A Nonhomothetic Price Index and Cost-of-Living Inequality

Philipp Hochmuth
Oesterreichische Nationalbank

Markus Pettersson
Stockholm University

Christoffer Jessen Weissert
Danmarks Nationalbank

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Do changes in the cost of living depend on income? Yes! But how much?

$$\textbf{Change in cost of living} = \begin{pmatrix}\text{price change}\\ \text{of good 1}\end{pmatrix}^{\delta_1} \times \cdots \times \begin{pmatrix}\text{price change}\\ \text{of good } N\end{pmatrix}^{\delta_N}$$

Weights $\delta_1, \ldots, \delta_N$ typically

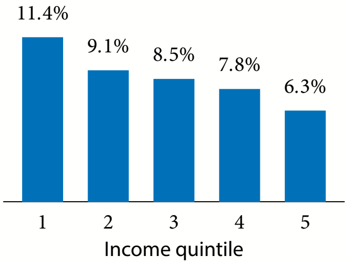
- Functions of budget shares
- Derived from utility maximisation

Standard case: homothetic preferences

- Budget shares independent of income
- $\delta_1, \ldots, \delta_N$ same for everyone
- **Change in cost of living** same for everyone

But: Engel's Law!

US budget shares for food at home, 2021



Source: US Bureau of Labor Statistics.

Consistent and **easy-to-compute** nonhomothetic cost-of-living index possible?

If so, what does it say about **inflation inequality**?

This paper: a nonhomothetic price index and cost-of-living inequality

1. Theoretically

- Nonhomothetic cost-of-living index
- Nests all superlative price indices
- Maintains consistent aggregate inflation measure
- Geometric-mean representation for decompositions

2. Methodologically

- Feasible estimation strategy: only two parameters
- Assumption: necessities and luxuries quasi-separable

3. Empirically

- Application to matched US CEX-CPI data
- Study inflation inequality between 1995 and 2020

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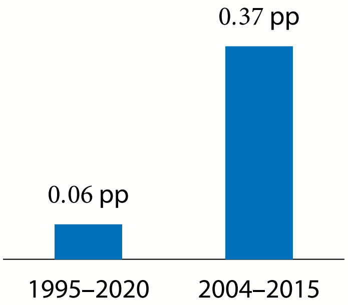
Empirical findings

Bottom vs. top expenditure deciles

Inflation rates:



Inflation volatility:
2.5 times larger for the poor

Previous approaches

$$\textbf{Change in cost of living} = \begin{pmatrix}\text{price change}\\ \text{of good } 1\end{pmatrix}^{\delta_1} \times \cdots \times \begin{pmatrix}\text{price change}\\ \text{of good } N\end{pmatrix}^{\delta_N}$$

Demand system estimation	**Grouped homothetic price indices**	**Nonparametric algorithms**
Deaton & Muellbauer (1980)	Jaravel (2019)	Baqaee, Burstein & Koike-Mori (2024)
Banks, Blundell & Lewbel (1997)	Argente & Lee (2021)	Jaravel & Lashkari (2024)
✓ Theoretically consistent	✗ Theoretically consistent	✓ Theoretically consistent
✗ Easy to estimate	✓ Easy to estimate	✓ Easy to estimate
✓ Timely inflation inequality	✗ Timely inflation inequality	✗ Timely inflation inequality
✓ Product decompositions	✓ Product decompositions	✗ Product decompositions

Theoretical framework

Consumer preferences: distinction between *necessities* and *luxuries*

(Muellbauer, 1975, 1976; Boppart, 2014)

$$V\left(e,\boldsymbol{p}\right) = \frac{1}{\varepsilon}\left[\left(\frac{e}{B(\boldsymbol{p})}\right)^{\varepsilon} - 1\right] - \frac{\nu}{\gamma}\left[\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} - 1\right]$$

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- e **Consumption expenditures** (nominal)
- $B(p)$ **Price aggregator of luxuries** (linearly homogenous)
- $D(p)$ **Price aggregator of necessities** (linearly homogenous)

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Budget share of necessities declines with the **expenditure level**:

$$w_D = \nu\left(\frac{B(\boldsymbol{p})}{e}\right)^{\varepsilon}\left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})}\right)^{\gamma} \longrightarrow 0 \quad \text{as} \quad e \longrightarrow \infty$$

Cost-of-living index:

Change in minimum expenditures needed to maintain a *fixed* standard of living

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Change in minimum expenditures needed to maintain a *fixed* standard of living

$$P(u, \boldsymbol{p}_s, \boldsymbol{p}_t) \equiv \frac{c(u, \boldsymbol{p}_t)}{c(u, \boldsymbol{p}_s)} = \frac{1}{e_s}\left[\left(1 - \frac{\varepsilon \nu}{\gamma} + \varepsilon u\right) B(\boldsymbol{p}_t)^{\gamma} + \frac{\varepsilon \nu}{\gamma} D(\boldsymbol{p}_t)^{\gamma}\right]^{\frac{1}{\varepsilon}} B(\boldsymbol{p}_t)^{1-\frac{\gamma}{\varepsilon}}$$

Cost-of-living index: higher sensitivity to necessity prices at low incomes

Change in minimum expenditures needed to maintain a *fixed* standard of living

$$P(u, \boldsymbol{p}_s, \boldsymbol{p}_t) = \left[\left(1 - \frac{\varepsilon\, w_{Ds}}{\gamma}\right) P_{Bt}{}^{\gamma} + \frac{\varepsilon\, w_{Ds}}{\gamma} P_{Dt}{}^{\gamma} \right]^{\frac{1}{\varepsilon}} P_{Bt}{}^{1-\frac{\gamma}{\varepsilon}}$$

Function of . . .

- Base period **necessity budget share**

  (pins down standard of living)

- Price indices $P_{Ct} = C(\boldsymbol{p}_t) \,/\, C(\boldsymbol{p}_s)$

  (standard homothetic price index formulae)

- Parameters ε, γ

Cost-of-living index: higher sensitivity to necessity prices at low incomes

Change in minimum expenditures needed to maintain a *fixed* standard of living

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Function of . . .

