

Idiosyncratic Risk, Government Debt and Inflation

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Introduction

This paper

- Government debt: A concern for monetary policy?
 - High and growing debt levels in many developed countries
 - Standard models: Not if public debt **funded**

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- Government debt: A concern for monetary policy?
 - High and growing debt levels in many developed countries
 - Standard models: Not if public debt **funded**
- However, if government bonds useful for private sector *self-insurance*, the “neutral” rate changes with public debt
- If central bank does not sufficiently adapt their reaction function, *more public debt implies higher inflation*
 - Just reacting to inflation is not enough to offset variation in “neutral rate”
- This does **not** require Fiscal Dominance/FTPL \implies **Funded** public debt can be inflationary too

This paper (2)

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- In modern 2-asset HANK models, these effects can be substantial in short/medium run
 - I illustrate this by studying debt & inflation dynamics after a fiscal expansion
- Potentially even **too** strong: Under standard assumptions, stark effects of public debt on real interest rates in state-of-the-art 2-asset HANK
 - Structure of asset market is crucial
 - ..cannot be pinned down using household micro data alone
- Important for inflation results but also other outcomes
 - e.g. fiscal costs and -sustainability, investment dynamics
- I propose a simple extension to address this issue

Agenda

1. Literature
2. Simple Model & Sanity check
3. Quantitative model & Calibration
4. Model results: Role of the asset market
5. Model results: Fiscal shock
 - Isolating the “debt inflation”
 - Implications for monetary policy
6. (Soon: Analysis of post-Covid period)

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 - FTPL literature: Focusses on *unfunded* debt/deficits
 - Bayer et al. (2023), Auclert et al. (2023): *Fiscal policy in 2-asset HANK*
 - Chiang and Zoch (2023): *Role of financial intermediation in 2-asset HANK*
 - Ascari and Rankin (2013), Aguiar et al. (2023): *Related results in tractable OLG frameworks*
 - Additionally, independent working paper by Campos et al. (2024) has related analysis in 1-asset HANK

Simple Intuition & Intuition

- Household problem:

$$\max_{\{c_{it}\}_{t=0}^{\infty}, \{N_{it}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\log(c_{it}) + \gamma \log(1 - N_{it})]$$

- Budget constraint in real terms:

$$w_t z_t N_{it} + \frac{1 + i_t}{\pi_t} b_{t-1} + T_t = c_t + b_t + z_t \tau_t$$

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- No aggregate risk, all households start with $b_{-1} = 0$
- Income risk in period 0 *only*:
 - Households initially identical with $z_0 = 1$
 - With prob. ρ^h , $z_t = z^h \forall t \geq 1$, $z_t = z^l$ otherwise
 - Normalize $z_0 = \rho^h z^h + (1 - \rho^h) z^l = 1$

Supply Side and Government

- Supply side: Off-the-shelf CES & Rotemberg setup [Details](#)
- Monetary authority determines nominal bond rate according to Taylor rule

$$i_{t+1} = r_t^* + \theta_\pi(\pi_t - 1)$$

with $\theta_\pi > 1$. r_t^* are parameters with $r_t^* = \beta^{-1} - 1 \quad \forall t \geq 1$

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- Fiscal authority does the following:
 1. In $t = 0$, issue b_0^g bonds, rebate proceeds back to households
 2. In $t = 1$, raise taxes $\tau = \frac{1+i_{t+1}}{\pi_{t+1}} b_0^g$ to pay back debt
 3. $\forall t > 1$: does nothing

Model results: Summary

- In the presence of idiosyncratic risk, the neutral rate of interest increases in the level of public debt [Details](#)

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- In the presence of idiosyncratic risk, the **neutral rate of interest increases in the level of public debt** [Details](#)
- For given interest rate rule, **higher public debt increases Inflation** [Details](#)
- This requires neither
 - Fiscal dominance (b_0^g is fully funded)
 - Ex-ante redistribution (HHs are initially identical)
 - Distortionary transfers/taxes

How can a higher neutral rate cause inflation? Simple Intuition

- “Textbook” Taylor rule:

$$1 + i_t = \pi^* R^* + \theta_\pi (\pi_t - \pi^*)$$

π^* : Inflation target; R^* : Gross “natural rate”

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- What if real rate changes to $R(b^g)$? Can rewrite as follows:

$$1 + i_t = \tilde{\pi} R(b^g) + \theta_\pi (\pi_t - \tilde{\pi}) \quad \text{with} \quad \tilde{\pi} := \pi^* \frac{\theta_\pi - R^*}{\theta_\pi - R(b^g)} .$$

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- If $R(b^g) > R^*$ and $\theta_\pi > 1$, then $\tilde{\pi} > \pi^*$.
- Looks like a rule with a higher inflation target!

Is the channel plausibly relevant?

