Monetary and Exchange Rate Policies in a Global Economy*

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Abstract

This paper provides a general macroeconomic framework that integrates monetary and exchange rate policies and studies their optimality, interaction, and trade-offs. The model breaks down the conventional dichotomy in an open economy that monetary policy and foreign exchange intervention (FXI) should separately stabilize the inflation and exchange rate. Instead, I find that, under cooperation, optimal FXI mitigates the inflation-output trade-off of monetary policy and improves monetary autonomy by allowing central banks to stabilize the inflation and exchange rates without a large increase in the monetary policy rate. However, this comes at the cost of terms-of-trade distortion and international resource misallocation in favor of the domestic economy over the foreign. This trade-off renders the combination of monetary policy and FXI the optimal policy. Moreover, I find that capital flow in international finance can be driven by dollar dominance in international trade.

Keywords: Capital flows, International risk sharing, Foreign exchange intervention, Optimal targeting rules, International policy cooperation.

JEL Classification Codes: E58, F31, F32, F41, F42.

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1 Introduction

How should central banks assign different objectives to monetary and exchange rate policies? A classical "trilemma" (Mundell 1957, Fleming 1962) suggests that monetary policy should target domestic inflation stabilization and let the exchange rate float. However, according to a modern view (Rey 2015, Kalemli-Özcan 2019, Miranda-Agrippino and Rey 2020), financial globalization and volatile capital flows turned this trilemma into a "dilemma": independent monetary policy is feasible if and only if the capital account is managed appropriately. Central banks across the globe use various unconventional policy tools to limit capital mobility. In particular, foreign exchange intervention (FXI), i.e., the purchase or sale of foreign currency reserves to manipulate the exchange rate, is an increasingly popular policy tool. Recent data suggests that FXI is used in more than 120 countries (Adler et al. 2023) and growing empirical evidence supports its effectiveness (Fratzscher et al. 2019).

However, little attention has been given to the interaction and trade-off between monetary and exchange rate policies. Monetary policy and FXI are often considered to have two separate objectives: monetary policy should target inflation and FXI should target the exchange rate. However, in practice, policymakers combine monetary policy and FXI to target inflation. For example, when COVID-19 and the Russian-Ukraine war drove worldwide inflation, central banks responded by selling the dollar using FXI without a large increase in monetary policy rate. This paper studies both monetary policy and FXI in an integrated framework and studies their optimality, interaction, and trade-off.

First, I use data on FXI and statements by central banks and currency officials to suggest the following stylized facts. First, not only small open economies but also large open economies use FXI on a massive scale. Second, countries cooperate to achieve exchange rate stability and excessive intervention can cause disagreements across countries. Third, countries combine monetary policy and FXI to stabilize inflation. Fourth, in countries with FXI against dollars, exports and imports are mainly denominated in dollars but assets and liabilities are not necessarily in dollars relative to the countries without FXI.

Motivated by these observations, I construct a general macroeconomic framework that incorporates both monetary and exchange rate policies and study their optimality, interaction, and trade-offs. The model features two large countries (US and local), two policies (monetary policy and FXI), and two frictions (nominal and financial frictions). In the goods market, costly adjustment of prices (and wages) leads to a non-neutrality of monetary policy (Rotemberg

1982). In the financial market, households can only trade domestic currency bonds and their net foreign asset position must be intermediated by global financial intermediaries with limited capacity to bear the exchange rate risk (Gabaix and Maggiori 2015, Itskhoki and Mukhin 2021), resulting in equilibrium deviation from the uncovered interest rate parity (UIP).[1] When central banks purchase the local currency using FXI, investors cannot take the opposite position to buy the dollar. Hence, FXI affects the exchange rate.[2]

I provide a full analytical characterization of optimal targeting rules for monetary policy and FXI, focusing on a cooperation and commitment case. When monetary policy can achieve both the first-best inflation and output, FXI achieves the first-best exchange rate. When the number of inefficiencies (nominal and financial frictions) is equal to the number of policies, monetary policy and FXI have two separate objectives. However, this "dichotomy" breaks down when monetary policy cannot achieve the first-best and faces an inflation-output trade-off (for example, due to cost-push inflation driven by a pandemic or war). Instead, FXI improves the monetary policy trade-off by stabilizing inflation but distorts the exchange rate by strengthening the local currency and improving the local demand over the US. More generally, when the number of inefficiencies is greater than the number of policies, FXI trades off internal and external objectives (inflation and exchange rate stabilization).

The key to understanding this result is the different transmission channels of monetary policy and FXI. Local central banks have two policy tools to stabilize inflation. First, by raising the nominal interest rate, both inflation and domestic consumption demand decrease. Second, by purchasing the local currency and selling the dollar bond, inflation decreases but domestic consumption increases. Intuitively, FXI has two main transmission mechanisms. Suppose the central bank buys the local currency and the local currency appreciates. Since the US demand for local goods decreases, the domestic good inflation decreases. Moreover, since the dollar is cheaper, the price of imported goods decreases. Hence, FXI is used to stabilize inflation. At the same time, by purchasing the local currency, FXI increases the price of the local bond and reduces its return relative to the dollar bond. However, due to the limits to arbitrage, local households cannot immediately take an opposite position to invest in the dollar bond with higher returns. Since local households face a lower rate of return on savings, local households increase

[1]Models of limits to arbitrage are motivated by empirical literature on forward premium puzzle (Fama 1984). Data shows that cross-currency interest differentials are not offset by expected exchange rate depreciation, resulting in positive excess returns on currency carry trades.

[2]Without financial friction, the UIP condition always holds and FXI has no effect on the exchange rate (Backus and Kehoe 1989).

consumption demand relative to the US. Hence, FXI stabilizes inflation at the cost of distorting the exchange rate and terms of trade and exacerbating international resource misallocation in favor of the local economy.

Recent evidence suggests that the majority of world trade is invoiced in a small number of dominant currencies, particularly the US dollar (Gopinath et al. 2020). This paper establishes a general relationship between capital flow management in international finance and the US dollar's dominance in international trade, which are often discussed in two separate contexts. The model delivers two main implications. First, optimal FXI targets currency misalignments. When the local currency is undervalued and the price of an identical local good is lower locally than in the US, the optimal FXI under cooperation is to buy the local currency and fix the cross-currency price dispersion. Second, the transmission of FXI is asymmetric across countries. Since the exchange rate pass-through on import prices is complete only for US goods, FXI stabilizes the local inflation with limited spillover effects on the US inflation. This result explains cross-country patterns of why some countries intervene in the FX market and others do not, related to the fourth stylized fact.

The next step of this research is to study the non-cooperative (Nash) equilibrium and quantify the gains and losses from deviating from the cooperation (Bodenstein et al. 2019; 2023).

Literature. This paper is related to two main strands of literature. First, this paper builds on models of exchange rate determination in an imperfect financial market (Gabaix and Maggiori 2015, Itskhoki and Mukhin 2021, Maggiori 2022, Fukui et al. 2023). Their models have been used to study the exchange rate disconnect from macroeconomic fundamentals, deviation from UIP, and the effects of FXI (Fanelli and Straub 2021, Davis et al. 2023, Ottonello et al. 2024).[3] More recent literature studies both monetary policy and FXI in a small open economy (Cavallino 2019, Amador et al. 2020, Basu et al. 2020, Itskhoki and Mukhin 2023).[4] The contribution of

[3]Other strands of literature model the deviation from uncovered or covered interest rate parity as an exogenous UIP shock (Devereux and Engel 2002, Jeanne and Rose 2002, Kollmann 2005) or convenience yield on the dollar bond (Jiang et al. 2021; 2023, Engel and Wu 2023, Kekre and Lenel 2023)

[4]Cavallino (2019) shows that FXI is costly for a central bank since FX purchase lowers the FX return while it is profitable for intermediaries as they take an opposite carry trade position against the central bank. When the domestic households do not own the entire share of the intermediaries, FXI trades off the carry cost with exchange rate stabilization. Amador et al. (2020) show that the zero lower bound of the nominal interest rate generates capital inflow since the expected appreciation of local currency is not offset by the lower interest rate. FXI absorbs the capital flows by accumulating foreign reserves but generates a resource cost. Basu et al. (2020) builds an "integrated policy framework" that jointly studies monetary policy, FXI, capital control, and macroprudential regulation. When banks face a sudden stop, a lower policy rate relaxes the domestic borrowing constraint but tightens the external borrowing constraint due to currency depreciation. FXI limits this depreciation and improves the monetary trade-off. Itskhoki and Mukhin (2023) show that unrestricted use of monetary policy and FXI

my paper is to study cooperative monetary policy and FXI in a large two-country model and find that FXI faces a trade-off between the internal objective (inflation and output gap) and the external objective (exchange rate and consumption misalignments across countries).

Second, this paper is based on the literature on optimal monetary policy. A large body of literature studies optimal monetary policy in a small open economy (Clarida et al. 2001, Schmitt-Grohé and Uribe 2001, Kollmann 2002, Galí and Monacelli 2005, Faia and Monacelli 2008). Another strand of papers study international monetary policy transmission or international cooperation in a large two-country economy (Obstfeld and Rogoff 2000, Corsetti and Pesenti 2001, Clarida et al. 2002, Benigno and Benigno 2003, Corsetti and Pesenti 2005, Benigno and Benigno 2006, Devereux and Engel 2003, Corsetti et al. 2010; 2020; 2023, Engel 2011). These papers study monetary policy independently of FXI. My paper contributes to the literature by providing a joint analysis of monetary policy and FXI.[5]

The effectiveness of FXI is backed by its empirical analysis (Dominguez and Frankel 1993, Dominguez 2003, Fatum and Hutchison 2010, Blanchard et al. 2015, Kuersteiner et al. 2018, Adler et al. 2019, Fratzscher et al. 2019, Hofmann et al. 2019, Fratzscher et al. 2023, Rodnyansky et al. 2023). My paper contributes to the literature by providing a normative analysis based on a full analytical characterization of optimal FXI.

Finally, this paper is related to recent literature on the dominance of the US dollar in trade invoicing (Gopinath 2016, Gopinath et al. 2020, Gopinath and Stein 2021, Mukhin 2022, Egorov and Mukhin 2023). My contribution is to bridge the gap between the literature on international trade and finance and suggests a novel mechanism where the capital flow management is motivated by dollar pricing.

2 Background

To motivate the theoretical model, this section presents four stylized facts on the objectives of FXI and the characteristics of countries that frequently use FXI.

Fact 1: Large countries conduct FXI. Figure 1, panel (a) shows the top 15 largest volumes of FXI by country. Panel (b) shows the FXI over GDP ratio for the same set of countries as in

eliminates both inflation and UIP deviation separately. However, when FXI is constrained, monetary policy faces a trade-off between inflation and UIP stabilization.

[5] Table 1 provides a summary of the literature on monetary policy and FXI by classifying them into six different categories. Each row corresponds to a small open economy or two large open economies, and each column corresponds to monetary policy only, FXI only, or both monetary policy and FXI.

panel (a). I use the quarterly data by Adler et al. (2023) and took the average over the sample period 2000-24. We learn that not only small open economies, such as Switzerland but also large open economies conduct FXI on massive scales. China, which has the second largest GDP in the world, intervenes most actively with around 50 billion dollars (1.2% of GDP) per quarter. China was identified as a "currency manipulator" by the United States in 2020 for accumulating the dollar reserves to obtain unfair trade advantage (Dominguez 2020). Other large emerging economies, such as India and Russia, and even advanced economies, Japan and Korea, are also active users of FXI. India is estimated to have intervened with 78 billion dollars from December 2022 to October 2023 to slow down the depreciation of rupee (Roy and Mazumdar 2023). Japan sold around 43 billion dollars in October 2022 to support the yen. Korea's foreign exchange reserves dropped by 6 billion dollars in April 2024, which was the biggest monthly fall since September 2022 (Lee 2024).

Fact 2: Cooperative Intervention. Countries intervene cooperatively to promote exchange rate stability. A classical example is the Plaza Accord, where G5 economies (United States, United Kingdom, Germany, France, and Japan) agreed to intervene cooperatively to halt the over-appreciation of the US dollar. More recently, following the sharp depreciation of the Japanese yen and Korean won, the Finance Ministers of Japan, Korea, and the United States made the following joint press statement in the trilateral meeting on April 2024: *"We will continue to cooperate to promote sustainable economic growth, financial stability, as well as orderly and well-functioning financial markets. We will also continue to consult closely on foreign exchange market developments in line with our existing G20 commitments, while acknowledging serious concerns of Japan and the Republic of Korea about the recent sharp depreciation of the Japanese yen and the Korean won."* (U.S. Department of the Treasury 2024) This suggests that FXI is set cooperatively to offset excessive fluctuations in the exchange rate.

However, at the same time, currency manipulation by one country can cause disagreements and opposition across countries. Janet Yellen, the Secretary of the US Treasury, mentioned the following in an interview: *"The Treasury chief said China's yuan is among the currencies that she monitors, along with the euro and yen. Yellen continued to register her discomfort with government intervention in currency markets — especially among Group of Seven countries. "It's possible for countries to intervene... but we believe that it should happen very rarely and be communicated to trade partners if it does."* (Condon et al. 2024) This suggests that actual FXI needs to take into account not only domestic but also foreign policymakers' objectives. In particular, in a large-economy context, FXI that manipulates the exchange rate for the sake of its own country may harm the rest of the world, which explains why the G7 economies disagree with frequent interventions.

Fact 3: Combination of Monetary Policy and FXI. The objectives of monetary and exchange rate policies are not separate from each other but central banks and treasuries combine the two policy tools to stabilize the inflation. For example, Rhee Chang-yong, the Governor of the Bank of Korea, told in an interview that intervention is used to slow down the inflation in import prices: *Rhee also said South Korea's central bank is ready to take steps, including intervention to stabilize the won against the dollar. ... "This depreciation pressure due to the dollar strength actually is a bad factor for our inflation, because our imported prices increase a lot."* (Schneider et al. 2022) Another example is Switzerland, which sold 22 billion Swiss francs (2.8% of GDP) in 2022. At the same time, it raised the policy interest rate from −0.75% to −0.25% in June and 0.5% in September 2022 but kept the interest rate lower than abroad. Martin Schlegel, the Vice Chairman of the Governing Board of the Swiss National Bank, mentioned in his speech: *"The combination of rising interest rates and foreign currency sales was effective in quickly bringing inflation back into the range of price stability. The appreciation of the Swiss franc dampened imported inflation in particular, which initially played a significant role. ... Without the use of foreign currency sales, the SNB would have had to raise the policy rate to a higher level. Our decisive action contributed to keeping medium-term inflation expectations anchored."* (Schlegel 2024)

To understand how monetary and exchange rate policies are combined, I focus on a worldwide inflationary event triggered by supply chain disruption and import price increase due to COVID-19 and the Russian-Ukraine war in 2022 and compare the transmission of inflation in countries with and without FXI. I use the quarterly FXI dataset constructed by Adler et al. (2023) and divide countries into "large FXI" and "small or no FXI" groups, defined by whether the FXI volume is larger or smaller than the median. To focus on large open economies, the sample excludes relatively small economies, defined as those whose annual GDP is smaller than 500 million dollars. The sample also excludes countries with a fixed exchange rate regime, which is classified as "pre-announced peg or currency board arrangement" in Ilzetzki et al. (2019), and countries with extremely high inflation, such as Argentina and Turkey. The final sample includes 18 advanced and emerging economies.[6] I also combine data from BIS, OECD, Global

[6]Large FXI group includes Brazil, India, Indonesia, Japan, Korea, Mexico, Russia, Saudi Arabia, and Switzerland. Small or no FXI group includes Australia, Canada, China, the Euro area, Israel, Norway, Poland, Sweden,

Financial Data, and Bloomberg to study changes in policy interest rate, consumer price index (CPI) inflation, and real exchange rate in 2022.

Figure 2, panel (a) compares the median volumes of dollar sales by each group in 2022Q3 and Q4 when the inflationary pressure was the strongest. The median volume of dollar sales is 23 billion dollars (1.1% of GDP) for the large FXI group and zero for the small or no FXI group. Panel (b) compares the change in policy interest rate from December 2021 to December 2022 around the inflationary event. The large FXI group experienced a slightly smaller increase in the policy rate (2.25pp for the large FXI group and 3pp for the small or no FXI group). Panel (c) compares the median CPI inflation and panel (d) compares the median real exchange rate depreciation in each group from 2021Q4 to 2022Q4. The large FXI group experiences both smaller inflation and exchange rate depreciation than the small or no FXI group. The inflation rate is 5.6% for the large FXI group and 7.8% for the small or no FXI group. The depreciation is 54% for the large FXI group and 9.5% for the small or no FXI group.[7] These graphs inform us that, by selling the dollar reserves using FXI, countries manage inflation and exchange rate depreciation without a large increase in interest rate using monetary policy.

Fact 4: Dollar Dominance in International Trade. Finally, data suggests a strong relationship between capital flow management in international finance and dollar dominance in international trade, which were often studied in two separate contexts. As documented by Gopinath et al. (2020), a majority of world trade is dominated by a small number of currencies, most often the US dollar, and the pass-through of the dollar exchange rate to inflation is increasing in the share of imports invoiced in dollars. I find that FXI is often used in countries where exports and imports are invoiced in US dollars. Figure 3, panels (a) and (b) compares the shares of exports and imports invoiced in dollars over total exports and imports, respectively (Boz et al. 2022). In the large FXI group, 85% of exports and 74% of imports are invoiced in dollars, while in the small or no FXI group, 40% of exports and 29% of imports are invoiced in dollars.

