Feedback Effects, Market Valuations, and Real Efficiency*

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Abstract

This paper studies the interdependence of myopic corporate behavior and the so-called feedback effect, where financial prices contain useful information for corporate decision making. We model the feedback effect in a mostly standard trading environment, except that its tractable analysis is ensured by Pareto distribution of productivity. The analysis shows that the feedback effect causes a price inflation and the resulting long-term productive inefficiency, which can be understood in the context of innovation strategies. It sheds light on the negative side of learning from financial prices, and, at the same time, explains its prevalence, which requires the availability of superior information in financial markets.

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1 Introduction

Despite the absence of capital issuance directly involved, secondary financial markets appear to play a crucial role in the real economy. Their "real" effect may arise from the information channel where corporate decision makers learn from prices, thereby allocating the resources more efficiently in the real economy. Dating long back to Hayek (1945), the basic idea that prices guide production and allocation decisions by aggregating information across market participants is recognized as one of the main channels whereby the establishment of equity markets can improve the efficiency of the real economy.

However, empirical evidence on learning from stock price and its contribution to firm productivity is mixed. On the one hand, firms behave as if they recognize the informativeness of stock price, presumably due to the availability of superior information in the price compared with internal sources (e.g., Chen, Goldstein and Jiang, 2007).[1] On the other hand, learning from stock prices appears to be only a small part of total learning at the firm level, even in the United States, despite substantial losses in productivity and output due to limited information about demand conditions (David, Hopenhayn, and Venkateswaran, 2016). Rather, learning occurs primarily from internal sources. Even if stock price informativeness increases firm productivity, such effect appears to be mainly driven by a channel other than the purely informational one, such as CEO turnover (Bennett, Stulz, and Wang, 2020).

To tackle this issue, we note that the interaction between real and financial sectors occurs over time in the long run. Accordingly, the real efficiency reflects many important corporate decisions involving long-term commitments, such as strategic planning and market positioning of firms, making it relatively debatable whether secondary financial markets indeed improve the real efficiency via the information channel. One popular counterargument is that secondary financial markets promote myopic corporate behaviors, thereby hurting the real efficiency in the long run. Despite the prevalence of informed speculation in financial markets, which may value future cash flows even in the long run, such "short-termism" may happen when short-term prices incorrectly reflect the long-term consequences of certain types of actions possibly chosen by firms (e.g., Stein, 1989).

Building on the above insight, this paper studies the interdependence of myopic corporate behavior and the feedback effect, where financial prices contain useful information for corporate decision making. In particular, we show that the feedback effect leads to an inflated equity price through risk-driven asymmetric trading behavior of speculators in response to noise trade. Combined with the short-term incentive for decision makers, this feature explains why the feedback effect itself, which requires superior information of financial markets compared with internal sources, is prevalent through ex-ante project choice. At the same time, it can explain the weak contribution of learning from prices to firm productivity.

To do so, this paper proposes a modeling framework to analyze the interaction between corporate decision making and financial prices via information and risk channels in a tractable manner by assuming a Pareto distribution of productivity. The model considers a single firm whose decision maker first chooses between two mutually exclusive projects. While these projects are ex ante identical in terms of the (Pareto) distribution of initially unknown productivity, they differ on which side has an informational advantage between the decision maker and the financial market. Then informed speculators with CARA preferences and noise traders trade the shares in the financial market. Subsequently, the decision maker determines the scale of operations on the chosen project based on his own information and the information in the price.

Our model emphasizes the choice facing the firm's decision maker between two long-term projects. Despite their ex ante identical values, which represent the productivity of operations, these projects differ on whether the decision maker or the financial market has superior information regarding their values. We consider two scenarios depending on which project is initially chosen: In the scenario of the former project, the decision maker optimally chooses the scale of operations based on his information. In contrast, in the scenario of the latter project, its value is partially revealed through the financial

[1] See Subsection 5.2 for the evidence in more details.

price so that the decision maker learns from the price. While the former project is the first-best in terms of the firm's profit, it may not be chosen in equilibrium if the decision maker pursues to maximize the financial price, which may deviate from the firm's expected profit.

To fix ideas, the two long-term projects can be an innovation strategy of pursuing new knowledge and that of building on the firm's existing knowledge, respectively. For example, one may think of corporate managers determining the extent to which they invest in producing patents in technology classes unknown to yet and those in the narrow scope of the knowledge domain. The demand-related information, which is likely to be feasible from public data and can be acquired in a less biased manner in the financial market, is more useful for the latter strategy, which tends to meet the needs of existing customers. On the other hand, the quality-related information, which tends to be proprietary, is more important for the former strategy, which serves emergent markets and involves radical technological departures. Accordingly, the decision maker (the financial market) tends to have an informational advantage in the former (latter).

In the scenario with superior information for the decision maker, there is no feedback effect where the decision maker learns from the price. In the financial market, informed speculators use their relatively noisy signals to make the inference on the project value and the scale of operations, ultimately valuing the firm's profit from operations. We show that the expected price approaches the firm's expected profit under fat-tailed project-value distribution. This stems from two features: (i) the asset supply is zero so that informed speculators are equally likely to buy or sell the asset, and (ii) their trading aggressiveness is symmetric between when they buy and when they sell conditional on large project values. These features lead to a symmetric impact of noise trade on the price conditional on large project values. Then the expected price predominantly reflects those conditional on large project values, where (symmetric) noise trades are averaged out, under fat-tailed project-value distribution.

In the scenario with superior information for the financial market, the scale of operations depends on the information in the price. This results in the feedback effect in that the price reflects the firm's profit and noise trade, the former of which in turn reflects the (price-dependent) scale of operations. We find that the expected price is significantly higher than the firm's expected profit, in contrast to what occurs in the scenario with superior information for the decision maker. Such price inflation is not necessarily specific to fat-tailed project-value distribution. Rather, it results from the fact that the second feature in the first scenario above does not hold, leading to an asymmetric impact of noise trade on the price conditional on large project values. This is the case due to two standard assumptions: (i) speculators are risk-averse, and (ii) the firm uses the price as an informative signal to determine the scale of operations. By the second assumption, high (low) prices lead to high (low) operation scales, resulting in high (low) remaining project-value risk for speculators. Combined with the first assumption, speculators trade asymmetrically due to differential risks across prices; they are aggressive for low prices, but they are less so for high prices. This leads to an average price inflation of equity because noise-driven low (high) prices face aggressive buying (mild selling) pressure by informed speculators.

Combining these results together, we can identify a type of myopic behavior of the decision maker naturally arising from the feedback effect and the resulting asymmetry in trading behavior. In particular, while learning from the price unambiguously improves the efficiency on the scale of operations given the chosen project, the resulting (ex ante) price inflation incentivizes the short-term-oriented decision maker to choose the inefficient project which is more informative to the financial market. Apart from formalizing the idea of interdependence of myopic corporate behavior and the feedback effect, the analysis provides the following additional insights: First, the analysis can endogenously explain the availability of superior information in financial markets compared with internal sources, which is typically assumed in the theoretical literature to ensure the validity of feedback effect. Second, the highlighted price inflation may result in smaller scale of operations by the firm, suggesting that an overall underspending in the real economy may occur together with an overvaluation of equity in the financial market.

The analysis captures corporate innovation strategies (Subsection 5.1), and its main results are consistent with mixed empirical findings regarding the informational role of financial prices and its contribution to the productive efficiency (Subsection 5.2). Some other trading environments are discussed in Subsection 5.3, leading us to compare the mechanism behind the price inflation with related findings in Boleslavsky, Kelly and Taylor (2017) and Edmans, Goldstein and Jiang (2015).

The rest of the paper is as follows. The following subsection reviews the related literature, focusing on analytic studies related to the main points of the paper. Section 2 introduces the model, and Section 3 motivates the use of fat-tailed distribution of productivity in the model. Section 4 presents the analysis of the model, and Section 5 discusses relevant examples, empirical evidence, and the robustness to other trading environments. Section 6 concludes. Proofs can be found in the Appendix.

1.1 Related literature

First and foremost, this paper contributes to the theoretical literature on feedback effects between secondary financial markets and real investments. The literature builds on the premise that prices guide production and investment decisions in the real economy via the information channel, which is especially relevant for speculations about future synergies, competition, and demand. Early studies identify the informational role of financial prices in the allocation of resources and its welfare consequences (e.g., Dow and Gorton, 1997; Dow and Rahi, 2003; Subrahmanyam and Titman, 1999), whereas the recent literature highlights various interesting features of financial markets that arise from feedback effects (e.g., Boleslavsky, Kelly, and Taylor, 2017; Dow, Goldstein, and Guembel, 2017; Edmans, Goldstein, and Jiang, 2015; Goldstein, Ozdenoren, and Yuan, 2013; Goldstein and Yang, 2019).[2] Taking the viewpoint outside of a single firm, other recent studies consider product market competition in the presence of feedback effects (e.g., Rondina and Shim, 2015; Xiong and Yang, 2021) as well as the interaction between financial markets and policymakers (e.g., Bond and Goldstein, 2015; Siemroth, 2019; Siemroth, 2021).

As seen in Section 4, the analysis in this paper generally features an interaction between investors' valuation risk and corporate decisions, which would not be captured by the existing frameworks of feedback effects. Many previous studies consider CARA-preference-based frameworks with financial derivatives whose payoffs are essentially linear in productivity rather than profit from operations (e.g., Rondina and Shim, 2015; Siemroth, 2019; Xiong and Yang, 2021). These frameworks abstract from the valuation risk facing stock-market investors, which is inherent in the uncertainty over the profit from operations, and the resulting asymmetry in their trades. Alternatively, other studies adopt the modeling framework based on lognormal distribution of productivity but abstract from risk-averse preferences of investors and the endgenous determination of their trading aggressiveness (e.g., Goldstein, Ozdenoren, and Yuan, 2013; Goldstein and Yang, 2019). In contrast, our framework tractably analyzes feedback effect maintaining risk-averse preferences of investors by adopting the assumption of Pareto distribution of productivity.[3] In this regard, our analysis offers a novel approach to tractably analyze the interaction between real and financial sectors through information and risk channels, providing an explanation for the prevalence of learning from stock price and its relatively small contribution to firm productivity.

One well-known insight of feedback effects is that a decision maker's intervention based on financial price may weaken the link between the initial state and the asset payoff, thereby making the financial price itself less informative (e.g., Bond, Goldstein, and Prescott, 2010; Boleslavsky, Kelly, and Taylor, 2017; Siemroth, 2021). It could be due to the binary nature of intervention and/or the strategic interaction between a large informed speculator and the decision maker. This paper abstracts from these potential concerns by considering a competitive financial market with continuous real investments and then focusing on the equilibrium where the link between the initial state and the asset payoff persists despite the feedback effect. Instead, it sheds light on the channel whereby the feedback effect influences the average stock price, leading to the possibility that the informational role of financial prices may have adverse consequences which dominate its direct real effect via learning from prices.

[2] See also Bond, Edmans and Goldstein (2012) and Goldstein (2022) for comprehensive reviews of the large literature.
[3] Section 3 formally describes differences between the existing frameworks and ours.

The asymmetry in trading aggressiveness present in this paper also appears in Boleslavsky, Kelly and Taylor (2017) and Edmans, Goldstein and Jiang (2015). However, in contrast to this paper, they both consider a large informed trader's strategic incentive to buy or sell a firm's shares. Combined with the feedback effect, their results go in the same direction as in this paper but arise from a different mechanism. In their mechanism, the large trader's price impact is essential, creating the "limit to arbitrage," whereas the large trader is assumed to be risk neutral, thereby abstracting from risk consideration.[4] In the current paper, the asymmetry in trading aggressiveness arises from competitive and risk-averse traders' response to different risks coming from different prices via operation scales.[5] At this point, their risk aversion effectively creates the limit to arbitrage. In this regard, our theory explains the asymmetry in trading aggressiveness in a new perspective, drawing on different premises on the formation of price and the resulting trading opportunity. They come down to different modeling frameworks of financial markets, as detailed in Subsection 5.3.

In addition, this paper is related to the literature on the role of secondary financial markets in corporate short-termism. The literature highlights various types of short-termism drawing on the premise that firms may choose to boost their stock prices by manipulating what shareholders can observe, while these shareholders also correctly conjecture that there will be such manipulation, taking into account in making their predictions (e.g., Aghion and Stein, 2008; Brandenburger and Polak, 1996; Edmans, 2009; Stein, 1989). These types of short-termist behavior could be attributed to agency problem, including executive compensation (e.g., Peng and Roell, 2014) and learning via short-term performance (e.g., Burkart and Dasgupta, 2021), or shareholders' incentive to boost stock prices in the short run (e.g., Bolton, Scheinkman, and Xiong, 2006; Hackbarth, Rivera, and Wong, 2022). While abstracting from the sources of short-term incentives, such as stock-based compensation, this paper highlights a novel inefficiency of these short-term incentives, which operates through learning from stock prices and the resulting price inflation and has the following two distinctive features: (i) The short-termist inefficiency in this paper arises from publicly observable actions, rather than hidden ones; (ii) It occurs together with the overvaluation of equity in the financial sector. The first feature appears only in Brandenburger and Polak (1996) among the previous studies listed above, and the second one appears only in Bolton, Scheinkman and Xiong (2006). These features potentially fit with certain contexts of corporate decisions including that in Subsection 5.1.

2 Model

There is a firm whose shares are traded in a financial market. The firm has access to the following production technology for projects $d \in \{A, B\}$:

$$\pi_d = a_d \left(\theta_d + \epsilon_d\right) - \frac{1}{2} a_d^2,$$

where θ_d and ϵ_d represent the forecastable and unforecastable parts of productivity factors, respectively, and a_d is the scale of operations on project d. The first and second terms can be viewed as the (linear) gross revenue and (quadratic) cost of operations, respectively, given the chosen project d and its scale a_d. As the resulting profit π_d is strictly concave in a_d with maximum at $\theta_d + \epsilon_d$, this specification boils down to speculating about the productivity factors $\theta_d + \epsilon_d$. While two long-term projects A and B are identical in terms of the distribution of productivity factors θ_d and ϵ_d, they differ on whether the firm or the financial market has superior information on the productivity factors, as detailed below. In what follows, we call θ_d the project value.

[4]Their specific mechanism is as follows: When observing "bad news" that noise trade is positive so that the price is higher than the true value of the firm, the large trader refrains from selling the shares because his trade would then decrease the price, thereby reducing the firm's investments and thus destroying his trading opportunity from superior information. This results in his asymmetric trading behavior, which can lead to an increase in the expected price as well (Boleslavsky, Kelly, and Taylor, 2017).

[5]The intuition in the current paper is as follows: When observing the same "bad news" (i.e., positive noise trade), informed speculators refrain from selling the shares because they take the current higher-than-fair price as given, which results in a larger operation scale and thus a higher remaining project-value risk.

The model has four dates, $t = 0, 1, 2$, and 3. At $t = 0$, a decision maker chooses one and only one of two long-term projects A and B. The chosen project $d \in \{A, B\}$ is common knowledge. At $t = 1$, the financial market opens: A continuum of informed speculators trade the firm's shares based on their information. At $t = 2$, the decision maker decides the scale of operations. At $t = 3$, the firm generates cash flows from operations (i.e., π_A and π_B) and thus all agents get paid and consume.

The initial choice of long-term projects gives rise to two different scenarios as follows: In the scenario of project A, the decision maker holds superior information about the project value θ_A at $t = 2$ so that he decides the scale of operations a_A without learning from the financial price. In contrast, in the scenario of project B, informed speculators in the financial market hold superior information at $t = 2$ about the project value θ_B. Accordingly, the decision maker learns from the price to decide the scale of operations a_B at $t = 2$.

We assume that the decision maker is short-term-oriented. In particular, at $t = 0$, the decision maker pursues to maximize the financial price at the subsequent period (i.e., $t = 1$). This sort of "short-termism" may follow from various exogenous concerns, including stock-based compensation and the possibility of turnovers. In the next period ($t = 2$), the decision maker determines the scale of operations a_d. At this point, he faces no discrepancy between short-term and long-term incentives (i.e., the financial price and the firm's cash flows at $t = 3$, respectively) so that the firm's cash flows from operations (i.e., π_A and π_B) are maximized. Such short-termist preferences can be viewed as the limit of more general case of maximizing a weighted average of financial price and cash flows, which would operate similarly as long as the weight on the financial price is sufficiently large.

2.1 Productivity of projects

For each project $d \in \{A, B\}$, we assume that the project value $\theta_d \geq 0$ commonly follows a Pareto distribution whose power coefficient is $\lambda \in (1, 3]$. Formally, we set the probability distribution of project value θ_d as

$$g(\theta_d) = \frac{\lambda - 1}{\gamma} \left(\frac{\theta_d}{\gamma} + 1 \right)^{-\lambda}$$

for constant $\gamma > 0$.