- Positive medium-/long run association supported by various empirical work
 - Summary by Rachel and Summers (2019): Debt-GDP ratio \uparrow 1 p.p. $\implies r^g \uparrow$ 3-6 basis points
 - In cross-country data, I find relationship of similar size if controlling for time trends

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- How much might “implicit target” $\tilde{\pi} := \pi^* \frac{\theta_{\pi} - R^*}{\theta_{\pi} - R(b^g)}$ plausibly change due to public debt?
- $\theta_{\pi} = 1.5$, $R^* = 1.02^{1/4}$ (both quarterly), $\pi^* = 1.02^{1/4}$ (2% annual)
- Assume: B^g / Y ratio \uparrow 1 p.p. \implies ann. real rate 4 basis points \uparrow
- B^g / Y ratio \uparrow 10%:

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- Assume: B^g / Y ratio \uparrow 1 p.p. \implies ann. real rate 4 basis points \uparrow
- B^g / Y ratio \uparrow 10%: annualized $\tilde{\pi} \approx 1.0281$ (2.81% annual)

Details

Quantitative HANK model

The model: Big picture

- “Back-of-the-envelope” calculation suggests quantitative relevance, but is it?
 - P_Ubl_ic debt \implies inflation : Many reasons for comovement, potentially other mechanisms, ...
 - ..however, we can control them in a structural model.

The model: Big picture

- “Back-of-the-envelope” calculation suggests quantitative relevance, but is it?
 - Public debt \implies inflation : Many reasons for comovement, potentially other mechanisms, ...
 - ..however, we can control them in a structural model.
- I use a 2-asset HANK framework:
 - Features idiosyncratic labor- and profit income risk
 - 2-asset setup argued to be important for effects of fiscal policy on real returns (Bayer et al., 2023)
 - Standard frictions from “medium-scale” DSGE models
 - Calibration in line with micro-moments emphasized by literature

- Model features a unit mass of households $i \in [0, 1]$ with utility function:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{it}^{1-\zeta} - 1}{1-\zeta} - \zeta \frac{N_{it}^{1+\gamma}}{1+\gamma} \right)$$

- Households differ by a range of individual state variables:
 - liquid asset** holdings $a_{it} \geq \underline{a}$
 - illiquid asset** holdings $k_{it} \geq 0$; can only be adjusted with prob. λ
 - labor productivity** s_{it} ; follows discrete Markov process
 - “worker” / “entrepreneur” status** $\Xi_{it} \in \{0, 1\}$, switch with exog. probs. ζ and ι
- N_{it} not chosen individually but by labor union. Individual (non-asset) income:

$$y_{it}(s_{it}, \Xi_{it}) = \begin{cases} (1 - \tau_t)(w_t s_{it} N_t)^{1-\omega} & \text{if } \Xi_{it} = 0 \\ (1 - \tau_t \tau^\Xi) \Pi_t^\Xi & \text{if } \Xi_{it} = 1 \end{cases}$$

Model: Firms

Supply Side: Standard “medium scale” DSGE

- Final goods firms: CES aggregator, purchase inputs from retailers
- Retailers: Produce using intermediate goods; subject to Calvo-type nominal frictions [Details](#)
- Intermediate goods firm: Produce using capital K and labor services H ; choose capital utilization [Details](#)
- Investment goods firm: Turn final goods into capital; subject to real investment adjustment cost [Details](#)
- Labor packer and unions: Produce labor services; set wages nominal wages subject to nominal rigidity [Details](#)

Fiscal authority:

- supplies any amount of bonds B_{t+1}^g necessary to balance budget

$$B_{t+1}^g + (\text{tax revenue}) = \frac{R_t^B}{\pi_t} B_t^g + G_t + T_t$$

- Baseline: $G_t = G_{ss} \forall t$ and $T_t = 0$, budget consolidation through tax rule

$$\left(\frac{\tau_t}{\tau_{ss}} \right) = \left(\frac{\tau_{t-1}}{\tau_{ss}} \right)^{\rho_\tau} \left(\frac{B_t^g}{B_{ss}^g} \right)^{(1-\rho_\tau)\psi_B}$$

Model: Government

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Monetary authority:

- Sets R_{t+1}^B according to standard Taylor rule with rate smoothing:

$$R_{t+1}^B = (R_t^B)^{\rho_R} (R_{ss}^*)^{1-\rho_R} \left(\frac{\pi_t}{\pi_{ss}} \right)^{(1-\rho_R)\theta_\pi}$$

Model: Asset market

- Centralized market for capital. Liquid assets provided by **Liquid Asset Funds** (LAF):
 - Collect HH liquid assets A_t^l , can invest in bonds B_t^l and capital K_t^l
 - Not subject to liquidity frictions but face cost $\varphi + \frac{\Psi}{2} \left(1 - \frac{B_t^l}{A_t^l}\right)^2$ per unit of liquidity provided.

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- **Why?** Set-up nests different assumption on asset market in HANK literature
 - $\Psi \rightarrow \infty$ Segmented asset markets as in Kaplan et al. (2018), Bayer et al. (2023)
 - $\Psi \rightarrow 0$ Integrated asset markets as in Auclert et al. (2023)
 - Simple set-up allows moving in between without changing SS
- Ex-post return to liquid assets given by

$$R_t^a = \frac{\frac{R_t^B}{\pi_t} B_t^l + \frac{r_t + q_t}{q_{t-1}} (A_t^l - B_t^l)}{A_t^l} - \varphi - \frac{\Psi}{2} \left(1 - \frac{B_t^l}{A_t^l}\right)^2$$

