Information in Decentralised Evaluations *

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Abstract

A risky opportunity is assessed by many evaluators until one seizes it. Evaluations are decentralised: no evaluator knows how many others rejected the opportunity, nor why they did so. An adverse selection problem ensues: evaluators recognise that merely receiving the opportunity is suggestive of its past rejections, and is therefore bad news. Better information might leave evaluators worse off by exacerbating this adverse selection problem. I characterise which Blackwell improvements of evaluators' information improve their decisions (payoffs) and which ones harm them. This characterisation informs a regulator who wants to improve evaluators' decision quality by restricting their information. I show that she must strive to prevent any regret that adverse selection might beget: ideally, no evaluator takes the opportunity unless it would be optimal despite all her competitors' rejections.

1 Introduction

A risky opportunity is often considered by many interested parties before one eventually seizes it. A bank might be receiving a loan application turned down by all its competitors. A venture capitalist might be seeking to invest in an idea other investors judged to be unviable. A record label might have been the last hope of the band they just signed. Indeed, the question might be *how many* rather than *if*: reportedly "*all* VCs in Europe and North America" turned Spotify down before it secured its first investment[1]. This breeds an *adverse selection* problem. Her *mere receipt* of the opportunity suffices to worry an evaluator that her favourable assessment of it is the exception among her competitors' cynicism.

Better information about the value of this opportunity might, one could hope, mitigate this worry by allowing these evaluators to be more confident in their assessments. However, this is

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[1] See Parson, 2018 and Faull, 2023.

no rose without thorns: when *everyone* is better informed, each might grow more fearful of their peers' past rejections; amplifying this adverse selection problem. Better information can thus even backfire by exacerbating the adverse selection problem to the extent of crowding out the confidence it instils in evaluators about their own assessments.

I study how better information shapes the quality of evaluators' decisions – and therefore their payoffs – against the backdrop of this adverse selection problem. I focus on settings where evaluations are *decentralised*: an evaluator can freely seize the opportunity when it presents itself, but she can only rely on her own private information to evaluate it. In particular, she remains ignorant both about *how many* of her peers already rejected the opportunity and *why* they did so. Examples abound: lenders do not see where else their applicant applied to within his 45 day "shopping period"[2]. Record labels or VCs considering an opportunity can hardly rely on their competitors – and even less so on the bearer of the opportunity – to disclose whether and why they already rejected it.

To study this problem, I build a simple model where an applicant of unknown quality sequentially visits multiple evaluators, pursuing an approval from one of them. He might be seeking a loan, offering an asset, or requesting funding for his enterprise. He visits each evaluator on his list until either one of them approves him, or they all reject him. Approving the applicant entails a fixed cost, but yields a positive return when he has high quality. So, each evaluator strives to approve a high quality applicant, but reject a low quality one.

Once she receives the applicant, an evaluator privately observes the outcome of a Blackwell experiment informative about the applicant's quality. She is disclosed no information about her applicant's past rejections, but understands that his *mere visit* is already a forewarning. How receiving this visit shapes the evaluator's beliefs about the applicant depends on whether she believes many evaluators already rejected him, and what those rejections would mean about his quality. In equilibrium, her approval decisions must be optimal given these beliefs, which she forms endogenously.

Evaluators' equilibrium payoffs depend on how well they can detect and approve high quality applicants while keeping low quality applicants out. How do these payoffs change when each evaluator observes the outcome of a *(Blackwell) more informative* experiment about the applicant's quality? Blackwell (1953) asked and answered this question for a *single* evaluator who faces *no* adverse selection, for a lack of competitors. He showed that that evaluator *always* benefits from better information, as it allows her to judge the applicant's quality more accurately and fine tune her decisions.

With multiple evaluators, another, potentially countervailing effect of better information emerges. By affecting *how likely* the applicant received a multitude of rejections earlier and

[2] A loan seeker's credit history displays any applications within this shopping period as a single inquiry. See Consumer Financial Protection Bureau, 2023.

what those rejections mean for his quality, better information might leave each evaluator more pessimistic about their visitor from the get go. This effect might overwhelm the improved judgement it affords each individual evaluator, eventually leaving each of them worse off.

In this paper, I characterise *when* – against the backdrop of this adverse selection problem – evaluators benefit from more information, and when they are harmed by it. To grasp the intuition behind my main takeaway, let evaluators' Blackwell experiment be particularly simple: one that delivers either a "good news" signal or a "bad news" signal. Initially, "good news" sway the evaluator to an approval, and "bad news" to a rejection. But imagine that just before she strikes her verdict, the evaluator receives a *second* piece of information about her applicant's quality. If this piece of information follows "good news", the evaluator uses it either to reaffirm her optimism or revisit her initial approval intention. As it double-checks "good news", I say the evaluator gets *stronger good news*. If this piece of information otherwise follows "bad news", it either consolidates the evaluator's pessimism or challenges her initial intention to reject. I then say she get *stronger bad news*.

Stronger good news jeopardise an applicant who – absent that second piece of information – would have been approved by some evaluator. Once evaluators receive stronger good news, this applicant might find that *every* evaluator who would otherwise approve him revised her verdict to a rejection. Evaluators benefit from his reversal of fortune – one of them would otherwise have approved this applicant who is now unfavourably assessed by *all* of them. With *stronger bad news*, in contrast, an applicant otherwise every evaluator would reject gets another chance. *Any* of those evaluators might receive favourable evidence following the initial "bad news", overturning her initial rejection verdict. Oblivious, that evaluator might be alone in her renewed optimism, perhaps dominated by the – stronger – bad news her competitors received. When so, her revised approval decision leaves her worse off.

This intuition drives my main takeaway: evaluators *always* benefit from *stronger good news*, but are hurt by *stronger bad news* when adverse selection is too acute. I formalise these intuitive forms "more information" can take as *local mean preserving spreads* of different signals. When formalised as such, stronger good and bad news become the building blocks of any more complex Blackwell improvement of evaluators' experiment. I also show that the pressure adverse selection levies on the benefits of better information is substantial. Stronger bad news hurt evaluators *whenever* an evaluator would regret overturning a rejection upon learning that her worst case scenario had materialised: that *every* other evaluator rejected the applicant.

This is a valuable lesson for a regulator who wishes to restrict evaluators' information to improve their payoffs – equivalently, decision quality. I next turn to this regulator's problem. At the outset of the game, the regulator chooses a *garbling* of evaluators' experiment. Thereafter, an evaluator observes only the garbled outcome of her experiment when the applicant visits her. By coarsening evaluators' information as such, the regulators wishes to maximise evaluators' expected payoffs in equilibrium. Strikingly, I show that the regulator's optimal garbling focuses on the *last* evaluator the applicant can visit. This optimal garbling strives to prevent an approval *whenever* the evaluator would regret doing so upon discovering that she was the *last* evaluator the applicant could apply to. As so, the regulator wishes evaluators to behave as if they were *last* in the applicant's list when approving him. This aligns with our previous finding: *whenever* adverse selection is acute enough that an evaluator would regret approving in her worst-case scenario, the regulator tries to withhold favourable information from her.

This exercise has direct implications for interpreting recent financial policy, especially in lending markets. In the wake of the 2008 subprime mortgage crisis, global financial regulation sought to improve lending quality – especially in mortgage markets – by *restricting* both what *kind* of information lenders can use to assess borrowers, and *how* they can use it[3]. Such regulation often followed a "distrust" in the ability – or willingness – of lenders to assess the underlying risks of their decisions prudently, and a fear that they were consequently lending too much[4] This idea that lending practices could be "disciplined" by curbing the information lenders could use did not go unopposed. Critics quickly pointed out *coarsening* lenders' information would only *worsen* the quality of their decisions, contrary to regulators' stated goals[5].

I find that regulation that curbs the information available to evaluators *can*, in fact, increase the quality of evaluators' decisions. The reason is not rooted in evaluators' inability or unwillingness to interpret the information they receive correctly, but rather in a simple but fundamental adverse selection problem endemic to settings with decentralised evaluations. When employed correctly, coarsening evaluators' information alleviates this adverse selection burden. Such policy, I find, is particularly likely to be successful when it *constrains approvals*, precisely the aim – and criticism behind – the Ability-to-Repay Rule and Basel III accords. However, my analysis also presents a cautionary note. Regulations or technological advances that allow evaluators to screen their applicant better may backfire, *worsening* the quality of their decisions instead of improving them.

I structure the rest of the paper as follows. Section 1.1 lays out the related literature and discusses closely related work. Section 2 introduces the model. Section 3 characterises the equilibria of the model, and explains how evaluators form beliefs about the applicant they receive. In Section 4 I develop my main result, characterising when more information benefits

[3]The Ability-to-Repay Rule prescribes minimum requirements a borrower must meet, preventing lenders from financing him based on other evidence they possess; see CFPB, 2013 and 2019. The Basel III Accords severely limited the use of "advanced internal ratings systems" to determine credit risk exposure, overturning Basel II. These systems allowed banks to use their private information to evaluate their risk exposure. See BCBS, 2017.

[4]The ATR Rule was motivated by "too many mortgages [having been] made to consumers without regard to [their] ability to repay", and sought to ensure "a reasonable and good faith determination" of consumers' creditworthiness (CFPB, 2013). The Basel Committee believed they could not trust banks to model certain risks "in a robust and prudent manner" (BCBS, 2017).

[5]The chairman of the Global Financial Markets Association stated that "[a]ssessments that are made internally by banks ... enable banks to make the most efficient capital allocation ... Relying on regulators' assessments of risks ... rough approximations at best ... would be a major step backwards." See Bentsen, Jr., 2020.

evaluators, and when it hurts them. I do this in two stages: in Section 4.1, I focus on evaluators with simple, binary experiments to develop intuition and pave the building blocks of my general result. In Section 4.2, I generalise the insight I develop there, using the ideas and techniques I build in Section 4.1. I then turn to the regulator's problem and study the optimal garbling of evaluators' experiment in Section 5. Unless I state otherwise, I reserve the proofs of my results to Section 6.3, choosing instead to provide a proof outline for the most important results. Section 6.2 has some additional results that might spark the reader's interest along the way.

1.1 Related Literature

to be added!

2 The Model

An *applicant* (he) with either *High* or *Low* quality, $\theta \in \{H, L\}$, seeks an approval from any one of n *evaluators* (she). To obtain the approval he seeks, he sequentially visits these n evaluators, each at most once. Any evaluator who receives his visit decides whether to *approve* or *reject* the applicant. Once he is approved or has visited all n evaluators, the applicant stops his visits. Otherwise he visits the evaluator labelled $\tau(k)$ after his $k-1^{\text{th}}$ rejection, where $\tau(.)$ is a permutation of the set of evaluators' labels $\{1, 2, ..., n\}$.

If an evaluator approves the applicant, she pays a fixed cost of $c \in [0, 1]$. She also receives a benefit of 1 whenever the applicant has *High* quality. With her approval, the game ends; the applicant stops and the remaining evaluators walk away with a payoff of 0. On the other hand, if she rejects the applicant, she receives a payoff of 0. The applicant then proceeds to visit the remaining evaluators, unless none remain.

At the outset of the game, the applicant and all n evaluators share the *prior belief* that the applicant has *High* quality with probability $\rho \in (0, 1)$. Moreover, the evaluators commonly believe that they are equally likely to be anywhere in the applicant's visit order; i.e. that $\mathbb{P}(\tau(k) = i) = \frac{1}{n}$ for all $k, i \in \{1, 2, ..., n\}$. No further information about the applicant's visits is disclosed to evaluators; neither the order $\tau(.)$ he follows, nor the number of evaluators who rejected him already.

Crucially however, an evaluator receiving a visit understands that the applicant was rejected in all his previous visits, however many there might have been. With this information the applicant's visit conveys, she updates her prior belief ρ about the applicant's quality to an *interim belief* ψ. She uses Bayes' Rule to do so.

Subsequently, she costlessly and privately observes the outcome of a Blackwell experiment $\mathcal{E} = (\mathbf{S}, p_L, p_H)$. The outcome s of this experiment, the evaluator's *signal*, is an element of the finite set $\mathbf{S}$, and has a distribution p_θ over this set given the applicant's quality. Conditional on the applicant's quality, different evaluators' signals are IID. After she observes the signal $s \in \mathbf{S}$, the evaluator updates her beleif about the applicant's quality again; now from the interim belief ψ to a *posterior belief* $\mathbb{P}_\psi(\theta = H \mid S = s)$. For this update too, she uses Bayes' Rule.

An evaluator's strategy $\sigma : \mathbf{S} \to [0, 1]$ prescribes a probability of approval $\sigma(s)$ to every possible signal $s \in \mathbf{S}$ she might observe. I call a strategy σ *optimal against the evaluator's interim belief* ψ if, given this interim belief ψ, it maximises her expected payoff. Under such a strategy σ, the evaluator approves after any signal $s \in \mathbf{S}$ which raises her posterior belief that the applicant has *High* quality above c. Likewise, she rejects whenever this posterior belief sinks below c:

$$\sigma(s) = \begin{cases} 0 & \mathbb{P}_\psi(\theta = H \mid S = s) < c \\ 1 & \mathbb{P}_\psi(\theta = H \mid S = s) > c \end{cases}$$

A signal that sets her posterior belief exactly equal to c leaves her indifferent between approving and rejecting the applicant. Any approval probability her strategy dictates after such a signal realisation is consistent with its optimality.

I focus on the *symmetric Bayesian Nash Equilibria* of this game. Hereafter, I reserve the word "equilibrium" for such equilibria unless I state otherwise. A strategy and belief pair (σ^*, ψ^*) is an *equilibrium* of this game if and only if it satisfies the two conditions below:

1. The interim belief ψ^* is *consistent* with the strategy σ^*. That is, an evaluator receiving a visit forms the interim belief ψ^* given others' strategies are σ^*.

2. The strategy σ^* is optimal given the interim belief ψ^*.

I call any strategy σ^* which constitutes part of an equilibrium an *equilibrium strategy.*

3 Belief Formation and Equilibria

Before her verdict, the evaluator who receives a visit must assess the probability that she faces a *High* quality applicant. Her privately observed signal about this applicant's quality plays a crucial part in this assessment. But she obtains her first piece of information even earlier, through *her mere receipt of the applicant's visit.*

The applicant visits this evaluator only if he was rejected by every evaluator he visited earlier. Any such rejections are themselves bad news about the applicant's quality, as they reveal his past evaluators' negative assessments. No information about the number of these past rejections is disclosed to our evaluator. Nonetheless, she is aware of the adverse selection problem she faces: the likelier her peers are to reject the applicant, the likelier she is to be visited by him. Therefore, she interprets the applicant's mere visit as bad news about his quality already.

In particular, when all her peers have the strategies σ, our evaluator understands that an applicant with quality θ faces a probability $r_\theta(\sigma; \mathcal{E})$ of getting rejected from any of his visits, where this probability is given by:

$$r_\theta(\sigma; \mathcal{E}) = 1 - \sum_{j=1}^{m} p_\theta(s_j)\sigma(s_j)$$

She – ex-ante – believes she is equally likely to be anywhere in the applicant's visit order $\tau(.)$. So, she believes that an applicant with quality θ will visit her with probability $\nu_\theta(\sigma; \mathcal{E})$ before any of her peers approves him:

$$\nu_\theta(\sigma; \mathcal{E}) = \frac{1}{n} \times \sum_{k=1}^{n} r_\theta(\sigma; \mathcal{E})^{k-1}$$

Our evaluator's *interim belief* ψ that the applicant who visited her has *High* quality must be *consistent* with these beliefs she holds. Through Bayes Rule, this consistency requirement pins her interim belief down uniquely:

$$\psi := \mathbb{P}\left(\theta = H \mid \text{visit received}\right) = \frac{\mathbb{P}\left(\text{visit received} \mid \theta = H\right) \times \mathbb{P}(\theta = H)}{\mathbb{P}\left(\text{visit received}\right)}$$
$$= \frac{\rho \times \nu_H\left(\sigma; \mathcal{E}\right)}{\rho \times \nu_H\left(\sigma; \mathcal{E}\right) + (1-\rho) \times \nu_L\left(\sigma; \mathcal{E}\right)}$$

After the evaluator updates her prior belief to this interim belief, she observes her private signal $s \in \mathbf{S}$. From this signal, she distils further information about the applicant's quality and updates her interim belief ψ to a final *posterior belief* $\mathbb{P}_\psi\left(\theta = H \mid s\right)$:

$$\mathbb{P}_\psi\left(\theta = H \mid s\right) = \frac{\psi \times p_H(s)}{\psi \times p_H(s) + (1-\psi) \times p_L(s)}$$

The information packed in the signal $s \in \mathbf{S}$ is determined exclusively by the conditional probabilities of this outcome, $p_H(s)$ and $p_L(s)$. So for notational convenience, I label every signal $s \in \mathbf{S}$ after a ratio of these conditional probabilities:

$$s = \frac{p_H(s)}{p_H(s) + p_L(s)}$$

I call this ratio the *normalised belief* signal s induces. I merge signals with equal normalised beliefs as there is no reason to distinguish between them. Likewise without loss, I enumerate every signal the evaluator might observe, $\mathbf{S} = \{s_1, s_2, ..., s_m\}$, with indices strictly increasing in the normalised beliefs they induce; $s_1 < s_2 < ... < s_m$.

Using this notation, we can re-express the evaluator's posterior belief upon observing the signal $s_j \in \mathbf{S}$ as simply:

$$\mathbb{P}_\psi\left(\theta = H \mid s_j\right) = \frac{\psi \times s_j}{\psi \times s_j + (1-\psi) \times (1-s_j)}$$

As this expression clarifies, an evaluator's posterior belief is also increasing in the index of the signal $s_i \in \mathbf{S}$ she observes: her posterior belief is a simple rescaling of the *normalised belief* this signal induces with the evaluator's interim belief. Note that the normalised belief equals the posterior belief when the evaluator's interim belief assigns equal probability to either quality; $\psi = 0.5$.

Whenever a strategy σ^* is optimal against the unique interim belief ψ^* consistent with it, the pair (σ^*, ψ^*) forms an equilibrium. In principle, there might be many such pairs, or none. I set the ground in Proposition 1 by ruling this last possibility out: an equilibrium is always guaranteed to exist. Also in Proposition 1, I describe some properties of these equilibria that are fundamental to the rest of our analysis.

Proposition 1. Let Σ be the set of evaluators' equilibrium strategies. Then:

1. Σ is non-empty and compact.

2. Any equilibrium strategy σ^* is *monotone*: $\sigma^*(s) > 0$ for some $s \in \mathbf{S}$ implies that $\sigma^*(s') = 1$ for every $s' \in \mathbf{S}'$ such that $s' > s$.

3. All equilibria exhibit adverse selection: $\psi^* \leq \rho$ for any interim belief ψ^* consistent with an equilibrium strategy $\sigma^* \in \Sigma$.

Proof outline: To establish the existence of an equilibrium, I construct a *best response correspondence* Φ for evaluators. Φ maps any strategy σ to the set of all strategies optimal against the unique interim belief consistent with σ. Put differently, $\Phi(\sigma)$ gives the set of strategies maximising an evaluator's expected payoff when all her peers use the strategy σ. Note that a strategy σ^* is an equilibrium strategy if and only if it is a fixed point of this best response correspondence; $\sigma^* \in \Phi(\sigma^*)$. I show that Φ indeed has a fixed point, by a routine application of Kakutani's Fixed Point Theorem. To this end, I prove that Φ is upper semicontinuous, which establishes the compactness of the set of equilibrium strategies as well.

Monotonicity is a simple consequence of optimality. Higher signals induce higher posterior beliefs; so if an evaluator (weakly) prefers approving her applicant after a signal, she (strictly) prefers it after a higher one too. A crucial consequence of monotonicity is that a *Low* quality applicant is likelier to be rejected after any of his visits, as his evaluators are likelier to observe lower signals. Thus, each evaluator risks getting visited by an applicant who was *adversely selected* through his past rejections. This pushes their interim beliefs ψ below their prior ρ.

$\square$

Though an equilibrium is guaranteed to exist, it need not be unique. I illustrate this with a simple example which I modify and revisit on occasion throughout this paper. Consider two evaluators who have the prior belief $\rho = 0.5$ about the applicant's quality, and a cost $c = 0.2$ of approving him. The experiment $\mathcal{E}$ whose outcome they observe is binary; $\mathbf{S} = \{0.2, 0.8\}$. This outcome has the distribution:

$$p_L(s) = \begin{cases} 0.8 & s = 0.2 \\ 0.2 & s = 0.8 \end{cases} \qquad\qquad p_H(s) = \begin{cases} 0.2 & s = 0.2 \\ 0.8 & s = 0.8 \end{cases}$$

One equilibrium strategy for evaluators in this example is to approve every applicant who visits them. Doing so eliminates adverse selection: evaluators never receive an applicant with a past rejection, so their interim belief ψ always equals their prior $\rho = 0.5$. However, at this interim belief, the low signal $s = 0.2$ still implies a 20% probability that the applicant has *High* quality, rendering his approval against the cost 0.2 optimal.

This equilibrium, however, is not unique. There is yet another equilibrium where evaluators approve the applicant only upon the high signal, $s = 0.8$. Their selectivity triggers adverse selection: each evaluator risks being visited by a past reject, thus revises her interim belief ψ below her prior $\rho = 0.5$:

$$\psi = \frac{1 + 0.2}{(1 + 0.2) + (1 + 0.8)} = 0.4$$

Consequently, she places a 1/7 probability on the applicant having *High* quality upon observing the low signal $s = 0.2$; justifying his rejection. She still finds approving the applicant optimal upon the high signal $s = 0.8$, though. Even at this interim belief, she places a probability greater than 70% on him having *High* quality when she observes this signal. In this latter equilibrium, an applicant – regardless of his quality – faces higher rejection chances in any of his visits. His evaluators are *more selective*; any signal they might observe leads to a (weakly) higher chance of rejection.

Whenever we have multiple equilibria, comparing them in their *selectivity* is instinctive.

Definition 1. Where σ' and σ are two strategies for evaluators, σ' is *more selective than* σ (or, σ is *less selective than* σ') if $\sigma'(s) \leq \sigma(s)$ for all $s \in \mathbf{S}$.

While natural, this *selectivity* (or *pointwise*) order between strategies might initially appear restrictive. This impression is misleading. In fact, the set of equilibrium strategies is *totally ordered* (or, a *chain*) under this order; any two equilibrium strategies can be compared under it.