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Separability between necessities/luxuries:

- Observable in data
  (given necessity/luxury split)
- Observable in data
  (using prices and within-basket budget shares)
- Estimable

Cost-of-living index: a generalized decomposition

$B(\boldsymbol{p}), D(\boldsymbol{p})$: expenditure functions that generate geometric-mean price indices, for example

$$B(\boldsymbol{p}) = \left(\sum_{j\in J} \omega_j p_j^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \implies P_{Bt} = \prod_{j\in J}\left(\frac{p_{jt}}{p_{js}}\right)^{\frac{w_{jt}-w_{js}}{\ln w_{jt}-\ln w_{js}} / \sum_i \frac{w_{it}-w_{is}}{\ln w_{it}-\ln w_{is}}}$$

CES **Sato-Vartia index**

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CES **Sato-Vartia index**

Then the rate of change of the cost-of-living index can be decomposed according to

$$\frac{P(u, \boldsymbol{p}_t, \boldsymbol{p}_s)}{P(u, \boldsymbol{p}_{t-1}, \boldsymbol{p}_s)} = \prod_{j \in J} \left(\frac{p_{jt}}{p_{jt-1}}\right)^{\left(\text{weight on necessities}\right) \times \left(\text{weight on } j \text{ in necessities}\right) + \left(\text{weight on luxuries}\right) \times \left(\text{weight on } j \text{ in luxuries}\right)}$$

Cost-of-living index: a generalized decomposition

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Household-specific weights across
vary with expenditure level

Homothetic weights within
same for everyone

Empirical Implementation

Data

Expenditures

US consumer expenditure survey (CEX)

- 1995–2020 (26 years)
- $\approx$ 3,000 households per year
- 21 broad **non-durable commodities** (food at home, public transport, . . .)
- Equivalence adjustment of income and expenditures

Prices

Consumer price index (CPI)

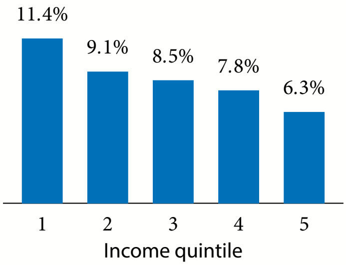
- Group-specific CPI sub-indices
- Match to each of the 21 goods

Overall implementation strategy

Overall implementation strategy

1. Classify individual goods as **necessity** or **luxury**
   (slope of Engel curves)

US budget shares for food at home, 2021



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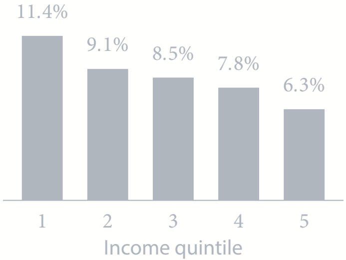

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   (from **necessity budget share** equation)

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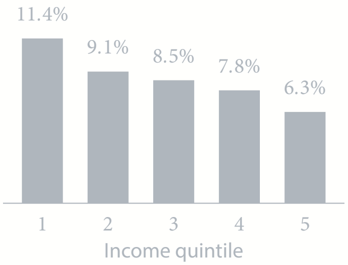

$$w_D = \nu \left(\frac{B(\boldsymbol{p})}{e} \right)^{\varepsilon} \left(\frac{D(\boldsymbol{p})}{B(\boldsymbol{p})} \right)^{\gamma}$$

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   (for some base-period **expenditure level**)

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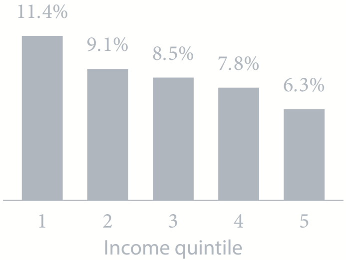

$$w_D = \nu \left(\frac{B(p)}{e}\right)^{\varepsilon} \left(\frac{D(p)}{B(p)}\right)^{\gamma}$$

$$\left[\left(1 - \frac{\varepsilon\, w_{Ds}}{\gamma}\right) P_{Bt}^{\gamma} + \frac{\varepsilon\, w_{Ds}}{\gamma} P_{Dt}^{\gamma}\right]^{\frac{1}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}}$$

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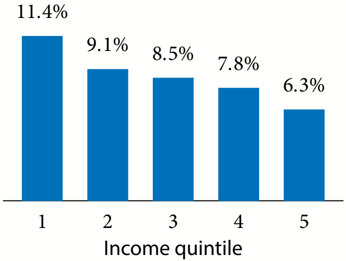

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   ($\boldsymbol{P_D}$, $\boldsymbol{P_B}$, standard price index formulae)

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$$\left[\left(1 - \frac{\varepsilon\, w_{Ds}}{\gamma} \right) P_{Bt}^{\gamma} + \frac{\varepsilon\, w_{Ds}}{\gamma} P_{Dt}^{\gamma} \right]^{\frac{1}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}}$$

Empirical Results

Cost-of-living changes similar on average . . .

