Bilateral Trade with Costly Information Acquisition

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Introduction

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- Which objectives can be implemented?

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Bilateral trade problem with information acquisition.

- Principal proposes a trading mechanism,
- Buyer and Seller privately acquire payoff-relevant information,
- Information acquisition is costly and flexible,
- Information acquisition can be arbitrarily correlated across players.

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- Information acquisition is costly and flexible,
- Information acquisition can be arbitrarily correlated across players.

What do we do?

- Provide implementability conditions,
- Characterize info structures consistent with allocational efficiency.
  - Application: subsidy minimization for efficient trade.

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Endogenous information acquisition can help.

- Bikhchandani (2010): FSE $\Rightarrow$ incentives to acquire info about others.
- Bikhchandani and Obara (2017): "inflexible" info $\Rightarrow$ FSE (not always).
  - "inflexible" = finitely many conditionally independent signals.

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  - "inflexible" $=$ finitely many conditionally independent signals.

Flexible endogenous information acquisition addresses the challenge.

- Also interesting in its own right.
- Growing literature on flexible info in fixed games.

Preview of results

Tractable characterization of implementabilty.

- Finite dimensional system of equations and inequalities.

Information structures consistent with allocational efficiency.

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- Perfectly correlated signals $\Rightarrow$ allocational efficiency.
- *Allocational efficiency $\Rightarrow$ (essentially) perfectly correlated signals*.

Application: subsidy minimization for efficient trade.

- Perfect correlation forces the designer to give up surplus.
  - Compensate for the cost of information acquisition $\Rightarrow$ no *gross* FSE.
  - Prevent further information acquisition $\Rightarrow$ no *net* FSE.

Model: setup

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Principal collects revenue from/subsidizes trade.

Model: information

The true quality $v \in V$ is unknown to anyone at the beginning.

We need a model where players jointly determine info structrure:

State v	s_1^s	s_2^s	$\ldots$	s_J^s
s_1^b	$\alpha_{11}(v)$	$\alpha_{12}(v)$	$\ldots$	$\alpha_{1J}(v)$
s_2^b	$\alpha_{21}(v)$	$\alpha_{22}(v)$	$\ldots$	$\alpha_{2J}(v)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	$\alpha_{I1}(v)$	$\alpha_{I2}(v)$	$\ldots$	$\alpha_{IJ}(v)$

$\Rightarrow$ **player's actions $=$ random variables.**

Model: information acquisition

Commonly known to everyone at the beginning:

- Probability space $(X, \mathcal{F}, \mathbb{P})$, where $X = [0,1] \ni x$ and $\mathbb{P}$ is uniform.
- A random variable $\mathbf{V} : X \to V$, induces a common prior μ_0 on V.

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Players acquire info about $v \in V$ by choosing other random var's.

- Players have access to a countably infinite set of signal realizations.
- A signal of player $p \in \{b, s\}$ is a pair $\sigma^p = (S^p, \mathbf{S}^p)$, where
  - S^p is a finite non-empty subset of $\mathbb{N}$, $\mathbf{S}^p : X \to S^p$ is a random variable.

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  - S^p is a finite non-empty subset of $\mathbb{N}$, $\mathbf{S}^p : X \to S^p$ is a random variable.

$(\mathbf{V}, \sigma^b, \sigma^s)$ induces a joint distribution α over $V \times S^b \times S^s$.

- Any Bayes-plausible (i.e. $\text{marg}_V \alpha = \mu_0$) α can be induced.

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Signals are costly; $C(\sigma^p)$ is posterior separable, ...

- **Today:** $C(\sigma^p)$ is proportional to reduction in entropy.

Model: timing

1. Nature draws $x \in X$ uniformly, but nobody observes it.
2. Principal designs a trading mechanism (M, q, t).
   - $M = M^b \times M^s$; M^p is the message space of player p.
   - $q = (q^b, q^s)$; $q^p : M \to [0,1]$ is the allocation function of player p.
   - $t = (t^b, t^s)$; $t^p : M \to \mathbb{R}$ is the payment function of player p.
3. Each player p privately chooses $\sigma^p = (S^p, \mathbf{S}^p)$.
4. Each player p privately observes $s^p = \mathbf{S}^p(x)$ and sends $m^p \in M^p$.
5. Allocations and payments are determined according to (q, t);
   Quality $v = \mathbf{V}(x)$ is realized.

