College as human capital investments or tournament: A macroeconomic analysis *

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Abstract

A college education is commonly viewed as a *productive* investment in the human capital theory. However, it also serves a *competitive* purpose to stand out in the job market, like a tournament contest. In this paper, I study the relative importance of these two channels in determining college attendance. I build a general equilibrium life-cycle model with a college decision, human capital investments, and skill allocation. The model's novelty is that workers are allocated to different occupations based on their relative ranking of human capital, so college education also has a competitive value. Results show that the competitive channel accounts for 39% of college attendance and decreases aggregate output by 1.3%. In addition, I evaluate the optimal policy of taxation and college subsidy, and find that lowering college subsidy and the progressivity of labor income tax would increase social welfare by 5.9%. This policy system mitigates over-investment in human capital and alleviates negative externalities brought by the competitive margin.

Keywords: Human Capital, Taxation, Education Policy

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1 Introduction

A college education is typically viewed as an instrument to accumulate human capital before entering the labor market, which makes workers more *productive*. Meanwhile, it also serves the *competitive* purpose as a way of standing out in the job market. For instance, people who graduate from prestigious colleges are more likely to find better jobs.[1]

In this paper, I study college education with both *productive* and *competitive* values, and assess the relative importance of these two channels. The quantification is crucial for policy implications, especially for the college subsidy. If the productive channel is more important, then subsidizing college could be beneficial as it encourages more people to accumulate human capital. However, if the competitive channel dominates, then college subsidy could encourage over-investment in college education and hence lead to negative externalities.

I first use a static model of college education to illustrate the competition mechanism and its associated externality. Workers with heterogeneous costs make human capital investment to increase their efficiency units. They are allocated to different occupations based on their accumulated human capital: workers with higher human capital are matched to more productive occupations.[2] As a result, the relative ranking of one's "after-college" human capital determines the marginal productivity of his efficiency units, so education also has a *competitive* value.

I show analytically that this competitive margin encourages over-investment in human capital and leads more people to attend college compared to the case without competition. The intuition is very similar to the rat race model in Akerlof (1976). Workers would accumulate excessive human capital in order to keep up with their peers and avoid falling behind.

[1] This is well explored in the literature through different mechanisms such as tournament (Lazear and Rosen (1981)) or signaling (Spence (1973)).

[2] This is micro-founded by a frictionless two-sided matching market as in Hopkins (2012) and Chade and Eeckhout (2016). The allocation satisfies positive assortative matching such that the worker with the highest human capital is matched with the occupation with the highest productivity.

To assess this competitive channel quantitatively, I build up a general equilibrium life-cycle model with college decisions, human capital investments, and skill allocation. Workers decide to attend college and make human capital investments or go to work directly based on their initial human capital and learning ability. Upon entering the labor market, they will be allocated to different occupations through a job market based on their relative ranking of human capital. During the working stage, people also make human capital investments to maximize lifetime earnings. I embed the model in a general equilibrium environment where the government collects progressive taxes to finance the college subsidy, a lump-sum transfer to those who attend college. Furthermore, the wage rates across occupations are endogenously determined in the equilibrium.

The model is then parameterized to match the facts in several dimensions: the incidence of matching between education and occupation, the earnings structure across education and occupation, life-cycle earnings patterns as well as the moments related to college subsidies.

Findings My quantitative analysis suggests that the competitive channel accounts for 39% of the college attendance while the productive channel accounts for 53%, with the rest explained by the interaction. Furthermore, the competitive (productive) channel decreases (increases) aggregate output by 1.3% (0.4%).

To shut down the productive channel, I assume human capital investment at college only affects skill allocation but not efficiency units supplied to the labor market. College attainment drops from 31.6% to 12.4%, which represents the fraction of college attendance purely driven by the competitive incentive. Moreover, output per worker drops 2.5% and aggregate output decreases by 0.4%. The wedge is caused by the increase in the workforce as more people skip the college stage and enter the labor market at an early age. As a result, the total labor inputs increased though the average labor quality declined due to the lack of human capital investments in college.

To isolate the competitive channel, I assume that skill allocation is based on initial human capital so human capital accumulation at college only affects the amount of efficiency units. The college attainment rate drops from 31.6% to 16.8%, which means the human capital channel solely explains 53% of the college attendance.

Moreover, the interaction of these two channels completely changes the patterns of college attendance. In the absence of the competitive channel, only workers with low initial conditions choose to attend college since their time cost is relatively low. People with high initial conditions prefer to accumulate human capital at work instead. However, when combined with the competitive channel, workers with low initial conditions will skip the college stage and those with high learning ability will attend college to acquire more human capital. The competitive channel brings additional incentives for college attendance and crowds out people with low initial conditions from the college education.

I also evaluate the optimal policy system that maximizes average social welfare in the steady state. I find that eliminating college subsidies with lower progressivity of labor income tax will increase social welfare by 5.9% and the aggregate output by 11.3%. The intuition is that such a combination of policies will alleviate the unnecessary competition among the bulk of the population and only incentivize people with high initial conditions to attend college. Therefore, it minimizes negative externalities brought by the competitive margin of the college education. Specifically, 15.6% of people would switch from the college track to the non-college track after the policy reform, and their welfare increases by 9.6% on average. The elimination of college subsidies frees them up from the rat race competition in the college stage so that they can enter the labor market directly.

Related literature This paper contributes to the macroeconomic literature on college attainment by adding the competitive incentive to human capital accumulation. Previous studies mainly focus on the productive value of a college education, like Ionescu (2009), Restuccia and Vandenbroucke (2013), Hendricks and Leukhina (2018) and Donovan and Herrington (2019). Workers accumulate human capital not only to increase efficiency units but also to match with high-skill occupations.

My work also contributes to the discussion on the tradeoff between college subsidy and taxation as in Benabou (2002), Bohacek and Kapicka (2008), and Matsuda and Mazur (2022). My finding is contrary to Krueger and Ludwig (2016) where they find college subsidies should be increased. The reason is that they only focus on the productive value of college education. When the competitive margin of a college education is taken into account, college subsidies could also generate inefficiency through the rat race competition and encourage people to over-invest in human capital. Therefore, the optimal policy should mitigate negative externalities generated by this competition channel.

In addition, my paper links to the literature on overeducation with similar empirical findings as in Leuven and Oosterbeek (2011). In particular, my work complements the literature by explaining overeducation through the lens of skill allocation in a general equilibrium framework. Overeducation occurs because of the shortage of high-skill occupations and frictions generated by skill allocation. The novelty is to endogeneize the allocation process by introducing a two-sided matching mechanism.

The paper is organized as follows. Section 2 introduces a static model of college with competition. Section 3 describes the general equilibrium life-cycle model. Section 4 dicusses empirical findings and parameterization. In section 5, I conduct counterfactual experiments to quantify the competition channel. In section 6, I derive the optimal taxation. Section 7 introduce financial frictions as an extension and Section 8 concludes.

2 College Education with Competition

In this section, I introduce a static model of college education with competition. Workers are heterogenous in their initial human capital, which affects their cost of human capital accumulation. They are allocated to different occupations based on their relative rankings in the distribution of "after-college" human capital, which directly determines their marginal productivity of human capital. So human capital accumulation not only affects their efficiency units but also affects their wage rate of efficiency units.

The purpose of this exercise is to illustrate that this competition channel generates negative externalities and leads to over-investment in human capital accumulation, in comparison with the model without competition, like the rat race model in Akerlof (1976). In addition, the introduction of competition encourages more people to attend college.

2.1 Matching

There is a continuum of workers with heterogenous human capital h which follows a cumulative distribution function $F(h)$. There is also a measure one of occupations with productivity w drawn from the uniform distribution $U[\underline{w}, \bar{w}]$. Workers and occupations meet with each other in a frictionless market and form matches. The matching surplus follows the functional form $y(w,h) = w \cdot h$ therefore the stable allocation satisfies positive assortative matching as in Becker (1973).

The matching function $\mu(.)$ maps worker with human capital h to occupation w such that

$$\mu(h) = F(h)(\bar{w} - \underline{w}) + \underline{w} \tag{1}$$

For example, if a worker ranks 50th percentile in the distribution ($F(h) = 1/2$), he will be matched to the occupation with median productivity $w = (\bar{w} + \underline{w})/2$.

For simplicity I assume there is no outside option for each side and the surplus is split through Nash barganing with equal shares. Therefore the payoff (or earnings) of the worker with human capital h is given by

$$\pi(h) = \frac{1}{2} \cdot h \cdot (F(h)(\bar{w} - \underline{w}) + \underline{w}) \tag{2}$$

Alternatively, we can re-write the earnings as the product of human capital h and a wage rate function $w(h)$:

$$w(h) = \frac{1}{2} \cdot F(h)(\bar{w} - \underline{w}) + \underline{w} \tag{3}$$

Hence, the relative ranking of one's human capital in the distribution also determines one's wage rate.