Calibration & Solution

Calibration & Solution method

- Micro-calibration consistent with micro moments from literature Details
 - Liquid asset targets to relate to domestically held public debt.
 - Model features high MPCs ($\sim 19\%$ quart.) and reasonable distribution Moments
- Various aggregate parameters set exogenously Details
 - Standard steady state parameters (depreciation, capital share, markup...)
 - Adjustment- and utilization costs from Bayer et al. (forthcoming)
- Policy:
 - Standard **active** Taylor rule: $\rho_R = 0.75, \theta_\pi = 1.5$
 - Slow fiscal consolidation: $\rho_\tau = 0.9, \psi_B = 0.75$
- Numerical Solution: Bayer et al. (2024) linearization method Details

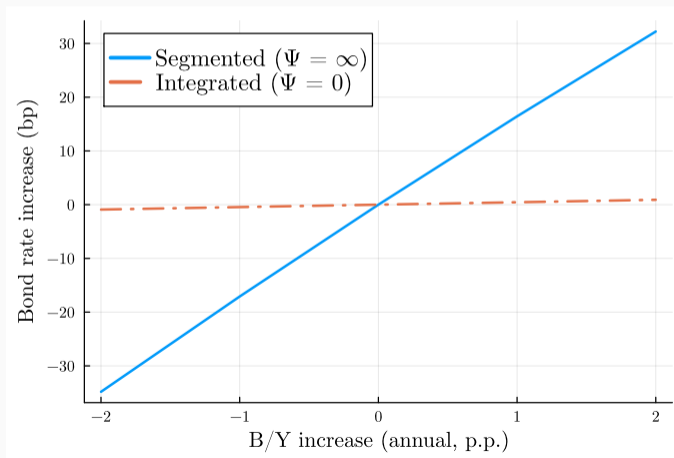
Model Results: Gov't debt and real rates

Public Debt and real rates in the long run

- Can 2-asset HANK relate to empirical magnitudes under standard assumptions?
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Public Debt and real rates (2)

- Can 2-asset HANK relate to empirical magnitudes under standard assumptions?

No!

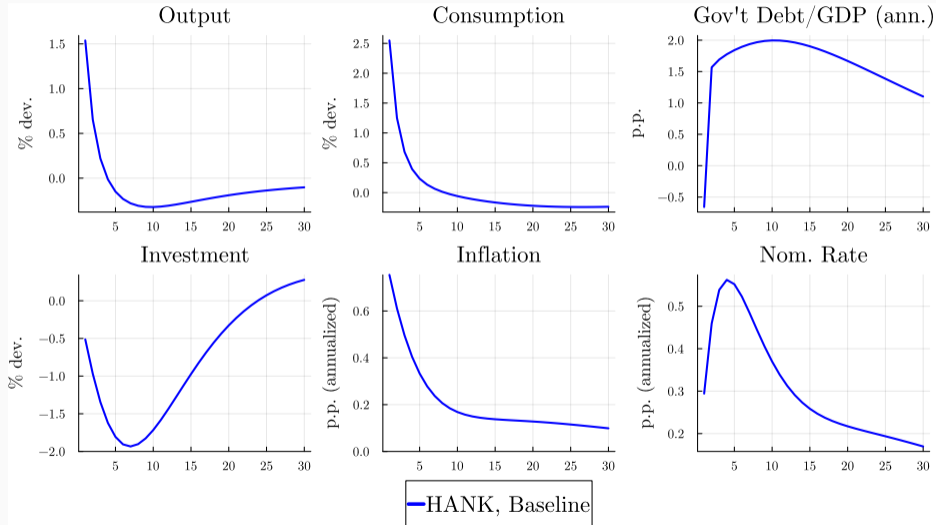
- Segmented asset markets: *Effect much stronger*
- Integrated asset markets: *Effect much weaker*

- Thus: Model can be consistent with very different magnitudes

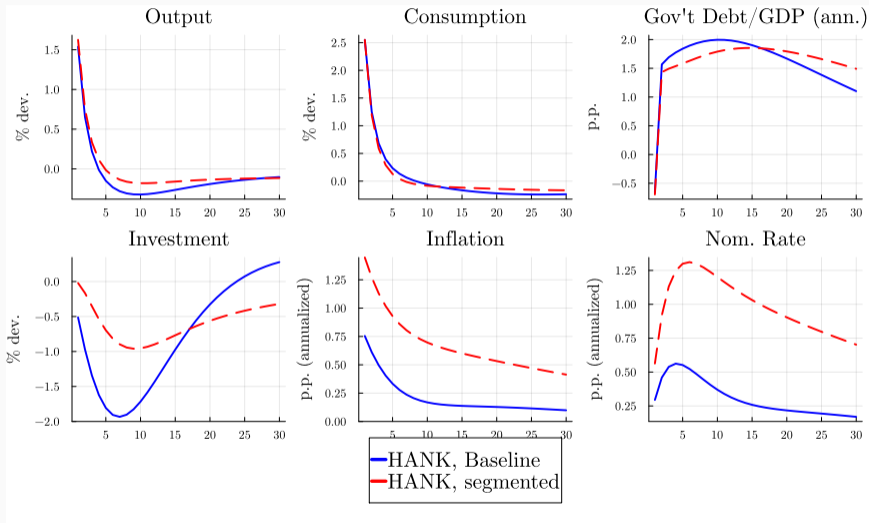
- Solution here: Choose Ψ to be in line with empirical values Graph
 - Segmented model with few HtM / low MPCs can do well, but obvious calibration tradeoff
 - Potentially other solutions..

