

A Firm Link:
Overall, Between- and Within-Firm Inequality
Through the Lens of a Sorting Model

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Empirical Trends: US

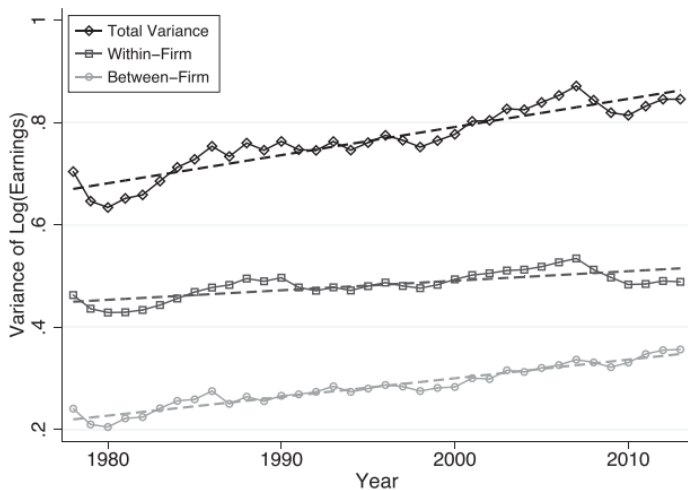


Figure: Decomposition of log annual earnings variance in US 1978-2013 (from Song et al. (2018))

Empirical Trends: Norway

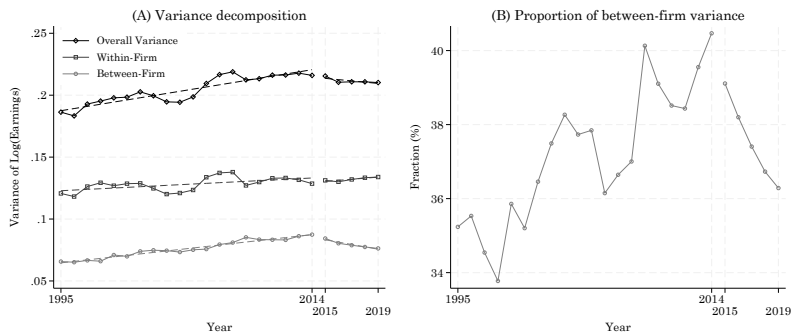


Figure: Decomposition of log annual earnings variance in Norway (1995-2019)

Research Questions

1. What types of technological change can increase wage inequality predominantly through its between-firm component?
 - ▶ Can skill-biased technological change account for it?
 - ▶ **Challenge:** Many standard models either have no within-firm inequality or no notion of firm
 - ▶ Need for a new model!
2. What do these new stylised facts imply for the evolution of welfare inequality?
 - ▶ Sorkin (2018) finds that up to 70% of wage premia variance caused by compensating differentials
 - ▶ **A priori** possible that much of the increase in wage inequality does not reflect increases in welfare inequality
 - ▶ Again, need a model!

Model Preview

Model builds on Costrell and Loury (2004) and Eeckhout and Kircher (2018)

