

Model Uncertainty as Partial-Identification Problems: Application to Policy Promises during Crises

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Financial Crises and Uncertainty

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- agents may have uncertainty about the key features of distressed financial markets.
- e.g. capitalization of the financial sector, dividends from distressed risky assets

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Policymakers announce policy promises to reduce such uncertainty.

- **“whatever it takes”** during European debt crisis by Mario Draghi
- altering pessimistic asset valuation **without actual implementation of policy**

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Policy promises **could affect the inferred set of models agents consider as plausible.**

Outline

- ① Model descriptions
- ② Equilibrium implications of subjective beliefs
- ③ Equilibrium effects of government policy promises

Technology, Markets, Agents

Endowment economy, infinite horizon, continuous-time in spirit of He and Krishnamurthy (2013)

Two assets traded in Walrasian markets:

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Two types of agents: a continuum of households and financial intermediaries.

Each household invests in an intermediary's portfolio subject to a margin constraint

$$\underbrace{H_{j,t}}_{\text{Household } j\text{'s investment}} \leq \underbrace{m_j}_{\text{Inverse tightness of constraint}} \times \underbrace{W_{i,t}}_{\text{Intermediary } i\text{'s wealth}}$$

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and rests in risk-free asset.

Tighter constraint or lower intermediary wealth leads to capital scarcity in risky asset market.

⇒ **Profitable investment opportunity in terms of individual investors during crises.**

Intermediaries' Preferences: Maximization Part

Given a subjective belief, an intermediary $i \in [0, 1]$ solves a standard Merton-type problem:

$$\max_{\{c_{i,t}, \alpha_{i,t}\}} \mathbb{E}^S \left[\int_0^\infty e^{-\rho t} \frac{c_{i,t}^{1-\gamma}}{1-\gamma} dt \right]$$

s.t.

$$\frac{dw_{i,t}}{w_{i,t}} = \alpha_{i,t}(dR_t - r_t dt) + r_t dt - \frac{c_{i,t}}{w_{i,t}} dt.$$

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Optimal portfolio choice crucially depends on **subjective expected returns**:

$$\alpha_{i,t} = \underbrace{\frac{1}{\gamma} \frac{\mathbb{E}_t^S(dR_t - r_t dt)}{\sigma_{R,t}^2}}_{\text{myopic}} + \text{intertemporal hedging}.$$

Information Set

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- tightness of margin constraints faced by other intermediaries: $m_j = m, (j \neq i)$
- long-run dividend growth and aggregate dividend growth shock: g and dZ_t

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Observable:

- individual wealth $w_{i,t}$;
- individual margin constraint: m_i ;
- asset market information about return volatility and risk-free rate $(\sigma_{R,t}, r_t)$.

Subjective Belief Formation

Understand mapping between (x, m, g) and $(\pi_{R,t}, \sigma_{R,t}, r_t)$.

Infer combinations of (x, m, g) consistent with observable information:

$$\underbrace{\sigma_{R,t}}_{\text{Observed}} = \underbrace{\sigma_R(x, m, g)}_{\text{Model}};$$

$$\underbrace{r_t}_{\text{Observed}} = \underbrace{r(x, m, g)}_{\text{Model}}.$$

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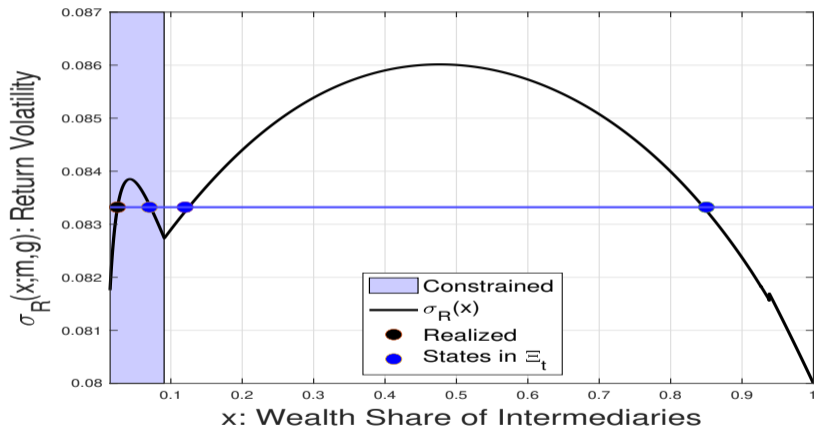
$$\underbrace{r_t}_{\text{Observed}} = \underbrace{r(x, m, g)}_{\text{Model}}.$$

Partial identification problem: many combinations of (x, m, g) are consistent.