Under classical Mundell (1957) and Fleming (1962) framework, the flexible exchange rate is optimal since the expenditure switching channel works both on the export and import sides. However, when both exports and imports are denominated in dollars, currency depreciation increases the import price but has a limited effect on the export side. If central banks sell the dollar, it is likely that currency appreciation benefits importers by lowering the import price inflation but does not harm exporters. This explains why actual central banks use FXI for a

and the United Kingdom.

[7]For each panel, I excluded the top and bottom observations to remove the outliers.

price stabilization objective, as documented in *Fact 3*.

Another possible hypothesis for why countries intervene is the balance sheet channel: if firms or banks issue liability in the US dollar, an appreciation of the dollar increases the liability in terms of local currency (Akinci and Queralto 2024). While this is the case for large emerging economies such as Brazil and Mexico, it does not necessarily apply for advanced economies such as Japan and Korea. Panels (c) and (d) show the share of external assets and liabilities denominated in US dollars over the total assets and liabilities, respectively (Bénétrix et al. 2019). While the large FXI group has a greater share of dollar assets and liabilities than the small or no FXI group, the difference is less pronounced than the exports and imports.[8]

3 Optimal Policy

To capture the key intuition, I conduct a step-by-step construction of the model. First, I derive the optimal monetary policy (without FXI) under nominal friction and show that the first-best allocation is achieved. Second, I derive the optimal monetary policy and FXI under nominal and financial frictions and again show that the first-best allocation is achieved. Third, I introduce cost-push shocks so that monetary policy faces an inflation-output trade-off. I show that the optimal FXI improves the monetary policy trade-off but creates a cross-country misalignment in demand.

3.1 Optimal Monetary Policy

I begin the analysis with the simplest possible case with only monetary policy and nominal friction (no FXI and financial friction). There are two symmetric large open economies, local and US, and the US variables are denoted with an asterisk. I refer to the US unit of account as the dollar.

Households. In each country, there is a continuum of households that maximize the expected discount value of their lifetime utility. I assume the households have a constant relative risk

[8] In Figure A3, I replace panels (c) and (d) of Figure 3 with debt asset and liability share and the difference between large FXI and small or no FXI groups is small.

aversion (CRRA) utility in consumption. The maximization of local households is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \zeta_l \frac{L_t^{1+\eta}}{1+\eta} \right],$$

where C_t and L_t are the consumption and labor supply, σ is the inverse intertemporal elasticity of substitution, β is the discount factor, and η is the inverse Frisch elasticity of labor. The households' consumption basket C_t is a constant elasticity of substitution (CES) aggregator of local and US goods:

$$C_t = \left[a C_{Lt}^{\frac{\phi-1}{\phi}} + (1-a) C_{Ut}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{1-\phi}},$$

where C_{Lt} and C_{Ut} are the consumption of local and US goods, a is the weight of the local good, and ϕ is the elasticity of substitution between the local and US goods. I assume $a \in (1/2, 1]$ so that households exhibit home bias of consumption. C_{Lt} and C_{Ut} are the bundles of differentiated goods:

$$C_{Lt} = \left[\int_0^1 C_t(l)^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{\zeta}{1-\zeta}}, \quad C_{Ut} = \left[\int_0^1 C_t(u)^{\frac{\zeta-1}{\zeta}} du \right]^{\frac{\zeta}{1-\zeta}}$$

where $C_t(l)$ and $C_t(u)$ are the local households' consumption of the local good l and imported good u, respectively.

The local households' budget constraint is:

$$P_{Lt}C_{Lt} + P_{Ut}C_{Ut} + \frac{B_t}{R_t} + \frac{\mathcal{E}_t B_{Ut}}{R_t^*} = B_{t-1} + \mathcal{E}_t B_{Ut-1} + W_t L_t + \Pi_t + T_t, \tag{1}$$

where P_{Lt} and P_{Ut} are the prices of local and US goods faced by local households, B_t and B_{Ut} are the local households' investments in one-period state non-contingent bonds denominated in local currency and US dollars, R_t and R_t^* are the interest rates on the local and US bonds ($1/R_t$ and $1/R_t^*$ are the bond prices), $\mathcal{E}_t$ is the nominal exchange rate in terms of the local unit of account per dollar (an increase in $\mathcal{E}_t$ implies a depreciation of the local currency), W_t is the wage, Π_t is the lump-sum transfer of firms' profit, and T_t is the government transfer.

The price index of the local good is given by:

$$P_{Lt} = \left[\int_0^1 P_t(l)^{1-\zeta} dl\right]^{\frac{1}{1-\zeta}},$$

and the consumer price index (CPI) associated with the consumption basket C_t is given by:

$$P_t = \left[aP_{Lt}^{1-\phi} + (1-a)P_{Ut}^{1-\phi}\right]^{\frac{1}{1-\phi}}. \tag{2}$$

The real exchange rate is defined as the ratio of CPIs: $e_t \equiv \mathcal{E}_t P_t^* / P_t$. The terms-of-trade is defined as the relative price of local imports over exports: $\mathcal{T}_t = P_{Lt}/\mathcal{E}_t P_{Ut}^*$.

The households' intratemporal consumption allocation problem gives the following demand for local and US goods:

$$C_{Lt} = a\left(\frac{P_{Lt}}{P_t}\right)^{-\phi} C_t, \quad C_{Ut} = (1-a)\left(\frac{P_{Ut}}{P_t}\right)^{-\phi} C_t,$$

and the demand for differentiated goods produced within each country:

$$C_t(l) = \left(\frac{P_t(l)}{P_{Lt}}\right)^{-\zeta} C_{Lt}, \quad C_t(u) = \left(\frac{P_t(u)}{P_{Ut}}\right)^{-\zeta} C_{Ut}.$$

The households' Euler equation for local currency bond and labor supply equation:

$$\beta R_t E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} = 1,$$
$$C_t^{\sigma} L_t^{\eta} = \frac{W_t}{P_t},$$

Since households can trade bonds in two currencies, combining the Euler equations for the local currency bond faced by the local and US households, I obtain:

$$E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right] = E_t\left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \frac{\mathcal{E}_t P_t^*}{\mathcal{E}_{t+1} P_{t+1}^*}\right]. \tag{3}$$

When the asset market is complete and the households can trade state-contingent bonds, Equation (3) holds state-by-state. However, when the market is incomplete and households can only trade state non-contingent bonds, Equation (3) only holds in expectation. I define the demand gap $\mathcal{D}_t$ as the ratio of the marginal utility of consumption across the two countries:

$$\mathcal{D}_t \equiv \frac{\left(C_t^*\right)^{-\sigma}/\mathcal{E}_t P_t^*}{C_t^{-\sigma}/P_t} = \left(\frac{C_t}{C_t^*}\right)^{\sigma}\frac{1}{e_t}. \tag{4}$$

When $\mathcal{D}_t = 1$, consumption risk is efficiently shared across the two countries. When $\mathcal{D}_t > 1$, the marginal utility of the US households is higher than that of the local households so the local households have an excess demand over the US households. $\mathcal{D}_t = 1$ for every state of the economy under a complete market but $\mathcal{D}_t \neq 1$ in general under an incomplete market. However, it is well known that in the limiting case of $\sigma = 1$ (log-consumption utility) and $\phi = 1$ (Cobb-Douglas aggregator) and the only source of shocks is productivity, consumption risk is efficiently shared across countries regardless of asset market structure. I call this case "CO preference" after Cole and Obstfeld (1991). As discussed later, monetary policy and FXI have two separate objectives under this special case.

Firms. Firms use domestic labor to produce a differentiated good l following a production function:

$$Y_t(l) = A_t L_t(l),$$

where $Y_t(l)$ is the output and $L_t(l)$ is the labor input for the producer of good l. A_t is a technology shock common to all firms and follows an AR(1) process: $\log(A_t) = \rho_a \log(A_{t-1}) + \sigma_a \epsilon_{at}$, where ρ_a and σ_a are the persistence and the standard deviation, respectively. Let $Y_{Lt} = \left[\int_0^1 Y_t(l)^{\frac{\zeta-1}{\zeta}} dl\right]^{\frac{\zeta}{\zeta-1}}$ be the final output of the local good. The demand for the differentiated good l is given by:

$$Y_t(l) = \left(\frac{P_t(l)}{P_{Lt}}\right)^{-\zeta} Y_{Lt}.$$

Firms are subject to nominal rigidity (Rotemberg 1982) so that firms set the price $P_t(l)$ but must pay a quadratic adjustment cost $\frac{\nu}{2}\left(\frac{P_t(l)}{P_{t-1}(l)} - 1\right)^2 P_t Y_t$. To capture the key intuition, I assume producer currency pricing (PCP) so that the export price is sticky in the exporters' currency (Section 4 derives the optimal policy under dollar pricing). The firms' maximization problem is as follows:

$$\max_{\{P_t(l)\}_{t=0}^{\infty}} E_0 Q_{0,t} \left[(1+\tau) P_t(l) Y_t(l) - W_t L_t(l) - \frac{\nu}{2} \left(\frac{P_t(l)}{P_{t-1}(l)} - 1 \right)^2 P_{Lt} Y_{Lt} \right], \tag{5}$$

where $Q_{0,t} = \beta^t \left(\frac{C_t}{C_0}\right)^{-\sigma} \frac{P_0}{P_t}$ is the households' stochastic discount factor and τ_t is the sales subsidy. In a symmetric steady state where all firms choose the same price ($P_t(l) = P_{Lt}$), defining $\pi_{Lt} = P_{Lt}/P_{Lt-1} - 1$ as the net inflation, the New Keynesian Phillips Curve (NKPC) can be written as:

$$\pi_{Lt}(1+\pi_{Lt}) = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Y_{Lt+1}}{Y_{Lt}} \pi_{Lt+1}(1+\pi_{Lt+1}) \right] + \frac{\zeta - 1}{\nu} \left[\frac{\zeta}{\zeta - 1} \frac{W_t}{A_t P_{Lt}} - (1+\tau_t) \right].$$

The output gap $\tilde{Y}_{Lt}$ as the deviation of log output $\hat{Y}_{Lt}$ from the natural (flexible price) level $\hat{Y}_{Lt}^n$. Formally,

$$\tilde{Y}_{Lt} = \hat{Y}_{Lt} - \hat{Y}_{Lt}^n.$$

Similarly, the terms-of-trade gap is defined as:

$$\tilde{\mathcal{T}}_t = \hat{\mathcal{T}}_{Lt} - \hat{\mathcal{T}}_{Lt}^n.$$

Using log-linearization, the NKPC for the local firms (and similarly for the US firms) can be written in terms of the terms-of-trade gap and the demand gap in addition to the output gap:

$$\pi_{Lt} = \beta \pi_{Lt+1} + \kappa \{ (\sigma + \eta) \tilde{Y}_{Lt} - (1-a)[2a(\sigma\phi - 1)\tilde{\mathcal{T}}_t - \tilde{D}_t] + \mu_t \}, \tag{6}$$

$$\pi_{Ut}^* = \beta \pi_{Ut+1}^* + \kappa \{ (\sigma + \eta) \tilde{Y}_{Ut} + (1-a)[2a(\sigma\phi - 1)\tilde{\mathcal{T}}_t - \tilde{D}_t] + \mu_t^* \}, \tag{7}$$

where π_{Lt} and π_{Ut}^* are the inflation of local (US) goods faced by the local (US) households, respectively, $\kappa = (1+\tau_t)(\zeta - 1)/\nu$ is the slope of the NKPCs, and $\mu_t = \zeta/((\zeta - 1)(1+\tau_t))$ is the equilibrium markup. Intuitively, when there is a local terms-of-trade improvement ($\mathcal{T}_t$ decreases) or a local excess demand ($\mathcal{D}_t$ increases), the local inflation rate increases due to an increase in the local production cost. Under an appropriate subsidy so that there are no cost-push or markup shocks ($\hat{\mu}_t = \hat{\mu}_t^* = 0$), the efficient allocation is satisfied when $\pi_{Lt} = 0$.

Central Banks. Central banks in the two countries use monetary policy to set the nominal interest rate. I focus on the cooperation and commitment case, in which central banks maximize the sum of expected discounted utility in the two countries. This is equivalent to minimizing the quadratic loss function which is approximated around the efficient flexible-price equilibrium:[9]

$$\mathcal{L} = -E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left[\begin{array}{c} (\sigma + \eta) \left(\tilde{Y}_{Lt}^2 + \tilde{Y}_{Ut}^2 \right) + \frac{\zeta}{\kappa} \left(\pi_{Lt}^2 + \pi_{Ut}^{*2} \right) \\ -\frac{2a(1-a)(\sigma\phi - 1)\sigma}{4a(1-a)(\sigma\phi - 1) + 1} \left(\tilde{Y}_{Lt} - \tilde{Y}_{Ut} \right)^2 + \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi - 1) + 1} \tilde{\mathcal{D}}_t^2 \end{array} \right]. \tag{8}$$

Importantly, under cooperation, the loss function not only depends on the internal objective (inflation and output gap in each country) but also the external objective (demand gap across countries).

Optimal Policy. Solving the central banks' maximization problem, I obtain the following well-known result in the literature (Corsetti et al. 2010).

Lemma 1 (Optimal Monetary Policy)**.** *Under cooperation and commitment, CO preference, PCP, and without cost-push shocks, optimal monetary policy closes all gaps in the economy:*

$$\pi_{Lt} = \pi_{Ut}^* = \tilde{Y}_{Lt} = \tilde{Y}_{Ut} = \tilde{\mathcal{D}}_t = 0.$$

Proof. See Appendix B.2.

Without cost-push shocks, by setting the nominal interest rate at the natural level, the monetary policy achieves zero inflation and zero output gap in each country. Moreover, the demand gap also remains zero. Intuitively, since the intertemporal elasticity of substitution and the trade elasticity of substitution between the local and US goods are both unity ($\sigma = \phi = 1$), the terms-of-trade movements automatically pool the production risk regardless of the asset market structure. When the local output increases relative to the US, the local price decreases proportionally. Hence, the total wealth defined as the price times output remains unchanged ($P_{Lt}Y_{Lt} = P_{Ut}Y_{Ut}$) so there is no demand gap.

[9] See Corsetti et al. (2023) for the detailed derivation.

3.2 Optimal Monetary Policy and FXI when the First-Best is Achieved

Next, I consider the two-policy (monetary policy and FXI) and two-friction (nominal and financial) environment. I model the financial sector based on Jeanne and Rose (2002), Gabaix and Maggiori (2015), and Itskhoki and Mukhin (2021). Figure 4 shows the basic model structure. The key departure from the previous section is currency market segmentation. Households can only trade bonds in their own currency and their net foreign asset position must be intermediated by financiers (global financial intermediaries) who are averse to exchange rate risk, which creates limits to arbitrage.[10] The local central bank has two instruments. In addition to setting the nominal interest rate using monetary policy, they can use FXI to trade bonds in two currencies to affect their relative demand and supply. Finally, liquidity (noise) traders generate exogenous capital flow (UIP) shocks due to the special role of dollars such as liquidity and safety, or investors' irrationality and sentiment. This capital flow shock explains the lack of correlation between exchange rates and other macroeconomic fundamentals (Itskhoki and Mukhin 2021).[11]

International Financial Market. The local households can invest only in the local currency bond (B_t) and the US households can invest only in the dollar bond (B_t^*). However, due to currency market segmentation, the local and US households cannot directly trade any assets with each other.

In addition to the households, there is a measure m_n of liquidity (noise) traders who generate an exogenous capital flow (UIP) shock. The liquidity traders hold a zero-net portfolio (N_t, N_t^*) so the investment N_t^* in dollar bonds is matched by the investment $N_t/R_t = -\mathcal{E}_t N_t^*/R_t^*$ in the local currency bonds. The positive N_t^* implies that the noise traders take a long position in the dollar and a short position in the local currency, and vice versa. I assume that the liquidity traders' position follows an AR(1) process: $N_t = \rho_n N_{t-1} + \sigma_n \epsilon_{nt}$.

The local central bank uses sterilized intervention and trades bonds in the two currencies.

[10] For tractability, I assume that households cannot access foreign currency bonds ($B_{Ut} = 0$ in Equation (1)), following Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021). Fukui et al. (2023) generalize this setup so that households and firms can borrow and invest in foreign currency but it is costly to access foreign currency bonds.

[11] Since the main focus of this paper is the economic consequence of UIP shocks and the role of monetary and exchange rate policies, the model is agnostic about the source of UIP shocks to keep tractability. There is an extensive discussion on the drivers of UIP shocks, including investors' heterogeneous beliefs (Bacchetta and Van Wincoop 2006) and cognitive bias (Burnside et al. 2011), rare disaster risk (Farhi and Gabaix 2016), interbank friction (Bianchi et al. 2023a), and special role of US Treasury bonds (Bianchi et al. 2023b). How different sources of UIP shocks affect the optimal policy design is beyond the scope of this paper and is left for future research.

The local central bank holds a zero-net portfolio (F_t, F_t^*) given by $F_t/R_t = -\mathcal{E}_t F_t^*/R_t^*$ and its profits and losses are transferred to the local households in a lump-sum way.[12]

There is a measure m_d of financiers who intermediate the portfolio positions of the households, the liquidity traders, and the local central bank. The financiers hold a zero-net portfolio (D_t, D_t^*) given by $D_t/R_t = -\mathcal{E}_t D_t^*/R_t^*$. Following Itskhoki and Mukhin (2021) and Fukui et al. (2023), I assume that the financiers maximize the following constant absolute risk aversion (CARA) utility:

$$\max_{D_t^*} E_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \bar{R}_t^* \frac{D_t^*}{P_t^*} \right) \right\}, \tag{9}$$

where $\omega \geq 0$ is a risk-aversion parameter and

$$\bar{R}_t^* = R_t^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}},$$

is the unhedged return on the carry trade.[13] In a limiting case where $\omega = 0$, arbitrageurs are risk-neutral and take a carry trade position without charging a risk premium. Hence, the UIP holds and the expected excess return is zero: $E_t \bar{R}_t^* = 0$. However, when $\omega > 0$, arbitrageurs are risk-averse and require a risk premium for taking the risky carry trade position, which drives the UIP deviation: $E_t \bar{R}_t^* \neq 0$.

