Pollution diffusion, limited production factors and non monotonic growth.

Carmen Camacho, Alexandre Cornet, Weihua Ruan, Benteng Zou

The paper, very briefly:

- A **spatial growth model** for an agricultural economy where pollution diffuses in the soil.

- At each location, the **only production factor is fertile soil**, which is at the same time naturally bounded by the amount of available land, and eventually exposed to local pollution, and that diffused from neighboring locations.

- Results crucially depend on **impatience**. In the homogeneous case:
  - When agents are **very patient**, the policy maker starts by making fully fertile all land. Consumption increases with time.
  - When agents are **moderately patient**, the policy maker abates some pollution but privileges short-term consumption, which decreases with time.
  - For **highly impatient agents**, there is no abatement at any time, all land becomes fully polluted in the long-term.

Outline

1. Introduction. Motivation. Literature.

2. Modeling spatial diffusion.

3. Results.

4. Numerical Illustrations.

5. Conclusions.

1. Introduction. Motivation.

- The Status of the World's Soil Resources Report identified **soil pollution** as one of the **main threats to all the services provided by soils ecosystems** (FAO and ITPS, 2015).

- **Soil pollution**:"the presence of a chemical or substance out of place and/or present at a higher than normal concentration that has adverse effects on any non-targeted organism"

- Generally speaking, **anthropogenic activities** are the main source of soil pollution.

- We will focus here on **diffuse soil pollution**, which is mainly originated from intensive agricultural practices, and in particular from the use of **nitrogenous fertilizers**.

- **Question:** How should a policy maker **balance consumption and pollution abatement** when we acknowledge that land is bounded and that pollution is not a local matter?

1.2. Nitrogenous fertilizers, arsenic and more.

- **Nitrogenous fertilizers** are the most common fertilizer nowadays, and on top of the contain of arsenic, they **contribute to global warming, via the release of N_2O to the atmosphere, pollution of groundwater, and acidification of soils.**

- Among all heavy metals, **arsenic** deserves attention because of its mobility, toxicity and longevity. Arsenic accumulates in the soil, and according to some studies, it could remain in the soil for 9000 years (see Mc Curdy, 1986, and references therein).

- Arsenic is not used any longer as a main fertilizer, but nitrogenous fertilizers include arsenic in minimal amounts, which still present a danger to human health.

- Despite all this, the **use of nitrogenous fertilizers in agriculture keeps increasing every year**. If the worldwide consumption of nitrogenous fertilizer attained 60 million tonnes in 1980, it reached 110 million tonnes in 2014 (FAO, 2015). As reported by Martinez et al. (2021), by 2050 the nitrogen pollution level is expected to be 150% higher than in 2010.

1.3. Literature on soil conservation.

- Economics of soil conservation has a long tradition of **interdisciplinary** thinking.
- In the **early days** of soil economics, Ciriacy-Wantrup (1968) borrowed from ecology the concept of damage thresholds to study irreversible damages due to agricultural production.
- **A second wave** of theoretical research in the 1980's analyzed interactions between agricultural practices and soil fertility (soil loss, topsoil depth, net farm income, and technological progress), Pope et al. (1983), Saliba (1985), region specific Segarra et al. (1987) and Barbier (1990).
- **In the 2000's**, the question of soil conservation shifted to the context of developing countries where it is thought that better soil management practices could lead to the highest potential gains (Antle et al. (2006), Hagos et al. (2006), de Graaf et al. (2008), Stephens et al. (2012), Barrett et al. (2015), Bevis et al. (2017), or Berazneva et al. (2018)).

1.4. Literature on diffusion in spatial economics:

- Technically, our paper belongs to the literature in **diffusion in spatial economics.**
- Building on Puu (1999), Brito (2004) wrote the first Ramsey-type model with **diffusion** of capital across space, in which k is spatially distributed over a region $R \subseteq \mathbb{R}$