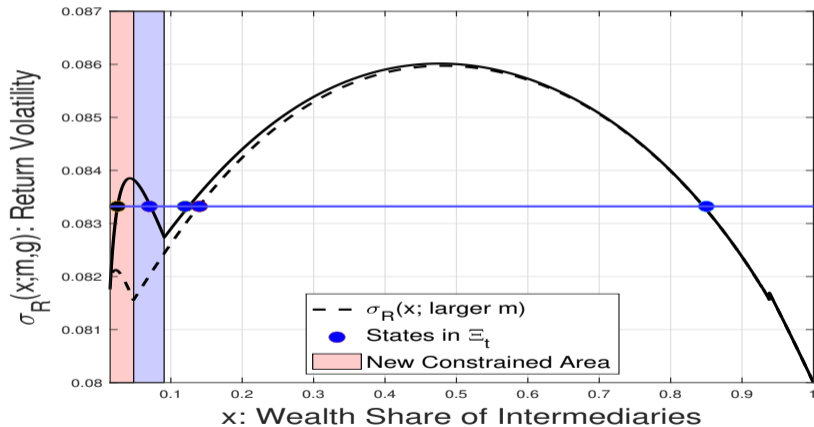
Denote the set of those combinations as Ξ_t .

Each combination has different implication for $\pi_{R,t}(x, m, g)$.

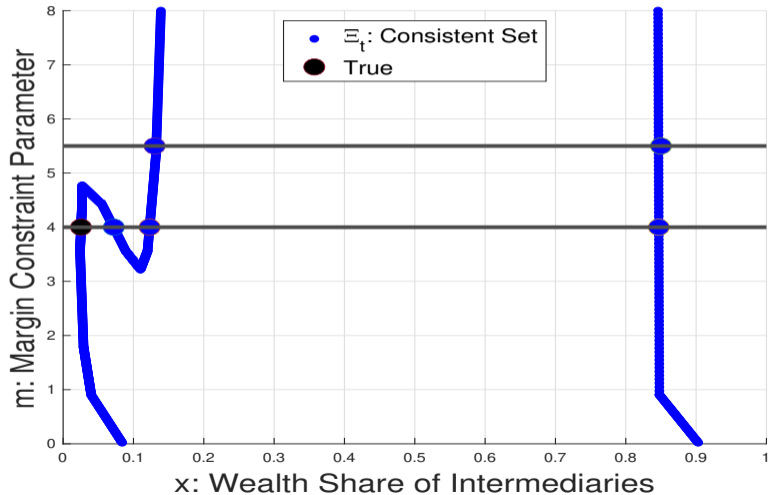
Multiple x 's Consistent with $\sigma_{R,t}$



Multiple (x, m) 's Consistent with $\sigma_{R,t}$



Partially Identified Set of (m, x)



Intermediaries' Minimization

$$\max_{\{c_{i,t}, \alpha_{i,t}\}} \min_{(x_t, m_t, g_t) \in \Xi_t} \mathbb{E}^S \left[\int_0^\infty e^{-\rho t} \frac{c_{i,t}^{1-\gamma}}{1-\gamma} dt \right]$$

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$$dx_t = \mu_x(x_t, m_t, g_t) dt + \sigma_{x,t} dZ_t^S.$$

Choose the worst-case combination instant by instant $(x_t^{worst}, m_t^{worst}, g_t^{worst})$

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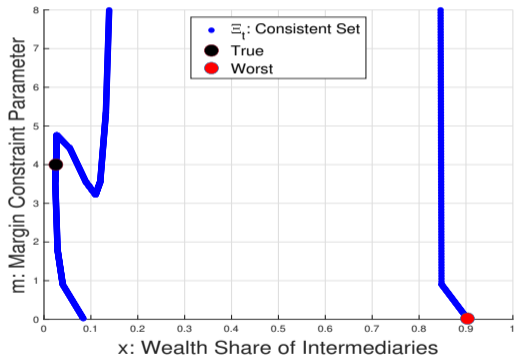
Expected return affects both individual and aggregate state evolution.

- in equilibrium, lower expected return implies lower utility.

