Density forecasts of inflation: a quantile regression forest approach*

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Abstract

Inflation density forecasts are a fundamental input for a medium-term oriented central bank, such as the European Central Bank (ECB). We show that a quantile regression forest, capturing a general non-linear relationship between euro area (headline and core) inflation and a large set of determinants, is competitive with state-of-the-art linear benchmarks and judgemental survey forecasts. The median forecasts of the quantile regression forest are very collinear with the Eurosystem inflation point forecasts, displaying similar deviations from "linearity". Given that the Eurosystem modelling toolbox is overwhelmingly linear, this finding suggests that the expert judgement embedded in the projections may be characterized by some mild non-linearity. Finally, we provide an example of how the model is used in real-time to gauge the risks to the Eurosystem inflation projections.

Keywords: Inflation, Non-linearity, Quantile Regression Forest.

JEL Codes: C52, C53, E31, E37

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1 Introduction

The mandate of the European Central Bank (ECB) is to maintain price stability *over the medium-term*. The medium-term orientation implies that sources of fluctuations with temporary effects on inflation are likely to be looked through, while driving forces of a more persistent nature may influence monetary policy decisions. Consequently, the inflation projections, which synthesize the views of the Eurosystem staff on inflation dynamics, play a pivotal role in shaping monetary policy decisions. However, economic projections are inherently surrounded by uncertainty, and policy decisions hinge on a careful evaluation of the likelihood of various hypothetical current and future scenarios, defined as "risk assessment", rather than solely relying on point forecasts.

Modelling inflation dynamics in the euro area continues to pose a challenge, due to the many potential driving factors of inflation, the difficulty in capturing their relationship with inflation dynamics and the relatively short historical sample available for econometric analysis (see, for example, Koester et al., 2021). One of the core debates centers on whether a linear or non-linear relationship better characterizes the interaction of inflation with its numerous determinants. In this respect, the expanding literature on macroeconomics@risk underscores the importance of considering non-linearity in the dynamics of key policy-relevant variables, such as GDP and inflation, for the soundness of the risk assessment conducted by central banks. In particular, Adrian et al. (2019) show that the dynamics in the upper and lower quantiles of the GDP predictive density are associated with the dynamics of comprehensive indices of financial conditions. Carriero et al. (2016), Chavleishvili and Manganelli (2019), López-Salido and Loria (2020), Adams et al. (2021), Korobilis et al. (2021), Goulet Coulombe et al. (2022), Kiley (2022), Amburgey and McCracken (2023a), Amburgey and McCracken (2023b), Botelho et al. (2023), Boyarchenko et al. (2023), Chavleishvili et al. (2023) and Chavleishvili and Kremer (2023) show that similar considerations also concern the density forecasts of inflation and other macroeconomic variables.

The comprehensive overview of the Eurosystem economic analysis conducted in Darracq Pariès et al. (2021) as part of the recent ECB strategy review, reveals the predominant utilization of linear models within the Eurosystem modeling toolkit. In this paper, we introduce a novel non-linear model for euro area inflation density forecasting. Specifically, to capture the relationship of a set of potential predictors and inflation, we employ the quantile regression forest (QRF) of Meinshausen (2006), which is a variant of the random forest of Breiman (2001). The random forest is an ensemble technique, combining a number of non-linear predictive models, called regression trees. Regression trees split the predictor sample into (potentially many) subsets, called "leaves", and predict the target variable by computing the average value within each leaf. Quantile regression forests are based on the same principles but additionally estimate empirical quantiles of the target variable's distribution, enabling density forecasting.

Compared to the literature cited above, the QRF can capture more general forms of non-linearity because it does not assume any specific parametric relationship between predictors and the target variables. For example, unlike many @risk models, we do not impose linearity on the relationship between predictive quantiles and their potential determinants. Additionally, the QRF can easily accommodate a large number of predictors, enabling us to include all potentially relevant information for inflation. This flexibility is a significant advantage over current @risk applications, which are typically limited to a handful of variables and, to manage a sufficiently comprehensive information set, have to rely on summary measures of financial or general economic conditions.

We measure headline inflation as the rate of change of the Harmonized Index of Consumer Prices (HICP), and we also consider a measure of "core inflation", specifically the rate of change of the Harmonized Index of Consumer Prices excluding Energy and Food prices (HICPex).[1] The potential determinants of headline and core inflation in our model are broadly inspired by the Phillips Curve framework and include measures of inflation expectations, cost pressures, real activity and financial variables.

In our empirical application, we evaluate the QRF in a recursive out-of-sample exercise over an evaluation sample spanning the last twenty years and ending in December 2022. We focus on density forecasts up to the one-year-ahead horizon and we compare the accuracy of the QRF predictions with those from a state-of-the-art linear benchmark (a combination of a large number of Bayesian VAR models, VARCOMB) and survey forecasts (the ECB Survey of Professional Forecasters, SPF). Additionally, we compare the median QRF predictions with institutional forecasts (the published Eurosystem Inflation Projections, BMPE); for this comparison, we use only the median QRF, as the institutional forecasts are publicly available as point forecasts. We conduct tests of correct calibration of the forecasts as developed by Rossi and Sekhposyan (2019). We then use the continuous ranked probability score (CRPS) of Gneiting and Raftery (2007), a proper scoring rule that evaluates both calibration and sharpness, allowing us to rank the different forecasting methods.

We find that the QRF produces well-calibrated density forecasts for euro area inflation and is competitive with state-of-the-art linear and survey forecasting methods for headline and, in particular, for core inflation. The QRF's predictive performance is notably strong at horizons of up to six months, but declines compared to linear models and survey forecasts at longer horizons. For instance, when assessing the predictions for current year inflation conducted over the last two quarters of the year, QRF density forecasts are much sharper around the true value than the SPF counterparts. Moreover, the relative forecasting performance of the QRF and VARCOMB varies over time. The QRF did not perform well during and immediately after the Great Financial Crisis of 2007-2009, but it outperformed VARCOMB during the extended period of low inflation in the euro area before the COVID pandemic. These results indicate that euro area inflation dynamics may exhibit some degree of non-linearity. However, the differences in accuracy between QRF and VARCOMB are relatively small, suggesting that the non-linearity is relatively mild.

Interestingly, the evidence of non-linearity is stronger for core than headline inflation. Since our measure of core inflation excludes the energy and the food components of headline inflation, our results imply that the evidence of non-linearity in energy and food prices is much less compelling than for the other inflation components. The stronger non-linearity of core inflation is further highlighted when we examine the contribution of different predictors to the inflation forecasts using Shapley values (see Shapley, 1952; Strumbelj and Kononenko, 2010; Lundberg and Lee,

[1] In the rest of the paper, we refer to HICPex inflation as to core inflation. Despite the popularity of "exclusion measures" of consumer prices to proxy core inflation, there are numerous other measures of core inflation, with advantages and shortcomings. Policy-makers typically consider a range of different measures rather than settling on a single one. For more details, see for example Lenza (2011) and Ehrmann et al. (2018).

2017; Buckmann and Joseph, 2022). Our analysis of the functional forms relating the different predictors to inflation shows that most relevant predictors have an essentially linear relationship with inflation. However, measures of inflation expectations exhibit a state-dependent relationship, particularly with core inflation. We conclude that the QRF is a valuable addition to the Eurosystem forecasting toolbox. However, it should be seen more as a complement rather than a substitute for existing methods used to forecast inflation.

Turning to the comparison of the QRF median forecasts with the judgemental Eurosystem inflation forecasts, we find that both sets of forecasts exhibit similar accuracy and dynamics. Specifically, when examining the gaps between our median VARCOMB forecasts and both the median QRF and the Eurosystem forecasts - a rough measure of "distance from linearity" - we observe a strong positive correlation. This suggests that both the QRF and the Eurosystem forecasts exhibit similar deviations from linearity, despite the Eurosystem's reliance on an overwhelmingly linear modelling toolbox. This finding implies that the judgemental component of the Eurosystem forecasts tends to incorporate a mild non-linearity in the projected inflation dynamics. Overall, it is remarkable that the QRF produces predictions competitive with those of the SPF and the Eurosystem forecasts, considering that both the SPF and Eurosystem forecasts also incorporate expert judgement informed by news on likely future events, such as VAT changes, fiscal plans or geopolitical developments like the invasion of Ukraine.

We also discuss how the QRF is used to assess the risks surrounding the Eurosystem projections and illustrate how this assessment played out in real-time over the period of euro area disinflation that began at the end of 2022 and continued until the first quarter of 2024, the last quarter for which inflation data is available at the time of writing. Notably, we highlight how the QRF signalled some downside risks to the Eurosystem inflation projections in the second half of 2023, which eventually materialized.

In addition to the @risk literature cited above, this paper contributes to the extensive literature on inflation forecasting and the broader literature on potential non-linearity in inflation dynamics. Regarding the former, comprehensive surveys of the literature can be found in Faust and Wright (2013) and, more recently, in Banbura et al. (2024). For the latter, a substantial body of research examines the likelihood of changes in the shape of the Phillips Curve and the factors that may explain such changes. For an extensive survey and a systematization of the debate, see Del Negro et al. (2020). Several studies in this literature highlight the differing relationships between inflation and its determinants in high and low inflation regimes, or generally emphasize the state-dependence nature of these relationships of inflation with its determinants (see, for example Akerlof et al., 1996; Fahr and Smets, 2010; Benigno and Ricci, 2011; Lindé and Trabandt, 2019; Forbes et al., 2021; Clark et al., 2022; Costain et al., 2022; Cavallo et al., 2023; Benigno and Eggertsson, 2023).