$$k_t(x,t) = k_{x,x}(x,t) + f(k(x,t)) - \delta k(x,t) - c(x,t) \tag{1}$$

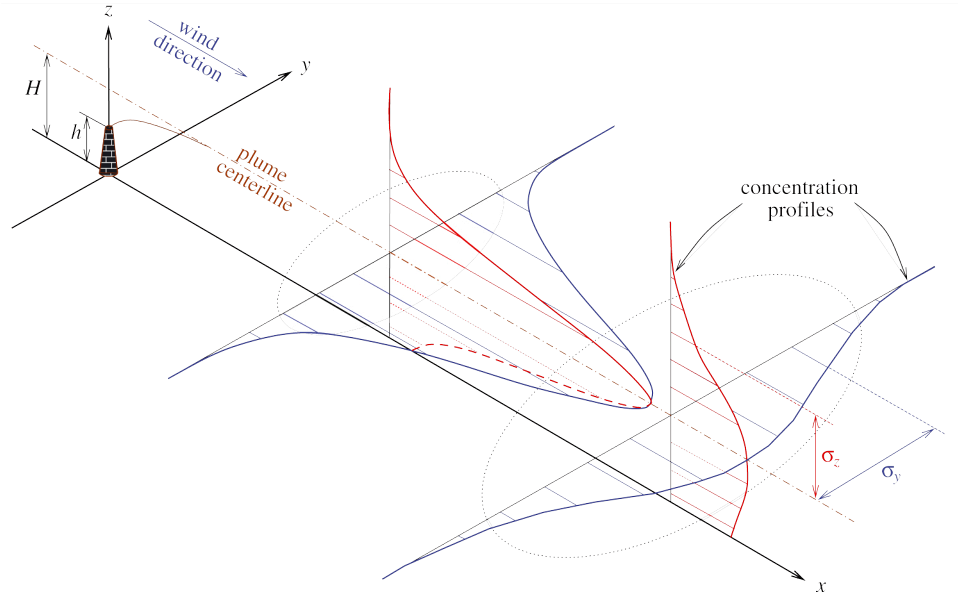
  where $k_{x,x}(x,t)$ is the net *trade (capital) balance* at location $x \in R$ and at time $t \geq 0$.
- To summarize the 'challenges' faced by these spatial/diffusion optimization models:
  - In general, one cannot proof there exists a **unique** solution;
  - In general, one cannot provide with an **analytical** solution;
  - It is a challenge even to perform **numerical simulations**, they are sensitive...

Technical Contributions:

- The **Solow version** can be solved analytically (Camacho and Zou, 2003, Neto and Claeyssen, 2015, Capasso et al., 2015, Xepapadeas and Yannacopoulos, 2016);
- The **Spatial Ramsey** model has been solved and studied using **Optimal control** in Boucekkine et al. (2009) with linear utility; and in Boucekkine et al. (2013) and (2019) using an AK production function and **Dynamic Programming** in infinite dimension for impatient enough agents.

Contributions in Environmental Economics:

- Beyond physical capital, diffusion models have been applied to study the **diffusion of air pollution** (Camacho and Perez-Barahona, 2015, Frutos and Martin-Herran, 2019, De la Torre et al., 2015). These papers rely on the Gaussian plume:



In one dimension, the Gaussian plume writes as

$$p_t = dp_{xx} + E(x).$$

- The closest works to our paper are Augeraud-Veron et al. (2019) and (2021), which study **ground water pollution** due to agriculture activities. Main difference is that in these works, productivity does not decrease with pollution.

2. Soil pollution diffusion in a linear growth model.

- We consider a closed economy, where both land and households are distributed over the unit circle on the plane, $\mathcal{S} = \{(\sin\theta, \cos\theta) \in \mathbb{R}^2 : \theta \in [0, 2\pi]\}$.

- Each location $\theta \in [0, 2\pi]$ is populated by $N(\theta)$ individuals and is endowed with an amount of land $L(\theta)$, constant in time.

- Land is made of polluted and fertile land $L(\theta) = L_P(t, \theta) + L_F(t, \theta)$.

- Production is linear in fertile land, $Y(t, \theta) = A(\theta) L_F(t, \theta)$.

- Y can be consumed or invested in abatement, with $\phi(\theta)$ the local abatement technology.

- We assume that polluted land at location θ at time t evolves according to

$$\frac{\partial L_P}{\partial t} = D\frac{\partial^2 L_P}{\partial \theta^2} + \nu A L_F - \phi(A L_F - C)$$

where

* D: diffusion parameter, $D \geq 0$;

* AL_F: production, that we can also write $A(L - L_P)$;

* ν: local sensitivity of fertile soil to pollution;

* $C(t, \theta)$: total consumption;

* $\phi(\theta)$: local pollution abatement efficiency.

- The policy maker aims at maximizing overall welfare over $\mathcal{S}$, a function of consumption per capita, $c\,(t,\theta) = \frac{C(t,\theta)}{N(\theta)}$.

- Knowing that the **policy maker** discounts time at a constant rate ρ, her problem writes as

$$\max_{c} \int_0^{\infty} e^{-\rho t} \left[\int_0^{2\pi} \frac{c(t,\theta)^{1-\sigma}}{1-\sigma} N(\theta) d\theta \right] dt, \tag{2}$$

subject to

$$\frac{\partial L_P}{\partial t} = D \frac{\partial^2 L_P}{\partial \theta^2} + A\,(\phi - \nu)\,(L_P - L) + C\phi,$$

and

$$\begin{cases} 0 \leq L_P(t,\theta) \;\leq\; L(\theta), \\ 0 \leq c(t,\theta) \leq \frac{A(\theta)[L - L_P(t,\theta)]}{N(\theta)}. \end{cases} \tag{3}$$

- We also assume that $L_P(t,0) = L_P(t,2\pi)$ and that the initial distribution of L_P is known.