Roadmap

1. Revelation principle
2. Implementability
3. Information structures consistent with efficient trade
4. Application: subsidy minimization for efficient trade
5. Concluding remarks

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- Principal selects equilibrium $\Rightarrow$ correctly anticipates players' choice of signals $\Rightarrow$ can ask about their signal realizations directly.

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- Principal selects equilibrium ⇒ correctly anticipates players' choice of signals ⇒ can ask about their signal realizations directly.

Revelation principle: it is w.l.o.g. to consider direct mechanisms.

- Players could report one of their signal realizations or abstain:

$$M^b = S^b \cup \{m_\emptyset^b\}, \qquad M^s = S^s \cup \{m_\emptyset^s\},$$

where S^b and S^s are endogenously determined.

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Implementability lemma

Lemma (Implementability for the buyer)

(α, q, t) is implementable for the buyer iff there are multipliers $\lambda_j^b(v)$ for all $s_j^s \in S^s$ and $\phi_{ij}^b(v)$ for all $(s_i^b, s_j^s) \in S^b \times S^s$ and all $v \in V$:

$$(\mathrm{ST}^b) \quad \underbrace{q_{ij}^b u^b(v) - t_{ij}^b}_{\frac{\partial U^b}{\partial \alpha_{ij}(v)}} - \underbrace{\log\left(\mu_i^b(v)\right)}_{\frac{\partial C^b}{\partial \alpha_{ij}(v)}} - \lambda_j^b(v) + \phi_{ij}^b(v) = 0,$$

$$(\mathrm{DF}^b) \quad \phi_{ij}^b(v) \geq 0,$$

$$(\mathrm{CS}^b) \quad \alpha_{ij}(v)\phi_{ij}^b(v) = 0,$$

$$(\mathrm{NA}^b) \quad \sum_{v \in V} \exp\left(-\min_j \{\lambda_j^b(v)\}\right) \leq 1.$$

- Analogous conditions apply to the seller. Seller's implementability

Buyer's problem

Consider a candidate (α, q, t) with α induced by some (σ^b, σ^s).

- Does Buyer have a profitable deviation $\tilde{\sigma}^b$?
- $I+1$ actions under $\sigma^b \Rightarrow \tilde{\sigma}^b$ with $\leq I+1$ realizations are w.l.o.g.
- $(\tilde{\sigma}^b, \sigma^s)$ will induce an alternative information structure $\tilde{\alpha}$.
- Can rewrite the best deviation problem in terms of $\tilde{\alpha}$:

$$\mathcal{BD}^b(\alpha, q, t) = \underset{\tilde{\alpha}, \tilde{S}^b}{\operatorname{argmax}} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{v \in V} \tilde{\alpha}_{ij}(v) \big(q_{ij}^b u^b(v) - t_{ij}^b\big) - c^b(\tilde{\alpha}),$$

$$(1) \quad \tilde{S}^b = S^b \cup \{s_\emptyset^b\}, \quad \tilde{\alpha} \in \Delta\big(\tilde{S}^b \times S^s \times V\big);$$

$$(2) \quad \operatorname{marg}_{S^s \times V} \tilde{\alpha} = \operatorname{marg}_{S^s \times V} \alpha.$$

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$$(1) \quad \tilde{S}^b = S^b \cup \{s^b_\emptyset\}, \quad \tilde{\alpha} \in \Delta\big(\tilde{S}^b \times S^s \times V\big);$$

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Implementability condition for the buyer: $(\alpha, S^b) \in \mathcal{BD}^b(\alpha, q, t)$.

Solution to Buyer's problem

We split Buyer's deviations into two classes:

- **Class 1:** induce different $\tilde{\alpha}$'s over the same signal realizations S^b.
- **Class 2:** augment S^b with $s^b_\emptyset$ with positive probability.