Two long-term projects A and B differ on which type of agents has an informational advantage in determining the scale of operations (i.e., a_A and a_B) at $t = 2$. In the scenario of project A, the decision maker at $t = 2$ has perfect information about θ_A, whereas informed speculators in the financial market commonly observe a noisy signal s about the project value θ_A given by

$$s = \theta_A + \eta,$$

where η is defined as a random variable whose support is a finite set $\{\eta_1, \cdots, \eta_K\}$ with probabilities $r_1, \cdots, r_K$, respectively. The signal error η is independent of all other variables in the model, and, without loss of generality, we assume that $\mathbb{E}[\eta] = \sum_{k=1}^{K} r_k \eta_k = 0$. Formally, the information sets of the decision maker and informed speculators are given by $\mathcal{I}_{IA} = \{\theta_A, p_A\}$ and $\mathcal{I}_{OA} = \{s, p_A\}$, respectively. In the scenario of project B, the decision maker at $t = 2$ has no private information about θ_B apart from the price p_B, whereas informed speculators in the financial market observe perfect information about θ_B. Formally, the information sets of the decision maker and informed speculators are given by $\mathcal{I}_{IB} = \{p_B\}$ and $\mathcal{I}_{OB} = \{\theta_B, p_B\}$, respectively.

In addition, the unforecastable part of productivity factors ϵ_d is normally distributed with mean zero and variance σ_ϵ^2 and it is independent of the chosen project $d \in \{A, B\}$ and the project value θ_d as well as any other variable in the model.

2.2 Financial sector

At $t = 1$, a continuum of informed speculators $i \in [0, 1]$ and noise traders participate in the financial market to trade the firm's shares. As in Grossman and Stiglitz (1980) and many others, they can simultaneously buy or sell any amount of the shares.

We denote informed speculator i's demand for the shares by x_{iA} and x_{iB} in the scenarios of projects A and B, respectively, conditional on the price and other observable variables. Given project $d \in \{A, B\}$, every informed speculator i pursues to maximize CARA utility with coefficient $\varphi > 0$ as follows:

$$U_{id} = -\frac{1}{\varphi} \exp\left[-\varphi \left\{x_{id}\left(\pi_d - p_d\right)\right\}\right].$$

Noise traders submit random demands. In particular, the amount of noise trade is represented by a random variable ω whose support is a finite set $\{\omega_1, \cdots, \omega_J\}$ with probabilities $q_1, \cdots, q_J$, respectively. We assume that it is independent of all other variables (i.e., θ_d, ϵ_d, and η) and satisfies $\mathbb{E}[\omega] = \sum_{j=1}^{J} q_j \omega_j = 0$ without loss of generality.

2.3 Feedback from financial price

In the scenario where project A is chosen at $t = 0$, the decision maker observes interim information about the project value θ_A at $t = 2$ (i.e., $\mathcal{I}_{IA} = \{\theta_A, p_A\}$), which is more informative than that available in the financial market. It naturally follows that the price p_A is redundant for the decision maker's inference about the project value θ_A. In contrast, in the scenario where project B is chosen at $t = 0$, the decision maker does not have any private information at $t = 2$ (i.e., $\mathcal{I}_{IB} = \{p_B\}$). Therefore, he learns from the price p_B, which partially reveals informed speculators' information about the project value θ_B.

As is well-recognized in the literature on feedback effects (e.g., Goldstein, 2022; Goldstein and Yang, 2019), insiders (e.g., corporate executives) and financial markets may have comparative advantages in different types of information about the productivity of a firm. While insiders can have superior information about the quality of the firm's products, which they get internally, they find it more difficult to evaluate the competition that the firm faces in the product market. This type of information requires aggregation from different sources to be precise, and thus, learning from prices is valuable. The additional insight described in the current model is that different projects may differ by nature on the relative importance between quality- and demand-related information, which determines the relative importance of learning from prices. This point is discussed in Subsection 5.1.

2.4 Equilibrium

In defining the notion of equilibrium, one technical issue is that the expectation of the price (i.e., $\mathbb{E}[p_d]$) can be infinite due to the fact that the firm's profit is roughly proportional to the square of the project value θ_d, whose tail distribution is approximately $\theta_d^{-\lambda}$, for each project $d \in \{A, B\}$.[6] We sidestep the nonexistence of finite expectation by using the expectation of price with respect to the truncated project-value distribution $\hat{g}_{(0,M)}$, i.e. $\hat{g}_{(0,M)}(\theta) := \left(\int_0^M g(\theta) d\theta\right)^{-1} g(\theta)$ for $\theta \in (0, M)$. For every finite M, the expected price $\mathbb{E}_{\hat{g}}[p_d]$ is finite for each project $d \in \{A, B\}$. This enables the decision maker to compare between projects A and B, provided that such comparison is consistent for sufficiently large M as is the case in our model. This criterion can be viewed as extending the standard maximization of expected price to the case where its expectation diverges. Indeed, it is particularly convincing for $\lambda = 3$, which corresponds to the limit of the non-fat-tail case (i.e., $\lambda > 3$), where the unconditional expectation of price is finite even under the untruncated distribution g and thus the standard maximization of expected price applies.

Denote by $x_{iA} = x_{iA}(\mathcal{I}_{OA})$ and $x_{iB} = x_{iB}(\mathcal{I}_{OB})$ informed speculator i's demands in the scenarios of projects A and B, respectively, and, by $p_A = p_A(\mathcal{I}_{OA}, \omega)$ and $p_B = p_B(\mathcal{I}_{OB}, \omega)$ price functions in the scenarios of projects A and B, respectively. In addition, the decision maker's investment strategies conditional on the scenarios of projects A and B are denoted by $a_A = a_A(\mathcal{I}_{IA})$ and $a_B = a_B(\mathcal{I}_{IB})$, respectively. We define the notion of equilibrium of the model as follows:

[6] Formally, the expectation of the price is approximately proportional to $\int_1^\infty \theta_d^2 g(\theta_d) d\theta_d \approx (\lambda - 1)\gamma^{\lambda-1} \int_1^\infty \theta_d^{2-\lambda} d\theta_d$, which goes to infinity as long as $\lambda \leq 3$.

Definition 1. An equilibrium consists of a long-term project $d \in \{A, B\}$, informed speculators' demands (x_{iA}^*, x_{iB}^*), price functions (p_A^*, p_B^*), and the decision maker's investment strategies (a_A^*, a_B^*) such that the following conditions are satisfied:

1. At $t = 0$, the decision maker chooses project $d \in \{A, B\}$ whenever it maximizes

$$\mathbb{E}_{\hat{g}}\left[p_d^*\left(\mathcal{I}_{Od}, \omega\right)\right] = \int_0^M p_d^*\left(\mathcal{I}_{Od}, \omega\right) \hat{g}_{(0,M)}\left(\theta_d\right) d\theta_d$$

for sufficiently large $M \in \mathbb{R}_+$.

2. At $t = 1$, for each $d \in \{A, B\}$, informed speculators submit demands $x_{id}^*\left(\mathcal{I}_{Od}\right)$ to maximize their expected utility $\mathbb{E}[U_{id}|\mathcal{I}_{Od}]$ conditional on the information set $\mathcal{I}_{Od}$, where $\mathcal{I}_{OA} = \{s, p_A\}$ and $\mathcal{I}_{OB} = \{\theta_B, p_B\}$. At the end of this stage, the price $p_d = p_d^*\left(\mathcal{I}_{Od}, \omega\right)$ clears the market, i.e.

$$\int_{i \in [0,1]} x_{id}^*\left(\mathcal{I}_{Od}\right) di + \omega = 0.$$

3. At $t = 2$, for each $d \in \{A, B\}$, the decision maker chooses $a_d^*\left(\mathcal{I}_{Id}\right)$ to maximize the firm's expected profit $\mathbb{E}[\pi_d|\mathcal{I}_{Id}]$ conditional on the information set $\mathcal{I}_{Id}$, where $\mathcal{I}_{IA} = \{\theta_A, p_A\}$ and $\mathcal{I}_{IB} = \{p_B\}$.

Throughout the analysis in Section 4, we use an important property of fat-tailedness that the expected price $\mathbb{E}_{\hat{g}}\left[p_d^*\left(\mathcal{I}_{Od}, \omega\right)\right]$ predominantly reflects those (i.e., $\mathbb{E}\left[p_d^*\left(\mathcal{I}_{Od}, \omega\right)|\theta_d\right]$) conditional on large values of θ_d as M becomes large. Accordingly, the decision maker's project choice at $t = 0$ boils down to comparing these expected prices conditional on large values of θ_d. To see this, we note that these conditional expectations are roughly proportional to θ_d^2, as formally verified for each project $d \in \{A, B\}$ in the analysis in Section 4. Then we get the property by using the below lemma:

Lemma 1. *For $m = 1, 2$, let K_m be a bounded function of $\theta \geq 0$ such that $K_m(\theta) \to \bar{K}_m > 0$ as $\theta \to \infty$. Then*

$$\frac{\mathbb{E}_{\hat{g}}\left[K_1(\theta)\theta^2\right]}{\mathbb{E}_{\hat{g}}\left[K_2(\theta)\theta^2\right]} \to \frac{\bar{K}_1}{\bar{K}_2} \text{ as } M \to \infty.$$

This property of the "dominance" of tail values in the expectation of approximately-quadratic function substantially simplifies our analysis in Section 4 by allowing us to focus on the tail values of productivity. Apart from its technical convenience, it is in line with the informal observation justifying fat-tailed distribution that a few large firms account for a disproportionate share of overall economic activity.

3 Preliminary analysis of feedback effect without fat-tailed distribution

In this section, we show how our main question on the interdependence between myopic corporate behavior and the feedback effect, which requires modeling the feedback effect in a trading environment with CARA preferences, naturally motivates the use of fat-tailed distribution of productivity. Throughout this section, we focus on project B, where only informed speculators know the project value so that the feedback effect occurs. Instead of fat-tailed distribution, we assume that the project value $\tilde{\theta}_B$ is normally distributed with mean zero and variance σ_θ^2. Denote by $\tilde{\omega}$ the noise trade following a discrete or continous distribution where $\mathbb{E}[\tilde{\omega}] = 0$.

To begin with, we assume that the per-unit asset payoff is equal to the project value $\tilde{\theta}_B$, as it would be natural without feedback effect. That is, informed speculators earn the project value $\tilde{\theta}_B$ per unit of the security rather than the firm's profit from operations. The first-order condition of each speculator i's profit implies that his demand is $x_{iB}^{*}=\frac{\mathbb{E}[\tilde{\theta}_B+\epsilon|\tilde{p}_B]-\tilde{p}_B}{\varphi\mathrm{Var}[\tilde{\theta}_B+\epsilon|\tilde{p}_B]}$. Combined with the market-clearing condition $\int x_{iB}^{*}di+\tilde{\omega}=0$, this implies that the equilibrium price is equal to his (per-unit) valuation $\tilde{\theta}_B$ plus a "mispricing" term, which reflects the relative proportion of noise trade, as follows:

$$\tilde{p}_B=\tilde{\theta}_B+\varphi\sigma_\epsilon^2\tilde{\omega}. \tag{1}$$

A few standard properties of the equilibrium price include:

1. The existence and uniqueness of equilibrium (price) are ensured by Equation (1).
2. The price is $\tilde{\theta}_B$ on average: The mispricing term comes from the noise trade $\tilde{\omega}$ but is symmetric across its realization so that it is averaged out. Intuitively, as informed speculators are risk-averse, they might require a premium for the remaining project-value risk (i.e., variance of ϵ). However, such risk premium does not exist in Equation (1). To see this, note that the project-value risk can affect the price in both directions: If informed speculators buy the asset, their risk makes them buy less, causing the typical risk premium (i.e., lowering the price). However, if they sell the asset, their risk makes them sell less, thereby influencing the price in the opposite direction (i.e., raising the price). In the current environment, informed speculators trade symmetrically so that these effects offset each other, leading to the symmetric mispricing term in Equation (1). The reason is twofold: (i) The asset supply is zero so that they are generally equally likely to buy or sell the asset; (ii) Their trading aggressiveness is symmetric between when they buy (i.e., $\tilde{\theta}_B-p_B>0$) and when they sell (i.e., $\tilde{\theta}_B-p_B<0$) as their risk aversion is symmetric (i.e., constant) and they symmetrically face the same project-value risk.
3. The conditional expectation of $\tilde{\theta}_B$ can be expressed as a function Φ_p of the price as follows:

$$\Phi_p\left(\tilde{p}_B\right):=\mathbb{E}\left[\tilde{\theta}_B|\tilde{p}_B\right].$$

   Accordingly, it can also be a function of $\tilde{\theta}_B$ and $\tilde{\omega}$ since $\mathbb{E}[\tilde{\theta}|\tilde{p}_B]=\Phi_p\left(\tilde{\theta}_B+\varphi\sigma_\epsilon^2\tilde{\omega}\right)$ by Equation (1). In the case where the noise trade $\tilde{\omega}$ is normally distributed, say with mean zero and variance σ_ω^2, Φ_p is linear.[7] Such linearity of conditional expectation, which is a special property of normal distribution, is not essential per se in the existence and analysis of equilibrium (e.g., Breon-Drish, 2015). However, it is at the heart of tractability of standard CARA-normal framework (without feedback effect).

To model feedback effect, we now consider the case of asset payoff reflecting the firm's profit as in Section 2, while maintaining normally-distributed productivity (i.e., the same normal distribution of $\tilde{\theta}_B$). In particular, informed speculators earn the firm's profit per unit of the security. By the market-clearing condition, we have

$$\tilde{p}_B=\underbrace{\tilde{a}_B\tilde{\theta}_B-\frac{1}{2}\left(\tilde{a}_B\right)^2}_{\mathbb{E}[\pi_B^*|\tilde{\theta}_B,\tilde{p}_B]}+\underbrace{\varphi\sigma_\epsilon^2\left(\tilde{a}_B\right)^2\tilde{\omega}}_{\varphi\omega\mathrm{Var}[\pi_B^*|\tilde{\theta}_B,\tilde{p}_B]}, \tag{2}$$

provided that an equilibrium exists where the scale of operations $\tilde{a}_B$ satisfies $\tilde{a}_B=\Phi_p\left(\tilde{p}_B\right)$, leading to an interaction among the scale of operations, the firm's profit, and the noise term in Equation (2). Specifically, as the scale of operations $\tilde{a}_B$ increases, the firm's profit may change proportionally at most, while the noise term increases quadratically. The latter effect arises from the fact that informed speculators face higher remaining project-value risk so that they trade less aggressively, driving the price noisier. Moreover, the scale of operations $\tilde{a}_B$ in turn depends on the price p_B, which is the sum of the firm's profit and the noise term, as the decision maker learns from the price. This chain of influence is what is termed the feedback effect in the literature reviewed in Subsection 1.1.

[7] In particular, we get $\Phi_p(x)=\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\omega^2}x$ by the standard Bayesian rule.

In the Appendix, we formally describe how the above chain of influence undermines the linearity of conditional expectation Φ_p in the standard CARA-normal framework. Indeed, this issue has been recognized and informally discussed in the literature (Bai, Philippon, and Savov, 2016 (p.647)). The existing CARA-preference-based framework of the feedback effect reviewed in Subsection 1.1 effectively maintains the linearity by focusing on financial derivatives whose payoffs are approximately proportional to $\tilde{\theta}_B$, rather than stocks whose payoffs corresponds to profits from operations (e.g., Rondina and Shim, 2015; Siemroth, 2019; Xiong and Yang, 2021). Other existing frameworks of the feedback effect maintain the linearity of the conditional expectation Φ_p with a different set of assumptions, including risk neutrality of informed speculators combined with an effectively binary choice of trades, thereby abstracting from the endogenous determination of their trading aggressiveness via risk channel (e.g., Goldstein, Ozdenoren, and Yuan, 2013; Goldstein and Yang, 2019). Indeed, the above chain of influence would be shut down in both of these existing frameworks via these simplifying assumptions.[8]

In the main analysis in the following section, we analyze the above chain of influence requiring risk-averse preferences and learning from prices, while maintaining the tractability of analysis by assuming Pareto distribution of productivity to ensure the linearity of conditional expectation. This mathematical feature of linear conditional expectation stems from the invariance of fat-tailed distribution to scale-independent random growth, corresponds to a transformation from $\tilde{\theta}_B$ to its conditional expectation $\tilde{a}_B = E[\tilde{\theta}_B|\tilde{p}_B]$ in the current model. It is recognized in the literature that such invariance to scale-independent random growth mathematically characterizes the class of power-function distribution (e.g., Gabaix, 1999).[9]

4 Analysis

Now we come back to the current model where the asset payoff reflects the profit from operations whose productivity is Pareto-distributed. Following backward induction, we first solve two scenarios (i.e., subgames following projects A and B). The equilibrium characterization for each subgame allows us to determine the expected price given the corresponding project. Then we examine the decision maker's choice between two projects A and B at $t = 0$.