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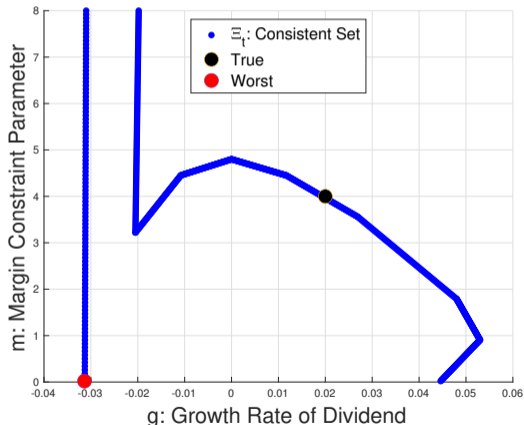
Worst-Case (m, x) : Higher Capitalization and Tighter Constraint



The worst-case $x^{worst} > x^{true}$ implies:

$$\pi_R(x^{worst}, m^{worst}, g^{true}) < \pi_R(x^{true}, m^{true}, g^{true})$$

Worst-case (m, g) : Lower Long-Run Dividend Growth



The worst-case long-run dividend growth would induce stronger saving motive:

$$r(x^{worst}, m^{worst}, g^{worst}) = r(x^{true}, m^{true}, g^{true}),$$

offsetting effect of less precautionary saving of x^{worst} on risk-free rate.

Calibration

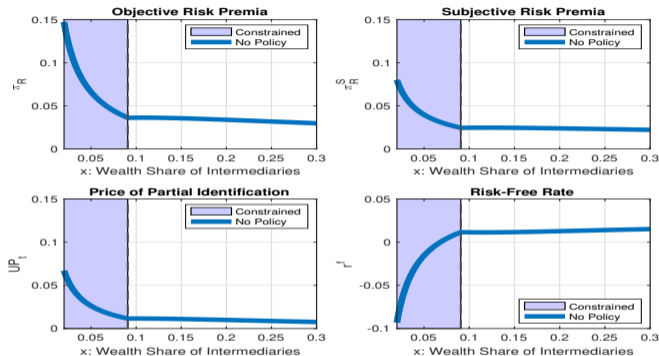
Parameter	Description	Value	Target	Target value	Model
γ	Relative Risk Aversion	1.8	Average expected excess return of MBS	3.4	3.8
ρ	Discount Rate	0.08	Average risk-free rate	1	0.78
σ	Dividend Volatility	0.08	Return volatility of MBS	0.81	0.83
λ	Debt Household Share	0.6	Average Debt-to-Asset ratio in 2007	0.52	0.55
l	Labor Income Ratio	1.84	Share of Labor Income in Total Income	0.66	0.64

Table: Matched Moments and Internally Calibrated Parameters

Parameter	Description	Value	Source
m	Intermediation multiplier	4	HK (Share of managers compensation in intermediaries' profit)
g	Dividend Growth	2%	HK (Average real output growth in the U.S.)

Table: Fixed Parameters

Equilibrium Returns: Pessimism in Tail



$$UP_t = \pi_R(x_t^{true}, m_t^{true}, g_t^{true}) - \pi_R(x_t^{worst}, m_t^{worst}, g_t^{worst}).$$

- compensation for uncertainty about expected returns

Around 40% from UP_t , amplified during crises survey evidence

Survey Evidence for Subjective Beliefs

Worst-case subjective belief in model is consistent with analysts' forecasts in survey.

- Price-dividend ratio is driven by subjective dividend forecasts, rather than return forecasts.
 - consistent with evidence from analysts' survey forecasts by [De la O and Myer \(2021\)](#)
 - REE model predicts return forecasts drive P-D ratio, not dividend forecasts. [details](#)

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Natural laboratory for policy analysis with subjective beliefs consistent with survey evidence

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Belief Management: Policy Promises in Crisis