The market clearing conditions for the bond market imply the net demand for local currency and dollar bonds is zero:

$$B_t + N_t + D_t + F_t = 0, \quad \text{and} \quad B_t^* + N_t^* + D_t^* + F_t^* = 0. \tag{10}$$

The competitive equilibrium is defined as the set of prices, quantities, and policy variables that solve the maximization problems of households, firms, and arbitrageurs under the constraints and market clearing conditions.

[12] I assume that only the local central bank conducts FXI since data shows that interventions by the Federal Reserve Board are infrequent. Moreover, I assume that FXI is unconstrained for simplicity. In reality, central banks face a zero lower bound on FX reserves, which creates an additional policy trade-off. Davis et al. (2023) show that, when reserves cannot be borrowed, the optimal policy is to accumulate the FX reserves during normal times and sell them during crisis times.

[13] The assumption of a CARA utility improves the traceability since their portfolio decision does not depend on the wealth, allowing us to avoid an additional state variable. The potential ways to microfound the banks' risk-aversion are to introduce occasionally binding borrowing constraints, costs of currency hedging, or liquidity holdings by banks (Bianchi et al. 2023a). Moreover, I assume that the financiers' profit is transferred to the local households as a lump-sum payment. As discussed in Appendix B.3, the profits and losses generated by carry trade positions do not affect the first-order dynamics of the model.

Solving the households' and financiers' maximization problems gives the equilibrium relationship between the demand gap, UIP deviation, and the demand for bonds in two currencies.

Lemma 2. *The equilibrium condition in the financial market, which is log-linearized under a symmetric steady state, can be written as:*

$$E_t\tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t = \tilde{r}_t - \tilde{r}_t^* - E_t\Delta\tilde{e}_{t+1} = \chi_1(n_t^* - f_t) - \chi_2 b_t, \tag{11}$$

where $r_t \equiv R_t - E_t\pi_{t+1}$*,* $r_t^* \equiv R_t^* - E_t\pi_{t+1}^*$*,* $f_t \equiv F_t/\overline{Y}$*,* $N_t \equiv N_t/\overline{Y}$*,* $b_t \equiv B_t/\overline{Y}$*,* $\chi_1 \equiv m_n(\omega\sigma_{et}^2/m_d)$ *and* $\chi_2 \equiv \overline{Y}(\omega\sigma_{et}^2/m_d)$ *for finite* $\omega\sigma_{et}^2/m_d$*, where* $\overline{Y} \equiv \overline{Y}_L = \overline{Y}_U$ *is GDP under the symmetric steady state and* $\sigma_{et}^2 \equiv var(\Delta\log\mathcal{E}_{t+1})$ *is the standard deviation of the change in log exchange rate* $(\Delta\log\mathcal{E}_{t+1} \equiv \log\mathcal{E}_{t+1} - \log\mathcal{E}_t)$.

Proof. See Appendix B.3.

Intuitively, suppose that the liquidity traders increase their demand for the dollar bond (positive n_t^*). To provide the dollar bonds to liquidity traders, financiers take a short position in the dollar and a long position in the local currency. In a limiting case where $\omega\sigma_{et}^2/m_d \to 0$, financiers' risk-bearing capacity is sufficiently high so that UIP holds in equilibrium. However, when $\omega\sigma_{et}^2/m_d > 0$, financiers have limited risk-bearing capacity and require a risk premium as compensation for exchange rate risk in carry trade.[14] This results in the positive UIP deviation ($\widetilde{UIP}_t \equiv \tilde{r}_t - \tilde{r}_t^* - E_t\Delta\tilde{e}_{t+1} > 0$) so that the rate of return on the local currency bond is higher than that of the dollar bond. Since households are restricted from trading assets internationally, they cannot take an opposite carry trade position against the noise traders. This implies that the local households face a higher rate of return on savings, so they have more incentive to invest in bonds and postpone their consumption than the US households. As a result, the home households' demand is expected to increase in the future ($E_t\tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t > 0$). Similarly, the households' net foreign debt position ($b_t < 0$) is associated with the positive UIP deviation. To focus on the role of financial sectors in driving the UIP deviation, I consider the limiting case

[14] The risk aversion parameter ω is scaled so that the risk premium $\omega\sigma_{et}^2/m_d$ is finite and nonzero and the variance of the exchange rate σ_{et}^2 affects the first-order dynamics of the model. See discussion by Hansen and Sargent (2011) and Itskhoki and Mukhin (2021).

where $\chi_2 = 0$, so that:[15]

$$E_t \tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t = \tilde{r}_t - \tilde{r}_t^* - E_t \Delta \tilde{e}_{t+1} = \chi_1 (n_t^* - f_t). \tag{12}$$

The local central bank can use FXI to eliminate the distortion due to the segmented currency market. If the central bank takes an offsetting position against the liquidity traders and demands the local currency bond ($f_t = n_t^*$), the right-hand side of Equation (11) becomes zero so that the UIP deviation becomes zero. In other words, FXI effectively shifts the exchange rate risk away from risk-averse financiers to central banks' balance sheets. Since households in the two countries face equal rates of return on savings, the demand gap is zero in expectation ($E_t \tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t = 0$). The resulting allocation is identical to that when the asset market is incomplete but the currency market is not segmented (Corsetti et al. 2010; 2023). This does not necessarily imply $\tilde{\mathcal{D}}_t = 0$ in general but under the assumption of CO preference, the real exchange rate automatically insures the countries from consumption risk so that $\tilde{\mathcal{D}}_t = 0$ holds for every state of the economy, as discussed in the previous section.

Optimal Policy. The following proposition characterizes the optimal monetary policy and FXI in two countries.

Proposition 1 (Optimal Monetary Policy and FXI)**.** *Under cooperation and commitment, CO preference, PCP, and without cost-push shocks, optimal monetary policy closes the inflation and output gap in the two countries:*

$$\pi_{Lt} = \pi_{Ut}^* = \tilde{Y}_{Lt} = \tilde{Y}_{Ut} = 0.$$

Optimal FXI, by setting $f_t = n_t^*$, closes the demand gap in the two countries:

$$\tilde{\mathcal{D}}_t = 0.$$

Proof. See Appendix B.4.

Intuitively, when there are two policies (monetary policy and FXI) and two frictions (nominal

[15] This assumption can be interpreted so that the size of the financial sector, including noise traders (m_n) and financiers (m_d), are sufficiently large relative to the real sector. See Itskhoki and Mukhin (2021). In the later quantitative section, I will relax this assumption and consider the case where both χ_1 and χ_2 are positive.

and financial), the optimal monetary policy targets the internal objectives (zero inflation and output gap) and optimal FXI targets the external objectives (zero demand gap and UIP deviation). I call this result a well-known "dichotomy" in the open economy since the monetary policy and FXI have two separate targets.[16]

3.3 Optimal Monetary Policy and FXI when the First-Best is Not Achieved

Next, I consider the case where the "dichotomy" between monetary policy and FXI does not hold, that is, the separation of the two policies is incomplete. I modify two assumptions in the previous section. First, I allow for cost-push shock ($\mu_t, \mu_t^* \neq 0$) so the first-best allocation cannot be achieved, and monetary policy trades off inflation and output gap stabilization. Second, I remove the assumption of CO preference and assume general CRRA and CES utility ($\sigma, \phi \neq 1$). $1/\sigma$ denotes the intertemporal elasticity of substitution between current and future consumption (C_t, C_{t+1}), and ϕ denotes the intratemporal elasticity of substitution between the local and US goods (C_{Lt}, C_{Ut}). As long as $\sigma\phi > 1$, the local and US goods are substitutes rather than complements.[17]

I compare the optimal policies when the central bank only uses monetary policy and when it combines monetary policy and FXI. First, the following lemma characterizes the optimal monetary policy when FXI is not available.

Lemma 3 (Optimal Monetary Policy Trade-offs)**.** *Under cooperation and commitment, PCP, and when FXI is not available ($f_t = 0$), optimal monetary policy rules for the local and US central banks are characterized by:*

$$0 = \theta\pi_{Lt}^* + \left(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}\right) + \psi_D(\tilde{D}_t - \tilde{D}_{t-1}), \tag{13}$$

$$0 = \theta\pi_{Ut}^* + \left(\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}\right) - \psi_D(\tilde{D}_t - \tilde{D}_{t-1}), \tag{14}$$

[16] Itskhoki and Mukhin (2023) show a similar result in a small-open-economy model instead of a two-country model.

[17] Qualitatively, the first assumption of cost-push shock is enough to generate a policy trade-off of monetary policy and FXI. However, quantitatively, the effect of FXI on inflation and output gap is larger under the second assumption since cost-push shock has a negative international spillover effect when $\sigma\phi > 1$, and FXI closes the cross-country differential of inflation and output gap. The condition $\sigma\phi > 1$ holds under standard calibration of international business cycle literature (Corsetti et al. 2008). Intuitively, this condition is understood that the goods in two large economies (e.g., manufacturing goods) tend to be substitutes rather than complements. The optimal monetary policy and FXI can be characterized in an analytically tractable but general case on household preferences.

where:

$$\psi_D = \frac{4a(1-a)\phi}{\sigma + \eta\{4a(1-a)(\sigma\phi - 1) + 1\}} \frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1}, \tag{15}$$

which hold without imposing restrictions on σ and ϕ.

Proof. See Appendix B.2.

The result is isomorphic to the one under an incomplete asset market without currency market segmentation (Corsetti et al. 2023). The optimal monetary policy faces a trade-off of inflation, output gap, and demand gap, the last of which is absent under a complete asset market.[18]

The next lemma characterizes the transmission of an inefficient cost-push shock under the optimal monetary policy without FXI based on Corsetti et al. (2010).

Lemma 4 (Transmission of Cost-Push Shock)**.** *Suppose that the local and US goods are substitutes ($\sigma\phi > 1$), FXI is not available, and monetary policy follows the optimal rule in Lemma 3. Up to the first order, the elasticities of inflation, output gap, and real exchange rate to a period-0 US cost-push shock satisfy:*

$$\frac{\partial \pi^*_{U0}}{\partial \mu^*_0} > 0, \quad \frac{\partial \pi^*_{U1}}{\partial \mu^*_0} < \frac{\partial \pi^*_{U2}}{\partial \mu^*_0} < \cdots < 0, \quad \frac{\partial \tilde{Y}_{U0}}{\partial \mu^*_0} < \frac{\partial \tilde{Y}_{U1}}{\partial \mu^*_0} < \cdots < 0, \tag{16}$$

$$\frac{\partial \pi_{L0}}{\partial \mu^*_0} < 0, \quad \frac{\partial \pi_{L1}}{\partial \mu^*_0} > \frac{\partial \pi_{L2}}{\partial \mu^*_0} > \cdots > 0, \quad \frac{\partial \tilde{Y}_{L0}}{\partial \mu^*_0} > \frac{\partial \tilde{Y}_{L1}}{\partial \mu^*_0} > \cdots > 0, \tag{17}$$

$$\frac{\partial \tilde{Q}_0}{\partial \mu^*_0} > \frac{\partial \tilde{Q}_1}{\partial \mu^*_0} > \cdots > 0. \tag{18}$$

Proof. See Appendix B.5.

The optimal US monetary policy is to commit to tightening, which lowers the inflation expectation and output gap over time. Hence, the United States faces temporary inflation due to the initial impact of the cost-push shock, followed by mild and persistent deflation due to the monetary tightening. The decrease in the US output depreciates the local currency and worsens

[18]Under an incomplete asset market and without CO preference, a US cost-push shock makes the demand gap positive (local excess demand) but the effect is quantitatively small, as shown in Figure 5. Under the CO preference, Equation (13) reduces to $0 = \theta\pi_{Lt} + \left(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}\right)$, which is the same under the optimal monetary policy under a complete asset market.

the local terms of trade. As shown in the NKPC (6) for the local firms, as long as the local and US goods are substitutes ($\sigma\phi > 1$), the local terms-of-trade worsening (an increase in $\mathcal{T}_0$) has a similar transmission mechanism to a negative local cost-push shock (a decrease in μ_t) and generates a negative comovement of inflation and output gap across countries. The local currency depreciation causes an increase in demand for local goods, so the local output gap is positive. The optimal local monetary policy is to commit to tightening, so the local economy faces temporary deflation due to tightening, followed by mild and persistent inflation due to higher demand. Figure 5, panel (a) provides a graphical representation of this transmission mechanism. I plot the impulse response to a one-percentage increase in the US markup μ_0^*. The parameter values for the benchmark calibration are listed in Table 2.[19]

Next, I consider the case where both monetary policy and FXI can be implemented optimally. The following proposition characterizes the optimal monetary policy and FXI.

Proposition 2 (Optimal Monetary Policy and FXI with Cost-Push Shock)**.** *Under cooperation and commitment, PCP, optimal monetary policy rules for the local and US central banks are characterized by:*

$$0 = \theta\pi_{Lt} + \left(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}\right) + \psi_\pi\theta(\pi_{Lt} - \pi^*_{Ut}) + \psi_D(\tilde{D}_t - \tilde{D}_{t-1}), \quad (19)$$

$$0 = \theta\pi_{Ut} + \left(\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}\right) - \psi_\pi\theta(\pi_{Lt} - \pi^*_{Ut}) - \psi_D(\tilde{D}_t - \tilde{D}_{t-1}), \quad (20)$$

where:

$$\psi_\pi = (1-a)\frac{2a(\sigma\phi - 1) + 1}{\sigma + \eta\{4a(1-a)(\sigma\phi - 1) + 1\}}\frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1},$$

and ψ_D is given in Equation (15). The optimal FXI for the local central bank is characterized by:

$$f_t = n^*_t + \frac{(1-a)}{\chi_1}\frac{2a(\sigma\phi - 1) + 1}{2a(1-a)\phi}\theta(E_t\pi_{Lt+1} - E_t\pi^*_{Ut+1}). \quad (21)$$

These optimal rules hold without imposing the restrictions on σ and ϕ.

[19] There is a wide range of estimates for the trade elasticity. Bernard et al. (2003) estimate the value around 4 using US plant-level data, while Corsetti et al. (2008) use 0.85 to generate an empirically relevant correlation between the real exchange rate and the relative consumption across countries (Backus and Smith 1993). I follow Backus et al. (1994) and set $\phi = 1.5$, which is widely used in international finance literature. I set the elasticity of UIP to FXI at $\chi_1 = 0.43$ to match the observed response of the UIP deviation to Japan's dollar sales from September to October 2022. Following Itskhoki and Mukhin (2021), I set the elasticity of UIP to households' net foreign asset position in Equation (11) at $\chi_2 = 0.001$ to match its observed persistence.

Proof. See Appendix B.4.

The key difference from Lemma 3 is that, with cost-push shock, optimal FXI does not perfectly offset the capital flow shock, i.e., $f_t = n_t^*$ is no longer optimal. Instead, when the local inflation expectation is higher than the US ($E_t\pi_{Lt+1} > E_t\pi^*_{Ut+1}$), the optimal FXI is to buy the local currency and sell the US dollar.

The next proposition compares the transmission of a cost-push shock where both monetary policy and FXI are set optimally with the case where only monetary policy is set optimally but FXI is unavailable.

Proposition 3 (Transmission of Cost-Push Shock when FXI is available)**.** *Assume PCP, cooperation, and commitment, and suppose that the local and US goods are substitutes ($\sigma\phi > 1$). Up to the first order, comparing the cases where both monetary policy and FXI follow the optimal rule in Proposition 1 (denoted with FXI) and where FXI is unavailable and monetary policy follows the optimal rule in Lemma 4, the elasticities of inflation, output gap, real exchange rate, and UIP deviation to the cost-push to a period-0 US cost-push shock satisfy:*

$$\frac{\partial \pi^{*FXI}_{U0}}{\partial \mu_0^*} < \frac{\partial \pi^*_{U0}}{\partial \mu_0^*} (> 0), \quad \frac{\partial \pi^{*FXI}_{Ut}}{\partial \mu_0^*} > \frac{\partial \pi^*_{Ut}}{\partial \mu_0^*} (< 0), \quad \frac{\partial \tilde{Y}^{FXI}_{Ut}}{\partial \mu_0^*} > \frac{\partial \tilde{Y}_{Ut}}{\partial \mu_0^*} (< 0), \tag{22}$$

$$\frac{\partial \pi^{FXI}_{L0}}{\partial \mu_0^*} > \frac{\partial \pi_{L0}}{\partial \mu_0^*} (< 0), \quad \frac{\partial \pi^{FXI}_{Lt}}{\partial \mu_0^*} < \frac{\partial \pi_{Lt}}{\partial \mu_0^*} (> 0), \quad \frac{\partial \tilde{Y}^{FXI}_{Lt}}{\partial \mu_0^*} < \frac{\partial \tilde{Y}_{Lt}}{\partial \mu_0^*} (> 0), \tag{23}$$

$$\frac{\partial \tilde{Q}^{FXI}_t}{\partial \mu_0^*} < \frac{\partial \tilde{Q}_t}{\partial \mu_0^*} (> 0), \quad \frac{\partial \widetilde{UIP}^{FXI}_t}{\partial \mu_0^*} < \frac{\partial \widetilde{UIP}_t}{\partial \mu_0^*} (\fallingdotseq 0). \tag{24}$$

Proof. See Appendix B.6.