Model Results: Fiscal Shock

Aggregate response to a transfer shock



Fiscal Shock: The role of the asset market



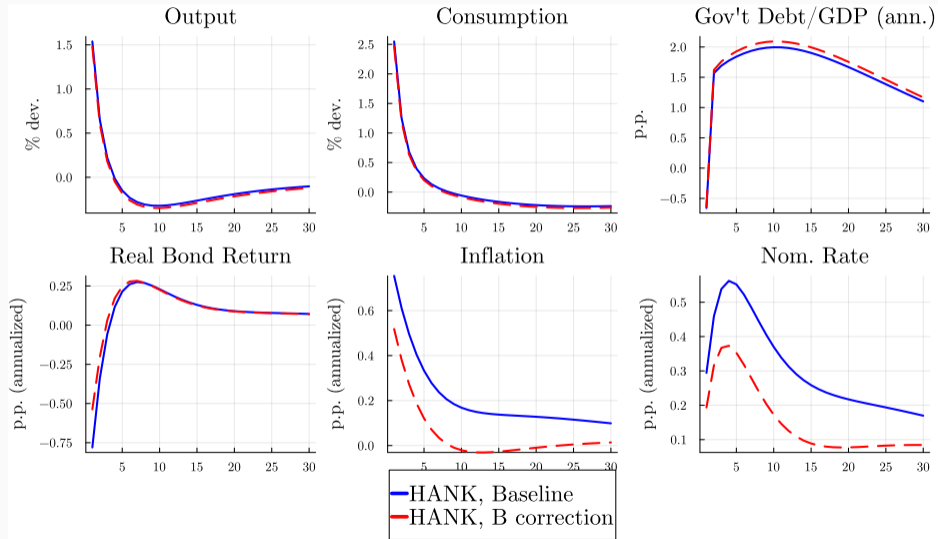
The role of public debt

- Response seem in line with intuition
- But in complex DSGE model, couldn't there be other explanations?
- Consider the following:
 1. Approximate corrected “ R^* ” using steady state elasticity $\gamma_r = 0.0056$:

$$\log(R_t^*) = \log(R_{ss}^*) + \gamma_r(\log(B_t/Y_t) - \log(B_{ss}/Y_{ss}))$$

2. Compare with interest rule using R_t^* instead R_{ss}^*
 - In this case, CB reacts to public debt even if *no inflation*
3. Can also compare with implicit $\tilde{\pi} := \pi_{ss} \frac{\theta_{\pi} - R_{ss}^*}{\theta_{\pi} - R_t^*}$ from simple formula Graph

Fiscal Shock: "Debt" inflation



Implications for monetary policy

What can monetary policy do?

- Central banks might want to counteract inflationary pressure of public debt
 - However, “neutral rate” or relevant elasticities hard to measure in practice
 - Effectiveness requires public to know CB stance w.r.t. public debt [Details](#)

What can monetary policy do?

- Central banks might want to counteract inflationary pressure of public debt
 - However, “neutral rate” or relevant elasticities hard to measure in practice
 - Effectiveness requires public to know CB stance w.r.t. public debt [Details](#)
- Can simpler rules work as well? Various alternatives, e.g.
 1. A “hawkish” rule (e.g. $\theta_\pi = 2$) [Details](#)
 - Reduces overall magnitude, but not persistence
 2. Use *realized* real rate $\frac{R_t^B}{\pi_t}$ as R^* in interest rate rule [Details](#)
 - Implication: Keep raising R^B as long as π_t over target

Concluding remarks

- In the presence of idiosyncratic risk, public debt can create inflation
 - Consistent with active monetary policy
 - Key channel: $B^g \uparrow \implies r^l \uparrow$
- Magnitude depends on structure of the asset market
 - Baseline calibration: Magnitude moderate
 - But persistently elevated debt \implies persistently elevated inflation.
- CB can counteract “debt inflation” by reacting to public debt or real bond returns
- We should think more about modelling asset markets in rich HANK models

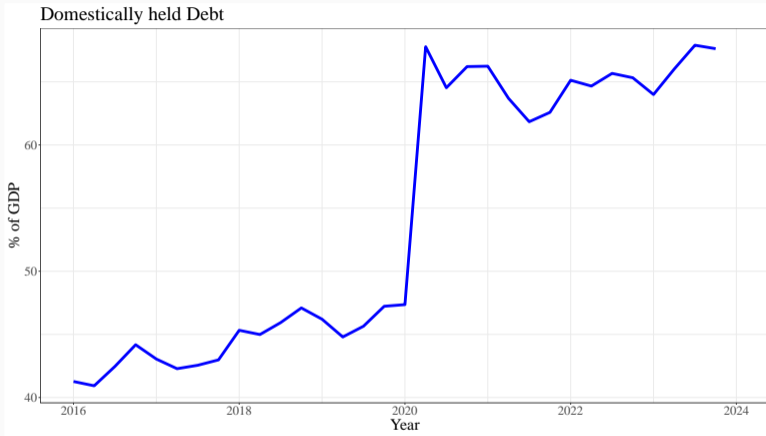
THANK you for your attention!

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[mhaense1.github.io](https://github.com/mhaense1)

Appendix

More US gov't debt



[back](#)

[calibration](#)

Source: FRED. Defined as FYGFDPUN (federal debt held by the public) minus FDHBFIN (fed. debt held by foreign and international investors) over GDP.