Study unanticipated **policy promises** aimed at resolving agents' uncertainty

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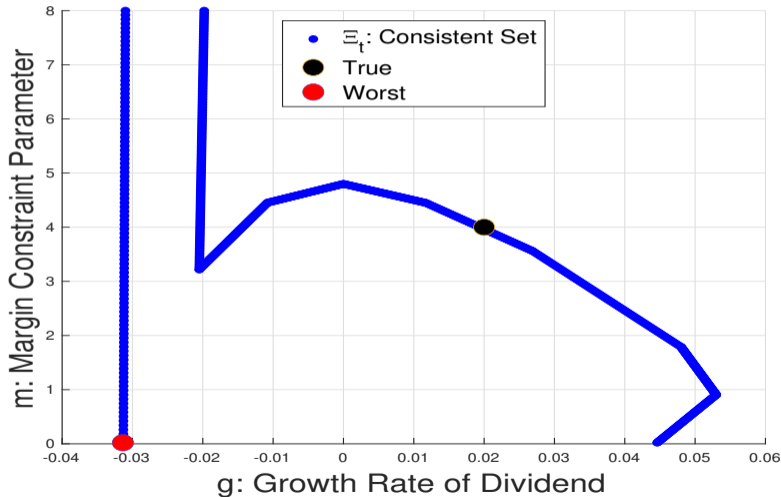
Promises eliminate some beliefs inconsistent with announcements.

Promises work through pronouncement **without actual implementations**:

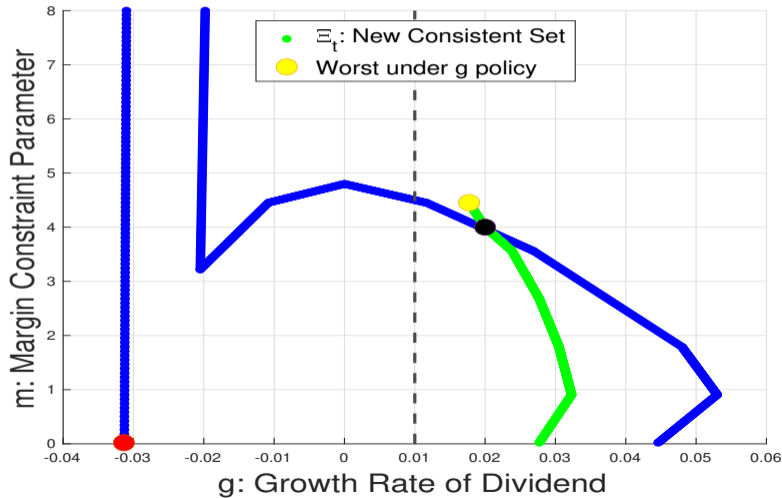
- **guarantee cash flow from risky asset** (g policy):
 - federal government guaranteed cash flow from MBS during 2007-2009 crises.
 - eliminating overly pessimistic view on g , restriction on the set $g > 0.01$

Policy announcement changes **entire equilibrium dynamics**.

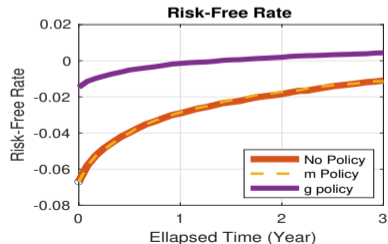
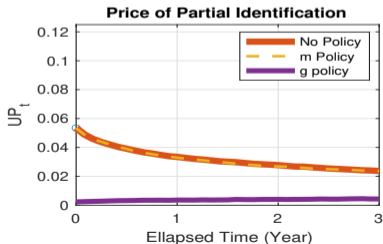
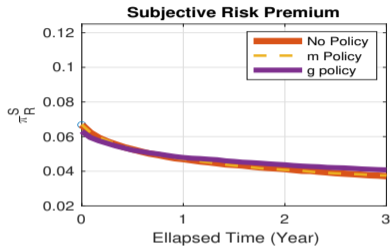
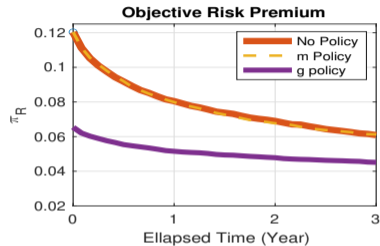
Consistent Set of (m, g) : Lower Expected Dividend Growth



Consistent Beliefs (m, g) : g Policy



Mean Transition Dynamics: g Policy Reduces Risk Premia



Contribution

Theoretical contribution: uncertainty over **endogenous** variables using **endogenous** signal

- Gilboa and Schmeidler (1989); Chen and Epstein (2002); Hansen and Sargent (2021,2022), etc.
 - uncertainty over **exogenous** variables, disciplined by **exogenous** signals

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Applied contribution: incorporating subjective beliefs into a model with financial frictions

- Brunnermeier and Sannikov (2014); Di-Tella (2017); He and Krishnamurthy (2012, 2013, 2019), etc
 - rational expectations equilibrium (REE), no model uncertainty
- subjective beliefs in line with survey evidence of De la O and Meyer (2021), Nagel and Xu (2023)

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Policy implications: theoretical framework to analyze equilibrium effects of policy promises

- Haddad, Moreira, and Muir (2023)
 - no structural model to study equilibrium feedback
- promises could work by affecting subjective beliefs without actual implementations.