In addition, our paper relates to a growing literature on the virtues of ensemble methods, which are becoming more and more popular in the econometric literature for prediction (Fernandez et al., 2001; Avramov, 2002; Sala-I-Martin et al., 2004; Inoue and Kilian, 2008; Bai and Ng, 2009; Wright, 2009; Rapach and Strauss, 2010; Faust et al., 2013; Ng, 2013; Jin et al., 2014; Varian, 2014; Wager and Athey, 2018; Athey et al., 2019; Clark et al., 2021). Giannone et al. (2021) demonstrates that ensemble methods can be particularly successful due to their ability to effectively manage model uncertainty. Medeiros et al. (2021) shows that the random forest performs well for US inflation prediction. Compared to that paper, we focus on euro area inflation and, more importantly, on density forecasts, which are crucial for the risk assessment at the core of monetary policy decisions. Despite their relevance for policy-making institutions, density forecasts remain relatively underexplored in the machine learning literature,

The rest of the paper is organized as follows. In section 2, we discuss our empirical strategy. Section 3 presents the results of our out-of-sample forecasting accuracy assessment. Section 4 examines the contribution of different predictors to our inflation forecasts using Shapley values. Section 5 analyzes how the QRF was utilized at the ECB for policy analysis and how it performed in real-time during the period of rapid disinflation that began at the end of 2022 in the euro area. Section 6 concludes.

2 Empirical models, data and out-of-sample evaluation

2.1 The quantile regression forest

We adopt a "direct" forecasting scheme, which requires to estimate the relationship of inflation at time t with its determinants at time t-h, for a generic forecasting horizon h. Then, we apply the estimated model on the data at time t to produce an inflation forecast at time t+h. The variable that we fit in our model is π_t^h, i.e. the annualized growth rate of the Harmonized Index of Consumer Prices or of the Harmonized Index of Consumer Prices excluding energy and food prices (HICP or HICPex, defined as P_t below), at the forecast horizons of 3, 6, 9 and 12 months ahead (h = 3, 6, 9 and 12):

$$\pi_t^h = (12/h) \times [ln(P_t) - ln(P_{t-h})]$$

Formally, we would like to estimate a non-linear relationship between our target concept of inflation π_t^h, its lags and a set of determinants x_{t-h}:

$$\pi_t^h = m(\pi_{t-h}^1 ... \pi_{t-h-p}^1; x_{t-h} ... x_{t-h-k}) + \varepsilon_t$$

and then obtain an inflation forecast as

$$\hat{\pi}_{t+h}^h = m(\pi_t^1 ... \pi_{t-p}^1; x_t ... x_{t-k})$$

Rather than tightly parameterizing $m(.)$, we capture quite general forms of non-linearity by resorting to machine learning techniques. In particular, we estimate the potentially non-linear relationship of inflation with its determinants by means of the quantile regression forest (QRF) developed by Meinshausen (2006), which is a variant of the random forest of Breiman (2001), allowing for density forecasting.

A quantile regression forest is an ensemble method which combines the results from a certain (potentially large) number of non-linear models, called regression trees. A regression tree fits a specific target variable (headline or core inflation, in our case) by repeatedly splitting the sample of the potential predictors in different sub-samples. Once the final split is achieved, the predicted value of the target variable associated with a specific sub-sample is represented by the sample mean or median of the target variable in that sub-sample, called “leaf”, for point prediction. In our paper, we focus on density prediction, which can be carried out by computing the empirical quantiles of the target variable associated with each leaf, as suggested in Meinshausen (2006). The sub-sample splits in a regression tree are obtained through a process defined as binary recursive partitioning, an iterative process that splits the data into partitions. The process continues until the splits achieve an improvement in terms of a statistical criterion, such as the mean squared error in the fit for inflation (our target variable) or, alternatively, until the splitting process hits a stopping rule which, in our case, is that any leaf contains at least ten data points. Trees are simple models yet they tend to overfit, which makes them bad predicting tools. Many “relatively” uncorrelated regression trees are built to maximize the advantages of combining them, via the following two steps. First, the observations from the original data are bootstrapped with replacement before constructing any new tree. Notice that inflation may be auto-correlated, and we also include two lags of inflation in the inflation determinants, so that the bootstrap procedure does not impair the ability of our model to account for the potential autoregressive dynamics of inflation. Second, the splits are computed, at each node, only by looking at a randomly selected set of the regressors. The default choice for the size of the latter set, which we take in this paper, is to draw a third of the variables for each split. Finally, we set the number of combined regression trees, i.e. the size of the forest, to the default value of 500.[2]

2.2 Benchmark models

We compare the predictions from the quantile regression forest to several benchmarks.

First, we consider a state-of-the-art linear model, i.e. an equally weighted combination of 500 VAR models, which we define as VARCOMB. We choose as benchmark the combination of individual BVAR models to be as close as possible to the QRF, which is also a combination of models, i.e. regression trees. The main difference between VARCOMB and QRF lies in the possible non-linearity captured by the latter. Each individual VAR model includes inflation (headline or core), plus four randomly selected indicators from our dataset. The data are stationarized, before entering the VAR models, and the latter are specified with two lags to mimic the procedures we follow in the QRF. The models are estimated using bayesian techniques. The prior distributions for the lag coefficients and error variances are in the Normal-Inverse Wishart class and are parameterized to shrink the model estimates toward the parameters of a random walk model, in the tradition

[2] Probst et al. (2019) discusses the default specification choices for random forests and quantile regression forests and also elaborates on the techniques to tune the model.

of the Minnesota prior (Litterman, 1979; Doan et al., 1984; Banbura et al., 2010).[3] The prior hyperparameters are treated as random variables and their value is drawn from their posterior, following Giannone et al. (2015).

Our second benchmark consists in the headline and core inflation density forecasts from the ECB Survey of Professional Forecasters (SPF).[4] The SPF is conducted on a quarterly basis and its participants are experts affiliated with financial or non-financial institutions based within Europe. For this paper, we gathered the historical vintages, aggregated across experts, appearing at the beginning of February 2002 until November 2022, for headline inflation, and February 2017 to November 2022, for core inflation. The density forecasts are available for two definitions of inflation, i) year-on-year inflation[5] and ii) inflation in the current year.[6] In terms of methodology, the experts use a mix of models and expert judgement and provide a probabilistic assessment of inflation falling in certain pre-specified ranges of values.

Our third benchmark consists in the Eurosystem headline and core inflation projections (BMPE, in short).[7] These institutional forecasts are also prepared on a quarterly basis and are published at the beginning of the third month of each quarter. For these forecasts, we look at the vintages published from March 2002 to December 2022, both for headline and core inflation. Notice that the BMPE do not provide a density forecast for a large part of the sample, so we only compare these projections to the point forecasts of the QRF. In terms of methodology, the BMPE are also based on model analysis complemented by expert judgement.

In our forecasting evaluation, we adapt the data availability of the QRF and the VARCOMB models to mimic the data availability of the SPF and the BMPE forecasters. Appendix A provides more details on how we match the timing across the different sets of forecasts.

For our assessment of the density forecasts, it is convenient to work with a probability density function. Therefore, for all our density forecasts (QRF, VARCOMB and SPF), we follow the practice in the literature (see, for example, Adrian et al., 2019) and fit a skew-t distribution (Azzalini and Capitanio, 2003).[8] As also shown in Montes Galdon et al. (2022), the skew-t distribution is an appropriate choice because it is a flexible parametric density that allows for fat tails, as well as asymmetries.

In the appendix B, we also provide a comparison of the median QRF forecasts with the forecasts from a random walk model (RW), a popular benchmark of non-forecastability, which as in Atkeson

[3]The data are stationary, so we center the prior on all the lag coefficients to zero.

[4]Details on the survey and the historical data are available at https://www.ecb.europa.eu/stats/ecb_surveys/survey_of_professional_forecasters/html/index.en.html.

[5]For example, for the vintage in the first quarter of the year "t", the experts provide an assessment of inflation between the fourth quarter of year "t-1" and the fourth quarter of year "t".

[6]The concept of inflation in the current year "t" is, effectively, the average year-on-year growth rate of inflation over the four quarters of year "t".

[7]See https://www.ecb.europa.eu/pub/projections/html/index.en.html for more information on the BMPE projections.

[8]See the details in appendix A. Strictly speaking, we would not need to fit a distribution to the VARCOMB forecasts, which are already a draw from the posterior distribution of VARCOMB. We fit the skew-t distribution also for VARCOMB only for comparability purposes, but all the results in the paper are robust to using the original posterior draws.

and Ohanian (2001), forecasts inflation at time "t+h" as

$$\hat{\pi}^{12}_{t+h} = \pi^{12}_t$$

2.3 Data

Beside headline HICP and HICP excluding energy and food, our two target variables, our database contains 60 variables. The data is obtained from the ECB Statistical Data Warehouse (SDW) and comes from a variety of original sources. Broadly speaking, the dataset is inspired by the Phillips Curve framework, covering different areas of the economy, and the choice of the variables is similar to de Bondt et al. (2018).