Proposition 2. The set of evaluators' equilibrium strategies Σ is *totally ordered* under the selectivity order. Moreover, Σ contains a *most selective* and *least selective* strategy, $\hat{\sigma} \in \Sigma$ and $\check{\sigma} \in \Sigma$ respectively; for any $\sigma^* \in \Sigma$:

$$\hat{\sigma}(s) \leq \sigma^*(s) \leq \check{\sigma}(s) \qquad \text{for all } s \in \mathbf{S}$$

Proof. By Proposition 1, the set of equilibrium strategies Σ is a subset of the set of monotone strategies. The latter is a chain under the *selectivity* order; for two monotone strategies σ and σ', we have:

$$\sigma'(s_j) > \sigma(s_j) \implies \begin{array}{ll} 1 = \sigma'(s_{j'}) \geq \sigma(s_{j'}) & \text{for any } s_{j'} > s_j \in \mathbf{S} \\ \sigma'(s_{j'}) \geq \sigma(s_{j'}) = 0 & \text{for any } s_{j'} < s_j \in \mathbf{S} \end{array}$$

Since any subset of a chain is also a chain, Σ is a chain too.

By Proposition 1, Σ is a compact set. Since it is also a chain, applying a suitably general Extreme Value Theorem[6] to the identity mapping on Σ reveals that Σ has a *minimum* and *maximum* element with respect to this order; i.e. there are two strategies $\hat{\sigma}, \check{\sigma} \in \Sigma$ such that for any other strategy $\sigma^* \in \Sigma$ we have $\hat{\sigma}(s) \leq \sigma^*(s) \leq \check{\sigma}(s)$ for all $s \in \mathbf{S}$. □

[6]See, for instance, Theorem 27.4 in Munkres, 2000

Evaluators' *most* and *least* selective equilibrium strategies are critical in the sequel. I dub them the *extreme equilibria*. It were, in fact, these equilibria we identified in our previous example. The importance of these extreme equilibria for the applicant is clear. Regardless of his quality, he is worse off when his evaluators get more selective; therefore he is *best* off under the *least* selective equilibrium, and *worst* off under the *most* selective one. How moving to more selective equilibria affects evaluators' payoffs is less clear. Their payoffs are determined by how they balance their two key objectives: identifying and approving a *High* quality applicant, and rejecting a *Low* quality one. The expression $\Pi(\sigma;\mathcal{E})$, the *sum* of evaluators' payoffs when each use the strategy σ, highlights this:

$$\begin{aligned}\Pi(\sigma;\mathcal{E}) := \rho \times (1-c) \times \mathbb{P}\,(\text{some ev. approves when all use strategies } \sigma \mid \theta = H) \\ -(1-\rho) \times c \times [1 - \mathbb{P}\,(\text{all ev.s reject when all use strategies } \sigma \mid \theta = L)]\end{aligned} \tag{3.1}$$

Each evaluator expects simply $\left(\frac{1}{n}\right)^{\text{th}}$ of this sum of course, as the equilibrium is symmetric.

Increased selectivity has counteracting effects on these two objectives. It mitigates their risk of approving a *Low* quality applicant when they face one, sparing them a cost of c. However, this comes at the expense of curbing the approval chances of a *High* quality applicant too, which means forsaking a payoff of $1-c$. In principle, increased selectivity can therefore both be a vice and a virtue.

Our previous example, where we had identified two equilibria, illustrates these competing effects of increased selectivity. In the least selective equilibrium, evaluators approve all applicants; either *High* quality, or *Low*. Their payoffs, therefore, sum to:

$$\Pi(\check{\sigma};\mathcal{E}) = 0.5 \times [(1-c) - c] = 0.3$$

In the most selective equilibrium on the other hand, an evaluator rejects an applicant for whom she observes the low signal $s = 0.2$. This depresses the approval chances of any applicant. A *High* quality applicant faces a probability $p_H^2(0.2) = 0.04$ of getting rejected by both his evaluators. This probability is higher for a *Low* quality applicant, $p_L^2(c) = 0.64$. Under these more selective strategies, evaluators' payoffs sum to:

$$\Pi(\hat{\sigma};\mathcal{E}) = 0.5 \times \left[(1-c) \times (1 - p_H(0.2)^2) - c \times \left(1 - p_L(0.2)^2\right)\right] = 0.348$$

Despite reducing the approval chances of both *Low* and *High* quality applicants, selectivity pays off for our evaluators.

Why increased selectivity ends up helping evaluators is clear in this example. When they switch to the more selective equilibrium, evaluators push out only applicants for whom *both* of them saw low signals. The probability that such an applicant has *High* quality is less than 0.06.

By rejecting him, they save a cost of 0.2 against an expected benefit of 0.06, which raises their payoffs.

In more intricate examples, judging whether more selectivity will benefit evaluators might become difficult. Nonetheless, this phenomenon our simple example illustrates is in fact, general. Proposition 3 establishes that evaluators' trade-off between more and less selective equilibria are *always* resolved in favour of the former. Notably, this brings the welfare of the applicant, who is unambiguously harmed by selectivity, into conflict with the evaluators'.

Proposition 3. Where σ^* and σ^{**} are two equilibrium strategies such that σ^{**} is more selective than σ^*, evaluators' expected payoffs under σ^{**} exceed those under σ^*; $\Pi(\sigma^{**};\mathcal{E}) \geq \Pi(\sigma^*;\mathcal{E})$.

Thus, besides being a natural one, selectivity is a very powerful comparison of equilibria. We can use it to compare any two equilibria, and to determine both the applicant's and evaluators' relative welfare. *Extreme* equilibria deserve particular focus. The *most selective* equilibrium maximises evaluators payoffs across all equilibria while minimising the applicant's approval chances. The *least selective* equilibrium, vice versa. They remain under my spotlight in the remainder of this paper.

Proposition 3 follows as a corollary of Lemma 1, which establishes that in fact deviating to *any* less selective strategy hurts evaluators' payoffs when they start from an equilibrium strategy.

Lemma 1. Let σ^* and σ be two monotone strategies, where σ^* is more selective than σ. If σ^* is an equilibrium strategy, then $\Pi(\sigma^*;\mathcal{E}) \geq \Pi(\sigma;\mathcal{E})$.

Proof. See Section 6. □

Besides birthing Proposition 3, Lemma 1 highlights an important contrast between the problem a *single* evaluator with no peers faces, and the one *multiple* evaluators do. A single evaluator with no peers faces no adverse selection; no evaluator could have received her applicant earlier. So her interim belief *is* her prior belief, regardless of the strategy she chooses. In equilibrium, her strategy must be optimal against this belief. Except for how she breaks ties when indifferent, this equilibrium strategy is unique. Any equilibrium strategy gives her the same expected payoff, and deviating from it can leave her only worse off.

With multiple evaluators, a non-trivial multiplicity of equilibria becomes a possibility. There might be multiple strategies which induce an interim belief they are optimal against. Crucially, evaluators' payoffs vary between these different equilibria; joint deviations to *more selective* equilibria benefit them. Lemma 1 highlights that only increased selectivity might pay off, though. Deviating – individually or jointly – from any equilibrium strategy to a *less selective* one hurts an evaluator's payoffs, just as it would if she had no peers.

When evaluators move to a more selective equilibrium, they push two kinds of applicants out. Some fall through the cracks: their evaluators' signals do not definitively evidence *Low* quality, but cannot outweigh evaluators' adverse selection fear either. The evaluators might have revised their negative verdicts for such applicants, had they known that *no* evaluator saw a definitive bad signal. On the other hand, they also reject applicants who – when the evaluators were less selective – would have secured an approval *despite* decisive rejections by many evaluators. Once evaluators become more selective, they lose this bnefit of doubt a few unsuspecting evaluators would grant them. I prove Lemma 5 by showing that the overall effect is always driven by this latter group of applicants, whose rejection benefits evaluators.

4 More Informative Experiments and Equilibrium Payoffs

As we discussed in the previous section, evaluators' payoffs are determined fundamentally by how well they can distinguish and approve a *High* quality applicant while rejecting a *Low* quality one. The information they obtain about the applicant's quality from their experiment $\mathcal{E}$ lies at the heart of this exercise. One might, perhaps naturally, conjecture that a *more informative* experiment is the key to improving evaluators' welfare. After all, more information helps evaluators identify their applicant's quality better. This ought to ease the tension between their two fundamental objectives, allowing them to boost their payoffs.

This hypothesis would indeed be correct if we only had *one* evaluator. With no peers, she faces a simple decision problem: given her fixed prior belief about her applicant's quality, she must choose whether to approve him given the outcome of her experiment $\mathcal{E}$. As Blackwell's seminal result (1953) establishes, observing instead the outcome of an experiment $\mathcal{E}'$ that is *(Blackwell) more informative* than $\mathcal{E}$ would indeed leave her better off[7]. This is precisely because more information relaxes our evaluator's key trade-off; she can reject *Low* quality applicants more frequently without compromising *High* quality applicants more often, and (or) vice versa [8]. I illustrate this in Figure 1 in the next subsection, in the context of *binary* experiments where $\mathbf{S} = \{s_1, s_2\}$.

Nevertheless, this naïve hypothesis fails in our current setting. Re-expressing an individual evaluator i's payoff $\pi_i(\sigma; \mathcal{E})$ showcases what goes wrong:

$$\pi_i(\sigma;\mathcal{E}) = \underset{\sim\sim\sim}{\mathbb{P}\left(\text{applicant visits } i\right)} \times \left[\underset{\sim}{\psi} \times (1-c) \times \mathbb{P}\left(i \text{ approves} \mid \theta = H\right) + \underset{\sim\sim}{(1-\psi)} \times (-c) \times \mathbb{P}\left(i \text{ approves} \mid \theta = L\right)\right]$$

[7]Blackwell's Theorem 1953 states that a decision maker prefers observing the outcome of experiment $\mathcal{E}'$ over $\mathcal{E}$ *regardless* of the decision problem she faces if and only if the former is *Blackwell more informative than* the latter. The implication is that for certain decision problems, there might be weaker conditions under which the DM prefers $\mathcal{E}'$ over $\mathcal{E}$. However, for *dichotomies* – where the state of nature is binary – the *necessity* direction still remains valid. I present a self contained proof for this in Section 6.2, Lemma 3 for completeness.

[8]See Blackwell and Girshick, 1954's Theorems 12.2.2 and 12.4.2 for a textbook exposition of these classic results.

Evaluator i's payoff, like others', is determined by how well she can tailor her decisions to the applicant's quality. However, she is also affected by the extent of *adverse selection* she faces. The applicant might not visit her at all, and if he does, he might be very unlikely to have *High* quality given no other evaluator approved him so far.

The extent of adverse selection evaluator i faces is shaped by *other* evaluators' strategies. These evaluators do not account for the adverse selection their decisions impose on her; just as she disregards the adverse selection she imposes on them. More information might inadvertently accentuate this externality they inflict on each other. The quality an evaluator expects the applicant who visits her to have might deteriorate from the get go, either as she expects the applicant's visit after more rejections or as each of those rejections become more damning. This effect might eclipse evaluators' improved ability to evaluate the applicant, and leave all of them worse off.

Our simple example illustrates this possibility. To simplify this illustration, I take evaluators' approval cost to be $c = 0.5$ now, instead of the 0.2 earlier. Under that previous cost, evaluators approved every applicant in the least selective equilibrium. That equilibrium does not survive this cost increase: even without adverse selection, evaluators find it optimal to reject upon the low signal $s = 0.2$. In fact, the most selective equilibrium we identified earlier now becomes the unique one: evaluators approve the applicant upon the high signal $s = 0.8$, but reject him upon the low signal, $s = 0.2$. With this modified cost, evaluators' payoff in this equilibrium becomes:

$$0.5 \times \left[(1-c) \times (1 - p_H(0.2)^2) - c \times \left(1 - p_L(0.2)^2\right)\right] \Big|_{c=0.5} = 0.15$$

Now, consider swapping our evaluators' experiment $\mathcal{E}$ with a more informative binary experiment $\mathcal{E}^g$ with the set of possible outcomes $\mathbf{S}^g = \{0.2, 1\}$. *Good news* carried by the high signal in $\mathcal{E}^g$ is now *conclusive*; since no evaluator observes it when the applicant has *Low* quality, any evaluator to observe it concludes definitively that he has *High* quality. The evidence the low signal carries for *Low* quality, however, is no stronger than before. For the *same* interim belief, observing the low signal from $\mathcal{E}^g$ leads to the same posterior belief as observing the low signal from $\mathcal{E}$ does. The outcomes of experiment $\mathcal{E}^g$ have the distribution:

$$p_L^g(s) = \begin{cases} 1 & s = 0.2 \\ 0 & s = 1 \end{cases} \qquad\qquad p_H^g(s) = \begin{cases} 0.25 & s = 0.2 \\ 0.75 & s = 1 \end{cases}$$

Under this more informative experiment, the unique equilibrium remains one where evaluators approve upon the high signal $s = 1$, but reject upon the low signal $s = 0.2$. Evaluators thus manage to avoid approving *any Low* quality applicant, but they forsake *High* quality applicants more often than earlier, too. That any visit becomes likelier to result in a rejection exacerbates the adverse selection evaluators are exposed to as well; while they previously attached a probability 0.4 to their visitor having *High* quality, this now drops below 0.385:

$$\psi^g = \frac{1+0.25}{(1+0.25)+(1+1)} \approx 0.385$$

Nonetheless, evaluators benefit from this improvement in their information. Their payoffs now sum up to 0.23, surpassing their payoffs of 0.15 under their original experiment $\mathcal{E}$:

$$0.5 \times \left[(1-c) \times \left(1 - p_H^g(0.2)^2\right)\right] \Big|_{c=0.5} \approx 0.23$$

Now consider another binary experiment, $\mathcal{E}^b$, again more informative than evaluators' original experiment $\mathcal{E}$. The possible outcomes of $\mathcal{E}^b$ lie in the set $\mathbf{S}^b = \{0, 0.8\}$ this time. They have the distribution:

$$p_L^b(s) = \begin{cases} 0.75 & s = 0 \\ 0.25 & s = 0.8 \end{cases} \qquad p_H^b(s) = \begin{cases} 0 & s = 0 \\ 1 & s = 0.8 \end{cases}$$

This time, it is the *bad news* carried by the low signal $s = 0$ that becomes *conclusive*: any evaluator concludes that the applicant has *Low* quality upon observing it. Would our evaluators benefit if we swapped their experiment not with $\mathcal{E}^g$ but with $\mathcal{E}^b$ instead?

The answer, it turns out, is no this time. There is still a unique equilibrium: evaluators approve upon the high signal $s = 0.8$ and reject upon the low, $s = 0$. Their more informative experiment now guarantees that evaluators never miss a *High* quality applicant, though at the expense of approving *Low* quality applicants more often than before. Rejections thus happen less frequently, meaning evaluators become less likely to be visited by an applicant who has previously received one. However, a past rejection now spells *certain* news that the applicant has *Low* quality. Once again, this accentuates the adverse selection evaluators are exposed to, dragging their interim belief down to 0.36:

$$\psi^b = \frac{1}{1+(1+0.75)} \approx 0.36$$

This strengthening of adverse selection overwhelms the improvement in evaluators' information, and leaves them worse off. Their equilibrium payoffs now sum up to 0.14, falling behind those under the original experiment $\mathcal{E}$:

$$0.5 \times \left[(1-c) - c \times \left(1 - p_L(0)^2\right)\right] \Bigg|_{c=0.5} \approx 0.14$$

The pattern in this example is a general one: in the appropriate sense, *stronger good news* always benefits evaluators, while *stronger bad news* eventually hurts them.

In the remainder of this section, I uncover and explore this general pattern. I start by restricting evaluators to binary experiments in Section 4.1. There, Theorem 1 characterises how evaluators payoffs change when their binary experiment becomes Blackwell more informative. Section 4.1 is centered around Theorem 1, and the intuition that surrounds it. Section 4.2 rises on that groundwork. It centers around Theorem 2, which generalises Theorem 1 to Blackwell improvements of an *arbitrary* experiment evaluators might hold.

Throughout, I focus on evaluators' payoffs across *extreme equilibria*. They delineate the boundaries of both the evaluators' and the applicant's welfare across equilibria, hence command the highest importance. This is also a (arguably, the most) natural comparison of the *sets* of equilibrium payoffs different experiments support.

4.1 Evaluators with Binary Experiments

In this section, I consider evaluators who observe the outcome of a *binary* experiment. Such an experiment has two possible outcomes, $\mathbf{S} = \{s_1, s_2\}$, which I respectively rename $\mathbf{S} = \{s_L, s_H\}$ for notational convenience. The low outcome induces the normalised belief $s_L \in [0, 0.5]$, carrying *bad news* about the applicant's quality. In contrast, the high outcome induces the normalised belief $s_H \in [0.5, 1]$, carrying *good news* about the applicant's quality.

How do evaluators' equilibrium payoffs change when instead, they observe the outcome of a more informative binary experiment? Here, I answer this question. This answer also lays the building block and key intuition for the next section, where I ask how Blackwell improving an *arbitrary* experiment evaluators might hold affects their payoffs.

Let $\mathcal{E}$ and $\mathcal{E}'$ be two binary experiments, with possible outcomes in $\mathbf{S} = \{s_L, s_H\}$ and $\mathbf{S}' = \{s'_L, s'_H\}$ respectively. Recall that these outcomes are labelled after the normalised beliefs they induce. $\mathcal{E}'$ is *(Blackwell) more informative than* (or *(Blackwell) improves on*) $\mathcal{E}$ if and only if it carries both *stronger good news* and *stronger bad news* than $\mathcal{E}$[9]; i.e.:

$$s'_L \leq s_L \qquad\qquad s'_H \geq s_H$$

For any fixed interim belief $\psi \in [0, 1]$, the experiment $\mathcal{E}'$ helps an evaluator form more confident assessments of the applicant's quality. Its low outcome leaves her more confident that her applicant has *Low* quality, and its high outcome that he has *High*:

$$\mathbb{P}_\psi\left(\theta = H \mid s'_L\right) \leq \mathbb{P}_\psi\left(\theta = H \mid s_L\right) \qquad\qquad \mathbb{P}_\psi\left(\theta = H \mid s'_H\right) \geq \mathbb{P}_\psi\left(\theta = H \mid s_H\right)$$

Figure 1 illustrates how switching to the more informative experiment $\mathcal{E}'$ transforms evalu-

[9]See Section 12.5 in Blackwell and Girshick, 1954 for a textbook exposition of this classic result.

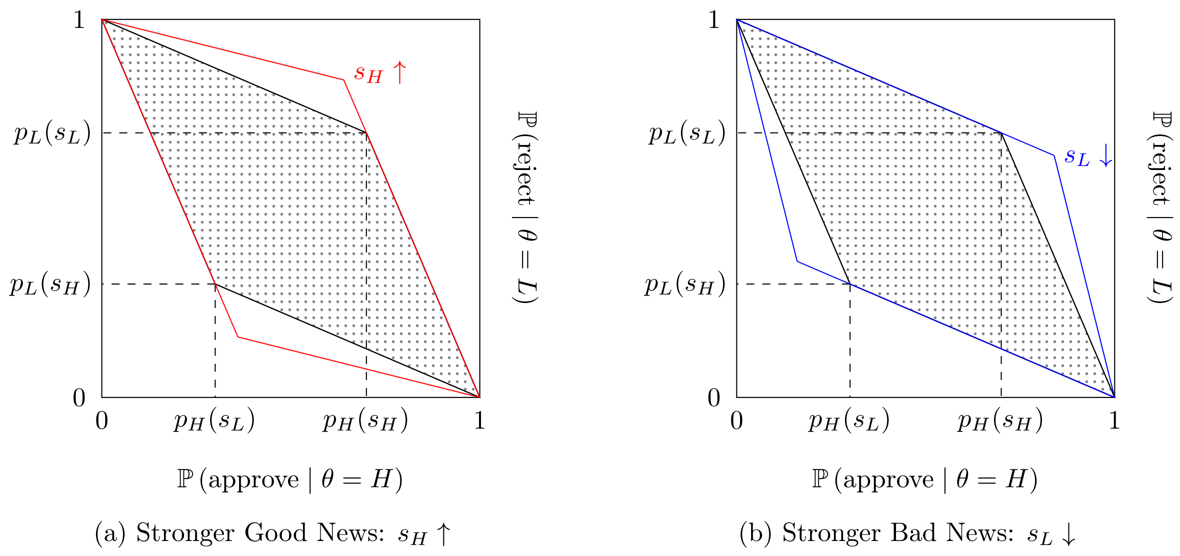


Figure 1: Evaluators' Trade-Offs with More Informative Experiments

ators key trade-off. Irrespective of her interim belief, any strategy an evaluator adopts yields a probability that she approves the applicant if he has *High* quality, and a probability that she approves him if he has *Low* quality. I call this probability pair the *consequence* of her strategy, for short. In the unit squares Figure 1 depicts, each point depicts one such possible consequence: with the former probability on the horizontal axis, and the latter on the vertical.

Of course, which of these consequences the evaluator *can* implement depends which experiment she holds. The top right corner of the square, for instance, where an evaluator certainly approves *High* quality applicants and rejects *Low* quality ones, requires $\mathcal{E}$ to *fully reveal* the applicant's quality. Instead, the dotted region depicts the subset consequences the evaluator can implement with the experiment $\mathcal{E}$ by freely varying her strategy. For example, the top corner of that dotted parallelogram corresponds to the consequence she can implement by "approving upon the high signal, and rejecting upon the low".

As the Figure illustrates, this region of implementable consequences expands with a more informative experiment $\mathcal{E}'$; whether it carries *stronger good news* (left panel) or *stronger bad news* (right panel). A *single* evaluator can never suffer from this expansion. However, an evaluator who is surrounded by peers who enjoy the same expansion, might. Those peers' choices determine the extent of adverse selection our evaluator is exposed to, and their new choices might aggravate it.

How, then, does more information affect evaluators' *equilibrium* payoffs? As the preceding discussion foreshadowed, Theorem 1 reveals that this hinges precisely on whether evaluators receive *stronger good news* or *stronger bad news*. Stronger good news – higher s_H – always increase evaluators' payoffs, both in the *most selective* equilibria and the *least*. The effect of stronger bad news – lower s_L – in contrast, is more delicate. Initially, evaluators benefit from stronger bad news. However, once s_L falls below a cutoff, their payoffs fall as bad news get even stronger. This effect occurs across both the most and the least extreme equilibria; though the cutoff for these two equilibria may differ.