2.2 College education with competition

I first introduce a model of college education without competition as the benchmark. A worker with initial human capital h_0 maximize utility by choosing effort x:

$$\begin{aligned} \max_{x \geq 0} \quad & h' \cdot w(h_0) - x - \mathbb{1}\{x > 0\} \cdot C \\ \text{s.t.} \quad & h' = h_0 + (h_0 x)^\gamma \end{aligned} \tag{4}$$

where h' is the human capital after college and C is the fixed cost of college education. A worker can also skip the college stage and avoid the fixed cost C by choosing $x = 0$. The wage function w is described in Equation (3) where $F(h)$ is the CDF of the initial human capital. One can interpret it as if the matching process happens before the college stage.

Given the functional form of human capital production, it can be easily shown that there exists a cutoff value h^* such that that worker is indifferent between attending college and not attending college

$$h'(h^*) \cdot w(h^*) - x(h^*) - C = h^* \cdot w(h^*) \tag{5}$$

where $x(h^*)$ is the optimal effort and $h'(h^*) = h^* + (h^* x(h^*))^\gamma$ is "after-college" human capital. Workers with initial condition higher than h^* will attend college and make human capital investment.

Taking first order condition of Equation (4) generates the optimal effort for those who attend college:

$$x(h) = w(h)^{\frac{1}{1-\gamma}} \cdot \gamma^{\frac{1}{1-\gamma}} \cdot h^{\frac{\gamma}{1-\gamma}} \tag{6}$$

The optimal effort is strictly increasing in initial human capital as $\gamma \in (0,1)$

Next, I introduce competition in the above model. In particular, I assume the matching process occurs after the college stage.

$$\begin{aligned} \max_{x \geq 0} \quad & h' \cdot w(h') - x - \mathbb{1}\{x > 0\}C \\ \text{s.t.} \quad & h' = h + (hx)^{\gamma} \end{aligned} \tag{7}$$

The difference comes from the new wage function:

$$w(h') = \frac{1}{2} \cdot G(h')(\bar{w} - \underline{w}) + \underline{w} \tag{8}$$

where $G(h')$ is the CDF of "after-college" human capital, which is an equilibrium outcome. Similarly, we can show that there also exists a cutoff value of h^* such that for any worker with initial human capital higher h^* he would attend college and make human capital investment.

Definition 1. The competitive equilibrium consists of college investment decisions $x(h)$, a wage function $w(h)$ and a "after-college" human capital distribution $G(h)$ such that[3]

1. College investments decisions $x(h)$ solve optimization problems.

2. The wage function satisfies: $w(h) = \frac{1}{2}(G(h) \cdot (\bar{w} - \underline{w}) + \underline{w})$

3. The "after-college" human capital distribution satisfies consistency:

$$F_0(h) = G(h'(h)) \quad \forall h$$

[3]Bodoh-Creed and Hickman (2018) proves there exists a monotone pure strategy nash equilibrium (PSNE) where each agent has a unique optimal response. Therefore I only focus on PSNE in the following analysis.

where $h'(h) = h + (h \cdot x(h))^\gamma$ is optimal "after-college" human capital and F_0 is the CDF of initial human capital.

2.3 Comparison with the model without competition

The question of interest is to look at human capital investments and college attendance across these two models. For illustration purpose, I use subscript 1 to denote functions and equilibrium outcomes in the model without comeptition and use subscript 2 for the model with competition.

Proposition 1. *The equilibrium allocation between workers and occupation are the same in both models. So the wage functions are the same in the equilibrium:*

$$w_1(h) = w_2\Big(h + (h \cdot x_2(h))^\gamma\Big) \quad \forall h \tag{9}$$

where $x_2(h)$ is the optimal investment in the model with competition.

This result comes from the fact that the optimal effort is monotone in initial condition. So workers with high h_0 will always accumulate more human capital. Therefore the ranking of "after-college" human capital is the same as the ranking of initial human capital.

Proposition 2. *For workers who attend college in both models, they spend more effort in human capital investment in the model with competition compared to the model without competition:*

$$x_2(h) > x_1(h) \tag{10}$$

The first order condition of Equation (7) generates the optimal effort with competition:

$$x_2(h) = \big(w_2(h') + h' \cdot g(h') \cdot (\bar{w} - \underline{w})\big)^{\frac{1}{1-\gamma}} \cdot \gamma^{\frac{1}{1-\gamma}} \cdot h^{\frac{\gamma}{1-\gamma}} \tag{11}$$

where $g(h')$ is the probability density function of the distribution of "after-college" human capital.

Recall that the optimal effort in the model without competition is given by:

$$x_1(h) = w_1(h)^{\frac{1}{1-\gamma}} \cdot \gamma^{\frac{1}{1-\gamma}} \cdot h^{\frac{\gamma}{1-\gamma}} \tag{12}$$

As mentioned in Proposition 1, the wage rate functions are the same in two cases as the relative rankings do not change. It is obvious that $x_2 > x_1$ when the term $h' \cdot g(h') \cdot (\bar{w} - \underline{w})$ is positive, which I denote as the *competitive margin*.

Human capital accumulation can not only increase efficiency units h' but also increase the wage rate through competition. Workers who attend college would like to spend more effort just to keep their relative advantage in the ranking competition. This competitive margin depends on the difference in the density of the distribution $g(h')$ as well as dispersion in payoff $(\bar{w} - \underline{w})$.

Furthermore, workers who over-invest in human capital are worse off compared to the benchmark case without competition. Since x_1 is the optimal effort derived from Equation (4), anything higher than x_1 would be sub-optimal. Therefore the optimized utility level in Equation (7) is lower than Equation (4) as the wage rates in the equilibrium are the same. Put it differently, the competition channel generates negative externalities among people who attend college.

Proposition 3. *Suppose the worker with h_1 is indifferent between college and non-college in the model without competition and the worker with h_2 is indifferent between college and non-college in the model with competition. Then we have:*

$$h_2 < h_1 \tag{13}$$

This proposition indicates that the introduction of competition leads to more college attendance as the cutoff value decreases. In the benchmark case without competition, workers face the same wage rate regardless of attending college or not. However, since the matching happens after the college stage, workers will face different wage rates depending on their human capital investments. In addition, as previous discussed, competition encourages over-investment in human capital which further distorts the distribution of “after-college” human capital. For the indifferent worker h_1, he would prefer to attend college even though over-investment results in a lower utility level in the model with competition. This is because his outside option of not attending college also decreases and the magnitude is even larger. Therefore the new cutoff value with competition is lower and college attendance increases.

3 A Life-cycle Model of College Education

The static model illustrates that mechanism of competition, where wage rates are determined by the relative ranking, leads to over-investment in human capital and encourages more college attendance. In this section, I build up a general equilibrium life-cycle model and quantify the relative imporant of the competitve margin in determining college attendance.

The model consists of three building blocks. The first one is a standard life-cycle model with a college decision and human capital investments. Workers, who are heterogeneous in initial human capital and learning ability, decide whether or not to attend college and make human capital investments, which serves the *productive* purpose. They also accumulate human capital over the life-cycle to maximize lifetime earnings.

The second block is a skill allocation process where workers are sorted into different occupations based on their relative rankings as discussed in Section 2. This is rationalized by frictionless two-sided matching job market as in Hopkins (2012). The introduction of this skill allocation adds the *competitive* value to the college education.

The model is then embeded in a general equilibrium framework where the wage rates across occupations are endogenously determined. The government runs a balanced budget constraint in each period. It collects taxes from labor earnings and corporate profits which are used for college subsidy and non-productivity purposes.

3.1 Demography

Time is discrete and the unit is four years. The economy consists of two types of agents: workers and firms. A measure one of workers is born in each period who live up to J periods. Workers are heterogenous in their initial human capital h_0 and learning ability k, which are drawn from two independent log normal distributions $LN(\mu_{h_0}, \sigma^2_{h_0})$ and $LN(\mu_k, \sigma^2_k)$. Workers maximize expected lifetime earnings by making human capital investments, which includes a college decision.

3.2 Workers

3.2.1 College decision

Workers are heterogenous in their initial human capital h_0 and learning ability k. At age 0 (biological age 18), workers decide: (1) whether or not attend college and (2) if so, how much to invest in human capital. If a worker does not attend college, he will enter the working stage directly. The value of the non-college path is given by:

$$V_{nc}(h_0, k) = \int_{\varepsilon} V(h_0, k, w(h_0 + \varepsilon), 0) dF(\varepsilon) \tag{14}$$

where V is the value of the working stage with human capital h_0, learning ability k, wage rate w at age 0. $h_0 + \varepsilon$ can be interpreted as the *observed* human capital in the matching process and ε stands for the noise that is drawn from the normal distribution $N(0, \sigma^2_\varepsilon)$.

The wage rate function $w(h_0 + \varepsilon)$ reflects the equilibrium allocation outcome of the two-sided matching market between works and occupations, which will be specified in Section 3.3. In short, workers are allocated to different occupations based on their *relative ranking* of their observed human capital $h_0 + \varepsilon$ in the distribution. The equilibrium allocation satisfies positive assortative matching such that worker with the highest observe human capital is matched with the occupation with the highest productivity. Therefore the wage function w is weakly increasing in its argument.