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Our approach:

1. Solve **Class 1**-problem, characterize solution in terms of λ, ϕ.
   - Convex problem $\Rightarrow$ KKT conditions are necessary and sufficient.
2. Show the following:

Lemma

If α solves **Class 1**-*problem, then (α, S^b) solves* **Class 2**-*problem iff*

$$(\mathrm{NA}^b) \qquad \sum_{v \in V} \exp\left(- \min_j \{\lambda^b_j(v)\}\right) \leq 1.$$

Class 2-lemma

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Proof sketch: illustrate the proof using a $2 \times 2 \times 2$ example.

State $\underline{v}$	s_1^s	s_2^s
s_1^b	$\underline{\alpha}_{11} - \epsilon\underline{\beta}_{11}$	$\underline{\alpha}_{12} - \epsilon\underline{\beta}_{12}$
s_2^b	$\underline{\alpha}_{21} - \epsilon\underline{\beta}_{21}$	$\underline{\alpha}_{22} - \epsilon\underline{\beta}_{22}$
$s_\emptyset^b$	$\epsilon(\underline{\beta}_{11} + \underline{\beta}_{21})$	$\epsilon(\underline{\beta}_{12} + \underline{\beta}_{22})$

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$G_\alpha(\epsilon\beta)$ **is the gain from deviation in direction** $\epsilon\beta$ **from** α**.**

$MG_\alpha(\beta) \equiv \lim_{\epsilon\to 0} \frac{1}{\epsilon} G_\alpha(\epsilon\beta)$ **is the corresponding marginal gain.**

Proof of Class 2-lemma

We prove the contrapositive statement:

- Suppose there is a deviation with a positive gain $G_\alpha(\beta) > 0$.
- Convexity of cost function $\Rightarrow \left[G_\alpha(\beta) > 0 \Rightarrow MG_\alpha(\beta) > 0\right]$.
- $MG_\alpha(\beta)$ can be computed in closed form:

$$MG_\alpha(\beta) = -\sum_{i,j,v} \overbrace{\beta_{ij}(v)}^{0 \text{ if } \alpha_{ij}(v)=0} \times \Big[\underbrace{q^b_{ij} u^b(v) - t^b_{ij} - \log\left(\mu^b_i(v)\right)}_{=\lambda^b_j(v) \text{ as long as } \alpha_{ij}(v)>0, \text{ by KKT}} \Big] - MCost(\beta)$$

$$= -\sum_{i,j,v} \beta_{ij}(v)\lambda^b_j(v) - MCost(\beta).$$

- Let $\beta^* = \operatorname{argmax}_\beta MG_\alpha(\beta; \lambda^b)$, then $MG_\alpha(\beta^*; \lambda^b) > 0 \Rightarrow \neg(\mathrm{NA}^b)$.

Roadmap

Information structures consistent with efficient trade

$V = \{\underline{v}, \overline{v}\}$ **and gains from trade at every quality level:** $u^b(v) > u^s(v)$.

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Proposition (Efficiency ⇒ Essentially perfect correlation)

If α is consistent with efficient trade, then α has the following form:

State v	s_1^s	$\dots$	s_k^s	$\dots$	$s_{I-\ell}^s$	$\dots$	s_I^s
s_1^b	α_{11}	$\dots$	α_{1k}	$\dots$	0	$\dots$	0
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
s_k^b	α_{k1}	$\dots$	α_{kk}	$\dots$	0	$\dots$	0
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$s_{I-\ell}^b$	0	$\dots$	0	$\dots$	$\alpha_{I-\ell,I-\ell}$	$\dots$	$\alpha_{I-\ell,I}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	0	$\dots$	0	$\dots$	$\alpha_{I,I-\ell}$	$\dots$	α_{II}

and the posteriors within each block are equal to each other.

