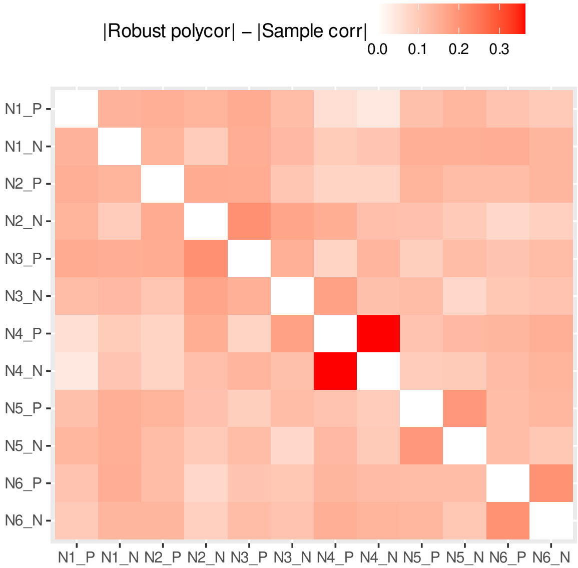


Analysis of "envious" vs. "not envious" in Arias et al. (2020)



back

Estimated correlation matrix for the *Neuroticism* scale



- Robustly estimated correlations are all stronger (ave. 0.130)
- MLE polyhoric correlations are similar to sample correlations (expected; cf. Rhemtulla et al., 2012)

back

Loadings of the neuroticism factor in data of Arias et al. (2020)

Item	Sample corr	Robust polycor
N1_P	0.70	0.80
N1_N	0.56	0.66
N2_P	0.76	0.86
N2_N	0.68	0.78
N3_P	0.77	0.88
N3_N	0.66	0.74
N4_P	0.35	0.46
N4_N	0.46	0.54
N5_P	0.69	0.77
N5_N	0.67	0.73
N6_P	0.57	0.66
N6_N	0.64	0.71
Proportion variance	0.40	0.53
Cronbach's α	0.89	0.93