Workers who choose to go to college spend resource cost s and one period to produce human capital. The resource cost s can be understood as the quality of a college and its associated tuition fee. The value of a college worker is described below:

$$\begin{aligned} V_c(h_0,k) = & \max_{s>0} \quad -s + \mathbb{1}\{s>0\}\phi + \beta \int_{\varepsilon} V(h',k,w(h'+\varepsilon),1)dF(\varepsilon) \\ & \text{s.t.} \quad h' = h + k \cdot (s \cdot h)^{\gamma} \end{aligned} \tag{15}$$

where h' is human capital after college education and ϕ is college subsidy.

Human capital accumulation at college serves two purposes in this setup. First, from the *productive* view, it directly increases one's efficiency units supplied in the labor market as h' in the value function. Second, from the *competitive* view, it also affects one's relative ranking in the distribution and determines the wage rate through the function $w(h'+\varepsilon)$.

The lifetime value of a worker at the first period is given as

$$W(h_0,k) = \max \quad \{V_{nc}(h_0,k), V_c(h_0,k)\} \tag{16}$$

Workers will be sorted into college and non-college paths given their initial conditions

3.2.2 Working stage

During the working stage, workers make human capital investments to maximize lifetime earnings. The value function of a worker with human capital h, learning ability k, wage rate w, and age j is presented as

$$\begin{aligned} V(h,k,w,j) = & \max_{s} \quad w \cdot h - T(w \cdot h) - s + \frac{1}{R} V(h',k,w,j+1) \\ & \text{s.t.} \quad h' = (1-\delta) \cdot h + k \cdot s^{\eta_1} \cdot h^{\eta_2} \end{aligned} \tag{17}$$

where R is the gross interest rate, T is progressive labor income tax, s is resource cost of investments and δ is human capital depreciation. The learning ability k

represents how fast one could accumulate human capital.

For tractability, I assume workers are not allowed to switch occupational types over the life-cycle.[4] In addition, I abstract away from the retirement stage so the value function in the last period is given by $V(h,k,w,J) = w \cdot h \cdot (1-\tau_l)$.

3.3 Skill allocation and technology

Next, I introduce the skill allocation process in which workers are allocated to different occupations based on their human capital upon entering the labor market. The purpose is to rationalize the mechanism where worker's relative rankings also determine their wage rate.

Firms There is a measure one of firms entering the economy in each period with productivity level z drawn from the uniform distribution $U[0,1]$. A firm can choose to become high-skill and provide an occupation position to workers with productivity z or become low-skill with fixed productivity $\tilde{z}$. A matching between a firm and a workers h generates per-period output $y(z,h) = z \cdot h$. The output is then sold to the final goods sector with price P_h or P_l.

Firms are endogenously sorted into high-skill or low-skill based on their initial productivity z and the price ratio, which is irrelevant of the matched worker. It is clear that there exists a cutoff value of $z^* = \tilde{z}\frac{P_l}{P_h}$ such that firms with productivity level $z < z^*$ will participate in the low-skill sector. Therefore the fraction of high-skill sector positions is determined endogenously by the price ratio $\frac{P_l}{P_h}$ and $\tilde{z}$.

The division of occupational types is to capture the general equilibrium effects on the demand side of the skill allocation. For instance, if the labor force consists of more college graduates, it might change the relative demand of high-skill occupations and hence affect the wage rate function. Furthermore, it is able to generate mismatches across occupations and education.

[4] This is not an important channel empirically. In particular, I find that only 0.7% of workers switch from low-skill occupations to high-skill occupations and 0.5% workers switch from the other direction within a year interval.

Matching Firms and workers will meet each other in a frictionless market and form matches. Workers know firms' productivity and type (high or low-skill) but firms can only observe noisy human capital from workers:

$$h^o = h + \varepsilon \tag{18}$$

where h^o is the observed human capital and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. One can interpret this process as interviews. Firms interview job applicants and decide to hire the ones who perform well. Their performance are noisy signals of their true human capital.

Once a match is formed, both sides exit the matching market and produce outputs until the worker retires. Both sides in an agreeable match are assumed to use the Nash bargaining solution to split the sequences of net surplus evenly until the worker retires.[5]

Since firms are risk-neutral and observed human capital are unbiased, firms will match with workers according to observed human capital h^o. In addition, as the match lasts for the entire life-cycle, firms also need to anticipate workers' human capital accumulation over the life-cycle. For tractability, I further assume that firms cannot observe workers' educational type and learning ability. As a result, they will only infer human capital trajectories from the observed human capital h^o.

Equilibrium allocation and wage The above assumptions are imposed to ensure that the matching surplus is supermodular in the observed human capital h^o and productivity z. Therefore the equilibrium allocation satisfies positive assortative matching as in Becker (1973).[6]

Formally, the matching function in the equilibrium $\Gamma(h^o) = z$ can be defined as:

$$\int_x^{\bar{x}} f_h(x)dx = \int_{\Gamma(x)}^{\bar{y}} f_z(y)dy \tag{19}$$

[5] For simplicity, I assume the value of the outside option for both sides is zero so the net surplus equals the output. Detailed discussion of the even split rule can be found in Dizdar and Moldovanu (2016) and Jia (2019).

[6] Chade and Eeckhout (2016) also provide conditions under which positive assortative matching is achieved in a similar setup in their Proposition 1.1.

where $f_h(h^o)$ denotes the distribution of observed human capital of workers who just enter the labor market and $f_z(z)$ denotes the distribution of firms.

This equation means that the highest type h^o is matched with the highest type of z. The worker with observed human capital x matches with an occupation with productivity y if the measure of workers above x is the same as the measure of occuaptions above y, i.e. their rankings are the same in their own distributions. It is worth mentioning that $f_h(h^o)$ is an equilibrium outcome in the economy that depends on individuals' college decisions and human capital investments.

Given the matching function, the wage rate function can be written as:

$$w(h+\varepsilon) = \begin{cases} \frac{1}{2} \cdot \Gamma(h+\varepsilon) \cdot P_h & \text{if} \quad \Gamma(h+\varepsilon) > z^* \\ \frac{1}{2} \cdot \tilde{z} \cdot P_l & \text{if} \quad \Gamma(h+\varepsilon) \leq z^* \end{cases} \tag{20}$$

In short, the relative ranking of one's human capital determines the marginal productivity through the matching function Γ. If one's ranking is lower than z^*, he will be allocated to low-skill occupations with homogeneous productivity $\tilde{z}$ so his wage rate is $\frac{1}{2} \cdot \tilde{z} \cdot P_l$. If his ranking is above the cutoff value, he will be matched to the occupation with corresponding productivity $z = \Gamma(h+\varepsilon)$. To summarize, the wage function is weakly increasing in one's observed human capital h^o.

Aggregate output To close the production side, I introduce the aggregate output function in the standard Cobb-Douglas form:

$$Y = H^{\alpha} L^{1-\alpha} \tag{21}$$

where H (L) is the aggregate intermediate inputs produced by high-skill (low-skill) occupations and α is the labor share of high-skill occupations. The first order conditions imply that the relative price ratio as:

$$\frac{P_h}{P_l} = \frac{\alpha}{1-\alpha} \frac{L}{H} \tag{22}$$

The relative price depends on the relative supply of high-skill and low-skill inputs, which in return shapes the endogenous division of occupational types in the equilibrium.

3.4 Government constraint and tax

The government runs a balanced budget constraint in each period. It imposes progressive taxes on all labor earnings and a proportional corporate tax τ_c on firm profits. A fraction of revenues θ_g is spent to finance college subsidy and the rest is for non-productive purposes G.

The progressive tax follows the functional form of Benabou (2002):

$$T(y) = (1 - \lambda(y/\bar{y})^{-\tau})y \tag{23}$$

where $\bar{y}$ is the mean labor earnings in the economy. The average tax rate of the individual with mean labor earnings is $1-\lambda$. This tax rate increases with labor earnings w in a concave pattern since $\tau > 0$.

The parameter λ controls for the level of the tax rate and the parameter τ stands for the progressivity in the tax schedule. In the case of $\tau = 0$, the average tax rate will not depend on labor income, i.e., it boils down to the standard proportional tax.

3.5 Competitive equilibrium

The competitive equilibrium consists of college investment decisions $s(h)$, prices P_h and P_l, the wage function w, the cutoff value z^*, the joint distribution of human capital and noise for labor market entrants $\lambda(h,\varepsilon)$, college subsidy ϕ such that

1. College and investments decisions solve optimization problems.

2. The matching function Γ is stable given the joint distribution $\lambda(h,\varepsilon)$.

3. Government runs a balanced budget constraint.

$$G+\int\phi\cdot\mathbb{1}\{s>0\}d\lambda(h,\varepsilon)=\int T(w(h,\varepsilon)h)d\lambda(h,\varepsilon)+\tau_c\cdot\int w(h,\varepsilon)hd\lambda(h,\varepsilon)$$

4. Prices are determined by the first order conditions as in Equation (22)

4 Stylized Facts and Parameterization

One important implication of the model is that not all college workers are allocated to high-skill occupations due to the presence of noise. I exploit the data on the mismatch between education and occupation to identify how noisy is the skill allocation.