More US gov't debt



Details “Back of the envelope” calculation

- Start with $R^* = R(b^g) = 1.01$
- Assumption: Annual $R^a(b^g)$ increases by 0.0004 for every 1% increase in b^g / Y
 \implies New quarterly $R(b^g) = (1.02 + 10 \times 0.0004)^{1/4}$
- Resulting annualized $\tilde{\pi}$:

$$\tilde{\pi} = \left(\pi^* \frac{\theta^\pi - R^*}{\theta^\pi - R(b^g)} \right)^4 \approx 1.0281$$

- Final good produced using intermediate inputs $y_t(j)$:

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$$

- Every variety j is produced by a monopolist
 - Owned by risk-neutral entrepreneurs discounting future at rate β
- Monopolists maximize profits subject to Rotemberg (1982) adjustment costs

$$\max_{\{p_t(j)\}_{t \geq 0}} \sum_{t=0}^{\infty} \mathbb{E}_0 \beta^t \left[(p_t(j) - w_t) \left[\frac{p_t(j)}{P_t} \right]^{-\epsilon} Y_t - \frac{\phi}{2} \left(\frac{p_t(j)}{p_{t-1}(j)} - 1 \right)^2 Y_t \right]$$

Analytical Model: "Natural rate"

Lemma

In period 0, the natural rate of interest r_0^n is implicitly characterized by

$$\frac{\epsilon}{\epsilon - 1} = \beta \rho^h \frac{1 + r_0^n}{\frac{\epsilon - 1}{\epsilon} z^h + \frac{r_1^*}{1 + r_1^*} (1 + r_0^n) b_0^g (1 - z^h)} + \beta (1 - \rho^h) \frac{1 + r_0^n}{\frac{\epsilon - 1}{\epsilon} z^l + \frac{r_1^*}{1 + r_1^*} (1 + r_0^n) b_0^g (1 - z^l)}$$

back

Cross-country regression results [back](#)

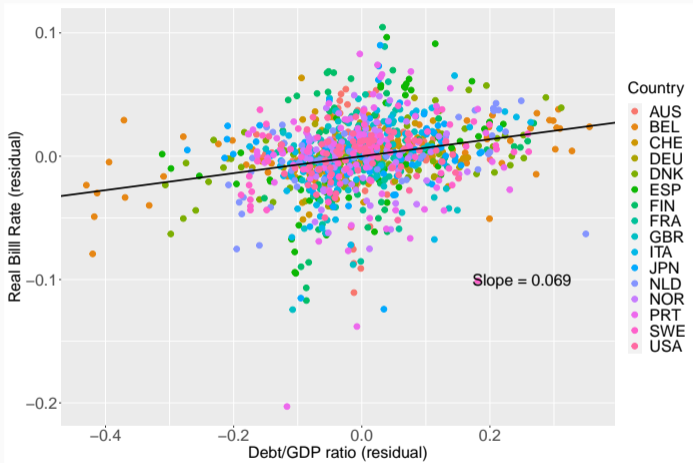


Figure 1: Data source: MacroHistory Database (Jorda et al., 2017)

Household problem in recursive form

- Only show capital adjuster problem:

$$V_t^a(a_{it}, k_{it}, s_{it}, \Xi_{it}) = \max_{a_{it+1}, k_{it+1}} \{u(c_{it}) - v(N_t) + \beta \mathbb{E}_t V_{t+1}(a_{it+1}, k_{it+1}, s_{it+1}, \Xi_{it+1})\}$$

$$\text{s.t. } c_{it} + a_{it+1} + q_t k_{it+1} = a_{it} R_t^a(a_{it}) + (q_t + r_t) k_{it} + y_{it}(N_t, s_{it}, \Xi_{it}) + T_t ,$$

$$a_{it+1} \geq \underline{a}, \quad k_{it+1} \geq 0$$

- Income is given by

$$y_{it}(s_{it}, \Xi_{it}) = \begin{cases} (1 - \tau_t)(w_t s_{it} N_t)^{1-\omega} & \text{if } \Xi_{it} = 0 \\ (1 - \tau_t \tau^\Xi) \Pi_t^\Xi & \text{if } \Xi_{it} = 1 \end{cases}$$

- Liquid return is given by

$$\tilde{R}_t^a(a_{it}) = R_t^a + \mathbb{1}_{a_{it} < 0} \bar{R}$$

Supply Side: Final Good & Retailers

- Final goods: As in simple model
- Retailers: Monopolistic competition, take into account demand curve

$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\epsilon_t} Y_t$$

- Produce using homogenous intermediate goods, choose price subject to Calvo-type adjustment frictions, i.e. solve

$$\max_{p_{jt}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_Y^t Y_t \left[\left(\frac{p_{jt} \bar{\pi}_Y^t}{P_t} - mc_t \right) \left(\frac{p_{jt} \bar{\pi}_Y^t}{P_t} \right)^{-\eta_t} \right] .$$

mc_t : Price of intermediate

Supply side: Intermediate goods firm

- Take capital rental rate r_t , price of labor services h_t and output price mc_t as given, solve

$$\max_{H_t, K_t, u_t} mc_t Z_t (u_t K_t)^\alpha H_t^{1-\alpha} - h_t H_t - (r_t + q_t \delta(u_t)) K_t$$