- Using that $L(\theta) = L_P(t, \theta) + L_F(t, \theta)$, we can write the problem in L_F instead:

$$\max_{c} \int_0^{\infty} e^{-\rho t} \left[\int_0^{2\pi} \frac{c(t,\theta)^{1-\sigma}}{1-\sigma} N(\theta) d\theta \right] dt, \tag{4}$$

subject to

$$\frac{\partial L_F}{\partial t} = D \frac{\partial^2 L_F}{\partial \theta^2} + A(\phi - \nu) L_F - \phi N c,$$

and

$$\begin{cases} 0 \leq L_F(t,\theta) \leq L(\theta), \\ 0 \leq c(t,\theta) \leq \frac{A(\theta) L_F(t,\theta)}{N(\theta)}. \end{cases} \tag{5}$$

3. Analytical results in the homogeneous economy

- Let us assume A, ϕ, ν, N and $L_F(0,\theta) \equiv L_F^0$ are positive constants.

- Optimal decision depends on the relative size of the time discount rate:

CASE 1: Small time discount. If $0 < \rho < \alpha = A(\phi - \nu)$ and $0 < \sigma < 1$, land becomes pollution free in finite time T, solution to the equation

$$P\left(\frac{L_{F,0}}{L}\right)^{-1/\sigma}\left[e^{\alpha T} - e^{gT}\right] = (\alpha - g)\left[\frac{L_{F,0}}{L}e^{\alpha T} - 1\right],$$

where P is the derivative of the value function and $g = \frac{\alpha-\rho}{\sigma} > 0$. **Optimal consumption increases at constant rate g from $t = 0$ to T**, and is given by

$$c(t) = \begin{cases} \frac{L}{\beta}P\left(\frac{L_{F,0}}{L}\right)^{-1/\sigma}e^{gt} & \text{for } 0 < t < T, \\ \frac{\alpha L}{\beta} & \text{for } t \geq T. \end{cases} \tag{6}$$

CASE 2: Intermediate discount. If $\alpha = A(\phi - \nu) \leq \rho < A[\phi - \nu(1-\sigma)]$, the optimal trajectories for consumption and associated polluted land are

$$c(t) = \frac{\sigma L_{F,0}}{L[\rho - \alpha(1-\sigma)]} e^{gt} \qquad \text{for } t > 0, \tag{7}$$

with $g = \frac{\alpha - \rho}{\sigma} < 0$, and

$$L_F(t) = L_{F,0} e^{gt} \qquad \text{for } t > 0. \tag{8}$$

There is always some abatement, consumption decreases from $t = 0$ and all land becomes polluted in the long term.

CASE 3: Large time discount. Let us assume $\rho > \alpha + \sigma A \nu = A\left[\phi - \nu(1-\sigma)\right]$, then the policy maker decides to consume all production at all times (i.e. no abatement ever):

$$c\left(t\right) = \frac{A}{N} L_F^0 e^{-A\nu t}, \quad L_F\left(t\right) = L_F^0 e^{-A\nu t} \qquad \text{for } t > 0.$$

Clearly, $\lim_{t\to\infty} L_P\left(t\right) = L$.

Initial c and L_F are at the highest in this case and they also decrease faster with time.

5. Heterogeneous economy. Numerical Illustrations.

ϕ	Abatement efficiency	0.3
ν	Pollution sensitivity	0.2
σ	Utility parameter	0.5
D	Diffusion parameter	0.1
L	Maximum Fertile land	1
$L_P(0)$	Initial polluted land	0.6

As in Boucekkine et al. (2019), let us assume there is a **technological pole** around location $\theta = 5\pi/4$ and a **population concentration** around $\theta = 3\pi/4$. In particular, we assume the following functional forms:

$$A(\theta) = \begin{cases} 0.4 \text{ if } \theta \in [0, \pi) \cup (3\pi/2, 2\pi], \\ 0.4 - 1.6(\theta - \pi)(2\theta - 3\pi)/\pi^2, \text{ elsewhere} \end{cases}$$