In particular, I find that more than one-third of college workers are in low-skill occupations and this fraction is quite stable across different age groups. Meanwhile, 8% of non-college people work in high-skill occupations. These fractions are used to quantify how noisy the allocation process is.

Moreover, I document there are earnings premiums for both high-skill occupations and college workers. These numbers are also targeted to identify the importance of human capital accumulation as well as the productivity gap across occupation types.

4.1 Data source

I utilize information from the O*NET data set to categorize occupations into low-skill and high-skill based on educational requirements. In particular, O*NET provides detailed information on the required level of education for each occupation. I denote occupations that require a college degree or above as high-skill occupations. Similarly, occupations that require less than a college degree are considered to be low-skill occupations.[7] Table 1 show some typical low-skill and high-skill occupations and their employment shares in the population.

[7] The detail is presented in Appendix A.

Table 1: Examples of occupations

	High-skill occupations		
Computer scientists (5.2%)	Postsecondary teachers (3.4%)	Accountants and auditors (3.4%)	financial managers (2.7%)
	Low-skill occupations		
Truck drivers (6.1%)	Chefs and cooks (2.6%)	Retail salespersons (2.5%)	Construction laborers (2.4%)

Note: The table shows typical high-skill and low-skill occupations and their employment shares in parenthese.

For the following analysis, I use the Current Population Survey (CPS) Annual Social and Economics Supplement (ASEC) over the period 2003-2023. I restrict my sample to full-time full-year male workers with earnings above 50% of the federal minimum wage in that year. I also exclude self-employed workers and people employed in the public sector. I harmonize occupational codes in both CPS and O*NET to the 2010 SOC code and link the constructed index from the O*NET to the CPS sample.

4.2 Mismatch between education and occupation

Table 2 shows the fraction of mismatch between education and occupations. At the aggregate level, 13% of population are college graduates but work in low-skill occupations. This is also denoted *overeducation* in the literature. Only 63% (=22/13+22) of college graduates work in high-skill occupations. Similarly, 13% (=8/8+57) of non-college workers are in high-skill occupations and the rest are working in low-skill occupations.

Surprisingly, the fraction of mismatch between education and occupations does not change significantly across different age groups. For instance, the fraction of overeducation is 10% among people between age 23 and 34, and this fraction slightly increases to 13% for people between age 35 and 46.[8]

[8] In the data, less than 5% of workers switch occupational types annually. Meroni and Vera-Toscano (2017) also document the persistence of overeducation for young people. Both findings justify the assumption that workers do not switch occupations over the life-cycle in the model.

Table 2: Mismatch between education and occupation

Occupation type	Worker Education level Non-college	College
Overall		
Low-skill	57%	13%
High-skill	8%	22%
Age group: 23-34		
Low-skill	61%	12%
High-skill	7%	20%
Age group: 35-46		
Low-skill	57%	13%
High-skill	8%	22%
Age group: 47-58		
Low-skill	57%	13%
High-skill	8%	22%

Source: Author's calculation from CPS ASEC 2003-2023 and O*NET.

4.3 Earnings structure

Next, I look at the earnings structure over education and occupation as shown in Table 3. In particular, I compute the mean log earnings conditional on each education and occupation type. For comparison purposes, I normalize the mean log earnings of non-college workers in low-skill occupations to 0.

Table 3 shows that there exist earnings premiums for both college workers and high-skill occupations. Among non-college workers, people in high-skill occupations earn 43% more relative to those in low-skill occupations on average. Similarly, among college workers, people in high-skill occupations earn 33% more than those in low-skill occupations. Furthermore, college workers earn 48% (38%) more than non-college workers in high-skill (low-skill) occupations.[9]

I also investigate how this earnings structure changes across different age groups. In Figure 1, I plot the age profiles of mean log earnings for each occupation and education category. For comparison purposes, I normalize the mean log earnings

[9] I also document that these patterns are robust for residualized earnings so it is not mainly driven by observables.

Table 3: Mean log earnings conditional on education and occupation

	Worker Education level	
Occupation type	Non-college	College
Low-skill	0	0.48
High-skill	0.43	0.81

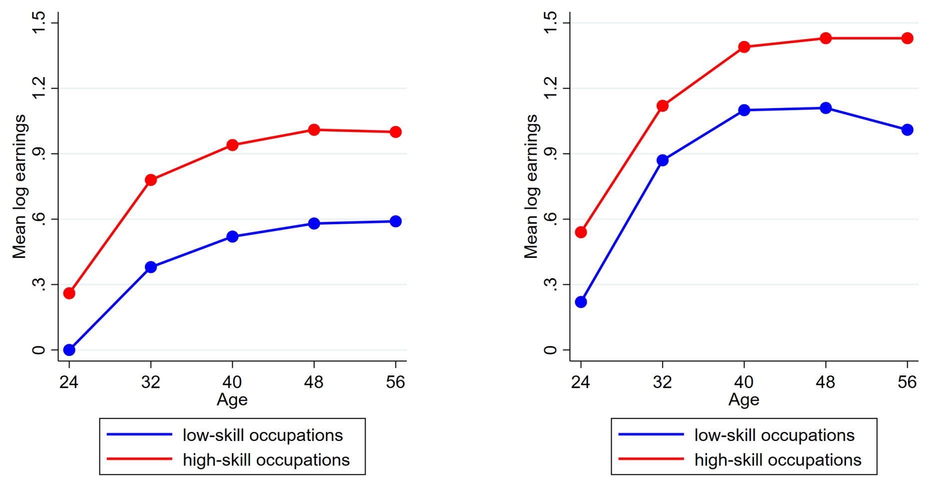
Note: The table shows mean log earnings conditional on education and occupation. Earnings of non-college workers in low-skill occupations are normalized to 0.

of non-college workers in low-skill occupations at age 24 to 0.

Figure 1: Age profiles of earnings by education and occupation

(a) Non-college workers

(b) College workers



Note: The figure shows the age profiles of mean log earnings conditional on education and occupation. Mean log earnings at age 24 for non-college workers in low-skill occupations are is normalized to 0.

Two features stand out from the age profiles. First, workers in high-skill occupations have higher earnings on average compared to their peers in low-skill occupations. Furthermore, this *high-skill premium* is relatively stable across different age groups. For non-college workers, the high-skill premium is 50% at age 32 and slightly decreases to 40% at age 56. Similarly, the high-skill premium for college workers is 32% at age 32 and increases to 42% at age 56.

Second, in spite of occupation type, the age profiles of earnings of college workers are steeper than non-college workers. The earnings difference between age 24 and 56 is 59% (74%) for non-college workers in low-skill (high-skill) occupations. For college workers, the difference in earnings between age 24 and 56 is 79% (89%) in low-skill (high-skill) occupations.

Loosely speaking, the difference in earnings between occupational types (conditional on education) implies the productivity gap across occupations. Moreover, the earnings gap across education (conditional on occupation type) indicates the importance of human capital accumulation at college.

4.4 Parameterization

In this subsection, I describe how to set parameters in the model as shown in Table 4. A subset of the parameters is chosen from external sources. The rest of the parameters are chosen jointly to match moments in four aspects: (1) fractions of skill allocation, (2) earnings structure across education and occupations, (3) life-cycle patterns and (4) educational expenditures.

Table 4: Parameterization

Parameter	Meaning	Value
	Internal	
μ_h, σ_h	distribution of initial human capital	1.61, 0.18
μ_k, σ_k	distribution of learning ability	-0.65, 0.13
γ	human capital production (college)	0.36
η_1, η_2	human capital production (work)	0.49, 0.43
σ_ε	noise in skill allocation	1.66
α	high-skill labor share in production	0.432
θ_g	fraction of taxes for college subsidy	0.018
z^*	threshold value of high-skill occupation	1.27
	External	
δ	human capital depreciation	0.077
R	discount factor	1.04^4
τ_c	corporate income tax	0.21
λ	labor income tax rate	0.9
τ	tax progressivity	0.1

4.4.1 Parameters chosen externally

The human capital depreciation rate is set to be 0.077 such that the annualized rate is 0.02, which is the ballpark of the literature.[10] Similarly, the discount factor is set to be 1.04^4 to match the real interest rate. The parameters of income labor tax are taken from Heathcote et al. (2017). The corporate tax is 0.21, which is consistent with the data in the U.S..

4.4.2 Parameters chosen internally

Initial distributions The initial distributions of human capital and learning ability are crucial to generating the earnings structure across education and occupation as well as the fractions of skill allocation. The distribution of human capital mainly affects the fractions of skill allocation. The initial distribution of learning ability largely affects earnings growth patterns. However, since I also target life-cycle earnings patterns for each occupation and education category (16 data points), I need extra degrees of freedom for human capital production functions in the working stage.