But differences arise in subperiods

- Change at median expenditure level

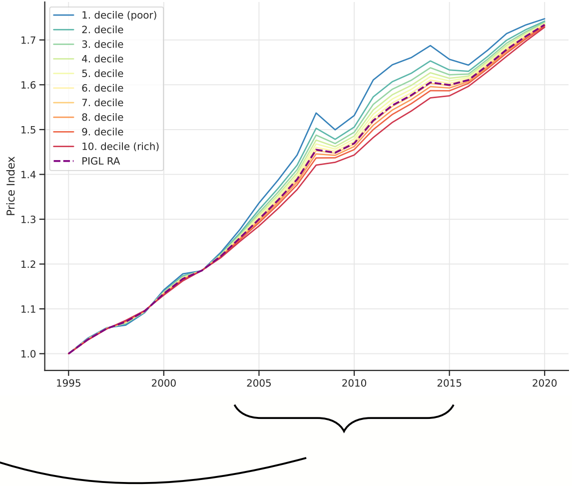
  2.24 % / year

- Top-bottom difference

  0.06 pp / year

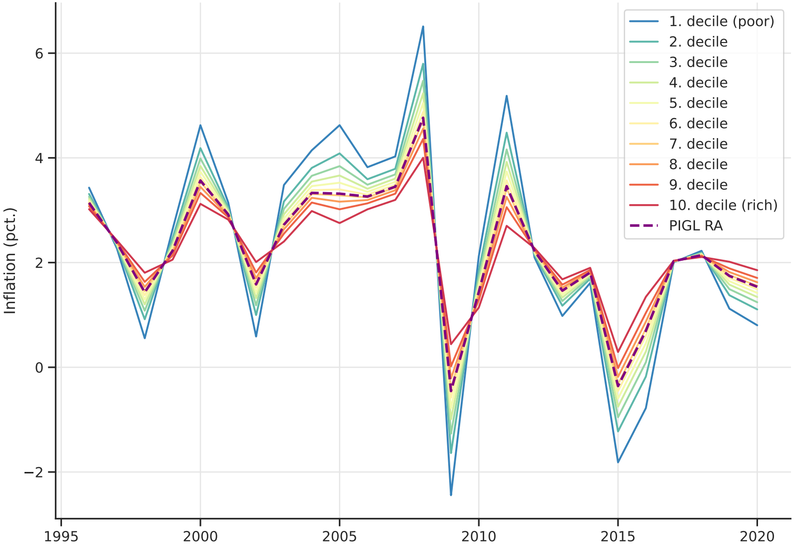
- Top-bottom difference **2004–2015**

  0.37 pp / year



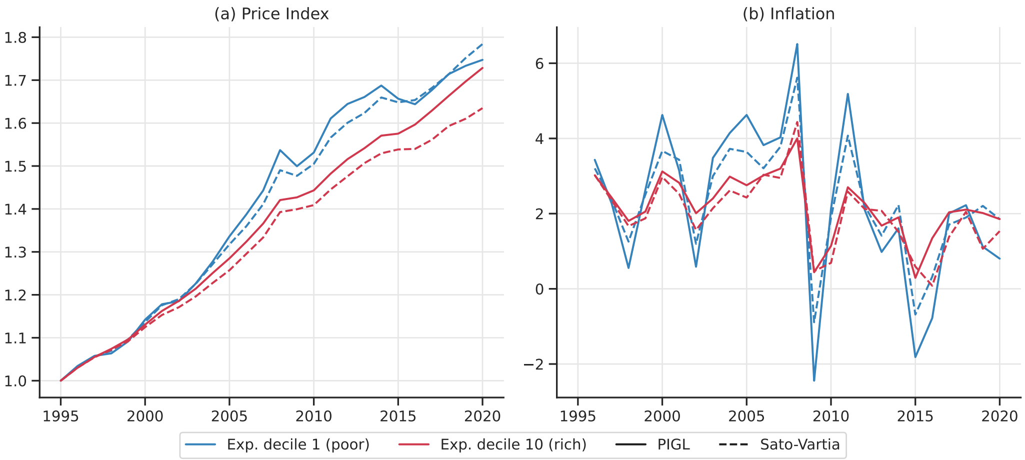
. . . but more volatile for low-expenditure consumers

Inflation 2.5 times more volatile for bottom decile than top decile



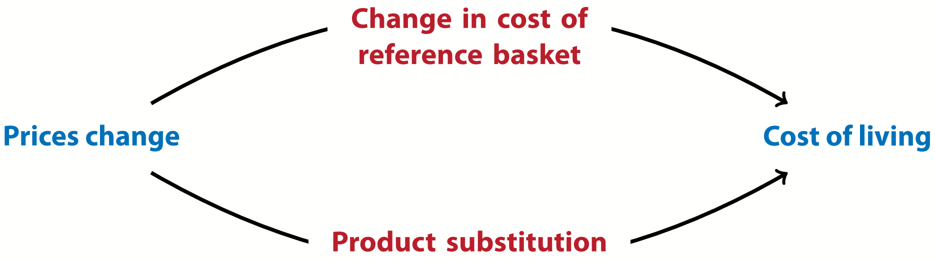
Income group-specific homothetic indices generate no convergence

Key difference: we maintain fixed reference standards of living



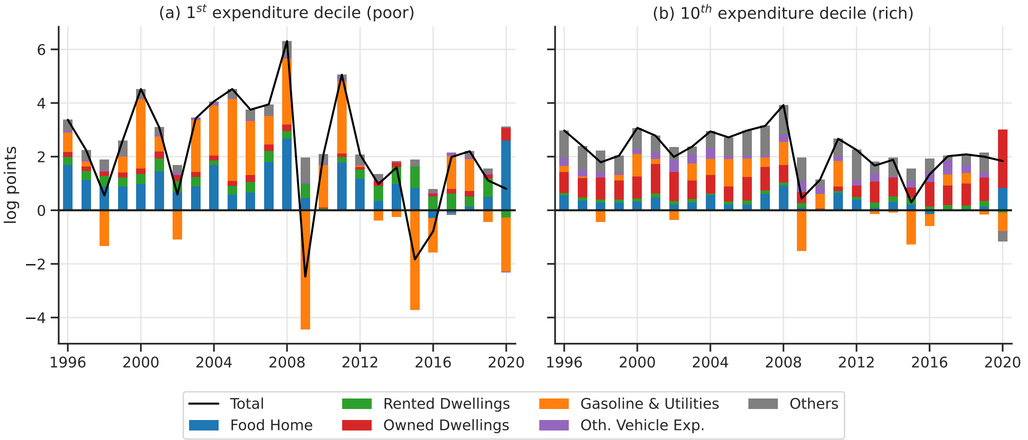
Top-bottom difference: **0.06 pp / year** vs **0.36 pp / year**

Where do the inflation differences ultimately come from?

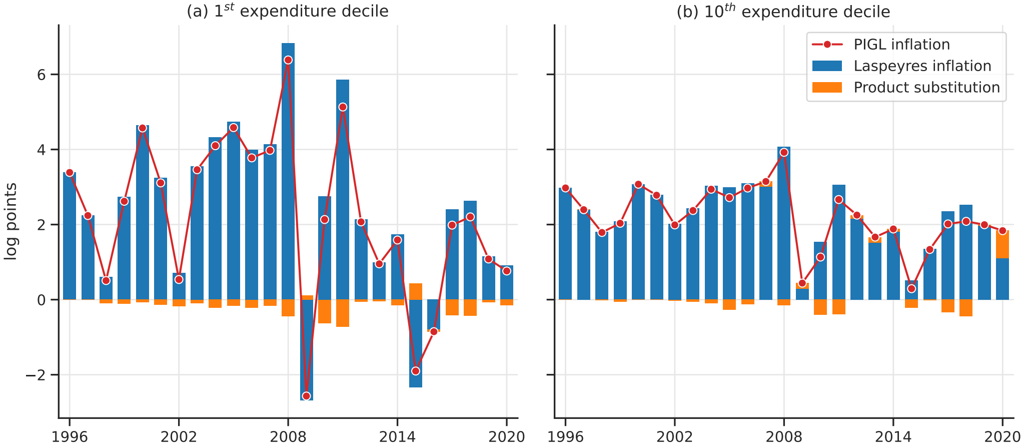


Reference baskets and **substitution behaviour** differ across the expenditure distribution!

Low-income consumers have higher exposure to food and energy . . .



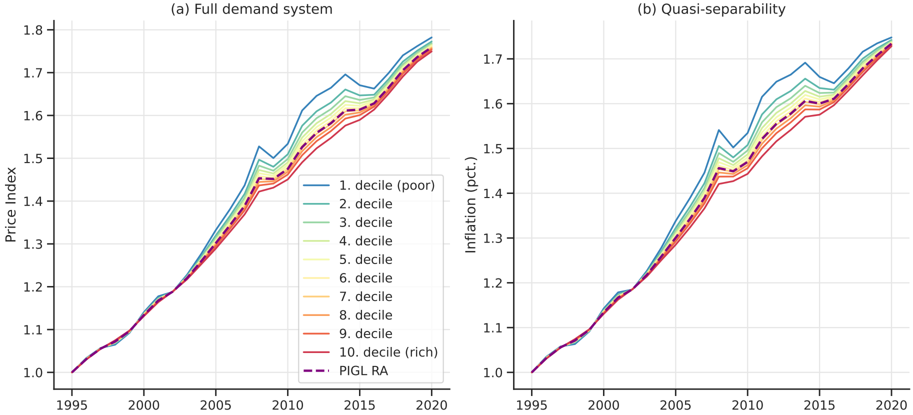
. . . meanwhile, product substitution does not matter



Robustness: estimation of fully parametrized demand model

Generates near-identical inflation response

Parametrization: $B(\boldsymbol{p}) = \left(\sum_j \omega_j p_j^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \quad D(\boldsymbol{p}) = \left(\sum_j \theta_j p_j^{1-\varphi}\right)^{\frac{1}{1-\varphi}}$



- Theoretical framework
- Empirical Implementation
- Empirical Results

Conclusion

Measuring cost-of-living changes under nonhomothetic preferences does not require a fully parametrized and estimated consumer demand model

Nonhomothetic cost-of-living index

PIGL preferences between necessities or luxuries

Generalizes standard homothetic indices

Easy implementation

Separability between necessities and luxuries

Consistent aggregation: can use macro data

Measuring cost-of-living changes under nonhomothetic preferences does not require a fully parametrized and estimated consumer demand model

Implications

Heterogeneous inflation

$\Longrightarrow$ heterogeneous real rates

monetary policy transmission?