4.1 Project A: Superior information for the firm

In the scenario where project A is chosen at $t = 0$, there is interim information about the project value θ_A for the decision maker deciding the scale of operations $a_A(\mathcal{I}_{IA})$ at $t = 2$. In the real sector, the decision maker chooses the optimal scale of operations (i.e., $a_A^*(\mathcal{I}_{IA}) = \theta_A$), leading to to the firm's profit given by

$$\pi_A^* = a_A^*(\mathcal{I}_{IA})(\theta_A + \epsilon_A) - \frac{1}{2}(a_A^*(\mathcal{I}_{IA}))^2 = \frac{1}{2}\theta_A^2 + \theta_A \epsilon_A.$$

In the financial market, the analysis is complicated by the fact that informed speculators take into account the valuation risk involved in the Bayesian inference on the firm's profit π_A^* conditional on their noisy signal $s = \theta_A + \eta$, which is generally neither normally nor symmetrically distributed. We can still generally obtain an expression of informed speculators' expected utility conditional on their

[8]Specifically, Corollary 2 would not hold in these existing frameworks due to the irrelevance of prices in traders' trading aggressiveness, which leads to the absence of their asymmetric trading behavior.

[9]More broadly, this sort of multiplicative functional forms of variables is common to the existing economic applications of power-function distribution (e.g., Landier and Gabaix, 2008; Edmans, Gabaix, and Landier, 2009).

information set $\mathcal{I}_{OA} = \{s, p_A\}$ as follows:

$$\begin{aligned}
\mathbb{E}\left[U_{iA}|\mathcal{I}_{OA}\right] &= \mathbb{E}\left[\mathbb{E}\left[U_{iA}|\theta_A, p_A\right]|\mathcal{I}_{OA}\right] \\
&= -\frac{1}{\varphi}\mathbb{E}\left[\exp\left\{-\varphi x_{iA}\left(\frac{\theta_A^2}{2} - p_A\right) + \frac{1}{2}\varphi^2 x_{iA}^2 \theta_A^2 \sigma_\epsilon^2\right\}|\mathcal{I}_{OA}\right] \\
&= -\frac{1}{\varphi}\exp\left\{-\varphi x_{iA}\left(\frac{\mathbb{E}[\theta_A^2|s]}{2} - p_A\right) + \frac{\varphi^2}{2} x_{iA}^2 \sigma_\epsilon^2 \mathbb{E}[\theta_A^2|s]\right\} \\
&\quad \times \mathbb{E}\left[\exp\left\{\frac{\hat{\eta}(s)}{2}\left(-\varphi x_{iA} + \varphi^2 x_{iA}^2 \sigma_\epsilon^2\right)\right\}|\mathcal{I}_{OA}\right],
\end{aligned} \tag{3}$$

where the third line is obtained by defining a random variable $\hat{\eta}(s) = \theta_A^2 - \mathbb{E}[\theta_A^2|s]$ for each realization of s, and the last expectation term is taken over the (discrete) distribution of $\hat{\eta}(s)$ conditional on the observed signal s. As detailed in the proof of Proposition 1, this expression of informed speculators' expected utility allows us to obtain the first-order condition with respect to x_{iA}, which, combined with the market-clearing condition $\int x_{iA}^* di + \omega = 0$, determines the equilibrium price p_A^* as follows:

$$p_A^* = \frac{\mathbb{E}[\theta_A^2|s]}{2} + \underbrace{\varphi\omega\sigma_\epsilon^2\mathbb{E}[\theta_A^2|s]}_{\substack{\text{Residual uncertainty} \\ \text{over } \pi_A^* \text{ given } \theta_A}} + \underbrace{\left(\frac{1}{2} + \varphi\omega\sigma_\epsilon^2\right)\frac{\sum_{k=1}^K \hat{\eta}_k(s) r_k \exp\left(\hat{\eta}_k(s)\left(\frac{\varphi}{2}\omega + \frac{\varphi^2}{2}\omega^2\sigma_\epsilon^2\right)\right)}{\sum_{k=1}^K r_k \exp\left(\hat{\eta}_k(s)\left(\frac{\varphi}{2}\omega + \frac{\varphi^2}{2}\omega^2\sigma_\epsilon^2\right)\right)}}_{\substack{\text{Uncertainty over } \pi_A^* \text{ from} \\ \text{the uncertainty over } \theta_A \text{ given } s}} \tag{4}$$

where $\hat{\eta}_k(s)$ is the (perfectly) inferred realization of $\hat{\eta}(s)$ conditional on s and $\eta = \eta_k$ for $1 \leq k \leq K$, whose detailed expression is given in the proof of Proposition 1.[10] In Equation (4), the first term represents the firm's expected profit, and the second one in Equation (4) arises from the residual uncertainty over the firm's profit π_A^* *given* the project value θ_A, and it is averaged out due to the symmetry across noise trade ω. The third term in Equation (4) arises from its uncertainty caused by the uncertainty over the project value θ_A given signal s. This term involves the interaction between informed speculators' misvaluations from signal error $\hat{\eta}(s)$ and noise trade ω, which might cause a bias in the price. While it is difficult to clearly identify the mechanics behind the last mispricing term in Equation (4), which can be either positive or negative on average, the proof of Corollary 1 shows that this term, which is bounded by informed speculators' possible misvaluations from signal error (i.e., $\hat{\eta}_k(s)$), is at most proportional to the project value θ_A. As a result, it becomes negligible compared with the firm's expected profit ($\propto \theta_A^2$) as the project value θ_A becomes large. Combined with the dominance of tail values by Lemma 1, this makes such mispricing negligible on average. Overall, these imply that the ex ante expected price is approximately equal to the firm's ex ante expected profit. This is the case for every equilibrium, though we cannot rule out the possibility of multiple equilibria. These results are summarized in the below proposition and corollary:

Proposition 1. *In the scenario where project A is chosen at $t = 0$, there is at least one equilibrium.*

Corollary 1. *In every equilibrium in the scenario where project A is chosen at $t = 0$, we have*

$$\lim_{M\to\infty} \frac{\mathbb{E}_{\hat{g}}[p_A^* - \pi_A^*]}{\mathbb{E}_{\hat{g}}[\pi_A^*]} \to 0.$$

The corollary tells us that the firm is ex ante valued identically to its expected profit. This suggests that the absence of information about project value θ_A for informed speculators in the financial market does not significantly affect the ex ante valuation of the firm. Recall that the price is equal to the firm's expected profit on average in the the CARA-normal environment in Equation (1) in Section 3 due to

[10] If the realization of signal s, which is equal to $\theta_A + \eta$, is close to zero, it may rule out some positive realizations of η. Still, it does not qualitatively change the results, which are driven by tail values of θ_A by Lemma 1.

two reasons: (i) The asset supply is zero; (ii) Speculators' trading aggressiveness is symmetric between when they buy and when they sell. In the current scenario under project A, the former (i) is still the case, and the latter (ii) also does apply at the tail of project value θ_A, where informed speculators are "almost" informed about θ_A given their signal s. Given their almost informedness about θ_A at the tail, they symmetrically face project-value risk over $\theta_A + \epsilon_A$ so that they trade symmetrically in response to θ_A. Then the corollary follows, combined with the dominance of tail values by Lemma 1.

4.2 Project B: Superior information for the financial market

In the scenario where project B is chosen at $t = 0$, there is no interim information about the project value θ_B for the decision maker at $t = 2$. Thus, the decision maker cannot choose the optimal operation scale in contrast to project A and the benchmark-case scenario. Instead, the decision maker decides the operation scale a_B based on the price p_B, which reveals the information held by informed speculators at $t = 1$ and thus can be regarded as a noisy signal about θ_B. That is, $a_B = a_B(\mathcal{I}_{IB}) = a_B(p_B)$. In this case, the equilibrium characterization boils down to a fixed-point problem of solving for $a^*_B = a^*_B(p_B)$ which maximizes the firm's profit π_B for each price p_B. Specifically, we first conjecture the decision maker's investment strategy $a^*_B(p_B)$ and determine informed speculators' trading strategy $x_{iB}(\mathcal{I}_{OB})$ following from their expectation on the firm's profit π_B. Then we use the market-clearing condition to determine the equilibrium price p^*_B and hence the information that the decision maker can learn from the price. Finally, we update the decision maker's belief to characterize his optimal strategy on the scale of operations and then compare it with the initial conjectured strategy $a^*_B(p_B)$ to solve for its underlying parameters.

As described above, we first conjecture an investment strategy of the decision maker a^*_B. Then, informed speculators recognize that the expected profit of the firm is given by $\mathbb{E}\left[\pi^*_B|\theta_B, p_B\right] = a^*_B\left(p_B\right)\theta_B - \frac{1}{2}\left(a^*_B\left(p_B\right)\right)^2$, noting that it depends not only on the project value θ_B but also on the price p_B via the scale of operations $a^*_B\left(p_B\right)$. As informed speculators face a normally distributed valuation risk (i.e., $a^*_B\left(p_B\right)\epsilon_B$), we obtain their optimal demand and then combine it with the market-clearing condition to get

$$p_B = \underbrace{a^*_B\left(p_B\right)\theta_B - \frac{1}{2}\left(a^*_B\left(p_B\right)\right)^2}_{\mathbb{E}\left[\pi^*_B|\mathcal{I}_{OB}\right]} + \underbrace{\varphi\sigma^2_\epsilon\left(a^*_B\left(p_B\right)\right)^2\omega}_{\varphi\omega\mathrm{Var}\left[\pi^*_B|\mathcal{I}_{OB}\right]}. \tag{5}$$

To determine the decision maker's investment strategy a^*_B, we start by considering what form of signal about θ_B can be extracted from the price p_B. In parallel with the linear conjecture on Φ_p in Section 3, we conjecture that a^*_B can be written as

$$a^*_B\left(p_B\right) = \alpha\left(p_B\right)\cdot\hat{\theta}\left(p_B\right), \tag{6}$$

where $\hat{\theta}$ is first defined by $\hat{\theta}\left(p_B\right) := \frac{p_B + \frac{1}{2}\left(a^*_B(p_B)\right)^2}{a^*_B(p_B)}$ and then coefficient α is defined by $\alpha\left(p_B\right) := \frac{a^*_B(p_B)}{\hat{\theta}(p_B)}$, which is generically price-dependent. Using the standard Bayesian rule, we have

$$\mathbb{E}[\theta_B|p_B] = \frac{\sum_{j=1}^J \theta_{Bj}\left(p_B\right)\Pr\left(\theta_B = \theta_{Bj}\left(p_B\right), \omega = \omega_j\right)}{\sum_{j=1}^J \Pr\left(\theta_B = \theta_{Bj}\left(p_B\right), \omega = \omega_j\right)} = \frac{\sum_{j=1}^J \theta_{Bj}\left(p_B\right) g\left(\theta_{Bj}\left(p_B\right)\right) q_j}{\sum_{j=1}^J g\left(\theta_{Bj}\left(p_B\right)\right) q_j}, \tag{7}$$

where $\theta_{Bj}\left(p_B\right)$ is defined as the (perfectly) inferred realization of θ_B conditional on p_B and $\omega = \omega_j$ for $1 \le j \le J$, which is given by[11]

$$\begin{aligned}\theta_{Bj}\left(p_B\right) &= \frac{1}{a^*_B\left(p_B\right)}\left\{p_B + \frac{1}{2}\left(a^*_B\left(p_B\right)\right)^2 - \varphi\sigma^2_\epsilon\left(a^*_B\left(p_B\right)\right)^2\omega_j\right\} \\ &= \hat{\theta}\left(p_B\right) - \varphi\sigma^2_\epsilon\omega_j a^*_B\left(p_B\right) = \hat{\theta}\left(p_B\right)\left(1 - \varphi\sigma^2_\epsilon\omega_j\alpha\left(p_B\right)\right),\end{aligned} \tag{8}$$

[11]In Equation (7), the discrete version of the Bayesian rule applies because the *conditional* distribution of the project value θ_B is discrete (i.e., over $\{\theta_{B1}(p_B), \cdots, \theta_{BJ}(p_B)\}$), even though its unconditional distribution $g(\theta_B)$ is continuous (i.e., over $[0, \infty)$).

where the first line is from Equation (5), the second line from Equation (6) and the definition of $\hat{\theta}\left(p_B\right)$. The remaining technical part includes plugging (8) into (7) and then using $a_B^*\left(p_B\right)=\mathbb{E}[\theta|p_B]$ to determine $\alpha(p_B)$ for each $p_B \geq 0$ and eventually the decision maker's investment strategy a_B^* and the equilibrium price p_B^*. It is detailed in the proof for Proposition 2 in the Appendix.

What is special with project-value distribution $g(\theta)$ being fat-tailed as assumed? Equations (5)-(8) would still hold for a general distribution of project value θ_B. Thus, provided that any equilibrium exists as conjectured, these equations would hold in the equilbrium. However, only under a power-function-tail distribution of project value θ_B, these equations determining $\alpha=\alpha\left(p_B\right)$ are scale-invariant (i.e., p_B being irrelevant) at the tail, as discussed in Section 3.[12] That is, the two-way relationship among the price and the scale of operations described by Equations (5)-(8) is invariant to the realized price p_B at the tail of project value θ_B. As a result, the equilibrium can be in closed form at the tail. Further, as long as $\lambda \leq 3$, Lemma 1 implies that the analysis at the tail predominantly determines the ex ante expected price following project B at $t=0$.

These results are summarized in the following proposition. The existence of equilibrium is guaranteed by the proportion of noise trade being not too large. It is consistent with the general intuition that an equilibrium involving the feedback effect is more likely to exist with higher-quality information in price (e.g., Edmans, Goldstein, and Jiang, 2015).

Proposition 2. *In the scenario where project B is chosen at $t=0$, if $\varphi\sigma_\epsilon^2\omega_j \in \left(-\frac{2}{\lambda-1},1\right)$ for every $j\in\{1,\cdots,J\}$, there is at least one equilibrium. Conditional on large project value θ_B, the decision maker's investment strategy a_B^* and the equilibrium price p_B^* are given by*

$$a_B^*\left(p_B\right)=\sqrt{\frac{2\bar{\alpha}p_B}{2-\bar{\alpha}}} \text{ and } p_B^*=\left(\bar{\alpha}-\frac{1}{2}\bar{\alpha}^2\right)\left(\frac{\theta_B}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^2,$$

where $\bar{\alpha}\in(0,1)$ satisfies $\bar{\alpha}=\frac{\sum_{j=1}^J q_j\left(1-\varphi\sigma_\epsilon^2\omega_j\bar{\alpha}\right)^{1-\lambda}}{\sum_{j=1}^J q_j\left(1-\varphi\sigma_\epsilon^2\omega_j\bar{\alpha}\right)^{-\lambda}}$. In this limit, the scale of operations is represented as a function of θ_B and ω as follows:

$$a_B^*\left(p_B^*\left(\theta_B,\omega\right)\right)=\frac{\bar{\alpha}\theta_B}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}.$$

Equilibrium multiplicity, which may arise from a strategic interaction across financial market participants and firms, is common in the literature on feedback effects (e.g., Dow and Gorton, 1997; Goldstein, Ozdenoren, and Yuan, 2013). Even if it is the case in the current model, all main results in what follows still continue to hold for every equilibrium.

The information in the price p_B is summarized by $\hat{\theta}(p_B)$ defined in Equation (6), which consists of the project value θ_B and a mispricing term (i.e., $\varphi\sigma_\epsilon^2 a_B^*\left(p_B\right)\omega$). Compared with the framework without framework effect, such as the environment seen in Equation (1) in Section 3, a notable feature of the information in the price p_B is that the mispricing term is asymmetric across the price. Indeed, this feature is inherent in the feedback effect rather than the use of Pareto distribution, given that it would be present under other project distributions as seen in Equation (2) in Section 3. Intuitively, as the price p_B is higher (lower), informed speculators trade less aggressively because they expect that they will face more (less) risk over the firm's project due to larger (smaller) scale of operations $a_B^*\left(p_B\right)$ chosen by the decision maker at $t=2$. As a result, the equilibrium price p_B^* involves a higher (lower) proportion of noise trade so that the information in the price (i.e., $\hat{\theta}(p_B^*)$) is noisier (less noisier) about the project value θ_B.