Noise The parameter σ_ε is identified from the fractions of mismatch, i.e. college (non-college) workers in low-skill (high-skill) occupations. When σ_ε is zero, the skill allocation is based on the true human capital level so there will be no non-college workers in high-skill occupations. σ_ε is set to 1.66 such that the model replicates the fractions of mismatches in the data.

Human capital production The parameter γ governs human capital accumulation at college, which is the key to generating college premiums at the beginning of the life-cycle. In the model, workers with high learning ability are more likely to attend college since they could accumulate more human capital.

[10] Dinerstein et al. (2022) shows the annual deprecation rate is 4.3% for non-employment. Rupert and Zanella (2015) argues that the depreciation rate is almost zero.

If college education has no effect on human capital accumulation($\gamma = 0$), then we would see the average earnings of non-college workers should be higher than college workers' in the first period conditional on occupation type. The reason is that for non-college workers in high-skill occupations, their average human capital is comparable to college workers in high-skill occupations but they have relatively lower learning ability, which discourages them from college education. Consequently, they have one more period to accumulate human capital at work whereas college workers cannot.

As shown in Figure 1, we know that college workers earn 28% (22%) more than non-college workers in high-skill (low-skill) occupations at age 24, which implies the importance of human capital accumulation at college.

The parameters η_1 and η_2 determine human capital accumulation at the working stage. As mentioned before, I also target life-cycle patterns of earnings for each education and occupation type. The extra degree of freedom from η_1 and η_2 allows me to match different earnings magnitudes across each category as well as the growth in life-cycle inequality (measured as the variance of log earnings).

Subsidy and production The parameter θ_g stands for the fraction of government revenues used for college subsidy (with the rest for non-productive purposes). It is set to 0.018 to match the fact that 38% of college expenditures are subsidized by the government.[11] In addition, the average resource cost of college education generated by the model is 65% of output per capita in the economy, which is quite close to the estimation from Krueger and Ludwig (2016).

The parameter α in the production function governs the price ratio between high-skill inputs and low-skill inputs which also determines the earnings premium across occupation. The parameter z is chosen to directly determine the fraction of high-skill occupations in the economy as discussed previously.

[11] See OECD (2023) table C3.2.

4.5 Benchmark economy

Table 5 shows the model's performance relative to the counterparts in the data. First, the model does a good job in replicating skill allocation patterns. For instance, 57% of non-college workers are allocated to low-skill occupations in the model, which is very close to the fraction in the data.

Table 5: Model fit

Moment	Model	Data
Skill allocation		
% non-college workers in high-skill occ	11%	8%
% non-college workers in low-skill occ	57%	58%
% college workers in high-skill occ	19%	22%
% college wokers in low-skill occ	12%	13%
Earnings structure (relative to non-college workers in low-skill occ)		
non-college workers in high-skill occ	0.36	0.43
college workers in low-skill occ	0.47	0.48
college workers in high-skill occ	0.84	0.81
Life-cycle patterns		
growth in mean log earnings (25-55)	0.633	0.627
growth in earnings inequality (25-55)	0.116	0.103
Government spending		
college subsidy/total college expenditure	36%	38%
non-productive government spending/GDP	16%	17%

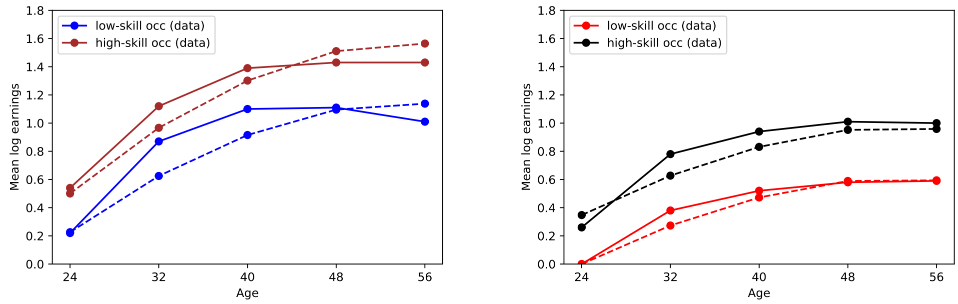
Second, the model is successful in generating the earnings structure by education and occupation at the aggregate level as shown in Table 5. The only drawback is that it slightly understates the average earnings for non-college workers in high-skill occupations. Furthermore, Figure 2 shows that the model can regenerate age profiles of earnings across education and occupation.

The benchmark economy also does an excellent job of replicating life-cycle patterns. In particular, the growth in mean log earnings between age 25 and 55 is 63.3 log points, which is quite close to the data (62.7 log points). Similarly, the growth in inequality, measured as the variance of log earnings, is 11.6 log points.

In terms of government spending, the share of total college expenditures subsidized by the government is 36%, which is close to 33% in the data. In addition, the non-productive government spending is 16% of the total output, which is consistent with the estimation from Krueger and Ludwig (2016).

Figure 2: Model fit: earnings dynamics

(a) College workers (b) Non-college workers



Note: This figure presents the age profiles of mean log earnings conditional on education and occupation type. Solid lines represent data and dotted lines represent model's counterpart.
Mean log earnings of non-college workers in low-skill occupations at age 24 is normalized to 0

5 Quantitative Analysis

In this section, I decompose the effects of productive and competitive channel for college attendance. I find that the competitive channel accounts for 39% of the college attendance and the productive channel accounts for 53%. Moreover, the combination of these two channels change the sorting pattern of college attendance.

5.1 Shut down the productive channel

The first experiment is to shut down the productive channel so people attend college only for the competitive purpose. One can think of an extreme case where individuals learn nothing at college but only use degrees as an instrument to stand out in the job market.

Particularly, I assume college education generates no human capital accumulation but only affects skill allocation after graduation through $w(h'+\varepsilon)$. So the

Table 6: Counterfactual experiments

	Benchmark	Only competitive	Only productive
Aggregate outcomes			
College attendance	31.6%	12.4%	16.8%
Aggregate output	100	99.6	101.3
Output per worker	100	97.5	99.5
Skill allocation mismatch			
non-college workers in high-skill occ	13%	29%	30%
college workers in low-skill occ	12%	2%	14%
Life-cycle patterns			
growth in mean log earnings	0.633	0.639	0.645
growth in earnings inequality	0.116	0.095	0.095

Note: Column (1) shows the benchmark economy. Column (2) presents the results when shutting down the productive channel and column (3) presents the results when shutting down the competitive channel.

continuation value of the college education in Equation (15) is re-written as:

$$\beta \int_{\varepsilon} V(h_0, k, w(h' + \varepsilon), 1) dF(\varepsilon)$$

The difference comes from the first argument which implies that college education does not change one's efficiency units at all.

After shutting down the productive channel, the fraction of college workers drops from 31.6% to 12.4% as shown in Table 6 column 2. This decrease implies that the competitive channel solely explains 39.2% of the college attendance. Furthermore, shutting down the productive channel lowers output per worker by 2.5% and aggregate output by 0.4%

It is obvious that output per worker would decrease if college does not serve the human capital purpose as it lowers the average human capital (labor quality) of workers. However, the impact on aggregate output of shutting down the productive channel seems be minor. The answer lies in the changes in the working population. When shutting down the human capital channel, around 20% of workers would skip the college stage and enter the labor market, which directly increases the size of the labor force. As a result, it offsets the drop in the average labor quality so the aggregate output decreases slightly.

Furthermore, shutting down the productive channel increases the growth in mean log earnings by 0.6 log points and marginally lowers the growth in inequality over the life-cycle by 2.1 log points. Though college does not provide the opportunity for human capital accumulation, workers could also make human capital investments at work, which compensate for the loss in the college stage.

5.2 Shut down the competitive channel

The second experiment is to shut down the competitive channel so college education only serves the role of human capital accumulation. To do so, I assume the skill allocation is based on the worker's initial human capital (plus the noise). Workers who enter college can still make human capital investments but they will compete with non-college workers based on their initial human capital. In particular, the continuation value in Equation (15) is modified as

$$\beta \int_{\varepsilon} V(h', k, w(h_0 + \varepsilon), 1) dF(\varepsilon)$$

The deviation comes from the third argument if the continuation value where the wage rate is determined by initial human capital h_0 plus noise instead of human capital after graduation h'.

After shutting down the competitive channel, the fraction of college workers drops to 16.8% as shown in column 3. Put differently, the productive channel accounts for 53% of the college attendance. Furthermore, shutting down the competitive channel increases aggregate output by 1.3% but decreases output per worker by 0.5%. Similarly, the change in the working population explains why the impact on aggregate output is larger than the impact on output per worker.