Specifically, we include measures of cost pressures (for example, commodity prices, exchange rates, wages and producer prices); survey and hard data on economic activity (for example, European Commission surveys on prices, employment expectations, confidence measures, industrial production, euro area business cycle indicators, various productivity measures); measures of inflation expectations (for example, survey and market-based measures over different forecast horizons); and financial variables (for example, interest rates, monetary aggregates, asset prices, bank lending).

Our sample ranges from December 1991 to December 2022 and the frequency of the data is monthly. We stationarize the data, when needed. We also de-seasonalize the data in accordance with our out-of-sample logic. Specifically, for all vintages of our out-of-sample exercise, we estimate the seasonal components by using only the data which would have been available to a forecaster in that vintage. See appendix A for more details.

2.4 Out-of-sample evaluation

Our out-of-sample exercise is based on a recursive updating scheme. When implementing the recursive scheme, we align with the date of publication of the survey and institutional forecasts, which have a quarterly frequency. In other words, for the sake of the comparison with those benchmarks, we run the QRF and VARCOMB only once per quarter, and with a data availability that is comparable to that of the SPF and the BMPE forecasters.[9]

Specifically, first, we estimate our models with data up to December 2001 (which is our first "t"), as the SPF first cut-off date is around mid-January 2002. Instead, when comparing to the BMPE, we estimate using data up to January 2002. We produce forecasts for inflation at the three, six, nine and twelve months horizon (t+h). Then, we continue to update the estimation sample by adding one quarter (effectively, three months) at a time, and we repeat all the steps

[9] Notice, however, that as our data is ex post revised, we are not able to reproduce the exact same data releases forecasters would have in real-time.

of the forecasting exercise until exhaustion of the sample. Our evaluation sample ranges until December 2022.[10]

The target variable for which we compute our measures of forecasting accuracy, both for headline and core HICP, is defined in terms of year-on-year growth rates. We adopt this convention because some of the benchmarks against which we compare (for example, the BMPE) are not seasonally adjusted. In other words, for the generic horizon h, we compute the measures of forecasting accuracy in terms of the variable[11]

$$\pi_{t+h}^{target} = ln(P_{t+h}) - ln(P_{t+h-12})$$

The only exception to this rule is for the exercise in which we compare our density forecasts with those of the SPF for the *current year*, where we conform to the practice of the SPF, which reports (headline and core) inflation in the current year in terms of the average year-on-year growth rate over the four quarters of the year.

In order to gauge the ability of the different models to capture the dynamics of inflation in different regimes, we focus both on the average accuracy over the whole sample and on the evolution of the measures of accuracy over time (Giacomini and Rossi, 2009, 2010; Rossi and Sekhposyan, 2016).

3 Results

Figure 1 reports the year-on-year growth rates of HICP (solid line) and HICPex (dashed line), our target measures of headline and core inflation.

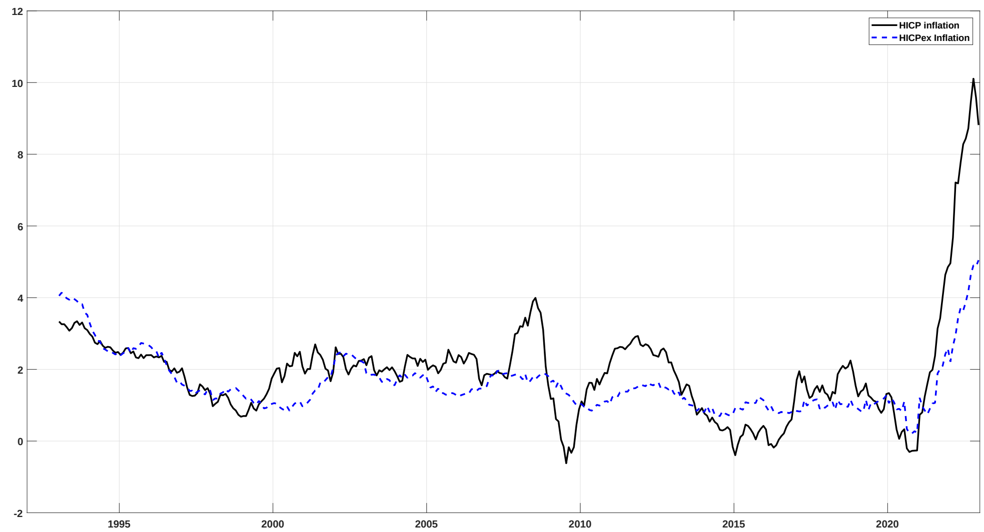
The figure shows the different regimes through which euro area inflation went over time. Notably, after the convergence toward the level of about 2% achieved in the early 90's, both headline and core inflation were quite stable until the financial crisis of 2007-2009. In the run-up to the financial crisis, headline inflation markedly increased, fueled by a large increase in commodity prices, while core inflation remained stable. The recession ensuing from the financial crisis led to a sudden and large drop in headline inflation and a more delayed slowdown in core inflation. After the Great Recession and the initial rebound of inflation, headline and core entered a protracted period of relatively low inflation. Finally, the post-pandemic environment has been characterized by a sudden increase in both headline and core inflation to levels which, especially for headline inflation, are unprecedented in the euro area sample. Kuik et al. (2022) describes the role played by the turmoil in energy markets caused by the Russian invasion of Ukraine for the increase in euro area inflation. After the initial boost to inflation affecting mainly the energy component of inflation, the upside pressure on consumer prices became more broad based and, in the course of 2022 has affected the whole basket of prices, leading to a strong increase also of core inflation[12].

[10] The evaluation of the QRF and VARCOMB forecasts, released at the monthly frequency, gives the same results as the evaluation based on the quarterly frequency and is available upon request.

[11] It may be worth reminding here that, as described above, to produce a forecast for "t+h" the variable we fit in our models is instead the annualized growth rate of prices between "t-h" and "t".

[12] See Giannone et al. (2014) for a quantification of the pass-through of commodity price shocks to core inflation components

Figure 1: Headline and Core Inflation



Note: Headline inflation: black solid line; Core inflation: blue dashed line. Inflation is defined in terms of year-on-year growth rates of prices and the sample ranges from January 1993 to December 2022.

3.1 A comparison of the density forecasts of QRF versus state-of-the-art linear and judgemental forecasts

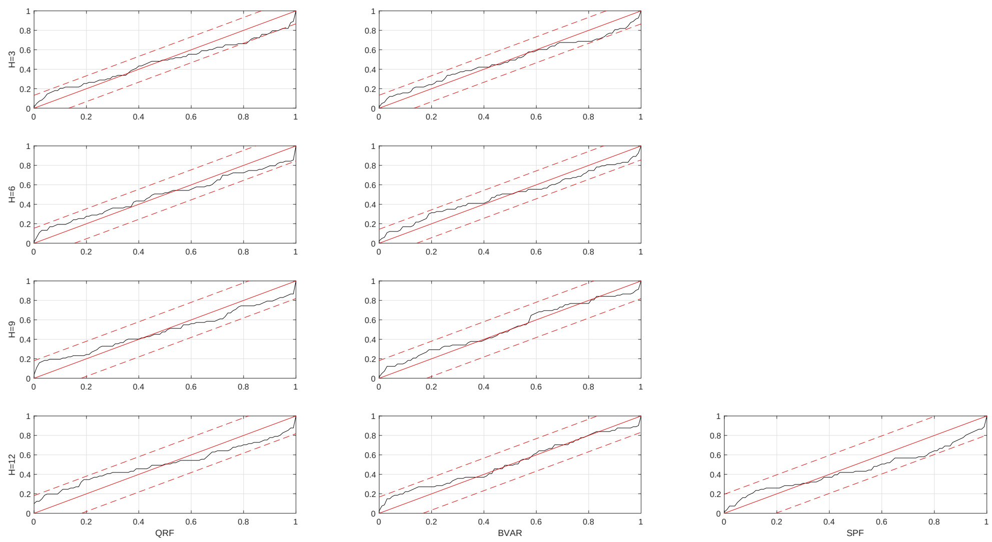
As a first step of our forecasting evaluation, we assess whether the density forecasts produced by the QRF, the VARCOMB and the SPF are correctly calibrated. A density forecast is correctly calibrated if, once it "assigns a certain probability to an event, then the event should occur with the stated probability over successive observations" (Elliott and Timmermann, 2016).

Defining as $p(y_t)$ a generic density forecast, we assess its correct calibration by testing whether the probability integral transform (the cumulative density function corresponding to $p(y_t)$, PIT in short) of the realizations of the y_t process is distributed as an $U(0,1)$ (Diebold et al., 1998). Several methods to test for correct calibration have been proposed in the literature (see, for example Diebold et al., 1998; Berkowitz, 2001; Corradi and Swanson, 2006; Hong et al., 2007; González-Rivera and Sun, 2015; Knüppel, 2015). We rely on the test procedure described in Rossi and Sekhposyan (2019), based on the Kolmogorov-Smirnov test, which has also a graphical representation.[13] Figure 2 presents the results for headline inflation and Figure 3 for core inflation.