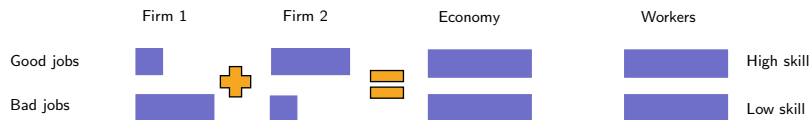
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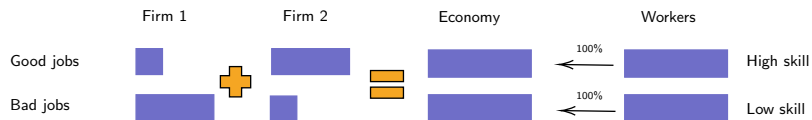
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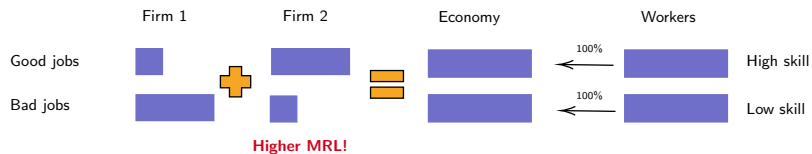
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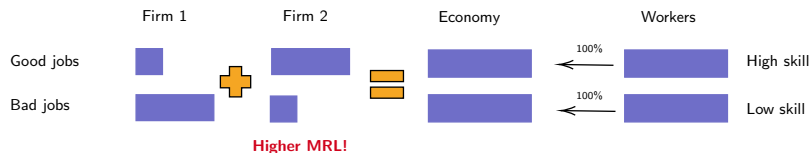
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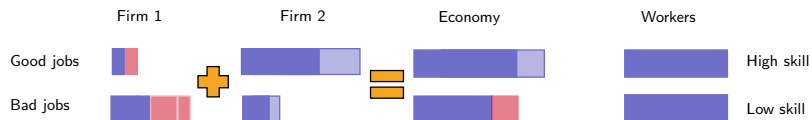
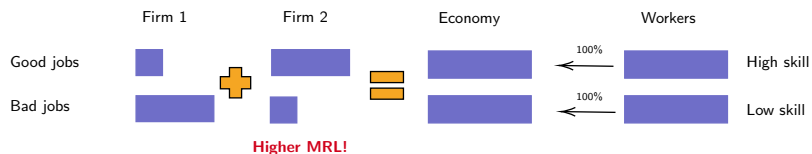
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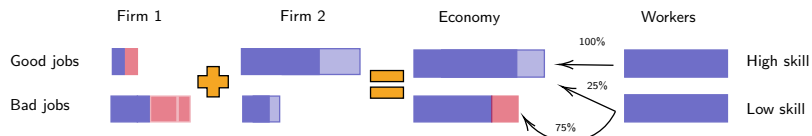
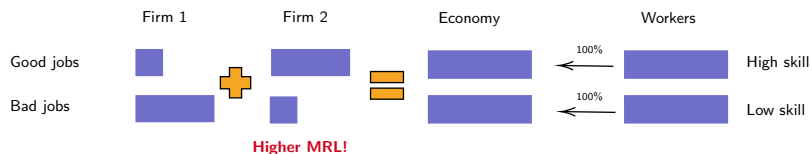
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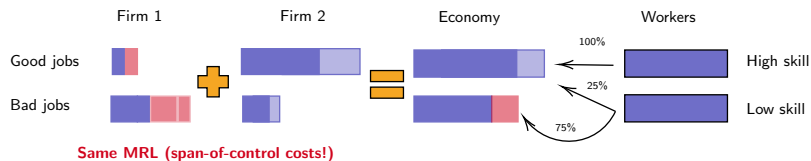
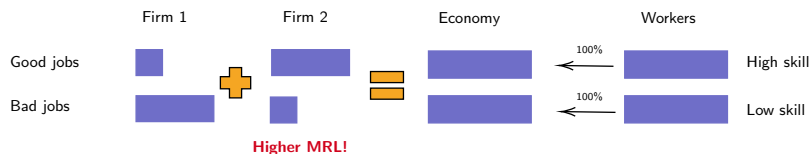
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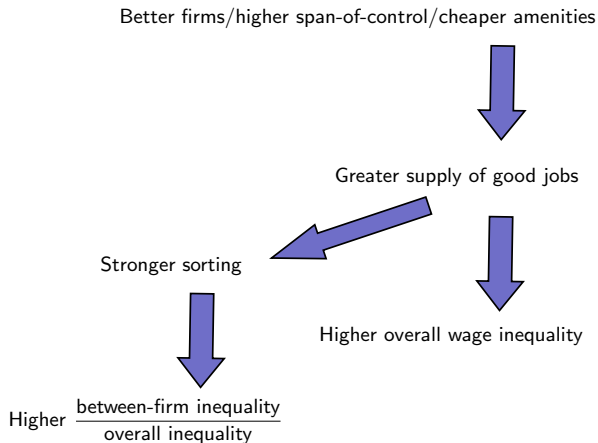


- ▶ Continuum of jobs, skills and firm → **very complicated fixed point problem**
- ▶ Yet, tractable because normal mixture of normals is normal