The above asymmetry in the mispricing term arises from the fact that informed speculators change their trading aggressiveness according to their correct expectations on the scale of operations $a_B^*\left(p_B\right)$

[12]To check this formally, we plug $g(\theta)=\theta^{-\lambda}$ into Equation (7) together with Equation (8) and can then see that $\mathbb{E}[\theta_B|p_B]$ is proportional to $\hat{\theta}_B\left(p_B\right)$, which is the only term by p_B. Combined with Equation (6) and $a_B^*\left(p_B\right)=\mathbb{E}[\theta|p_B]$, this implies that $\alpha\left(p_B\right)$ is irrelevant of p_B.

and the resulting change in their risk (i.e., $\mathrm{Var}[\pi_B^*|\theta_B, p_B]$ in Equation (5)). Such asymmetry in speculators' trade leads to the following corollary regarding the expected price:

Corollary 2. *In every equilibrium in the scenario where project B is chosen at $t = 0$, we have*

$$\lim_{M\to\infty} \frac{\mathbb{E}_{\hat{g}}\left[p_B^* - \pi_B^*\right]}{\mathbb{E}_{\hat{g}}\left[\pi_A^*\right]} = 2\varphi\sigma_\epsilon^2\bar{\alpha}^2 \sum_{j=1}^{J} \frac{q_j\omega_j}{\left(1 - \varphi\sigma_\epsilon^2\bar{\alpha}^2\omega_j\right)^2} > 0.$$

The corollary tells us that the firm is ex ante valued more compared with its (ex ante) expected profit. Somewhat counterintuitively, this suggests that the absence of information about project value θ_B for the decision maker actually causes the firm to be ex ante valued more. Note that the price would be equal to the firm's expected profit on average if the project value θ_B were known to both the decision maker and informed speculators because (i) the asset supply is zero, and (ii) speculators' trading aggressiveness is symmetric between when they buy and when they sell.[13] In the current scenario under project B in the absence of information for the decision maker, the former (i) is still the case, but the latter (ii) does not apply because informed speculators face different risks between when they buy and when they sell. In particular, they face more (less) risk when they observe a higher (lower) price so that they want to sell (buy) the asset. This comes from the feature that a higher price leads to a higher operation scale via the decision maker's learning from the price, thereby increasing the remaining project-value risk.

To see this point through the lens of equilibrium derivation in Equation (5), we first fix the project value θ_B and restrict attention to how the equilibrium price p_B^* changes with the presence of noise trade ω. In the absence of noise trade (i.e., $\omega \equiv 0$), the price is identical to the firm's expected profit $\mathbb{E}[\pi_B^*|\theta_B]$, as we can easily see from Equation (5). We then compare this with what occurs in the presence of noise trade ω, which can be either positive or negative. As the noise trade ω is positive (negative) so that it positively (negatively) impacts the price p_B^*, informed speculators trade against the noise trade by selling (buying) the shares. At this point, the aforementioned asymmetry in speculators' trade indicates that these speculators correctly expect larger (smaller) scale of operations $a_B^*(p_B^*)$ and the resulting increase (decrease) in their valuation risk $\mathrm{Var}[\pi_B^*|\theta_B, p_B]$ so that they sell less aggressively (buy more aggressively). As a result, the positive (negative) noise impacts the price more (less) than what would occur if these speculators do not take into account changes in the scale of operations. Taking the average over the realization of noise trade, the price increases on average.

It is noteworthy that the highlighted price inflation is a nonnegligible proportion of the firm's expected profit in contrast to the scenario of project A. In Equation (5), we can see that the second (mispricing) term causing the price inflation (i.e., $\varphi\sigma_\epsilon^2\left(a_B^*(p_B)\right)^2\omega$) is roughly proportional to the square of the scale of operations $a_B^*(p_B)$, which is in turn proportional to project value θ_B in equilibrium, as is the first term corresponding to the firm's expected profit. Intuitively, as the project value θ_B becomes large, the price inflation grows quadratically because the equilibrium scale of operations $a_B^*(p_B^*)$ grows proportionally so that the valuation risk (i.e., $\mathrm{Var}[\pi_B^*|\theta_B, p_B] = \varphi\sigma_\epsilon^2\left(a_B^*(p_B)\right)^2$) grows quadratically, thereby increasing the overall impacts of noise trade on the price at the same pace. This stands in contrast to what occurs in the scenario of project A, where informed speculators' misvaluations from their signal error (i.e., $\theta_A^2 - \mathbb{E}[\theta_A^2|s]$) and the resulting mispricing (i.e., the third term in Equation (4)) grow only proportionally as the project value θ_A becomes large.

The general point that asymmetric trading aggressiveness of informed and risk-averse investors leads to a gap between the price and the asset fundamentals is not new in the literature. For example, an overvaluation of assets has long been explained with short-selling constraints and heterogeneous beliefs (e.g., Harrison and Kreps, 1978; Bolton, Scheinkman, and Xiong, 2006), the former of which introduce a similar sort of trading asymmetry by exogenously limiting "selling". Rather, what is new in the corollary is identifying the sort of trading asymmetry that naturally comes from informed and

[13] This first-best benchmark corresponds to a special case of project-A scenario where informed speculators' signal s is precise about θ_A.

risk-averse investors' response to the feedback effect and the resulting correlation between the price and their risk.

This crucial point of the mechanism, which corresponds to informed and risk-averse investors' aggressiveness being responsive to different valuation risks (resulting from different prices and operation scales), is comparable with Buss and Sundaresan (2023). They develop a model of financial markets with active and passive investors to explain a positive relationship between passive investors' ownership and price informativeness. The mechanism behind their result is that higher passive ownership leads to higher stock-return variances, which increase the risk facing "active investors", who are equivalent to informed speculators in our model. This incentivizes these active investors to acquire more private information. Their mechanism based on endogenous costly information takes into account the possibility that the (project-value) risk can be reduced through a further effort to acquire information, potentially counteracting the mechanism behind Corollary 2 in our model, whereby such project-value risk makes informed speculators trade less aggressively *given* their private information. However, as long as we realistically assume that such information-acquiring activity occurs ex ante in that it is independent of the price, their mechanism would not apply straightforwardly to counteract the price inflation result in the current paper.[14] Still, it generally raises the possibility that the presence of ex-ante-information-acquiring activity would further complicate the equilibrium result.

4.3 Project choice

Now we consider the choice of long-term projects A and B facing the decision maker at $t = 0$ using the equilibrium results for given projects A and B described in Subsections 4.1 and 4.2. At this point, the (short-term-oriented) decision maker follows the criterion by Definition 1, which corresponds to maximization of the ex ante expected price under the (truncated) distribution of project value at $t = 1$. The complication comes from the fact that the firm's ex ante expected price, which corresponds to the decision maker's short-term objective, does not match with its expected profit. In particular, the decision maker recognizes that the ex ante expected price $\mathbb{E}_{\hat{g}}[p^*_B]$ ($\mathbb{E}_{\hat{g}}[p^*_A]$) is higher than (the same as) the firm's expected profit $\mathbb{E}_{\hat{g}}[\pi^*_B]$ ($\mathbb{E}_{\hat{g}}[\pi^*_A]$) under project B (A), as implied by Corollary 2 (1). Despite the fact that the expected profit is higher under project A (i.e., $\mathbb{E}_{\hat{g}}[\pi^*_A] > \mathbb{E}_{\hat{g}}[\pi^*_B]$) due to his interim information about the project value θ_A, the decision maker may expect to get a higher expected price at the subsequent period by choosing project B (i.e., $\mathbb{E}_{\hat{g}}[p^*_B] > \mathbb{E}_{\hat{g}}[p^*_A]$). This occurs when project B leads to price inflation of shares which is even larger than the loss of the firm's profit.

Building on Corollaries 1 and 2 to compare between projects A and B in terms of the firm's expected profit, the following proposition identifies a sufficient condition under which project B is indeed chosen by the decision maker.

Proposition 3. *In equilibrium, the decision maker chooses project B at $t = 0$ when λ is sufficiently close to* 2.

The following example shows that the condition $\lambda \to 2$ is not a necessary one:

Example 1. If $g(\theta_B)$ has power coefficient $\lambda = 3$, we can find a simple example where the decision maker chooses project B as follows: Consider $J = 2$, $\omega_1 = \left(\varphi\sigma_\epsilon^2\right)^{-1}\delta$, $\omega_2 = -\left(\varphi\sigma_\epsilon^2\right)^{-1}\delta$, and $q_1 = q_2 = \frac{1}{2}$, where $\delta \in (0,1)$. In the proof of Proposition 3, Equation (16) provides the condition under which the decision maker chooses project B. As $\lambda \to 3$, it is equivalent to

$$\left(1 - \frac{1}{2}\bar{\alpha}\right)\frac{\left\{\sum_{j=1}^{J} q_j\left(1 - \varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^{-2}\right\}^2}{\sum_{j=1}^{J} q_j\left(1 - \varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^{-3}} > \frac{1}{2}.$$

[14] To be specific, suppose that informed speculators may choose to costly acquire private information about the part of productivity which would otherwise be unforecastable (i.e., ϵ_d for each project $d \in \{A, B\}$ in the current paper's model). Then we reasonably assume that their information-acquiring activity occurs before the realization of the price so that it is symmetric across the price. As a result, it cannot counteract the main driving force behind our mechanism that these informed speculators may trade asymmetrically in response to different prices given their private information.

As Proposition 2 implies $\bar{\alpha} \in (0,1)$ in equilibrium, it suffices to show that

$$\frac{\left\{\sum_{j=1}^{J} q_j \left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^{-2}\right\}^2}{\sum_{j=1}^{J} q_j \left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^{-3}} = \frac{\left\{\frac{1}{2}\frac{1}{(1-\bar{\alpha}\delta)^2}+\frac{1}{2}\frac{1}{(1+\bar{\alpha}\delta)^2}\right\}^2}{\frac{1}{2}\frac{1}{(1-\bar{\alpha}\delta)^3}+\frac{1}{2}\frac{1}{(1+\bar{\alpha}\delta)^3}} = \frac{\left(1+\bar{\alpha}^2\delta^2\right)^2}{\left(1-\bar{\alpha}^2\delta^2\right)\left(1+3\bar{\alpha}^2\delta^2\right)} > 1,$$

which generically holds for every $\delta \in (0,1)$.

The proposition suggests that the decision maker's short-term incentive distorts his ex ante project choice toward a project whose information is superior in the financial market. It is consistent with two observable features: First, the superiority of information in financial markets is prevalent, making the feedback effect likely observed. In the model, two projects are ex ante available for the decision maker. Among them, only one of them (i.e., B) can cause the feedback effect. Yet, the ex ante project choice drives the prevalence of project B and thus that of the feedback effect. Second, the feedback effect, which is supposed to improve the functioning of the real sector, can actually cause an inefficiency in the real sector via price inflation of equity. In the model, the inefficiency in the real sector occurs because a significant inflation of price occurs only for a certain action (e.g., project B), thereby incentivizing the short-term-oriented decision maker to choose this action, and such action is far from the first-best in terms of the firm's profit.

What would be the natural "benchmark" when it comes to the second feature above? In line with the motivating idea behind the current model on the interdependence between myopic corporate behavior and the feedback effect, we may consider two benchmark cases as follows: First, the decision maker is long-term-oriented in the sense that he maximizes the firm's expected profit (i.e., $\mathbb{E}_{\hat{g}}[\pi_d^*]$). In this case, we can easily see that the decision maker chooses project A. In particular, conditional on the realization of project values $\theta_A = \theta$ and $\theta_B = \theta$, the difference in the firm's expected profit between projects A and B is represented by

$$\begin{aligned}\mathbb{E}[\pi_A^*|\theta_A]\theta] - \mathbb{E}[\pi_B^*|\theta_B = \theta] &= \frac{1}{2}\theta^2 - \mathbb{E}\left[a_B^*\left(p_B^*\right)\theta_B - \frac{1}{2}\left(a_B^*\left(p_B^*\right)\right)^2|\theta_B = \theta\right] \\ &= \frac{1}{2}\mathbb{E}\left[\left(\theta - a_B^*\left(p_B^*\right)\right)^2|\theta_B = \theta\right] > 0\end{aligned}$$

Its strictly positive sign comes from the fact that the productivity of project A is more informative to the decision maker, allowing him to decide more efficiently at $t = 2$. Second, the decision maker commits against learning from the price at $t = 2$ in any scenario. Such commitment intuitively makes the firm's expected profit very low in the scenario of project B due to the absence of information about the project value θ_B at $t = 2$, whereas it does not influence the firm's expected profit in the scenario of project A.[15] In the absence of learning from the price, the price also reflects the firm's expected profit on average under each project. Accordingly, the decision maker always chooses project A.

Compared with the benchmark where project A is chosen to maximize the firm's profit, the loss of expected profit under project B amounts to $\mathbb{E}_{\hat{g}}[\pi_A^*] - \mathbb{E}_{\hat{g}}[\pi_B^*]$. Conditional on large project value $\theta_A = \theta_B = \theta$, it is given by

$$\begin{aligned}\mathbb{E}[\pi_A^*|\theta_A = \theta] - \mathbb{E}[\pi_B^*|\theta_B = \theta] &= \frac{1}{2}\theta^2 - \mathbb{E}\left[a_B^*\left(p_B^*\right)\theta_B - \frac{\left(a_B^*\left(p_B^*\right)\right)^2}{2}|\theta_B = \theta\right] \\ &= \frac{1}{2}\theta^2\sum_{j=1}^{J} q_j\left(1 - \frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j}\right)^2,\end{aligned}$$

where the second line is obtained by representing the scale of operations $a_B^*\left(p_B^*\right)$ in equilibrium by Proposition 2 and constant α_0 is given by the same proposition. This loss is a fixed proportion of the firm's expected profit $\mathbb{E}[\pi_A^*|\theta_A = \theta] = \frac{1}{2}\theta^2$.

[15] Admittedly, the current model is intractable with this commitment under project B due to the absence of unconditional expectation about θ_B from the viewpoint of the decision maker. However, it is not unreasonable to assume that the decision maker simply chooses zero (so that the firm's expected profit is zero) without any information about θ_B.

The decision maker's initial project choice potentially relies on fat-tailed distribution of project value, which enables us to compare multiple economic forces at work in a tractable manner. To be specific, under project A, a potential bias arising from informed speculators' signal error is present in the price, but it becomes negligible for large project value θ_A (Corollary 1). Under project B, a price inflation arises from the feedback effect and the resulting risk-driven asymmetry in informed speculators' trades in response to noise trade, and it is significant compared with the firm's profit for large project value θ_B (Corollary 2). Combined with the dominance of tail values, these can lead to the inefficiency in the project choice as shown by Proposition 3. Overall, fat-tailedness of project-value distribution ensures that the price inflation from the feedback effect under project B, which is still significant at the tail of project value, can dominate other economic forces driving changes in the price.

4.4 Scale of operations in the real sector

In this subsection, we analyze the firm's scale of operations and the resulting expenditure on these operations in equilibrium. As noted in Section 2, the total cost of operations amounts to $\frac{1}{2}a_d^2$, given the scale of operations a_d on the chosen project $d \in \{A, B\}$. Given that these variables are empirically observable in various contexts (e.g., capital expenditure and R&D expenses), their analysis provides testable predictions regarding real investments. Compared with the first-best scale of operations (i.e., θ_A) and the resulting expenditure (i.e., $\frac{1}{2}\theta_A^2$) in the scenario of project A, the following proposition presents a different pattern of these observable variables in the scenario of proejct B:

Proposition 4. *Conditional on large project values θ_A and θ_B, project B tends to have a smaller scale of operations than project A (i.e., $a_A^*(\theta_A) = \theta_A$ and $\mathbb{E}\left[a_B^*(p_B^*)\,|\theta_B\right] < \theta_B$). Further, the firm tends to spend less resources on its operations under project B compared with project A. In particular, we have*

$$\lim_{M\to\infty} \frac{\mathbb{E}_{\hat{g}}\left[\frac{1}{2}\left(a_B^*(p_B^*)\right)^2\right]}{\mathbb{E}_{\hat{g}}\left[\frac{1}{2}\left(a_A^*(\theta_A)\right)^2\right]} = \sum_{j=1}^{J}\left(\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j}\right)^2 q_j < 1.$$

The possibility of bias in the scale of operations, which leads to underspending on these operations, occurs due to infinite skewness and the resulting absence of finite expectation under fat-tailed distribution of project values θ_A and θ_B. Otherwise, the equilibrium scale of operations $a_B^*(p_B^*)$ in the scenario of project B would be unbiased about the project value θ_B (i.e., $\mathbb{E}\left[a_B^*(p_B^*)\right] = \mathbb{E}\left[\mathbb{E}\left[\theta_B|p_B = p_B^*\right]\right] = \mathbb{E}\left[\theta_B\right]$) by the Law of Iterated Expectations, and this would likely result in larger expenditure on these operations.[16] However, under fat-tailed distribution, the Law of Iterated Expectations does not necessarily apply so that ex post efficient decisions may result in a systematic error. Generally, such systematic error (or bias) could be either negative or positive.

Proposition 4 documents a consistently negative sign of the bias, which might seem to be inconsistent with the price inflation in Proposition 3. In the scenario of project B, recall that a positive (negative) noise's positive (negative) effect on the price is greater (smaller) in magnitude (Proposition 3). However, the decision maker's inference about project value θ_B takes into account the fact that higher prices are noisier, thereby fully offsetting this sort of asymmetric effects of positive and negative noises on the price. Besides, high skewness of fat-tailed distribution tends to further weaken (strengthen) the former (latter) effect of positive (negative) noise, thereby leading to a negative bias in the scale of operations. The intuition is as follows: While the positive noise increases the scale of operations $a_B^*(p_B^*) = \mathbb{E}[\theta_B|p_B = p_B^*]$ via the inference about project value θ_B, such increase in the scale of operations is weakened by the strong prior of fat-tailed distribution concentrated on small project values. Also, the symmetric argument holds to strengthen the (negative) effect of the negative noise on the scale of operations.