The proposition shows that, by combining monetary policy and FXI, both inflation and output gap are smoothed out in both countries. The key underlying mechanism is the expenditure switching channel. If the central bank buys the local currency using FXI, the local currency appreciates. Hence, the US households decrease (increase) the relative demand of the local (US) goods. This change in demand composition narrows down the positive local output gap and the negative US output gap. Since FXI partially absorbs the output gap, the monetary policy can focus more on inflation stabilization. In other words, FXI improves the monetary policy trade-off between inflation and output gap stabilization.

However, at the same time, by buying the local currency using FXI, the local bond price increases, and its return decreases relative to the dollar bond. Due to limits to arbitrage, local households cannot invest in the dollar bond despite its higher return. Since the local households face a lower rate of return on savings than the US households, the local households have an excess demand over the US households (the demand gap becomes positive). Hence, FXI faces a trade-off in stabilizing the internal objective (inflation and output gap in each country) and the external objective (demand gap across countries).

Figure 5, panel (b) shows a graphical representation of this proposition. I compare the impulse response to a US cost-push shock with and without FXI. The red line shows the case where only monetary policy is available (same as panel a) and the blue line shows the case where both monetary policy and FXI are available.

4 Dollar Pricing

While PCP assumption provides the simplest analytical solution, data suggests that international trade is mainly invoiced in US dollars in countries with frequent FXI (*Fact 4*). Motivated by this fact, this section explores the novel interplay between dollar dominance in international trade and capital flow management in international finance. To this end, I extend the model by introducing the dominant currency pricing (DCP) in the spirit of Gopinath et al. (2020). Differently from the PCP case, I assume that both exports and imports are denominated in US dollars so that the law of one price (LOOP) does not hold and the exchange rate pass-through is incomplete for the local goods. The deviation from LOOP creates an inefficiency since, as long as the marginal cost of production is the same, it is inefficient to sell the good at different prices across countries (Engel 2011). Local exchange rate depreciation has limited effects on export competitiveness but expenditure switching mainly works via local import from the US.

For the local firms, the price-setting problem in local currency is given by Equation (5). The price-setting problem in the US dollar is:

$$\max_{\{P_t^*(l)\}_{t=0}^{\infty}} E_0 Q_{0,t}\left[(1+\tau)\mathcal{E}_t P_t^*(l)Y_t^*(l) - W_t L_t(l) - \frac{\nu}{2}\left(\frac{P_t^*(l)}{P_{t-1}^*(l)} - 1\right)^2 \mathcal{E}_t P_{Lt}^* Y_{Lt}^*\right], \tag{25}$$

where $P_t^*(l)$ and $Y_t^*(l)$ are the dollar price and quantity of local good l sold in the US. Let $\Delta_{Lt} \equiv \mathcal{E}_t P_{Lt}^*/P_{Lt}$ be the deviation from LOOP for the local goods ($\Delta_{Lt} = 1$ under PCP). A higher Δ_{Lt} implies that an identical local good is more expensive in the US dollar than in the local currency. Solving the maximization problems,

$$\pi_{Lt} = \beta\pi_{Lt+1} + \kappa\{(\sigma+\eta)\tilde{Y}_{Lt} - (1-a)[2a(\sigma\phi-1)(\tilde{\mathcal{T}}_t + \tilde{\Delta}_{Lt}) + (\tilde{D}_t + \tilde{\Delta}_{Lt})] + \mu_t\}, \tag{26}$$

$$\pi^*_{Lt} = \beta\pi^*_{Lt+1} + \kappa\{(\sigma+\eta)\tilde{Y}_{Lt} - (1-a)[2a(\sigma\phi-1)(\tilde{\mathcal{T}}_t + \tilde{\Delta}_{Lt}) + (\tilde{D}_t + \tilde{\Delta}_{Lt})] - \tilde{\Delta}_{Lt} + \mu^*_t\}, \tag{27}$$

and the NKPC for the US firms is similar to Equation (7).

The quadratic loss function in the DCP case can be characterized as:[20]

$$\mathcal{L} = -E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \begin{bmatrix} (\sigma+\eta)\left(\tilde{Y}^2_{Lt} + \tilde{Y}^2_{Ut}\right) + \frac{\zeta}{\kappa}\left(a\pi^2_{Lt} + (1-a)\pi^{*2}_{Lt} + \pi^{*2}_{Ut}\right) \\ -\frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}\left(\tilde{Y}_{Lt} - \tilde{Y}_{Ut}\right)^2 \\ +\frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\left(\tilde{\mathcal{D}}_t + \Delta_{Lt}\right)^2 \end{bmatrix}. \tag{28}$$

There are two key differences compared to the PCP case. First, the central banks take into account different prices of local goods sold in the two countries (π_{Lt}, π^*_{Lt}). Second, the loss depends on the deviation from the LOOP (Δ_{Lt}).

Under DCP, analytically tractable expressions for the optimal policy rule can be derived under the assumption of linear labor disutility (Engel 2011). The following lemma characterizes the optimal monetary policy under dollar pricing when FXI is not available.

Lemma 5 (Optimal Monetary Policy Trade-offs under DCP)**.** *Under cooperation and commitment, DCP, $\eta = 0$, and when FXI is not available ($f_t = 0$), optimal monetary policy rules for the local and US central banks are characterized by:*

$$0 = \theta a\pi_{Lt} + (\tilde{C}_t - \tilde{C}_{t-1}) + \frac{2a(1-a)\phi}{2a(\phi-1)+1}\frac{\sigma-1}{\sigma}(\tilde{\mathcal{D}}_t - \tilde{\mathcal{D}}_{t-1} + \tilde{\Delta}_t - \tilde{\Delta}_{t-1}), \tag{29}$$

$$0 = \theta[(1-a)\pi^*_{Lt} + \pi^*_{Ut}] - (\tilde{C}^*_t - \tilde{C}^*_{t-1}) - \frac{2a(1-a)\phi}{2a(\phi-1)+1}\frac{\sigma-1}{\sigma}(\tilde{\mathcal{D}}_t - \tilde{\mathcal{D}}_{t-1} + \tilde{\Delta}_t - \tilde{\Delta}_{t-1}). \tag{30}$$

Proof. See Appendix B.7.

The result is isomorphic to the one without currency market segmentation (Corsetti et al. 2020). Importantly, the optimal monetary policy rule is asymmetric across countries. The local central bank trades off the stabilization of domestic inflation (π_{Lt}) with growth rates of

[20] See Corsetti et al. (2020) for the details.

the demand gap and LOOP deviation. In contrast, the US central bank targets the international dollar price, which is the combination of the inflation of local goods prices in the dollars (π^*_{Lt}) and the US-produced goods (π^*_{Ut}).

Next, I study the case where both monetary policy and FXI are available. DCP has two key implications for the design and transmission mechanism of optimal FXI. First, Proposition 4 shows that the optimal FXI closes the inefficient cross-currency dispersion due to incomplete exchange-rate pass-through. The key analytical result can be characterized in the simple Cole and Obstfeld (1991) case. The full derivation of the policy rules is available in Appendix B.8.

Proposition 4 (Targeting the LOOP Deviation)**.** *Under cooperation and commitment, DCP, $\sigma = \phi = 1$, $\eta = 0$, and when both monetary policy and FXI follow the optimal rules,*

1. *Optimal local currency purchase f_t is an increasing function of the price dispersion Δ_{Lt}.*
2. *FXI reduces the elasticity of Δ_{Lt} to the US cost-push shock.*
3. *The elasticity of optimal local currency purchase to the US cost-push shock is larger under DCP than PCP.*

$$\frac{\partial f_t}{\partial \Delta_{Lt}} > 0, \quad \frac{\partial \Delta^{FXI}_{Lt}}{\partial \mu^*_t} < \frac{\partial \Delta_{Lt}}{\partial \mu^*_t} \; (> 0), \quad \left(\frac{\partial f_t}{\partial \mu^*_t}\right)^{DCP} > \left(\frac{\partial f_t}{\partial \mu^*_t}\right)^{PCP} \; (> 0). \tag{31}$$

Proof. See Appendix B.9.

Statements 1 and 2 show that optimal FXI addresses the inefficient cross-currency price dispersion due to incomplete exchange-rate pass-through. Under DCP, since the local exporters set the price in US dollars, a depreciation of the local currency increases the dollar price relative to the local currency price of an identical locally produced good, causing a deviation from the LOOP.[21] The proposition implies that the optimal FXI is to buy the local currency and respond to its undervaluation. Hence, the optimal FXI rule targets the LOOP deviation in addition to the UIP deviation and the inflation in the two countries. Hence, as implied by Statement 3, the optimal FXI volume is larger under DCP than under PCP.

Second, Proposition 5 characterizes the key difference in the transmission mechanism of FXI under different currency paradigms.

[21] In other words, since the price of the local good is sticky in local currency (P_{Lt}) in the local economy and sticky in the dollars (P^*_{Ut}) in the US. Hence, a depreciation of the local currency (an increase in $\mathcal{E}_t$) leads to an increase in the dollar price relative to the local currency price (an increase in $\Delta_{Lt} = \mathcal{E}_t P^*_{Ut}/P_{Lt}$).

Proposition 5 (Asymmetric Transmission)**.** *Assume cooperation and commitment, $\sigma = \phi = 1$, $\eta = 0$, and both monetary policy and FXI follow the optimal rules. In response to the US cost-push shock, under PCP, optimal FXI decreases the local CPI inflation and increases the US CPI inflation by the same degree:*

$$\left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{PCP} = -\left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{PCP} (< 0).$$

Under DCP, optimal FXI decreases the local CPI inflation more and increases the US CPI inflation less than the PCP case:

$$\left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{PCP} (< 0),$$
$$\left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{PCP} (> 0).$$

Proof. See Appendix B.10.

Under PCP, in response to US cost-push inflation, FXI has symmetric effects on local and US inflation. However, under DCP, the transmission is asymmetric across countries. On the one hand, FXI decreases local inflation more under DCP than PCP. Since the optimal FXI is larger under DCP (Proposition 4), FXI reduces the local import price of US goods and thus the local CPI inflation. On the other hand, FXI increases US inflation less under DCP. Since the US import price of local goods is sticky in dollars, local currency appreciation has a limited effect on the US import price. Hence, by purchasing the local currency, central banks can stabilize local inflation without causing a large upward pressure on US inflation. In other words, the international spillover effect of FXI is muted under DCP.[22]

These two propositions on currency misalignment and muted spillover effect rationalize why FXI is frequently used in countries under dollar pricing in international trade.

[22]Under local currency pricing (LCP) where both exports and imports are invoiced in the destination currency, optimal FXI is larger than the PCP case as it targets the LOOP deviation but the transmission is symmetric and FXI has muted effects on the import prices in both countries. This paper focuses on the DCP case as it is the most empirically relevant.

5 Quantitative Analysis

Finally, I build a quantitative DSGE framework to study the international transmission of the Fed's tightening and the interaction with monetary policy and FXI. I consider a case where monetary policy and FXI do not follow optimal policy rules since central banks in reality do not necessarily follow optimal rules but generate monetary surprises. First, I compare the transmission channels of a US monetary tightening shock when the local central bank uses FXI and when it does not use FXI. Next, I simulate the model and quantify the welfare gain of FXI.

I will make the following generalizations, which are the standard ways in the business cycle literature to match the model with empirical moments. I will focus on the local households. First, households have preference that features habit formation:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \zeta_l \frac{L_t^{1+\eta}}{1+\eta} \right],$$

where h is the parameter governing habit formation. This creates an empirical hump-shaped impulse response in consumption. Next, households are allowed to invest in capital so their budget constraint is:

$$P_{Lt}C_{Lt} + P_{Ut}C_{Ut} + I_t + \frac{B_t}{R_t} = B_{t-1} + W_t L_t + R_t^K K_t + \Pi_t + T_t,$$

where I_t is the investment, K_t is the capital stock, and R_t^K is the rate of return on capital. The law of motion of capital is:

$$K_{t+1} = (1-\delta)K_t + \left[1 - \frac{AC_k}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right],$$

where δ is the depreciation rate and AC_k controls the investment adjustment cost, which creates a hump-shaped response in investment. Finally, I introduce the assumption of wage rigidity to target the cyclicality of wages.

Firms use both capital and labor to produce a differentiated good h following a Cobb-Douglas production function:

$$Y_t(l) = A_t K_t(l)^{\alpha} L_t(l)^{1-\alpha},$$

where α is the capital share of income. The monetary policy follows a Taylor rule:

$$R_t = R_{t-1}^{\gamma} \left[\beta^{-1} \pi_{Lt}^{\phi_\pi} \left(\frac{Y_{Lt}}{Y_L^{SS}} \right)^{\phi_y} \right]^{1-\gamma} \upsilon_{rt},$$

where ϕ_π and ϕ_y are the reaction coefficients on domestic inflation and output and γ is the parameter on interest-rate smoothing. The interest rate shock υ_{rt} follows an AR(1) shock:

$$\upsilon_{rt} = \rho_r \upsilon_{rt-1} + \epsilon_t.$$

I consider three different FXI policy regimes, including no FXI, FXI that only stabilizes the risk sharing: $f_t = n_t^*$, and FXI that trades off stabilization of risk sharing and inflation, as characterized by Equation (21).

5.1 Calibration

Table 3 shows the list of calibrated parameters. I interpret each period as one quarter. To calibrate the parameters on business cycle moments, I used the BEA, Bureau of Labor Statistics, FRED, Penn World Table (Feenstra et al. 2015), and World Bank databases. I set $\beta = 0.994$ to match the annualized interest rate 2.3%. I use the estimates from the generalized method of moments to calibrate the relative risk aversion $\sigma = 1.70$, habit formation $h = 0.84$, and the persistence and standard deviations of technological shock: $\rho_a = 0.71$ and $\sigma_a = 0.24\%$. The labor disutility parameter $\zeta = 26.1$ is calibrated to match the steady-state labor supply $L = 1/3$. The capital share of income is set to $\alpha = 0.42$ to target the steady-state capital-output ratio. The capital depreciation rate is set to $\delta = 0.055$ to match the investment-capital ratio at the steady state. The investment adjustment cost parameter is set to $AC_k = 2.5$ following Christiano et al. (2005). The inverse of the Frisch elasticity of labor is set to $\eta = 1$ (Itskhoki and Mukhin 2021). The home bias of consumption is set to $a = 0.77$ to match the steady-state import share of consumption. The elasticity of substitution between local and US goods is set to $\phi = 1.5$ (Itskhoki and Mukhin 2021). The elasticities of substitution between differentiated goods and labor are set to $\theta = \theta_w = 6$ so that the steady-state markup is 1.2% (Galí and Monacelli 2005). For the monetary policy rule, I set the coefficients on inflation and output to $\phi_\pi = 1.5$ and $\phi_y = 0.125$ following the original estimates by Taylor (1993) and the interest rate smoothing parameter to $\gamma = 0.81$ following Smets and Wouters (2007). For persistence and standard deviations of monetary policy and price and wage markup shocks, I used the standard values in Smets and Wouters (2007).

5.2 International Transmission of US Monetary Tightening Shock

Using the calibrated model, I compare the impulse responses to a US monetary policy tightening shock when the local central bank uses FXI and when it does not. First, Figure 6, panel (a) plots the impulse responses to a US interest rate shock of 0.25pp (annualized 1pp) without FXI. When the US interest rate increases, the local exchange rate depreciates, and the inflation rate increases, which creates upward pressure on the local interest rate and decreases local consumption. The demand gap becomes negative, which implies the US excess demand over the local economy.[23] The local output increases in the short run due to the increased US demand for local goods, but the output decreases in the medium run since the local consumption decreases gradually due to intertemporal substitution.

Next, panel (b) compares the case without FXI (red line) and the case with FXI (blue line). By buying the local currency, the local currency appreciates, and the US demand for the local goods decreases. This alleviates the upward pressure on local inflation and interest rates. The local consumption increases and the local output is stabilized. The UIP deviation becomes negative (excess return on the dollar bond), so the US households have less incentive to save than the local households and the demand gap turns from negative to positive (local excess demand). The key implication of this result is that FXI improves the local monetary policy independence since the local monetary policy rate is less affected by the US monetary tightening shock. By combining monetary policy and FXI, the local central bank can stabilize the inflation, output gap, and exchange rate. However, this comes at the cost of generating consumption misalignment in favor of the local economy, which is disadvantageous for US households and can be the source of policy disagreements across countries.

5.3 Welfare Analysis

Finally, to study if FXI provides quantitatively meaningful welfare effects, I simulate the model under different FXI regimes and compare welfare gains. We first define the local

[23]Since I assume $\chi_2 \neq 0$, in general, the demand gap and UIP deviation depend on the households' net foreign asset position even without the noise traders' capital flow shocks. However, the effect on the UIP deviation is quantitatively small.

households' welfare $\mathcal{W}_t$ as:

$$\mathcal{W}_t = \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \zeta_l \frac{L_t^{1+\eta}}{1+\eta} + \beta E_t \mathcal{W}_{t+1},$$

and the US households' welfare $\mathcal{W}_t^*$ is defined analogously. We then take the sum of welfare in two countries and compute their unconditional expectation $\mathcal{W}_t + \mathcal{W}_t^*$. Table 4 shows the sum of unconditional welfare in the two countries measured in terms of consumption equivalence. In column (1), I treat the no FXI case as the benchmark. Column (2) shows that when the FXI only responds to the liquidity shock and stabilizes the UIP ($f_t = n_t^*$), the welfare gain is 0.70% relative to the no FXI benchmark. Column (3) shows that when the FXI responds to both UIP and cross-country inflation differential, the welfare gain is 1.17% relative to the benchmark. This implies that, under standard assumptions on parameters, FXI targets both UIP and inflation provides 1.7 times higher welfare than FXI that only targets UIP, suggesting that FXI targeting inflation leads to a quantitatively meaningful welfare gain.