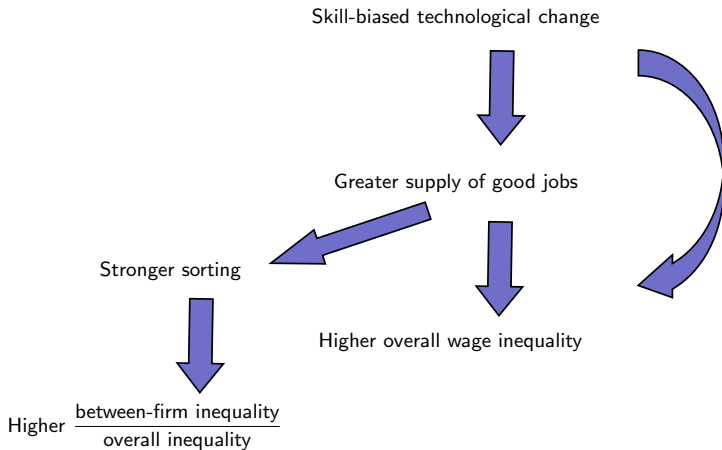
Model

Solution

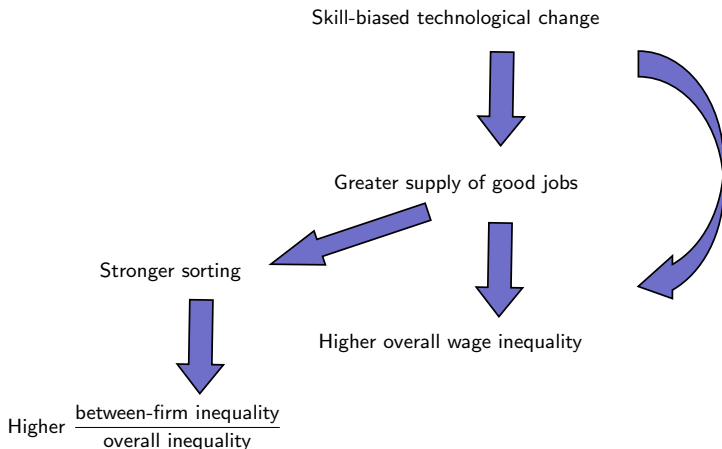
Results Preview



Results Preview



Results Preview



- ▶ Change in any of our parameters moves wage inequality and $\frac{\text{between-firm inequality}}{\text{overall inequality}}$ in the same direction
- ▶ Could move in opposite directions only if multiple parameters change simultaneously

Calibration Exercise Results

We calibrate the model using Norwegian administrative data for 1995-2019.

Data

We calibrate the full model (with effort provision and firm-level amenities) using five moments: average wages, variance of wages, within-firm inequality (weighted and unweighted by firm size) and the variance of within-firm variances.

Identification

Calibration

Calibration Results

1. Wage Inequality:

- ▶ 1995-2014: The increase in wage inequality mostly driven by falling span-of-control costs (62%) and SBTC (35%)
- Wage Inequality 1995-2014
- ▶ Post-2015: Decreased variance of firm productivity and increased amenity dominated SBTC and falling span-of-control costs
- Wage Inequality Post-2015

2. Welfare Inequality:

- ▶ 1995-2014: Welfare inequality increases, but much less than wage inequality. Main driver: SBTC (52%)
- Welfare Inequality 1995-2014
- ▶ Post-2015: Welfare inequality **continues to increase**. The decrease in wage inequality mostly reflects changes in compensating differentials.

Welfare Inequality Post-2015

References

Theoretical Sorting Models:

- ▶ Costrell and Loury (2004), Eeckhout and Kircher (2018), Sattinger (1975), Teulings (1995), Teulings (2005), Costinot (2009), Costinot and Vogel (2010)

Theory on Between- and Within-Firm Inequality:

- ▶ Boerma et al. (2023), Freund (2022), Trottner (2022), Cortes et al. (2023), Eeckhout and Pinheiro (2014)

Empirical Literature on Between-Firm Inequality:

- ▶ Rising influence: Tomaskovic-Devey et al. (2020), Barth et al. (2016), Barth et al. (2018), Song et al. (2018), Sorkin and Wallskog (2023), Haltiwanger et al. (2022), Haltiwanger et al. (2023), Håkanson et al. (2021), Card et al. (2013), Baumgarten et al. (2020), Freund (2022), Lochner and Schulz (2022), Helpman et al. (2016)
- ▶ Recent decreases: Aeppli and Wilmers (2022), Engbom et al. (2023), Lochner et al. (2020), Freund (2022), Landefeld et al. (2023)

Amenities and Inequality:

- ▶ Sorkin (2018), Ouimet and Tate (2023), Kristal et al. (2020), Sockin (2022), Bana et al. (2023), Maestas et al. (2023), Bagger et al. (2021), Taber and Vejlin (2020)

Model

Workers:

- ▶ Unit measure of workers with normally distributed skill x (mean 0, variance σ_x)
- ▶ Utility: $u(w, a) = \frac{w}{e}a$, where w is wage per unit effort e , and a is amenities provided by the firm
- ▶ Reservation utility: 0

Firms:

- ▶ Unit measure of profit-maximizing firms with normally distributed productivity θ (mean 0, variance σ_θ)
- ▶ Firm: collection of jobs h , each filled by a single worker. Distribution of jobs in firm $\theta: N(\theta, 1)$.
- ▶ Worker's contribution to output: $q(x, h, e) = A\sqrt{\exp(x(h))}e$
- ▶ Span-of-control cost: $C_l(L) = L^{1+c_l}$
- ▶ Amenity provision cost: $C_a(a, L) = \frac{L}{c_a} \left(\frac{c_a a}{1+c_a} \right)^{1+c_a}$
- ▶ Total output of firm θ :
 $Q(L, m, \theta, e) = L \int_{-\infty}^{\infty} q(m(h), h, e(m(t)))d\Phi(h - \theta) - C_l(L) - C_a(a, L)$

Firm's Problem and Equilibrium

Firm's problem:

- ▶ Choose number and types of workers, amenities level and the assignment of workers to jobs, to maximise profit
- ▶ Profit: $r(\theta) = Q(L, m, \theta, e(m)) - L \int_{-\infty}^{\infty} \frac{u(m(h))e}{a} d\Phi(t)$
- ▶ Demand for skill x : $D(x) = \int_{-\infty}^{\infty} L^*(\theta) \Pr(m^*(T) \geq x) d\Phi(\theta)$
- ▶ Supply of skill x : $S(x) = 1 - \Phi(x/\sigma_x)$

Equilibrium with full employment:

- ▶ Firms maximise profits given equilibrium utilities
- ▶ Markets clear: $D(x) = S(x)$
- ▶ Equilibrium wage: $w(x, \theta) = \frac{u(x)e^*(\theta)}{a^*(\theta)}$

Solution Strategy

- ▶ Guess that economy-wide job distribution is normal: $F(h) = \Phi(\frac{h}{\sigma})$, where $\sigma > \max\{\sigma_x, 1\}$
- ▶ Solve for equilibrium utility functions given this guess Eqm utility
- ▶ Derive firm size choices resulting from these equilibrium utilities Firm size
- ▶ Show that these firm size choice result in normally distributed jobs and solve for the equilibrium variance of jobs σ^2 (supply of quality jobs)
Equilibrium Model Preview

Effort Exertion, Assignment, and Utility

- ▶ First-order condition for effort schedule e : $e^*(\mu(h), \theta) = \frac{A^2 \exp(\mu(h)h)a^2}{4(u(\mu(h), \theta))^2}$
 - ▶ Firms offering higher amenities require disproportionately higher effort and compensate with higher wages
- ▶ Solve the assignment problem: $\frac{u'(\mu(h, \theta))}{u(\mu(h, \theta))} = h$
 - ▶ Optimal assignment $\mu(\cdot; \theta)$ is independent of θ
 - ▶ Market clearing implies $\mu(h) = \frac{\sigma_x}{\sigma} h$
- ▶ Equilibrium utility: $\ln(u(x)) = \frac{\sigma}{\sigma_x} \frac{x^2}{2} + \ln(u(0))$

Solution

Amenities and Firm Size

- ▶ Firm θ 's profit:

$$r(\theta) = \max_{L,a} L \left(\frac{aA^2}{4u(0)} \frac{\exp\left(\frac{\theta^2}{2\left(\frac{\sigma}{\sigma_x}-1\right)}\right)}{\sqrt{1-\frac{\sigma_x}{\sigma}}} - \frac{1}{c_a} \left(\frac{c_a a}{1+c_a}\right)^{1+c_a} - L^{c_l} \right)$$

- ▶ Optimal amenities: $a^*(\theta) = \left(\frac{1}{c_a} + 1\right) \left(\frac{A^2}{4u(0)} \frac{\exp\left(\frac{\theta^2}{2\left(\frac{\sigma}{\sigma_x}-1\right)}\right)}{\sqrt{1-\frac{\sigma_x}{\sigma}}}\right)^{\frac{1}{c_a}}$

- ▶ Optimal firm size: $L^*(\theta) = \frac{\exp\left[\alpha\left(\frac{\theta^2}{2\left(\frac{\sigma}{\sigma_x}-1\right)}+2\ln A-0.5\ln\left(1-\frac{\sigma_x}{\sigma}\right)-\ln(4u(0))\right)\right]}{(1+c_l)^{\frac{1}{c_l}}}$,

where $\alpha \equiv \frac{1+c_a}{c_l c_a}$

- ▶ Firm size depends exponentially on the square of firm productivity

Solution

Job Distribution and Equilibrium

- ▶ Density of equilibrium job distribution:

$$f(h) = \frac{L^*(0)}{\sqrt{1 + \sigma_\theta^2 \left(1 - \frac{\alpha}{\sigma/\sigma_x - 1}\right)}} \phi \left(h \sqrt{\frac{1}{1 + \frac{\sigma_\theta^2}{1 - \frac{\alpha \sigma_\theta^2}{\sigma/\sigma_x - 1}}}} \right)$$