Theorem 1. Let evaluators' experiment be binary, with outcomes inducing the normalised beliefs $s_L \in [0, 0.5]$ and $s_H \in [0.5, 1]$. Increasing s_H weakly increases evaluators' payoffs across the extreme equilibria. In contrast, as s_L decreases, evaluators' payoffs in the most (least) selective equilibrium:

1. weakly improve as long as s_L remains below a cutoff $\hat{s_L}$ ($\check{s_L}$),
2. weakly decrease once s_L falls below this cutoff.

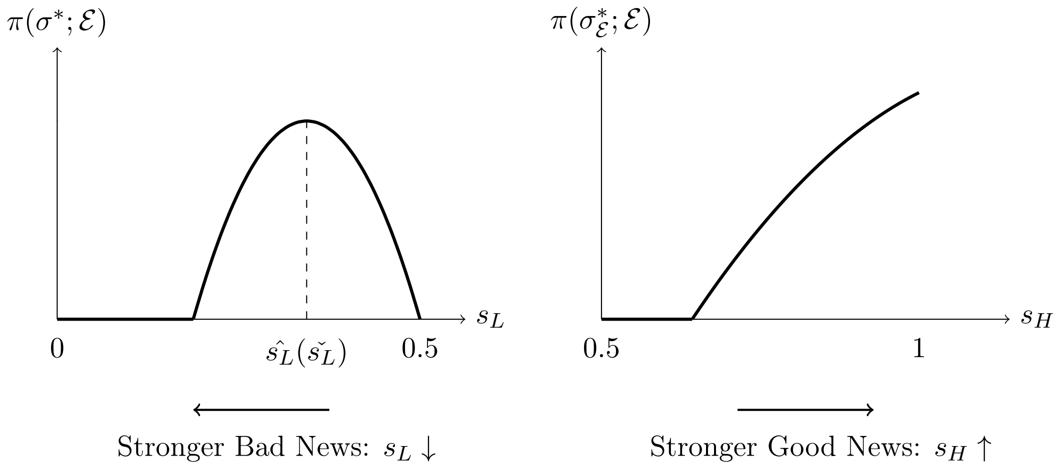


Figure 2: Theorem 1 illustrated

Figure 2 illustrates Theorem 1 for an example where evaluators' approval cost c weakly exceeds their prior belief ρ. This conveniently implies equilibrium uniqueness for every binary experiment $\mathcal{E}$, and I denote the associated strategies as $\sigma^*\mathcal{E}$. In examples with multiple equilibria, we would have two sets of plots that have the same shape as Figure 2, one for the most selective equilibrium and one for the least.

In the ensuing discussion, I explain the discrepancy between the effect of strengthening good news and strengthening bad news on evaluators' payoffs. I relegate the full proof of Theorem 1 to Section 6, which builds on the forces I lay out here. The eager reader will notice that Theorem 1 does not elaborate on *where* the cutoff below which stronger bad news harm evaluators' payoffs is. As I clarify the forces driving Theorem 1, I cast light on this cutoff as well. Building on that discussion, I characterise this cutoff in Proposition 4.

Any change in the evaluators' experiment ultimately affects who they approve and who they reject. A *High* quality applicant they would otherwise reject might this time receive an evaluator's approval, or a *Low* quality applicant who would otherwise slip through their net might instead face a wall of rejections. Other changes might be less welcome: evaluators might inadvertently dismiss more *High* quality applicants, or fail to do so with *Low* quality ones. It is the applicants they affect who determine how stronger good news or stronger bad news influence evaluators' payoffs.

Let us start with the case of *stronger bad news*. Consider switching evaluators' experiment from $\mathcal{E}$ to $\mathcal{E}'$, whose possible outcomes $\mathbf{S}' = \{s'_L, s'_H\}$ induce the normalised beliefs:

$$s'_L = \frac{p'_H(s'_L)}{p'_H(s'_L) + p'_L(s'_L)} = s_L - \delta \qquad\qquad s'_H = \frac{p'_H(s'_H)}{p'_H(s'_H) + p'_L(s'_H)} = s_H$$

for some small $\delta > 0$. Experiment $\mathcal{E}'$ thus offers *marginally stronger bad news* than $\mathcal{E}$, but the same the strength of good news as the latter. Evaluators' equilibrium strategies will of course react to this switch. Nevertheless, ignoring this strategic response allows for a clearer intuition. So instead, let us simply assume that under both experiments, evaluators approve whenever they observe the "high" outcome, and reject whenever they observe the "low" outcome. Which applicants' outcomes does the switch from $\mathcal{E}$ to $\mathcal{E}'$ affect?

The clearest way to answer this question is by reinterpreting this improvement in evaluators' information. Imagine, instead of replacing their original experiment $\mathcal{E}$ wholesale, that evaluators observe an *auxiliary signal* $\hat{S}$ in addition to their original S. We will construct this auxiliary signal $\hat{S}$ carefully so that it completes the information evaluators garner from their original experiment $\mathcal{E}$ to the one they could from $\mathcal{E}'$. I illustrate this construction in Figure 3. The reader might benefit from referring to it throughout the ensuing discussion.

This auxiliary signal $\hat{S}$ evaluators observe is also binary, with possible realisations $\hat{s} \in \{\hat{s_L}, \hat{s_H}\}$. Conditional on the applicant's quality θ, its outcome is independent both from the evaluator's original signal S and anything other evaluators observe. It has a distribution:

$$\hat{p_H}(\hat{s_H}) = \varepsilon \times \frac{s_H}{1 - s_H} \qquad\qquad \hat{p_L}(\hat{s_H}) = \varepsilon \times \frac{s_L}{1 - s_L}$$

ε, like δ, is a small positive number. It is related intimately to δ, as I explain shortly.

An evaluator observes the realisation of this auxiliary signal $\hat{s}$ *only* if the initial signal she observes is low, $s = s_L$. If she observes $\hat{s} = \hat{s_H}$ following this initial low signal, her belief that the applicant has *High* quality jumps to what it would be had she observed $s = s_H$ straightaway. This is most visible from the likelihood ratio for this signal pair:

$$\frac{\mathbb{P}\left(s_L, \hat{s_H} \mid \theta = H\right)}{\mathbb{P}\left(s_L, \hat{s_H} \mid \theta = L\right)} = \frac{p_H(s_L)}{p_L(s_L)} \times \frac{\hat{p_H}(\hat{s_H})}{\hat{p_L}(\hat{s_H})} = \frac{s_L}{1 - s_L} \times \frac{\frac{s_H}{1-s_H}}{\frac{s_L}{1-s_L}} = \frac{s_H}{1 - s_H}$$

If she instead observes $\hat{s} = \hat{s_L}$ though, she grows yet more confident that the applicant has *Low* quality. Again note this from the likelihood ratio for this signal pair, labelled $(L, \hat{L})$:

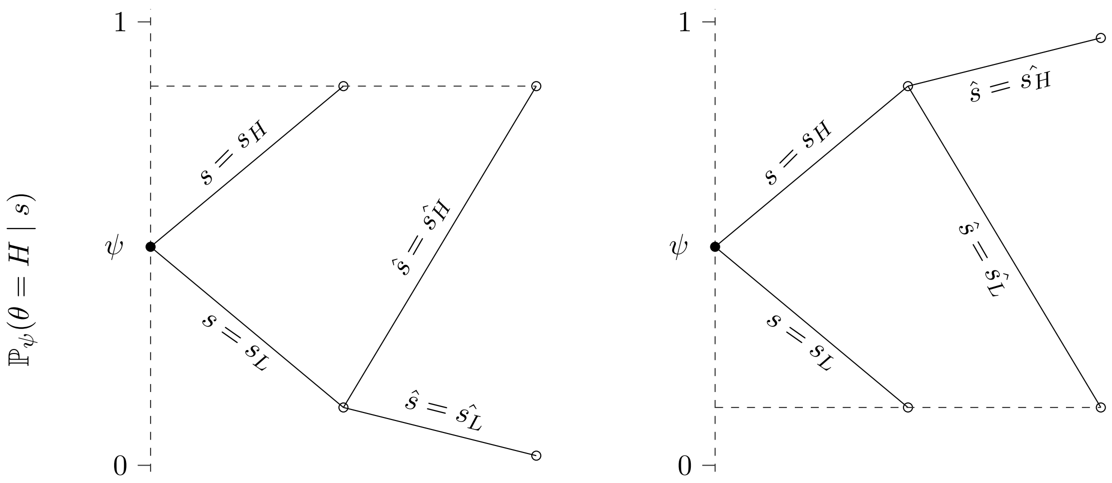
$$\frac{\mathbb{P}\left(s_L, \hat{s_L} \mid \theta = H\right)}{\mathbb{P}\left(s_L, \hat{s_L} \mid \theta = L\right)} = \frac{p_H(s_L)}{p_L(s_L)} \times \frac{\hat{p_H}(\hat{s_L})}{\hat{p_L}(\hat{s_L})} = \underbrace{\frac{s_L}{1 - s_L} \times \frac{1 - \frac{s_H}{1-s_H} \times \varepsilon}{1 - \frac{s_L}{1-s_L} \times \varepsilon}}_{(L,\hat{L})} < \frac{s_L}{1 - s_L}$$

The likelihood ratio $(L, \hat{L})$ decreases continuously and monotonically as ε rises from 0 to $\frac{1-s_H}{s_H}$. We can thus choose ε so that this likelihood ratio equals the one for the low outcome of experiment $\mathcal{E}'$, $s' = s'_L$. This latter likelihood ratio is labelled (L') below:

$$\frac{p'_H(s'_L)}{p'_L(s_L)} = \underbrace{\frac{s_L - \delta}{1 - (s_L - \delta)}}_{(L')}$$

Note that the value of ε equating these likelihood ratios is a continuous and strictly increasing function of δ.

When the likelihood ratios $(L, \hat{L})$ and (L') are equal, the information an evaluator obtains from observing the outcome of $\mathcal{E}'$ is equivalent to the one she does by observing the signal pair $(S, \hat{S})$. Receiving a high signal, either $s = s_H$ or $\hat{s} = \hat{s_H}$, carries the same information as the high outcome $s' = s'_H$ from experiment $\mathcal{E}'$. Receiving only the low signals $s = s_L$ and $\hat{s} = \hat{s_L}$ on the other hand, carries the same information as observing $s' = s'_L$ from $\mathcal{E}'$. Despite no longer observing the outcome of experiment $\mathcal{E}'$, our evaluator can use the information the signal pair $(s, \hat{s})$ yields her to replicate the consequence of approving upon the "high" and rejecting upon the "low" outcome under $\mathcal{E}'$. She simply approves the applicant if she observes a high signal, either s_H or $\hat{s_H}$, and rejects him otherwise.



(a) Stronger Bad News (b) Stronger Good News

Figure 3: Improving Binary Experiments with Auxiliary Signals

We can interpret this auxiliary signal as an evaluator's "re-evaluation" of her initial rejection decisions. Her initial approvals remain final; so this re-evaluation does not affect applicants who received some evaluator's approval anyway. Instead, it affects the applicants who were initially rejected by *all* n evaluators. Each of these n re-evaluations might overturn an evaluator's negative verdict, and grant this applicant the approval he seeks. It is this applicant, who some evaluator approves upon her re-evaluation, who drives the change in evaluators' payoffs.

How likely is this applicant to have *High* quality? All information we can harvest about this is contained in the signals evaluators would observe, if they *all* were to (re-)evaluate him. What is certain is that initially, they all observed low signals; this led to the applicant's initial rejections. Upon their re-evaluations, how many would see a high signal?

The applicant we are inspecting is approved upon these re-evaluations, so we know *at least one* evaluator must have seen a high signal. Discouragingly, however, she would *almost surely be the only one* to see it, for small δ and therefore ε. Recall from our construction that the probability any evaluator observes the signal $\hat{s} = \hat{s_H}$ for an applicant is proportional to ε; and so the probability that any k evaluators observe it, to ε^k. As ε shrinks to 0, the probability that multiple evaluators observed this signal vanishes in favour of the probability that just one of them did. Hence, that one evaluator who observes the signal $\hat{s} = \hat{s_H}$ approves the applicant against the backdrop of $n-1$ $\hat{s} = \hat{s_L}$ signals her peers observed. The stronger the bad news those signals carry, the less likely he is to have *High* quality. Inspecting his signals' likelihood ratios reveals precisely when bad news are *too strong* for evaluators to benefit from this applicant:

$$
\begin{aligned}
&\lim_{\delta\to 0}\ \mathbb{P}(\theta = H \mid\ n-1\ \hat{s_L}\text{ signals and one }\hat{s_H}) \geq c \\
\iff &\lim_{\delta\to 0} \frac{\mathbb{P}\left(\theta = H \mid\ n-1\ \hat{s_L}\text{ signals and one }\hat{s_H}\right)}{\mathbb{P}\left(\theta = L \mid\ n-1\ \hat{s_L}\text{ signals and one }\hat{s_H}\right)} \geq \frac{c}{1-c} \\
\iff &\lim_{\delta\to 0} \frac{\rho}{1-\rho} \times \left(\frac{\hat{s_L}}{1-\hat{s_L}}\right)^{n-1} \times \frac{s_H}{1-s_H} \geq \frac{c}{1-c} \\
\iff &\frac{\rho}{1-\rho} \times \underbrace{\left(\frac{s_L}{1-s_L}\right)^{n-1}}_{n-1\text{ low signals}} \times \underbrace{\frac{s_H}{1-s_H}}_{\text{a single high signal}} \geq \frac{c}{1-c}
\end{aligned}
$$

When the LHS above exceeds the RHS, *adverse selection causes no regret* for evaluators: even after $n-1$ low signals, a high signal justifies an applicant's approval. This threat subsides when evaluators have a lower bar for rejecting the applicant; either due to a favourable prior belief ρ about his quality, or a low approval cost c. Having less evaluators helps too, capping the number of low signals the applicant can accumulate. But fundamentally, whether adverse selection poses a threat depends on the strength of good and bad news. As bad news get stronger against good news, the applicant's $n-1$ low signals increasingly dominate over the single high signal he received. Once the strength of bad news exceeds a threshold, this high signal no longer vindicates the applicant. I denote the normalised belief which marks this threshold as s_L^{as}.

Definition 2. For a binary experiment $\mathcal{E}$ with given strength of good news s_H, s_L^{as} is the strongest level of bad news where *adverse selection causes no regret:*

$$\frac{\rho}{1-\rho} \times \left(\frac{s_L^{\text{as}}}{1-s_L^{\text{as}}}\right)^{n-1} \times \frac{s_H}{1-s_H} = \frac{c}{1-c}$$

This threshold is intimately related to the cutoff to which Theorem 1 points. Indeed, unless they already approve *every* applicant, stronger bad news always pushes evaluators to approve their *marginal rejects.* Whenever adverse selection poses a threat, these approvals hurt their payoffs. The example in the beginning of this section illustrates this. There, we have no equilibria where *every* applicant is approved; a low signal always warrants a rejection as evaluators' prior belief ρ is already below their approval cost c. Adverse selection poses a threat beyond the threshold $s_L^{\text{as}} = 0.2$; precisely the strength of bad news in their original experiment $\mathcal{E}$. Consequently, switching to $\mathcal{E}^b$ hurts evaluators' payoffs with each marginal reject it pushes them to approve.

The exception is equilibria where bad news are too weak to convince evaluators oblivious about the applicant's past to reject him. Until the strength of bad news exceeds a threshold s_L^{mute} I introduce below, its weakness causes evaluators to approve *every* applicant in the least selective equilibrium. This threshold is in general higher for the most selective equilibrium, where adverse selection might push evaluators to reject low signals earlier.

Definition 3. Where evaluators' experiment is binary, s_L^{mute} is the *strongest* level of bad news for which there is an equilibrium where evaluators approve upon any signal:

$$\frac{\rho}{1-\rho} \times \frac{s_L^{\text{mute}}}{1-s_L^{\text{mute}}} = \frac{c}{1-c}$$

Pushing evaluators away from such equilibria might, in general, require strengthening bad news beyond the threshold s_L^{as}. Once they do so however, further strengthening bad news is followed with the approval of marginal rejects, as before.

Proposition 4. Suppose the experiment $\mathcal{E}$ is binary with outcomes inducing the normalised beliefs $s_L \in [0, 0.5]$ and $s_H \in [0.5, 1]$. Stronger bad news decrease evaluators' payoffs when:

i s_L is below the cutoff $\check{s}_L := \min\{s_L^{\text{mute}}, s_L^{as}\}$ in the least selective equilibrium.

ii s_L is below a cutoff $\hat{s_L} \geq \check{s_L}$ in the most selective equilibrium.

Finally, let us turn to the case of *stronger good news.* Now, say the possible outcomes of experiment $\mathcal{E}'$ induce the normalised beliefs:

$$s'_L = \frac{p'_H(s'_L)}{p'_H(s'_L) + p'_L(s'_L)} = s_L \qquad\qquad s'_H = \frac{p'_H(s'_H)}{p'_H(s'_H) + p'_L(s'_H)} = s_H + \delta$$

for some small $\delta > 0$. This new experiment thus offers *marginally stronger good news* than the original experiment $\mathcal{E}$, but the same strength of bad news as the latter. As before, let evaluators approve upon the high outcome and reject upon the low in either experiment. Which applicants' outcome does evaluators' switch to this experiment with stronger news affect, then?

As before, we can interpret the additional information experiment $\mathcal{E}'$ provides as an auxiliary signal evaluators observe following their initial experiment $\mathcal{E}$. This time, they observe this signal $\hat{S}$ only if the outcome s of their initial experiment is high, s_H. This auxiliary signal $\hat{S}$, binary as before, has a distribution:

$$\mathbb{P}(\hat{s_L} \mid \theta = H) = \varepsilon \times \frac{s_L}{1 - s_L} \qquad\qquad \mathbb{P}(\hat{s_L} \mid \theta = L) = \varepsilon \times \frac{s_H}{1 - s_H}$$

and is, conditional on the applicant's quality θ, independent both from an evaluator's original signal S and anything other evaluators observe. Observing the low auxiliary signal, $\hat{s_L}$, is already equivalent to observing the low outcome s'_L from the new experiment $\mathcal{E}'$:

$$\frac{\mathbb{P}(s_H, \hat{s_L} \mid \theta = H)}{\mathbb{P}(s_H, \hat{s_L} \mid \theta = L)} = \frac{s_H}{1 - s_H} \times \frac{\frac{s_L}{1-s_L}}{\frac{s_H}{1-s_H}} = \frac{s_L}{1 - s_L}$$

We choose ε so that observing the high auxiliary signal $\hat{s_H}$ is equivalent to observing the high outcome s'_H from the new experiment $\mathcal{E}'$:

$$\frac{\mathbb{P}(s_H, \hat{s_H} \mid \theta = H)}{\mathbb{P}(s_H, \hat{s_H} \mid \theta = L)} = \frac{p'_H(s'_H)}{p'_L(s'_L)} \iff \frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \frac{s_L}{1-s_L}}{1 - \varepsilon \times \frac{s_H}{1-s_H}} = \frac{s_H + \delta}{1 - s_H - \delta}$$

as before, the value of ε which ensures this is a continuous and increasing function of δ. I illustrate this construction in the right panel of Figure 3.

This time, the auxiliary signal serves as a "re-evaluation" of an evaluator's initial approval decisions. Her initial rejections are final, but this re-evaluation might overturn an initial approval verdict. This might lose an applicant *any* approval he would otherwise secure, leaving him with a rejection from every evaluator.

Inferring the signals evaluators observed for these *marginal admits* they now reject is easier. This marginal admit is turned down by every evaluator once they observe the auxiliary signals. Thus, every evaluator must have seen a low signal, either s_L or $\hat{s_L}$. Evaluators' payoffs are guaranteed to improve when they reject such an applicant, unless bad news from the low signal are too weak to incentivise the rejection of any applicant. To prove this second part of Theorem 1, I show that strengthening good news pushes evaluators towards such equilibria. Once there, evaluators always benefit from the marginal admits it pushes out.

4.2 Blackwell Improvements of Experiments with Finite Outcomes

In the previous section, I characterised the effect of moving from one binary experiment, $\mathcal{E}$, to a more informative one, $\mathcal{E}'$, on evaluators' payoffs. I showed that the *direction* in which information improves, specficially whether *good news* or *bad news* get stronger, determines this effect. *Stronger good news* always benefits evaluators. It drives them to reject their marginal admits, re-assessed negatively by every evaluator upon richer information. In contrast, *stronger bad news* eventually hurt evaluators. It pushes them to approve their marginal rejects, re-assessed positively by some evaluators upon richer information. These applicants are adversely selected; their positive re-assessments come against the backdrop of other evaluators' negative verdicts. Once bad news are *too strong* against good news, adverse selection overpowers the positive re-assessment that led to their approval, and leaves evaluators worse off.

Binary experiments afford significant tractability. However, in many settings of interest, evaluators have richer sources of information. Traders of financial assets, for instance, might get recommendations of varying levels of strength such as "Strong Sell", "Sell", "Buy" and "Strong Buy". Likewise, a bank's credit scoring algorithm might output varying probabilities for a loan seeker's default rather than a simple "Good" score and a "Bad" one. With this motivation, here I investigate how the insight in Theorem 1 extends to improvements of *arbitrary* experiments.

Two complications challenge this exercise. Both stem from the fact that normalised beliefs are, in general, ill suited to characterise experiments. The distribution of their outcomes, described by $2m$ unknowns $\{p_L(s), p_H(s)\}_{s \in \mathbf{S}}$, cannot be determined by the $m+2$ linear equalities their normalised beliefs supply:

$$s = \frac{p_H(s)}{p_H(s) + p_L(s)} \quad \text{for } s \in \mathbf{S}$$

$$\sum_{s \in \mathbf{S}} p_\theta(s) = 1 \quad \text{for } \theta \in \{L, H\}$$

A binary experiment is the exception. Its normalised beliefs, which I dubbed s_H and s_L, uniquely determine its outcomes' distributions. Consequently, they describe its informational content. I reinterpreted these normalised beliefs as the strength of good news and bad news an experiment carries, respectively. A more informative binary experiment is simply one with stronger good news and stronger bad news; higher s_H and lower s_L.