- Depreciation determined by capacity utilization u_t according to

$$\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2$$

Details: Investment goods firm

- Investment firms take capital price q_t as exogenous, choose $\{I_t\}_{t=0}^{\infty}$ to solve

$$\max \mathbb{E} \sum_{t=0}^{\infty} \beta^t I_t \left\{ q_t \left[1 - \frac{\phi}{2} \left(\log \frac{I_t}{I_{t-1}} \right)^2 \right] - 1 \right\}$$

- Law of motion of aggregate capital stock

$$K_{t+1} = (1 - \delta(u_t))K_t + I_t \left[1 - \frac{\phi}{2} \left(\log \frac{I_t}{I_{t-1}} \right)^2 \right]$$

Details: Labor Packer and Unions

- Each HH is part of a union that supplies a specific variety of labor
 - Members of a union u are required to work the same hours N_{ut} .
- A competitive “labor packer” assembles the varieties with a CES technology

$$H_t = \int_0^1 N_{ut}^{\frac{\epsilon_h - 1}{\epsilon_h}} du \implies \text{Demand schedule: } N_{ut} = \left(\frac{w_{ut}}{h_t} \right)^{-\epsilon_h} H_t$$

- Union leadership chooses wage subject to Rotemberg-style nominal rigidity:

$$\max_{w_t} \sum_{t=0}^{\infty} \beta^t \left(\underbrace{\int \left[u(c_{ut}(w_{ut} N_{ut}, \dots)) - \zeta \frac{N_{ut} (w_{ut})^{1 + \frac{1}{\gamma}}}{1 + \frac{1}{\gamma}} \right] di}_{=\text{avg. felicity of union members}} - \frac{\kappa_w}{2} \left(\frac{w_t}{w_{t-1}} \pi_t \right)^2 \right)$$

F.O.C. gives rise to standard wage Philips curve

“Micro” calibration [back](#)

- Attempt to be consistent with micro moments from the literature
- Risk aversion ζ set to 1.5; income process as in Bayer et al. (forthcoming)
- $B^g / Y = 0.43$ target reflects US average 1950-2019
 - Also, *domestically held* public debt to GDP was around 45% pre-Covid [Graph](#)

- Endogenous calibrated parameters: [Details](#)

Parameter	Description	Target	Value
β	Discount factor	$K / Y = 11.44$	0.982
λ	Capital Liquidity	Annual $B^g / Y = 0.43$	0.034
ζ	“Entrepreneur” prob.	Top 10 Wealth Share 70%	0.0005
\bar{R}	Borrowing penalty	Borrower share 16%	0.038

- Model features high MPCs ($\sim 19\%$ quart.) and reasonable distribution [Moments](#)

Details Calibration: Externally set parameters

Parameter	Description	Value	Source
ζ	risk aversion	1.5	Standard
ι	Exit prob. entrepreneurs	1/16	Bayer et al. (2022)
α	Cobb-Douglas parameter	0.32	Standard
ϕ	investment adjustment cost	3.5	Bayer et al. (2022)
γ	labor supply parameter	1/0.5	Standard
μ_g	SS goods markup	1.1	Standard
κ_Y	Slope of NK Philips curve	0.08	Standard
ϵ_h/κ_w	Slope of NK wage Philips curve	0.04	
δ_0	Steady State depreciation	0.0155	Standard
δ_2/δ_1	utilization parameters	1.0	Bayer et al. (2022)
τ_{ss}	income tax level	0.2	Standard
ω	labor tax progrssivity	0.12	Ferriere and Navarro (2023)
(ρ_τ, ψ_B)	Gov't spending rule	(0.9, 0.75)	
(ρ_R, θ_π)	Taylor rule parameters	(0.75, 1.5)	Standard
(R_{ss}^a, π_{ss})	SS liquid rate& inflation	(1.0, 1.0)	Bayer et al. (2022)

Details Calibration: Internally calibrated

Parameter	Description	Value	Target
β	discount factor	0.982	$K/Y = 11.44$
ζ	entry prob. entrepreneurs	0.0005	Top 10 wealth share 70%
\bar{R}	borrowing penalty	0.038	16% borrower share
λ	illiquid asset adjustment prob.	0.034	$A^I/Y = 1.72$
\underline{a}	borrowing limit	-1.4	100% of avg. after-tax income
φ	Liquidity Wedge	0.0104	$B_{ss}^g = A_{ss}^I$
Ψ	Capital liquidity	0.0075	real rate response to B^g
ζ	Disutility of labor	0.591	$N_{ss} = 1$

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Fit distributional moments (1)

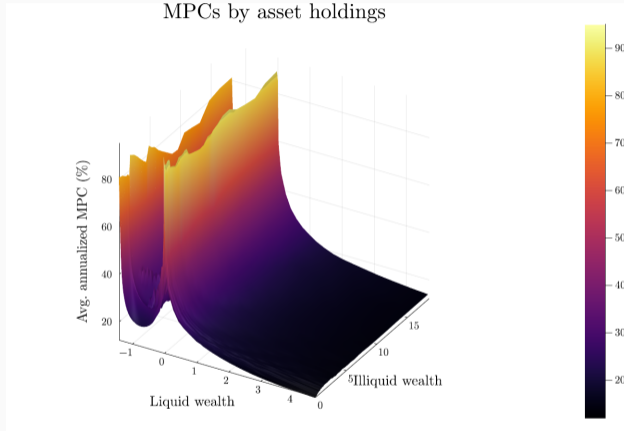
	Disposable Income		Net Worth	
	Model	Data	Model	Data
Quint. 1	6.7	4.5	0.0	-0.2
Quint. 1	10.7	9.9	1.1	1.2
Quint. 3	14.7	15.3	3.9	4.6
Quint. 4	20.4	22.8	10.3	11.9
Quint. 5	47.3	47.5	84.5	82.5
Gini	0.40	0.42	0.80	0.78

Note: "Data" refers to moments computed by Krueger et al. (2016) using PSID and SCF.