- ▶ Equilibrium conditions:

- ▶ $1 = \frac{L^*(0)}{\sqrt{1 - \frac{\alpha \sigma_\theta^2}{\sigma/\sigma_x - 1}}}$

- ▶ $\sigma^2 = \frac{\sigma_\theta^2}{1 - \frac{\alpha \sigma_\theta^2}{\sigma/\sigma_x - 1}} + 1$

- ▶ Solving for the equilibrium variance of jobs σ^2 :

- ▶ Define $t^{-1}(\sigma; \sigma_x, \sigma_\theta) \equiv \left(\frac{\sigma}{\sigma_x} - 1\right) \left(\frac{1}{\sigma_\theta^2} - \frac{1}{\sigma^2 - 1}\right)$

- ▶ $t(\alpha; \sigma_x, \sigma_\theta) = \sigma$ gives the standard deviation of the equilibrium distribution of jobs

- ▶ Utility level ensuring firms hire a measure one of workers:

$$\ln(u(0)) =$$

$$2 \ln(A) - 0.5 \ln\left(1 - \frac{\sigma_x}{\sigma}\right) - \ln 4 - \frac{1}{\alpha} \left(\ln(\sigma_\theta) - 0.5 \ln(\sigma^2 - 1)\right) - \frac{c_a}{1+c_a} \ln(1 + c_l)$$

Supply of Quality Jobs

$$\frac{d}{dc_a} \sigma = \frac{\partial \alpha}{\partial c_a} \frac{\partial}{\partial \alpha} t(\alpha; \sigma_x, \sigma_\theta) = - \frac{1}{c_l c_a^2} \frac{1}{\frac{\partial}{\partial \sigma} t^{-1}(\sigma; \sigma_x, \sigma_\theta)} < 0 \quad (1)$$

$$\frac{d}{d\sigma_x} \sigma = - \underbrace{\frac{\partial}{\partial \sigma_x} t^{-1}(\sigma; \sigma_x, \sigma_\theta)}_{<0} \underbrace{\frac{1}{\frac{\partial}{\partial \sigma} t^{-1}(\sigma; \sigma_x, \sigma_\theta)}}_{>0} > 0, \quad (2)$$

$$\frac{d}{d\sigma_\theta} \sigma = - \underbrace{\frac{\partial}{\partial \sigma_\theta} t^{-1}(\sigma; \sigma_x, \sigma_\theta)}_{<0} \underbrace{\frac{1}{\frac{\partial}{\partial \sigma} t^{-1}(\sigma; \sigma_x, \sigma_\theta)}}_{>0} > 0. \quad (3)$$

► Intuition:

- Changes in σ_x , c_a , and c_l cause all firms to hire more workers in partial equilibrium
 - Larger firms expand more due to the curvature of cost functions
- In general equilibrium, low-productivity firms contract and high-productivity firms expand, improving job quality
- Increase in σ_θ directly increases the number of quality jobs by increasing the number of high-productivity firms

Sorting

Sorting between workers and firms:

$$\text{Corr}(\theta^2, x^2) = 1 - \frac{1}{\sigma^2}$$

- ▶ Strength of sorting depends positively and exclusively on the supply of quality jobs [Result preview](#)

Intuition:

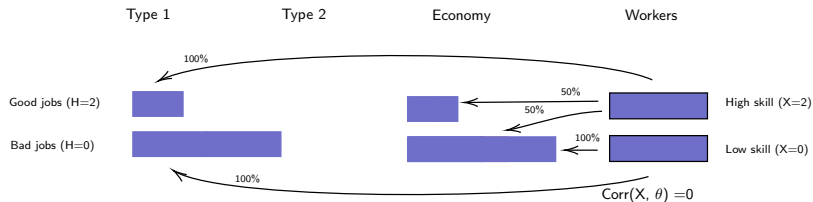
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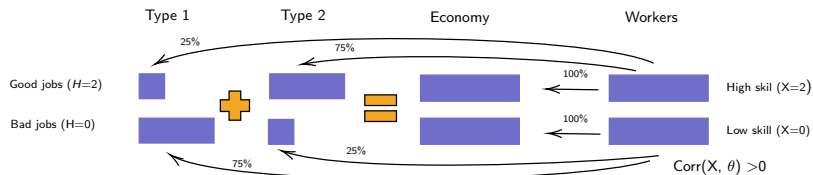
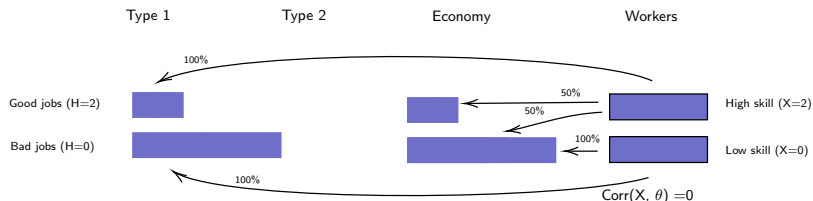
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Wage and Welfare Inequality

Suppose that $c_a \rightarrow \infty$, so that

$$\ln w(x) = \ln u(x) = \frac{\sigma}{2\sigma_x} x^2 + \ln u(0) \quad E(X^2|\theta^2) = \left(\frac{\sigma_x}{\sigma}\right)^2(1 + \theta^2)$$

so that

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From the law of total covariance we have that

$$\text{Var}_\theta \left(E(\ln w(x)|\theta^2) \right) = \text{Cov} \left(E(\ln w(x)|\theta^2), \ln w(x) \right),$$

and thus

$$\begin{aligned} \frac{\text{Var}_\theta \left(E(\ln w(x)|\theta^2) \right)}{\text{Var}_x(w(x))} &= \frac{\text{Var}_\theta \left(E(\ln w(x)|\theta^2) \right)^2}{\text{Var}_x(w(x)) \text{Var}_\theta \left(E(\ln w(x)|\theta^2) \right)} \\ &= \text{Corr} \left(E(\ln w(x)|\theta^2), \ln w(x) \right)^2. \end{aligned}$$

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$$\frac{\text{Var}_\theta \left(E(\ln w(x)|\theta^2) \right)}{\text{Var}_x (w(x))} = \text{Corr}(\theta^2, x^2)^2 = \left(1 - \frac{1}{\sigma^2}\right)^2.$$

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Overall welfare inequality: $\text{Var}(\ln w(x)) = 0.5\sigma_x^2\sigma^2$. [Result preview](#)

Wage Inequality and Differential Impact of Changes in Primitives

- ▶ Presence of amenities complicates the relationship between sorting and wage inequality
 - ▶ More productive firms pay higher wages to all workers, so a high-skill worker in a low-productivity firm may earn less than a low-skill worker in a high-productivity firm
- ▶ For realistic parameter values ($\sigma_x < 0.99$):
 - ▶ Changes in a single primitive affect overall wage inequality and its share explained by the between-firm component in the same direction
- ▶ Relative impact of changes in primitives (given equal impact on overall welfare inequality):
 - ▶ Changes in c_a have the strongest impact on $\frac{BFUI}{\text{Var}(\ln U)}$, $\frac{BFWI}{\text{Var}(\ln W)}$, and $\text{Var}(\ln W)$
 - ▶ If $\text{Var}(\ln U) < 0.5$ (true for most developed countries), changes in c_l have a stronger impact on $\frac{BFWI}{\text{Var}(\ln W)}$ and $\text{Var}(\ln W)$ than changes in σ_x

Data

- ▶ Matched employer-employee data in Norway from administrative registers (Statistics Norway)
 - ▶ Universe of workers and firms
 - ▶ Unique identifiers for workers and firms
 - ▶ Details: worker's date of birth, firm's industry sector, worker's wage at the firm
- ▶ Employment and wage data:
 - ▶ Annual basis before 2015
 - ▶ Monthly basis since 2015 (aggregated to annual earnings)
 - ▶ Sample divided into two parts due to reporting scheme change: 1995-2014 and 2015-2019
- ▶ Firms:
 - ▶ Businesses listed in the Central Register of Establishments and Enterprises
 - ▶ Subdivision of the legal unit (enterprise)
 - ▶ Consistent identification from 1995
- ▶ Baseline sample:
 - ▶ Individuals aged 20-60
 - ▶ Earning over two times the Basic Amount (legal amount for national insurance scheme applicability)
 - ▶ Firms with over 5 employees
 - ▶ Excluding sectors with preset wage schedules (e.g., public administration, education)
 - ▶ Average of 1.3 million workers and 54,000 firms per year