Under the hood

2.5 times higher inflation

volatility for the poor from

exposure to food, energy, utilities

Empirically

1995–2020: no differences

2004–2015: 0.37 pp/year

between top and bottom

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Thank you!

Appendix

Nonhomothetic CES as a special case when $\varepsilon = \gamma$

Parametrization:

$$B(\boldsymbol{p}) = \left(\sum_j \omega_j p_j^{\varepsilon}\right)^{\frac{1}{\varepsilon}}, \qquad D(\boldsymbol{p}) = \left(\sum_j \theta_j p_j^{\varepsilon}\right)^{\frac{1}{\varepsilon}}$$

Cost function:

$$c(u, \boldsymbol{p}) = \left[\sum_{j \in J} \Big(\theta_j \nu + \omega_j (1 - \nu + \varepsilon u)\Big) p_j^{\varepsilon}\right]^{\frac{1}{\varepsilon}}$$

Expenditure shares:

$$w_j = \omega_j \left(\frac{p_j}{B(\boldsymbol{p})}\right)^{\varepsilon} + \left[\theta_j \left(\frac{p_j}{D(\boldsymbol{p})}\right)^{\varepsilon} - \omega_j \left(\frac{p_j}{B(\boldsymbol{p})}\right)^{\varepsilon}\right] \nu \left(\frac{D(\boldsymbol{p})}{e}\right)^{\varepsilon}$$

Price index:

$$P(u, \boldsymbol{p}_s, \boldsymbol{p}_t) = \left[\sum_{j \in J} w_{js} \left(\frac{p_{jt}}{p_{js}}\right)^{\varepsilon}\right]^{\frac{1}{\varepsilon}}$$

Decomposition of the PIGL cost-of-living index

$$\frac{P(u,\boldsymbol{p}_s,\boldsymbol{p}_t)}{P(u,\boldsymbol{p}_s,\boldsymbol{p}_{t-1})} = \left(\frac{P_{Dt}}{P_{Dt-1}}\right)^{\frac{\gamma\phi_t}{\varepsilon}} \left(\frac{P_{Bt}}{P_{Bt-1}}\right)^{1-\frac{\gamma\phi_t}{\varepsilon}}, \qquad \phi_t = \frac{L\left(w^h_{Dt}, w^h_{Dt-1}\right)}{L\left(w^h_{Dt}, w^h_{Dt-1}\right) + L\left(\frac{\gamma}{\varepsilon} - w^h_{Dt}, \frac{\gamma}{\varepsilon} - w^h_{Dt-1}\right)}$$

w^h_{Dt} is the Hicksian necessity expenditure share associated with the reference utility level, $L(\cdot,\cdot)$ is the logarithmic mean:

$$w^h_{Dt} = w_{Ds}\left(\frac{P_{Dt}^{\frac{\gamma}{\varepsilon}} P_{Bt}^{1-\frac{\gamma}{\varepsilon}}}{P(u,\boldsymbol{p}_s,\boldsymbol{p}_t)}\right)^{\varepsilon} \quad \text{and} \quad L(x,y) = \begin{cases} \frac{x-y}{\ln x - \ln y} & \text{if } x \neq y, \\ x & \text{if } x = y. \end{cases}$$

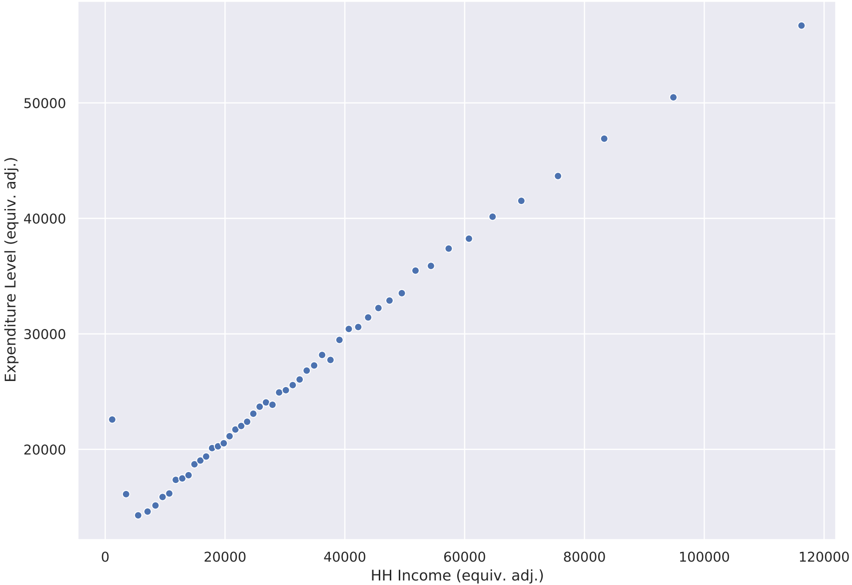
The basket price indices use prices p_j and within-basket expenditure shares w^C_j in standard index formulae, for example Sato-Vartia:

$$\frac{P_{Ct}}{P_{Ct-1}} = \prod_{j\in J_C}\left(\frac{p_{jt}}{p_{jt-1}}\right)^{L(w^C_{jt}, w^C_{jt-1}) \,/\, \sum_i L(w^C_{it}, w^C_{it-1})}, \qquad C \in \{B, D\}.$$

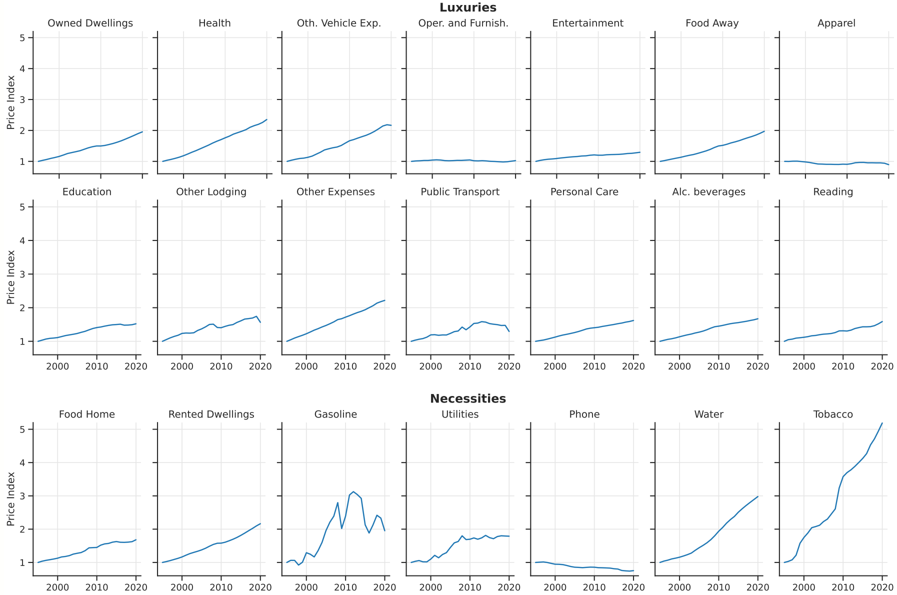
Consumption Expenditure Survey (CEX), 1995–2020