I exploited this exception in two ways. First, I characterised Blackwell improvements of a binary experiment through a dichotomy: those which *strengthen good news*, and others which *strengthen bad news*. Any improvement of a binary experiment combines these two forces. This dichotomy was the key to understanding the different Blackwell improvements' effect on applicants, and thus on evaluators' payoffs. How this categorisation extends beyond binary experiments is not immediately clear. Besides being insufficient to characterise improvements in information, normalised beliefs do not have natural interpretations as "good" and "bad" news in general.

Second, through marginal adjustments to these normalised beliefs, I constructed "small" improvements in evaluators' information. These small improvements in information formed the building blocks of "larger" Blackwell improvements. Most importantly, they helped me uncover how improving an experiment in either direction affects applicants, and ultimately evaluators. Beyond binary experiments, we cannot equate "small" movements in an experiment's normalised beliefs with "small" improvements in the information it carries.

I overcome these challenges by generalising the *auxiliary signals* we constructed in the previous section. There, I used these auxiliary signals to replicate and visualise Blackwell improvements of evaluators' experiments, and pin down the applicants they affect. Here, I formalise and generalise them under the name *local mean preserving spreads*. Adopting the role auxiliary signals played in the previous section, local mean preserving spreads serve as the building blocks of *any* Blackwell improvement an experiment might undergo.

Definition 4 (Local Mean Preserving Spread)**.** Take two experiments $\mathcal{E} = (\mathbf{S}, p_L, p_H)$ and $\mathcal{E}' = (\mathbf{S}', p'_L, p'_H)$ and let $s_1 < s_2 < ... < s_M$ be the normalised beliefs their joint outcome set $\mathbf{S} \cup \mathbf{S}'$ induces. Define the probability distributions p and p' over these outcomes as:

$$p(s) := \frac{p_H(s) + p_L(s)}{2} \qquad p'(s) := \frac{p'_H(s) + p'_L(s)}{2} \qquad \text{for all } s \in \{s_1, s_2, ..., s_M\}$$

Experiment $\mathcal{E}'$ differs from $\mathcal{E}$ by a *local mean preserving spread* at s_j if for some $j \in \{2, ..., M-1\}$:

$$p'(s_{j-1}) \geq p(s_{j-1}) \qquad 0 = p'(s_j) \leq p(s_j) \qquad p'(s_{j+1}) \geq p(s_{j+1})$$

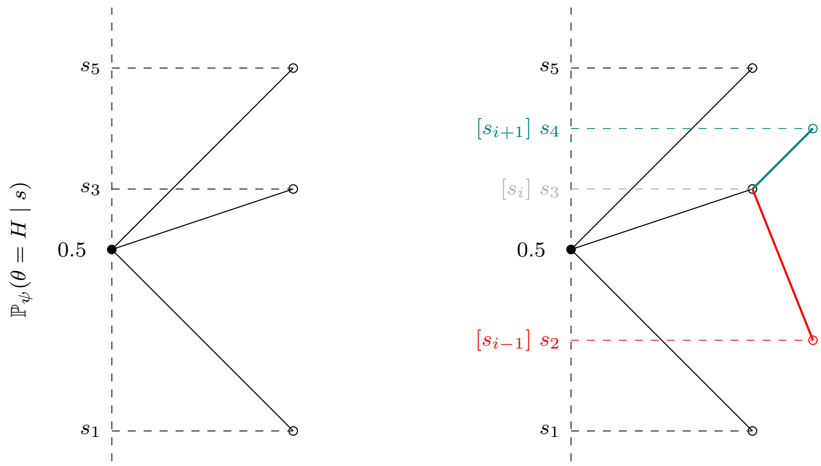
$$p'(s_k) = p(s_k) \quad \text{for all } j \notin \{j-1, j, j+1\}$$

$$\sum_{j=1}^{M} s_j \times p'(s_j) = \sum_{j=1}^{M} s_j \times p(s_j)$$

Much like an ordinary mean preserving spread (Rothschild and Stiglitz, 1970[10]), a *local mean preserving spread* distributes probability away from an *origin* point to two *destination* points, one above and one below it. It does so while preserving the mean of the original distribution. Crucially however, a mean preserving spread is *local* if and only if the destination points are the immediate neighbours of the origin point[11]. In other words, neither the original nor the

[10] Rothschild and Stiglitz, 1970 describe mean preserving spreads through *four* points in the support of the distribution. Here, I describe them through *three*. This is without loss of generality. In fact, mean preserving spreads were first characterised by Muirhead, 1900 in the context of majorisation, with *three* points. Rasmusen and Petrakis, 1992 show formally that these the three or four point characterisations of MPS are in fact equivalent.

[11] The reader will notice that this statement is ill-defined unless the signal structure is discrete. To the best of the author's knowledge, no counterpart for *local mean preserving spreads* exist for, say, atomless signal structures.



(a) Before Local Mean Preserving Spread (b) After Local Mean Preserving Spread

Figure 4: A Local Mean Preserving Spread
(for convenience, with $\psi = 0.5$ fixed)

resulting distribution assign positive probability to any other point between the origin and the two destination points[12].

The auxiliary signals I constructed in the previous section generate such local mean preserving spreads (hereafter, just *local spread*). To strengthen bad news, for instance, the auxiliary signal spreads all the probability mass the experiment $\mathcal{E}$ assigns to the origin point s_L to the neighbouring destination points s_H and s'_L, the former above, and the latter below it.

Local spreads are simple Blackwell improvements of an experiment. Like auxiliary signals, we can interpret them as an additional binary experiment an evaluator observes after a particular outcome of her original experiment. Despite their simplicity, local spreads are powerful enough to be pieced together into *any* Blackwell improvement of an experiment. I restate this result in Remark 1. Note that Remark 1 is a slight refinement of Rotschild and Stiglitz's classic result (1970) for experiments with finitely many outcomes.

Remark 1. [Müller and Stoyan, 2002, Theorem 1.5.29] An experiment $\mathcal{E}'$ is Blackwell more informative than another, $\mathcal{E}$, if and only if there is a finite sequence of experiments $\mathcal{E}_1, \mathcal{E}_2, ..., \mathcal{E}_k$ such that $\mathcal{E}_1 = \mathcal{E}$, $\mathcal{E}_k = \mathcal{E}'$, and $\mathcal{E}_{i+1}$ differs from $\mathcal{E}_i$ by a local mean preserving spread.

Like the auxiliary signals in the previous section, the power of local spreads lie in the way we reinterpret them. A local spread acts as a re-evaluation of an applicant for whom an evaluator initially observes the signal it spreads. Upon this re-evaluation, the evaluator might revise the verdict she would have reached after that signal. This jeopardises an applicant she would approve upon that signal, while offering one she would reject a second chance. In the case of binary experiments, we uncovered that stronger good news have the former influence, and stronger bad news the latter. These different influences drive their different consequences for

[12]The attentive reader will also realise that this definition also requires that *all* probability mass be spread away from the origin point. This difference is insignificant in our current setting.

evaluators' payoffs. The consequence of a local spread, too, depends on which of these influences it exerts on evaluators' verdicts.

Theorem 2. Let the experiment $\mathcal{E}'$ differs from $\mathcal{E}$ by a local mean preserving spread at s_j. Evaluators' payoffs under the most (least) selective equilibrium of $\mathcal{E}'$:

1. *weakly exceed* those under $\mathcal{E}$ if s_j leads to approvals under $\mathcal{E}$; i.e. $\hat{\sigma}_{\mathcal{E}}(s_j) = 0$ ($\check{\sigma}_{\mathcal{E}}(s_j) = 0$)

2. *fall weakly below* those under $\mathcal{E}$ if:

   i if s_j leads to rejections under $\mathcal{E}$; i.e. $\hat{\sigma}_{\mathcal{E}}(s_j) = 1$ ($\check{\sigma}_{\mathcal{E}}(s_j) = 1$), and

   ii the following condition holds:

$$\frac{\rho}{1-\rho} \times \left(\frac{r_H^*}{r_L^*}\right)^{n-1} \times \frac{s_{j+1}}{1-s_{j+1}} \leq \frac{c}{1-c}$$

   where $r_\theta^* := r_\theta(\hat{\sigma}_\varepsilon; \mathcal{E})$ ($r_\theta^* := r_\theta(\check{\sigma}_\varepsilon; \mathcal{E})$).

Evaluators' payoffs always increase when a signal they interpret as *good news* – and approve the applicant with – is locally spread. This new information helps evaluators re-assess an applicant who previously, some would have approved. If they *all* re-assess him negatively, that applicant might now be rejected by *all* of them. Of course, the evaluators can refuse to act on those negative re-assessments, sticking instead to the decisions they would have made before this additional information. Unless they do, this local spread makes them *more selective*, improving their equilibrium payoffs[13].

In contrast, locally spreading a signal evaluators interpret as *bad news* might hurt evaluators' payoffs. Upon such spreads, evaluators re-assess an applicant they would otherwise have rejected. This applicant thus gets a second chance from *every* evaluator, each of whom might approve him upon a positive re-assesment. That evaluator does so, however, unaware of some of her peers' (now reinforced) negative assessments of the applicant's quality. The bad news those negative assessments carry might overwhelm our evaluator's positive re-assessment, leaving her worse off from her approval decision in expectation. Theorem 2 confirms and generalises the tight limit adverse selection imposes on how strong those bad news can be before such spreads are guaranteed to harm evaluators. Unless an evaluator would *never* regret her positive re-assessment upon discovering how many of her peers rejected the applicant before her, evaluators are surely worse off after this spread of *bad news*.

The opposite equilibrium reactions these two *kinds* of improvements trigger ultimately drive their opposite effects on evaluators' payoffs. *Local spreads* are critical in offering the granularity needed to identify these improvements' distinct effects on evaluators' behaviour. The more

[13]Recall Lemma 5.

familiar "simple" mean preserving spreads can cause multiple and conflicting changes in evaluators' behaviour by creating new, "far away" signals. Spreading a rejection signal to a destination signal "far" above it, for example, motivates more approvals at that far destination, just as a local spread would. However, by doing so it might *also* motivate more rejections at lower signals, by pushing evaluators' interim beliefs too far down. Local spreads eliminate this worry, isolating the effect of each spread. This comes for free; like "simple" spreads, local spreads are sufficient building blocks for *any* larger Blackwell improvement.

Unlike Theorem 1, Theorem 2 demands knowledge of the equilibrium in concern to judge how a local spread will affect evaluators' payoffs. The analyst must both know how evaluators interpret the signal that is locally spread, and the extent of bad news carried by a rejection; ideally the probability with which a visit results in a rejection for *High* and *Low* quality applicants. In practice, she might wish to remain agnostic about these details. To alleviate such concerns, in Proposition 5 I offer a stronger sufficient condition for a local spread to harm evaluators. This sufficient condition reduces the analyst's dependance on her knowledge of the equilibrium being played.

Proposition 5. Let the experiment $\mathcal{E}'$ differ from $\mathcal{E}$ by a local mean preserving spread at s_j. Evaluators' payoffs in the most selective equilibrium of $\mathcal{E}'$ are below those of $\mathcal{E}$ if:

1. the signal s_j leads to rejections under the most selective equilibrium of $\mathcal{E}$, and

2. the following condition holds:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_j}{1-s_j}\right)^{n-1} \times \frac{s_{j+1}}{1-s_{j+1}} \leq \frac{c}{1-c}$$

The second condition in Proposition 5 offers a stronger adverse selection condition than Theorem 2 does. It still requires an evaluator who approves the applicant with the newly created signal s_{j+1} to be regret-free upon discovering all her peers rejected the applicant. However, it crucially allows her to retain "optimism" that their rejections were caused by the best signal possible that could lead to it, s_j. If even such optimism does not suffice to eliminate the evaluator's regret, this spread is guaranteed to hurt evaluators' payoffs.

This sufficient condition Proposition 5 offers is linked closely to the *adverse selection causes no regret* condition I defined in Section 4.1[14]. That condition characterised *exactly* when stronger bad news hurt evaluators, under the presumption that bad news is already strong enough to trigger a rejection. The auxiliary signal which generates that improvement is a local spread whose origin signal is s_L, and higher destination signal is s_H. When we substitute this origin and destination signal to Proposition 5's sufficient condition, we find that this sufficient condition coincides with the *adverse selection causes no regret* condition. This reveals that Proposition

[14] Definition 2.

5's sufficient condition is in some cases also *necessary*; for instance and notably, when improving the strength of bad news a binary experiment carries.

With Proposition 3, an analyst who wants to remain agnostic about the strength of bad news carried by evaluators' rejections can still study whether a local spread is guaranteed to harm evaluators. He, however, must still know whether the signal being spread is a rejection signal or not. Luckily, there is yet another sufficient condition which guarantees that this signal s_j *is* in fact a rejection signal. As any equilibrium features adverse selection, signals $s_j < s_L^{\text{mute}}$[15] are sure to be interpreted as bad news by evaluators, as I discussed earlier.

5 Designing Evaluators' Information

In Section 4, I showed that despite letting each individual evaluator tailor her decision better to the applicant's quality, more information might hurt evaluators' payoffs in equilibrium. This owes to an *adverse selection externality* inherent to decentralised evaluations: which applicant one evaluator rejects determines who the other is wary of receiving. With better informed peers, each evaluator might end up expecting a *worse* applicant; fearing some of those peers likely rejected him earlier. Eventually this might leave all evaluators worse off *despite* being better equipped to judge the applicant's quality.

Somewhat paradoxically then, evaluators could benefit from committing to ignore some information about their applicant's quality. Individually, they might be unable to do so. However, a regulator interested in the quality of evaluators' selection might have the power to prescribe both *what* information they must use, and *how* they must use it. In this Section, I study how this regulator can *best* coarsen evaluators' information about the applicant's quality, with the aim of maximising their equilibrium payoffs. Becoming less selective never benefits evaluators (Proposition 3), so in designing their information, the regulator seeks to boost evaluators' selectivity. Her approach is aggressive: she tries to steer evaluators away from *any* applicant they might regret approving upon learning his rejection history.

Formally, I consider a regulator who garbles evaluators' experiment $\mathcal{E}$ at the outset of the game. She can choose any garbling $S^G : \mathbf{S} \to \Delta\left(\mathbf{S}^G\right)$ which maps the original outcome $s \in \mathbf{S}$ of this experiment to a distribution over $\mathbf{S}^G$; an arbitrary finite set of garbled signals. Previously, I denoted the distribution of experiment $\mathcal{E}$'s outcomes conditional on the applicant's quality as p_θ. Following that notation, I denote the distribution of the garbled signal conditional on experiment $\mathcal{E}$'s outcome $s \in \mathbf{S}$ as $p_s^G := S^G(s)$. Likewise, I denote a representative element of $\mathbf{S}^G$ as s^G.

Once the regulator chooses the garbling S^G, the game proceeds as before. The applicant sequentially visits the n evaluators in the manner described earlier. The evaluator he visits no

[15] Recall Definition 3.

longer observes the outcome of $\mathcal{E}$, but instead the garbled signal s^G. She then decides whether to approve or reject the applicant. The game ends either when one evaluator approves the applicant, or they all reject him.

An *equilibrium*, as before, is a pair $\left(\sigma^G, \psi^G\right)$ such that (i) the strategy $\sigma^G : \mathbf{S}^G \to [0,1]$ is *optimal* given the interim belief ψ^G, and (ii) the interim belief ψ^G is *consistent* with the strategies σ^G. Throughout this section, I assume that evaluators always play the *most selective equilibrium* that the regulator's chosen garbling supports.

The regulator wishes to maximise evaluators' (most selective) equilibrium payoffs through her chosen garbling. I identify the garbling that achieves this objective – the *regulator-preferred* garbling – in this section.

I begin by introducing a special class of garblings which are crucial to this exercise.

Definition 5. A garbling S^G of $\mathcal{E}$ is *monotone binary* if:

1. The set $\mathbf{S}^G$ is binary, with elements labelled s_L^G and s_H^G without loss.

2. Either there is a threshold outcome $s^* \in \mathbf{S}$ such that:

$$p_s^G\left(s_H^G\right) = \begin{cases} 0 & s < s^* \\ \in (0,1] & s = s^* \\ 1 & s > s^* \end{cases}$$

   or $p_s^G(s_H^G) = 0$ for all outcomes $s \in \mathbf{S}$.

Monotone binary garblings are perhaps the simplest that come to mind. Rather than directly observing the outcome of her original experiment $\mathcal{E}$, the evaluator receives either a "high" or a "low" signal from the planner's garbling, which I denote as s_H^G and s_L^G. Outcomes which exceed the threshold $s^* \in \mathbf{S}$ trigger the "high" signal, whereas those below the threshold trigger the "low" one. The threshold outcome s^* is the lowest that can trigger the high signal. Moreover, it is the unique outcome which can *also* trigger the low signal with some probability.

The high signal from such a garbling serves as an "approval recommendation" for evaluators, while the low signal serves as a "rejection recommendation". Of course, in equilibrium, the evaluators will judge whether it is *optimal* for them to follow these recommendations. This depends both on what each recommendation tells evaluators about the applicant's quality, and what interim belief is consistent with all evaluators following them. When evaluators judge following these recommendations optimal, I say the planner's garbling *produces optimal recommendations.*

Definition 6. A monotone binary garbling S^G *produces optimal recommendations* if the strategy

σ^G which obeys its recommendations, defined as:

$$\sigma^G(s^G) := \begin{cases} 0 & s^G = s^G_L \\ 1 & s^G = s^G_H \end{cases}$$

is an equilibrium strategy under S^G.

Despite their simplicity, I establish in Lemma 2 that the regulator need not look beyond monotone binary garblings.

Lemma 2. Whenever it exists, the regulator-preferred garbling is monotone binary and produces optimal recommendations.

That the regulator can limit herself to binary garblings follows from a fundamental principle in information design. An evaluator ultimately distils the information relayed by the garbled signal into which action she ought to take. The regulator can distil that information herself, sending her only a simple "approve" or "reject" recommendation instead[16]. *Monotone* binary garblings recommend a rejection upon "low" outcomes, and an approval upon "high" outcomes. In doing so, they ensure a better alignment between evaluators' actions with the applicant's quality, in line with the regulator's objective. However, the regulator must still ensure that those recommendations remain optimal. Lemma 2's novelty lies there: I show that she need not depart from monotonicity to ensure the optimality of her recommendations. For any garbling, there is a monotone binary alternative with optimal recommendations that raises evaluators' equilibrium payoffs.

Much like monotone strategies, monotone binary garblings group the outcomes of $\mathcal{E}$ to an "approval" and "rejection" region. Thus, it is natural to use the "selectivity" comparison we previously employed for strategies between them as well. I call a monotone binary garbling *more selective* when it is less likely to send an approval recommendation to evaluators.

Definition 7. Where S^G and $S^{G'}$ are monotone binary garblings of $\mathcal{E}$, $S^{G'}$ is *more selective than* S^G if $p^{G'}_s(s^{G'}_H) \leq p_s(s^G_H)$ for any $s \in \mathbf{S}$.

Evaluators' equilibrium strategies, like the regulator's preferred garbling, are monotone. However, the regulator and the evaluators might disagree on *where* the line to approve an applicant should be drawn. As Lemma 1 established, the regulator would never wish that evaluators act less selectively, but she might rather they be *more* selective. Theorems 1 and 2 already foreshadowed that she might wish evaluators to "ignore" some information to that

[16]This elementary principle relies crucially on the evaluators coordinating on the most selective equilibrium (equivalently, the regulator's preferred one). Crucially, coarsening evaluators' experiment might also *create* new equilibrium outcomes. The regulator might dislike these new outcomes more than any original equilibrium outcome. This might push the regulator to providing *richer* information that eliminates some of these unfavourable outcomes.

end – particularly if, upon learning she was the *only* one not to reject him, an evaluator might regret approving her applicant. Garblings that *never* cause such regret – those with *regret-free approvals* – will be important for the regulator's problem.

Definition 8. The monotone binary garbling S^G is said to have *regret-free approvals* if it either:

i Recommends no approvals; i.e. $p_s(s_H^G) = 0$ for all $s \in \mathbf{S}$,

ii Recommends no rejections; i.e. $p_s(s_H^G) = 1$ for all $s \in \mathbf{S}$, and:

$$\frac{\rho}{1-\rho} \times \left(\frac{p_H(s_1)}{p_L(s_1)}\right)^n \geq \frac{c}{1-c}$$

or,

iii The following condition holds:

$$\frac{\rho}{1-\rho} \times \frac{p_H\left(s^*\right)}{p_L\left(s^*\right)} \times \left(\frac{\sum\limits_{s\in\mathbf{S}} p_H(s) \times p_s^G\left(s_L^G\right)}{\sum\limits_{s\in\mathbf{S}} p_L(s) \times p_s^G\left(s_L^G\right)}\right)^{n-1} \geq \frac{c}{1-c}$$

Otherwise, it is said to have *regret-prone approvals*.

Regret-free approvals is a stringent condition. It demands that the evaluator find it optimal to approve her applicant upon being recommended to do so regardless of (i) how many rejections she thinks the applicant had earlier, (ii) which outcome of the original experiment she thinks led to this recommendation. Nonetheless, Proposition **??** establishes that the regulator seeks to ensure precisely this condition when she is not constrained by the optimality of her recommendations.

Proposition 6. The regulator-optimal garbling is the *least selective* monotone binary garbling with *regret-free approvals* if it produces optimal recommendations. Otherwise, it is either:

- the least selective monotone binary garbling with regret-free approvals, or
- the most selective monotone binary garbling with regret-prone approvals

among those which produce optimal recommendations.

The regulator's aim is to maximise evaluators' *expected* payoffs. She strives for something much starker to achieve this: to leave the *last* evaluator posed to get the applicant best off. Evaluators will find this policy too cautious if anything; likelier than not, they are posed to receive the applicant earlier (the probability that they are the *last* evaluator is only $1/n$). However, only the *eventual* outcome of the applicant matters for the regulator – not whether any evaluator early in the applicant's sequence has favourable enough evidence to snatch him already. An evaluator's decision affects this eventual outcome only if *all* other evaluators reject him. The regulator aims to optimise this evaluator's decision through garbling her information.

Each evaluator, in contrast, cares only whether she could expect to profit from approving this applicant *now*. As such, evaluators might find the regulator's recommendations – optimised for the *last* evaluator posed to get the applicant – too stringent to obey. The regulator must then seek a second best garbling which can supply evaluators with optimal recommendations. Notably, she might choose a garbling that is even *more selective* than her first-best. Such a garbling supplies more stringent recommendations, but might exacerbate adverse selection too. Those recommendations might, this time, be optimal against this lower interim belief.