We accept the null hypothesis that the QRF, VARCOMB and SPF density forecasts for headline

[13]Notice that our forecasts are multi-step, since we look at forecast horizons ranging from three to twelve months ahead. Hence, for our tests we follow the suggestion of Rossi and Sekhposyan (2019) and we compute critical values from a block version of the weighted bootstrap of Inoue (2001). The computations are carried out by using the replication codes kindly provided in Rossi and Sekhposyan (2019).

Figure 2: Headline Inflation, test of uniformity of PITs



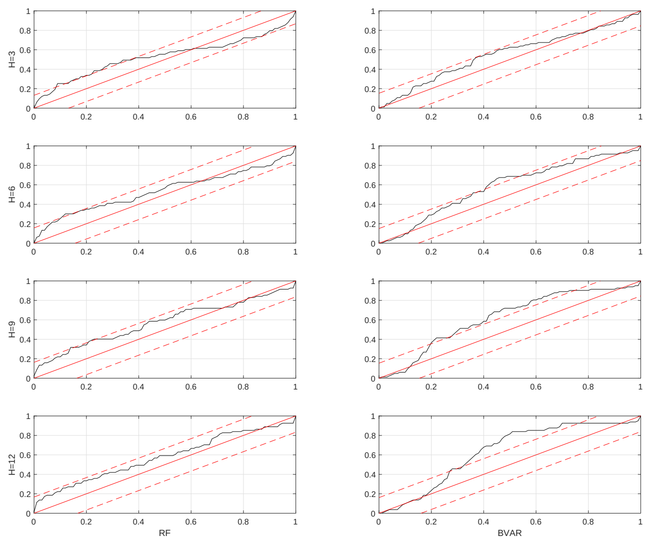
Note: Red lines: 1% critical values of the Kolmogorov-Smirnov test of PIT uniformity (dashed) and 45% degree line; Black line: Cumulative distribution function (CDF) of the PITs. If the probability density function of the PIT is a U(0,1), the CDF should be the 45% degree line. Left Column: QRF; Middle Column: VARCOMB; Right Column: SPF. The four rows correspond to the four forecasting horizons in the paper.

inflation are well calibrated. For core inflation, we have a slightly different picture. Both the QRF and VARCOMB[14] are less well calibrated, and for VARCOMB we also fail to accept the null of well calibrated forecasts for the forecasting horizons beyond three months. As in the empirical application of Rossi and Sekhposyan (2019), we find that the VARCOMB forecasts tend to be positively biased, especially in the last decade before the pandemic (see also figure 5).

Calibration is a desirable property for density forecasts. However, Hamill (2000) highlights how calibration is only a necessary condition for a model to mirror the ideal forecaster, i.e. to perfectly capture the actual cumulative distribution function. Gneiting et al. (2007) argues that maximizing sharpness, given calibration, helps to better approximate the ideal forecaster. For this reason, we also evaluate the relative accuracy of the density forecasts by a proper scoring rule, i.e. the continuous ranked probability score (CRPS) of Gneiting and Raftery (2007). The CRPS measures the "distance" of the predictive cumulative distribution function from the empirical cumulative distribution function associated with the observations of the target variable. The lower the CRPS, the more accurate a specific density forecast. Scoring rules such as the CRPS measure simultaneously calibration and sharpness (i.e. the concentration) of density forecasts. Hence, looking at the CRPS allows us to complement the assessment of calibration conducted above. Another advantage of scoring rules is that they also allow us to rank different models. Table 1 reports the results for headline (Panel A) and core inflation (Panel B).

[14]The SPF euro area core inflation forecasts are only available since 2017 and, hence the sample at our disposal is too short for a reliable assessment of the forecast accuracy.

Figure 3: Core Inflation, test of uniformity of PITs



Note: Red lines: 1% critical values of the Kolmogorov-Smirnov test of PIT uniformity (dashed) and 45% degree line; Black line: Cumulative distribution function (CDF) of the PITs. If the probability density function of the PIT is a U(0,1), the CDF should be the 45% degree line. Left Column: QRF; Middle Column: VARCOMB; Right Column: SPF. The four rows correspond to the four forecasting horizons in the paper.

Table 1: CRPS of different models for headline and core inflation

Horizon	**QRF**	**BVAR**	**SPF**
Panel a: Headline Inflation			
h=3	0.29	0.28	
h=6	0.50	0.49	
h=9	0.74	0.67	
h=12	0.93	0.88	0.87
Panel b: Core Inflation			
h=3	0.14	0.14	
h=6	0.23	0.24	
h=9	0.31	0.32	
h=12	0.37	0.39	

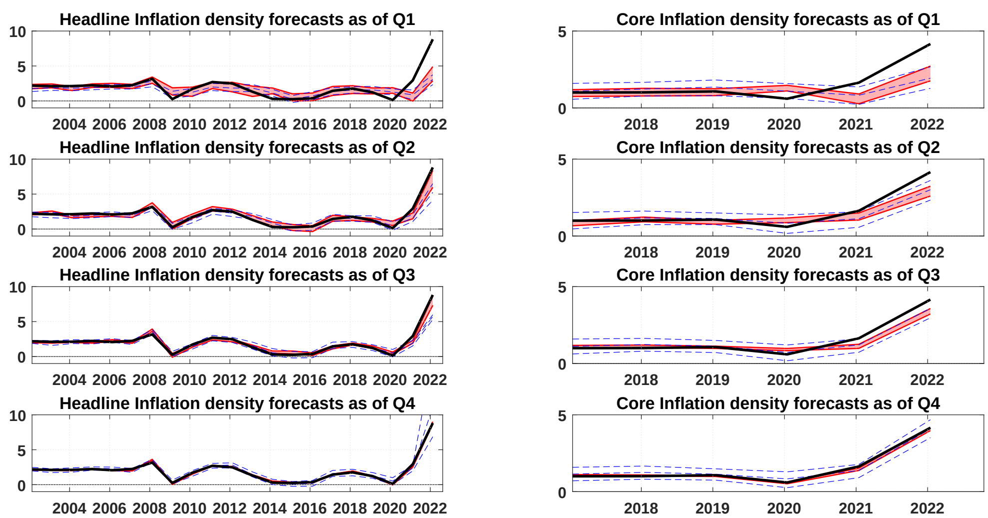
Note: CRPS for QRF (second column), VARCOMB (third column) and SPF (fourth column). The SPF are only available for the one-year-ahead forecasting horizon.

For headline inflation, we find that the QRF and VARCOMB have almost the same accuracy at the horizons of three and six months ahead. For the longer horizons, instead, the VARCOMB is more accurate than the QRF. For core inflation, instead, the QRF shows either a comparable or a slightly more accurate forecast accuracy than VARCOMB at all forecasting horizons. In general, our results suggest that the QRF is competitive with the state-of-the-art linear benchmark, particularly at the short horizons.

When we restrict our attention to the target defined in terms of year-on-year growth rates, the comparison with the SPF can only be carried out for the horizon of one year ahead and headline inflation. At that horizon, the accuracy of the SPF density forecasts is comparable to that of VARCOMB and it is superior to that of the QRF.

In order to get an idea of how the QRF compares to the SPF at shorter horizons than one year ahead, we assess the relative accuracy of the QRF and the SPF forecasts to predict *inflation over the current year*, as defined in sub-section 2.2. The prediction of *inflation over the current year* is reported in the SPF at each quarter of the year and, hence, it allows us to assess the SPF density forecasting accuracy at horizons which are shorter than one year ahead. Figure 4 below reports the charts with the observed average inflation (headline inflation, left and core inflation, right) for QRF and the SPF.

Figure 4: Headline and core inflation, density forecasts of QRF and SPF for the current year



Note: Red Area: 16th to 84th quantile of the QRF, current year for headline inflation (left panels) and core inflation (right panels); Dashed Lines: 16th to 84th quantile of the SPF, current year for headline inflation (left panels) and core inflation (right panels). The four rows correspond to the four quarters of each year in which the assessment is made.

For headline inflation, we have data over the 2002-2022 sample, while core inflation results are based on a shorter sample, ranging from 2017 to 2022. Interestingly, the QRF is as accurate or, especially in the third and fourth quarter of the year, more accurate than the SPF. In particular, QRF density forecasts are sharper around the actual value of inflation than the SPF. To appreciate the quantitative relevance of this point, Table 2 reports the CRPS for current year headline inflation forecasts of the QRF and the SPF.[15]

[15]For core inflation we have too few points, so we don't report CRPS.

Table 2: CRPS of QRF and SPF for current year headline inflation forecasts

Quarter	**QRF**	**SPF**
Q1	0.56	0.61
Q2	0.25	0.33
Q3	0.18	0.29
Q4	0.08	0.21

Note: First column: quarter of the year in which the forecast is produced; Second column: CRPS of QRF; Third column: CRPS of SPF.

Clearly, the QRF is more accurate than the SPF for short horizon forecasts. While this assessment has some limitations, because the SPF is conducted in real-time and we use ex post revised data for the QRF, it should also be noticed that the SPF is a judgemental forecast and it can make use of valuable information about the future which is not embedded in the QRF information set. Hence, it is quite remarkable that the QRF has a comparable, if not better, forecasting accuracy than the SPF.