- Focus on households:
  - between 25 and 65 years old
  - 4/4 quarters and full income reporter
  - positive household income
- Exclude durable goods
- Rent equivalence to impute service flows from owned housing
- Equivalence adjustment of expenditures and income
- Aggregation into 21 goods follows Hobijn and Lagakos (2005)
- Produce annual aggregate of group expenditures for each household
- Households enter/exit the panel each month
- Aggregate monthly price level for each expenditure group:
  - Expenditure weighted annual average of monthly price index
  - Household specific

Income against consumption expenditures in the CEX data

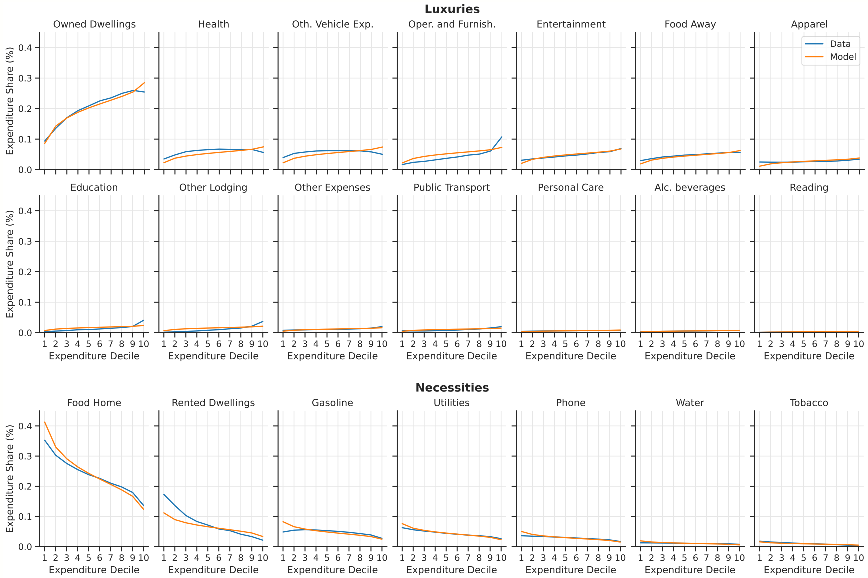


Individual CPI series



1. Classify individual goods as necessity or luxury

Implementation captures non-constant Engel curves



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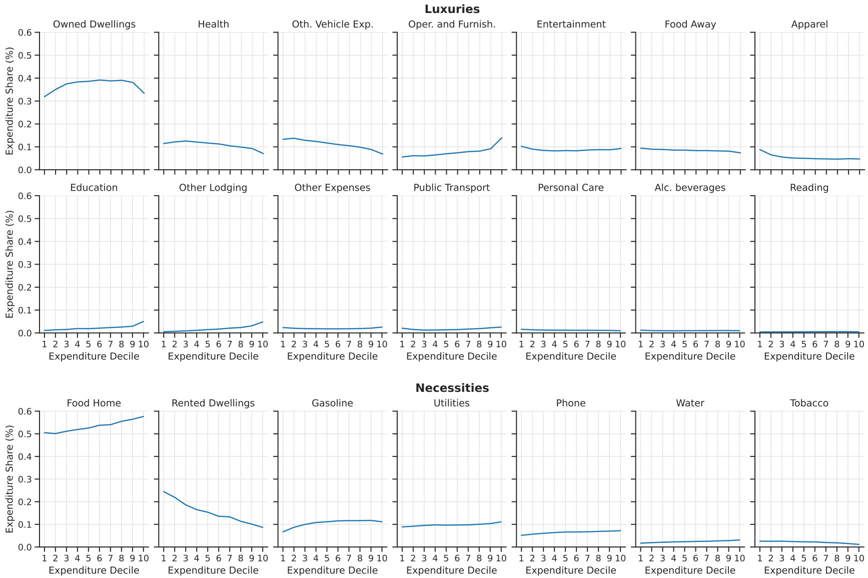
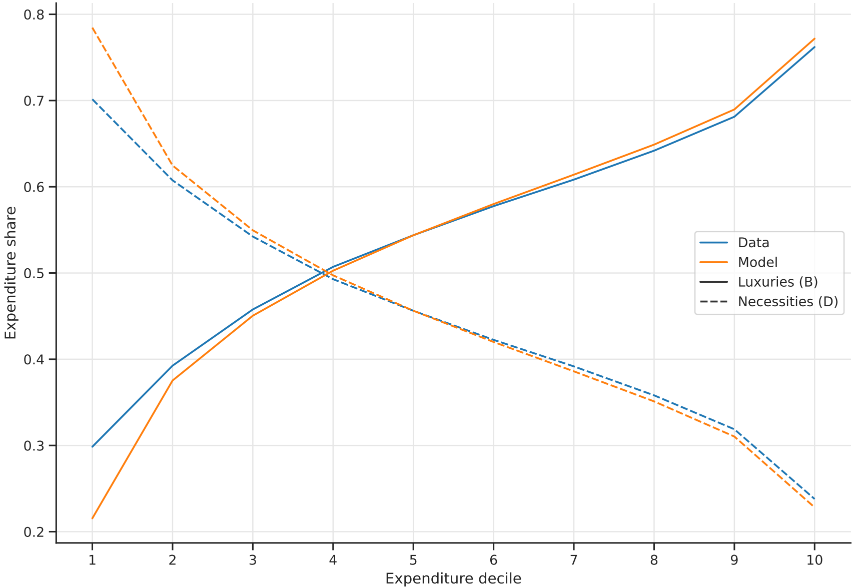
Expenditure shares within luxuries and necessities