Identification

- ▶ Five moments of the wage distribution used to identify exogenous parameters $\Sigma = (\sigma_x, \sigma_\theta, c_a, c_l, A)$:
 1. Within-firm wage inequality
 2. Variance of the within-firm wage variances
 3. Unweighted within-firm wage inequality
 4. Variance of log wages
 5. Expected (log) wage in the economy
- ▶ Recursive identification:
 - ▶ Moments 1-3 depend on c_a only through its impact on α , which affects the equilibrium supply of quality jobs σ^2
 - ▶ Moments 1-3 identify σ_x, σ_θ , and α
 - ▶ c_a identified from the overall wage variance (moment 4), together with α , identifies c_l
 - ▶ TFP parameter A identified from the average (log) wage in the economy (moment 5)
- ▶ Intuition for key moments:
 - ▶ Variance of within-firm wage variances: Large values imply a large equilibrium supply of quality jobs
 - ▶ Within-firm inequality: For a given σ , a larger degree of within-firm inequality requires a higher variance of skill
 - ▶ Ratio of unweighted to weighted within-firm wage inequality: Large weighted compared to unweighted inequality implies few high-productivity firms that are large

Calibrated parameters

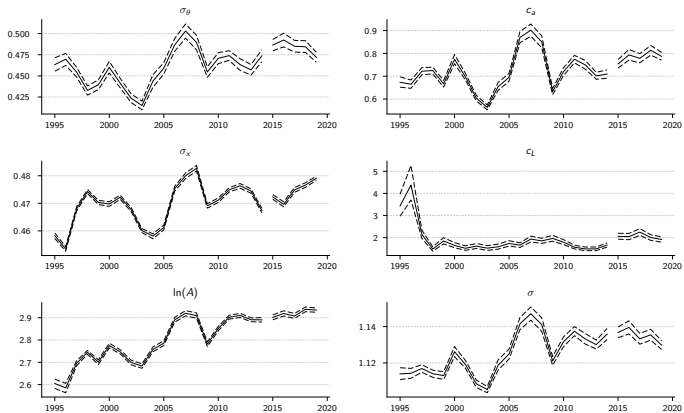


Figure: Calibrated parameters

Counterfactual Analysis: Wage Inequality 1995-2014

	σ_θ	σ_x	c_a	c_l	Overall
VarW	0.007	0.010	-0.006	0.018	0.030
conf. interval	[-0.001,0.015]	[0.009,0.012]	[-0.011,-0.001]	[0.015,0.022]	[0.029,0.030]
share explained	24.1	34.6	-20.8	62.2	100.0
conf. interval	[-1.7,49.0]	[29.9,39.4]	[-37.3,-3.7]	[50.9,72.9]	[100.0,100.0]
WFWI	0.001	0.005	-0.000	0.002	0.008
conf. interval	[-0.000,0.002]	[0.005,0.006]	[-0.000,-0.000]	[0.002,0.003]	[0.008,0.009]
share explained	10.9	62.2	-1.3	28.3	100.0
conf. interval	[-0.8,22.1]	[54.0,70.4]	[-2.3,-0.2]	[22.8,33.7]	[100.0,100.0]
BFWI	0.006	0.005	-0.006	0.016	0.021
conf. interval	[-0.000,0.013]	[0.004,0.006]	[-0.011,-0.001]	[0.013,0.019]	[0.021,0.022]
share explained	29.3	23.6	-28.5	75.6	100.0
conf. interval	[-2.1,59.8]	[20.3,27.2]	[-51.8,-5.0]	[61.6,89.2]	[100.0,100.0]
BFWI VarW	0.018	0.007	-0.019	0.047	0.054
conf. interval	[-0.001,0.037]	[0.006,0.008]	[-0.034,-0.003]	[0.039,0.055]	[0.052,0.055]
share explained	34.2	12.7	-35.5	88.6	100.0
conf. interval	[-2.4,69.7]	[10.7,14.8]	[-64.8,-6.3]	[72.0,104.4]	[100.0,100.0]