[16]This follows from $\mathbb{E}\left[\left(a_B^*\left(p_B^*\right)\right)^2\right] = \mathbb{E}\left[a_B^*\left(p_B^*\right)\right]^2 + \mathrm{Var}\left[a_B^*\left(p_B^*\right)\right]$, where $\mathrm{Var}\left[a_B^*\left(p_B^*\right)\right]$ may increase as project B is less informative to the decision maker.

4.5 Numerical example of equilibrium

We present an illustrative example of equilibrium involving the price and scale of operations. Here, we focus on the scenario of project B compared with the first-best, which is identical to the scenario of project A conditional on large project values in terms of the (ex ante) expected price and the equilibrium scale of operations by Corollary 1. We assume a Pareto distribution of project value $g(\theta) = 2(\theta+1)^{-3}$ for both projects. Also, suppose $J = 2$, $\omega_1 = \left(\varphi\sigma_\epsilon^2\right)^{-1}\delta$, $\omega_2 = -\left(\varphi\sigma_\epsilon^2\right)^{-1}\delta$, and $q_1 = q_2 = \frac{1}{2}$, where $\delta = \frac{3}{4}$ is fixed.[17] We closely follow the proof of Proposition 2 for numerical simulation. By Equation (13), α must satisfy

$$\frac{\left(\sqrt{\frac{2p_B}{2\alpha(p_B)-(\alpha(p_B))^2}}\left(1-\alpha\left(p_B\right)\delta\right)+1\right)^3}{\left(\sqrt{\frac{2p_B}{2\alpha(p_B)-(\alpha(p_B))^2}}\left(1+\alpha\left(p_B\right)\delta\right)+1\right)^3} = \frac{\alpha\left(p_B\right)\delta+\alpha\left(p_B\right)-1}{1-\alpha\left(p_B\right)+\alpha\left(p_B\right)\delta}$$

for every $p_B \geq 0$. Then, combined with Equations (15) and (16), it determines the decision maker's investment strategy a_B^* as a function of the price p_B and the equilibrium price p_B^* as a function of θ_B and ω.

Figure 1: Numerical results on the investment strategy of the decision maker

Figure 1 presents numerical results on the decision maker's investment strategy a_B^*. In particular, they illustrate how the decision maker chooses the scale of operations for small and large prices p_B, respectively. The scale of operations is concavely increasing in the price, as is consistent with the intuition that the price is approximately proportional to the firm's expected profit, which is quadratic to the project value θ_B.

[17]The choice of $\varphi > 0$ and $\sigma_\epsilon^2 > 0$ does not influence the equilibrium by itself in this example.

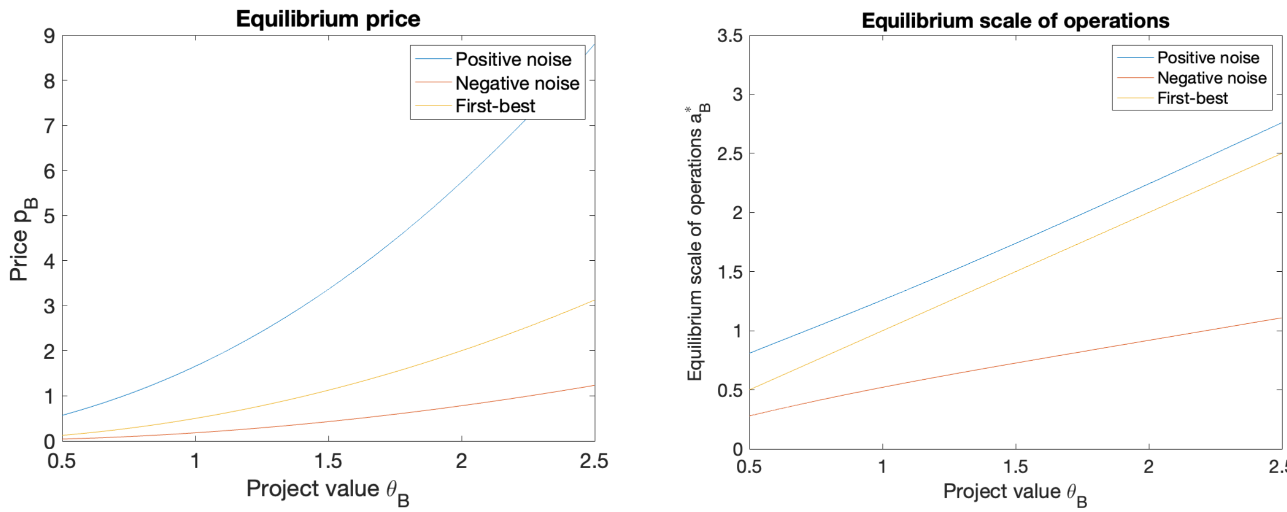

Figure 2: Numerical results on the equilibrium price and scale of operations

Figure 2 shows numerical results on the equilibrium price p_B^* and the equilibrum scale of operations $a_B^*(p_B^*)$ conditional on the realization of project value θ_B and noise trade ω. These results are presented with the corresponding ones in the first-best (i.e., θ_B is publicly known to the decision maker). They indicate that positive (negative) noise leads to a higher (lower) price, which in turn causes a larger (smaller) scale of operations compared with the corresponding price and operation scale in the first-best. They also confirm Corollary 2 in that the former (i.e., positive noise) is stronger than the latter (i.e., negative noise) in terms of its effect on the equilibrium price. Further, they confirm Proposition 4 in that the former (i.e., positive noise) is weaker than the latter (i.e., negative noise) in terms of its effect on the equilibrium scale of operations, which reflects the decision maker's Bayesian inference on project value θ_B based on the price p_B.

5 Discussion

5.1 Example of long-term corporate decision

Let us discuss an example of long-term corporate strategy captured by the current framework. As mentioned in the Introduction, a firm may choose between an innovation strategy of pursuing new knowledge and that of building on the firm's existing knowledge. In the former strategy which is called "exploration", the firm invests in patents in new technology classes, which potentially lead the firm to examine new products and markets. In contrast, in the latter strategy which is called "exploitatation", the firm focuses on producing patents in the firm's own knowledge domain, which can improve the existing products or those in proximate markets. For example, the Swiss-watch manufacturers' transition from hand-wound watches to automatic watches is a form of exploitation, as it builds on their extant mechanical engineering capabilities, whereas the emergence of battery-powered watches entails exploration from the standpoint of the mechanical energy and spring communalities (Lavie, Stettner, and Tushman, 2010). Though they are not necessarily a strict dichotomy in practice, there can be inherent substitutabilities between them arising from the scarcity of resources and their distinctive sets of skills and capabilities, which tend to make firms specialized in one side of them as in the choice between projects A and B in the model.[18]

Here, one may argue that the demand-related information is more useful for the strategy of exploitation, whose uncertainty tends to be on the demand side, whereas the quality-related information

[18]The distinction between exploration and exploitation has been extensively studied in the literature on corporate innovation. See Lavie, Stettner, and Tushman (2010) for a comprehensive review of the large literature.

on new technology and product is more important for the strategy of exploration. This argument follows from the observation that exploitative innovation meets the needs of existing customers, whereas explorative innovation is designed for emergent markets (Benner and Tushman, 2003). Combined with the fact that such demand-related information can be acquired and processed in a sophisticated way in the financial market, where informed speculators are experts in collecting publicly available data to value firms, this indicates that the informational advantage of the financial market is more pronounced for the strategy of exploitation. Symmetrically, the informational advantage of the firm's insider is more pronounced for the strategy of exploration due to its reliance on the quality-related information, which tends to be proprietary. In this regard, these strategies can be mapped into projects A and B in the model.

In this context, our analysis suggests that a firm's short-term-oriented decision maker may choose the strategy of exploitation to boost the stock price through its reliance on the demand-related information, which results in the feedback effect. Also, this can cause an overall underspending of resources in the real sector. Despite a potential challenge in quantifying different types of information involved in the strategies of exploration and exploitation, these results generally persist as long as (i) the strategy of exploitation leads to greater market feedback, thereby causing an inflation of equity price, compared with the strategy of exploration, and (ii) the distribution of productivity is fat-tailed or at least skewed so that its tail values are highly weighted.

On the empirical side, Flammer and Bansal (2017) find that executives' long-term incentives cause increases in explorative patents rather than exploitative ones, and that the total number of patents (i.e., explorative and exploitative ones) increases with these long-term incentives. In a similar vein, Gao, Hsu, and Li (2018) document that public firms' patents are more exploitative than those of private firms. They suggest that the shorter investment horizon associated with public firms is a key explanatory factor behind their findings. These are consistent with the main results in the current model, which highlight exploitation's inherent focus on serving existing customers. While the relationship between short-termism and exploitation might seem intuitive without the feedback effect due to the latter's viability in the short run, it cannot be explained straightforwardly with a financial market with informed speculation, which may value future cash flows even in the long run. Further, it is challenging to apply the existing hidden-action-based theories of short-termism (e.g., Stein, 1989; Bolton, Scheinkman and Xiong, 2006) as it is unclear whether the choice between exploration and exploitation can be regarded as a hidden action. In contrast, our analysis suggests that the strategy of exploitation can boost the short-term price, driving the inefficient focus on it, when the firm's strategy is publicly observed and the financial market contains agents who engage in informed speculation. Overall, the viability of our analysis hinges on its empirical prediction that such strategy of exploitation is publicly observable and leads to an inflated short-term price relative to that of exploration. What seems to be more direct and challenging to test is the prediction that this (average) price inflation is due to relatively sophisticated and informed portfolio investors more (less) actively trading the shares in the case of negative (positive) noise in stock price.

5.2 Empirical evidence on learning from stock prices and real investments

Our main results presented in Section 4 are generally consistent with previous empirical findings on learning from stock prices and real investments.

First, evidence on the sensitivity of corporate investment to stock price suggests that firms indeed recognize information in stock price, ruling out the possibility of ignorance or uninformative prices. Among many others, Chen, Goldstein and Jiang (2007) use two proxies of private information in stock price (i.e., price non-syncronicity and the probability of informed trading (PIN)) to document their positive effects on the sensitivity of corporate investment to stock price. In the context of M&A decisions, Luo (2005) finds a positive correlation between announcement date return and the completion of mergers. Moreover, recent studies address the causality issue inherent in these findings (e.g., Dessaint, Foucault, Fresard, and Matray, 2019; Edmans, Jayaraman, and Schneemeier, 2017). As such, there is abundant positive evidence despite its seemingly inconsistency with the general impression that corporate decision makers tend to be on the better-informed side. Our analysis at least partially explains the availability of superior information outside the firm, which is a necessary condition for the validity of feedback effect and thus is a long-standing concern in the literature. Taking for granted that there are projects for which outsiders have superior information together with projects for which they do not, it predicts the firm's choice toward the projects for which outsiders have superior information.

Second, mixed efficiency-based evidence suggests that learning from stock prices might not contribute so much to the productive efficiency (e.g., David, Hopenhayn, and Venkateswaran, 2016). This might stem from the fact that various important decisions in the real economy, such as innovation strategies of firms discussed in Subsection 5.1, feature long-term commitments, rather than allowing for immediate feedbacks from financial markets. Another related possibility is that management or shareholders may want to avoid relying on the stock price due to its potential side-effect, as shown by our model. Such commitment against relying on the stock price could be done by remaining private. Though an increase in CEO turnover driven by higher stock price informativeness appears to improve firm productivity (Bennett, Stulz, and Wang, 2020), it can also be an important factor contributing to managerial short-termism, which may essentially arise from investors' short-term incentive to boost stock prices. It is also possible that there are some long-term factors of firms' success which are not captured by measures of productivity, such as total factor productivity. More broadly, these suggest a challenge in deriving efficiency implications from improvements in price informativeness in stock markets documented in the literature (e.g., Bai, Philippon, and Savov, 2016).

Third, focusing on the short-termism side, Ladika and Sautner (2020) find that CEOs tend to cut investments when their incentives become more short-term. Similarly, Asker, Farre-Mensa, and Ljungqvist (2015) and Bernstein (2015) document that underinvestments occur in public firms compared with private firms. While underinvestments themselves could be explained with many alternative theories of short-termism (e.g., Stein, 1989), our main results connect such evidence on short-termism with learning from prices and the resulting inefficiency in long-term corporate decision.

5.3 Comparison with other trading environments

First, we may consider a large population of uninformed agents in the financial market, who behave as if they are market makers in that they trade on the firm's profit without any private information to ensure that the price is informationally efficient. Specifically, we can think of introducing a large mass of "uninformed speculators" in the sense of Kyle (1989), who trade on the firm's profit only by observing the price and recognizing its information content, into the financial market in the model, which originally consists of informed speculators and noise traders. Their trades ensure that the price is unbiased as their mass becomes large or they are risk-averse.[19] As the price is unbiased in both scenarios of projects A and B, the decision maker is incentivized to choose the long-term project that maximizes the firm's expected profit. Formally, for each project $d \in \{A, B\}$, the presence of large mass of uninformed speculators in the model ensures that the equilibrium price p_d satisfies $p_d^* = \mathbb{E}[\pi_d^* | p_d = p_d^*]$. By taking the expectation conditional on the project value θ_d, we have

$$\mathbb{E}[p_d^* | \theta_d] = \mathbb{E}\left[\mathbb{E}[\pi_d^* | p_d = p_d^*] | \theta_d\right] = E\left[\pi_d^* | \theta_d\right],$$

where the second equality comes from the Law of Iterated Expectations. That is, the price is unbiased about the firm's profit given the project value θ_d in every equilibrium. As the expected price correctly reflects the firm's expected profit, it naturally follows that the decision maker chooses project A, which delivers a higher expected profit for the firm.

Intuitively, these uninformed speculators' trades in a competitive market cause an asymmetry in "trading opportunity" for informed speculators, which tends to counteract the price-inflation result in Proposition 2 caused by an asymmetry in their trading aggressiveness. In particular, in the scenario of project B, higher (lower) noise-driven prices lead to higher (lower) operation scales, which in turn cause

[19] See Theorem 7.4 of Kyle (1989) for a formal argument of this point.

more (less) trading opportunity for informed speculators. As a result, these informed speculators tend to sell more (buy less) given their valuation risk. In a competitive market considered in the current model, this mechanism counteracts the price-inflation result in Proposition 2, thereby ensuring that the average price is unbiased as the population of uninformed speculators becomes large as above.

Second, we consider a non-competitive market, still maintaining a large population of uninformed speculators introduced above. The feedback effect in such non-competitive environment is considered in many previous studies including Boleslavsky, Kelly, and Taylor (2017) and Edmans, Goldstein, and Jiang (2015). Most notably, their equilibrium results indicate that the above intuition driving unbiased prices can be reversed. In particular, in the scenario of project B, such a large informed speculator additionally has an incentive to create more trading opportunity by selling less (buying more) in response to noise-driven higher (lower) prices, which correspond to "bad" ("good") news.[20] Such price-manipulating behavior can cause an average price inflation, as noted by Boleslavsky, Kelly, and Taylor (2017).

Combining these competitive and non-competitive cases together, one may generally argue that the effect of uninformed speculators' trades is two-faced: They weaken the risk-driven price inflation resulting from trades between informed speculators and noise traders in a competitive market, whereas they create the price-manipulation-driven price inflation resulting from trades between informed and uninformed speculators in the presence of price impact for informed ones.[21]

6 Concluding remarks

Motivated by different perspectives on secondary financial markets regarding their informational role and myopic corporate behavior, this paper offers a framework to analyze the interaction between real and financial sectors through information and risk channels and its long-term consequences in terms of market valuations and productive efficiency. We adopt Pareto distribution of productivity to ensure the tractability of analysis of feedback effect in a trading environment with CARA preferences. We show that the feedback effect and the resulting risk-driven asymmetry in speculators' trades across prices cause a price inflation of equity, thereby leading to an inefficiency in project choice. These results can be understood as inefficient focus on exploitative R&D activity, and are consistent with mixed empirical findings indicating the significance of learning from financial prices and its insignificant or relatively small contribution to firm productivity.