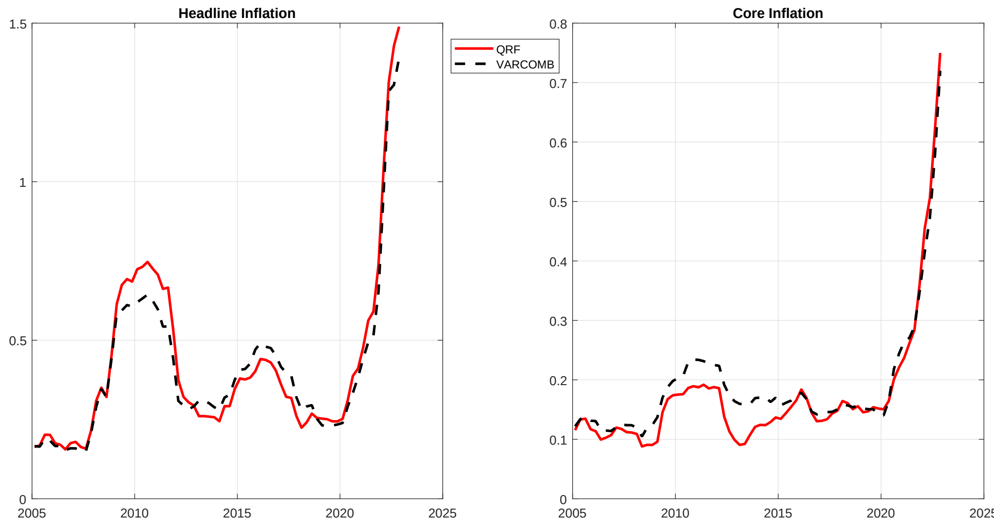
Overall, these results suggest that the QRF is a valid addition to the Eurosystem toolbox for inflation forecasting. Given the comparable accuracy with state-of-the-art linear models, the QRF is to be seen more as a complement rather than a substitute for the overwhelmingly linear Eurosystem forecasting toolbox. On the more general issue whether inflation dynamics are characterized by non-linearity, our results suggest that such non-linearity is probably not pervasive, but also that a mild non-linearity cannot be excluded, especially for core inflation.

In order to gauge the ability of the different models to capture the dynamics of inflation in different regimes (Giacomini and Rossi, 2009, 2010; Rossi and Sekhposyan, 2016) and, hopefully, to shed further light on the type of non-linearity in euro area inflation dynamics, figure 5 presents the CRPS of QRF and VARCOMB evaluated over rolling windows of three years. Again, we focus here on the horizon of six months ahead for both headline inflation (left panel) and core inflation (right panel), for brevity.

Focusing first on headline inflation, VARCOMB is better able than QRF to account for the quick inflation rebound post Great Recession, detecting earlier the inflation trough and having been less reactive than the QRF throughout the crisis period. This result is in line with Ferrara et al. (2015) and Bobeica and Jarociński (2019) which show that linear models (with potentially large shocks) are able to accurately describe the inflation dynamics around the Great Recession. At the same time, the QRF adapts much faster than the VAR forecasts to the prolonged period of low inflation characterizing the pre-COVID decade. The accuracy of the non-linear model in this episode suggests that low inflation regimes may be characterized by different inflation dynamics than high inflation regimes, as hinted in Forbes et al. (2021), although our evaluation sample is relatively short, making the identification of high and low inflation regimes potentially challenging. Over the most recent sample, both the QRF and VARCOMB had some difficulty to capture the high inflation regime.

Interestingly, the right panel tells a different story for core inflation. Notably, for core inflation the

Figure 5: CRPS, three years rolling window, headline inflation at six months horizon



Note: Red solid line: QRF; Black dashed line: VARCOMB. The value on the vertical axis at each point refers to the average CRPS over the current quarter and the previous eleven quarters.

QRF is superior to VARCOMB over most of the sample. The main difference between headline and core inflation is that the latter excludes the energy and the food prices from the HICP, two components which are obviously very affected by the dynamics of global commodity prices. Hence, considering together the results for headline and core inflation, our evidence suggests that the direct effects of commodity prices on headline inflation, especially via the energy components, are characterized by linear dynamics. When such direct effects of commodity prices are predominant for the dynamics of headline inflation, they dominate the non-linearity in the dynamics of the core inflation sub-component, making a linear model a competitive forecasting model for headline inflation in that regime.

3.2 Comparison of point forecasts with BMPE

The Eurosystem inflation forecasts (BMPE) have been reported as a point forecast for a large part of the sample under analysis and, hence, we will limit ourselves to a comparison of point forecasts. For the QRF, we consider the median of the density forecast distribution as point forecast. Table 3 reports the root mean squared errors (RMSE) for the QFR (left column) and the BMPE (right column), both for headline and core inflation.

The QRF point forecasts are generally comparable in accuracy to the BMPE forecasts at the short horizons and are less accurate at the nine and twelve month horizons. This result is remarkable, because the BMPE forecasts are the product of a very refined and sophisticated analysis by the Eurosystem forecasters and, via judgemental add-ons, they are flexible enough to embed all the

Table 3: RMSE of QRF and BMPE for headline and core inflation

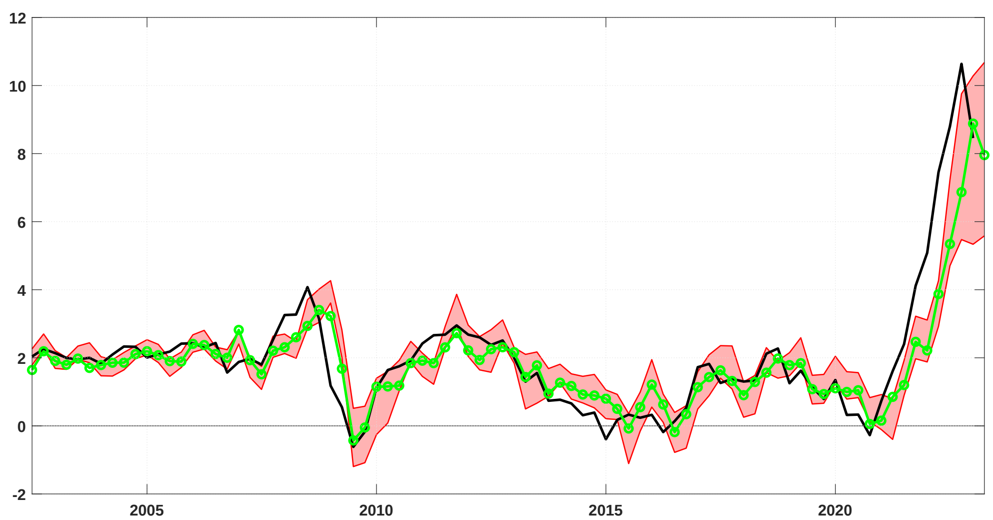
Horizon	**QRF**	**BMPE**
Panel a: Headline Inflation		
h=3	0.58	0.47
h=6	0.92	0.94
h=9	1.48	1.42
h=12	1.97	1.65
Panel b: Core Inflation		
h=3	0.21	0.22
h=6	0.36	0.38
h=9	0.64	0.58
h=12	0.82	0.68

Note: Column 2: QRF; Column 3: BMPE.

available information on future events of an economic relevance, which may fail to be incorporated in the variables used in the QRF.

In figure 6 we plot the headline QRF inflation forecasts together with the BMPE and observed inflation for h=6.

Figure 6: Headline Inflation, density forecasts of QRF and BMPE, h=6

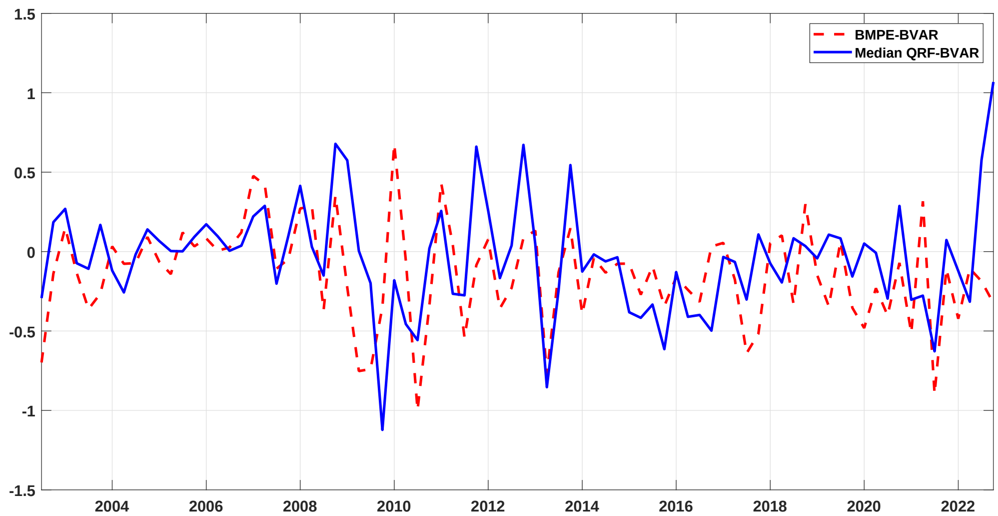


Note: Black solid line: year-on-year growth rate of HICP (headline inflation); Red area: 16th to 84th quantiles of the QRF density forecasts for the horizon of six months ahead, year on year growth rate of HICP; Green line with circles: BMPE projections for the horizon of six months ahead, year on year growth rate of HICP.