Fit distributional moments (2) [back](#)

Moments	Model	Data (incl. source)
<i>Illiquid asset shares</i>		Kaplan et al. (2018)
Top 10%	66.8	70
Next 40%	31.9	27
Bottom 50%	1.3	3
<i>Liquid asset shares</i>		Kaplan et al. (2018)
Top 10%	78.3	86
Next 40%	21.3	18
Bottom 50%	0.4	-4
<i>Hand-to-Mouth (HtM) Status</i>		Kaplan et al. (2014)
Share HtM	33.9	31.2
Share Wealthy HtM	20.9	19.2
Share Poor HtM	13.0	12.1

Consumption moments

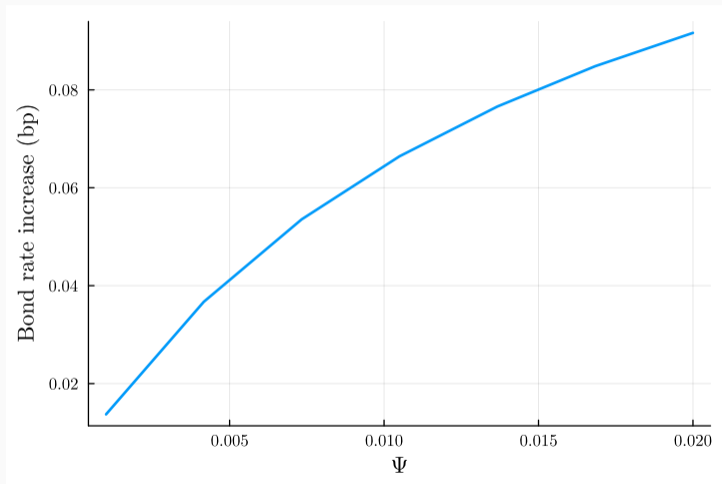


- Average MPC: 19.3 % quarterly, 41.6 % annualized

Solution method

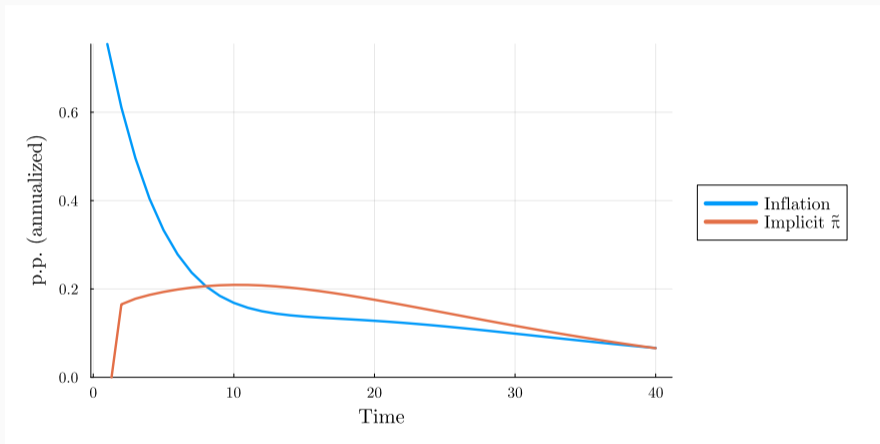
- For steady state, use multi-dimensional Endogenous Grid Method for HH problem and Young (2010) histogram method for distribution
- Obtain dynamic response with a variant of the Bayer and Luetticke (2020) method \implies First-Order Perturbation around non-stochastic steady state
- With my $80 \times 80 \times 16$ tensor grid for HH problem, discretized model has in principle huge system of equations ($> 300,000$)
- **Need dimension-reduction:**
 - Split joint distribution(s) into Copula(s) and marginal distributions
 - Sparse approximations of (marginal) value functions and Copula(s) using Discrete Cosine Transform (DCT)
 - \implies Reduce dimensionality to around 800

Public Debt and real rates by Ψ



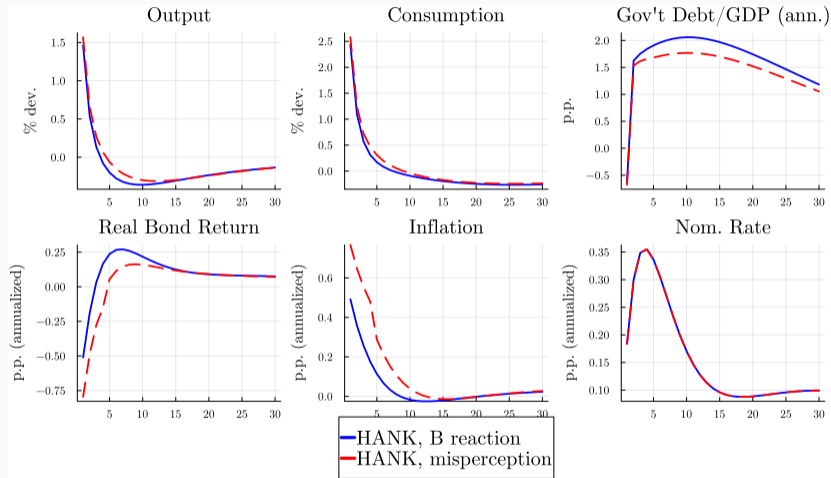
- Baseline model: $\Psi = 0.0075$ [back](#)