To derive policy implications from our analysis, it is debatable whether our efficiency criterion of firm productivity can proxy for social welfare. While firm productivity captures the use of resources in the production sector, it may conflict with other considerations in the welfare analysis. For example, hypothetically holding the shares, informed speculators would prefer a smaller scale of operations than the "first-best" in our model given their CARA utility. However, it is generally not distinguishable whether their utility comes from other investors' trading loss (e.g., less sophisticated ones captured by noise trade) or any sort of efficiency gain in the current framework with noise trade. Further, these informed speculators represent relatively short-term portfolio investors rather than the entire population of shareholders. Taking the neutral position on the welfare of financial market participants,

[20]This feature draws on the assumption that informed and uninformed agents trade with each other based on their available information. It is the case in their modeling frameworks where market makers effectively serve as the uninformed side of trades from the viewpoint of informed agents. In contrast, in our model, informed speculators trade only with noise traders, who do not respond to the price as well as any other variable in the model. This ensures that these speculators' trading opportunity does not change with the price, thereby shutting down the potential asymmetry in trading opportunity in contrast to these previous studies.

[21]Symmetrically, we can also say that the role of risk aversion is two-faced: On the one hand, it weakens the price-manipulation-driven price inflation in the presence of price impact and a large population of uninformed investors because the aforementioned price-manipulating behavior causes a large informed speculator to face more project-value risk, thereby making him profit less from his given trading opportunity. On the other hand, it creates the risk-driven price inflation in a competitive market where trades occur mainly between informed speculators and noise traders, as analyzed in the current paper.

our efficiency criterion of firm productivity can be regarded as one important policy consideration, rather than the full-fledged welfare criterion.

That being said, we can think of some policy implications concerning the inefficiency in the long-term project choice presented in our main results. First, a firm may choose to commit not to learn from the price, thereby increasing the long-term efficiency of their operations, as discussed in Subsection 4.3. Second, the financial sector involving a large population of uninformed speculators may prevent a price inflation, as seen more formally in Subsection 5.3. Third, the presence of underinvestments in the real sector does *not* imply that these underinvestments can be directly corrected with typical Pigouvian instruments such as subsidies per unit of firm investments, in contrast to what standard agency-based theories would suggest (e.g., Stein, 1989). In our main results (Proposition 4), underinvestments are ex post efficient.[22] Thus, providing subsidies may make firms even less efficient without guanranteeing a change in the ex ante project choice.

[22]In the scenario of project B in the model, the equilibrium scale of operations $a_B^*\left(p_B^*\right)$ is ex post optimal given the decision maker's available information. The sort of underinvestments here is manifested from the (ex ante) inefficiency in the project choice at $t = 0$, rather than any incentive misalignment at $t = 2$.

Appendix

Formal analysis of Section 3

In this Appendix, we present a formal analysis of feedback effect described in Section 3. Specifically, recall that Equation (2) describes the equilibrium price in the case of feedback effect where $\tilde{\theta}_B$ follows a *normal* distribution with mean zero and variance σ_θ^2, and $\tilde{\omega}$ follows a discrete or continuous distribution where $\mathbb{E}[\tilde{\omega}] = 0$. We first want to verify the intractability of analysis in this case without fat-tailed distribution. Then we move on to describe how the linearity of conditional expectation (i.e., Φ_p) is established with fat-tailed distribution of $\tilde{\theta}_B$.

Note that Equation (2) is valid only if an equilibrium exists with the feedback effect described above. To verify this formally, we want to see what form of signal about $\tilde{\theta}_B$ can be extracted from the price $\tilde{p}_B$. Assuming that informed speculators correctly recognize $\tilde{a}_B$ as a function of $\tilde{p}_B$, we can "normalize" $\tilde{p}_B$ to be a signal about $\tilde{\theta}_B$, i.e. $\hat{\theta} := \frac{\tilde{p}_B + \frac{1}{2}(\tilde{a}_B)^2}{\tilde{a}_B} = \tilde{\theta}_B + \varphi\sigma_\epsilon^2 \tilde{a}_B \tilde{\omega}$ by Equation (2). Recalling that a larger operation scale changes the firm's profit at most proportionally and the noise term quadratically in Equation (2), we can see that the latter dominates so that a larger operation scale leads to a noisier signal $\hat{\theta}$. Denote by Φ_p the conditional expectation of $\tilde{\theta}_B$ as a function of $\hat{\theta}$ in parallel with the previous case in Equation (1). Then the equilibrium scale of operations $\tilde{A}$ can be represented by a function of $\tilde{\theta}_B$ and $\tilde{\omega}$ via the above normalized signal $\hat{\theta}$, i.e.

$$\tilde{A}\left(\tilde{\theta}_B, \tilde{\omega}\right) = \mathbb{E}\left[\tilde{\theta}_B | \tilde{p}_B\right] := \Phi_p\left(\tilde{\theta}_B + \varphi\sigma_\epsilon^2 \tilde{A}\left(\tilde{\theta}_B, \tilde{\omega}\right)\tilde{\omega}\right). \tag{9}$$

In parallel with the previous case in Equation (1), we may conjecture that Φ_p is linear, say $\Phi_p(x) = kx$ for constant $k > 0$. Then we get $\tilde{A}\left(\tilde{\theta}_B, \tilde{\omega}\right) = \frac{k\tilde{\theta}_B}{1 - k\varphi\sigma_\epsilon^2\tilde{\omega}}$, provided that the support of $\tilde{\omega}$ is bounded above. How is the distribution of $\tilde{A}\left(\tilde{\theta}, \tilde{\omega}\right)$ related to the distribution of $\tilde{\theta}_B$ according to Equation (9)? As the "growth" process from $\tilde{\theta}_B$ to $\tilde{A}\left(\tilde{\theta}_B, \tilde{\omega}\right)$ is the same at all scales of $\tilde{\theta}_B$, higher (lower) values of $\tilde{\theta}_B$ tend to lead to even higher (lower) values of $\tilde{A}\left(\tilde{\theta}_B, \tilde{\omega}\right)$. As a result, the distribution of $\tilde{A}\left(\tilde{\theta}_B, \tilde{\omega}\right)$ becomes more scattered upward compared with that of $\tilde{\theta}_B$ so that the distribution of $\tilde{A}\left(\tilde{\theta}_B, \tilde{\omega}\right)$ has a "thicker" tail than that of $\tilde{\theta}_B$, rather than maintaining normality. However, this does not match with the fact that $\tilde{A}\left(\tilde{\theta}_B, \tilde{\omega}\right)$ is the conditional expectation of $\tilde{\theta}_B$ so that $\mathbb{E}\left[\tilde{A}\left(\tilde{\theta}_B, \tilde{\omega}\right)|\tilde{\theta}_B\right] = \tilde{\theta}_B$ by the Law of Iterated Expectations. Therefore, Φ_p cannot be linear under normal distribution of $\tilde{\theta}_B$.

What is different in Equations (2) and (9) due to the feedback effect (compared with the previous case without feedback effect described in Equation (1))? The scale independence described above is inherent in Equation (2), where the errror term $\varphi\sigma_\epsilon^2 \tilde{A}\left(\tilde{\theta}_B, \tilde{\omega}\right)\tilde{\omega}$ is proportional to the scale of operations $\tilde{a}_B(\tilde{p}_B) = \tilde{A}\left(\tilde{\theta}_B, \tilde{\omega}\right)$. As is mentioned in Section 3 in the main text, this property is due to the feedback effect, whereby an increase in the scale of operations makes informed speculators face higher remaining project-value risk. Combined with linear form of Φ_p, this leads to the scale independence so that a normal distribution of $\tilde{\theta}_B$ leads to a qualitatively different distribution of $\tilde{A}\left(\tilde{\theta}_B, \tilde{\omega}\right)$ having a thicker tail.

With linear form of Φ_p and the resulting scale independence, the above contradiction with normal distribution of $\tilde{\theta}_B$ comes from the fact that it is not invariant to scale-independent random growth. At this point, the only candidate distribution of $\tilde{\theta}_B$ is any distribution having a power-function tail. That is, under such power-function distribution of $\tilde{\theta}_B$ at the tail, we can see that $\tilde{A}\left(\tilde{\theta}_B, \tilde{\omega}\right) = \frac{k\tilde{\theta}_B}{1 - k\varphi\sigma_\epsilon^2\tilde{\omega}}$ also follows another power-function distribution at the tail. While a power-function distribution is mathematically undefined as a probability distribution for the entire range of $\tilde{\theta}_B$, we show in Section 4 that a Pareto distribution, which is well-defined as a probability distribution, can approximate such power function at the tail (i.e., large θ_B) and ensure the existence of equilibrium involving the feedback effect for the entire range of $\theta_B \in (0, \infty)$.

Proof of Lemma 1

Define $L(\theta) := \frac{\lambda-1}{\gamma}\left(\frac{\theta+\gamma}{\theta\gamma}\right)^{-\lambda}$ for $\theta > 0$. Then note that $g(\theta) = L(\theta)\theta^{-\lambda}$ holds, and that $L(\theta) \to (\lambda-1)\gamma^{\lambda-1}$, whose limit is defined as $\bar{L}$. For every $e > 0$, we can also define $\hat{M}(e)$ such that $|K_m(\theta) - \bar{K}_m| < e$ and $|L(\theta) - \bar{L}| < e$ for every $\theta > \hat{M}(e)$ and $m \in \{1,2\}$. Fix $e > 0$ (say, $e = 0.1$). Then, for a large number $M > \hat{M}(e)$, we have

$$\begin{aligned}
\mathbb{E}_{\hat{g}}[K_m(\theta)\theta^2] &= \int_0^M K_m(\theta)\theta^2 L(\theta_A)\theta^{-\lambda}d\theta \\
&= \int_0^M \left(\bar{K}_m + \left(K_m(\theta) - \bar{K}_m\right)\right)\left(\bar{L} + \left(L(\theta) - \bar{L}\right)\right)\theta^{2-\lambda}d\theta \\
&= \int_0^{\hat{M}(e)} \left[\bar{K}_m\bar{L} + \bar{K}_m\left(L(\theta) - \bar{L}\right) + \left(K_m(\theta) - \bar{K}_m\right)\bar{L} + \left(K_m(\theta) - \bar{K}_m\right)\left(L(\theta) - \bar{L}\right)\right]\theta^{2-\lambda}d\theta \\
&\quad + \int_{\hat{M}(e)}^M \left[\bar{K}_m\bar{L} + \bar{K}_m\left(L(\theta) - \bar{L}\right) + \left(K_m(\theta) - \bar{K}_m\right)\bar{L} + \left(K_m(\theta) - \bar{K}_m\right)\left(L(\theta) - \bar{L}\right)\right]\theta^{2-\lambda}d\theta
\end{aligned}$$

for each $m \in \{1,2\}$. On the last line above, there are 8 terms in total: 4 terms of them with integral over $\left[0, \hat{M}(e)\right]$ and the other 4 terms with integral over $\left[\hat{M}(e), M\right]$. Note that, as M becomes large, the former 4 terms are still finite, as $K_m(\theta) - \bar{K}_m$ and $L(\theta) - \bar{L}$ are bounded and the range of integral (i.e., $\left[0, \hat{M}(e)\right]$) is fixed. At the same time, the latter 4 terms diverge. Among these latter terms, the highest order of M is found in the following (first) term for each $m \in \{1,2\}$: If $\lambda < 3$, we have

$$\int_{\hat{M}}^M \bar{K}_m\bar{L}\theta^{2-\lambda}d\theta = \bar{K}_m\bar{L}\int_{\hat{M}}^M \theta_A^{2-\lambda}d\theta_A = \bar{K}_m\bar{L}\frac{M^{3-\lambda} - \hat{M}^{3-\lambda}}{3-\lambda},$$

which is of order $3 - \lambda > 0$ with respect to M and thus dominant over other terms in $\mathbb{E}_{\hat{g}}[K_m(\theta)\theta^2]$ as M becomes large. If $\lambda = 3$, the corresponding term is $\bar{K}_m\bar{L}\ln\left(\frac{M}{\hat{M}}\right)$, which is dominant similarly as M becomes large. As these hold for each $m \in \{1,2\}$, $\mathbb{E}_{\hat{g}}[K_1(\theta)\theta^2]$ and $\mathbb{E}_{\hat{g}}[K_2(\theta)\theta^2]$ have the same maximum order of M (i.e., $3 - \lambda$ or natural logarithm). This implies the lemma.

Proof of Proposition 1

At $t = 2$, the decision maker chooses $a_A^* = \theta_A$ to maximize the firm's expected profit $\mathbb{E}[\pi_A|\theta_A, p_A]$. This leads to $\pi_A^* = \frac{\theta_A^2}{2} + \theta_A\epsilon_A$ in equilibrium.

In the financial market, each informed speculator i's utility is given by Equation (3) in the main text by defining a random variable $\hat{\eta}(s) = \theta_A^2 - \mathbb{E}[\theta_A^2|s]$ for each realization of s. Note that its distribution depends on the realization of s and has zero mean (i.e., $\mathbb{E}[\hat{\eta}|s] = 0$) for every realization of s. We then plug $\theta_A^2 = E[\theta_A^2|s] + \hat{\eta}(s)$ into Equation (3) in the main text to get

$$\begin{aligned}
\mathbb{E}\left[U_{iA}|\mathcal{I}_{OA}\right] &= -\frac{1}{\varphi}\mathbb{E}\left[\exp\left\{-\varphi x_{iA}\left(\frac{\mathbb{E}[\theta_A^2|s] + \hat{\eta}(s)}{2} - p_A\right) + \frac{1}{2}\varphi^2 x_{iA}^2\sigma_\epsilon^2\left(\mathbb{E}[\theta_A^2|s] + \hat{\eta}(s)\right)\right\}|\mathcal{I}_{OA}\right] \\
&= -\frac{1}{\varphi}\exp\left\{-\varphi x_{iA}\left(\frac{\mathbb{E}[\theta_A^2|s]}{2} - p_A\right) + \frac{\varphi^2}{2}x_{iA}^2\sigma_\epsilon^2\mathbb{E}[\theta_A^2|s]\right\}\mathbb{E}\left[\exp\left\{\frac{\hat{\eta}(s)}{2}\left(-\varphi x_{iA} + \varphi^2 x_{iA}^2\sigma_\epsilon^2\right)\right\}|\mathcal{I}_{OA}\right]
\end{aligned} \tag{10}$$

Note that the last expectation term of the above equation is represented as $\exp\left(\varphi H(x_{iA}, s)\right)$, where

$$H(x_{iA}, s) = \frac{1}{\varphi}\ln\left[\sum_{k=1}^K \hat{\eta}_k(s)r_k\exp\left(\frac{1}{2}\left(-\varphi x_{iA} + \varphi^2 x_{iA}^2\sigma_\epsilon^2\right)\right)\right],$$

where $\hat{\eta}_k(s)$ is the realization of $\hat{\eta}(s)$ corresponding to the realization of s and η_k for $1 \leq k \leq K$ as follows: In particular, using the standard Bayesian rule, the conditional expectation $\mathbb{E}[\theta_A^2|s]$ is given by

$$\mathbb{E}[\theta_A^2|s] = \frac{\sum_{k=1}^{K} \theta_{Ak}^2 \Pr(\theta_{Ak}, \eta_k)}{\sum_{k=1}^{K} \Pr(\theta_{Ak}, \eta_k)} = \sum_{k=1}^{K} (s - \eta_k)^2 m_k,$$

where $\theta_{Ak} := s - \eta_k$ is defined as the (perfectly) inferred realization of θ_A for each possible value of signal noise $\eta = \eta_k$ for $1 \leq k \leq K$, which is realized with probability r_k as defined in Subsection 2.2, and m_k is defined as

$$m_k := \frac{g(\theta_{Ak}) r_k}{\sum_{k'=1}^{K} g(\theta_{Ak'}) r_{k'}}$$

for $1 \leq k \leq K$.[23] Noting that m_k can be viewed as a probability measure since $m_k \in [0, 1]$ and $\sum_{k=1}^{K} m_k = 1$, we define

$$\hat{\eta}_k(s) := \theta_{Ak}^2 - \mathbb{E}[\theta_A^2|s] = (s - \eta_k)^2 - \sum_{k=1}^{K} (s - \eta_k)^2 m_k. \tag{11}$$

Applying the above notation of $H(x_{iA}, s)$ to Equation (10), we have

$$\mathbb{E}[U_{iA}|\mathcal{I}_{OA}] = -\frac{1}{\varphi} \exp\left[-\varphi\left\{x_{iA}\left(\frac{\mathbb{E}[\theta_A^2|s]}{2} - p_A\right) - \frac{1}{2}\varphi x_{iA}^2 \sigma_\epsilon^2 \mathbb{E}[\theta_A^2|s] - H(x_{iA}, s)\right\}\right].$$

The first-order condition within the curly bracket above, which is equivalent to maximizing $E[U_i|\mathcal{I}_{OA}]$, is given by

$$\left(\frac{\mathbb{E}[\theta_A^2|s]}{2} - p_A\right) - \varphi x_{iA}\sigma_\epsilon^2 \mathbb{E}[\theta_A^2|s] - \frac{1}{\varphi}\frac{\sum_{k=1}^{K} \hat{\eta}_k(s) r_k \left(-\frac{\varphi}{2} + \varphi^2 x_{iA}\sigma_\epsilon^2\right) \exp\left(\hat{\eta}_k(s)\left(-\frac{\varphi}{2}x_{iA} + \frac{\varphi^2}{2}x_{iA}^2\sigma_\epsilon^2\right)\right)}{\sum_{k=1}^{K} r_k \exp\left(\hat{\eta}_k(s)\left(-\frac{1}{2}\varphi x_{iA} + \frac{1}{2}\varphi^2 x_{iA}^2 \sigma_\epsilon^2\right)\right)} = 0,$$

which determines $x_{iA} = x_{iA}^*$ in equilibrium. We can use the Intermediate Value Theorem to establish the existence of solution $x_{iA} = x_{iA}^*$ by taking $x_{iA} = -M'$ and $x_{iA} = M'$ for a large number M' on the left-hand side of the above equation and then showing that it is positive for $x_{iA} = -M'$ and negative for $x_{iA} = M'$.[24]

Now we can get the equilibrium price p_A^* which satisfies the market-clearing condition $\int x_{iA}^* di + \omega = 0$. Plugging $x_{iA} = -\omega$ into the above first-order condition, we have Equation (4) in the main text, where $\hat{\eta}_k(s)$ is given by Equation (11) above.