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch ($2 \times 2 \times 2$, distinct posteriors)

Consider the following special case:

State $\underline{v}$	s_1^s	s_2^s
s_1^b	$\underline{\alpha}_{11}$	$\underline{\alpha}_{12}$
s_2^b	$\underline{\alpha}_{21}$	$\underline{\alpha}_{22}$

State $\overline{v}$	s_1^s	s_2^s
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Our goal is to show perfect correlation:

State $\underline{v}$	s_1^s	s_2^s
s_1^b	$\underline{\alpha}_{11}$	0
s_2^b	0	$\underline{\alpha}_{22}$

State $\overline{v}$	s_1^s	s_2^s
s_1^b	$\overline{\alpha}_{11}$	0
s_2^b	0	$\overline{\alpha}_{22}$

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$$
\begin{aligned}
(\mathrm{ST}_{11}^b) \quad & \underline{u}^b - t_{11}^b - \log\left(\underline{\mu}_1^b\right) - \underline{\lambda}_1^b + \underline{\phi}_{11}^b = 0, \\
& \overline{u}^b - t_{11}^b - \log\left(\overline{\mu}_1^b\right) - \overline{\lambda}_1^b + \overline{\phi}_{11}^b = 0;
\end{aligned}
$$

$$
\overline{\lambda}_1^b - \underline{\lambda}_1^b - (\overline{u}^b - \underline{u}^b) = \overline{\phi}_{11}^b - \underline{\phi}_{11}^b - \log\left[\frac{\overline{\mu}_1^b}{\underline{\mu}_1^b}\right].
$$

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$$(\mathrm{ST}^b_{11}) \quad \begin{aligned} &\underline{u}^b - t^b_{11} - \log\left(\underline{\mu}^b_1\right) - \underline{\lambda}^b_1 + \underline{\phi}^b_{11} = 0, \\ &\overline{u}^b - t^b_{11} - \log\left(\overline{\mu}^b_1\right) - \overline{\lambda}^b_1 + \overline{\phi}^b_{11} = 0; \end{aligned}$$

$$\overline{\lambda}^b_1 - \underline{\lambda}^b_1 - (\overline{u}^b - \underline{u}^b) = \overline{\phi}^b_{11} - \underline{\phi}^b_{11} - \log\left[\frac{\overline{\mu}^b_1}{\underline{\mu}^b_1}\right].$$

$$(\mathrm{ST}^b_{21}) \quad \begin{aligned} &\underline{u}^b - t^b_{21} - \log\left(\underline{\mu}^b_2\right) - \underline{\lambda}^b_1 + \underline{\phi}^b_{21} = 0, \\ &\overline{u}^b - t^b_{21} - \log\left(\overline{\mu}^b_2\right) - \overline{\lambda}^b_1 + \overline{\phi}^b_{21} = 0; \end{aligned}$$

$$\overline{\lambda}^b_1 - \underline{\lambda}^b_1 - (\overline{u}^b - \underline{u}^b) = \overline{\phi}^b_{21} - \underline{\phi}^b_{21} - \log\left[\frac{\overline{\mu}^b_2}{\underline{\mu}^b_2}\right].$$

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch ($2 \times 2 \times 2$, distinct posteriors)

$$\overline{\phi}_{11}^{b} + \underline{\phi}_{21}^{b} = \overline{\phi}_{21}^{b} + \underline{\phi}_{11}^{b} + \underbrace{\log\left[\frac{\overline{\mu}_1^b}{\underline{\mu}_1^b}\right] - \log\left[\frac{\overline{\mu}_2^b}{\underline{\mu}_2^b}\right]}_{>0 \text{ by ordering assumption}}.$$

Efficiency ⇒ Essentially perfect correlation

Proof sketch ($2 \times 2 \times 2$, distinct posteriors)

$$\overline{\phi}^b_{11} + \underline{\phi}^b_{21} = \overline{\phi}^b_{21} + \underline{\phi}^b_{11} + \underbrace{\log\left[\frac{\overline{\mu}^b_1}{\underline{\mu}^b_1}\right] - \log\left[\frac{\overline{\mu}^b_2}{\underline{\mu}^b_2}\right]}_{>0 \text{ by ordering assumption}}.$$