Fiscal Shock: Implicit “Debt Inflation”



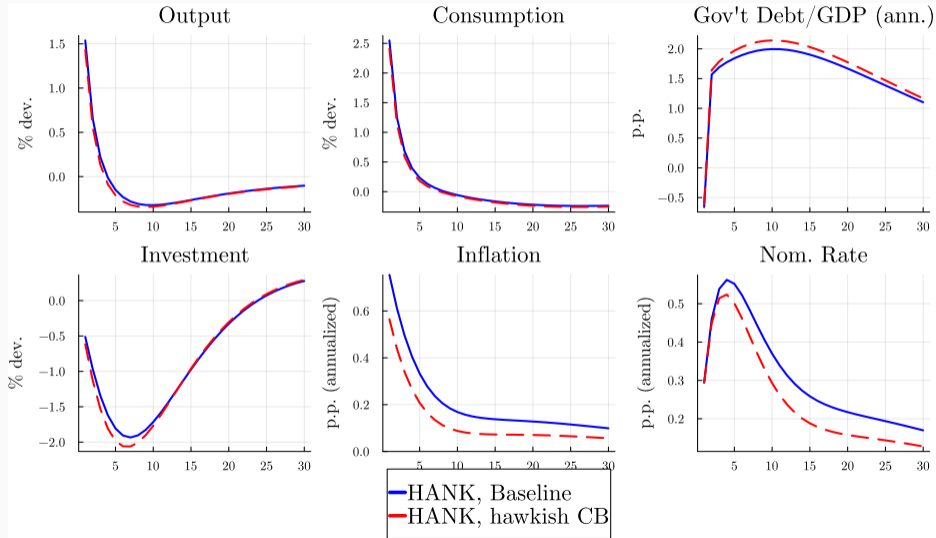
- Steady State liquid return elasticity predicts medium term inflation well

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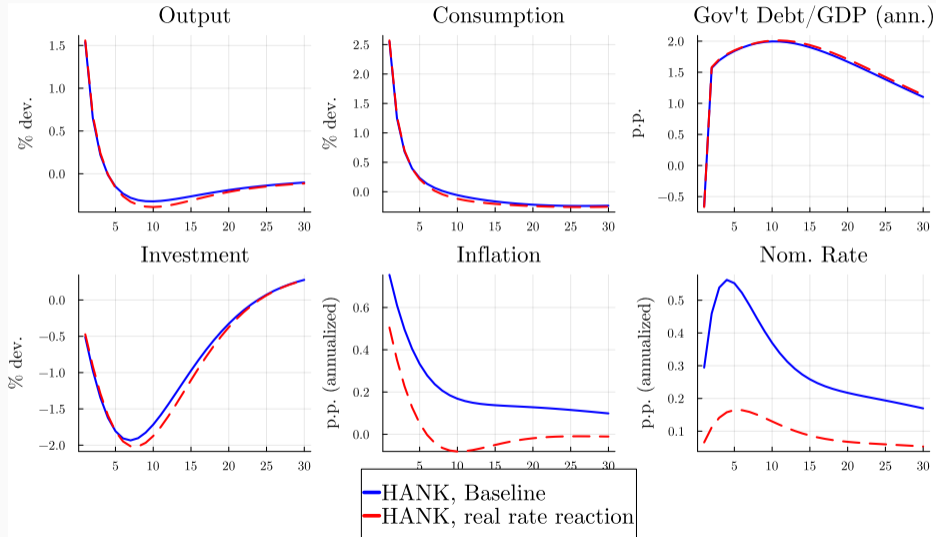


- Red dashed line: Public understands CB “debt reaction” only after 4 quarters

Effects of Hawkish rule ($\theta_\pi = 2$)

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Effects of Real Rate Reaction [Back](#)



Appendix: Analytical Model

Model Results (1)

- “Neutral rate” r_0^n : Nominal rate so that $\pi_0 = 1$.

Proposition (1)

Assume $b_g^0 \in \left[0, \frac{\epsilon-1}{\epsilon} \frac{\beta}{1-\beta}\right)$. In that case, the neutral rate implicitly defined in [Lemma 1](#) fulfills

$$\frac{\partial r_0^n}{\partial b_0^g} > 0 \quad ,$$

i.e. the neutral rate of interest in period 0 is increasing in the level of government debt issued.

Model Results (2)

Proposition (2)

Assume that r_0^* is fixed at the neutral rate $r_0^n(\bar{b})$, as implicitly defined in [Lemma 1](#), for some given level $\bar{b} \in \left[0, \frac{\epsilon-1}{\epsilon} \frac{\beta}{1-\beta}\right)$ of government debt to be issued in period 0. Then,

$$\left. \frac{\partial \pi_t}{\partial b_0^g} \right|_{b_0^g = \bar{b}} > 0 ,$$

i.e. inflation increases in the amount of government debt.

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