Even if the similarity between the two sets of forecasts is magnified by reporting year-on-year growth rates, the BMPE forecasts seem genuinely very collinear to the QRF forecasts. The

Eurosystem forecasts are produced by a toolkit that is essentially linear and, therefore, this result suggests that the BMPE judgemental component tends to introduce a non-linearity in the projections. Figure 7 shows the gaps of the median QRF forecast versus the median VARCOMB and the BMPE versus VARCOMB, which is a rough measure of "distance from linearity" of the two forecasts.

Figure 7: Gaps BMPE and QRF (median) versus linear BVAR



Note: Solid blue line: six months ahead (median) QRF forecast of headline inflation minus corresponding VARCOMB forecast; Dashed red line: six months ahead BMPE forecast of headline inflation minus corresponding VARCOMB forecast.

The two gaps are obviously correlated. Indeed, the correlation coefficient is about 0.4 for the three and the six month horizons and about 0.3 for the nine and twelve month horizons. This result suggests that judgement adds some element of mild non-linearity in the Eurosystem projections, rather consistently over time.

4 Shapley Values: interpretation of the forecasts and their functional forms

In this section, we study the drivers of our QRF predictions. For our analysis, we exploit recent advances in the machine learning literature (see Strumbelj and Kononenko, 2010; Lundberg and Lee, 2017; Buckmann and Joseph, 2022) which suggest to adopt the concept of Shapley values (Shapley, 1952) to define the contributions of the different variables to our predictions. In short, the Shapley value of a specific variable[16] for a forecast consists in the average marginal contribution of that variable to the forecast with respect to all the so called "coalitions" among variables, i.e.

[16]In the literature on machine learning, the variables used as predictors are also defined as "features".

all possible combinations of variables in the predictor set. The marginal contribution of a specific variable to a coalition is the additional contribution from adding the variable to the coalition, once all variables not in the coalition have been integrated out. In the special case of a linear regression in which all the predictors are orthogonal with each other, the Shapley values of a variable are given by the regression coefficient times the deviation of that variable from its mean.[17]

Generally, estimating Shapley values in presence of a large set of potentially correlated predictors is a rather complicated task, both for the large set of coalitions to be accounted for and the difficulties to correctly model the cross-correlation among variables when integrating out the effects of "off-coalition" variables. We rely on the method developed in Lundberg et al. (2019), which exploits the structure of regression trees underlying the QRF to speed up the computation of Shapley values and handle the issue of correlated predictors.[18]

We use the Shapley values for two main goals. First, we aim to define which predictors drive the inflation outlook. The predictors in our database are correlated among themselves, as it is generally the case for macroeconomic and financial variables. Hence, we do not aim to provide a fully fledged "narrative", i.e. a causal account of why our inflation predictions evolve in a specific direction, we only wish to study from which variables our model is extracting the signal for the inflation outlook.

As an illustration, figure 8 shows the results of the Shapley value analysis for headline (left panel) and core inflation (right panel), at the horizon of six months ahead, over the 2019-2022 period. The individual variables have been classified in groups (see appendix A) and their Shapley values have been aggregated to compute the contribution of the specific group to each prediction.[19]

In the period 2019-2022, the QRF extracts the signal that inflation would be raising over the subsequent six months mostly from measures of inflation expectations and producer price indices. The other variable groups have a negligible contribution to the forecasts. In particular, commodity prices do not seem to play a very large role for headline and core inflation. This is explained by the fact that some other measures (for example, inflation expectations) may have captured the signal that the boost in commodity prices would lead to higher inflation.

The Shapley values of individual variables are also useful to dig deeper on the question of which functional forms are captured by the QRF, suggesting whether inflation dynamics are characterized by non-linearity. Table 4 reports the top seven[20] individual contributors to the forecasts of headline and core inflation, at the horizon of six months ahead. The ranking is formulated on the basis of the mean absolute value of the contributions over the out-of-sample evaluation period.

We find that short-term interest rates, measures of inflation expectations and real activity are important predictors of inflation. Which functional forms best characterize the relationship be-

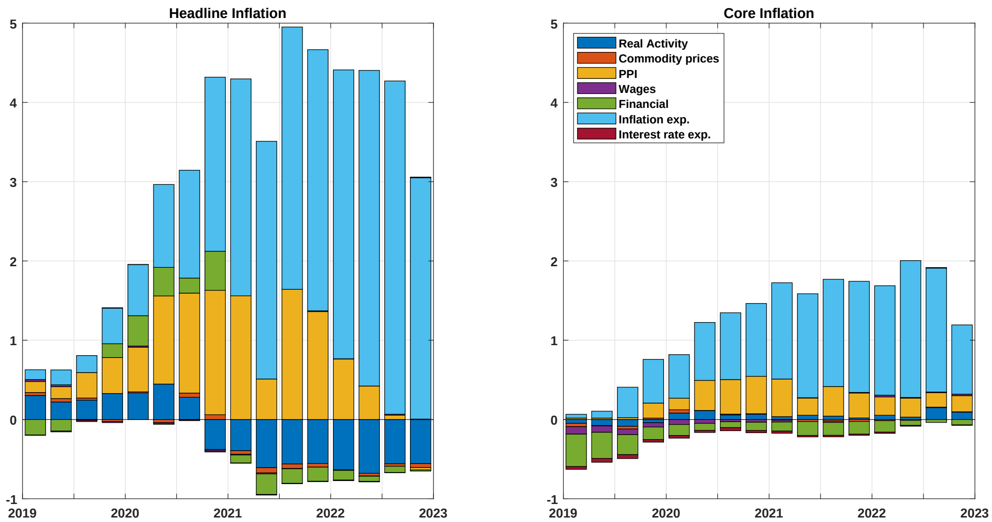
[17] See Aas et al. (2020) for a derivation of this result.

[18] In practice, we carry out the computation of Shapley values by using the Tree SHAP package of Lundberg et al. (2019).

[19] The Shapley values of all the variables in the predictor set sum up to the deviation of the predicted value from the average value of the target variable.

[20] Seven is roughly 10% of the variables in our set of predictors.

Figure 8: Decomposition of the h=6 QRF median forecast



Note: Shapley values associated with different groups of predictors, sample 2019 - 2022, horizon of six months ahead. Left panel: Headline inflation; Right panel: Core inflation.

Table 4: Top contributors to six-months ahead forecast

	Headline Inflation	**Core inflation**
1	Euribor 3-Months	Euribor 3-Months
2	Building permits	Ten-Year Govt Bond Yield
3	Industry survey - selling price expectations for the 3 months ahead	Consumer survey - price trends over next 12 month
4	Unemployment rate	Long-term interest rate future (6 months, DE)
5	Consumer survey - price trends over next 12 month	Unemployment rate
6	Ten-Year Govt Bond Yield	Inflation rate future (6 months, DE)
7	Industry survey - selling price expectations, Intermediate Goods	Indicator of negotiated wage rates - total excluding bonuses

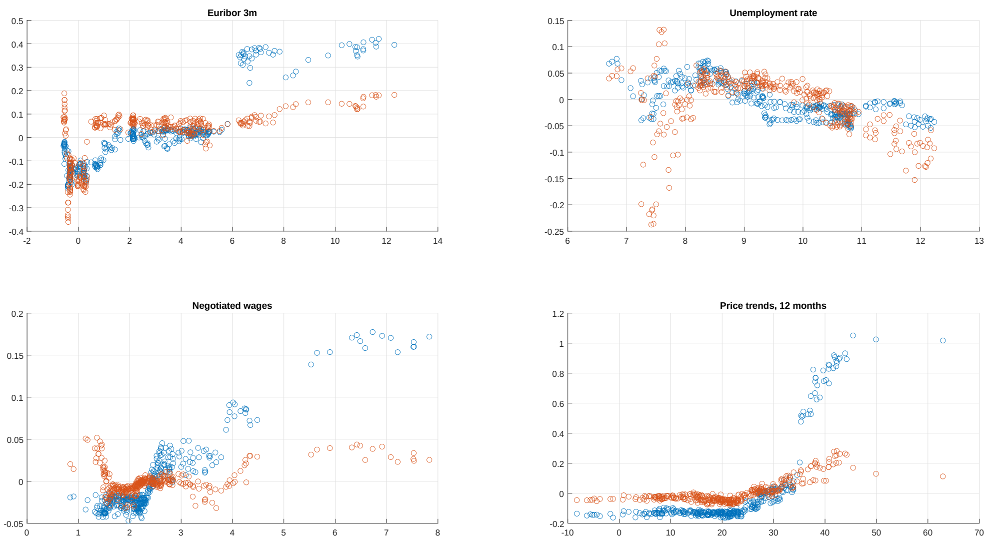
Note: Ranking of top contributors in terms of absolute mean of Shapley value over the evaluation sample, six months ahead horizon. Left column: Headline inflation; Right column: Core inflation.

tween these predictors and headline/core inflation? Figure 9 shows the Shapley values of some selected variables for headline (red circles) and core inflation (blue circles), plotted against the values of the variables over the historical sample. The variables are chosen among the most relevant indicators for inflation in the full sample, as reported in Table 4. These scatter plots give a rough indication of the type of the relationship that the QRF estimates between the variables and headline and core inflation for the six months forecasting horizon.[21]

For three of these variables, the relationship captured by the QRF seems roughly linear. For example, for the Euribor the relationship with inflation is essentially linear and, as expected, positive: high short-term interest rates signal that inflation is expected to be high in the future and needs some "leaning against the wind" from the central bank. The only non-linearity appearing in the Euribor charts has to do with the attainment of the effective lower bound of interest rates

[21]Notice that, differently from the rest of the paper, here we are estimating the model for inflation six month ahead only on the full sample and, hence, figure 9 reports the in-sample Shapley values.