There is a crucial condition which ensures that the regulator's first best garbling will have optimal recommendations. Whenever the cost of approving is higher than evaluators' prior belief – $c \geq \rho$ – *any* rejection recommendation will be found optimal by the evaluators; their interim belief is guaranteed to be below their prior, and a rejection recommendation is always bad news.

Corollary 7. When evaluators' approval cost c weakly exceeds their prior ρ, the regulator-optimal garbling is the *least selective* monotone binary garbling with regret-free approvals.

6 Proof Appendix

6.1 Useful Definitions and Notation

In what follows, I occasionally operate with the likelihood ratios of beliefs for convenience. The reader can easily verify the identities:

$$\frac{\psi}{1-\psi} = \frac{\rho}{1-\rho} \times \frac{\nu_H(\sigma;\mathcal{E})}{\nu_L(\sigma;\mathcal{E})} \qquad\qquad \frac{\mathbb{P}_\psi(\theta = H \mid s_i)}{1-\mathbb{P}_\psi(\theta = H \mid s_i)} = \frac{\psi}{1-\psi} \times \frac{s_i}{1-s_i}$$

Through similar reasoning, the reader can verify that it is optimal to approve the applicant when:

$$\frac{\mathbb{P}_\psi(\theta = H \mid s_i)}{1-\mathbb{P}_\psi(\theta = H \mid s_i)} > \frac{c}{1-c}$$

Some strategies require evaluators to randomise when approving their applicant upon observing a particular signal realisation. To facilitate the technical discussion, I assume that each evaluator observes the realisation of a *tie-breaking signal* $u \sim U[0,1]$ alongside the outcome of her experiment. This signal is not informative about the applicant's quality: it is distributed independently from it conditional on the experiment's outcome. I denote the outcome of evaluator i's experiment as s^i and her tie-breaking signal as u^i. Without loss, evaluator i approves the applicant if and only if $\sigma(s^i) \leq u^i$; where σ is her strategy. I call the pair (s^i, u^i) the *score* evaluator i observes for the applicant.

Definition 9. The tuple $Z^i = (s^i, u^i)$, where $u^i \overset{IID}{\sim} U[0,1]$ is the *score* evaluator i observes for the applicant. The applicant's *score profile* $\mathbf{z}$ is the set of scores each evaluator would observe if he were to visit them all; $\mathbf{z} = \{(s^i, u^i)\}_{i=1}^n$. Analogously, the applicant's *signal profile* $\mathbf{s} = \{s^i\}_{i=1}^n$ is the set of outcomes of each evaluator's experiment.

Some proofs in Section 6.3 require comparing interim beliefs across pairs of strategies and experiments; $(\sigma, \mathcal{E})$. For convenience, I define the mapping from such a pair to the interim belief consistent with them as $\Psi(.;\mathcal{E}) : [0,1]^n \to [0,1]$:

$$\Psi(\sigma;\mathcal{E}) := \frac{\rho \times \nu_H(\sigma;\mathcal{E})}{\rho \times \nu_H(\sigma;\mathcal{E}) + (1-\rho) \times \nu_L(\sigma;\mathcal{E})}$$

Wherever necessary, I treat each strategy $\sigma : \mathbf{S} \to [0,1]$ for an experiment $\mathcal{E}$ as a vector in the compact set $[0,1]^m \subset \mathbb{R}^n$. This is a finite dimensional vector space, so I endow it with the metric induced by the taxicab norm without loss of generality (see Kreyszig, 1978 Theorem 2.4-5):

$$||\sigma' - \sigma|| = \sum_{j=1}^{m} |\sigma'(s_j) - \sigma(s_j)| \qquad \text{for any two strategies } \sigma' \text{ and } \sigma$$

Note that the interim belief function $\Psi(.;\mathcal{E})$ is thus a continuous function of evaluators' strategies[17].

6.2 Omitted Results

Lemma 3. Suppose there is a single evaluator, $n = 1$. Her equilibrium expected payoff under experiment $\mathcal{E}'$ exceed that under $\mathcal{E}$ *regardless of her approval cost* $c \in [0,1]$ *and prior belief* $\rho \in [0,1]$ if and only if $\mathcal{E}'$ is (Blackwell) more informative than $\mathcal{E}$.

Proof. The sufficiency part of this Lemma follows from Blackwell's Theorem (Blackwell and Girshick, 1954, Theorem 12.2.2). To show necessity, I fix an arbitrary prior belief ρ for the evaluator.

Let q_j be the posterior belief the evaluator forms about the applicant's quality upon observing the outcome $s_j \in \mathbf{S}$:

$$q_j = \frac{\rho \times s_j}{\rho \times s_j + (1-\rho) \times (1-s_j)}$$

Furthermore, let $F(.)$ and $F'(.)$ be the CDFs of posterior beliefs $\mathcal{E}$ and $\mathcal{E}'$ induce, respectively,

[17] r_θ is a continuous function of σ, thus both the nominator and denominator are strictly positive continuous functions of σ.

for this prior belief ρ:

$$F(q) = (1-\rho) \times \sum_{s \in \mathbf{S}: s \leq q} p_L(s) + \rho \times \sum_{s \in \mathbf{S}: s \leq x} p_H(s)$$
$$F'(q) = (1-\rho) \times \sum_{s \in \mathbf{S}: s \leq q} p'_L(s) + \rho \times \sum_{s \in \mathbf{S}: s \leq x} p'_H(s)$$

The evaluator's expected payoff under $\mathcal{E}$ is given by:

$$\int_c^1 (q-c) dF(q) = \int_c^1 q dF(q) - c \times (1 - F(c)) = (1-c) - \int_c^1 F(q) dq$$

An analogous expression gives her expected payoff under $\mathcal{E}'$. Therefore, for her expected payoffs under $\mathcal{E}'$ to exceed those under $\mathcal{E}$ for any $c \in [0,1]$, we must have:

$$\int_c^1 \left(F(q) - F'(q)\right) dq \geq 0$$

which is equivalent to $\mathcal{E}'$ being Blackwell more informative than $\mathcal{E}$[18].

□

Lemma 4 proves useful when proving Theorem 2, the main result of Section 4.2. This Lemma can also be used towards an alternative and direct proof for the equilibrium existence claim of Proposition 1.

Lemma 4. For each $j \in \{1, 2, ..., m\}$, let σ_j be the strategy defined as:

$$\sigma_j(s) = \begin{cases} 0 & s < s_j \\ 1 & s \geq s_j \end{cases}$$

and ψ_j be the interim belief consistent with this strategy. Unless σ_j is itself an equilibrium strategy:

i There is an equilibrium strategy σ^* that is *more selective than* σ_j if $\mathbb{P}_{\psi_j}(\theta = H \mid s_j) < c$.

ii There is an equilibrium strategy σ^* that is *less selective than* σ_j if $\mathbb{P}_{\psi_j}(\theta = H \mid s_{j-1}) > c$.

Proof. Abusing notation slightly, I add two fully revealing outcomes s_0 and s_{m+1} to the set $\mathbf{S}$ (duplicating s_1 and s_m if either of them are already fully revealing), and denote the strategy

[18] See Müller and Stoyan, 2002, Theorem 1.5.7. The Blackwell order between signal structures is equivalent to the convex order between the posterior belief distributions they induce; see Gentzkow and Kamenica, 2016.

which approves *no* applicant as σ_{m+1}:

$$\frac{s_{m+1}}{1-s_{m+1}}=\infty \qquad \frac{s_0}{1-s_0}=0 \qquad \frac{\psi_{m+1}}{1-\psi_{m+1}}=\frac{\rho}{1-\rho}$$

Claim i.

The strategy σ_{m+1} is the most selective strategy evaluators can adopt, and is an equilibrium strategy unless:

$$\frac{s_m}{1-s_m}\times\frac{\psi_{m+1}}{1-\psi_{m+1}}>\frac{c}{1-c}$$

So, assume this condition is satisfied. Likewise, the strategy σ_k for $k\geq j$ is an equilibrium if the following inequality is satisfied:

$$\frac{s_k}{1-s_k}\times\frac{\psi_k}{1-\psi_k}\geq\frac{c}{1-c}\geq\frac{s_{k-1}}{1-s_{k-1}}\times\frac{\psi_k}{1-\psi_k} \tag{6.1}$$

So, assume inequality 6.1 is violated for every $k\geq j$. This gives us:

$$\frac{s_{m+1}}{1-s_{m+1}}\times\frac{\psi_{m+1}}{1-\psi_{m+1}}>\frac{c}{1-c}>\frac{s_j}{1-s_j}\times\frac{\psi_j}{1-\psi_j} \tag{6.2}$$

where the last part of this inequality is by hypothesis.

Now, let $k^*\in\{j,j+1,...,m\}$ be the first index for which the following inequality is satisfied:

$$\frac{s_{k^*+1}}{1-s_{k^*+1}}\times\frac{\psi_{k^*+1}}{1-\psi_{k^*+1}}\geq\frac{c}{1-c}\geq\frac{s_{k^*}}{1-s_{k^*}}\times\frac{\psi_{k^*}}{1-\psi_{k^*}}$$

such a k^* must exist due to inequality 6.2. But since inequality 6.1 is violated, we must have:

$$\frac{s_{k^*}}{1-s_{k^*}}\times\frac{\psi_{k^*+1}}{1-\psi_{k^*+1}}>\frac{c}{1-c}\geq\frac{s_{k^*}}{1-s_{k^*}}\times\frac{\psi_{k^*}}{1-\psi_{k^*}}$$

But since the function $\Psi(\sigma;\mathcal{E})$ is continuous in evaluators' strategy σ, we can then find some strategy σ^*:

$$\sigma^*(s)=\begin{cases}1 & s>s_{k^*}\\ \in[0,1] & s=s_{k^*}\\ 0 & s<s_{k^*}\end{cases}$$

such that:

$$\frac{s_{k^*+1}}{1-s_{k^*+1}}\times\frac{\Psi(\sigma^*;\mathcal{E})}{1-\Psi(\sigma^*;\mathcal{E})}\geq\frac{c}{1-c}=\frac{s_{k^*}}{1-s_{k^*}}\times\frac{\Psi(\sigma^*;\mathcal{E})}{1-\Psi(\sigma^*;\mathcal{E})}$$

The strategy σ^* is thus an equilibrium strategy. It is clearly more selective than σ_j; since it is more selective than σ_{k^*}, where $k^*\geq j$.

Claim ii.

For any $k \in \{1, 2, ..., j\}$, the strategy σ_k is an equilibrium if the inequality 6.1 is satisfied. So, as earlier, assume 6.1 is violated for every such k. Then, we get:

$$\frac{s_j}{1-s_j} \times \frac{\psi_j}{1-\psi_j} \geq \frac{s_{j-1}}{1-s_{j-1}} \times \frac{\psi_j}{1-\psi_j} > \frac{c}{1-c} > \frac{s_1}{1-s_1} \times \frac{\psi_1}{1-\psi_1}$$

where the second inequality in the chain follows by hypothesis and the last inequality follows from the violation of inequality 6.1 for $k = 1$. We can now repeat the argument we constructed after inequality 6.2 to prove Claim i, to prove the existence of an equilibrium strategy σ^* that is less selective than σ_j.

□

6.3 Omitted Proofs

Proposition 1. Let Σ be the set of evaluators' equilibrium strategies. Then:

1. Σ is non-empty and compact.
2. Any equilibrium strategy σ^* is *monotone*: $\sigma^*(s) > 0$ for some $s \in \mathbf{S}$ implies that $\sigma^*(s') = 1$ for every $s' \in \mathbf{S}'$ such that $s' > s$.
3. All equilibria exhibit adverse selection: $\psi^* \leq \rho$ for any interim belief ψ^* consistent with an equilibrium strategy $\sigma^* \in \Sigma$.

Proof. In what follows, I treat each strategy $\sigma : \mathbf{S} \to [0, 1]$ as a vector in the compact set $[0, 1]^m \subset \mathbb{R}^n$, endowed with the taxicab metric (see the end of Section 6.1). I start by proving that any equilibrium strategy must be monotone and all equilibria exhibit adverse selection. Using these observations, I prove that the set of equilibrium strategies is non-empty and compact.

2. Any equilibrium strategy is monotone.

Any equilibrium strategy σ^* must be optimal against the interim belief ψ^* consistent with it. Whenever $\rho \in (0, 1)$, $\psi^* = \Psi(\sigma^*; \mathcal{E}) \in (0, 1)$, and so $\mathbb{P}_{\psi^*}(\theta = H \mid s') > \mathbb{P}_{\psi^*}(\theta = H \mid S = s)$ for $s', s \in \mathbf{S}$ such that $s' > s$.

3. All equilibria exhibit adverse selection.

A fortiori, $\Psi(\sigma; \mathcal{E}) \leq \rho$ for any monotone strategy σ. To see this, note that $p_H(.)$ first order stochastically dominates $p_L(.)$ since it likelihood ratio dominates it[19]. Therefore, $\nu_L(\sigma; \mathcal{E}) \geq \nu_H(\sigma; \mathcal{E})$. The result then follows since $\frac{\Psi(\sigma;\mathcal{E})}{1-\Psi(\sigma;\mathcal{E})} = \frac{\rho}{1-\rho} \times \frac{\nu_H(\sigma;\mathcal{E})}{\nu_L(\sigma;\mathcal{E})}$.

1. The set of equilibrium strategies is non-empty and compact.

[19]Theorem 1.C.1 in Shaked and Shanthikumar, 2007.

i The set of equilibrium strategies is non-empty.

Define $\Phi(.) : [0,1]^m \to 2^{[0,1]^m}$ to be the evaluators' *best response correspondence.* $\Phi(.)$ maps any strategy σ to the set of strategies that are optimal against the interim belief $\Psi(\sigma;\mathcal{E})$ it induces:

$$\Phi(\sigma) = \left\{\sigma' \in [0,1]^m : \sigma' \text{ is optimal against } \Psi(\sigma;\mathcal{E})\right\}$$

A strategy σ^* is an equilibrium strategy if and only if it is a fixed point of evaluators' best response correspondence; $\sigma^* \in \Phi(\sigma^*)$. I establish that the correspondence Φ has at least such fixed point through Kakutani's Fixed Point Theorem.

Φ is trivially non-empty; every interim belief has some strategy optimal against it. It is also convex valued; if two distinct approval probabilities are optimal after some outcome $s \in \mathbf{S}$, *any* approval probability is optimal upon that outcome.

The only task that remains is to prove that Φ is upper-semi continuous. For this, take an arbitrary sequence of strategies $\{\sigma_n\}$ such that $\sigma_n \to \sigma_\infty$. Denote the interim beliefs consistent with these strategies as $\psi_n := \Psi(\sigma_n;\mathcal{E})$. Since $\Psi(.;\mathcal{E})$ is continuous in evaluators' strategies, we also have $\psi_n \to \psi_\infty$ where $\psi_\infty = \Psi\left(\sigma_\infty;\mathcal{E}\right)$. Now, take a sequence of strategies $\{\sigma_n^*\}$ where $\sigma_n^* \in \Phi(\sigma_n)$. Note that every σ_n^* is monotone since optimality against any interim belief $\psi \in (0,1)$ requires monotonicity. We want to show that Φ is upper semicontinuous; i.e.:

$$\sigma_n^* \to \sigma_\infty^* \implies \sigma_\infty^* \in \Phi(\sigma_\infty)$$

By the Monotone Subsequence Theorem, the sequence $\{\sigma_n^*\}$ has a subsequence $\sigma_{n_k}^* \to \sigma_\infty^*$ of strategies whose norms $||\sigma_{n_k}^*||$ are monotone in their indices n_k. Here I take the case where these norms are increasing, the proof is analogous for the opposite case. Since σ_∞^* is the limit of a subsequence of monotone strategies, it must be a monotone strategy too. Assuming otherwise leads to a contradiction; for any $s, s' \in \mathbf{S}$ such that $s' > s$:

$$\sigma_\infty^*(s) > 0 \ \&\ \sigma_\infty^*(s') < 1 \quad \implies \quad \exists N \in \mathbb{N} \text{ s.t. } \forall\ n_k \geq N \quad \sigma_{n_k}^*(s) > 0 \ \&\ \sigma_{n_k}^*(s') < 1$$

Now let $\bar{s}$ be the highest outcome for which $\sigma_\infty^*(\bar{s}) > 0$. I show that:

- If $\sigma_\infty^*(\bar{s}) \in (0,1)$, then:
$$\frac{\psi_\infty}{1-\psi_\infty} \times \frac{\bar{s}}{1-\bar{s}} = \frac{c}{1-c}$$

- If $\sigma_\infty^*(\bar{s}) = 1$, then:
$$\frac{\psi_\infty}{1-\psi_\infty} \times \frac{s}{1-s} \begin{cases} \leq \frac{c}{1-c} & s < \bar{s} \\ \geq \frac{c}{1-c} & s \geq \bar{s} \end{cases}$$

The first case easily follows by noting that:

$$\sigma^*_\infty(\bar{s}) \in (0,1) \implies \sigma^*_{n_k}(\bar{s}) \in (0,1) \implies \frac{\psi_{n_k}}{1-\psi_{n_k}} \times \frac{\bar{s}}{1-\bar{s}} = \frac{c}{1-c} \implies \frac{\psi_\infty}{1-\psi_\infty} \times \frac{\bar{s}}{1-\bar{s}} = \frac{c}{1-c}$$

for all $n_k \geq N' \in \mathbb{N}$. The second case follows similarly, by noting that $\sigma^*_\infty(\bar{s}) = 1$ and $\sigma^*_\infty(s') = 0$ for all $s' < \bar{s}$ implies $\sigma^*_{n_k}(\bar{s}) > 0$ and $\sigma^*_{n_k}(s') = 0$ for all $n_k \geq N'' \in \mathbb{N}$.

ii The set of equilibrium strategies is compact.

Σ is a subset of $[0,1]^m$ and therefore bounded, hence it suffices to show that is closed. Let $\{\sigma^*_n\}$ be a sequence of equilibrium strategies. Note that this means $\sigma^*_n \in \Phi(\sigma^*_n)$. Since $\Phi(.)$ is upper semicontinuous, $\sigma^*_n \to \sigma_\infty$ implies $\sigma_\infty \in \Phi(\sigma_\infty)$, and therefore an equilibrium strategy itself.

□

Proposition 3. Where σ^* and σ^{**} are two equilibrium strategies such that σ^{**} is more selective than σ^*, evaluators' expected payoffs under σ^{**} exceed those under σ^*; $\Pi(\sigma^{**};\mathcal{E}) \geq \Pi(\sigma^*;\mathcal{E})$.

Proof. This is an immediate corollary to Lemmas 1 and 5 below; both of independent interest.

□

Lemma 5. Take three monotone strategies σ'', σ' and, σ, ordered from the least selective to the most. If $\Pi(\sigma';\mathcal{E}) \leq \Pi(\sigma;\mathcal{E})$, then $\Pi(\sigma'';\mathcal{E}) \leq \Pi(\sigma';\mathcal{E})$.

Proof. For the three strategies σ'', σ', and σ, consider three sets $Z, Z', Z'' \subset (S \times [0,1])^n$ where the applicant's score profile $\mathbf{z}$ might lie:

$$\mathbf{z} \in \begin{cases} Z & \text{if } \mathbf{z} \text{ is eventually approved under } \sigma'' \text{ but not } \sigma \\ Z' & \text{if } \mathbf{z} \text{ is eventually approved under } \sigma' \text{ but not } \sigma \\ Z'' & \text{if } \mathbf{z} \text{ is eventually approved under } \sigma'' \text{ but not } \sigma' \end{cases}$$

Notice that $Z' \cap Z'' = \emptyset$ and $Z' \cup Z'' = Z$. We can write the difference between the sum of evaluators' payoffs under different strategies as:

$$\Pi(\sigma';\mathcal{E}) - \Pi(\sigma;\mathcal{E}) = \mathbb{P}\left(\mathbf{z} \in Z'\right) \times \left[\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z'\right) - c\right]$$

and:

$$\Pi(\sigma'';\mathcal{E}) - \Pi(\sigma';\mathcal{E}) = \mathbb{P}\left(\mathbf{z} \in Z''\right) \times \left[\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z''\right) - c\right]$$

Therefore we want to prove that:

$$\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z'\right) \leq c \implies \mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z''\right) \leq c$$

Now, note that $\mathbb{P}(\theta = H \mid \mathbf{z} \in Z)$ is a convex combination of $\mathbb{P}(\theta = H \mid \mathbf{z} \in Z')$ and $\mathbb{P}(\theta = H \mid \mathbf{z} \in Z'')$. Furthermore:

$$\mathbb{P}(\theta = H \mid \mathbf{z} \in Z) \geq \mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z \cap Z''\right) = \mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z''\right)$$

which then implies:

$$\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z''\right) \leq \mathbb{P}(\theta = H \mid \mathbf{z} \in Z) \leq \mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z'\right) \leq c$$

□

Lemma 1. Let σ^* and σ be two monotone strategies, where σ^* is more selective than σ. If σ^* is an equilibrium strategy, then $\Pi(\sigma^*; \mathcal{E}) \geq \Pi(\sigma; \mathcal{E})$.

Proof. Let $\mathbf{z}$ be the applicant's *score profile*. Take an equilibrium strategy σ^* and a less selective strategy σ such that:

$$\sigma(s) - \sigma^*(s) = \begin{cases} \varepsilon & s = \underline{s} \\ 0 & \text{otherwise} \end{cases}$$

for some $\varepsilon > 0$, where $\underline{s} := \min\{s \in S : \sigma^*(s) < 1\}$. I show that:

$$\lim_{\varepsilon \to 0} \Pi(\sigma; \varepsilon) - \Pi(\sigma^*; \varepsilon) \leq 0$$

By Lemma 5, this establishes the result.

Now, let $Z \subset (S \times [0,1])^n$ be the set of score profiles with which at least one evaluator approves the applicant with σ, but all reject him with σ^*:

$$\mathbf{z} \in Z \iff \begin{matrix} \sigma^*(s^i) > u^i & \text{for all } i \in \{1, 2, \ldots, n\}, \\ & \text{and} \\ \sigma(s^i) \leq u^i & \text{for some } i \in \{1, 2, \ldots, n\}. \end{matrix}$$

Furthermore, for a given score profile $\mathbf{z}$, let # be the number of evaluators whose observed scores are such that $\sigma(s^i) \geq u^i > \sigma^*(s^i)$. These evaluators would approve the applicant under the strategy σ, but not under σ^*.