Consideration of (ST^b_{11}) and (ST^b_{21}) therefore implies:

$$\underbrace{\overline{\phi}^b_{11} + \underline{\phi}^b_{21}}_{\Rightarrow \text{at least one term is } >0} > \overline{\phi}^b_{21} + \underline{\phi}^b_{11}.$$

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$$\overline{\phi}_{11}^{b} + \underline{\phi}_{21}^{b} = \overline{\phi}_{21}^{b} + \underline{\phi}_{11}^{b} + \underbrace{\log\left[\frac{\overline{\mu}_1^b}{\underline{\mu}_1^b}\right] - \log\left[\frac{\overline{\mu}_2^b}{\underline{\mu}_2^b}\right]}_{>0 \text{ by ordering assumption}}.$$

Consideration of (ST_{11}^{b}) and (ST_{21}^{b}) therefore implies:

$$\underbrace{\overline{\phi}_{11}^{b} + \underline{\phi}_{21}^{b}}_{\Rightarrow \text{at least one term is } >0} > \overline{\phi}_{21}^{b} + \underline{\phi}_{11}^{b}.$$

Analogous consideration of (ST_{12}^{b}) and (ST_{22}^{b}) implies:

$$\underbrace{\overline{\phi}_{12}^{b} + \underline{\phi}_{22}^{b}}_{\Rightarrow \text{at least one term is } >0} > \overline{\phi}_{22}^{b} + \underline{\phi}_{12}^{b}.$$

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch ($2 \times 2 \times 2$, distinct posteriors)

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch ($2 \times 2 \times 2$, distinct posteriors)

Suppose $\overline{\phi}^b_{11} > 0$ and $\overline{\phi}^b_{12} > 0$, then CS implies:

State $\underline{v}$	s^s_1	s^s_2
s^b_1	$\underline{\alpha}_{11}$	$\underline{\alpha}_{12}$
s^b_2	$\underline{\alpha}_{21}$	$\underline{\alpha}_{22}$

State $\overline{v}$	s^s_1	s^s_2
s^b_1	0	0
s^b_2	$\overline{\alpha}_{21}$	$\overline{\alpha}_{22}$

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch ($2 \times 2 \times 2$, distinct posteriors)

Suppose $\overline{\phi}^b_{11} > 0$ and $\overline{\phi}^b_{12} > 0$, then CS implies:

State $\underline{v}$	s^s_1	s^s_2
s^b_1	$\underline{\alpha}_{11}$	$\underline{\alpha}_{12}$
s^b_2	$\underline{\alpha}_{21}$	$\underline{\alpha}_{22}$

State $\overline{v}$	s^s_1	s^s_2
s^b_1	0	0
s^b_2	$\overline{\alpha}_{21}$	$\overline{\alpha}_{22}$

Suppose $\underline{\phi}^b_{21} > 0$ and $\underline{\phi}^b_{22} > 0$, then CS implies:

State $\underline{v}$	s^s_1	s^s_2
s^b_1	$\underline{\alpha}_{11}$	$\underline{\alpha}_{12}$
s^b_2	0	0

State $\overline{v}$	s^s_1	s^s_2
s^b_1	$\overline{\alpha}_{11}$	$\overline{\alpha}_{12}$
s^b_2	$\overline{\alpha}_{21}$	$\overline{\alpha}_{22}$

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch ($2 \times 2 \times 2$, distinct posteriors)

Suppose $\overline{\phi}^b_{11} > 0$ and $\underline{\phi}^b_{22} > 0$, then CS implies:

State $\underline{v}$	s_1^s	s_2^s
s_1^b	$\underline{\alpha}_{11}$	$\underline{\alpha}_{12}$
s_2^b	$\underline{\alpha}_{21}$	0

State $\overline{v}$	s_1^s	s_2^s
s_1^b	0	$\overline{\alpha}_{12}$
s_2^b	$\overline{\alpha}_{21}$	$\overline{\alpha}_{22}$

Efficiency ⇒ Essentially perfect correlation

Proof sketch ($2 \times 2 \times 2$, distinct posteriors)