Figure 9: Top contributors in terms of Shapley Values



Note: Vertical axis: in-sample Shapley values for the variable indicated in the title for headline inflation (red) and core inflation (blue). Horizontal axis: value of the variable indicated in the title

by the ECB. The panel on the unemployment rate effectively suggests the existence of a Phillips correlation with a mild negative slope. The pass-through from wages to inflation is also roughly linear. For what concerns the measure of inflation expectations reported in the chart, i.e. “price trends expected over the next twelve months”, the relationship with headline and, above all, core inflation is clearly nonlinear. The measure of expectations is defined in terms of the balance of survey respondents. A positive number indicates that there is a larger share of respondents who expect an increase rather than a decrease in prices. Hence, the figure suggests that the increase in the share of survey respondents who expect higher inflation beyond a certain threshold is an indication of a marked acceleration in inflation. The fact that the non-linearity in core inflation is estimated to be more quantitatively relevant is in line with our finding in the previous section that the QRF is a better predictor for core rather than headline inflation, compared to VARCOMB, our state-of-the-art linear model.

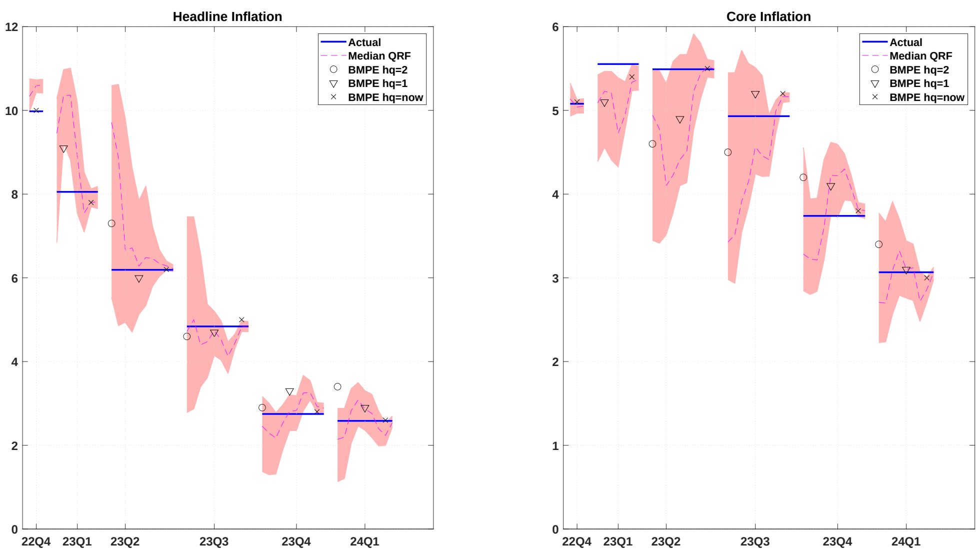
5 The euro area disinflation in 2023-2024 through the lenses of the QRF analysis in real-time

In the last quarter of 2022, the QRF was integrated into the ECB’s modeling toolbox for preparing the short-term inflation projections . The model generates an inflation density forecast, which aids in assessing the risks to the official projections. An example of the model’s application can be found in Lane (2024), chart 9.

Since the fourth quarter of 2022, euro area headline inflation has declined, with core inflation following suit in the second half of 2023. How did the QRF perform in capturing this disinflation path and as a gauge of the risks surrounding the Eurosystem official projections, in real-time?

In figure 10, we report the observed year-on-year growth rates of headline HICP (left panel, blue solid line) and our measure of core inflation (right panel, blue line) for the quarters 2022Q4 until 2024Q1. These are shown alongside the Eurosystem projections for those quarters, produced over different horizons (ranging from two quarters ahead to the current quarter, different markers) and the 16th to 84th quantiles of the QRF projections produced at the time of the Eurosystem projections and in the intervening weeks (orange areas).[22]

Figure 10: Headline and core inflation, real-time forecasts 2022Q4-2024Q1



Note: Blue solid lines: Headline (left panel) and Core (right panel) inflation outcomes, 2022Q4-2024Q1; Orange areas: 16th-84th range of QRF density forecasts, computed twice per month; Circle: Eurosystem projections, 2 quarters ahead; Triangle: Eurosystem projections, 1 quarter ahead; Star: Eurosystem projections, current quarter;

Figure 10 illustrates the disinflation path for headline and core inflation. Over the six quarters in the picture, the QRF has generally been quite accurate. For headline inflation, during 2023Q1 and 2023Q2, the disinflation process had just begun to materialize, and both the QRF and the Eurosystem projections made two quarters ahead were still significantly higher than the final release, but they adjusted relatively quickly over time. Since then, both the QRF and the Eurosystem forecasts have been close to the final outcomes already two quarters before the first release. Interestingly, the QRF has consistently signaled downside risks to the Eurosystem projections over the last two quarters in the sample, and these risks have eventually materialized. Similar observations apply to core inflation although, for this variable, both the QRF and the Eurosystem forecasts tended to underpredict in the early stages of the disinflation process, especially at the two-quarter-ahead horizon. For the last two quarters, as with headline inflation,

[22] The QRF forecasts are updated twice per month.

the QRF consistently signalled downside risks to the Eurosystem projections, which subsequently materialized.

6 Conclusion

In this paper, we show that the quantile regression forest, a non-linear model, could be a valuable addition to the current toolbox for forecasting euro area inflation and assessing the risks surrounding the central tendency of the Eurosystem projections, which predominantly rely on linear models.

The quantile regression forest exhibits comparable accuracy to state-of-the-art linear models for density forecasting. Additionally, it competes effectively with institutional (BMPE) and survey (SPF) forecasts, despite the reliance of the latter on expert judgement.

The similar accuracy of linear and non-linear models over the full sample analyzed suggests the quantile regression forest serves as a complement rather than a substitute for the Eurosystem modeling toolbox for forecasting inflation.

On the broader question of whether euro area inflation exhibits non-linear dynamics, our analysis concludes that while non-linearity is present, it tends to be mild. Furthermore, this non-linearity is more pronounced for core inflation compared to headline inflation.

A Database of the QRF, SPF and BMPE

The target variables in our exercise are the euro area Harmonized Index of Consumer Prices (HICP) and the Harmonized Index of Consumer Prices excluding Energy and Food (HICPex). The former is what we consider our "headline" measure, while the latter is our "core" measure in this paper. Both measures and their dynamics are described at length in the main body of the paper. Table 5 reports the name (column 4) of the variables we use in our QRF and VARCOMB as predictors, the group of variables to which they belong (column 2) for the computation of the grouped Shapley values and the transformation we apply to make the variable stationary (column 3).

A.1 Survey of Professional Forecasters

The Survey of Professional Forecasters (SPF) for the European Union has been taking place quarterly since the beginning of 1999. The survey asks to a panel of professional forecasters within the EU to give an estimate on the future values for euro area gross domestic product growth, HICP inflation, and the unemployment rate (de Vincent-Humphreys et al., 2019; Kenny et al., 2013). We focus here on inflation forecasts, for two separate horizons, namely current year and one-year-ahead[23]. The target for the two assessments are different. While the one-year-ahead concept (which, in the main body of the paper is defined as year-on-year inflation at h=12 months ahead) measures the change in prices from the quarter preceding the assessment to four quarters later, the current year assessment pertains, in each quarter in which the survey is released, to the "average" inflation over the current year (average means, roughly, the average of the four year-on-year inflation rates in the four quarter of the current year). Respondents are asked to give both a point forecast and to assign probabilities for each variable's future outcome falling within pre-determined ranges. The individual responses are then aggregated, and a histogram of average probabilities for the economic outlook results. We do not focus on individual responses, following the results in Genre et al. (2013), where the simple average is proven to be the best combination method. Other aggregation methods include optimal pooling like in Conflitti et al. (2015), and a more recent work by Diebold et al. (2020), where the authors propose to build regularised mixtures of individual densities.

The first SPF vintage for headline inflation corresponds to February 2002, while the first one for core inflation is February 2017. The forecasts for those vintages are supposed to be produced aroundmid- of the previous month. We assume that forecasters had data up to the previous December when matching the information provided to the QRF. Then, we continue to update the estimation sample by adding one quarter (effectively, three months) at a time, and we repeat all the steps of the forecasting exercise until exhaustion of the sample.

[23] For each round, the target quarter refers to the current or one year after the latest official release available at the time of the questionnaire.