Conclusion

Policy implications of policy promises:

- resolving uncertainty about financial frictions m is not so effective in current framework (in paper).
- how alternative announcements alter beliefs and affect entire equilibrium dynamics.

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Work in progress: empirically disciplining the set of (x, m, g) , in particular x

- Subjective belief about x implied from option price data on financial institutions' stock.
e.g. OptionMetrics

Thanks

Thank you!

Appendix: Households' Problem

Two types of households:

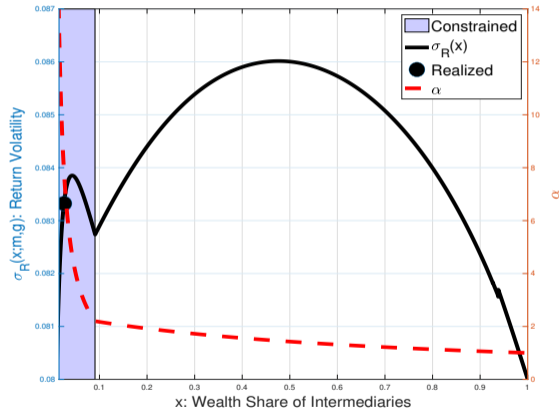
- debt households with fraction λ invest only in the riskfree asset
- risky households with remaining fraction invest both in intermediaries and riskfree asset

Risky households' portfolio choice:

$$\max_{\alpha_t^h \in [0,1]} \alpha_t^h E_t[dR_t^I - r_t dt] - \frac{1}{2} (\alpha_t^h)^2 \text{Var}_t[dR_t^I - r_t dt],$$

where $dR_t^I \equiv \alpha_t dR_t + (1 - \alpha_t)r_t dt$ is a return for intermediaries' portfolio and subject to the margin constraint. [Back](#)

Appendix: Return Volatility



- Return volatility

$$\sigma_R = \frac{p'}{p} \underbrace{\sigma_x}_{\text{Wealth share volatility}} + \sigma$$

- Volatility of wealth share of intermediaries

$$\sigma_x = \frac{\underbrace{x}_{\text{Size of intermediaries}} \underbrace{(\alpha - 1)\sigma}_{\text{Portfolio heterogeneity}}}{\underbrace{1 - x(\alpha - 1)\sigma}_{\text{Amplification}}},$$

where α is portfolio weight of other intermediaries on risky asset.

Appendix: Equilibrium Definition

Definition

The equilibrium parameterized by a baseline value of (\hat{m}, \hat{g}) must satisfy the following conditions. It comprises price processes $\{P_t\}$ and $\{r_t\}$, decisions $\{c_t, c_t^h, \alpha_t^l, \alpha_t^h\}$, and the set of alternative beliefs $\{\Xi_t\}$ such that:

- 1 Given the price processes and beliefs, decisions solve the consumption-savings problems of the debt household, the risky asset households and the intermediaries;
- 2 Decisions satisfy the intermediation constraint;
- 3 The risky asset market clears

$$\frac{\alpha_t^l(w_t + \alpha_t^h(1 - \lambda)w_t^h)}{P_t} = 1;$$

- 4 The goods market clears;

$$c_t + c_t^h = D_t(1 + l);$$

- 5 The alternative models Ξ_t must be consistent with the observed return volatility and the risk-free rate. $\sigma_R(x_t, m, g)$ and $r(x_t, m, g)$ must be implied by an equilibrium in the set of alternative economies parameterized by some (m, g) .