Suppose $\overline{\phi}^b_{11} > 0$ and $\underline{\phi}^b_{22} > 0$, then CS implies:

State $\underline{v}$	s^s_1	s^s_2
s^b_1	$\underline{\alpha}_{11}$	$\underline{\alpha}_{12}$
s^b_2	$\underline{\alpha}_{21}$	0

State $\overline{v}$	s^s_1	s^s_2
s^b_1	0	$\overline{\alpha}_{12}$
s^b_2	$\overline{\alpha}_{21}$	$\overline{\alpha}_{22}$

- Bayes-plausibility then implies:

$$\overline{\mu}_0 < \overline{\mu}^b_1 = \frac{\overline{\alpha}_{12}}{\underline{\alpha}_{11} + \underline{\alpha}_{12} + \overline{\alpha}_{12}} \leq \frac{\overline{\alpha}_{12}}{\underline{\alpha}_{12} + \overline{\alpha}_{12}},$$

$$\overline{\mu}_0 > \overline{\mu}^s_2 = \frac{\overline{\alpha}_{12} + \overline{\alpha}_{22}}{\underline{\alpha}_{12} + \overline{\alpha}_{12} + \overline{\alpha}_{22}} \geq \frac{\overline{\alpha}_{12}}{\underline{\alpha}_{12} + \overline{\alpha}_{12}}.$$

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch ($2 \times 2 \times 2$, distinct posteriors)

The only remaining possibility is $\overline{\phi}^b_{12} > 0$ and $\underline{\phi}^b_{21} > 0$:

State $\underline{v}$	s^s_1	s^s_2
s^b_1	$\underline{\alpha}_{11}$	$\underline{\alpha}_{12}$
s^b_2	0	$\underline{\alpha}_{22}$

State $\overline{v}$	s^s_1	s^s_2
s^b_1	$\overline{\alpha}_{11}$	0
s^b_2	$\overline{\alpha}_{21}$	$\overline{\alpha}_{22}$

Efficiency $\Rightarrow$ Essentially perfect correlation

Proof sketch ($2 \times 2 \times 2$, distinct posteriors)

The only remaining possibility is $\overline{\phi}^b_{12} > 0$ and $\underline{\phi}^b_{21} > 0$:

State $\underline{v}$	s^s_1	s^s_2
s^b_1	$\underline{\alpha}_{11}$	$\underline{\alpha}_{12}$
s^b_2	0	$\underline{\alpha}_{22}$

State $\overline{v}$	s^s_1	s^s_2
s^b_1	$\overline{\alpha}_{11}$	0
s^b_2	$\overline{\alpha}_{21}$	$\overline{\alpha}_{22}$

Analogous argument for Seller gives $\underline{\phi}^s_{12} > 0$ and $\overline{\phi}^s_{21} > 0$:

State $\underline{v}$	s^s_1	s^s_2
s^b_1	$\underline{\alpha}_{11}$	0
s^b_2	$\underline{\alpha}_{12}$	$\underline{\alpha}_{22}$

State $\overline{v}$	s^s_1	s^s_2
s^b_1	$\overline{\alpha}_{11}$	$\overline{\alpha}_{12}$
s^b_2	0	$\overline{\alpha}_{22}$

Roadmap

Subsidy minimization $\Rightarrow$ perfect correlation is w.l.o.g.

Corollary (Perfect correlation)

If $(\alpha', I', J'; t'; \phi', \lambda')$ is feasible in the subsidy minimization problem, then there is $(\alpha, I, J; t; \phi; \lambda)$, which is also feasible and achieves the same objective value, but $I = J$ and $\underline{\alpha}_{ij} = \overline{\alpha}_{ij} = 0$ for $i \neq j$.