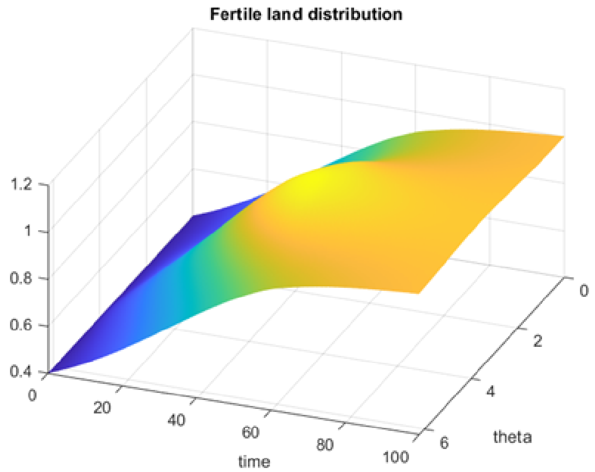
and

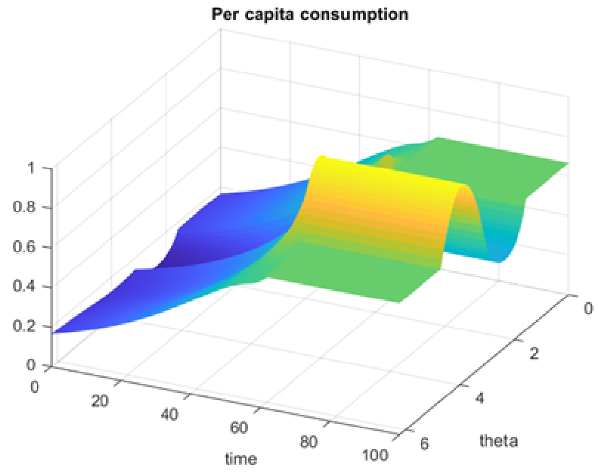
$$N(\theta) = \begin{cases} 0.2 \text{ if } \theta \in [0, \pi/2) \cup (\pi, 2\pi], \\ 0.2 - 0.8(\theta - \pi)(2\theta - \pi)/\pi^2, \quad \text{elsewhere.} \end{cases}$$

- As in the homogeneous case, there exists a key **threshold for** ρ, which is ≈ 0.0436.

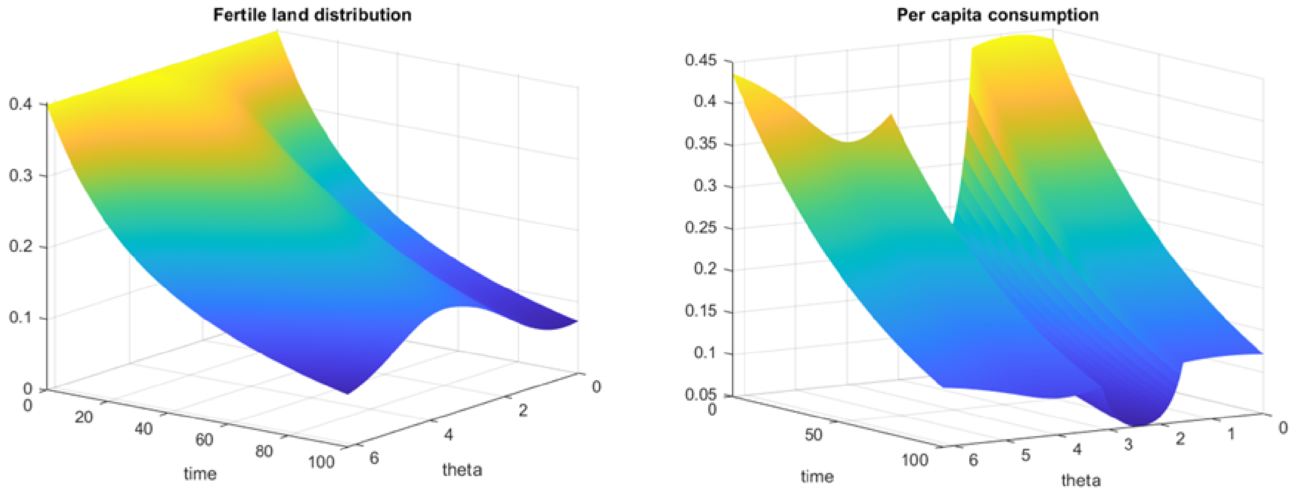
- We set ρ to 3% and 5% so that one is less than and the other greater than λ_0. Note that 3% time discount is also used in Boucekkine et al. (2019) and Lopez (2008).

Small time discount, $\rho = 0.03$.





Large time discount, $\rho = 0.05$**.**



4. Summary and Conclusions

- We develop a **spatial growth model with bounded production factors** to account for the diffusion of pollution in agricultural soils;
- The optimal solution depends on the time discount, and it may be non-monotonic;
- Moreover, the limitedness of the production factor shapes the optimal solution.