6 Conclusion

Data shows that FXI is combined with monetary policy and frequently implemented by large advanced and emerging economies on a massive scale. This paper studies the optimality and international transmission mechanism of monetary policy and FXI in a general two-country model with nominal price stickiness in the goods market and intermediation friction in the international financial market.

I first provide a full analytical characterization of optimal targeting rules for cooperative monetary policy and FXI. When the number of policy instruments is equal to the number of frictions, the separation between monetary policy and FXI is complete: monetary policy closes the inflation and the output gap and FXI closes the UIP deviation and achieves the first-best exchange rate, thereby achieving the first-best allocation. However, this "dichotomy" breaks down with a cost-push shock. FXI improves the inflation-output trade-off of monetary policy by appreciating the local currency and narrowing down the inflation and the output gap. However, FXI creates misalignments in exchange rate and purchasing power in favor of the local economy over the United States. Hence, FXI faces a trade-off in stabilizing the internal objective (inflation) and the external objective (international resource allocation). Finally, using a quantitative model, I find that FXI in response to a US monetary tightening shock stabilizes the inflation without raising the domestic interest rate. FXI improves the monetary autonomy and provides insurance from US monetary policy spillover. The key insight of my paper is that monetary policy and FXI are not two separate policy tools but central banks should combine them appropriately to stabilize the exchange rate and the inflation.

An important direction for future research is to study the role of FXI in a dollarized world (Gopinath et al. 2020). Another important agenda for future research is to understand how to combine FXI with other capital account management policies, including capital control and macroprudential policy, in the context of IMF's integrated policy framework (Basu et al. 2020).

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Table 1: Literature on Monetary and Exchange Rate Policies in Open Economy

	(1) Monetary Policy	(2) FX Intervention	(3) Both MP and FXI
(a) Small Open Economy	Clarida et al. (2001) Schmitt-Grohé and Uribe (2001) Kollmann (2002) Galí and Monacelli (2005) Faia and Monacelli (2008) Egorov and Mukhin (2023)	Fanelli and Straub (2021) Davis et al. (2023) Ottonello et al. (2024)	Cavallino (2019) Amador et al. (2020) Basu et al. (2020) Itskhoki and Mukhin (2023) Devereux and Wu (2023)
(b) Large Open Economies (Two-country or multi-country)	Corsetti and Pesenti (2001) Clarida et al. (2002) Benigno and Benigno (2003; 2006) Devereux and Engel (2003) Engel (2011) Corsetti and Pesenti (2005) Corsetti et al. (2010; 2020; 2023)	Backus and Kehoe (1989) Gabaix and Maggiori (2015) Maggiori (2022)	This Paper

Table 2: Benchmark Parameters

	Description	Value	Notes
β	Discount factor (local)	0.995	Annual interest rate = 2%
σ	Relative risk aversion	5	Cole and Obstfeld (1991)
η	Inverse Frisch elasticity	1.5	Itskhoki and Mukhin (2021)
ζ_l	Labor disutility (local)	1	$\overline{L} = 1$
a	Home bias of consumption	0.88	Bodenstein et al. (2023)
ϕ	CES Local & US goods	1.5	Cole and Obstfeld (1991)
θ	CES differentiated goods	10	Ottonello and Winberry (2020)
ρ_a	Persistence of productivity shock	0.95	Bodenstein et al. (2023)
χ_1	Elasticity of UIP to FXI	0.43	$\Delta \log \text{UIP}_t / \Delta \log \text{FXI}_t$
χ_2	Elasticity of UIP to NFA	0.001	UIP/NFA ratio

Note: The table shows the benchmark parameter settings for Figure 5.

Table 3: Calibration

Description	Parameter
Discount factor	$\beta = 0.994$
Relative risk aversion	$\sigma = 1.70$
Habit formation	$h = 0.84$
Labor disutility	$\zeta_l = 26.1$
Capital share	$\alpha = 0.42$
Capital depreciation rate	$\delta = 0.055$
Investment adjustment cost	$AC_k = 2.5$
Inverse Frisch elasticity	$\eta = 1$
Home bias of consumption	$a_H = 0.77$
CES local & US goods	$\phi = 1.5$
CES demand for goods	$\theta = 6$
CES demand for labor	$\theta_w = 6$
Elasticity of UIP to NFA	$\chi = 0.0022$

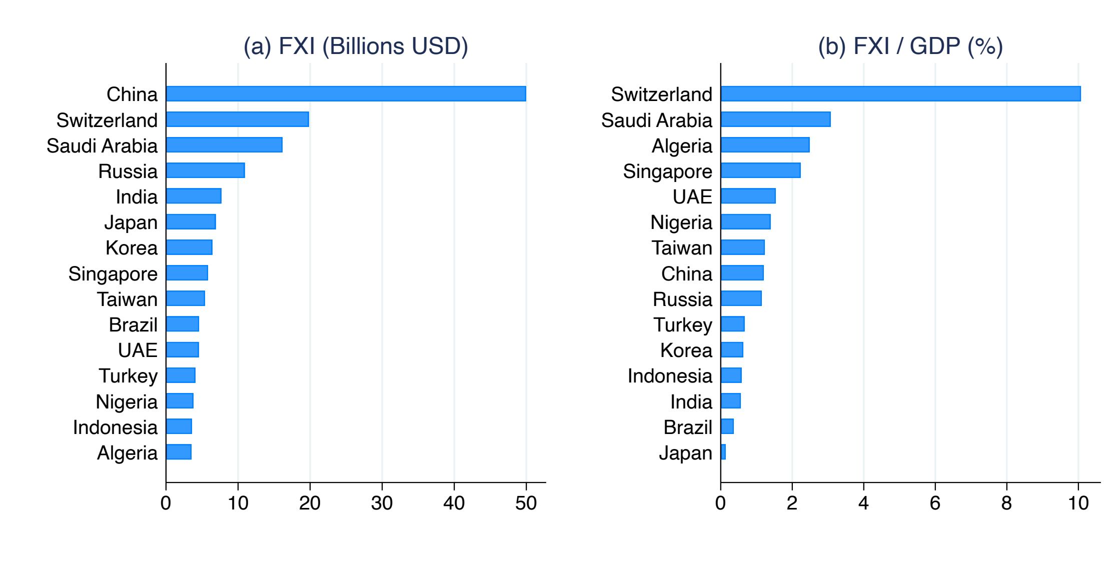
Description	Parameter
Taylor coefficient on inflation	$\phi_\pi = 1.5$
Taylor coefficient on output	$\phi_\pi = 0.125$
Interest rate smoothing	$\gamma = 0.81$
Persistence of UIP shock	$\rho_n = 0.71$
Persistence of FXI shock	$\rho_f = 0.82$
Persistence of technology shock	$\rho_a = 0.86$
Persistence of monetary shock	$\rho_r = 0.43$
Persistence of price markup shock	$\rho_\theta = 0.90$
Persistence of wage markup shock	$\rho_{\theta_w} = 0.97$
SD of UIP shock	$\sigma_n = 0.87\%$
SD of FXI shock	$\sigma_f = 0.82\%$
SD of technology shock	$\sigma_a = 0.42\%$
SD of monetary shock	$\sigma_r = 0.24\%$
SD of price markup shock	$\sigma_\theta = 0.14\%$
SD of wage markup shock	$\sigma_{\theta_w} = 0.24\%$

Table 4: Welfare Analysis

(1) No FXI	(2) FXI targets UIP	(3) FXI targets UIP + inflation
0% (benchmark)	0.70%	1.17%

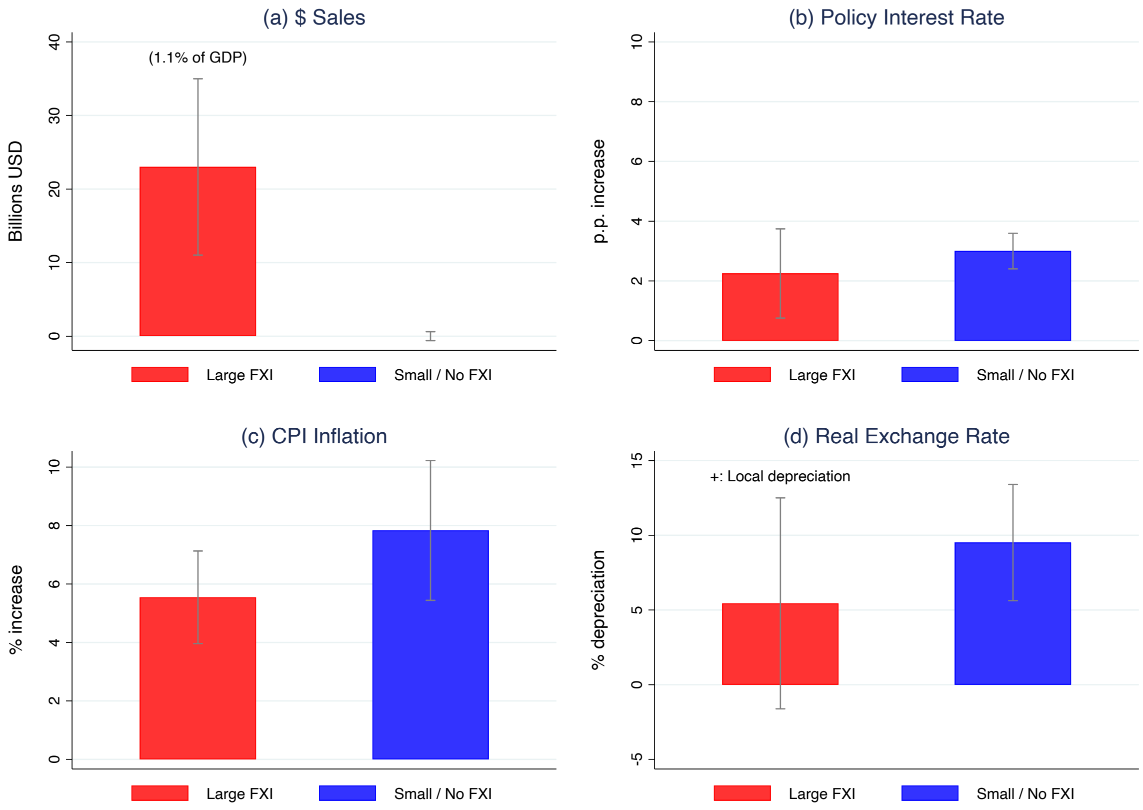
Note: The table shows the sum of unconditional welfare in the two countries measured in terms of consumption equivalence relative to the no FXI case. I simulated the model for 1,100 periods and dropped the first 100 observations. I assume the Taylor rule for both local and US central banks. I allow for productivity, markup, nominal interest rate, and capital flow shocks. In column (1), central banks do not use FXI ($f_t = 0$). In column (2), FXI only stabilizes the UIP ($f_t = n_t^*$). In column (3), FXI stabilizes both UIP and cross-country inflation differential ($f_t = n_t^* + (\theta/2a_H)E_t(\pi_{Lt+1} - \pi^*_{Ut+1})$).

Figure 1: Top-15 Largest FXI Volumes by Country



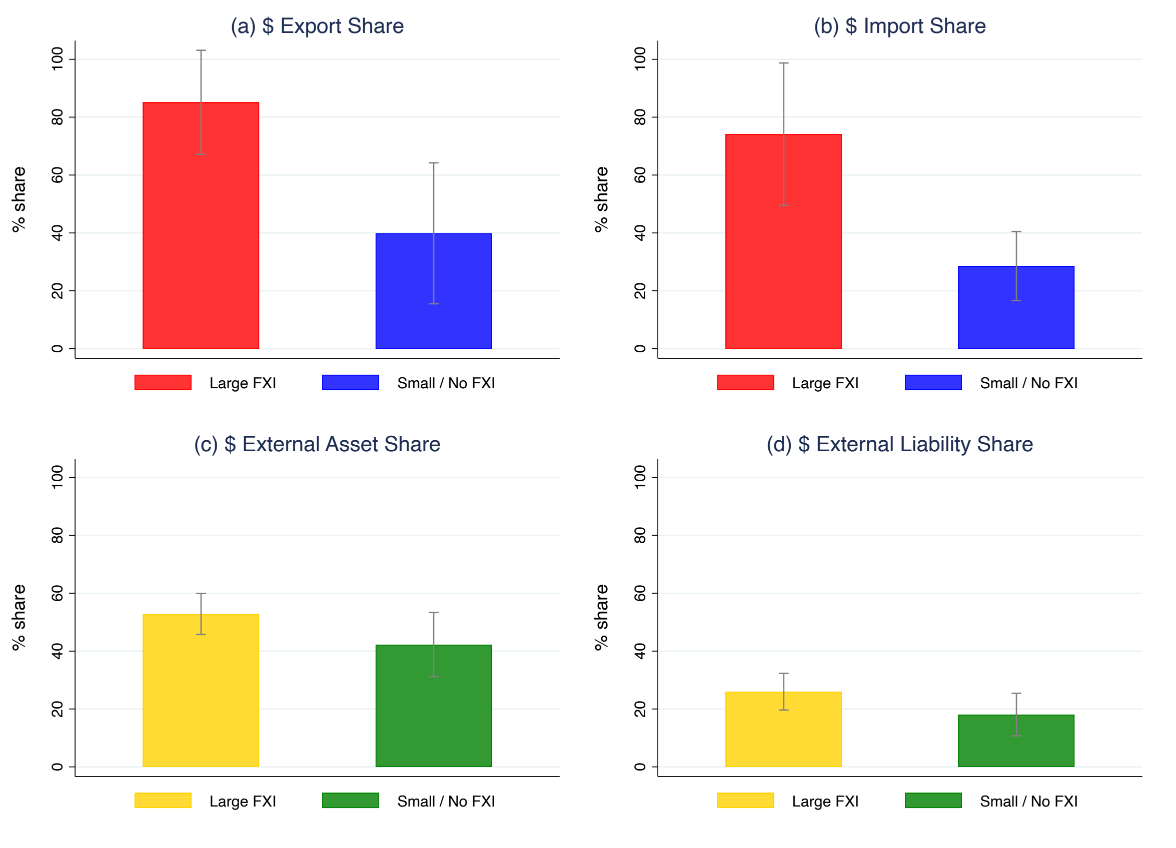
Note: Panel (a) plots the 15 largest FXI volumes (in billions of US dollars) by country. Panel (b) shows the FXI over GDP ratio for the same set of countries as in panel (a). I combine published FXI and FXI proxy constructed by Adler et al. (2023) using foreign currency reserves and balance-of-payments data. I use quarterly data between 2000 and 2024 and took the average over the sample period. The data includes both purchases and sales of foreign currency. The data is available in 122 countries in total. The sample excludes countries that mainly intervene against the euro.

Figure 2: FXI around 2022 Russian-Ukraine War



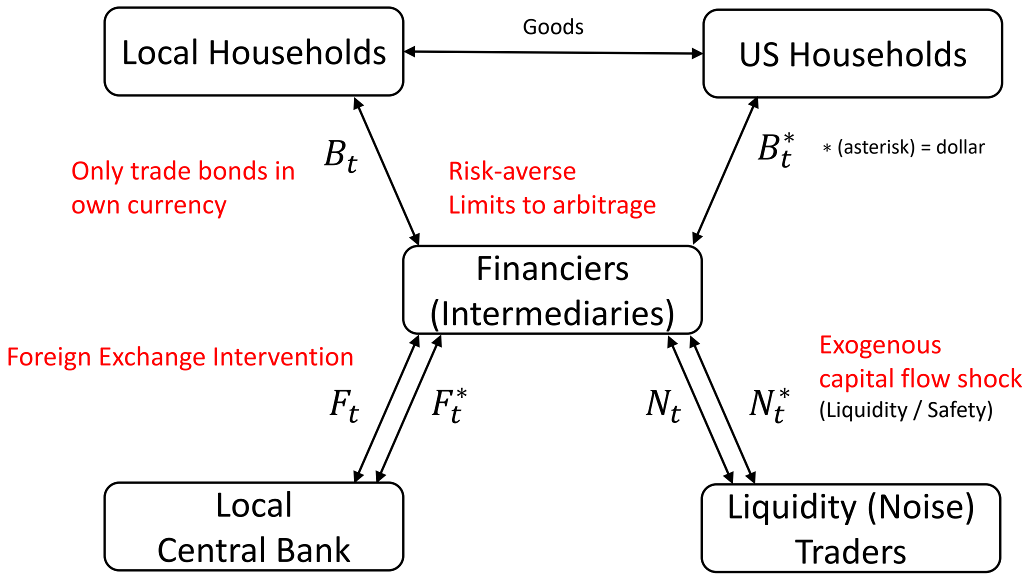
Note: Panel (a) plots the median volume of dollar sales by the large FXI group and the small or no FXI group in 2022Q3 and Q4. Panel (b) plots the median change in policy interest rate from December 2021 to December 2022 in each group. Panel (c) plots the median CPI inflation from 2021Q4 to 2022Q4 in each group. Panel (d) plots the median real exchange rate depreciation from 2021Q4 to 2022Q4 in each group. The large FXI and the small or no FXI groups are defined as countries with dollar sales larger or smaller than the median. The error bars show one standard deviation above and below the median. Data source: Adler et al. (2023), BIS, OECD, Global Financial Data, and Bloomberg.