State $\underline{v}$	s_1^s	s_2^s	$\dots$	s_I^s
s_1^b	$\underline{\alpha}_1$	0	$\dots$	0
s_2^b	0	$\underline{\alpha}_2$	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	0	0	$\dots$	$\underline{\alpha}_I$

State $\overline{v}$	s_1^s	s_2^s	$\dots$	s_I^s
s_1^b	$\overline{\alpha}_1$	0	$\dots$	0
s_2^b	0	$\overline{\alpha}_2$	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	0	0	$\dots$	$\overline{\alpha}_I$

Proof: merge signal realizations with equal posteriors.

Subsidy minimization $\Rightarrow$ perfect correlation is w.l.o.g.

Corollary (Perfect correlation)

If $(\alpha', I', J'; t'; \phi', \lambda')$ *is feasible in the subsidy minimization problem, then there is* $(\alpha, I, J; t; \phi; \lambda)$*, which is also feasible and achieves the same objective value, but* $I = J$ *and* $\underline{\alpha}_{ij} = \overline{\alpha}_{ij} = 0$ *for* $i \neq j$.

State $\underline{v}$	s_1^s	s_2^s	$\dots$	s_I^s
s_1^b	$\underline{\alpha}_1$	0	$\dots$	0
s_2^b	0	$\underline{\alpha}_2$	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	0	0	$\dots$	$\underline{\alpha}_I$

State $\overline{v}$	s_1^s	s_2^s	$\dots$	s_I^s
s_1^b	$\overline{\alpha}_1$	0	$\dots$	0
s_2^b	0	$\overline{\alpha}_2$	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	0	0	$\dots$	$\overline{\alpha}_I$

Proof: merge signal realizations with equal posteriors.

Two design concerns for the principal: IC and total cost of info.

- **IC:** More correlated signals $\Rightarrow$ easier to incentivize truthful reporting.
- **Total cost:** Less correlated signals $\Rightarrow$ more info at lower cost.

Subsidy minimization $\Rightarrow$ perfect correlation is w.l.o.g.

Corollary (Perfect correlation)

If $(\alpha', I', J'; t'; \phi', \lambda')$ is feasible in the subsidy minimization problem, then there is $(\alpha, I, J; t; \phi; \lambda)$, which is also feasible and achieves the same objective value, but $I = J$ and $\underline{\alpha}_{ij} = \overline{\alpha}_{ij} = 0$ for $i \neq j$.

State $\underline{v}$	s_1^s	s_2^s	$\dots$	s_I^s
s_1^b	$\underline{\alpha}_1$	0	$\dots$	0
s_2^b	0	$\underline{\alpha}_2$	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	0	0	$\dots$	$\underline{\alpha}_I$

State $\overline{v}$	s_1^s	s_2^s	$\dots$	s_I^s
s_1^b	$\overline{\alpha}_1$	0	$\dots$	0
s_2^b	0	$\overline{\alpha}_2$	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_I^b	0	0	$\dots$	$\overline{\alpha}_I$

Proof: merge signal realizations with equal posteriors.

Two design concerns for the principal: IC and total cost of info.

- **IC:** More correlated signals $\Rightarrow$ easier to incentivize truthful reporting.
- **Total cost:** Less correlated signals $\Rightarrow$ more info at lower cost.

IC overwhelmingly dominates $\Rightarrow$ pay for the same info twice!

Subsidy minimization as Bayesian persuasion

$$\max_{\{\tau,\mu;I;\Lambda\}} \sum_{i=1}^{I} \tau_i T(\underline{\mu}_i, \overline{\mu}_i; \Lambda^b, \Lambda^s)$$

$$\text{(BP)} \quad \sum_{i=1}^{I} \tau_i \underline{\mu}_i = \underline{\mu}_0, \quad \sum_{i=1}^{I} \tau_i \overline{\mu}_i = \overline{\mu}_0;$$

$$(\text{NA}^b) \quad \exp\left(-\underline{\Lambda}^b\right) + \exp\left(-\overline{\Lambda}^b\right) = 1,$$

$$(\text{NA}^s) \quad \exp\left(-\underline{\Lambda}^s\right) + \exp\left(-\overline{\Lambda}^s\right) = 1.$$

where $\tau_i = \underline{\alpha}_i + \overline{\alpha}_i$, and $\underline{\Lambda}^p = \min_i \left\{\underline{\lambda}_i^p\right\}$ and $\overline{\Lambda}^p = \min_i \left\{\overline{\lambda}_i^p\right\}$.