Counterfactual Analysis: Wage Inequality 2015-2019

	σ_{θ}	σ_x	c_a	c_l	Overall
VarW	-0.010	0.008	-0.005	0.002	-0.005
conf. interval	[-0.017,-0.004]	[0.007,0.009]	[-0.009,-0.001]	[-0.001,0.005]	[-0.006,-0.005]
share explained	194.3	-150.1	99.5	-43.7	100.0
conf. interval	[73.9,312.3]	[-185.4,-117.8]	[23.3,181.4]	[-99.8,12.3]	[100.0,100.0]
WFWI	-0.001	0.004	-0.000	0.000	0.003
conf. interval	[-0.002,-0.001]	[0.003,0.005]	[-0.000,-0.000]	[-0.000,0.001]	[0.002,0.003]
share explained	-49.8	142.4	-3.8	11.1	100.0
conf. interval	[-85.4,-17.0]	[120.1,167.7]	[-6.5,-1.0]	[-3.1,25.7]	[100.0,100.0]
BFWI	-0.009	0.004	-0.005	0.002	-0.008
conf. interval	[-0.014,-0.003]	[0.003,0.005]	[-0.009,-0.001]	[-0.001,0.005]	[-0.009,-0.008]
share explained	109.5	-47.7	62.8	-24.6	100.0
conf. interval	[40.4,177.5]	[-55.6,-39.7]	[15.5,110.1]	[-56.0,6.8]	[100.0,100.0]
BFWI VarW	-0.025	0.005	-0.015	0.006	-0.030
conf. interval	[-0.040,-0.009]	[0.004,0.006]	[-0.027,-0.004]	[-0.002,0.013]	[-0.031,-0.028]
share explained	84.3	-17.5	52.0	-18.8	100.0
conf. interval	[30.7,137.6]	[-20.2,-14.8]	[13.0,91.2]	[-43.1,5.3]	[100.0,100.0]

Counterfactual Analysis: Welfare Inequality 1995-2014

	σ_θ	σ_x	c_a	c_l	Overall
VarU	0.001	0.006	-0.000	0.004	0.011
conf. interval	[-0.000,0.003]	[0.005,0.006]	[-0.000,-0.000]	[0.003,0.004]	[0.010,0.011]
share explained	13.7	52.2	-1.7	35.8	100.0
conf. interval	[-1.1,27.0]	[43.7,61.1]	[-2.9,-0.3]	[28.6,43.0]	[100.0,100.0]
BFUI	0.001	0.000	-0.000	0.001	0.002
conf. interval	[-0.000,0.001]	[0.000,0.000]	[-0.000,-0.000]	[0.001,0.002]	[0.002,0.003]
share explained	24.0	14.1	-3.0	64.9	100.0
conf. interval	[-2.2,44.2]	[10.4,19.0]	[-4.8,-0.7]	[48.6,84.7]	[100.0,100.0]
$\frac{BFUI}{VarU}$	0.004	0.000	-0.000	0.009	0.013
conf. interval	[-0.000,0.007]	[0.000,0.001]	[-0.001,-0.000]	[0.008,0.011]	[0.010,0.016]
share explained	26.6	3.8	-3.3	72.9	100.0
conf. interval	[-2.6,48.3]	[2.6,5.3]	[-5.2,-0.8]	[53.5,98.4]	[100.0,100.0]

Preview

Counterfactual Analysis: Welfare Inequality 2015-2019

	σ_θ	σ_x	c_a	c_l	Overall
VarU	-0.002	0.004	-0.000	0.001	0.002
conf. interval	[-0.004,-0.001]	[0.004,0.005]	[-0.000,-0.000]	[-0.000,0.001]	[0.002,0.003]
share explained	-103.9	188.4	-7.4	22.9	100.0
conf. interval	[-204.3,-29.9]	[134.8,266.9]	[-11.5,-2.4]	[-6.0,57.2]	[100.0,100.0]
BFUI	-0.001	0.000	-0.000	0.000	-0.001
conf. interval	[-0.001,-0.000]	[0.000,0.000]	[-0.000,-0.000]	[-0.000,0.000]	[-0.001,-0.000]
share explained	188.8	-65.0	20.5	-44.3	100.0
conf. interval	[127.7,304.1]	[-164.1,-34.9]	[1.8,81.5]	[-137.6,15.9]	[100.0,100.0]
$\frac{BFUI}{VarU}$	-0.005	0.000	-0.000	0.001	-0.004
conf. interval	[-0.009,-0.002]	[0.000,0.001]	[-0.001,-0.000]	[-0.000,0.003]	[-0.007,-0.002]
share explained	128.7	-10.9	12.4	-30.2	100.0
conf. interval	[84.8,170.2]	[-19.4,-7.2]	[1.6,38.1]	[-77.5,9.8]	[100.0,100.0]

Preview

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