Figure 3: Invoicing Currencies in Trade and Finance



Note: Panels (a) and (b) plot the share of exports (imports) denominated in US dollars over total exports (imports) for large FXI and small or no FXI groups, respectively. Panels (c) and (d) plot the share of assets (liabilities) denominated in US dollars over total assets (liabilities) for large FXI and small or no FXI groups, respectively. The error bars show one standard deviation above and below the median. Data source: Boz et al. (2022) and Bénétrix et al. (2019).

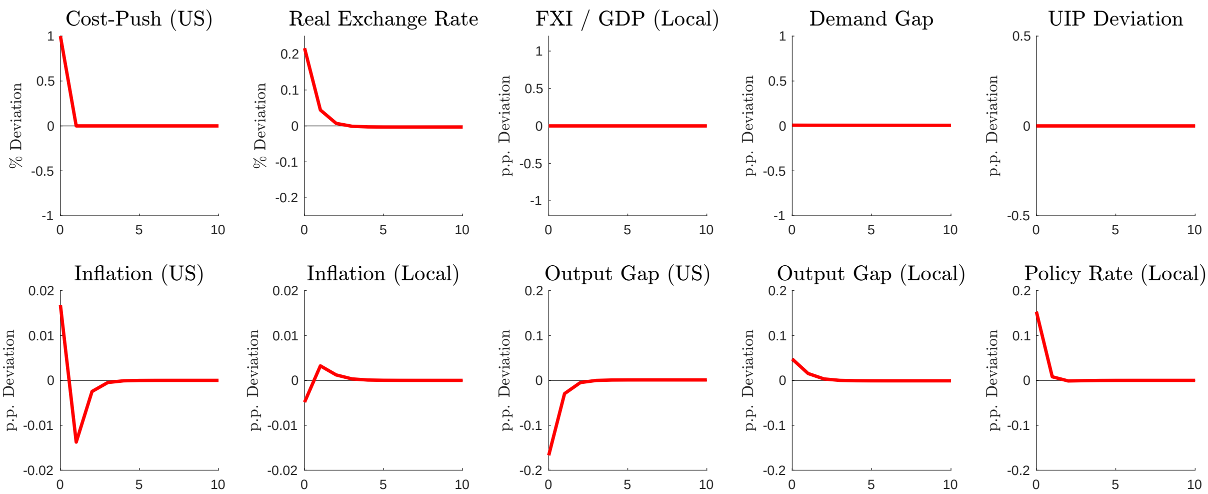
Figure 4: The Basic Model Structure



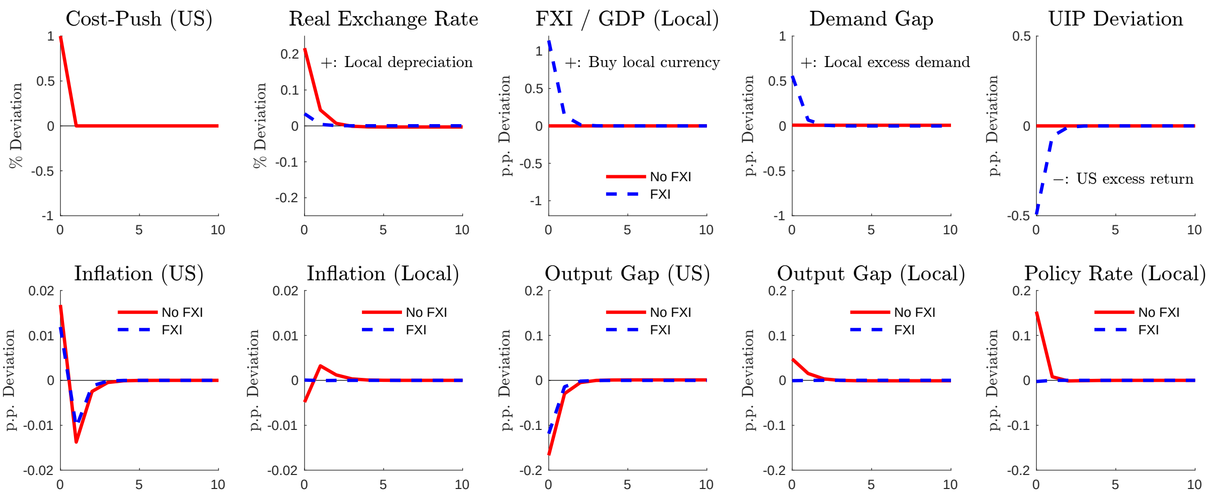
Note: The figure shows the basic structure of the model with both nominal and financial frictions. Local and US households can only trade bonds in their own currency (B_t, B_t^*). The local central bank uses foreign exchange intervention to trade bonds in two currencies (F_t, F_t^*). Liquidity (noise) traders generate an exogenous capital flow shock (N_t, N_t^*). Financiers (global financial intermediaries) intermediate the net foreign asset positions of the households, the local central bank, and the liquidity traders.

Figure 5: Impulse Response to a US Cost-Push Shock

(a) No Intervention



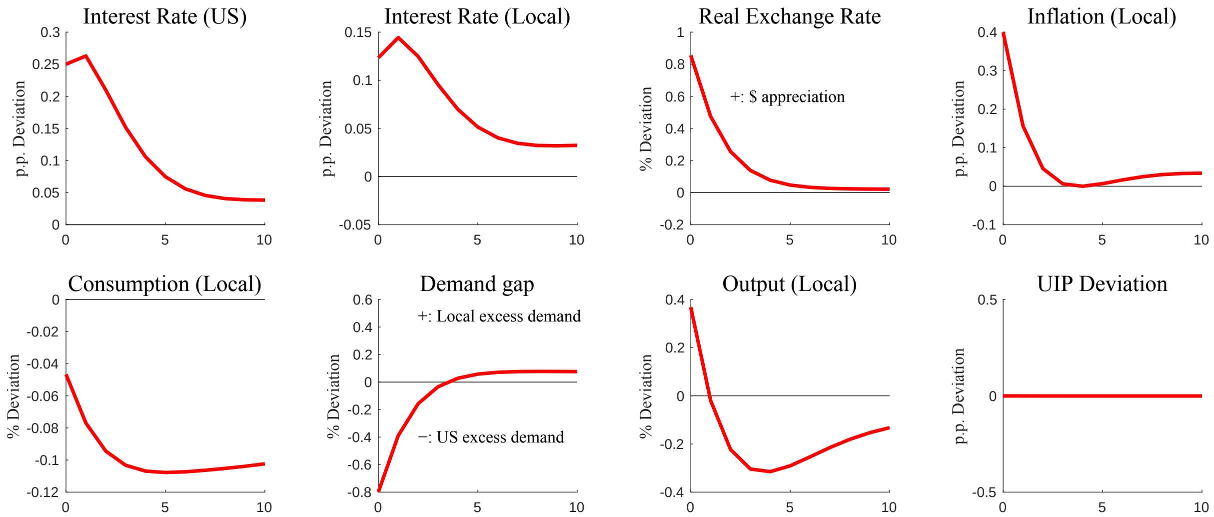
(b) No Intervention vs. Intervention



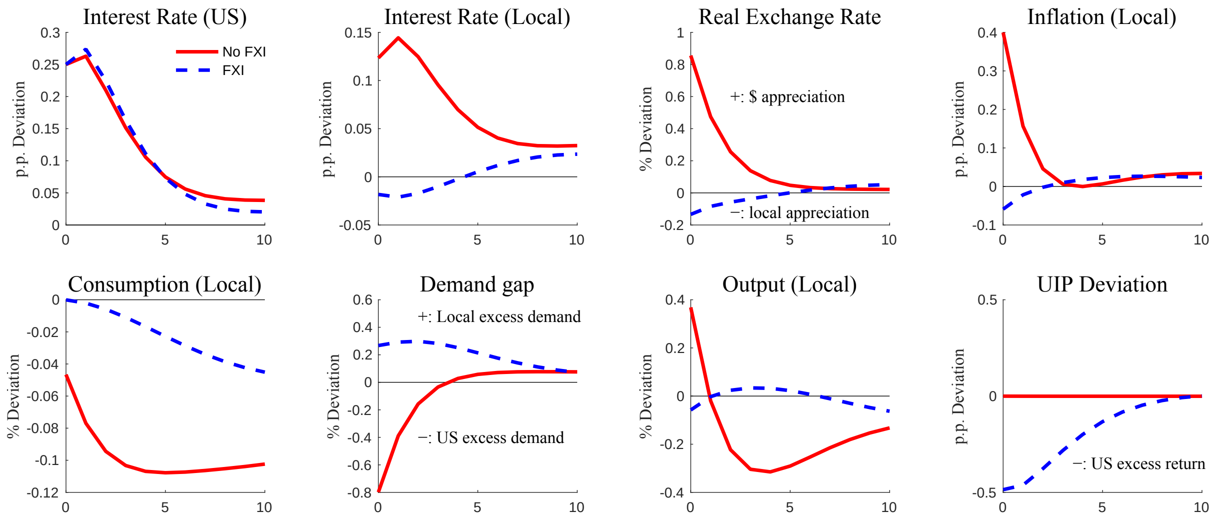
Note: The figure plots the impulse responses to a one-percentage increase in the US markup. Panel (a) plots the case without FXI and panel (b) compares the case with and without FXI. Red: the local central bank uses only monetary policy. Blue: the local central bank combines monetary policy and FXI (local bond purchase).

Figure 6: Impulse Response to a US Monetary Tightening Shock

(a) No Intervention



(b) No Intervention vs. Intervention



Note: The figure plots the impulse responses to an annualized one-percentage-point increase in the US interest rate. Panel (a) plots the case without FXI and panel (b) compares the case with and without FXI. Red: the local central bank uses only monetary policy. Blue: the local central bank combines monetary policy and FXI (local bond purchase).

Appendix

A Additional Figures for Section 2

Figure A1: Policies around 2022 Russian-Ukraine War: Raw Data

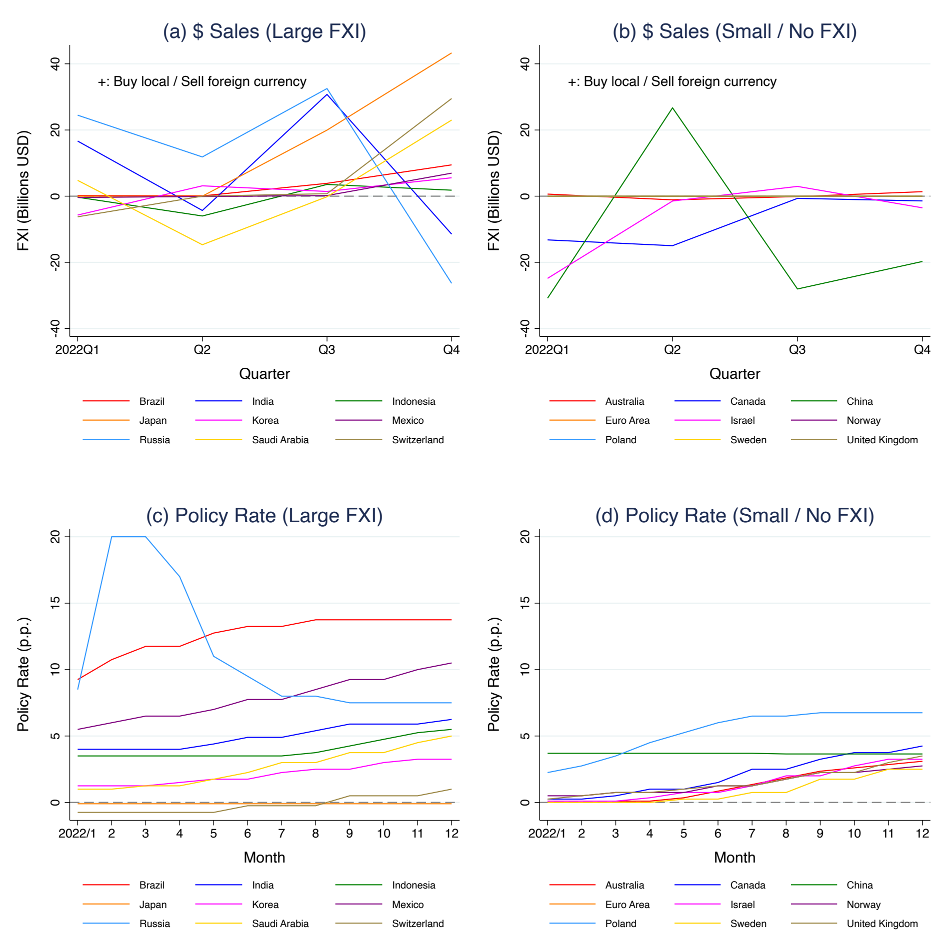
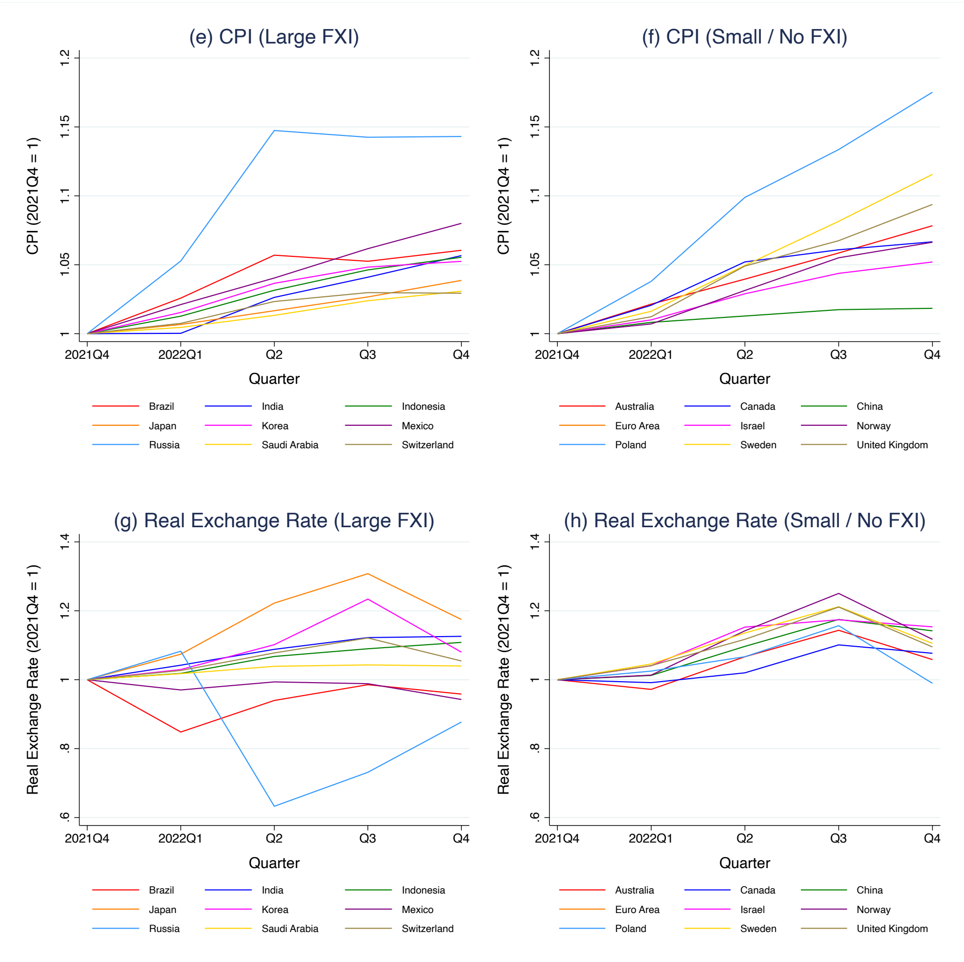


Figure A1: Policies around 2022 Russian-Ukraine War: Raw Data (Continued)



Note: The figure plots the raw data for Figure 2. Panel (a) and (b) plot the volume of dollar sales by the large FXI group and the small or no FXI group in 2022 Q3 and Q4, respectively. Panel (c) and (d) plot the change in policy interest rate from December 2021 to December 2022 for each group. Panel (e) and (f) plot the CPI inflation from 2021Q4 to 2022Q4 for each group. Panel (g) and (h) plot the real exchange rate depreciation from 2021 Q4 to 2022 Q4 for each group. The large FXI and the small or no FXI groups are defined as countries with dollar sales larger or smaller than the median. Data source: Adler et al. (2023), BIS, OECD, Global Financial Data, and Bloomberg.