Table 5: Database - predictors

	Group	Transf.	Variable description
1	G1	0	EU Commission, DG-ECFIN, Retail trade survey - expected business situation for 3 months ahead - Percentage balances
2	G1	0	EU Commission, DG-ECFIN, Consumer survey - financial situation over next 12 months - Percentages
3	G1	0	EU Commission, DG-ECFIN, Business climate indicator - Points of standard deviation
4	G1	0	EU Commission, DG-ECFIN, Consumer survey - general economic situation over next 12 months - Percentages
5	G1	0	EU Commission, DG-ECFIN, Consumer survey - major purchases over next 12 months - Percentages
6	G1	0	EU Commission, DG-ECFIN, Consumer survey - savings over next 12 months - Percentages
7	G1	0	EU Commission, DG-ECFIN, Consumer survey - unemployment expectations over next 12 months - Percentages
8	G1	0	EU Commission, DG-ECFIN, Consumer survey - consumer confidence indicator - Percentages
9	G1	0	EU Commission, DG-ECFIN, Economic sentiment indicator - Percentage balances
10	G1	0	EU Commission, DG-ECFIN, Industry survey - employment expectations for 3 months ahead - Percentage balances
11	G1	0	EU Commission, DG-ECFIN, Industry survey - production expectations for the 3 months ahead - Percentage balances
12	G6	0	EU Commission, DG-ECFIN, Industry survey - selling price expectations for the 3 months ahead - Percentage balances
13	G6	0	EU Commission, DG-ECFIN, Industry survey - selling price expectations for the months ahead, Intemediate Goods - Percentage balances
14	G6	0	EU Commission, DG-ECFIN, Industry survey - selling price expectations for the months ahead, Consumer Goods - Percentage balances
15	G6	0	EU Commission, DG-ECFIN, Consumer survey - price trends over next 12 months - Percentages
16	G7	0	ZEW, Short-term interest rate future (6 months) - Percentage balances
17	G1	0	Germany, ZEW, Economic situation future (6 months) - Percentage balances
18	G6	0	Germany, ZEW, Inflation rate future (6 months) - Percentage balances
19	G7	0	Germany, ZEW, Long-term interest rate future (6 months) - Percentage balances
20	G2	1	Equity/index - Baltic DRY Index (BDI) - Historical close, average of observations through period
21	G2	2	Bloomberg European Dated Brent Forties Oseberg Ekofisk (BFOE) Crude Oil Spot Price - Historical close - US dollar
22	G2	2	WORLD-MKT PRICES, RAW MATERIALS, EXCL.ENERGY(MU17), EUR-BASIS - HWWA . Euro area - HAMBURG WORLD ECONOMIC ARCHIVE
23	G2	2	WORLD-MKT PRICES, ENERGY RAW MATERIALS(MU17), EUR-BASIS - HWWA. Euro area - HAMBURG WORLD ECONOMIC ARCHIVE
24	G2	2	World market prices of raw materials, Index total, euro
25	G2	2	World market prices of raw materials, Index Total excluding energy, euro
26	G2	2	World market prices of raw materials, Energy, euro
27	G2	2	World market prices of raw materials, Crude oil, euro
28	G2	2	World market prices of raw materials, Industrial raw materials, euro
29	G2	2	World market prices of raw materials, Food and tropical beverages, euro
30	G2	2	ECB Commodity Price index Euro denominated, import weighted, Non-food
31	G2	2	ECB Commodity Price index Euro denominated, import weighted, Agricultural raw materials
32	G2	0	EXCH.RATE: US DOLLARS/1 EUR,SPOT AT 2:15 PM (CET) D,W,M,Q,A-AVG
33	G2	0	ECB Nominal effective exch. rate of the Euro against, EER-12 group of trading partners: AU,CA,DK,HK,JP,NO,SG,KR,SE,CH,GB,US,EA excluding the Euro
34	G3	2	Producer Price Index, domestic sales, Consumer goods industry
35	G3	2	Producer Price Index, domestic sales, MIG Durable Consumer Goods Industry
36	G3	2	Producer Price Index, domestic sales, MIG Non-durable Consumer Goods Industry
37	G3	2	Producer Price Index, domestic sales, MIG Intermediate Goods Industry
38	G3	2	Producer Price Index, domestic sales, MIG Capital Goods Industry
39	G3	2	Producer Price Index, domestic sales, MIG Energy
40	G3	2	Producer Price Index, domestic sales, MANUFACTURING
41	G3	2	Producer Price Index, domestic sales, Total Industry (excluding construction)
42	G4	0	Indicator of negotiated wage rates, Total - Annual rate of change
43	G4	0	Indicator of negotiated wage rates - total excluding bonuses, Total - Annual rate of change
44	G1	2	Industrial Production Index, Total Industry (excluding construction)
45	G1	1	Building Permits / dwellings, Residential buildings except residences for communities
46	G1	0	European Labour Force Survey; Unemployment rate; Total; Age 15 to 74
47	G1	1	EA19 Leading Indicators OECD ¿ Leading indicators ¿ CLI ¿ Amplitude adjusted / Level. rate or national currency
48	G1	0	United States; European Labour Force Survey; Unemployment rate; Total; Age 15 to 74
49	G5	0	Euribor 3-month - Last trade price or value
50	G5	0	Benchmark bond - Euro area 10-year Government Benchmark bond yield - Yield
51	G5	2	European Monetary Union Market Index. Equity Index.
52	G5	0	IBES MSCI EMU Index Earnings. Weighted average long term growth EPS (Earnings per share) forecast expressed as a percentage
53	G5	2	Euro area - Equity/index - European Monetary Union Consumer Goods Index (EUR)
54	G5	2	Equity/index - European Monetary Union Consumer Services Index (EUR) - Historical close
55	G5	2	Monetary aggregate M1
56	G5	2	Monetary aggregate M3
57	G5	2	Monetary aggregate M2
58	G5	2	Loans, Total maturity, All currencies combined - Euro area (changing composition) counterpart
59	G6	2	US - CONSUMER PRICES, ALL ITEMS (ALL URBAN CONSUMERS)
60	G6	2	US - CONSUMER PRICES, CORE INFLATION (URBAN CONSUMERS)

Note: G1: real activity, G2 : commodity prices, G3: PPI, G4: wages, G5: financial, G6: inflation expectations, G7: interest rate expectations. Transformations for stationarity: 0 = no transformation, 1 = natural logarithm, 2 = first difference of natural logarithm.

A.2 Inflation projections from the BMPE

Eurosystem and ECB staff produce macroeconomic projections (BMPE) that cover the outlook for the euro area and the wider global economy. These contribute to the ECB Governing Council's assessment of economic developments and risks to price stability.

They are published four times a year (in March, June, September and December).

The first BMPE vintage corresponds to March 2002. We assume that forecasters had data up to January 2002 when matching the information provided to the QRF. Then, we continue to update the estimation sample by adding one quarter (effectively, three months) at a time, and we repeat all the steps of the forecasting exercise until exhaustion of the sample.

A.3 Fitting the skew-t distribution to our density forecasts

The skew-t distribution is a flexible, parametric density that allows for fat tails as well as asymmetries, controlled by the parameters defining the distribution.

We define the skew-t (ST) for a variable Y as:

$$Y \sim ST(\xi, \omega, \alpha, \nu)$$

where ξ is a location parameter, ω is the scale, α is the slant parameter that determines the skewness of the distribution, and ν is the degrees of freedom.

In order to fit a skew-t to our density forecasts, we match the empirical quantiles of our forecasts. For the QRF, quantiles are directly available. For the SPF, we derive them from the SPF histograms.[24]

Specifically, we consider, for each release of the SPF, the histogram based on the reported probabilities, for the horizons of interest, i.e. for the forecasts of the current year HICP and HICPex inflation and of the year-on-year growth rates of HICP and HICPex one year ahead. We obtain the quantiles of the empirical cdf from the bin edges of the SPF histogram, and we match the closest possible quantiles to the 5th/16th/84th/95th quantiles we used for the QRF.

Once we have the quantiles, we follow Montes Galdon et al. (2023) and fit the pdf of a skew-t distribution.[25] Note however that we need to keep the degrees of freedom of the distribution, ν, as a discrete value. Therefore, we proceed as follows. We construct first a grid for the degrees of

[24]Notice that for VARCOMB we have already the draws from the posterior, which can be used to compute all our accuracy statistics. However, to produce the results in our tables, we also fit a skew-t to match the quantiles of the VARCOMB forecast. We do that for comparability purposes, but the results are basically unchanged if we use the original posterior draws.

[25]Notice that there are alternative approaches as in Engelberg et al. (2009), which assumes a normal or a beta distribution for the SPF histograms and Billio et al. (2013), which produces a continuous SPF distribution, as well as draws from this distribution, using a kernel smoother.

freedom. For each value of the grid, we find the location, scale and slant parameters with the best match of the quantiles provided from the pdf. At this stage, we have a set of parameters matching the quantiles we have chosen, given a certain value of ν. In this set, we select the parameters with the minimum squared 2-norm distance from the empirical quantiles.

The skew-t distribution we obtain is then used to draw the density forecasts which enter of out-of-sample evaluation.

B Comparison of RMSE betweek median QRF and Random Walk forecasts

Table 6: RMSE of QRF and RW for headline and core inflation

Horizon	**QRF**	**RW**
Panel a: Headline Inflation		
h=3	0.58	0.72
h=6	0.92	1.11
h=9	1.48	1.51
h=12	1.97	1.87
Panel b: Core Inflation		
h=3	0.21	0.31
h=6	0.36	0.45
h=9	0.64	0.61
h=12	0.82	0.75

Note: Column 2: QRF; Column 3: RW.

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