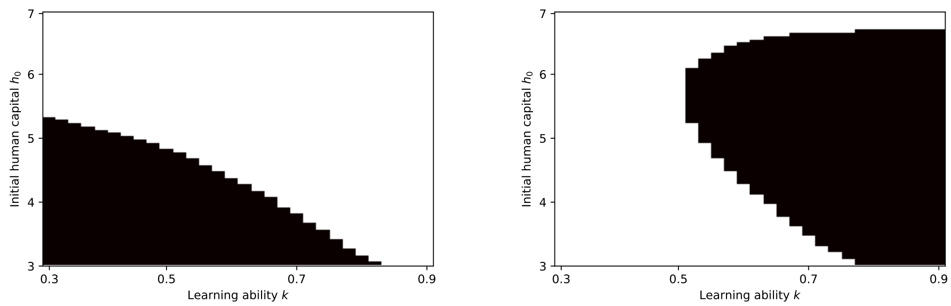
In addition, shutting down the competitive channel increases the growth in mean log earnings by 1.2 log points and decreases the growth in inequality over the life-cycle by 2.1 log points.

5.3 College decisions

To better understand how the competitive channel shapes the college attendance and the skill allocation, it is important to look at who are sorted into the college path. Figure 3 shows college decisions (represented by black areas) based on the combination of initial human capital h_0 and learning ability κ.

Figure 3: College decisions

(a) Only productive (b) Benchmark



Note: This figure presents college decisions based on the combination of initial human capital h_0 and learning ability k. Black area represents people who attend the college in each scenario.

Only productive channel Panel (a) shows the college policy when the competitive channel is shut down, i.e., people go to college only to accumulate human capital. Surprisingly, only workers with low initial human capital h_0 and learning ability k choose to attend college for the productive purpose. People with high initial conditions do not benefit from human capital accumulation at the college stage, which is contrary to the standard view.

For workers with high initial conditions, their time cost of college education exceeds the benefits of a college education even though it provides an extra opportunity to make human capital investments. Therefore they would be better off if enter the labor directly and make human capital at work at a earlier stage.

Workers with low initial conditions would like to attend college only to collect the college subsidy. As the college attainment goes down and the fraction of taxes used to finance college subsidy is fixed, they will collect a large amount of subsidy, which is 93% of the average earnings of workers of the same age.

Combined with competitive channel Panel (b) presents the college decision in the benchmark economy where both channels are included. Interestingly, the combination of these two channels completely crowds out people with low initial conditions to attend college and only encourages people with high learning ability or human capital to acquire a college education in the equilibrium.

These changes illustrate how the competitive margin distorts college attendance as a rat race competition. For mediocre workers, their optimal path is to skip college in the absence of the competitive channel. However, when the skill allocation depends on human capital accumulation, it leads to a rat race competition in the college education, which encourages these workers to attend college in fear of falling behind. When these marginal workers attend college, it further encourages more workers with similar initial conditions to attend college as ripple effects. In the equilibrium, the fraction of college attendance increases to 31.6% with a totally different composition of college workers compared to the scenario only with the productive channel.

For those with extremely high initial human capital h_0, they will still skip the college stage for three reasons. First, they do not need to signal their human capital through the college stage as their relative rankings are already at the top. Second, the time cost of a college education is too high compared to the benefits. In addition, they can also make human capital investments at work to compensate for the potential loss in the college stage.

6 Policy Implications

This section evaluates the policy implications with the presence of the competitive channel. I first assess the roles of college subsidy and tax progressivity separately without tax neutrality. Then I derive the optimal policy of labor taxation and college subsidies that maximizes the social welfare under the balanced budget constraint.

The optimal policy combination is to eliminate college subsidy and lowering tax progressivity, which will increase output by 11.3% and social welfare by 4.9% in the steady state. This policy combination will alleviate the unnecessary competition among popualtion and only incentivize people with high initial conditions to attend college. Hence it will mitigate the negative externality brought by the competitive margin.

6.1 College subsidy

The first experiment is to evaluate the role of college subsidy. The parameter θ_g stands for the fraction of taxes collected to finance college subsidy. Table 7 shows how aggregate outcomes change with different θ_g while tax progressivity τ and the level of tax rate λ are unchanged.

Table 7: Policy experiments: college subsidy

Policy parameters			Benchmark		
% taxes for college subsidy θ_g	0.040	0.030	0.019	0.010	0
College subsidy ϕ	1.84	1.72	1.31	1.07	0
Aggregate outcomes					
College attendance	47.9%	39.0%	31.6%	20.6%	7.7%
Level of income inequality	0.232	0.237	0.237	0.238	0.231
Output per worker	100.7	100.1	100	98.7	97.8
Aggregate output	98.8	99.4	100	100.3	100.7
Average social welfare	99.9	100.7	100	101.1	103.1

Note: This table shows the results of policy experiments with changes in the fraction of taxes collected spent in college subsidy θ_g.

First of all, as θ_g rises, the level of college subsidy and the fraction of college attendance increase as shown in Table 7. However, the overall level of income inequality does not change significantly as shown in the forth row.

The fifth and sixth rows present changes in output. Output per worker increases with college subsidy but aggregate output decreases with college subsidy. As more people attend college, the average level of human capital is higher so the quality of workers improves. On the other hand, since more people attend college, the size of labor force shrinks which leads to a drop in aggregate output. Furthermore, the decline driven by changes in the labor force dominates the increase resulting from the improvement on labor quality so aggregate output decreases with the level of college subsidy.

The last row of Table 7 shows that how average welfare changed with respect to college subsidy. Since workers in the economy maximize lifetime earnings, individual's welfare can be understood as the net lifetime value, which is the lifetime earnings minus the cost of human capital investment. Formally, the utilitarian social welfare function is defined as

$$SFW = \int \int W(h_0, k) dF_{h_0}(h_0) dF_k(k) \tag{24}$$

where $F_{h_0}(h_0)$ and $F_k(k)$ are the CDF of initial conditions and $W(h_0, k)$ is the lifetime value of a worker at the first period as described in Equation (16).

Average social welfare responses in a non-symmetric way to the changes in college subsidy. When the college subsidy is elinminated, the average social welfares increases by 3.1% compared to the benchmark case. On the other hand, increasing college subsidy from 1.31 to 1.72 also slightly increases social welfare by 0.7%. However, the improvement will diminish when college subsidy is raised to 1.84.

Table 8: Relative social welfare changes: college subsidy

College subsidy ϕ	High subsidy 1.84	Zero subsidy 0
By education groups		
Always college	+0.8%	+2.1%
Always non-college	-0.3%	+2.4%
C to NC	/	+5.0%
NC to C	-0.4%	/

Note: This table shows changes in mean welfare with different college subsidies relative to the benchmark ($\theta_g = 0.019$). Output and average social welfare are normalized to 100 in the benchmark case.

To better understand the mechanism, I investigate welfare changes conditional on different education groups in two cases: high subsidy and zero subsidy. I look at changes in mean welfare on (1) workers who always attend college, (2) workers who always skip college, and (3) workers who switch educational groups when subsidy is changed.

The key finding is that the increase in college subsidy hurts people who switch from non-college to college path because of the subsidy. As shown in the first two columns of Table 8, the average welfare increases 0.8% for workers who are already in the college track as they now enjoy a higher subsidy. Surprisingly, for those who switch to the college track, their average welfare decreases 0.4% compared to the benchmark case where they did not attend college.

Similarly, when the subsidy is eliminated, workers who switch from college to non-college track experience an increase of 5% in average welfare compared to the benchmark case. Meanwhile, for those who are already in the college track, their average welfare also increase by 2.1%. This is because as fewer people attend college, they would face less competition in the skill allocation process so they are more likely to be matched with high-skill occupations. These results together indicate that college subsidy encourages over-investment and negative externalities through the competition channel and lowers average social welfare.

6.2 Tax progressivity

Next, I examine the role of tax progressivity τ while leave college subsidy and tax rate constant and the results are presented in Table 9.

As progressivity increases, the college attainment rate drops and the overall level of income inequality decreases. When labor income tax becomes more progressive, it distorts the incentive to accumulate human capital and hence compresses the wage structure. As a result, the average quality of workers drops so both aggregate output and output per worker decreases drastically with the progressivity parameter τ.

Table 9: Policy experiments: tax progressivity

Policy parameters			Benchmark		
Tax progressivity τ	-0.1	0	0.1	0.2	0.3
Aggregate outcomes					
College attendance	40.5%	34.7%	31.6%	22.8%	17.6%
Level of income inequality	0.326	0.284	0.237	0.192	0.152
Output per worker	120.7	110.3	100	89.1	80.0
Aggregate output	119.6	109.9	100	90.1	81.2
Average social welfare	95.8	98.6	100	101.3	100.1

Note: This table shows the results of policy experiments with changes in the labor income tax progressivity τ. Output and average social welfare are normalized to 100 in the benchmark case. C to NC stands for workers who attend college in the benchmark but switch to non-college path in the counterfactual.

On the other hand, a more progressive tax slightly increases average social welfare. In particular, when tax progressivity increases from 0.1 to 0.2, the average social welfare increases by 1.3% even though aggregate output drops by 10%. This is because a more progressive tax redistributes income from top earners to those of lower income.