Expenditure shares across luxuries and necessities



Formal classification into necessities and luxuries

Regression equation

$$w_{jh} = \alpha_j + \beta_j \mathbf{d}_h + \epsilon_{jh}$$

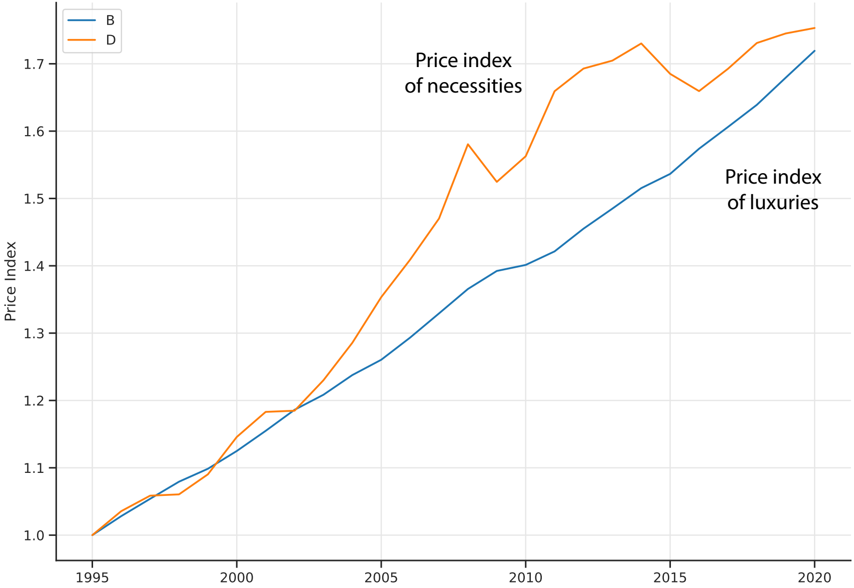
w_{jh} budget share on product j for household h

$\mathbf{d}_h$ expenditure decile of household h

Classification	Goods category	Marginal effect (in p.p.)	SE
Luxuries	Owned dwellings	1.757	(0.019)
	Household operations and furnishing	0.690	(0.010)
	Entertainment	0.386	(0.006)
	Other lodging	0.303	(0.005)
	Food away from home	0.299	(0.005)
	Education	0.280	(0.006)
	Health	0.215	(0.007)
	Public transport	0.137	(0.003)
	Other vehicle expenses	0.124	(0.006)
	Other expenses	0.105	(0.002)
	Apparel	0.098	(0.004)
	Alcoholic beverages	0.044	(0.001)
	Personal care	0.031	(0.001)
	Reading	0.028	(0.001)
Necessities	Food at home	−2.031	(0.012)
	Rented dwellings	−1.496	(0.017)
	Utilities	−0.362	(0.004)
	Gasoline	−0.219	(0.004)
	Phone	−0.186	(0.003)
	Tobacco	−0.158	(0.003)
	Water	−0.045	(0.002)
Expenditure category dummies		Yes	
Observations		1,562,211	
Adjusted R^2		0.579	

2. Construct prices for necessities and luxuries

Increasing relative price of necessities



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3. Estimate parameters ε, γ

- Estimate ε, γ from expenditure share on necessities:

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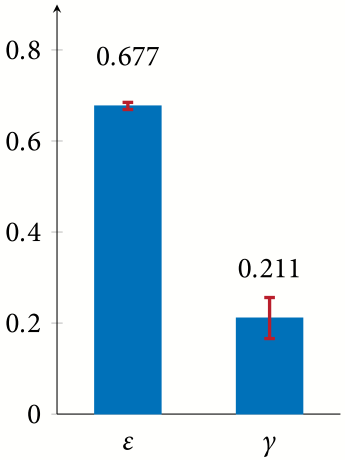
Homotheticity rejected: $\varepsilon > 0$



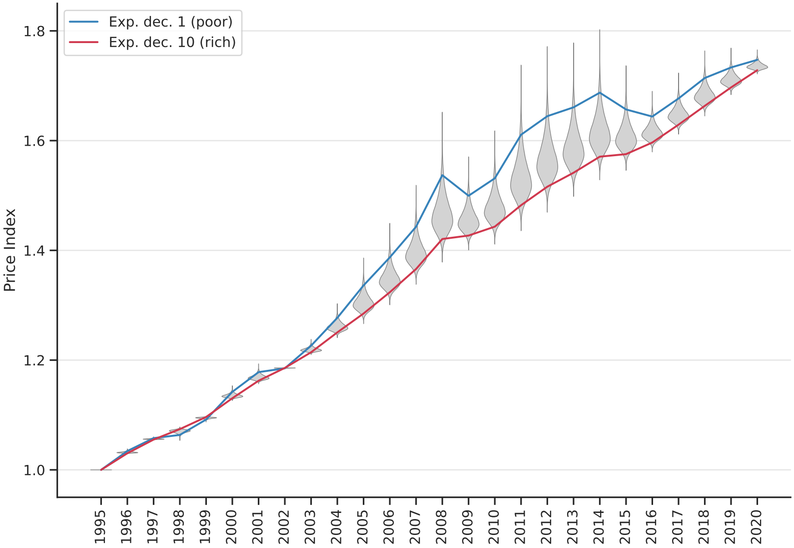
Parameter estimates for different price index formulae

Table 1. GMM estimates of the PIGL parameters under quasi-separability.

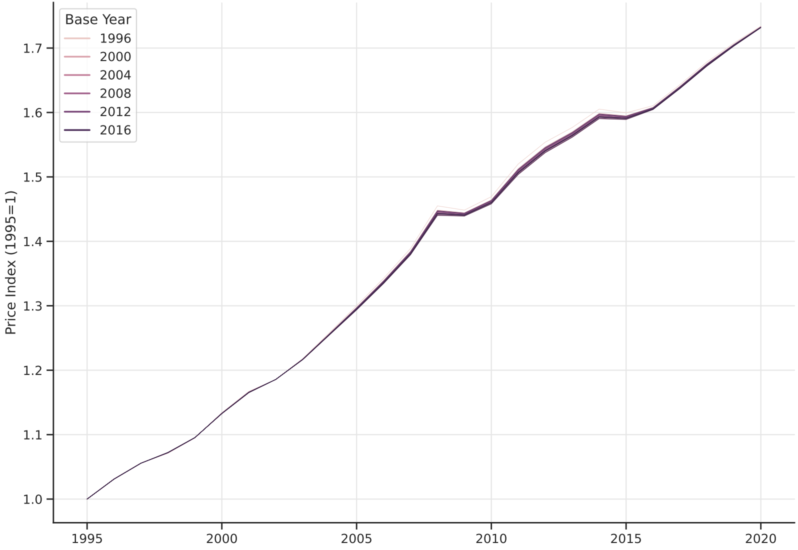
	Sato-Vartia	Törnqvist	Geometric Walsh	Theil	Fisher's Ideal	Implicit Walsh
ε	0.677	0.677	0.677	0.677	0.677	0.677
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
γ	0.211	0.211	0.211	0.211	0.211	0.211
	(0.023)	(0.023)	(0.023)	(0.023)	(0.023)	(0.023)
$\widetilde{\nu}$	327.271	327.437	327.173	327.273	324.273	324.600
	(13.358)	(13.365)	(13.354)	(13.358)	(13.217)	(13.233)
Observations	74,372	74,372	74,372	74,372	74,372	74,372
RMSE	0.1487	0.1486	0.1487	0.1487	0.1486	0.1486

Notes. Robust standard errors in parentheses. "RMSE" refers to the root-mean-square error of the expenditure share on the D good: $\sqrt{\sum_h (w_{Dh} - \widehat{w}_{Dh})^2 / N}$. Observations are weighted by their CEX sampling weights.