Proof of Corollary 1

The only non-trivial part is to show that the last term in Equation (4) in the main text is bounded by a constant times θ_A. We first define

$$\bar{m}_k(s, \omega) := \frac{r_k \exp\left(\hat{\eta}_k(s)\left(\frac{\varphi}{2}\omega + \frac{\varphi^2}{2}\omega^2\sigma_\epsilon^2\right)\right)}{\sum_{k'=1}^{K} r_{k'} \exp\left(\hat{\eta}_{k'}(s)\left(\frac{1}{2}\varphi\omega + \frac{1}{2}\varphi^2\omega^2\sigma_\epsilon^2\right)\right)},$$

[23] In case where s is small so that we rule out some positive realizations of η, we may redefine $\eta_1, \cdots, \eta_{K'}$, where $K' < K$, so that they are possible realizations of η given the realization of s. Then we can also redefine $\hat{\eta}_{k'}(s)$ and $m_{k'}$ in parallel with the above expressions. Even in this corner case, the analysis in what follows is still valid.

[24] Precisely, this statement follows from the fact that the last term in the above equation is bounded by a constant times θ_A given x_i, whereas the second term in the above equation is approximately proportional to θ_A^2. This is verified in the below proof of Corollary 1.

which can be viewed as a probability measure since $\bar{m}_k \in [0,1]$ and $\sum_{k=1}^{K} \bar{m}_k = 1$. Then, for each realization of ω and s, the last term in Equation (4) in the main text is given by

$$\left(\frac{1}{2} + \varphi\omega\sigma_\epsilon^2\right) \sum_{k=1}^{K} \hat{\eta}_k(s)\bar{m}_k(s,\omega) \quad \leq \quad \left(\frac{1}{2} + \varphi\omega\sigma_\epsilon^2\right) 2s\left(\bar{\eta} - \underline{\eta}\right) + \max\left(\bar{\eta}, -\underline{\eta}\right),$$

where $\bar{\eta}$ and $\underline{\eta}$ are the maximum and minimum of η_k, respectively, for $1 \leq k \leq K$, since it holds for each k and each realization of s that

$$\begin{aligned} |\hat{\eta}_k(s)| &= |\left(s - \eta_k\right)^2 - \sum_{k'=1}^{K} \left(s - \eta_{k'}\right)^2 m_{k'}| \\ &\leq \max_{k'_1,k'_2} |\left(s - \eta_{k'_1}\right)^2 - \left(s - \eta_{k'_1}\right)^2| = \max_{k'_1,k'_2} |2s\left(\eta_{k'_2} - \eta_{k'_1}\right) + \eta_{k'_1}^2 - \eta_{k'_2}^2| \\ &\leq 2s\left(\bar{\eta} - \underline{\eta}\right) + \max\left(\bar{\eta}, -\underline{\eta}\right). \end{aligned}$$

Taking the expectation of the term over the realization of ω and η given the realization of θ_A, we have

$$\mathbb{E}\left[\left(\frac{1}{2} + \varphi\omega\sigma_\epsilon^2\right) \sum_{k=1}^{K} \hat{\eta}_k(s)\bar{m}_k(s,\omega)|\theta_A\right] \leq s\left(\bar{\eta} - \underline{\eta}\right) + \frac{1}{2}\max\left(\bar{\eta}, -\underline{\eta}\right).$$

By symmetric argument, we also have

$$\mathbb{E}\left[\left(\frac{1}{2} + \varphi\omega\sigma_\epsilon^2\right) \sum_{k=1}^{K} \hat{\eta}_k(s)\bar{m}_k(s,\omega)|\theta_A\right] \geq -s\left(\bar{\eta} - \underline{\eta}\right) - \frac{1}{2}\max\left(\bar{\eta}, -\underline{\eta}\right).$$

We can then use the above bounds and Equation (4) in the main text to show that

$$|\mathbb{E}\left[p_A|\theta_A\right] - \frac{1}{2}\theta_A^2| \quad \leq \quad \left(\bar{\eta} - \underline{\eta}\right)\theta_A + \frac{1}{2}\max\left(\bar{\eta}, -\underline{\eta}\right) \tag{12}$$

for each realization of $\theta_A > 0$.

To prove the corollary, we define

$$\Delta(\theta_A) := \mathbb{E}\left[p_A|\theta_A\right] - \frac{1}{2}\theta_A^2,$$

which satisfies $|\Delta(\theta_A)| \leq \left(\bar{\eta} - \underline{\eta}\right)\theta_A + \frac{1}{2}\max\left(\bar{\eta}, -\underline{\eta}\right)$ by Equation (12). Then we set

$$K_1(\theta_A) = \frac{1}{\theta_A^2}\mathbb{E}\left[p_A|\theta_A\right] - \frac{1}{2} = \Delta(\theta_A)\theta_A^{-2} \text{ and } K_2(\theta_A) = \frac{1}{2},$$

and we can see that $K_1(\theta_A)$ is bounded for every $\theta_A \geq 0$. We apply these to Lemma 1 to get the corollary.

Proof of Proposition 2

We initially conjecture $a_B^* = a_B^*(p_B)$ and consider the first stage. Given that informed speculators correctly recognize the decision maker's investment strategy a_B^* in equilibrium, and that we can use the well-known linear property, i.e.,

$$\mathbb{E}\left[-\frac{1}{\varphi}e^{-\varphi x_{iB}(\pi_B^* - p_B)}|\theta_B, p_B\right] = -\frac{1}{\varphi}\exp\left[-\varphi x_{iB}\left(\mathbb{E}\left[\pi_B^*|\theta_B, p_B\right] - p_B\right) + \frac{1}{2}\varphi^2 x_{iB}^2 \mathrm{Var}\left[\pi_B^*|\theta_B, p_B\right]\right],$$

we can see that each informed speculator's expected utility is equivalent to

$$
\begin{aligned}
U_{iB} &\equiv x_{iB}\left(\mathbb{E}[\pi_B^*|\theta_B]-p_B\right)-\frac{\varphi}{2}x_{iB}^2\mathrm{Var}[\pi_B^*|\theta_B]\\
&= x_{iB}\left[a_B^*\left(p_B\right)\theta_B-\frac{1}{2}\left(a_B^*\left(p_B\right)\right)^2-p_B\right]-\frac{\varphi}{2}x_{iB}^2\left(a_B^*\left(p_B\right)\right)^2\sigma_\epsilon^2.
\end{aligned}
$$

Using the first-order condition, informed speculator i's optimal demand is given by

$$
x_{iB}^*\left(\theta_B,p_B\right)=\frac{a_B^*\left(p_B\right)\theta_B-\frac{1}{2}\left(a_B^*\left(p_B\right)\right)^2-p_B}{\varphi\sigma_\epsilon^2\left(a_B^*\left(p_B\right)\right)^2}.
$$

Applying this to the market-clearing condition $\int x_{iB}^* di+\omega=0$, we get Equation (5) in the main text.

Then, following the main text throughout Equations (5)-(8), we define α and then pin down it by plugging (8) into (7) and then using $a_B^*(p_B)=\mathbb{E}[\theta|p_B]$ to obtain

$$
\begin{aligned}
\alpha\left(p_B\right) &= \frac{\sum_{j=1}^J\left(1-\varphi\sigma_\epsilon^2\omega_j\alpha\left(p_B\right)\right)g\left(\hat{\theta}\left(p_B\right)\left(1-\varphi\sigma_\epsilon^2\omega_j\alpha\left(p_B\right)\right)\right)q_j}{\sum_{j=1}^J g\left(\hat{\theta}\left(p_B\right)\left(1-\varphi\sigma_\epsilon^2\omega_j\alpha\left(p_B\right)\right)\right)q_j}\\
&= \frac{\sum_{j=1}^J\left(1-\varphi\sigma_\epsilon^2\omega_j\alpha\left(p_B\right)\right)g\left(\sqrt{\frac{2p_B}{2\alpha(p_B)-(\alpha(p_B))^2}}\left(1-\varphi\sigma_\epsilon^2\omega_j\alpha\left(p_B\right)\right)\right)q_j}{\sum_{j=1}^J g\left(\sqrt{\frac{2p_B}{2\alpha(p_B)-(\alpha(p_B))^2}}\left(1-\varphi\sigma_\epsilon^2\omega_j\alpha\left(p_B\right)\right)\right)q_j}, \qquad (13)
\end{aligned}
$$

where the second line is obtained by

$$
\hat{\theta}\left(p_B\right)=\sqrt{\frac{2p_B}{2\alpha\left(p_B\right)-\left(\alpha\left(p_B\right)\right)^2}}
$$

using Equations (6) and (7).

Now we want to show the existence of solution $\alpha\left(p_B\right)\in(0,1)$ for Equation (13) for every realization of $p_B\geq 0$. Note that Equation (13) is equivalent to

$$
\begin{aligned}
&\alpha\left(p_B\right)\sum_{j=1}^J L\left(\sqrt{\frac{2p_B}{2\alpha\left(p_B\right)-\left(\alpha\left(p_B\right)\right)^2}}\left(1-\varphi\sigma_\epsilon^2\omega_j\alpha\left(p_B\right)\right)\right)\left(1-\varphi\sigma_\epsilon^2\omega_j\alpha\left(p_B\right)\right)^{-\lambda}q_j\\
= &\sum_{j=1}^J L\left(\sqrt{\frac{2p_B}{2\alpha\left(p_B\right)-\left(\alpha\left(p_B\right)\right)^2}}\left(1-\varphi\sigma_\epsilon^2\omega_j\alpha\left(p_B\right)\right)\right)\left(1-\varphi\sigma_\epsilon^2\omega_j\alpha\left(p_B\right)\right)^{1-\lambda}q_j, \qquad (14)
\end{aligned}
$$

where $L\left(\theta\right)=\frac{\lambda-1}{\gamma}\left(\frac{1}{\theta}+\frac{1}{\gamma}\right)^{-\lambda}$ as defined in the proof of Lemma 1.

Claim 1. For every $p_B>0$, there exists at least one solution $\alpha(p_B)\in(0,1)$ satisfying Equation (14), as long as $\varphi\sigma_\epsilon^2\omega_j\in\left(-\frac{2}{\lambda-1},1\right)$ for every $j\in\{1,\cdots,J\}$ as assumed in the proposition.

Proof. To prove this claim, it suffices to fix p_B and then show the following two statements: (i) at $\alpha(p_B)=0$, the left-hand side of Equation (14) is zero, which is lower than its right-hand side (i.e., $L\left(\sqrt{\frac{2p_B}{2\alpha(p_B)-(\alpha(p_B))^2}}\right)\to\left(\lambda-1\right)\gamma^{\lambda-1}$), and that (ii) as $\alpha\left(p_B\right)$ increases toward 1, the left-hand side of Equation (14) is higher than its right-hand side. Then we can apply the Intermediate Value Theorem to prove the existence of solution α for every realization of p_B.

The first statement (i) is straightforward by applying $\alpha\left(p_B\right)\to 0$ to both sides of Equation (14) and then using the assumption that $g(\theta)=L(\theta)\theta^{-\lambda}$ is finite for every $\theta\geq 0$. To prove the second statement (ii), we first note that

$$\begin{aligned}\text{LHS-RHS of Eq. (14)}|_{\alpha=1} &= \sum_{j=1}^{J} q_j L\left(\sqrt{2p_B}\left(1-\varphi\sigma_\epsilon^2\omega_j\right)\right)\left(1-\varphi\sigma_\epsilon^2\omega_j\right)^{-\lambda}\varphi\sigma_\epsilon^2\omega_j \\ &= \varphi\sigma_\epsilon^2\left(\sqrt{2p_B}\right)^{\lambda}\sum_{j=1}^{J} q_j g\left(\sqrt{2p_B}\left(1-\varphi\sigma_\epsilon^2\omega_j\right)\right)\omega_j.\end{aligned}$$

Here, we define $\hat{g}(w,p_B)=w\cdot g\left(\sqrt{2p_B}\left(1-\varphi\sigma_\epsilon^2 w\right)\right)$ for every $w\in(-\infty,\infty)$. To ensure that the above expression is positive, we want to show that $\hat{g}\left(w,p_B\right)$ is convex in ω for the range of w where $\varphi\sigma_\epsilon^2 w\in\left(-\frac{2}{\lambda-1},1\right)$. This enables us to apply Jensen's inequality to get $\sum_{j=1}^{J} q_j\hat{g}(\omega_j)>\hat{g}\left(\sum_{j=1}^{J} q_j\omega_j\right)=0$, which implies that the above expression is positive. That is, it suffices to show

$$\begin{aligned}\frac{\partial^2}{\partial w^2}\left[\hat{g}\left(w,p_B\right)\right] &= \frac{\partial}{\partial w}\left[g\left(\sqrt{2p_B}\left(1-\varphi\sigma_\epsilon^2 w\right)\right)-w\cdot\sqrt{2p_B}\varphi\sigma_\epsilon^2 g'\left(\sqrt{2p_B}\left(1-\varphi\sigma_\epsilon^2 w\right)\right)\right] \\ &= -2\sqrt{2p_B}\varphi\sigma_\epsilon^2 g'\left(\sqrt{2p_B}\left(1-\varphi\sigma_\epsilon^2 w\right)\right)+w\left(\sqrt{2p_B}\varphi\sigma_\epsilon^2\right)^2 g''\left(\sqrt{2p_B}\left(1-\varphi\sigma_\epsilon^2 w\right)\right) \\ &> 0.\end{aligned}$$

Combined with $g'<0$, it is equivalent to

$$\frac{\varphi\sigma_\epsilon^2 w}{2}\sqrt{2p_B}\frac{g''\left(\sqrt{2p_B}\left(1-\varphi\sigma_\epsilon^2 w\right)\right)}{g'\left(\sqrt{2p_B}\left(1-\varphi\sigma_\epsilon^2 w\right)\right)}<1$$

for $\varphi\sigma_\epsilon^2 w\in\left(-\frac{2}{\lambda-1},1\right)$. This inequality can be shown by dividing into two cases: First, if $\varphi\sigma_\epsilon^2 w\in(0,1)$, then the left-hand side of the inequality is negative by $g''<0$ so that the inequality always holds. Second, if $\varphi\sigma_\epsilon^2 w\in\left(-\frac{2}{\lambda-1},0\right)$, the left-hand side of the inequality is positive but it is still less than one since $\frac{\varphi\sigma_\epsilon^2 w}{2(1-\varphi\sigma_\epsilon^2 w)}\in\left(-\frac{1}{\lambda+1},0\right)$ and $\theta_B\frac{g''(\theta_B)}{g'(\theta_B)}\in[-\lambda-1,0]$. These establish the second statement (ii) above.

As mentioned above, combining the two statements (i) and (ii) together, we can apply the Intermediate Value Theorem to prove the claim. □

By the above claim, we can determine $\alpha\left(p_B\right)\in(0,1)$ for every realization of $p_B\geq 0$.Then, Equations (6) and (7) imply

$$a_B^*(p_B^*)=\sqrt{\frac{2\alpha(p_B^*)p_B^*}{2-\alpha(p_B^*)}}. \tag{15}$$

Also, plugging this into Equation (5) gives

$$p_B^*=\frac{\alpha\left(p_B^*\right)\left(1-\frac{1}{2}\alpha\left(p_B^*\right)\right)}{\left(1-\varphi\sigma_\epsilon^2\alpha\left(p_B^*\right)\omega\right)^2}\theta_B^2. \tag{16}$$

This equation implicitly determines p_B^* as a function of θ_B and ω because (i) the left-hand side is lower than the right-hand side at $p_B=0$; (ii) the left-hand side goes to infinity as $p_B\to\infty$ while the right-hand side still converges because $\alpha\left(p_B\right)\to\alpha_0\in(0,1)$; and (iii) the Intermediate Value Theorem applies.