Figure A2: Invoicing Currencies in Trade and Finance: Raw Data

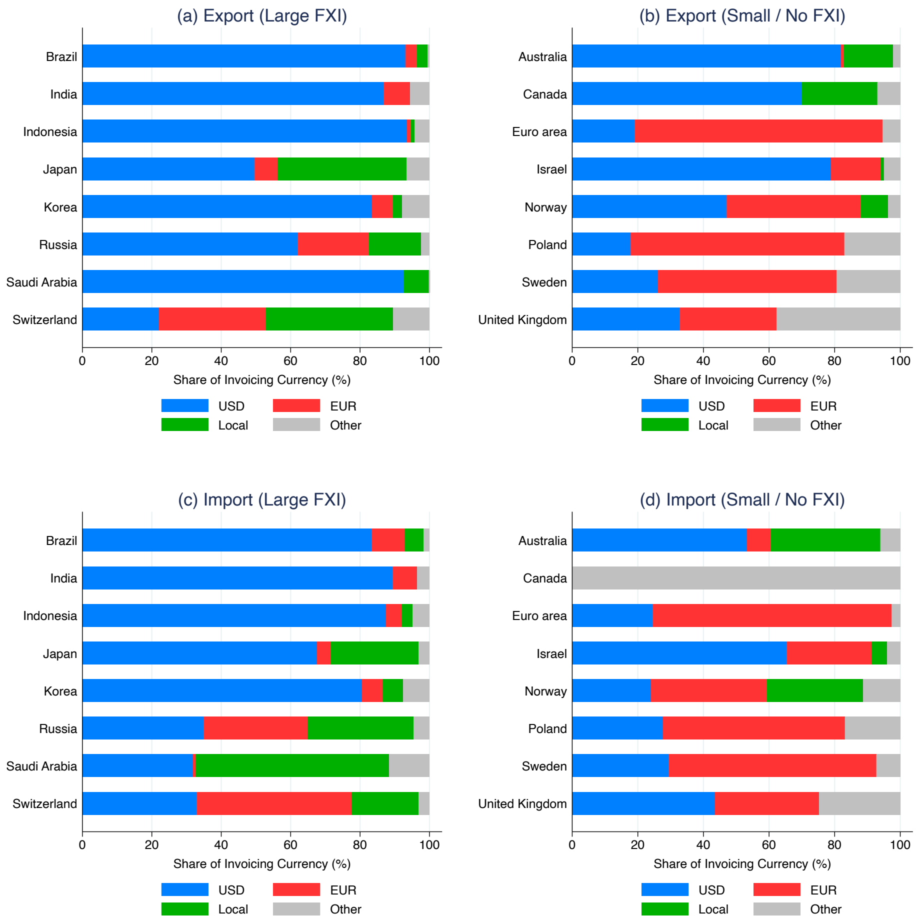
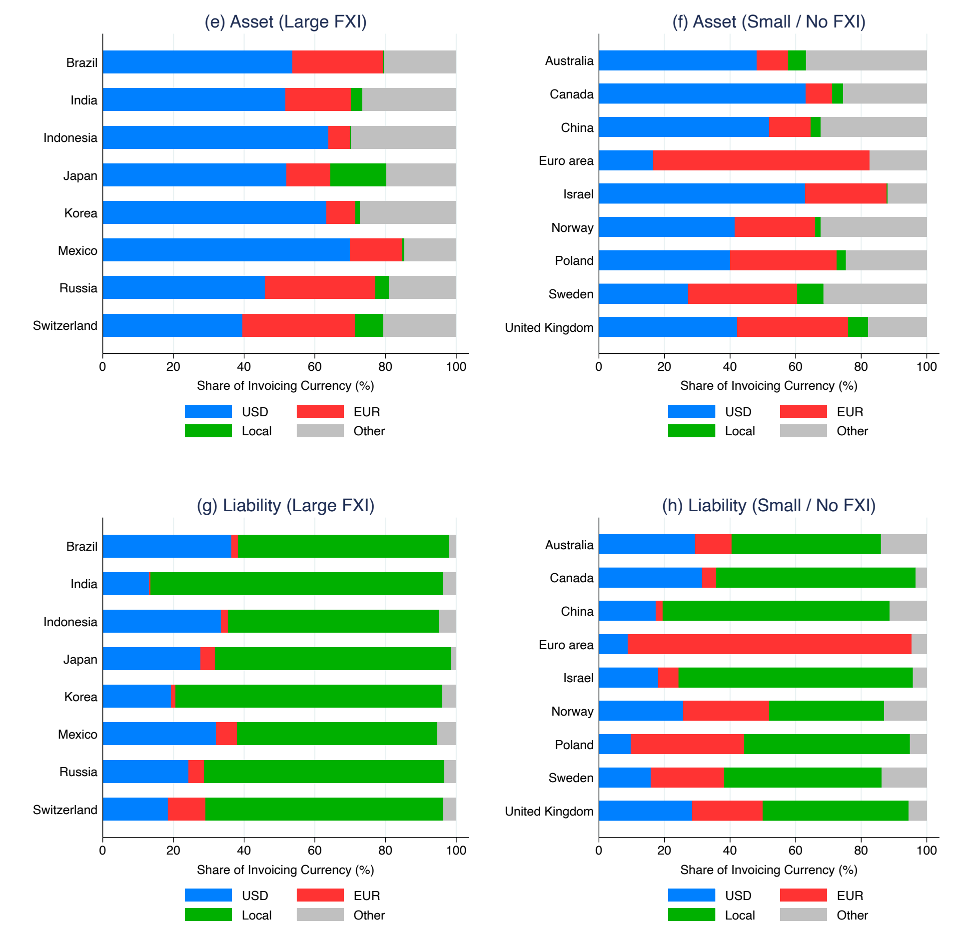
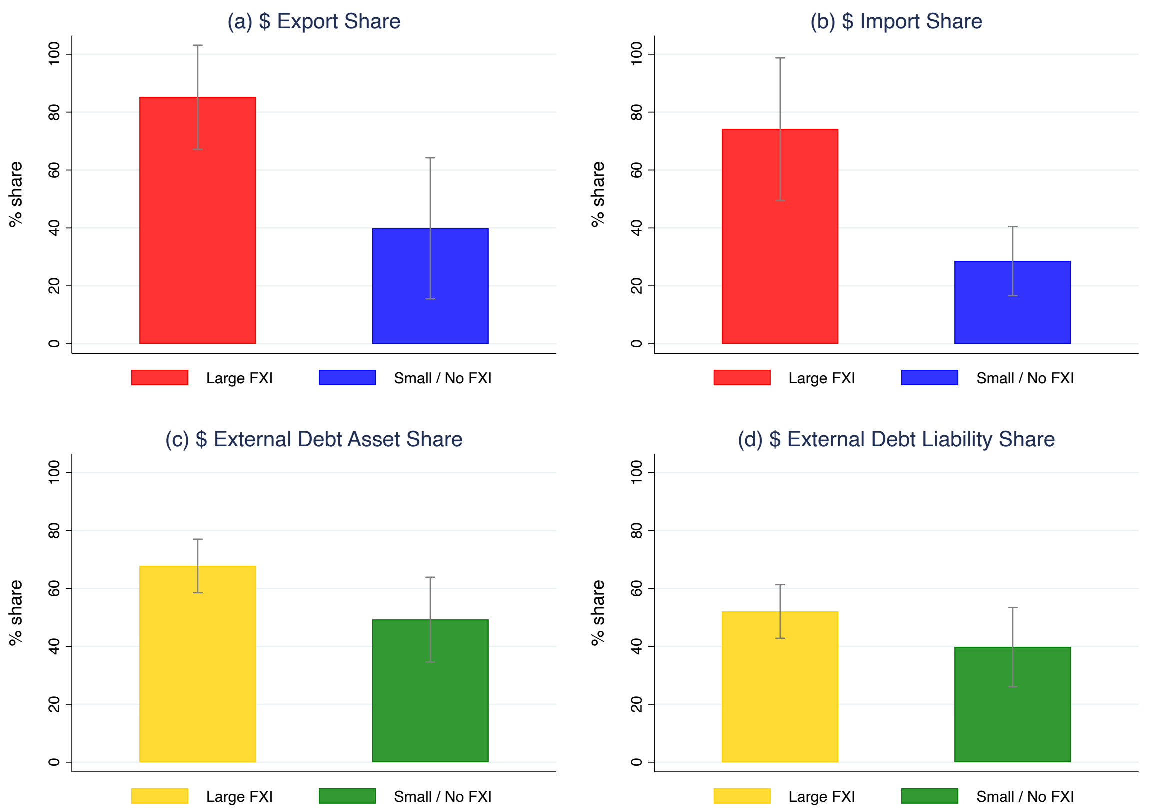


Figure A2: Invoicing Currencies in Trade and Finance: Raw Data (Continued)



Note: The figure plots the raw data for Figure 3. Panels (a) and (b) plot the share of exports denominated in US dollars over total exports for large FXI and small or no FXI groups, respectively. Panels (c) and (d) plot the share of imports denominated in US dollars over total imports for each group. Panels (e) and (f) plot the share of assets denominated in US dollars over total assets for each group. Panels (g) and (h) plot the share of assets denominated in US dollars over total assets for each group. The blue, red, and green bars represent the proportions of the US dollar, the euro, and the local currency, respectively. The large FXI and the small or no FXI groups are defined as countries with dollar sales larger or smaller than the median. Data source: Boz et al. (2022) and Bénétrix et al. (2019).

Figure A3: Invoicing Currencies in Trade and Finance (Debt Asset and Liability)



Note: Panels (a) and (b) plot the share of exports (imports) denominated in US dollars over total exports (imports) for large FXI and small or no FXI groups, respectively. Panels (c) and (d) plot the share of debt assets (debt liabilities) denominated in US dollars over total debt assets (debt liabilities) for large FXI and small or no FXI groups, respectively. The error bars show one standard deviation above and below the median. Data source: Boz et al. (2022) and Bénétrix et al. (2019).

B Derivations and Proofs

B.1 Useful Equilibrium Relationships

This section provides equilibrium first-order relationships which are useful for proofs of propositions in Section 3. The derivation follows Corsetti et al. (2010; 2023).

I focus on the LCP case (the PCP case can be derived analogously by setting $\Delta_t = 0$). Let the variables with hat denote the deviation from the steady state. For simplicity, assume symmetry so that $\mathcal{E}_t P^*_{Lt}/P_{Lt} = \mathcal{E}_t P^*_{Ut}/P_{Ut} = \Delta_t$. By the definition of real exchange rate, it is expressed in terms of terms of trade and price dispersion:

$$e_t = \frac{\mathcal{E}_t P^*_t}{P_t} = \frac{\mathcal{E}_t \left[a(P^*_{Lt})^{1-\phi} + (1-a)(P^*_{Ut})^{1-\phi}\right]^{\frac{1}{1-\phi}}}{\left[a(P_{Lt})^{1-\phi} + (1-a)(P_{Ut})^{1-\phi}\right]^{\frac{1}{1-\phi}}}$$

$$= \frac{\left[a\left(\frac{\mathcal{E}_t P^*_{Lt}}{P_{Lt}}\right)^{1-\phi} + (1-a)\left(\frac{\mathcal{E}_t P^*_{Ut}}{P_{Lt}}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}}{\left[a + (1-a)\left(\frac{P_{Ut}}{P_{Lt}}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}}, \tag{A1}$$

where:

$$\frac{\mathcal{E}_t P^*_{Ut}}{P_{Lt}} = \frac{\mathcal{E}_t P^*_{Lt}}{P_{Lt}} \frac{P_{Ut}}{\mathcal{E}_t P^*_{Lt}} \frac{\mathcal{E}_t P^*_{Ut}}{P_{Ut}} = \Delta_t^2 \mathcal{T}_t, \tag{A2}$$

$$\frac{P_{Ut}}{P_{Lt}} = \frac{\mathcal{E}_t P^*_{Lt}}{P_{Lt}} \frac{P_{Ut}}{\mathcal{E}_t P^*_{Lt}} = \Delta_t \mathcal{T}_t. \tag{A3}$$

Log-linearizing Equation (A1), we obtain:

$$\hat{\mathcal{Q}}_t = (2a_H - 1)\hat{\mathcal{T}}_t + 2a_H \tilde{\Delta}_t. \tag{A4}$$

Next, I approximate the aggregate demand. Under the assumption of symmetry, we have:

$$\hat{Y}_{Lt} + \hat{Y}_{Ut} = \hat{C}_t + \hat{C}^*_t = 0. \tag{A5}$$

Combining Equations (4) and (A5) gives:

$$\hat{Y}_{Lt} - \hat{C}_t = \hat{C}^*_t - \hat{Y}_{Ut} = \frac{1}{2}[\hat{Y}_{Lt} - \hat{Y}_{Ut} - \sigma^{-1}(\hat{\mathcal{Q}}_t + \tilde{\mathcal{D}}_t)]. \tag{A6}$$

Substituting Equation (3.1) into the aggregate demand $Y_{Lt} = C_{Lt} + C^*_{Lt}$ for local good gives:

$$Y_{Lt} = \left(\frac{P_{Lt}}{P_t}\right)^{-\phi} [a_H C_t + (1-a)(e_t \Delta_t^{-1})^{\phi} C^*_t]. \tag{A7}$$

Log-linearizing the CPI (2) gives:

$$\hat{P}_t - \hat{P}_{Lt} = (1-a)(\hat{\mathcal{T}}_t + \tilde{\Delta}_t). \tag{A8}$$

Using Equation (A8), (A7) can be log-linearized as:

$$\hat{Y}_{Lt} - \hat{C}_t = (1-a)\sigma^{-1}[\sigma\phi(\hat{e}_t + \hat{\mathcal{T}}_t) - \hat{e}_t - \tilde{\mathcal{D}}_t]. \tag{A9}$$

Using Equations (A4), (A9) can be rewritten as:

$$\hat{Y}_{Lt} - \hat{C}_t = (1-a)\sigma^{-1}[2a_H\sigma\phi(\hat{\mathcal{T}}_t + \tilde{\Delta}_t) - \hat{e}_t - \tilde{\mathcal{D}}_t]. \tag{A10}$$

Combining the two expressions (A6) and (A10) for the aggregate demand, the terms of trade can be expressed as:

$$\hat{\mathcal{T}}_t + \tilde{\Delta}_t = \frac{\hat{Y}_{Lt} - \hat{Y}_{Ut} - (2a_H - 1)(\tilde{\mathcal{D}}_t + \tilde{\Delta}_t)}{4a_H(1-a)(\sigma\phi - 1) + 1}. \tag{A11}$$

B.2 Proof of Lemmas 1 and 3

The proof follows the appendix of Corsetti et al. (2023). Note that Lemma 1 is a special case of Lemma 3 where $\sigma = \phi = 1$ and there is no cost-push shock. Under cooperation and commitment, the central banks in the two countries minimize the loss (8) subject to the NKPCs (6) and (7) and the UIP condition (12). Let γ_{Lt}, γ^*_{Ut}, and λ_t be the Lagrange multipliers for the local and US NKPCs and the UIP condition, respectively.

The first-order conditions can be written as:

$$\begin{aligned}
\tilde{Y}_{Lt}: \quad 0 = &-(\sigma+\eta)\tilde{Y}_{Lt} + \frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}(\tilde{Y}_{Lt} - \tilde{Y}_{Ut}) \\
&- \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}\tilde{\mathcal{D}}_t \\
&+ \left[\sigma + \eta - \frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\right]\kappa\gamma_{Lt} + \frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\kappa\gamma^*_{Ut}
\end{aligned}$$

$$
- \frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1}(\lambda_t - \beta^{-1}\lambda_{t-1}), \tag{A12}
$$

$$
\begin{aligned}
\tilde{Y}_{Ut}: \quad 0 = & -(\sigma + \eta)\tilde{Y}_{Ut} - \frac{2a(1-a)(\sigma\phi - 1)\sigma}{4a(1-a)(\sigma\phi - 1) + 1}(\tilde{Y}_{Lt} - \tilde{Y}_{Ut}) \\
& + \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi - 1) + 1}\frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1}\tilde{\mathcal{D}}_t \\
& + \left[\sigma + \eta - \frac{(1-a)(\sigma - 1)}{2a(\phi - 1) + 1}\right]\kappa\gamma^*_{Ut} + \frac{(1-a)(\sigma - 1)}{2a(\phi - 1) + 1}\kappa\gamma_{Lt} \\
& + \frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1}(\lambda_t - \beta^{-1}\lambda_{t-1}),
\end{aligned} \tag{A13}
$$

$$
\pi_{Lt}: \quad 0 = -\frac{\theta}{\kappa}\pi_{Lt} - \gamma_{Lt} + \gamma_{Lt-1}, \tag{A14}
$$

$$
\pi^*_{Ut}: \quad 0 = -\frac{\theta}{\kappa}\pi^*_{Ut} - \gamma^*_{Ut} + \gamma^*_{Ut-1}, \tag{A15}
$$

$$
\begin{aligned}
\mathcal{B}_t: \quad 0 = & \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi - 1) + 1}(E_t\tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t) \\
& + (1-a)\frac{2a(\sigma\phi - 1) + 1}{4a(1-a)(\sigma\phi - 1) + 1}\kappa[(E_t\gamma_{Lt+1} - \gamma_{Lt}) - (E_t\gamma^*_{Ut+1} - \gamma^*_{Ut})] \\
& - [(E_t\lambda_{t+1} - \beta^{-1}\lambda_t) - (\lambda_t - \beta^{-1}\lambda_{t-1})].
\end{aligned} \tag{A16}
$$

Under the assumption of $f_t = n^*_t = 0$, we have $E_t\tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t = 0$. By taking the sum of the FOCs for the output gap in the two countries and combining it with the FOCs for inflation, we obtain:

$$
\begin{aligned}
0 &= \tilde{Y}_{Lt} + \tilde{Y}_{Ut} + \theta(p_{Lt} + p^*_{Ut}) \\
&= (\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}) + (\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}) + \theta(\pi_{Lt} + \pi^*_{Ut}).
\end{aligned} \tag{A17}
$$

Next, taking the difference of FOCs for the output gap,

$$
\begin{aligned}
0 = & \left[\sigma + \eta - \frac{4a(1-a)(\sigma\phi - 1)\sigma}{4a(1-a)(\sigma\phi - 1) + 1}\right](\tilde{Y}_{Lt} - \tilde{Y}_{Ut}) \\
& + \frac{4a(1-a)\phi}{4a(1-a)(\sigma\phi - 1) + 1}\frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1}\tilde{\mathcal{D}}_t \\
& + \left[\sigma + \eta - \frac{(1-a)(\sigma - 1)}{2a(\phi - 1) + 1}\right]\theta(\tilde{p}_{Lt} - \tilde{p}^*_{Ut}) \\
& + 2\frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1}(\lambda_t - \beta^{-1}\lambda_{t-1}).
\end{aligned}
$$

The FOC for the net foreign asset implies:

$$-(\lambda_t - \beta^{-1}\lambda_{t-1}) = (1-a)\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}\theta(p_{Lt} - p^*_{Ut}).$$

Combining the FOCs, we obtain the difference rule:

$$0 = \left[\sigma + \eta - \frac{4a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}\right]\left[(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}) - (\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}) + \theta(\pi_{Lt} - \pi^*_{Ut})\right]$$
$$+ \frac{4a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\tilde{\mathcal{D}}_t - \tilde{\mathcal{D}}_{t-1}).$$

Combining the sum and difference rules, we obtain the country-specific monetary policy rules (13) and (14) (Lemma 3).