For a fixed Λ, this is a Bayesian persuasion problem
$\Rightarrow$ look at concave closure of T. Concave closure of T

Roadmap

Concluding remarks

- Bilateral trade problem with information acquisition.
- Information acquistion is costly and flexible.
- Tractable characterization of implementability.
- Characterization of info structures consistent with efficient trade.
- Subsidy minimization for efficient trade.

Appendix

Implementability for the seller

Lemma (Implementability for the seller)

(α, q, t) is globally implementable for the seller iff there are multipliers $\lambda_i^s(v)$ for all $s_i^b \in S^b$ and $\phi_{ij}^s(v)$ for all $(s_i^b, s_j^s) \in S^b \times S^s$ and all $v \in V$:

$$\text{(ST}^s\text{)} \quad \underbrace{t_{ij}^s - q_{ij}^s u^s(v)}_{\frac{\partial U^s}{\partial \alpha_{ij}(v)}} - \underbrace{\log\left(\mu_j^s(v)\right)}_{\frac{\partial C^s}{\partial \alpha_{ij}(v)}} - \lambda_i^s(v) + \phi_{ij}^s(v) = 0,$$

$$\text{(DF}^s\text{)} \quad \phi_{ij}^s(v) \geq 0,$$

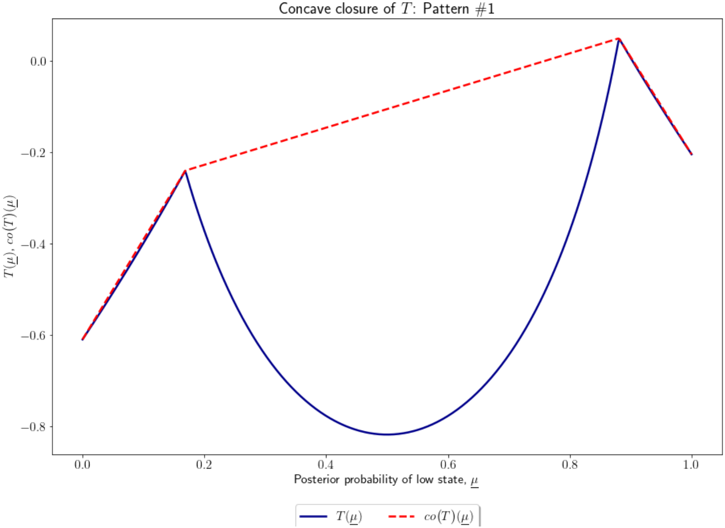
$$\text{(CS}^s\text{)} \quad \alpha_{ij}(v)\phi_{ij}^s(v) = 0,$$

$$\text{(NA}^s\text{)} \quad \sum_{v \in V} \exp\left(-\min_i\{\lambda_i^s(v)\}\right) \leq 1.$$

- Analogous conditions apply to the buyer. Buyer's implementability

Appendix

Concave closure of $T(\underline{\mu}, 1-\underline{\mu}; \Lambda^b, \Lambda^s)$



Optimality conditions

Proposition (Optimality conditions)

If the subsidy minimization problem achieves a minimum, then we can set $I = 2$ *w.l.o.g., and moreover the optimal posteriors satisfy*

$$(\text{Opt}^b) \quad \underline{u}^b - \log(\underline{\mu}_1) - \underline{\Lambda}^b = \overline{u}^b - \log(\overline{\mu}_1) - \overline{\Lambda}^b,$$

$$(\text{Opt}^s) \quad \underline{u}^s + \log(\underline{\mu}_2) + \underline{\Lambda}^s = \overline{u}^s + \log(\overline{\mu}_2) + \overline{\Lambda}^s.$$

Combine (Opt) **with** (NA) **to solve for** Λ **and plug into the objective** $\Rightarrow$ **unconstrained problem for posteriors**.

Go back