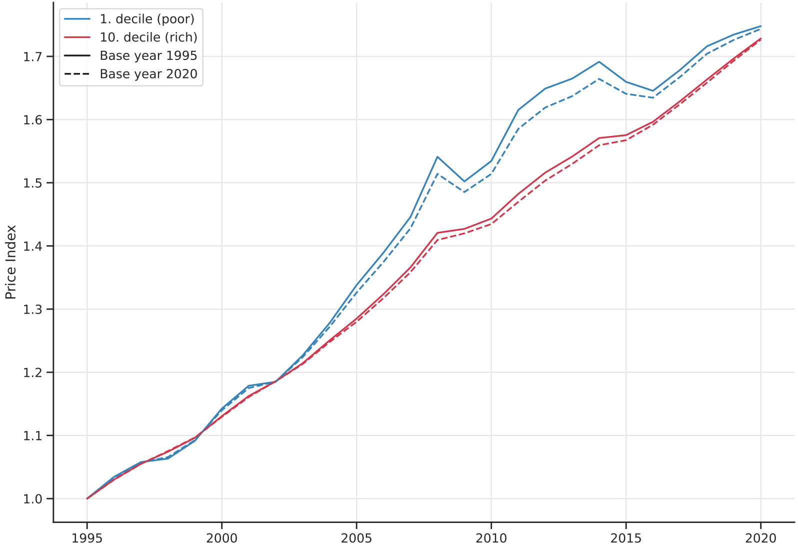
Full price index distribution



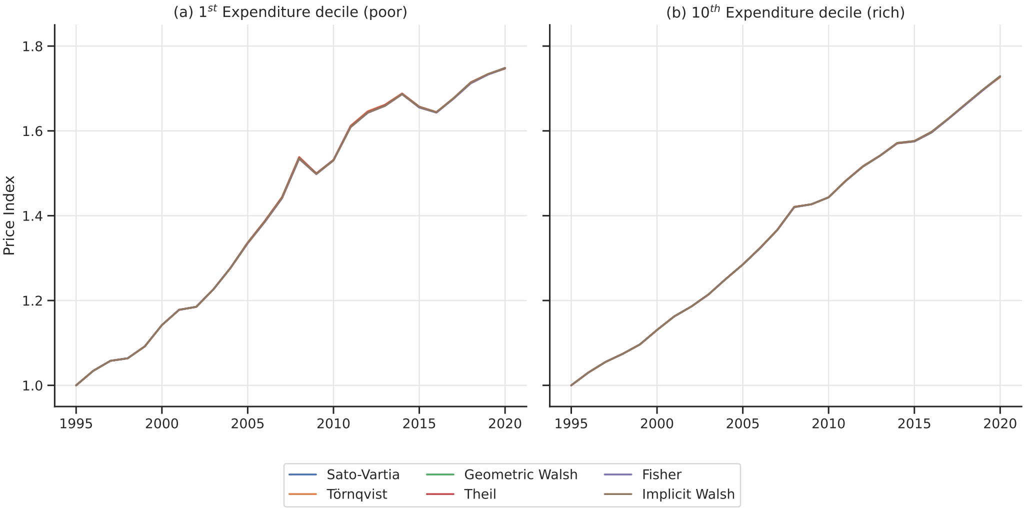
Different base years: representative agent



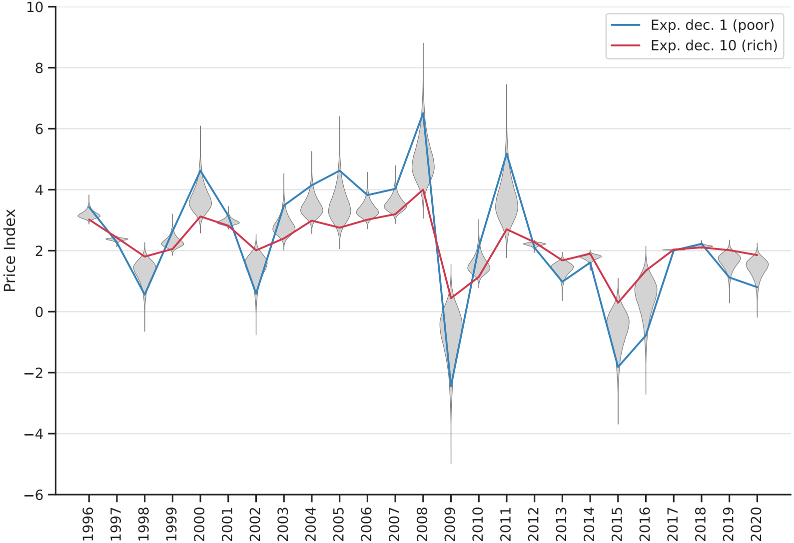
Different base years: top vs. bottom expenditure deciles



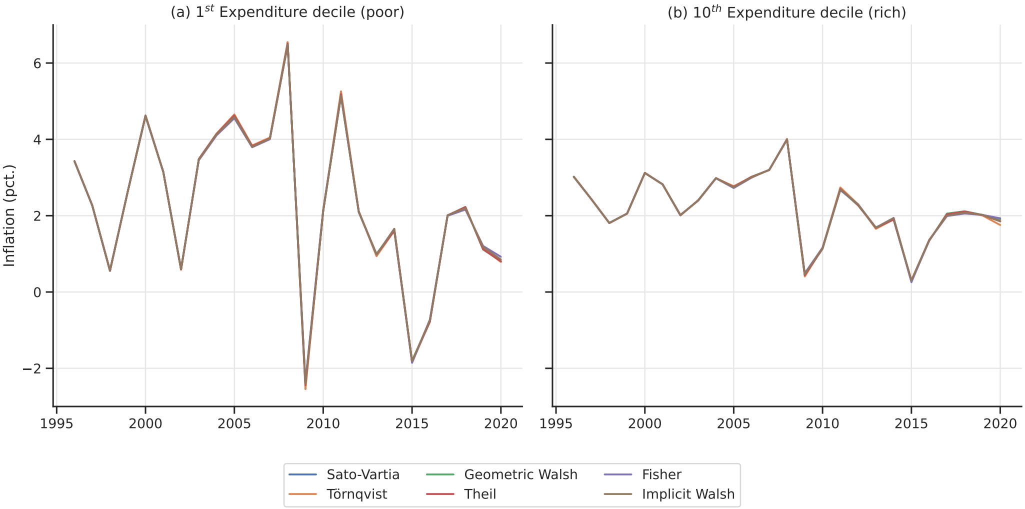
Cost-of-living indices for different superlative price index formulae



Full inflation distribution



Inflation for different superlative price index formulae



Inflation rate levels and dispersion across the distribution

Table 2. Inflation rate levels and dispersion over time across the expenditure distribution.