An applicant's eventual outcome differs between the strategy profiles σ and σ^* if and only if his score profile $\mathbf{z}$ lies in Z. Furthermore, his eventual outcome can only change from a rejection by all evaluators in σ^* to an approval by some evaluator in σ. Thus:

$$\begin{aligned} \Pi(\sigma; \mathcal{E}) - \Pi(\sigma^*; \mathcal{E}) &= [\mathbb{P}(\theta = H \mid \mathbf{z} \in Z) - c] \times \mathbb{P}(\mathbf{z} \in Z) \\ &\propto \mathbb{P}(\theta = H \mid \mathbf{z} \in Z) - c \end{aligned}$$

Focus therefore, on the probability that $\theta = H$ given the applicant's signal profile lies in Z:

$$\mathbb{P}(\theta = H \mid \mathbf{z} \in Z) = \sum_{i=1}^{n} \mathbb{P}(\theta = H \mid \# = i) \times \frac{\mathbb{P}(\# = i)}{\mathbb{P}(\mathbf{z} \in Z)}$$

Now note:

$$\mathbb{P}(\# = i \mid \theta) = (p_\theta(\underline{s}))^i \times (1 - p_\theta(\underline{s}))^{n-i} \times \varepsilon^i$$

and thus $\mathbb{P}(\# = i) \propto \varepsilon^i$. Since $\mathbb{P}(\mathbf{z} \in A) = \sum_{i=1}^{n} \mathbb{P}(\# = i)$, we have $\lim_{\varepsilon \to 0} \frac{\mathbb{P}(\#=i)}{\mathbb{P}(\mathbf{z} \in A)} = 0$ for any $i > 1$. Thus:

$$\lim_{\varepsilon \to 0} \mathbb{P}(\theta = H \mid \mathbf{z} \in A) - \mathbb{P}(\theta = H \mid \# = 1) = 0$$

I conclude the proof by showing that $\mathbb{P}(\theta = H \mid \# = 1) \leq c$ as $\varepsilon \to 0$:

$$\begin{aligned}
\lim_{\varepsilon \to 0} \frac{\mathbb{P}(\theta = H \mid \# = 1)}{\mathbb{P}(\theta = L \mid \# = 1)} &= \lim_{\varepsilon \to 0} \frac{\mathbb{P}(\theta = H)}{\mathbb{P}(\theta = L)} \times \frac{\mathbb{P}(\# = 1 \mid \theta = H)}{\mathbb{P}(\# = 1 \mid \theta = L)} \\
&= \lim_{\varepsilon \to 0} \frac{\mathbb{P}(\theta = H)}{\mathbb{P}(\theta = L)} \times \left(\frac{r_H(\sigma; \mathcal{E})}{r_L(\sigma; \mathcal{E})}\right)^{n-1} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \\
&= \frac{\mathbb{P}(\theta = H)}{\mathbb{P}(\theta = L)} \times \left(\frac{r_H(\sigma^*; \mathcal{E})}{r_L(\sigma^*; \mathcal{E})}\right)^{n-1} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \\
&\leq \frac{\psi^*}{1 - \psi^*} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \leq \frac{c}{1 - c}
\end{aligned}$$

where $\psi^* = \Psi(\sigma^*; \mathcal{E})$ is the interim belief of the evaluators induced by σ^*. The penultimate inequality holds due to the straightforward fact that:

$$\frac{\psi^*}{1 - \psi^*} = \frac{\rho}{1 - \rho} \times \frac{1 + r_H^* + ... + (r_H^*)^{n-1}}{1 + r_L^* + ... + (r_L^*)^{n-1}} \leq \frac{\rho}{1 - \rho} \times \left(\frac{r_H^*}{r_L^*}\right)^{n-1}$$

where $r_\theta^* := r_\theta(\sigma^*; \mathcal{E})$. The last inequality is due to the fact that $\underline{s} \in S$ is optimally rejected under σ^*.

□

Theorem 1. Let evaluators' experiment be binary, with outcomes inducing the normalised beliefs $s_L \in [0, 0.5]$ and $s_H \in [0.5, 1]$. Increasing s_H weakly increases evaluators' payoffs across the extreme equilibria. In contrast, as s_L decreases, evaluators' payoffs in the most (least) selective equilibrium:

1. weakly improve as long as s_L remains below a cutoff $\hat{s_L}$ ($\check{s_L}$),

2. weakly decrease once s_L falls below this cutoff.

I will use Lemmas 6, 7, and 8 below, possibly of independent interest, to prove Theorem 1. Throughout, I denote the most and least selective equilibrium strategies under the experiment $\mathcal{E}$ as $\hat{\sigma}_{\mathcal{E}}^*$ and $\check{\sigma}_{\mathcal{E}}^*$, respectively. I drop the subscript whenever the experiment in question is obvious.

Lemma 6. Let $\mathcal{E}$ be a binary experiment, with outcomes in $\mathbf{S} = \{s_L, s_H\}$, labelled after the respective normalised beliefs they induce. $\Psi(\sigma; \mathcal{E})$ is:

i strictly increasing in $\sigma(s_L)$, whenever $\sigma(s_H) = 1$,

ii strictly decreasing in $\sigma(s_H)$ whenever $\sigma(s_L) = 0$.

Proof. Part i:

Let $\sigma(s_L) \in (0, 1)$ and $\sigma(s_H) = 1$. The interim belief $\Psi(\sigma; \mathcal{E})$ is then given by:

$$\begin{aligned}\Psi(\sigma; \mathcal{E}) &= \mathbb{P}\left(\theta = H \mid \text{visit received}\right) \\ &= \sum_{i=0}^{n-1} \mathbb{P}(\text{visited after i}^{\text{th}} \text{ rejection } \mid \text{visit received}) \times \mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right] \\ &= \sum_{i=0}^{n-1} \frac{\mathbb{P}(\text{visited after i}^{\text{th}} \text{ rejection })}{\mathbb{P}(\text{visit received})} \times \mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right]\end{aligned}$$

Note that $\mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right] < \mathbb{E}\left[\theta = H \mid \text{i+1 } s_L \text{ signals}\right]$; since every s_L signal is further evidence for $\theta = L$. We have:

$$\begin{aligned}\mathbb{P}(\text{ visited after i}^{\text{th}} \text{ rejection }) =& \mathbb{P}\left(\text{ev. was (i+1)}^{\text{st}} \text{ in order } \mid \text{applicant got i rejections}\right) \\ & \times \mathbb{P}\left(\text{applicant got i rejections}\right) \\ =& \frac{1}{n} \times \mathbb{P}(\text{i } s_L \text{ signals}) \times \left[1 - \sigma(s_L)\right]^i\end{aligned}$$

The proof is completed by noting that:

$$\frac{\mathbb{P}(\text{ visited after (i+1)}^{\text{st}} \text{ rejection })}{\mathbb{P}(\text{ visited after i}^{\text{th}} \text{ rejection })} = \frac{\mathbb{P}(\text{i+1 } s_L \text{ signals})}{\mathbb{P}(\text{i } s_L \text{ signals})} \times \left[1 - \sigma(s_L)\right]$$

decreases, and thus $\Psi(\sigma; \mathcal{E})$ increases, in $\sigma(s_L)$.

Part ii:

Now take $\sigma(s_L) = 0$. We then have:

$$r_H(\sigma; \mathcal{E}) = 1 - p_H(s_H)\sigma(s_H) \qquad\qquad r_L(\sigma; \mathcal{E}) = 1 - p_L(s_H)\sigma(s_H)$$

and:

$$\begin{aligned}\Psi(\sigma; \mathcal{E}) &\propto \frac{1 + r_H + ... + r_H^{n-1}}{1 + r_L + ... + r_L^{n-1}} \\ &= \frac{1 - r_H^n}{1 - r_L^n} \times \frac{1 - r_L}{1 - r_H} = \frac{1 - r_H^n}{1 - r_L^n} \times \frac{p_L(s_H)}{p_H(s_H)} \\ &\propto \frac{1 - r_H^n}{1 - r_L^n} = \frac{1 - \left(1 - p_H(s_H)\sigma(s_H)\right)^n}{1 - \left(1 - p_L(s_H)\sigma(s_H)\right)^n}\end{aligned}$$

Differentiating the last expression with respect to $\sigma(s_H)$ and rearranging its terms reveals that this derivative is proportional to:

$$\frac{s_H}{1-s_H} \times \left(\frac{r_H}{r_L}\right)^{n-1} - \frac{1-(r_H)^n}{1-(r_L)^n}$$

The positive term is the likelihood ratio of one s_H signal and $n-1$ rejections, and the negative term is the likelihood ratio from *at most* $n-1$ rejections. Since approvals only happen with s_H signals, the negative term strictly exceeds the positive term. This can be verified directly, too:

$$\begin{aligned}\frac{1-(r_H)^n}{1-(r_L)^n} > \frac{s_H}{1-s_H} \times \left(\frac{r_H}{r_L}\right)^{n-1} &\iff \frac{1-(r_H)^n}{1-(r_L)^n} \times \frac{1-r_L}{1-r_H} > \left(\frac{r_H}{r_L}\right)^{n-1} \\ &\iff \frac{1+...+(r_H)^{n-1}}{1+...+(r_L)^{n-1}} > \left(\frac{r_H}{r_L}\right)^{n-1}\end{aligned}$$

The last inequality can be verified easily. Thus, $\Psi(\sigma;\mathcal{E})$ decreases in $\sigma(s_H)$.

□

The Corollary below follows from Lemma 6. Let both $\mathcal{E}'$ and $\mathcal{E}$ are binary experiments, where the former is Blackwell more informative than the latter. If evaluators approve upon the high outcome and reject upon the low in both experiments, the interim belief under $\mathcal{E}'$ is lower.

Corollary 8. Let $\mathcal{E}'$ and $\mathcal{E}$ be two binary experiments, where the former is Blackwell more informative than the latter. Let the strategies σ' and σ for these respective experiments be defined as:

$$\sigma'(s') := \begin{cases} 0 & s' = s'_L \\ 1 & s' = s'_H \end{cases} \qquad\qquad \sigma(s) := \begin{cases} 0 & s = s_L \\ 1 & s = s_H \end{cases}$$

Then, $\Psi(\sigma';\mathcal{E}') \leq \Psi(\sigma;\mathcal{E})$.

Proof. Establishing that this holds for a pair $(\mathcal{E}',\mathcal{E})$ for which either (i) $s'_H > s_H$ and $s_L = s'_L$, or (ii) $s'_H = s_H$ and $s_L > s'_L$ suffices. I will only prove the first case, the second is analogous. Below I show that the outcome induced by σ under experiment $\mathcal{E}$ can be replicated by some strategy $\tilde{\sigma}$ under experiment $\mathcal{E}'$, where $\tilde{\sigma}(s_L) > 0$ and $\tilde{\sigma}(s_H) = 1$. Then, the desired conclusion follows from Lemma 6.

Take the pair $(\sigma,\mathcal{E})$. The probabilities that the applicant is rejected or approved upon a visit, conditional on θ, is given by:

$$\frac{\mathbb{P}(\sigma \text{ rejects } \mid \theta = H)}{\mathbb{P}(\sigma \text{ rejects } \mid \theta = L)} = \frac{s_L}{1-s_L} \qquad\qquad \frac{\mathbb{P}(\sigma \text{ approves } \mid \theta = H)}{\mathbb{P}(\sigma \text{ approves } \mid \theta = L)} = \frac{s_H}{1-s_H}$$

For the pair $(\tilde{\sigma}, \mathcal{E}')$ where $\tilde{\sigma}(s'_H) = 1$, we have:

$$\frac{\mathbb{P}(\tilde{\sigma} \text{ rejects} \mid \theta = H)}{\mathbb{P}(\tilde{\sigma} \text{ rejects} \mid \theta = L)} = \frac{s_L}{1 - s_L} \qquad \frac{\mathbb{P}(\tilde{\sigma} \text{ approves} \mid \theta = H)}{\mathbb{P}(\tilde{\sigma} \text{ approves} \mid \theta = L)} = \frac{p'_H(s_H) + \tilde{\sigma}(s_L)p'_H(s_L)}{p'_L(s_H) + \tilde{\sigma}(s_L)p'_L(s_L)}$$

where $\{p'_L, p'_H\}$ are the distributions pertaining to $\mathcal{E}'$. It is easy to verify that the expression on the right falls from $\frac{s'_H}{1-s'_H}$ to 1 monotonically and continuously as $\tilde{\sigma}(s_L)$ rises from 0 to 1. Thus, there is a unique interior value of $\tilde{\sigma}(s_L)$ that replicates the outcome of $(\sigma; \mathcal{E})$.

□

Lemma 7. Let $\mathcal{E}$ be a binary experiment. The equilibrium approval probability upon the low outcome s_L is either 0 or 1, both in the most and the least selective equilibrium.

Proof. I start by proving this for the least selective equilibrium; i.e. $\check{\sigma}^*(s_L) \in \{0, 1\}$. For s_L^{mute} defined in Definition 3, observe that when $s_L \geq s_L^{\text{mute}}$, $\sigma(s_L) = \sigma(s_H) = 1$ is an equilibrium; so we must have $\check{\sigma}^*(s_L) = 1$. The strategy σ defined by $\sigma(s_H) = \sigma(s_L) = 1$ gives rise to the interim belief $\Psi(\sigma; \mathcal{E}) = \rho$, which in turn renders approving upon the outcome s_L optimal. In turn, if $s_L < s_L^{\text{mute}}$, we must have $\sigma^*(s_L) = 1$ for any equilibrium strategy; since evaluators' interim beliefs always fall below their prior (Proposition 1).

Now consider the most selective equilibrium strategy; $\hat{\sigma}^*$. For contradiction, let $1 > \hat{\sigma}^*(s_L) > 0$ and $\hat{\sigma}^*(s_H) = 1$. Lemma 6 establishes that the interim belief falls as $\sigma(s_L)$ falls; which implies there must be another, more selective equilibrium strategy σ^* such that $\sigma^*(s_L) = 0$ and $\sigma^*(s_H) = 1$.

□

Lemma 7 establishes that when their experiment $\mathcal{E}$ is binary, evaluators *never* mix upon seeing the low outcome $s = s_L$ in extreme equilibria. Following up, Lemma 8 establishes that a more informative binary experiment pushes evaluators to reject upon the low outcome in extreme equilibria.

Lemma 8. Let $\mathcal{E}$ be a binary experiment, with outcomes in $\mathbf{S} = \{s_L, s_H\}$, labelled after the respective normalised beliefs they induce. Evaluators' approval probabilities upon the low outcome $s = s_L$ are given by:

$$\check{\sigma}^*(s_L) = \begin{cases} 1 & s_L \geq s_L^{\text{mute}} \\ 0 & s_L < s_L^{\text{mute}} \end{cases} \qquad \hat{\sigma}^*(s_L) = \begin{cases} 1 & s_L < s_L^{\dagger}(s_H) \\ 0 & s_L \geq s_L^{\dagger}(s_H) \end{cases}$$

where $s_L^{\dagger}(.)$ is an increasing function of s_H, and $s_L^{\dagger}(s_H) \geq s_L^{\text{safe}}$.

Proof. Note that there exists an equilibrium where $\sigma(s_L) = 1$ if and only if:

$$\frac{\rho}{1-\rho} \times \frac{s_L}{1 - s_L} \geq \frac{c}{1-c}$$

which, combined with Lemma 7, proves the part of the Lemma for the selective equilibrium.

Now, define the strategies σ_0 as σ_1 as:

$$\sigma_0(s) = \begin{cases} 0 & s = s_L \\ 0 & s = s_H \end{cases} \qquad \sigma_1(s) = \begin{cases} 0 & s = s_L \\ 1 & s = s_H \end{cases}$$

A necessary and sufficient condition for an equilibrium σ^* where $\sigma^*(s_L) = 0$ to exist is:

$$\frac{\Psi(\sigma_1; \mathcal{E})}{1 - \Psi(\sigma_1; \mathcal{E})} \times \frac{s_L}{1 - s_L} \leq \frac{c}{1 - c}$$

Sufficiency follows since either:

$$\frac{\Psi(\sigma_0; \mathcal{E})}{1 - \Psi(\sigma_0; \mathcal{E})} \times \frac{s_H}{1 - s_H} \leq \frac{c}{1 - c}$$

which implies σ_0 is an equilibrium, or there is an equilibrium strategy σ^* such that $\sigma^*(s_L) = 0$ and $\sigma^*(s_H) > 0$ by Lemma 6. The condition is necessary, since any strategy that is less selective than σ_1 induces a higher interim belief, by Lemma 6.

By Corollary 8, whenever this necessary and sufficient condition holds for an experiment $\mathcal{E}$, it also holds for a (Blackwell) more informative experiment $\mathcal{E}'$. Moreover, whenever the low signals are rejected in the least selective equilibrium, they must be in the most selective equilibrium. This concludes the proof.

□

Proof, Theorem 1: By Lemma 8, Blackwell improving evaluators' experiment shifts both their least selective and most selective equilibrium strategies once from approving *every* applicant to rejecting upon the low signal. By Lemma 5, this shift in evaluators' strategy increases evaluators' payoffs.

Let $\{\sigma_\alpha\}_{\alpha \in [0,1]}$ be the family of strategies where evaluators reject upon the low signal:

$$\sigma_\alpha(s) = \begin{cases} 0 & s = s_L \\ 1 & s = s_H \end{cases}$$

By Lemma 6, the interim belief ψ_α that the strategy σ_α induces is strictly decreasing in α. Thus, at most one of these can be an equilibrium strategy for a given experiment. Furthermore, whenever evaluators' expected payoffs from σ_1 is weakly positive, this must be the equilibrium strategy; decreasing α can only make approving upon the high signal *more* profitable. Hence, whenever evaluators reject upon the low signal in equilibrium, their payoffs are given by: $\Pi(\sigma^*; \mathcal{E}) = \max\{0, \Pi(\sigma_1; \mathcal{E})\}$. The Theorem then follows from the Claim below:

Claim. $\max\{0, \Pi(\sigma_1; \mathcal{E})\}$ is:

i weakly increasing in s_H whenever there is some equilibrium strategy σ^* s.t. $\sigma^*(s_L) = 0$.

ii hump-shaped in s_L. As s_L falls, it:

- weakly increases when $s_L \geq s_L^{as}$,
- weakly decreases when $s_L \leq s_L^{as}$

where s_L^{as} is defined in Definition 2.

Proof of the Claim. **Part i.** Increasing the strength of good news; i.e. s_H.
Let $\mathcal{E}$ and $\mathcal{E}'$ be two binary experiments with outcome sets $\mathbf{S} = \{s_L, s_H\}$ and $\mathbf{S}' = \{s'_L, s'_H\}$; each labelled after the normalised beliefs they induce. The experiment $\mathcal{E}'$ carries *marginally stronger good news* than experiment $\mathcal{E}$:

$$s'_L = s_L \qquad\qquad s'_H = s_H + \delta$$

for some small δ such that $1 - s_H \geq \delta > 0$. I show that $\Pi(\sigma'_1; \mathcal{E}') > \Pi(\sigma_1; \mathcal{E})$; where σ'_1 is defined analogously to σ_1 for experiment $\mathcal{E}'$.

Step 1. Replicating $\mathcal{E}'$ with a signal pair $(s, \hat{s})$.

Rather than observing the outcome of experiment $\mathcal{E}'$, say an evaluator initially observes her original signal s, and then potentially an additional auxilliary signal $\hat{s}$. The first signal she receives, s, records the outcome of $\mathcal{E}$. If the low outcome s_L materialises, the evaluator observes no more information. If, however, the high outcome s_H materialises, she then observes the additional auxiliary signal $\hat{s}$. This auxiliary signal records the outcome of *another* binary experiment, $\hat{\mathcal{E}}$. The outcome of $\hat{\mathcal{E}}$ is independent both from s and anything else any other evaluator observes. Conditional on the applicant's quality θ, the distribution over its outcomes is given by the pmf $p_\theta(.)$:

$$\hat{p}_H(\hat{s_H}) = 1 - \varepsilon \times \frac{s_L}{1 - s_L} \qquad\qquad \hat{p}_L(\hat{s_H}) = 1 - \varepsilon \times \frac{s_H}{1 - s_H}$$

The evolution of the evaluator's beliefs when she observes this signal pair is determined by the two likelihood ratios:

$$\frac{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_H}) \mid \theta = H)}{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_H}) \mid \theta = L)} = \frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}}{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}} \tag{6.3}$$

$$\frac{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_L}) \mid \theta = H)}{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_L}) \mid \theta = L)} = \frac{s_L}{1 - s_L} \tag{6.4}$$

Note that the likelihood ratio 6.3 increases continuously with ε.

The information from observing the pair $(s, \hat{s})$ as such is equivalent to observing the outcome of experiment $\mathcal{E}'$, when:

$$\frac{s_H}{1-s_H} \times \frac{1-\varepsilon \times \dfrac{s_L}{1-s_L}}{1-\varepsilon \times \dfrac{s_H}{1-s_H}} = \frac{s_H+\delta}{1-(s_H+\delta)} \tag{6.5}$$

for our chosen (δ, ε). I choose ε to satisfy this equality for our δ. As such, ε becomes a continuously increasing function of δ. Furthermore, note that by varying ε between 0 and $\frac{1-s_H}{s_H}$, we can replicate *any* experiment $\mathcal{E}'$ with $s'_L = s_L$ and $1 \geq s'_H \geq s_H$.

Step 2. $\pi(\sigma'_1; \mathcal{E}') \geq \pi(\sigma_1; \mathcal{E})$.

The evaluator who observes the signal pair $(s, \hat{s})$ obtains equivalent information to that from $\mathcal{E}'$. We now must identify the strategy $\tilde{\sigma} : \{s_L, (s_H, \hat{s_H}), (s_H, \hat{s_L})\} \to [0,1]$ for this signal pair that replicates the outcome of the strategy σ'_1 for experiment $\mathcal{E}'$. This strategy is defined as:

$$\tilde{\sigma}(s_H, \hat{s_H}) = 1 \qquad\qquad \tilde{\sigma}(s_L) = \tilde{\sigma}(s_H, \hat{s_L}) = 0$$

and replicates the likelihood ratios of an approval and rejection signal under $\mathcal{E}'$.

Now, fix the applicant's *signal profile* (defined in Section 6.1): the collection of signal draws each evaluator will observe if he visits them all: $\mathbf{s} = \{(s^i, \hat{s}^i)\}_{i=1}^n$. I call an applicant a *marginal admit* if his score profile is such that:

i for at least one $i \in \{1, 2, ..., n\}$, $s^i = s_H$, and

ii for *every* $i \in \{1, 2, ..., n\}$, either $s^i = s_L$, or $\hat{s}^i = \hat{s}_L$.