Table 10: Relative social welfare changes: progressivity

	Regressive tax	Progressive tax
Progressivity τ	-0.1	0.3
By education groups		
Always college	+6.2%	-6.4%
Always non-college	-11.6%	+4.0%
C to NC	/	-4.6%
NC to C	-0.3%	/

Note: This table shows changes in mean welfare with different college subsidies relative to the benchmark ($\tau = 0.1$). Output and average social welfare are normalized to 100 in the benchmark case. C to NC stands for workers who attend college in the benchmark but switch to non-college path in the counterfactual.

I also decompose the changes in social welfare by education groups in Table 10. As discussed above, a more progressive tax will increase the average tax rate of those who attend college. Therefore for those who always choose the college track, their average social welfare decreases with the level of progressivity. Similarly, the mean social welfare for people of low incomes (who always choose non-college track) increases with progressivity as their average tax rates become lower. Furthermore, a more progressive tax schedule encourages more people to attend college but their average welfare are lower than the benchmark case where they do not attend college.

6.3 Optimal policy

In this subsection, I search for the optimal combination of college subsidy and tax progressivity under a balanced budget constraint that maximize average social welfare. In particular, the level of tax rate λ is adjusted such that the non-productive purpose of government spending G is fixed over time.

Table 11: Optimal policy system

Policy parameters	Benchmark	Optimal
College subsidy ϕ	1.31	0
Progressivity τ	0.1	0.025
Tax rate λ	0.90	0.93
Aggregate outcomes		
College attendance	31.6%	16.0%
Level of income inequality	0.237	0.280
Output per worker	100	107.6
Aggregate output	100	113.6
Average social welfare	100	105.9

Note: This table compares outcomes in the steady state between the benchmark model and the optimal policy.

I find that the optimal policy is to eliminate college subsidy and make progressive tax almost flat ($\tau = 0.025$).[12] The intuition is that such policy combination mitigates over-investment in human capital and alleviates negative externalities brought by the competition in college education. Table 11 presents the equilibrium outcomes under optimal policy scheme. Specifically, the attendance rate drops from 31.6% to 16% due to the elimination of college subsidy. Meanwhile, as a

[12] To maintain the balanced budget constraint, the level of tax rate λ is adjusted to 0.93.

result of a less progressive tax schedule, the overall level of income inequality rises by 4.3 log points. Output per worker and aggregate output both increase significantly and the average social welfare increases by 5.9% under the optimal policy.

Table 12: Welfare changes by education groups: optimal policy

	Always college	Always non-college	C to NC
Fractions	16.0%	68.4%	15.6%
Changes relative to benchmark			
Average social welfare	+11.1%	+3.3%	+9.6%
Lifetime income	+26.8%	+9.4%	+15.1%
Human capital growth (log points)	+15.5	+8.2	+9.2

Note: This table shows the relative changes between the benchmark case and the optimal policy case. The population is divided into three education groups: (1) workers who attend college in both cases, (2) workers who skip college in both cases and (3) workers switch from college track to non-college track when the optimal policy is implemented.

To analyze where does the improvement come from, in Table 12 I decompose welfare changes by education groups as in previous exercise. The first column indicates that for those who always attend college, their social welfare increases by 11.1% on average with an increase of 26.8% in lifetime income. The increase in lifetime income is higher than the increase in social welfare because they also spend more resource costs to invest in human capital. Consequently, their human capital growth is 15.5 log points higher than the benchmark case. This result is driven by a less progressive tax as it incentivizes more human capital accumulation.

The second column shows welfare improvement for those who do not attend college in both cases. They experience a slight increase in average social welfare and lifetime income. This results from the fact that the level of tax rate is lower under the optimal policy.

The last column presents the changes for switchers, i.e. workers who attend college in the benchmark but decide to skip college under the optimal policy. Surprisingly, this group become better off without the support of college subsidy. Their social welfare increases 9.6% on average and their lifetime income increases 15.1% under the optimal policy. For them, the optimal path is to enter the labor market directly after high-school and accumulate human capital at work. However, due to the competition in the job market, they are forced to participate in the college competition. The elimination of college subsidy frees them up from the rat race competition.

7 Extension: financial friction

An important goal of college subsidies is to help financially constrained households, which is currently ignored from the baseline analysis. In this section, I extend the model with financial frictions and re-examine the optimal policy of college subsidies. Results indicate that such financial constraints do not change the intuition: the optimal policy is still to reduce college subsidies and tax progressivity in order to alleviate negative externalities.

7.1 Family wealth and college attendance

First of all, I present empirical evidence to show that people from rich families are more likely to attend college conditional on students' ability. I draw information from the National Longitudinal Surveys of Youth 97 (NLSY97), which contains information on both student ability and family wealth. Following Lochner and Monge-Naranjo (2011), I use scores on the Armed Forces Qualifying Test (AFQT) to approximate students' ability.

Table 13: College attainment rate by family wealth and ability

		Ability Quartiles			
		1	2	3	4
Family Wealth Quartiles	1	0.06	0.14	0.29	0.41
	2	0.08	0.19	0.34	0.56
	3	0.10	0.23	0.48	0.69
	4	0.11	0.39	0.60	0.80

Source: NLSY97 and author's calculation.

Table 13 shows conditional on ability quartiles, college attainment rate increases with family wealth level. For instance, in the first ability quartile, only 6% of people attend college in the lowest wealth quartile, and this number slightly increases to 11% in the highest wealth quartile. Similarly, In the fourth ability quartile, 41% of people have a college degree in the lowest wealth quartile, and this fraction almost doubles in the highest wealth quartile.

7.2 Introducing financial constrains

Motivated by the empirical evidence, I now introduce financial constrains to the benchmark model so that the level of family wealth also plays a role in shaping college decisions. In particular, I add another dimension of heterogeneity: family wealth quartiles $i \in [1,2,3,4]$, which affects the maximum amount of human capital investments at college. The value of a college worker with initial human capital h_0, learning speed k and family wealth quartile i is given by:

$$\begin{aligned} V_c(h_0,k,i) = \max_{s} \quad & -s + \mathbb{1}\{s>0\}\phi + \beta \int_{\varepsilon} V(h',k,w(h'+\varepsilon),1)dF(\varepsilon) \\ \text{s.t.} \quad & h' = h + k\cdot(s\cdot h)^{\gamma} \\ & 0 \leq s - \phi \leq s_i \end{aligned} \tag{25}$$

The modification here is that workers of different family wealth quartiles have investments constraints s_i, which is a reduced-form way to captures that low-wealth families face more financial frictions in the credit market.

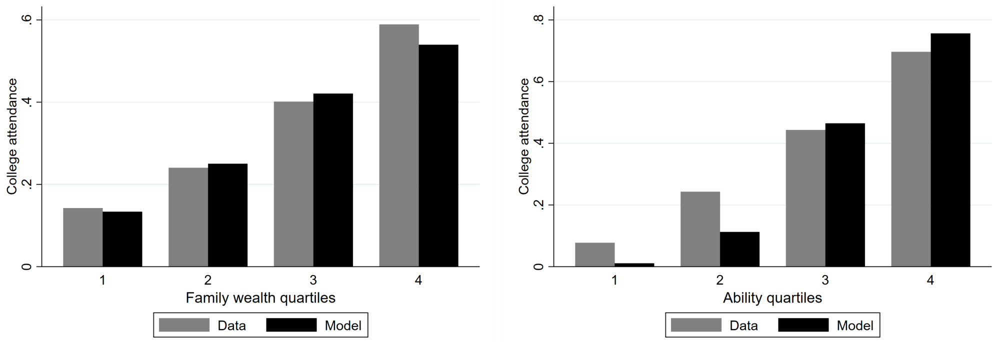
Due to this constraint, people from low-wealth families cannot make the optimal level of investment and therefore would skip the college education. As a result, college subsidies ϕ allow low-income families to make human capital investment that is higher than the constraint s_i.

The model is then re-parameterized to match a set of additional moments: college attainment rates by wealth quartiles. I assume the fourth wealth quartile families are not constrained so s_4 can be treated as infinity. I will calibrate s_1, s_2 and s_3

jointly such that it can match the college attainment rates in the each wealth quartile. Furthermore, I assume the initial distribution of learning speed k and wealth quartiles are jointly determined as suggested by the data.[13] The rest of parameters are unchanged as in the benchmark case.

Figure 4: College attainment rate by quartiles

(a) Wealth quartiles (targeted) (b) Ability quartiles (untargeted)



Note: The figure shows the college attainment rate by family wealth quartiles and learning speed (AFQT scores) quartiles.

Figure 4 panel (a) shows that the model does a great job in matching college attendance conditional on wealth quartiles. However, panel (b) shows that the model understates the attainment rate for the first two ability quartiles with the unchanged initial distributions. One possible reason is that the scores on AFQT are noisy signals of their true learning ability.

To further understand how investment constraints shape college decisions, I decompose the changes of college attendance in Table 14. In the baseline model without wealth heterogeneity, the college attainment rate is independent of wealth quartiles as shown in the first row. Similarly, the average ability of college graduates is 0.582 across all wealth quartiles.

The introduction of the correlation between ability and wealth directly generates the strong correlation between college attendance and wealth level as shown in the second row. By construction, people from rich families on average have higher

[13] As shown in NLSY97, the correlation between students' ability and and wealth level is 0.35. I interpret student ability as learning speed since it is highly correlated with the growth in life-cycle earnings as both in the model and in the data.