Next we want to determine a closed-form equilibrium in the limit where $\theta_B\to\infty$. We represent the equilibrium price as a function of θ_B and ω, i.e. $p_B^*=p_B^*\left(\theta_B,\omega\right)$. Given the realization of θ_B and

ω, Equation (13) implies

$$\begin{aligned} & \alpha\left(p_B^*\left(\theta_B,\omega\right)\right)\sum_{j=1}^{J} L\left(\frac{\theta_B\left(1-\varphi\sigma_\epsilon^2\omega_j\alpha\left(p_B^*\left(\theta_B,\omega\right)\right)\right)}{1-\varphi\sigma_\epsilon^2\alpha\left(p_B^*\left(\theta_B,\omega\right)\right)\omega}\right)\left(1-\varphi\sigma_\epsilon^2\alpha\left(p_B^*\left(\theta_B,\omega\right)\right)\omega_j\right)^{-\lambda} q_j \\ = & \sum_{j=1}^{J} L\left(\frac{\theta_B\left(1-\varphi\sigma_\epsilon^2\omega_j\alpha\left(p_B^*\left(\theta_B,\omega\right)\right)\right)}{1-\varphi\sigma_\epsilon^2\alpha\left(p_B^*\left(\theta_B,\omega\right)\right)\omega}\right)\left(1-\varphi\sigma_\epsilon^2\alpha\left(p_B^*\left(\theta_B,\omega\right)\right)\omega_j\right)^{1-\lambda} q_j. \end{aligned}$$

In the limit where $\theta_B\to\infty$, we can see that the L-terms converge for every realization of ω because $1-\varphi\sigma_\epsilon^2\omega_j\alpha\left(p_B^*\left(\theta_B,\omega\right)\right)$ is bounded (strictly) above zero for every $j\in\{1,\cdots,J\}$. It follows that

$$\begin{aligned} & \alpha\left(p_B^*\left(\theta_B,\omega\right)\right)\sum_{j=1}^{J}\left(1-\varphi\sigma_\epsilon^2\alpha\left(p_B^*\left(\theta_B,\omega\right)\right)\omega_j\right)^{-\lambda} q_j \\ = & \sum_{j=1}^{J}\left(1-\varphi\sigma_\epsilon^2\alpha\left(p_B^*\left(\theta_B,\omega\right)\right)\omega_j\right)^{1-\lambda} q_j. \end{aligned}$$

Here, the solution $\alpha\left(p_B^*\left(\theta_B,\omega\right)\right)$ is independent of the realized price $p_B^*\left(\theta_B,\omega\right)$. Also, as it is a special case of Equation (13), the solution $\bar{\alpha}$ exists by Claim 1. Then Equations (15) and (16) apply with $\alpha(p_B)=\bar{\alpha}$.

Proof of Corollary 2

Fix a large number M. Taking the ex ante expectation of Equation (5) with $\hat{g}_{(0,M)}(\theta_B)$, we have $\mathbb{E}_{\hat{g}}[p_B^*]=\mathbb{E}_{\hat{g}}[\pi_B^*]+\varphi\sigma_\epsilon^2\mathbb{E}_{\hat{g}}\left[\left(a_B^*\left(p_B^*\right)\right)^2\omega\right]$, which leads to

$$\frac{\mathbb{E}_{\hat{g}}[p_B-\pi_B^*]}{\mathbb{E}_{\hat{g}}[\pi_A^*]}=\varphi\sigma_\epsilon^2\frac{\mathbb{E}_{\hat{g}}\left[\left(a_B^*\left(p_B^*\right)\right)^2\omega\right]}{\mathbb{E}_{\hat{g}}[\pi_A^*]}. \tag{17}$$

Define

$$K_1(\theta_B):=\frac{\mathbb{E}_{\hat{g}}\left[\left(a_B^*\left(p_B^*\right)\right)^2\omega|\theta_B\right]}{\theta_B^2} \text{ and } K_2(\theta_B)=\frac{1}{2},$$

the former of which is well-defined by Equations (5)-(8) in the main text. Regarding $K_1(\theta_B)$, we can show that $K_1(\theta_B)=\mathbb{E}\left[\left(\frac{a_B^*(p_B^*)}{\theta_B}\right)^2\omega|\theta_B\right]$ is bounded above over $\theta_B\geq 0$. This follows from the fact that

$$\frac{a_B^*\left(p_B^*\right)}{\theta_B}=\frac{1}{\theta_B}\sqrt{\frac{2\alpha(p_B^*)p_B^*}{2-\alpha(p_B^*)}}=\sqrt{\frac{2\alpha(p_B^*)}{2-\alpha(p_B^*)}\frac{\alpha\left(p_B^*\right)\left(1-\frac{1}{2}\alpha\left(p_B^*\right)\right)}{\left(1-\varphi\sigma_\epsilon^2\alpha\left(p_B^*\right)\omega\right)^2}}=\frac{\alpha\left(p_B^*\right)}{1-\varphi\sigma_\epsilon^2\alpha\left(p_B^*\right)\omega}$$

by Equations (15) and (16), where $\alpha\left(p_B^*\right)$ can be redefined as a function of θ_B and ω (i.e., $\alpha\left(p_B^*\left(\theta_B,\omega\right)\right)$) because p_B^* can be represented as a function of θ_B and ω in equilibrium. We can see that it is bounded above by $\alpha\left(p_B^*\right)<1$ for every ω_j by our assumption that $\varphi\sigma_\epsilon^2\omega_j<1$ for every ω_j. Combined with the below claim, we can apply $K_1(\theta_B)$ to Lemma 1 to get the corollary.

Claim 2. As $\theta_B\to\infty$, we have $K_1(\theta_B)\to\bar{K}_1$, where

$$\bar{K}_1:=\bar{\alpha}^2\sum_{j=1}^{J}\frac{q_j\omega_j}{\left(1-\varphi\sigma_\epsilon^2\alpha_0\omega_j\right)^2}>0.$$

Proof. By Proposition 2, it suffices to prove that

$$K_1(\theta_B) = \frac{1}{\theta_B^2}\mathbb{E}\left[\left(a_B^*\left(p_B^*\right)\right)^2\omega|\theta_B\right] \to \mathbb{E}\left[\left(\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^2\omega\right] > 0$$

as $\theta_B \to \infty$. This is proven by noting that

$$\begin{aligned}\mathbb{E}\left[\left(\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^2\omega\right] &= \bar{\alpha}^2\sum_{j=1}^{J}\frac{\omega_j q_j}{\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^2} \\ &> \bar{\alpha}^2\left(\sum_{j=1}^{J}\frac{1}{\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^2}\right)\left(\sum_{j=1}^{J}\omega_j q_j\right)\frac{1}{J} = 0,\end{aligned}$$

where Chebyshev's sum inequality is applied on the second line with the rearrangement of ω such that $\omega_1 > \cdots > \omega_J$ without loss of generality. □

Proof of Proposition 3

By Proposition 2, as θ_B is large, we have

$$\frac{1}{\theta_B^2}\mathbb{E}\left[p_B^*|\theta_B\right] \to \left(\bar{\alpha}-\frac{1}{2}\bar{\alpha}^2\right)\sum_{j=1}^{J}\frac{q_j}{\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^2}. \tag{18}$$

Claim 3. As $M \to \infty$, we have

$$\frac{\mathbb{E}_{\hat{g}}\left[p_B^*\right]}{\mathbb{E}_{\hat{g}}\left[\pi_A^*\right]} \to 2\left(\bar{\alpha}-\frac{1}{2}\bar{\alpha}^2\right)\sum_{j=1}^{J}\frac{q_j}{\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^2}.$$

Proof. This comes from the dominance of large values of θ_A and θ_B in the distribution $\hat{g}_{(0,M)}$. In particular, following the notation of Lemma 1, we first define

$$K_1(\theta_B) := \frac{\mathbb{E}\left[p_B^*|\theta_B\right]}{\theta_B^2} = \left(\alpha\left(p_B^*\right)-\frac{1}{2}\alpha\left(p_B^*\right)^2\right)\sum_{j=1}^{J}\frac{q_j}{\left(1-\varphi\sigma_\epsilon^2\alpha\left(p_B^*\right)\omega_j\right)^2},$$

where $\alpha\left(p_B^*\right)$ can be redefined as a function of θ_B and ω (i.e., $\alpha\left(p_B^*\left(\theta_B,\omega\right)\right)$) because p_B^* can be represented as a function of θ_B and ω in equilibrium. The first term $\alpha\left(p_B^*\right)-\frac{1}{2}\alpha\left(p_B^*\right)^2$ is bounded because $\alpha\left(p_B^*\right)$ is bounded (i.e., less than one) by Claim 1 above. Also, the second term is bounded by

$$\sum_{j=1}^{J}\frac{q_j}{\left(1-\varphi\sigma_\epsilon^2\alpha\left(p_B^*\right)\omega_j\right)^2} \leq \frac{1}{1-\varphi\sigma_\epsilon^2\bar{\omega}},$$

where $\alpha\left(p_B^*\right) < 1$ and $\bar{\omega} < 1$ is the maximum realization of ω. These imply that $K_1(\theta_B)$ is bounded over $\theta_B \in [0,\infty]$. Also, by Equation (18), as $\theta_B \to \infty$, we have $K_1(\theta_B) \to \bar{K}_1$, where

$$\bar{K}_1 := \left(\bar{\alpha}-\frac{1}{2}\bar{\alpha}^2\right)\sum_{j=1}^{J}\frac{q_j}{\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^2}.$$

Then, define $K_2(\theta) = \frac{1}{2}$ and then apply these to Lemma 1 to get the claim. □

Note that

$$\frac{E_{\hat{g}}\left[p_B^*\right]}{E_{\hat{g}}\left[p_A^*\right]} = \frac{\frac{E_{\hat{g}}\left[p_B^*\right]}{E_{\hat{g}}\left[\pi_A^*\right]}}{\frac{E_{\hat{g}}\left[p_A^*\right]}{E_{\hat{g}}\left[\pi_A^*\right]}}.$$

By Claim 3, we have $\frac{E_{\hat{g}}\left[p_B^*\right]}{E_{\hat{g}}\left[\pi_A^*\right]} \to 2\left(\bar{\alpha} - \frac{1}{2}\bar{\alpha}^2\right)\sum_{j=1}^J \frac{q_j}{(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j)^2}$ as $M \to \infty$. On the other hand, Corollary 1 states that $\frac{\mathbb{E}_{\hat{g}}\left[p_A^*\right]}{\mathbb{E}_{\hat{g}}\left[\pi_A^*\right]} \to 1$ as $M \to \infty$. Combining these together, we have

$$\frac{\mathbb{E}_{\hat{g}}\left[p_B^*\right]}{\mathbb{E}_{\hat{g}}\left[p_A^*\right]} \to 2\left(\bar{\alpha} - \frac{1}{2}\bar{\alpha}^2\right)\sum_{j=1}^J \frac{q_j}{\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^2}$$

as $M \to \infty$. Therefore, project B is chosen if and only if

$$\left(\bar{\alpha} - \frac{1}{2}\bar{\alpha}^2\right)\sum_{j=1}^J \frac{q_j}{\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^2} > \frac{1}{2}.$$

Noting that Proposition 2 implies that $\bar{\alpha}$ satisfies

$$\bar{\alpha} = \frac{\sum_{j=1}^J q_j\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^{-\lambda+1}}{\sum_{j=1}^J q_j\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^{-\lambda}},$$

we plug this into the above inequality so that project B is chosen if and only if

$$\left(1 - \frac{1}{2}\bar{\alpha}\right)\frac{\sum_{j=1}^J q_j\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^{-\lambda+1}}{\sum_{j=1}^J q_j\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^{-\lambda}}\sum_{j=1}^J q_j\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^{-2} > \frac{1}{2}, \tag{19}$$

which always holds for $\lambda \to 2$ because Proposition 2 implies $\bar{\alpha} < 1$ and Jensen's inequality yields

$$\sum_{j=1}^J q_j\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)^{-1} > \left(\sum_{j=1}^J q_j\left(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega_j\right)\right)^{-1} = 1.$$

Proof of Proposition 4

Claim 4. The following inequality holds:

$$\mathbb{E}\left[\left(\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^{\lambda-1}\right] < 1.$$

Proof. Noting that $y = x^{\frac{\lambda}{\lambda-1}}$ is a convex transformation for $\lambda > 1$, we have

$$\mathbb{E}\left[\left(\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^{\lambda-1}\right]^{\frac{\lambda}{\lambda-1}} < \mathbb{E}\left[\left(\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^{\lambda}\right]$$

by using Jensen's inequality. Also, the equation determining $\bar{\alpha}$ in Proposition 2 yields

$$\mathbb{E}\left[\left(\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^{\lambda}\right] = \mathbb{E}\left[\left(\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^{\lambda-1}\right], \tag{20}$$

which, combined with the above inequality between expectation terms, implies

$$\mathbb{E}\left[\left(\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^{\lambda-1}\right]^{\frac{\lambda}{\lambda-1}} < \mathbb{E}\left[\left(\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^{\lambda-1}\right].$$

Dividing both sides by the right-hand side, we establish the claim. □

As $\theta_B \to \infty$, we have

$$\begin{aligned}
\frac{1}{\theta_B}\mathbb{E}[a_B^*|\theta_B] &\to \frac{1}{\theta_B}\mathbb{E}\left[\frac{\bar{\alpha}\theta_B}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}|\theta_B\right] = \mathbb{E}\left[\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right] \\
&< \mathbb{E}\left[\left(\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^{\lambda}\right]^{\frac{1}{\lambda}} \\
&= \mathbb{E}\left[\left(\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^{\lambda-1}\right]^{\frac{1}{\lambda}} \\
&< 1,
\end{aligned}$$

where the second line is obtained by using Jensen's inequality, the third line holds by Equation (20), and the fourth line comes from Claim 4 above. This implies the statement of the proposition regarding the scale of operations in the limit.

Now we move on to consider the total cost of operations, which corresponds to the square of the scale of operations. As $\theta_B \to \infty$, we have

$$\frac{1}{\theta_B^2}\mathbb{E}[(a_B^*(p_B^*))^2|\theta_B] \to \frac{1}{\theta_B^2}\mathbb{E}\left[\left(\frac{\bar{\alpha}\theta_B}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^2|\theta_B\right] = \mathbb{E}\left[\left(\frac{\bar{\alpha}}{1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega}\right)^2\right] < 1, \tag{21}$$

where the last inequality follows from Claim 4 for $\lambda = 3$. Now, following the notation of Lemma 1, we define

$$K_1(\theta_B) := \frac{1}{\theta_B^2}\mathbb{E}\left[\frac{1}{2}(a_B^*(p_B^*))^2|\theta_B\right] = \mathbb{E}\left[\frac{\frac{1}{2}\alpha(p_B^*)^2}{(1-\varphi\sigma_\epsilon^2\alpha(p_B^*)\omega)^2}|\theta_B\right],$$

where $\alpha(p_B^*)$ can be redefined as a function of θ_B and ω (i.e., $\alpha(p_B^*(\theta_B,\omega))$) because p_B^* is a function of θ_B and ω in equilibrium, and the latter equality follows from Equations (15) and (16). Then we can see that $K_1(\theta_B) \to \bar{K}_1 := \frac{\frac{1}{2}\bar{\alpha}^2}{(1-\varphi\sigma_\epsilon^2\bar{\alpha}\omega)^2} < \frac{1}{2}$ by Equation (21) as θ_B becomes large so that $\alpha(p_B^*) \to \bar{\alpha}$. We also note that $K_1(\theta_B)$ is bounded because

$$\left(\frac{a_B^*(p_B^*)}{\theta_B}\right)^2 = \left(\frac{\alpha(p_B^*)}{1-\varphi\sigma_\epsilon^2\alpha(p_B^*)\omega}\right)^2$$

is bounded above because $\alpha(p_B^*) < 1$ by Proposition 2 and $\varphi\sigma_\epsilon^2\omega_j < 1$ for every ω_j. For large $M > 0$, we have

$$\mathbb{E}_{\hat{g}}\left[\frac{1}{2}(a_B^*(p_B^*))^2\right] = \mathbb{E}_{\hat{g}}\left[K_1(\theta_B)\theta_B^2\right] \text{ and } \mathbb{E}_{\hat{g}}\left[\frac{1}{2}(a_A^*(\theta_A))^2\right] = \mathbb{E}_{\hat{g}}\left[K_2(\theta_A)\theta_A^2\right],$$

where $K_2(\theta_A) := \frac{1}{2}$ for every $\theta_A \geq 0$. Applying these to Lemma 1, we get

$$\lim_{M\to\infty}\frac{\mathbb{E}_{\hat{g}}\left[\frac{1}{2}(a_B^*(p_B^*))^2\right]}{\mathbb{E}_{\hat{g}}\left[\frac{1}{2}(a_A^*(\theta_A))^2\right]} = 2\bar{K}_1 < 1,$$

as stated in the proposition.

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