Appendix: Computational Algorithm

Algorithm 1: Fixed-Point Algorithm

Data: Guess for $\sigma_R(x, m, g)$ and $r(x, m, g)$, $x \in [0, 1]$, $m \in (0, \bar{m})$, $g \in (\underline{g}, \bar{g})$

Result: Equilibrium $\sigma_R(x, m, g)$ and $r(x, m, g)$

Initialization; Set $n = 1$ and $\sigma_R^{(0)}(x, m, g) = \sigma_R^{REE}(x; m, g)$ and $r^{(0)}(x, m, g) = r^{REE}(x; m, g)$

while do

for $(g_i, m_i) \in (\underline{g}, \bar{g}) \times (0, \bar{m})$ **do**

 Compute a competitive equilibrium where

- intermediaries form a set of beliefs $\{\Xi_t\}$ using $\sigma_R^{(n-1)}(x, m, g)$ and $r^{(n-1)}(x, m, g)$.
- (g_i, m_i) is true (baseline) parameter value in this equilibrium.

end

$\Rightarrow \{\sigma_R^{(n+1)}(x, m, g)\}, x \in [0, 1], m \in (0, \bar{m}), g \in (\underline{g}, \bar{g})$

if $\max_{(x, m, g) \in [0, 1] \times (0, \bar{m}) \times (\underline{g}, \bar{g})} |\sigma_R^{(n+1)}(x, m, g) - \sigma_R^{(n)}(x, m, g)| + |r^{(n+1)}(x, m, g) - r^{(n)}(x, m, g)| < \epsilon$

then

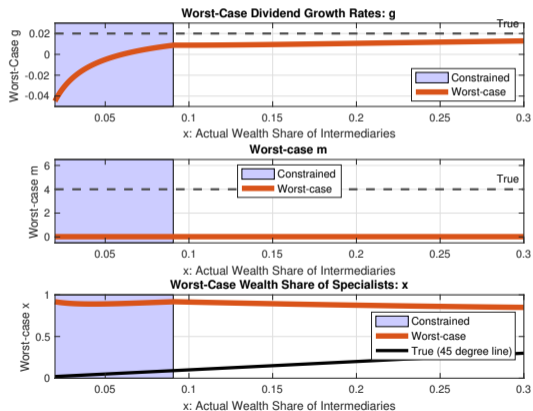
 | break;

end

 Set $n \Rightarrow n + 1$

end

Appendix: Worst-Case (g, m, x)



- Worst-case model of $\pi_{R,t}^S$ cannot be rejected statistically or distinguished from true model. [DEP details](#)

Appendix: Plausibility of Amount of Uncertainty

Verification of amount of uncertainty:

- employ detection error probability measuring statistical discrepancy between worst-case and baseline models.
- worst-case model is statistically hard to distinguish from the true DGP
- intermediaries' worst-case model is statistically admissible

Back

Appendix: Variance Decomposition

One-year ahead contribution of subjective expectations:

$$\underbrace{\frac{\text{cov}(\mathbb{E}_t^S(\log D_{t+1}/D_t), \log P_t/D_t)}{\text{var}(\log P_t/D_t)}}_{CF_1} + \underbrace{\frac{-\text{cov}(\mathbb{E}_t^S(R_{t+1} - R_t), \log P_t/D_t)}{\text{var}(\log P_t/D_t)}}_{DR_1} + \rho \frac{\text{cov}(\mathbb{E}_t^S(\log P_{t+1}/D_{t+1}), \log P_t/D_t)}{\text{var}(\log P_t/D_t)}$$

= 1.

	Subjective (Model)	Survey data (De la O (2021))	Objective (Model)	Rational Model
CF_1	0.19	0.41	0	0
DR_1	-0.13	-0.05	0.06	0.12

back

Appendix: Forecast Error Predictability

One-year ahead forecast error:

$$FE_{t+1}^X \equiv X_{t+1} - \mathbb{E}_t^S(X_{t+1}).$$

	Model	Survey data (De la O (2021))	Rational
$\text{Corr}(FE_{t+1}^R, P_t/D_t)$	-0.83	-0.25	0
$\text{Corr}(FE_{t+1}^{\log(D_{t+1}/D_t)}, P_t/D_t)$	-0.67	-0.52	0

back

Appendix: Cyclical Property of Risk Premium

Subjective risk premium is more acyclical than objective in the model.

One-year ahead predictive regressions:

$$\mathbb{E}_t^S(R_{t+1} - R_t - \int_0^1 r_{t+\tau} d\tau) = \beta_0^S - 0.17 \times \log(P_t/D_t) + u_t^S$$

$$\mathbb{E}_t(R_{t+1} - R_t - \int_0^1 r_{t+\tau} d\tau) = \beta_0 - 0.34 \times \log(P_t/D_t) + u_t$$

Consistent with survey evidence from [Nagel and Xu \(2023\)](#), though still overestimate

- slope coefficient for subjective is approximately 1/5 of that for objective
- cannot capture by rational expectations equilibrium

Pessimism is important to capture empirical properties of risk premia