Table 14: College attendance decomposition

	Family wealth quartiles			
	1	2	3	4
College attainment rate				
No wealth (benchmark)	0.31			
Only correlation, no constraints	0.18	0.27	0.30	0.41
Correlation and constraints	0.13	0.25	0.42	0.54
Average learning speed k (conditional on college graduates)				
No wealth (benchmark)	0.582			
Only correlation, no constraints	0.560	0.572	0.582	0.600
Correlation and constraints	0.583	0.585	0.578	0.596

Note: The upper panel shows the college attainment rate by wealth quartiles. The lower panel shows the average learning speed among college graduates by wealth quartiles. In the benchmark case, wealth is neutral and does not affect anything. The second row shows the case where initial ability k and wealth level s is positively correlated. The third row stands for the case with both correlation and investment constraints.

learning ability, so they are more likely to attend college, which is confirmed by the second row of the lower panel from Table 14 as the average learning speed increases with wealth quartiles. It is worth noticing that the correlation is not enough to match college attendance rates across wealth quartiles.

Investment constraints further strengthens the correlation between wealth and college attendance through the competitive margin. In particular, high ability students from low-wealth families cannot attend college due to constraints, which makes the college more attractive because it is less competitive. Therefore more students from high-wealth families will attend college instead. This is confirmed by the third row where the college attendance in the third (fourth) quartile increase by 12% (13%) after imposing the constraint. As presented in the lower panel, the average learning speed also drops after introducing the investment constraints, which implies that the threshold level of college attendance is lower once high-ability people from low-wealth families are ruled out by investment constraints.

7.3 Re-evaluate the optimal policy

With the modified model at hand, I re-evaluate the optimal policy that maximizes the average social welfare. Results suggest that the optimal policy with the presence of investment constraints is still to eliminate college subsidies and lower tax progressivity.

Table 15: Optimal policy system with investment constraints

Policy parameters	Benchmark	Optimal
College subsidy ϕ	1.24	0
Progressivity τ	0.1	-0.01
Tax rate λ	0.90	0.93
Aggregate outcomes		
College attendance	33.6%	12.5%
Level of income inequality	0.237	0.284
Output per worker	100	114.5
Aggregate output	100	116.6
Average social welfare	100	107.6

Note: This table compares outcomes in the steady state between the benchmark model and the optimal policy.

Table 15 presents the aggregate outcomes after the policy reform. Similarly, the optimal policy is to eliminate the college subsidy and lower tax progressivity. Unlike the benchmark case, the parameter of tax progressivity is -0.01, which means that the labor income tax is slightly regressive.

Under the new policy system, the college attainment rate drops from 33.6% to 12.5%. The average social welfare increases by 7.6% and aggregate output increases by 16.6%. Moreover, the level of income inequality, which is measured as the variance of log earnings, increases by 4.7 log points.

To better understand who benefit from this policy reform, I show changes in (discounted) lifetime earnings relative to the benchmark case conditional on both ability and wealth quartiles. As shown in the first row of Table 16, people from high-wealth families experience larger increase in lifetime earnings. In particular, people from the first (fourth) wealth quartile has experienced an 5.1% (10.8%)

Table 16: Changes in lifetime earnings by ability and wealth quartiles

	Quartiles		
1	2	3	4
By wealth quartiles			
+5.1%	+7.1%	+7.2%	+10.8%
By ability quartiles			
+1.4%	+4.6%	+8.5%	+14.1%

Note: This table presents the changes in lifetime earnings relative to the benchmark scenario conditional on wealth/ability quartiles.

increase in lifetime earnings on average. Similarly, the second row of Table 16 suggests that the benefit is larger for people of high ability.

8 Concluding remarks

In this paper, I study the competitive value of a college education and quantify its relative importance. I propose a theoretical framework where college education has both productive and competitive value. I find that the competitive channel accounts for 37% of the college attendance but has negative effects on output. Meanwhile, the productive channel accounts for 47% of the college attendance and also increases output. Furthermore, I evaluate the optimal tax system that maximizes social welfare. I find that the elimination of college subsidy with a flatter tax schedule would increase average social welfare by 5.9% and output by 13%. Such a policy combination will discourage over-investment in human capital and alleviate negative externalities brought by the competition in college education. The result is also robust with the inclusion of investment constraints.

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A Occupation required education level

The O*NET data set provides information on the required level of education at the occupational level. In particular, the survey interviews a sample of workers and asks them what is the minimum level of education required in that occupation. An occupation is considered to be high-skill if more than half of interviewees thinks that the occupation requires at least a bachelor's degree. Similarly, if less than 50% of interviewees think that the occupation does not require a bachelor's degree, then it is denoted as low-skill.

Another concern with the O*NET data set is that it updates a subset of its occupations annually so the educational requirement might also change over time. To rule out this possibility, I compare O*NET data sets between 2006 and 2023. The comparison shows that 90% of occupations does not change in its required educational level. 6.5% of occupations switched from low-skill to high-skill and 3.5% of occupations switched for the other direction.

B Proof of proposition 3

I re-write the optimization problem of college education as:

$$V(h) = \max\{V^c(h), V^{nc}(h)\}$$

where $V^{nc}(h) = h \cdot w(h)$ denotes the value of not attending college and $V^c(h)$ denotes the optimal value of attending college:

$$\begin{aligned} V^c(h) = &\max_{x>0} \quad h' \cdot w(h') - x - C \\ &\text{s.t.} \quad h' = h + (hx)^\gamma \end{aligned}$$

I use subscript 1 to denote functions and equilibrium outcomes in the model without comeptition and use subscript 2 for the model with competition.

First, we assume $h_2 > h_1$, i.e., the cutoff value of the indifferent worker is higher in the model with competition. Since the net value of college is strictly increasing in h, for any worker with $\hat{h} \in (h_1, h_2)$, he prefers the non-college path in the model with competition:

$$V_2^{nc}(\hat{h}) > V_2^c(\hat{h}) \tag{26}$$

Because workers below the cutoff value h_2 do not attend college and make no human capital investment, the relative ranking of $\hat{h}$ in both models are the same so we have: $w_1(\hat{h}) = w_2(\hat{h})$ As a result, the value of non-college is the same in both models:

$$V_1^{nc}(\hat{h}) = V_2^{nc}(\hat{h}) \tag{27}$$

We also know that the cutoff value is lower than $\hat{h}$ in the model without competition and again the net value of college is strictly increasing in h, we have

$$V_1^c(\hat{h}) > V_1^{nc}(\hat{h}) \tag{28}$$

In the model with competition, the worker with $\hat{h}$ can also achieve the same level of utility as in the model without competition by mimicing the investment behavior in the model without competition. By doing so, his relative ranking will not decrease $(w_2(\hat{h}_2) > w_1(\hat{h}))$. So we also have

$$V_2^c(\hat{h}) = \hat{h}_2 \cdot w_2(\hat{h}_2) - x_2(\hat{h}) - C \geq \hat{h}_1 \cdot w_1(\hat{h}) - x_1(\hat{h}) - C = V_1^c(\hat{h}) \tag{29}$$

where x_1 (x_2) is the optimal effort in the model without (with) competition and $\hat{h}_1$ ($\hat{h}_2$) is the "after-college" human capital in the model without (with) competition.

Combining equation Equation (27) to Equation (29) yields

$$V_2^c(\hat{h}) \geq V_2^{nc}(\hat{h})$$

which contradicts Equation (26) therefore we cannot have $h_2 > h_1$.

Next we assume $h_1 = h_2$, then we have two indifferent conditions

$$\tilde{h}_1 \cdot w_1(h_1) - x_1(h_1) - C = h_1 \cdot w_1(h_1) \tag{30}$$

$$\tilde{h}_2 \cdot w_2(\tilde{h}_2) - x_2(h_1) - C = h_1 \cdot w_2(h_1) \tag{31}$$

where $\tilde{h}_1$ ($\tilde{h}_2$) is "after-college" human capital in the model without (with) competition.

By Proposition 1, we know that $w_2(\tilde{h}_2) = w_1(h_1)$. In addition, as workers under h_1 do not attend college, the relative rankings of this marginal worker are also the same in both models, i.e., $w_2(h_1) = w_1(h_1)$. Proposition 2 indicates that this marginal worker will over-invest in human capital compared to the case without competition. Therefore given the same wage rate, the value of college with competition is strictly smaller than the value of college without competition.

$$\tilde{h}_1 \cdot w_1(h_1) - x_1(h_1) - C > \tilde{h}_2 \cdot w_2(\tilde{h}_2) - x_2(h_1) - C$$

On the other hand, the values of non-college are the same in both cases:

$$h_1 \cdot w_1(h_1) = h_1 \cdot w_2(h_1)$$

Consequently the marginal worker h_1 would strictly prefer non-college path to college path in the model with competition, which contradicts to our assumption that the worker h_1 is different between college and non-college paths.