These marginal admits drive the wedge between evaluators' payoffs under $\mathcal{E}'$ and $\mathcal{E}$: while one of their evaluators approves them under $\mathcal{E}$, they *all* reject him under $\hat{\mathcal{E}}$. So:

$$\Pi(\sigma'_1; \mathcal{E}') - \Pi(\sigma_1; \mathcal{E}) = \mathbb{P}\,(\text{marginal admit}) \times \underbrace{[c - \mathbb{P}\,(\theta = H \mid \text{marginal admit})]}_{(1)}$$

A marginal admit only has signal realisations $(s, \hat{s}) = (s_H, \hat{s_L})$ or $s = s_L$. These carry equivalent information about θ. Thus, the expression (1) above equals:

$$c - \mathbb{P}\left[\theta = H \mid s^1 = ... = s^n = s_L\right]$$

In the relevant region where there is an equilibrium strategy that leads to rejections after the low outcome s_L, the expression above must be weakly positive:

$$c - \mathbb{P}\left[\theta = H \mid s^1 = ... = s^n = s_L\right] \propto \frac{c}{1-c} \times \frac{\rho}{1-\rho} \times \left(\frac{s_L}{1-s_L}\right)^n$$
$$\leq \frac{c}{1-c} - \frac{\rho}{1-\rho} \times \frac{\sum_{k=0}^{n-1} p_H(s_L)^k}{\sum_{k=0}^{n-1} p_L(s_L)^k} \times \frac{s_L}{1-s_L}$$
$$= \frac{c}{1-c} - \frac{\Psi(\sigma_1;\mathcal{E})}{1-\Psi(\sigma_1;\mathcal{E})} \times \frac{s_L}{1-s_L} \leq 0$$

where the last inequality follows from the necessary and sufficient condition the proof of Lemma 8 introduced for such an equilibrium to exist.

Part ii. Increasing the strength of bad news; i.e. decreasing s_L.

Now, let the experiment $\mathcal{E}'$ carry *marginally stronger bad news* than experiment $\mathcal{E}$ instead; for some arbitrarily small $\delta \in [0, s_L]$:

$$s'_L = s_L - \delta \qquad\qquad s'_H = s_H$$

Where σ'_1 and σ_1 are defined as before, I show that:

i $\Pi(\sigma'_1;\mathcal{E}) - \Pi(\sigma_1;\mathcal{E}) \geq 0$ when $s_L \geq s_L^{as}$, and

ii $\Pi(\sigma'_1;\mathcal{E}) - \Pi(\sigma_1;\mathcal{E}) \leq 0$ when $s_L \leq s_L^{as}$

Step 1. Replicating $\mathcal{E}'$ with a signal pair $(s, \hat{s})$.

As before, let the evaluator potentially observe *two* signals, s and $\hat{s}$. She first observes s, which records the outcome of $\mathcal{E}$. If the high outcome s_H materialises, she receives no further information. If, however, the low outcome s_L materialises, she then observes the additional auxiliary signal $\hat{s}$, which records the outcome of *another* binary experiment, $\hat{\mathcal{E}}$. As before, the outcome of this experiment is independent both from s and anything else any other evaluator observes. Its distribution conditional on the applicant's quality θ is given by the pmf $p_\theta(.)$:

$$\hat{p}_H\left(\hat{s}_H\right) = \varepsilon \times \frac{s_H}{1-s_H} \qquad\qquad \hat{p}_L\left(\hat{s}_H\right) = \varepsilon \times \frac{s_L}{1-s_L}$$

The evolution of the evaluator's beliefs upon seeing the signal pair $(s, \hat{s})$ is then determined by the two likelihood ratios:

$$\frac{\mathbb{P}\left((s,\hat{s})=(s_L,\hat{s_H})\mid\theta=H\right)}{\mathbb{P}\left((s,\hat{s})=(s_L,\hat{s_H})\mid\theta=H\right)}=\frac{s_H}{1-s_H} \tag{6.6}$$

$$\frac{\mathbb{P}\left((s,\hat{s})=(s_L,\hat{s_L})\mid\theta=H\right)}{\mathbb{P}\left((s,\hat{s})=(s_L,\hat{s_L})\mid\theta=H\right)}=\frac{s_L}{1-s_L}\times\frac{1-\varepsilon\times\dfrac{s_H}{1-s_H}}{1-\varepsilon\times\dfrac{s_L}{1-s_L}} \tag{6.7}$$

Note that 6.7 is continuously and strictly decreasing with ε, taking values between $\frac{s_L}{1-s_L}$ and 0 as ε varies between 0 and $\frac{s_H}{1-s_H}$. The signal pair $(s,\hat{s})$ is informationally equivalent to $\mathcal{E}'$ when:

$$\frac{s_L}{1-s_L}\times\frac{1-\varepsilon\times\dfrac{s_H}{1-s_H}}{1-\varepsilon\times\dfrac{s_L}{1-s_L}}=\frac{s_L-\delta}{1-(s_L-\delta)}$$

I choose ε to satisfy this equality. As before, ε then becomes a continuously increasing function of δ.

Step 2. $\Pi(\sigma_1';\mathcal{E}')-\Pi(\sigma_1;\mathcal{E})\begin{cases}\geq 0 & s_L\geq s_L^{\text{as}}\\ \leq 0 & s_L\leq s_L^{\text{as}}\end{cases}$

The evaluator who observes the signal pair $(s,\hat{s})$ obtains equivalent information to that from $\mathcal{E}'$. We now must identify the strategy $\tilde{\sigma}:\{(s_L,\hat{s_H}),(s_L,\hat{s_L}),s_H\}\to[0,1]$ for this signal pair that replicates the outcome of the strategy σ_1' for experiment $\mathcal{E}'$. This strategy is defined as:

$$\tilde{\sigma}(s_L,\hat{s_H})=\tilde{\sigma}(s_H)=1 \qquad\qquad \tilde{\sigma}(s_L,\hat{s_L})=0$$

and replicates the likelihood ratios of an approval and rejection signal under $\mathcal{E}'$.

Now, fix the applicant's *score profile*; the collection of signal draws evaluators will see for him were he to visit them all: $\mathbf{s}=\{(s^i,\hat{s}^i)\}_{i=1}^n$. I call an applicant a *marginal reject* if:

i for every $i\in\{1,2,...,n\}$, $s^i=s_L$, and

ii for at least one $i\in\{1,2,...,n\}$, $\hat{s}^i=\hat{s}_H$.

These marginal rejects drive the wedge between evaluators' payoffs under $\mathcal{E}'$ and $\mathcal{E}$: while *no* evaluator approves them under $\mathcal{E}$, *at least one* evaluator does under $\mathcal{E}'$. So:

$$\Pi(\sigma_1';\mathcal{E}')-\Pi(\sigma_1;\mathcal{E})=\mathbb{P}\left(\text{marginal reject}\right)\times\underbrace{\left[\mathbb{P}\left(\theta=H\mid\text{marginal reject}\right)-c\right]}_{(2)}$$

The evaluator of a marginal reject has either observed $(s^i,\hat{s}^i)=(s_L,\hat{s}_L)$, or $(s^i,\hat{s}^i)=(s_L,\hat{s}_H)$. Denote the latter number of evaluators as $\#$. Since the applicant is a marginal reject, $\#\geq 1$.

Then, (2) equals:

$$\sum_{i=1}^{n} \underbrace{\frac{\mathbb{P}\left(i\ \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)}{\sum_{j=1}^{n} \mathbb{P}\left(j\ \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)}}_{(3)} \times \mathbb{P}\left(\theta = H \mid \# = i\right) - c$$

where:

$$\mathbb{P}\left(i\ \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right) = k \times \binom{n}{i} \times \left(\frac{s_H}{1-s_H} \times \varepsilon\right)^i \times \left(1 - \frac{s_H}{1-s_H} \times \varepsilon\right)^{n-i}$$
$$+ (1-k) \times \binom{n}{i} \times \left(\frac{s_L}{1-s_L} \times \varepsilon\right)^i \times \left(1 - \frac{s_L}{1-s_L} \times \varepsilon\right)^{n-i}$$

and $k = \mathbb{P}\left(\theta = H \mid s^1 = ... = s^n = s_L\right)$. Thus, the limit of expression (3) as $\varepsilon \to 0$ (and therefore, $\delta \to 0$) for any $i > 1$ is:

$$\lim_{\varepsilon \to 0} \frac{\frac{1}{\varepsilon} \times \mathbb{P}\left(i\ \hat{s} = \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)}{\frac{1}{\varepsilon} \times \sum_{j=1}^{n} \mathbb{P}\left(j\ \hat{s} = \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)} = 0 \tag{6.8}$$

Therefore, we get:

$$\lim_{\varepsilon \to 0} \sum_{i=1}^{n} \frac{\mathbb{P}\left(i\ \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)}{\sum_{j=1}^{n} \mathbb{P}\left(j\ \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)} \times \mathbb{P}\left(\theta = H \mid \# = i\right) - c$$
$$= \mathbb{P}\left(\theta = H \mid \# = 1\right) - c$$
$$\propto \frac{\rho}{1-\rho} \times \left(\frac{s_L}{1-s_L}\right)^{n-1} \times \frac{s_H}{1-s_H} - \frac{c}{1-c}$$

proving the claim. □

□

Proposition 4. Suppose the experiment $\mathcal{E}$ is binary with outcomes inducing the normalised beliefs $s_L \in [0, 0.5]$ and $s_H \in [0.5, 1]$. Stronger bad news decrease evaluators' payoffs when:

i s_L is below the cutoff $\check{s}_L := \min\{s_L^{\text{mute}}, s_L^{as}\}$ in the least selective equilibrium.

ii s_L is below a cutoff $\hat{s_L} \geq \check{s_L}$ in the most selective equilibrium.

Proof. i The least selective equilibrium:

By Lemma 7, the probability that evaluators' approve upon the low outcome in the least selective equilibrium is:

$$\check{\sigma}^*(s_L) = \begin{cases} 1 & s_L \geq s_L^{\text{mute}} \\ 0 & s_L < s_L^{\text{mute}} \end{cases}$$

Thus, the sum of their payoffs equals (i) the expected payoff from always approving the applicant when $s_L \geq s_L^{\text{mute}}$, and (ii) $\max\{0, \Pi(\sigma_1; \mathcal{E})\}$ when $s_L < s_L^{\text{mute}}$ (established in the proof of Theorem 1):

$$\Pi(\check{\sigma}^*; \mathcal{E}) = \begin{cases} \rho - c & s_L \geq s_L^{\text{mute}} \\ \max\{0, \Pi(\sigma_1; \mathcal{E})\} & s_L < s_L^{\text{mute}} \end{cases}$$

Since approving all applicants is always feasible, we have $\max\{0, \Pi(\sigma_1; \mathcal{E})\} \geq \rho - c$ when $s_L < s_L^{\text{mute}}$ by Lemma 5. Furthermore, the final Claim in Theorem 1's proof establishes that as s_L falls, the expression $\max\{0, \Pi(\sigma_1; \mathcal{E})\}$ weakly increases (decreases) when $s_L \geq s_L^{\text{as}}$ ($s_L \leq s_L^{\text{as}}$). Thus the desired conclusion is established.

ii The most selective equilibrium:

By Lemma 8, the most selective equilibrium shifts from one where every applicant is approved to one where evaluators reject upon the low signal when $s_H \geq s_H^{\dagger}(s_L)$, where $s_H^{\dagger}(.)$ is an increasing function of s_L. Following the arguments made for the least selective equilibrium then, evaluators' most selective equilibrium payoffs:

- weakly increase as s_L decreases, when $s_L \geq \min\left\{s_L^{\text{as}}, s_L^{\dagger}(s_H)\right\}$
- weakly decrease as s_L decreases, when $s_L \leq \min\left\{s_L^{\text{as}}, s_L^{\dagger}(s_H)\right\}$.

The desired result follows by noting that $s_L^{\dagger}(s_H) \geq s_L^{\text{safe}}$, and therefore $\min\left\{s_L^{\dagger}, s_L^{\text{as}}(s_H)\right\} \geq \min\left\{s_L^{\text{safe}}, s_L^{\text{as}}(s_H)\right\}$.

□

Theorem 2. Let the experiment $\mathcal{E}'$ differs from $\mathcal{E}$ by a local mean preserving spread at s_j. Evaluators' payoffs under the most (least) selective equilibrium of $\mathcal{E}'$:

1. *weakly exceed* those under $\mathcal{E}$ if s_j leads to approvals under $\mathcal{E}$; i.e. $\hat{\sigma}_{\mathcal{E}}(s_j) = 0$ ($\check{\sigma}_{\mathcal{E}}(s_j) = 0$)
2. *fall weakly below* those under $\mathcal{E}$ if:

   i if s_j leads to rejections under $\mathcal{E}$; i.e. $\hat{\sigma}_{\mathcal{E}}(s_j) = 1$ ($\check{\sigma}_{\mathcal{E}}(s_j) = 1$), and

   ii the following condition holds:

$$\frac{\rho}{1-\rho} \times \left(\frac{r_H^*}{r_L^*}\right)^{n-1} \times \frac{s_{j+1}}{1 - s_{j+1}} \leq \frac{c}{1-c}$$

   where $r_\theta^* := r_\theta(\hat{\sigma}_\varepsilon; \mathcal{E})$ ($r_\theta^* := r_\theta(\check{\sigma}_\varepsilon; \mathcal{E})$).

Proof. The Theorem focuses either on the least, or the most selective equilibrium strategies under both experiments. In the discussion below, I let σ^* and $\sigma^{*'}$ denote whichever equilibria we are focusing on under the respective experiments $\mathcal{E}$ and $\mathcal{E}'$. When I need to distinguish between the least and most selective equilibria, I denote them as $(\check{\sigma}, \check{\sigma}')$ and $(\hat{\sigma}, \hat{\sigma}')$, respectively. Following the notation introduced in Definition 4, let $\mathbf{S} \cup \mathbf{S}' = \{s_1, s_2, ..., s_M\}$ be the joint support of the experiments $\mathcal{E}$ and $\mathcal{E}'$, with elements increasing in their indices as usual. Since $\mathcal{E}'$ is obtained by a *local* mean preserving spread of $\mathcal{E}$, there is a monotone strategy $\sigma' : \mathbf{S}' \to [0,1]$ whose outcome under $\mathcal{E}'$ replicates the outcome of σ^* under $\mathcal{E}$:

$$\sigma'(s) = \begin{cases} \sigma^*(s_j) & s \in \{s_{j-1}, s_{j+1}\} \\ \sigma^*(s) & s \notin \{s_{j-1}, s_{j+1}\} \end{cases}$$

Claim 1. Evaluators' payoffs under the most (least) selective equilibrium of $\mathcal{E}'$ weakly exceed those under $\mathcal{E}$ when $\hat{\sigma}(s_j) = 1$ ($\check{\sigma}(s_j) = 1$).

Now suppose s_j leads to approvals under σ^*; $\sigma^*(s_j) = 1$. Therefore, $\sigma'(s_{j-1}) = \sigma'(s_{j+1}) = 1$. Below, I show that $\sigma^{*'}$ is *more selective than* σ'. By Lemma 1, it follows that $\Pi\left(\sigma^{*'}; \mathcal{E}'\right) \geq \Pi\left(\sigma'; \mathcal{E}'\right) = \Pi\left(\sigma; \mathcal{E}\right)$.

If $s_{j-1} = \min \mathbf{S} \cup \mathbf{S}'$ or $\sigma^{*'}(s_{j-2}) = 0$, $\sigma^{*'}$ must necessarily be more selective than σ'; and we are done. So, for contradiction, I assume the following:

- $s_{j-1} > \min \mathbf{S} \cup \mathbf{S}'$
- $\sigma^{*'}(s_{j-2}) > 0$
- $\sigma^{*'}$ is *less* selective than σ', where the two strategies differ.

Case i. σ^* and $\sigma^{*'}$ are the least selective equilibrium strategies; i.e. $\sigma^* = \check{\sigma}$ and $\sigma^{*'} = \check{\sigma}'$.

I will prove the contradiction by constructing a strategy $\tilde{\sigma} : \mathbf{S} \to [0,1]$ for experiment $\mathcal{E}$, such that:

i $\tilde{\sigma}$ replicates the outcome $\check{\sigma}'$ induces in $\mathcal{E}'$,

ii That $\check{\sigma}'$ is an eqm. strategy under $\mathcal{E}'$ implies that $\tilde{\sigma}$ is an eqm. strategy under $\mathcal{E}$,

iii But $\tilde{\sigma}$ is less selective than $\check{\sigma}$, contradicting that $\check{\sigma}$ is the least selective equilibrium strategy under $\mathcal{E}$.

I define the strategy $\tilde{\sigma} : \mathbf{S} \to [0,1]$ for $\mathcal{E}$ as:

$$\tilde{\sigma}(s) := \begin{cases} 1 & s = s_i \\ \sigma'(s) & s \neq s_i \end{cases}$$

it is seen easily that $\tilde{\sigma}$ replicates the outcome of $\check{\sigma}'$. Furthermore, $\check{\sigma}'$ is an equilibrium under $\mathcal{E}'$ if and only if $\tilde{\sigma}$ is an equilibrium under $\mathcal{E}$: they induce the same interim belief ψ, and share the following necessary and sufficient condition for optimality:

$$\mathbb{P}_{\psi}\left(\theta = H \mid s_{j-2}\right) \begin{cases} = c & \sigma'(s_{j-2}) < 1 \\ \geq c & \sigma'(s_{j-2}) = 1 \end{cases}$$

The strategy $\tilde{\sigma}$ under experiment $\mathcal{E}$ replicates the outcome of $\check{\sigma}'$ under experiment $\mathcal{E}'$, and σ' under $\mathcal{E}'$ replicates the outcome of $\check{\sigma}$ under experiment $\mathcal{E}$. Since we assumed that $\check{\sigma}'$ is less selective than σ', it must be that $\tilde{\sigma}$ is less selective than $\check{\sigma}$.

Case ii. σ^* and $\sigma^{*'}$ are the most selective equilibrium strategies; i.e. $\sigma^* = \hat{\sigma}$ and $\sigma^{*'} = \hat{\sigma}'$.

Since strategy σ' for experiment $\mathcal{E}'$ replicates the outcome of $\hat{\sigma}$ for experiment $\mathcal{E}$, the two strategies induce the same interim belief ψ. Therefore, if $\mathbb{P}_{\psi}\left(\theta = H \mid s_{j-1}\right) \geq c$, σ' is an equilibrium under $\mathcal{E}'$; meaning $\hat{\sigma}'$ must be more selective than σ'.

Otherwise, say $\mathbb{P}_{\psi}\left(\theta = H \mid s_{j-1}\right) < c$. Then, by Lemma 4, there must be an equilibrium strategy that is more selective than σ' under $\mathcal{E}'$.

Claim 2. Evaluators' payoffs under the most (least) selective equilibrium of $\mathcal{E}'$ fall weakly below those under $\mathcal{E}$ if:

i. s_j leads to rejections under $\mathcal{E}$; i.e. $\hat{\sigma}(s_j) = 0$ ($\check{\sigma}(s_j) = 0$), and

ii. the following condition holds:

$$\frac{\rho}{1-\rho} \times \left(\frac{r_H(\sigma;\mathcal{E})}{r_L(\sigma;\mathcal{E})}\right)^{n-1} \times \frac{s_{j+1}}{1-s_{j+1}} \leq \frac{c}{1-c}$$

Now, suppose s_j leads to rejections under σ^*; $\sigma^*(s_j) = 0$. Consequently, we have $\sigma'(s_{j-1}) = \sigma'(s_{j+1}) = 0$. I establish Claim 2 in two steps:

Step 1. $\sigma^{*'}$ is less selective than σ'; evaluators approve *more often* when s_j is mean preserving local spread.

Step 2. This decreases evaluators' payoffs when the condition in Claim 2 is met; $\Pi(\sigma^{*'};\mathcal{E}') \leq \Pi(\sigma';\mathcal{E}') = \Pi(\sigma^*;\mathcal{E})$.

Step 1.

If $s_{j+1} = \max \mathbf{S} \cup \mathbf{S}'$ or $\sigma^{*'}(s_{j+1}) > 0$, it must be the case that $\sigma^{*'}$ is less selective than σ', and we are done. So instead, I assume that $s_{j+1} < \max \mathbf{S} \cup \mathbf{S}'$ and $\sigma^{*'}(s_{j+1}) = 0$.

Case i. σ^* and $\sigma^{*'}$ are the least selective equilibrium strategies; i.e. $\sigma^* = \check{\sigma}$ and $\sigma^{*'} = \check{\sigma}'$

Since σ' replicates the outcome of $\breve{\sigma}$, we have $\Psi(\breve{\sigma};\mathcal{E}) = \Psi(\sigma';\mathcal{E}') = \psi$. Thus, σ' must be an equilibrium strategy under $\mathcal{E}'$ if $\mathbb{P}_\psi(\theta = H \mid s_{j+1}) \leq c$: the optimality conditions for all signals below s_{j+1} are satisfied *a fortiori*, and those for the signals above s_{j+1} are satisfied since $\breve{\sigma}$ has the same optimality conditions under $\mathcal{E}$. So, $\breve{\sigma}'$ must be less selective than σ', since the former is the least selective equilibrium. If on the other hand, $\mathbb{P}_\psi(\theta = H \mid s_{j+1}) > c$, there must be an equilibrium strategy under experiment $\mathcal{E}'$ that is *less* selective than σ', by Lemma 4.

Case ii. σ^* and $\sigma^{*'}$ are the most selective equilibrium strategies; i.e. $\sigma^* = \hat{\sigma}$ and $\sigma^{*'} = \hat{\sigma}'$.

$\hat{\sigma}'$ is the most selective equilibrium strategy under experiment $\mathcal{E}'$, and we assumed that $\hat{\sigma}'(s_{j+1}) = 0$. The strategy $\tilde{\sigma}$ defined below for experiment $\mathcal{E}$ replicates the outcome $\hat{\sigma}'$ generates under experiment $\mathcal{E}'$:

$$\tilde{\sigma}(s) = \begin{cases} 0 & s \leq s_j \\ \hat{\sigma}'(s) & s > s_j \end{cases}$$

Note that $\tilde{\sigma}$ must be an equilibrium under experiment $\mathcal{E}$, since the interim belief it induces is the same as the one $\hat{\sigma}'$ does, and its optimality constraints are a subset of the latter's. But since $\hat{\sigma}$ is the *most* selective equilibrium strategy under $\mathcal{E}$, $\tilde{\sigma}$ must be less selective than it.