	1996–2020				2004–2015			
	Level		Dispersion		Level		Dispersion	
Decile	Mean (%)	Relative to top decile (pp. diff.)	Standard deviation (%)	Relative to top decile (x Std Dev)	Mean (%)	Relative to top decile (pp. diff.)	Standard deviation (%)	Relative to top decile (x Std Dev)
1	2.28	0.06	2.14	2.51	2.57	0.37	2.71	2.38
2	2.26	0.04	1.76	2.06	2.46	0.26	2.25	1.98
3	2.25	0.03	1.59	1.86	2.41	0.21	2.05	1.79
4	2.24	0.03	1.46	1.71	2.38	0.18	1.89	1.66
5	2.24	0.02	1.36	1.60	2.35	0.15	1.78	1.56
6	2.24	0.02	1.28	1.50	2.33	0.13	1.67	1.46
7	2.23	0.02	1.20	1.40	2.30	0.10	1.57	1.38
8	2.23	0.01	1.12	1.31	2.28	0.08	1.47	1.29
9	2.22	0.01	1.02	1.20	2.25	0.05	1.35	1.18
10	2.22	0.00	0.85	1.00	2.20	0.00	1.14	1.00

Notes. Arithmetic mean and standard deviation of annual inflation over the time periods 2004–2015 and 1996–2020. Under inflation rate levels, the "Relative to top decile" columns show the percentage point difference in the average annual inflation rate to that of the tenth expenditure decile. Under inflation rate dispersion, the same columns show the standard deviation of annual inflation as a multiple of that of the tenth expenditure decile.

Decomposing the index into reference basket and product substitution

$$\ln P_t = \sum_{j \in J} w_{jt}^L \ln\left(\frac{p_{jt}}{p_{js}}\right) + \sum_{j \in J} \left(\chi_{jt} - w_{jt}^L\right) \ln\left(\frac{p_{jt}}{p_{js}}\right)$$

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- PIGL decomposition weights

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- PIGL decomposition weights
- Laspeyres weights with fixed base-period quantities

Decomposing the index into reference basket and product substitution

$$\ln P_t = \underbrace{\sum_{j \in J} w_{jt}^{L} \ln\left(\frac{p_{jt}}{p_{js}}\right)}_{\text{Laspeyres price index}} + \underbrace{\sum_{j \in J} \left(\chi_{jt} - w_{jt}^{L}\right) \ln\left(\frac{p_{jt}}{p_{js}}\right)}_{\text{Product substitution}}$$

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- Laspeyres weights with fixed base-period quantities

Decomposing the index into reference basket and product substitution

$$\ln P_t = \underbrace{\sum_{j \in J} w^L_{jt} \ln\left(\frac{p_{jt}}{p_{js}}\right)}_{\text{Laspeyres price index}} + \underbrace{\sum_{j \in J} \left(\chi_{jt} - w^L_{jt} \right) \ln\left(\frac{p_{jt}}{p_{js}}\right)}_{\text{Product substitution}}$$

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- Laspeyres weights with fixed base-period quantities

Laspeyres weights

The Laspeyres price index keeps **quantities** q_j fixed to some base period s:

$$P_t^L \equiv \frac{\sum_{j\in J} p_{jt} q_{js}}{\sum_{j\in J} p_{js} q_{js}}$$

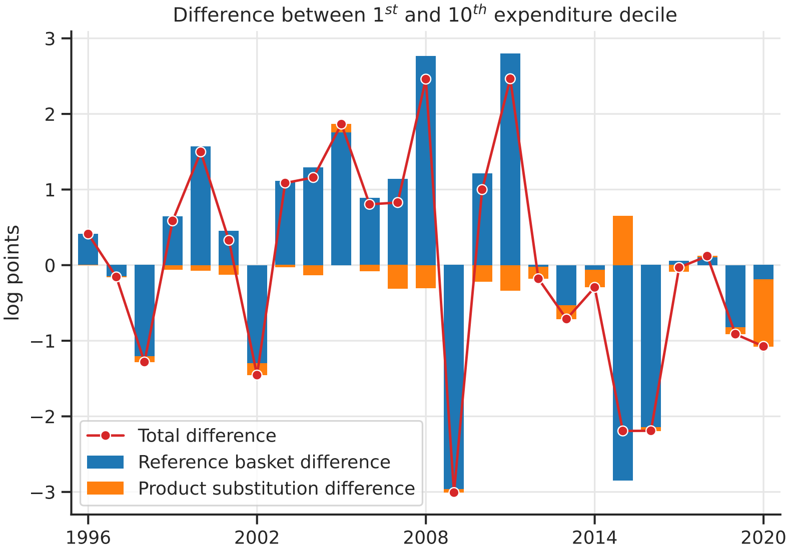
A bit of algebra allows us to rewrite this as

$$P_t^L = \prod_{j\in J} \left(\frac{p_{jt}}{p_{js}}\right)^{w_{jt}^L},$$

where

$$w_{jt}^L \equiv \frac{w_{js}\, L\left(\frac{p_{jt}}{p_{js}},\ P_{Lt}\right)}{\sum_{i\in J} w_{is}\, L\left(\frac{p_{jt}}{p_{js}},\ P_{Lt}\right)}.$$

Product substitution: difference between top and bottom deciles



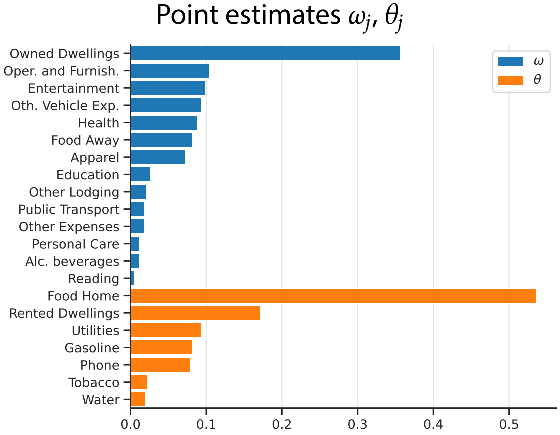
Full demand system parameter estimates

Parametrization: $B(\boldsymbol{p}) = \left(\sum_j \omega_j p_j^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \quad D(\boldsymbol{p}) = \left(\sum_j \theta_j p_j^{1-\varphi} \right)^{\frac{1}{1-\varphi}}$

Table 3. GMM estimates of the preference parameters.

	Quasi-separability (Sato-Vartia)	Full demand system (CES)
ε	0.677 (0.004)	0.685 (0.004)
γ	0.211 (0.023)	0.505 (0.018)
$\widetilde{\nu}$	327.271 (13.358)	346.736 (11.793)
σ		0.050 (0.012)
φ		0.360 (0.006)
Observations	74,372	74,372
RMSE	0.1487	0.1494

Notes. Robust standard errors in parentheses. "RMSE" refers to the root-mean-square error of the expenditure share on the D good: $\sqrt{\sum_h (w_{Dh} - \widehat{w}_{Dh})^2 / N}$. Observations are weighted by their CEX sampling weights.



Full demand system estimation: inflation rates