Step 2.

The statement is trivially true when $\sigma' = \sigma^{*'}$, so I focus on the case where these two strategies differ. As Step 1 established, $\sigma^{*'}$ must be less selective than σ'. This implies that $\sigma^{*'}(s_{j+1}) > 0$. To see why, say we had $\sigma^{*'}(s_{j+1}) = 0$ instead. We can then construct a strategy $\tilde{\sigma}$ for experiment $\mathcal{E}$, which replicates the outcome $\sigma^{*'}$ generates under experiment $\mathcal{E}'$:

$$\tilde{\sigma}(s) = \begin{cases} 0 & s \leq s_j \\ \sigma^{*'}(s) & s > s_j \end{cases}$$

As they induce the same interim belief and the optimality constraints of the latter are a subset of the former's, $\tilde{\sigma}$ must be an equilibrium under $\mathcal{E}$. This contradicts with σ^* and $\sigma^{*'}$ being the least selective strategies; since $\sigma^{*'}$ being less selective than σ' implies that $\tilde{\sigma}$ must be less selective than σ^*. It also contradicts with σ^* and $\sigma^{*'}$ being the most selective strategies; since it would imply that σ', more selective than $\sigma^{*'}$, should be an equilibrium under $\mathcal{E}'$.

Given that $\sigma^{*'}(s_{j+1}) > 0$, I now take another strategy $\sigma^\delta_{\mathcal{E}'} : \mathbf{S}' \to [0,1]$ for experiment $\mathcal{E}'$:

$$\sigma^\delta_{\mathcal{E}'}(s) = \begin{cases} 1 & s > s_{j+1} \\ \delta & s = s_{j+1} \\ 0 & s < s_{j+1} \end{cases}$$

where $\delta > 0$ is small enough so that $\sigma^{\delta}_{\mathcal{E}'}$ is more selective than $\sigma^{*'}$, but less selective than σ'. I will show that, when the condition stated in Claim 2 holds, we have $\Pi\left(\sigma^{\delta}_{\mathcal{E}'}; \mathcal{E}'\right) \leq \Pi\left(\sigma'; \mathcal{E}'\right)$ for $\delta \to 0$. Lemma 5 then implies that $\Pi\left(\sigma^{*'}; \mathcal{E}'\right) \leq \Pi\left(\sigma^{\delta}_{\mathcal{E}'}; \mathcal{E}'\right)$, which coins the result.

I show this slightly circuitously. I construct another experiment $\mathcal{E}^{\text{re}}$ under which I will use compare two strategies, σ_{re} and $\sigma^{\delta}_{\text{re}}$, that replicate the outcomes of the strategies σ' and $\sigma^{\delta}_{\mathcal{E}'}$, respectively. The experiment $\mathcal{E}^{\text{re}}$ has three possible outcomes, $\{s^{\text{re}}_L, s^{\text{re}}_\delta, s^{\text{re}}_H\}$. Conditional on the applicant's quality θ, its outcome distribution is independent from any other information any evaluator sees, and is given by the following pmf p^{re}_θ:

$$p_\theta(s^{\text{re}}) = \begin{cases} 1 - r_\theta(\sigma^*; \mathcal{E}) & s = s^{\text{re}}_H \\ \delta \times p'_\theta(s_{j+1}) & s = s^{\text{re}}_\delta \\ r_\theta\left(\sigma^*; \mathcal{E}\right) - \delta \times p'_\theta(s_{j+1}) & s = s^{\text{re}}_L \end{cases}$$

Define the strategies σ_{re} and $\sigma^{\delta}_{\text{re}}$ for this experiment as follows:

$$\sigma_{\text{re}}(s) = \begin{cases} 1 & s = s^{\text{re}}_H \\ 0 & s = s^{\text{re}}_\delta \\ 0 & s = s^{\text{re}}_L \end{cases} \qquad \sigma^{\delta}_{\text{re}} = \begin{cases} 1 & s = s^{\text{re}}_H \\ 1 & s = s^{\text{re}}_\delta \\ 0 & s = s^{\text{re}}_L \end{cases}$$

Now note that these two strategies replicate the outcomes of the strategies σ' and $\sigma^{\delta}_{\mathcal{E}'}$, respectively. Under $\sigma_{\text{re}}(s)$, the probability that an applicant is approved upon a visit, conditional on his quality, is the same as it is under strategy σ' (or σ^*, which it replicates), and under $\sigma^{\delta}_{\text{re}}$, it is the same as it is under $\sigma^{\delta}_{\mathcal{E}'}$.

So, the difference between evaluators' payoffs under these two strategies is determined by the *marginal reject* who:

- is rejected by *every* evaluator under the strategy σ_{re}.
- is approved by *at least one* evaluator under the strategy $\sigma^{\delta}_{\text{re}}$.

Where $\mathbf{s^{re}} = \left\{s^1, ..., s^n\right\}$ is the applicant's signal profile under the experiment $\mathcal{E}^{\text{re}}$, he has:

- *no* s^{re}_H signals; $s^i \neq s^{\text{re}}_H$ for all $i \in \{1, 2, ..., n\}$ and
- *at least one* s^{re}_δ signal; there exists some $i \in \{1, 2, ..., n\}$ such that $s^i = s^{\text{re}}_H$.

Thus we have:

$$\begin{aligned} \Pi(\sigma^{\delta}_{\mathcal{E}'}; \mathcal{E}') - \Pi(\sigma'; \mathcal{E}') &= \Pi(\sigma^{\delta}_{\text{re}}; \mathcal{E}^{\text{re}}) - \Pi(\sigma_{\text{re}}; \mathcal{E}^{\text{re}}) \\ &= \mathbb{P}\left(\text{marginal reject}\right) \times \underbrace{\left[\mathbb{P}\left(\theta = H \mid \text{marginal reject}\right) - c\right]}_{(2)} \end{aligned}$$

The expression labelled (2) above equals:

$$\sum_{i=1}^{n} \frac{\mathbb{P}\left(i\ s_\delta^{\text{re}} \text{ and } n-i\ s_L^{\text{re}} \text{ signals}\right)}{\sum_{k=1}^{n} \mathbb{P}\left(k\ s_\delta^{\text{re}} \text{ and } n-k\ s_L^{\text{re}} \text{ signals}\right)} \times \mathbb{P}\left(\theta = H \mid i\ s_\delta^{\text{re}} \text{ and } n-i\ s_L^{\text{re}} \text{ signals}\right) - c$$

Since the probability that an evaluator observes the s_δ^{re} signal is proportional to δ, we have[20]:

$$\lim_{\delta \to 0} \frac{\mathbb{P}\left(i\ s_\delta^{\text{re}} \text{ and } n-i\ s_L^{\text{re}} \text{ signals}\right)}{\sum_{k=1}^{n} \mathbb{P}\left(k\ s_\delta^{\text{re}} \text{ and } n-k\ s_L^{\text{re}} \text{ signals}\right)} = 0$$

Therefore, we get:

$$\lim_{\delta \to 0} \sum_{i=1}^{n} \frac{\mathbb{P}\left(i\ s_\delta^{\text{re}} \text{ and } n-i\ s_L^{\text{re}} \text{ signals}\right)}{\sum_{k=1}^{n} \mathbb{P}\left(k\ s_\delta^{\text{re}} \text{ and } n-k\ s_L^{\text{re}} \text{ signals}\right)} \times \mathbb{P}\left(\theta = H \mid i\ s_\delta^{\text{re}} \text{ and } n-i\ s_L^{\text{re}} \text{ signals}\right) - c$$

$$\lim_{\delta \to 0} \mathbb{P}\left(\theta = H \mid \text{one } s_\delta^{\text{re}} \text{ signal and } n-1\ s_L^{\text{re}} \text{ signals}\right) - c$$

$$\propto \lim_{\delta \to 0} \frac{\rho}{1-\rho} \times \frac{p_H'(s_{j+1})}{p_L'(s_{j+1})} \times \left(\frac{r_H(\sigma^*;\mathcal{E}) - \delta \times p_H'(s_{j+1})}{r_L(\sigma^*;\mathcal{E}) - \delta \times p_L'(s_{j+1})}\right)^{n-1} - \frac{c}{1-c}$$

$$= \frac{\rho}{1-\rho} \times \frac{s_{j+1}}{1-s_{j+1}} \left(\frac{r_H(\sigma^*;\mathcal{E})}{r_L(\sigma^*;\mathcal{E})}\right)^{n-1} - \frac{c}{1-c}$$

□

Proposition 5. Let the experiment $\mathcal{E}'$ differ from $\mathcal{E}$ by a local mean preserving spread at s_j. Evaluators' payoffs in the most selective equilibrium of $\mathcal{E}'$ are below those of $\mathcal{E}$ if:

1. the signal s_j leads to rejections under the most selective equilibrium of $\mathcal{E}$, and

2. the following condition holds:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_j}{1-s_j}\right)^{n-1} \times \frac{s_{j+1}}{1-s_{j+1}} \leq \frac{c}{1-c}$$

Proof. First, I let $\hat{\sigma}(s_{j+2}) < 1$. I show that this implies $\hat{\sigma}$ and $\hat{\sigma}'$ induce equivalent outcomes under their respective experiments. The strategy $\sigma' : \mathbf{S}' \to [0,1]$ which replicates the outcome of $\hat{\sigma}'$ under experiment $\mathcal{E}'$:

$$\sigma'(s) = \begin{cases} \hat{\sigma}(s) & s \geq s_{j+2} \\ 0 & s < s_{j+2} \end{cases}$$

[20] See expression 6.8 and the surrounding discussion in the proof of Theorem 1 for a more detailed explanation of this.

must then be an equilibrium strategy under experiment $\mathcal{E}'$. This is because these strategies induce the same interim belief, that $\hat{\sigma}$ is an equilibrium strategy under $\mathcal{E}$ ensures that the optimality conditions of σ' for signals below s_{j+2} are satisfied, and for signals above s_{j+2}, the optimality conditions are the same as those for σ'. This means that $\hat{\sigma}'$ must be more selective than σ'. However, when proving Theorem 2, we established that σ' must be more selective than $\hat{\sigma}'$. So it must be that $\sigma' = \hat{\sigma}'$, and we are done.

So instead, let $\hat{\sigma}(s_{j+2}) = 1$. But then, it is easily established that:

$$\frac{r_H(\hat{\sigma}; \mathcal{E})}{r_L(\hat{\sigma}; \mathcal{E})} \leq \frac{s_j}{1 - s_j}$$

since $r_\theta(\hat{\sigma}; \mathcal{E}) = \sum_{k=1}^{j} p_\theta(s_k)$. So, the condition Proposition 5 supplies is sufficient for the one Theorem 2 does.

□

Lemma 2. Whenever it exists, the regulator-preferred garbling is monotone binary and produces optimal recommendations.

Proof. To prove this statement, I take some garbling S^G and an equilibrium $\sigma^G : \mathbf{S}^G \to [0, 1]$ it supports. I then construct a monotone binary garbling S^{G*} which produces optimal recommendations, and show that evaluators' payoffs under S^{G*} and the strategy which obeys its recommendations, σ^{G*}, are higher than those under S^G and σ^G.

For any monotone binary garbling S^{G*} and the garbling S^G in question, define the expressions:

$$f^*(s) := p_L(s) \times p_s^{G*}\left(s_L^{G*}\right) \qquad f(s) := p_L(s) \times \sum_{s^G \in S^G} p_s^G\left(s^G\right) \times \left(1 - \sigma^G(s^G)\right)$$

for each $s \in \mathbf{S}$. Given the applicant has *Low* quality, $f^*(s)$ is the probability that (i) his evaluator would observe the signal $s \in \mathbf{S}$ in her original experiment, *and* (ii) the garbling S^{G*} would issue her a "rejection recommendation". Similarly, $f(s)$ is the probability that (i) his evaluator would observe the signal $s \in S$ in her original experiment, *and* (ii) he would be rejected under the equilibrium strategies σ^G. Then, r_L^{G*} and r_L^G below are the probabilities that the (*Low* quality) applicant recommended rejection under S^{G*}, and rejected under σ^G:

$$r_L^{G*} := \sum_{s \in \mathbf{S}} f^*(s) \qquad r_L^G := \sum_{s \in \mathbf{S}} f(s)$$

Now, take the least selective (see Definition 7) monotone binary garbling S^{G*} such that $r_L^{G*} = r_L^G$. Evidently, this garbling exists.

Clearly, one can treat f^* and f as probability density functions over $\mathbf{S}$ when normalised.

Furthermore, the distribution the former describes is first order stochastically dominated by the one the latter does; $\frac{f^*(s_j)}{\sum\limits_{s\in\mathbf{S}} f^*(s)}$ crosses $\frac{f(s)}{\sum\limits_{s\in\mathbf{S}} f(s)}$. Therefore we get:

$$r_H^* := \sum_{s\in\mathbf{S}} \frac{p_H(s)}{p_L(s)} \times \frac{f^*(s)}{\sum\limits_{s\in\mathbf{S}} f^*(s)}$$
$$\leq \sum_{s\in\mathbf{S}} \frac{p_H(s)}{p_L(s)} \times \frac{f(s)}{\sum\limits_{s\in\mathbf{S}} f(s)} =: r_H$$

where r_H^* and r_H are the probabilities that a *High* quality applicant is rejected from a visit under the strategies σ^{G*} and σ^G, respectively.

Since $r_H^* \geq r_H$ and $r_L^* = r_L$, evaluators' payoffs are higher under σ^* than they are under σ. It only remains to show that the strategy σ^* is optimal against the interim belief ψ^* consistent with it.

The interim belief ψ^* consistent with S^{G*} and σ^{G*} is below ψ – the interim belief consistent with S^G and σ^G:

$$\frac{\psi^*}{1-\psi^*} = \frac{\sum\limits_{k=0}^{n-1} (r_H^*)^k}{\sum\limits_{k=0}^{n-1} \left(r_L^*\right)^k} = \frac{\sum\limits_{k=0}^{n-1} (r_H)^k}{\sum\limits_{k=0}^{n-1} (r_L)^k} \leq \frac{\psi}{1-\psi}$$

Under the interim belief ψ^*, it is optimal for an evaluator to reject an applicant upon the signal s_L^{G*} if and only if:

$$\frac{\psi^*}{1-\psi^*} \times \frac{r_H^*}{r_L^*} \leq \frac{c}{1-c}$$

But this inequality must hold; since $\frac{r_H^*}{r_L^*} \leq \frac{r_H}{r_L}$, $\psi^* \leq \psi$, and σ^G is optimal against ψ:

$$\frac{\psi^*}{1-\psi^*} \times \frac{r_H^*}{r_L^*} \leq \frac{\psi}{1-\psi} \times \frac{r_H}{r_L} \leq \frac{c}{1-c}$$

Furthermore, that $\Pi(\sigma^{G*}; S^{G*}) \geq \Pi\left(\sigma; S^G\right) \geq 0$ suggests that the expected payoff of approving an applicant upon the "approve" recommendation must be weakly positive; hence optimal. Thus, the strategy σ^{G*} is optimal against ψ^*.

□

Proposition 6. The regulator-optimal garbling is the *least selective* monotone binary garbling with *regret-free approvals* if it produces optimal recommendations. Otherwise, it is either:

- the least selective monotone binary garbling with regret-free approvals, or
- the most selective monotone binary garbling with regret-prone approvals

among those which produce optimal recommendations.

Proof. **Step 1:** The following are well-defined:

- the least selective monotone binary garbling with regret-free approvals,
- the least (most) selective monotone binary garbling with regret-free (regret-prone) approvals among those which produce optimal recommendations.

I first show the least selective monotone binary garbling with regret-free approvals is well defined. For each monotone binary garbling S^G, define:

$$\mathbf{p^G} := \left(p^G_{s_1}\left(s^G_H\right), ..., p^G_{s_m}\left(s^G_H\right)\right) \qquad\qquad d(S^G) := ||\mathbf{p^G}||$$

Evidently, $d(.)$ is a bijection between the space of monotone binary garblings of $\mathcal{E}$ and $[0, m]$. Also, where both are monotone binary garblings of $\mathcal{E}$, S^G is more selective than $S^{G'}$ if and only if $d(S^G) \leq d(S^{G'})$. Thus, we seek the monotone binary garbling $d^{-1}(D^*)$ where $D^* := \max\left\{D \in [0, m] : d^{-1}(D) \text{ has regret-free approvals}\right\}$. We only ought to show that D^* is well defined. To that end, define the Real valued function F over the space of monotone binary garblings, where:

$$F(S^G) = \begin{cases} \frac{\rho}{1-\rho}\frac{p_H(s^*)}{p_L(s^*)} \times \left(\frac{r^G_H}{r^G_L}\right)^{n-1} - \frac{c}{1-c} & d(S^G) \in (0, m) \\ \lim\limits_{D \downarrow 0} F \circ d^{-1}(D) & d(S^G) = 0 \\ +\infty & d(S^G) = m \end{cases} \qquad r^G_\theta := \sum_{s \in \mathbf{S}} p^G_s(s^G_L) \times p_\theta(s)$$

Consequently, we seek $D^* := \max\left\{D \in [0, m] : F \circ d^{-1}(D) \geq 0\right\}$. But this maximiser exists because the function $F \circ d^{-1}$ is upper semicontinuous: $F \circ d^{-1}$ is a decreasing function, and for any $\bar{D} \in [0, m]$ and $\varepsilon > 0$, we can find some δ_ε such that $D \in (\bar{D} - \delta_\varepsilon, \bar{D}) \cap [0, m]$ implies $d^{-1}(\bar{D})$ and $d^{-1}(D)$ have the same threshold signal s^* and thus $F \circ d^{-1}(D) < F \circ d^{-1}(\bar{D}) + \varepsilon$ since r^G_H / r^G_L is continuous in $d(S^G)$.

Now say this garbling does not provide optimal recommendations. Denote the interim belief that is consistent with evaluators following S^G's recommendations as ψ^G. The garbling S^G provides optimal recommendations if:

$$\underbrace{\frac{r^G_H}{r^G_L} \times \frac{\psi^G}{1 - \psi^G}}_{:= f_1(d(S^G))} \leq \frac{c}{1-c} \qquad\qquad \underbrace{\frac{1 - r^G_H}{1 - r^G_L} \times \frac{\psi^G}{1 - \psi^G}}_{:= f_2(d(S^G))} \geq \frac{c}{1-c}$$

As defined above, both $f_1(.)$ and $f_2(.)$ are continuous. Therefore, the set of monotone binary garblings with optimal recommendations – $\left\{D \in [0, m] : f_1(D) \leq \frac{c}{1-c} \text{ and } f_2(D) \geq \frac{c}{1-c}\right\}$ – is compact. Thus, both objects below are well-defined:

$$\max\left\{D \in [0,m] \text{ and } d^{-1}(D) \text{ has optimal rec.s } : F \circ d^{-1}(D) \geq 0\right\}$$
$$\min\left\{D \in [0,m] \text{ and } d^{-1}(D) \text{ has optimal rec.s } : F \circ d^{-1}(D) \leq 0\right\}$$

Among those with optimal recommendations, the former gives us the least selective garbling with regret-free approvals. The latter gives us the most selective garbling with regret-prone approvals among such garblings, since the least-selective garbling with regret-free approvals does *not* have optimal recommendations (the minimiser of this set *must* have $F \circ d^{-1}(D) < 0$).

Step 2: Proving the statement of Proposition 6.

The sum of evaluators' payoffs under a monotone binary garbling S^G and strategies σ^G that obey its recommendations is given by:

$$\Pi\left(\sigma^G; S^G\right) = \rho - c - \rho \times \left(r_H^G\right)^n \times (1-c) + (1-\rho) \times \left(r_L^G\right)^n \times c$$

As a function of $d^{-1}(.)$, these payoffs are continuous and therefore attain their maxima over the set $[0,m]$. I show that this maximum is attained with the least selective garbling with regret-free approvals.

For the garbling S^G, define $S^G_{+\varepsilon} := d^{-1}(d(S^G)+\varepsilon)$ and $S^G_{-\varepsilon} := d^{-1}(d(S^G)-\varepsilon)$. Likewise, let $s^*_{+\varepsilon}$ and $s^*_{-\varepsilon}$ be the threshold signals of these experiments, and $r^*_{\theta;+\varepsilon}$, $r^*_{\theta;-\varepsilon}$ be their relative rejection rates for each quality of applicant. From our earlier reasoning about the impact of making evaluators strategies marginally more (less) selective, we observe that:

$$\lim_{\varepsilon \to 0} \Pi\left(\sigma^G_{+\varepsilon}; S^G_{+\varepsilon}\right) - \Pi\left(\sigma^G; S^G\right) \propto \lim_{\varepsilon \to 0} \frac{\rho}{1-\rho} \times \frac{p_H(s^*_{+\varepsilon})}{p_L(s^*_{+\varepsilon})} \times \left(\frac{r^G_{H;+\varepsilon}}{r^G_{L;+\varepsilon}}\right)^{n-1} - \frac{c}{1-c} \leq 0$$

where the last inequality follows since S^G is the least selective regret-free garbling. We conclude that giving evaluators a marginally less selective garbling, and therefore (Lemma 5) any garbling that is less selective than S^G, cannot improve their payoffs. Likewise, for a marginally more selective garbling we have:

$$\lim_{\varepsilon \to 0} \Pi\left(\sigma^G_{+\varepsilon}; S^G_{-\varepsilon}\right) - \Pi\left(\sigma^G; S^G\right) \propto -\lim_{\varepsilon \to 0} \frac{\rho}{1-\rho} \times \frac{p_H(s^*)}{p_L(s^*)} \times \left(\frac{r^G_{H;+\varepsilon}}{r^G_{L;+\varepsilon}}\right)^{n-1} - \frac{c}{1-c} \geq 0$$

where the term on the RHS is now negative because an applicant is *lost* when strategies become more selective. By a reasoning similar to that behind Lemma 5, this reveals that *no* garbling that is more selective can improve evaluators' payoffs either.

This also proves that among those with optimal recommendations, the least selective regret-proof garbling cannot be improved with a more selective garbling and the most selective regret-prone garbling cannot be improved with a less selective garbling.

□

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