When $\sigma = \phi = 1$, it is possible to show that when $\sigma = \phi = 1$, the constrained optimal allocation under PCP satisfied $\tilde{\mathcal{D}}_t = 0$ (see Appendix 2.2.2 of Corsetti et al. (2023) for detailed derivation). The monetary policy rules reduce to:

$$0 = \theta\pi_{Lt} + \left(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}\right), \tag{A18}$$

$$0 = \theta\pi^*_{Ut} + \left(\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}\right), \tag{A19}$$

and the NKPCs with $\tilde{\mathcal{D}}_t = \tilde{\Delta}_t = 0$ reduce to:

$$\pi_{Lt} = \beta\pi_{Lt+1} + \kappa(\sigma+\eta)\tilde{Y}_{Lt}, \tag{A20}$$

$$\pi^*_{Ut} = \beta\pi^*_{Ut+1} + \kappa(\sigma+\eta)\tilde{Y}_{Ut}. \tag{A21}$$

Hence, the equilibrium is the first-best: $\pi_{Lt} = \pi^*_{Ut} = \tilde{Y}_{Lt} = \tilde{Y}_{Ut} = \tilde{\mathcal{D}}_t = 0$. □

B.3 Proof of Lemma 2

The proof follows the appendix of Itskhoki and Mukhin (2021). Differently from their paper, the central bank can use FXI in addition to monetary policy.

To begin with, I show the first equality of Equation (11), which describes the relationship between demand gap and UIP deviation. The Euler equations of local and US households are characterized in log-linearized form:

$$\tilde{r}_t = \sigma E_t\left[\tilde{C}_{t+1} - \tilde{C}_t\right], \tag{A22}$$

$$\tilde{r}_t^* = \sigma E_t \left[\tilde{C}_{t+1}^* - \tilde{C}_t^* \right] \tag{A23}$$

Taking the difference of Equations (A22) and (A23) and subtracting $\Delta\tilde{e}_{t+1} = \tilde{e}_{t+1} - \tilde{e}_t$ from both sides,

$$E_t \left[\sigma \left\{ (\tilde{C}_{t+1} - \tilde{C}_t) - (\tilde{C}_{t+1}^* - \tilde{C}_t^*) \right\} - \Delta\tilde{e}_{t+1} \right] = \tilde{r}_t - \tilde{r}_t^* - E_t \Delta\tilde{e}_{t+1}.$$

Using the definition of demand gap (4), we obtain the first equality:

$$E_t \tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t = \tilde{r}_t - \tilde{r}_t^* - E_t \Delta\tilde{e}_{t+1}. \tag{A24}$$

Next, I show the second equality of Equation (11), which describes the relationship between financial flows and UIP deviation. The maximization problem (9) of arbitrageurs can be rewritten as:

$$\max_{d_t^*} E_t \left\{ -\frac{1}{\omega} \exp \left(-\omega \bar{R}_t^* (1 - e^{x_t^*}) \frac{D_t^*}{P_t^*} \right) \right\}, \tag{A25}$$

where $x_t^* = \tilde{r}_t^* - \tilde{r}_t - \Delta\tilde{e}_{t+1}$ is the nominal carry trade return. When the time period is short, x_t^* can be expressed as the normal diffusion process:

$$d\mathcal{X}_t^* = x_t^* dt + \sigma_{et} dB_t,$$

where $x_t^* = \tilde{r}_t^* - \tilde{r}_t - \Delta\tilde{e}_{t+1}$ is the nominal carry trade return and B_t is a standard Brownian motion. Note that the excess return is equal in nominal and real terms when log-linearized:

$$\begin{aligned}
x_t^* &= \tilde{r}_t^* - \tilde{r}_t - \Delta\tilde{e}_{t+1} \\
&= (\tilde{R}_t - E_t \pi_{t+1}) - (\tilde{R}_t^* - E_t \pi_{t+1}^*) - E_t(\Delta\tilde{e}_{t+1} + \pi_{t+1}^* - \pi_{t+1}) \\
&= \tilde{R}_t^* - \tilde{R}_t - \Delta\tilde{e}_{t+1}.
\end{aligned}$$

The maximization problem (A25) can be rewritten as:

$$\max_{D_t^*} E_t \left\{ -\frac{1}{\omega} \exp \left(-\omega \bar{R}_t^* (1 - e^{d\mathcal{X}_t^*}) \frac{D_t^*}{P_t^*} \right) \right\}. \tag{A26}$$

Using Ito's lemma, the objective function can be rewritten as:

$$E_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \overline{R}_t^* \left(-d X_t^* - \frac{1}{2} (d X_t^*)^2 \right) \frac{D_t^*}{P_t^*} \right) \right\}$$
$$= -\frac{1}{\omega} \exp\left(\left[\omega \left(x_t^* + \frac{1}{2}\sigma_{et}^2 \right) \frac{D_t^*}{P_t^*} - \frac{1}{2}\omega^2 \sigma_{et}^2 \left(\frac{D_t^*}{P_t^*} \right)^2 \right] dt \right).$$

Solving the maximization problem, the optimal portfolio decision is:

$$\frac{D_t^*}{P_t^*} = -m_d \frac{\tilde{R}_t^* - \tilde{R}_t - \Delta \tilde{e}_{t+1} + \sigma_{et}^2}{\omega \sigma_{et}^2}. \tag{A27}$$

Substituting Equation (A27) and $N_t^* = m_n n_t^*$, $F_t^* = m_n f_t^*$ into the market clearing condition (10) for the dollar bond, we obtain:

$$\frac{B_t^*}{P_t^*} + \frac{1}{P_t^*} m_n n_t^* - m_d \frac{\tilde{R}_t^* - \tilde{R}_t - \Delta \tilde{e}_{t+1} + \sigma_{et}^2}{\omega \sigma_{et}^2} + \frac{1}{P_t^*} m_n f_t^* = 0. \tag{A28}$$

Since the arbitrageurs, noise traders, and central bank (FXI) takes zero net positions,

$$\frac{D_t + N_t + F_t}{R_t} = -\mathcal{E}_t \frac{D_t^* + N_t^* + F_t^*}{R_t^*}.$$

Using (10), we obtain $B_t/R_t + \mathcal{E}_t B_t^*/R_t^* = 0$. Substituting the zero net positions for households and central bank into Equation (A28) yields:

$$\frac{\tilde{R}_t^* - \tilde{R}_t - \Delta \tilde{e}_{t+1} + \sigma_{et}^2}{\omega \sigma_{et}^2 / m_d} = \frac{1}{P_t^*} m_n n_t^* - \frac{R_t^*}{R_t} \frac{1}{e_t} \frac{1}{P_t^*} m_n f_t^* - \frac{R_t^*}{R_t} \frac{Y_t}{e_t} \frac{B_t}{P_t Y_t}. \tag{A29}$$

Log-linearizing this gives the second equality:

$$\tilde{r}_t - \tilde{r}_t^* - E_t \Delta \tilde{e}_{t+1} = \chi_1 (n_t^* - f_t) - \chi_2 b_t. \tag{A30}$$

Combining Equations (A24) and (A30), we obtain the UIP equation (11). □

Incomes and Losses of Carry Trade Positions. For simplicity, I assume that the profits and losses of carry-trade positions by the financiers and noise traders and interventions by the local central bank are transferred to the local households in a lump-sum way. However, the assumption on the ownership structure does not affect the first-order dynamics of the model, as discussed in Itskhoki and Mukhin (2021). To see this, combining the positions of the financiers, the noise traders, and the local central bank, the total carry trade profit can be written as:

$$\bar{R}_t^*(D_t + N_t + F_t) = -\bar{R}_t^* B_t = \bar{R}_t^* \bar{Y} b_t.$$

The combined carry trade profit equals the product of the UIP deviation ($\bar{R}_t^*$) and the households' net foreign asset position ($\bar{Y} b_t$). Each of them is first-order but their product is second-order and small enough relative to the size of the countries' budget constraint.

B.4 Proof of Propositions 1 and 2

Note that Proposition 1 is a special case of Proposition 2 where $\sigma = \phi = 1$ and there are no cost-push shocks. The central banks in the two countries face a similar minimization problem to the case without FXI (Appendix B.2), except that the local central bank chooses f_t optimally. The first-order condition for f_t implies:

$$\lambda_t = 0. \tag{A31}$$

First, combining Equation (A31) with Equations (12), (A12), (A13), (A14), and (A15), we obtain the difference rule:

$$\begin{aligned}
0 = &(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}) - (\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}) + \theta(\pi_{Lt} - \pi_{Ut}^*) \\
&+ 2(1-a)\frac{2a(\sigma\phi - 1) + 1}{\sigma + \eta\{4a(1-a)(\sigma\phi - 1) + 1\}} \frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1} \theta(\pi_{Lt} - \pi_{Ut}^*) \\
&+ \frac{4a(1-a)\phi}{\sigma + \eta\{4a(1-a)(\sigma\phi - 1) + 1\}} \frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1} (\tilde{\mathcal{D}}_t - \tilde{\mathcal{D}}_{t-1}).
\end{aligned}$$

Combining this with the sum rule (A17), we obtain the country-specific monetary policy rules (19) and (20). Next, combining Equation (A31) with Equations (A14), (A15), and (A16), we obtain the optimal FXI rule (21). This proves Proposition 2.

In the special case where $\sigma = \phi = 1$, since $\psi_\pi = \psi_D = 0$, the optimal output gap and inflation are pinned down by Equations (A18) through (A21), which are the same as in the no FXI case. Hence, the optimal FXI is to set $f_t = n_t^*$ and the first-best equilibrium $\pi_{Lt} = \pi_{Ut}^* = \tilde{Y}_{Lt} = \tilde{Y}_{Ut} = \tilde{\mathcal{D}}_t = 0$ is achieved.

B.5 Proof of Lemma 4

When $f_t = n_t^* = 0$, the demand gap $\tilde{\mathcal{D}}_t$ is zero on average and at most second order. I first consider the US inflation and output gap. Combining the NKPC (7) for the US firms and the optimal monetary policy rule (14) and assuming the economy is initially at the steady state ($\tilde{Y}_{L,-1} = 0$):

$$\frac{\partial \pi_{U0}^*}{\partial \mu_0^*} = \frac{1}{1+\theta\kappa} > 0, \quad \frac{\partial \tilde{Y}_{U0}}{\partial \mu_0^*} = -\frac{\theta}{1+\theta\kappa} < 0.$$

in period 0 and:

$$\frac{\partial \pi_{U0}^*}{\partial \mu_0^*} = -\frac{\theta\kappa}{(1+\theta\kappa)^{t+1}} < 0, \quad \frac{\partial \tilde{Y}_{U0}}{\partial \mu_0^*} = -\frac{\theta}{(1+\theta\kappa)^{t+1}} < 0.$$

in period $t \geq 1$. This confirms Equation (16).

Next, I consider the transmission of the US cost-push shock to the real exchange rate and the local inflation and output gap. Using Equations A4 (with $\tilde{\Delta}_{Lt} = 0$), the elasticity of the terms-of-trade satisfies:

$$\frac{\partial \tilde{\mathcal{T}}_0}{\partial \mu_0^*} > \frac{\partial \tilde{\mathcal{T}}_1}{\partial \mu_0^*} > \cdots > 0.$$

Using the relationship (A11) between the real exchange rate and the terms of trade, we obtain Equation (18). To simplify the proof, I consider the case where $1+\theta\kappa$ is large enough so that $\mathcal{T}_0$ has a first-order effect on the local inflation while $\mathcal{T}_t$ ($t \geq 1$) does not. As shown in Equation (6) $\sigma\phi > 1$, an increase in $\mathcal{T}_t$ is analogous to a decrease in μ_t. Hence, Equation (17) can be proven similarly to Equation (16). □

B.6 Proof of Proposition 3

From Equations (16), (17), and (21), optimal FXI satisfies $\partial \tilde{f}_t / \partial \mu_0^* > 0$ for all t when $\mu_0^* > 0$ and $\mu_t^* = 0$ for all $t \geq 1$. From Equation (11),

$$\frac{\partial \widetilde{UIP}_t^{FXI}}{\partial \tilde{\mathcal{D}}_t} < 0, \quad \text{and} \quad \frac{\partial \tilde{\mathcal{D}}_t}{\partial f_t} > 0,$$

for a given value of $\tilde{\mathcal{D}}_{t+1}$. Since $\partial \tilde{Y}_{Lt}/\partial \tilde{\mathcal{T}}_t = 2a(\phi - 1) + 1$ and:

$$\frac{\partial \tilde{\mathcal{T}}_t}{\partial \tilde{\mathcal{D}}_t} = -\frac{2a-1}{4a(1-a)(\sigma\phi - 1) + 1} < 0, \tag{A32}$$

from Equation (A11), we have:

$$\frac{\partial \tilde{Y}_{Lt}}{\partial \tilde{\mathcal{D}}_t} = \frac{\partial \tilde{Y}_{Lt}}{\partial \tilde{\mathcal{T}}_t}\frac{\partial \tilde{\mathcal{T}}_t}{\partial \tilde{\mathcal{D}}_t} = -\frac{(2a-1)[2a(\phi-1)+1]}{4a(1-a)(\sigma\phi-1)+1}$$

Hence, we have:

$$\frac{\partial \tilde{Y}_{Lt}}{\partial \mu_0^*} > \frac{\partial \tilde{Y}_{Lt}^{FXI}}{\partial \mu_0^*}.$$

Combining this result with the optimal policy rule (13), we obtain:

$$\frac{\partial \pi_{L0}}{\partial \mu_0^*} < \frac{\partial \pi_{L0}^{FXI}}{\partial \mu_0^*}, \quad \frac{\partial \pi_{Lt}}{\partial \mu_0^*} > \frac{\partial \pi_{Lt}^{FXI}}{\partial \mu_0^*}.$$

This proves Equation (17). Equation (16) can be proved analogously. Combining Equations (A4) (with $\tilde{\Delta}_{Lt} = 0$) and (A32), we obtain:

$$\frac{\partial \tilde{Q}_t}{\partial \mu_0^*} > \frac{\partial \tilde{Q}_t^{FXI}}{\partial \mu_0^*}.$$

This proves Equation (24). □

B.7 Proof of Lemma 5

The proof follows (Corsetti et al. 2020; 2023). Under cooperation and commitment, the central banks in the two countries minimize the loss (28) subject to the NKPCs (26), (27), and (7), the UIP condition (12), and the condition that relates the relative price to the terms-of-trade and the LOOP deviation:

$$\pi_{Ut} - \pi_{Lt} = \tilde{\mathcal{T}}_t - \tilde{\mathcal{T}}_{t-1} + \Delta_{Lt} - \Delta_{Lt-1}.$$

Let γ_{Lt}, γ_{Lt}^*, and γ_{Ut}^* be the Lagrange multipliers for the local and US NKPCs, λ_t for the UIP condition, and γ_t for the terms-of-trade equation.

The first-order conditions can be written as:

$$\tilde{Y}_{Lt}: \quad 0 = -(\sigma+\eta)\tilde{Y}_{Lt} + \frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}(\tilde{Y}_{Lt}-\tilde{Y}_{Ut})$$
$$-\frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t)$$
$$+\left[\sigma+\eta-\frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\right]\kappa(\gamma_{Lt}+\gamma^*_{Lt}) + \frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\kappa\gamma^*_{Ut}$$
$$+\frac{1}{2a(\phi-1)+1}(\beta E_t\gamma_{t+1}-\gamma_t) - \frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\lambda_t-\beta^{-1}\lambda_{t-1}), \quad \text{(A33)}$$

$$\tilde{Y}_{Ut}: \quad 0 = -(\sigma+\eta)\tilde{Y}_{Ut} - \frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}(\tilde{Y}_{Lt}-\tilde{Y}_{Ut})$$
$$+\frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t)$$
$$+\left[\sigma+\eta-\frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\right]\kappa\gamma^*_{Ut} + \frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\kappa(\gamma_{Lt}+\gamma^*_{Lt})$$
$$-\frac{1}{2a(\phi-1)+1}(\beta E_t\gamma_{t+1}-\gamma_t) + \frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\lambda_t-\beta^{-1}\lambda_{t-1}), \quad \text{(A34)}$$

$$\pi_{Lt}: \quad 0 = -\frac{\theta}{\kappa}\pi_{Lt} - \gamma_{Lt} + \gamma_{Lt-1} - \gamma_t, \quad \text{(A35)}$$

$$\pi^*_{Lt}: \quad 0 = -\frac{\theta}{\kappa}\pi^*_{Lt} - \gamma^*_{Lt} + \gamma^*_{Lt-1}, \quad \text{(A36)}$$

$$\pi^*_{Ut}: \quad 0 = -\frac{\theta}{\kappa}\pi^*_{Ut} - \gamma^*_{Ut} + \gamma^*_{Ut-1}, \quad \text{(A37)}$$

$$\mathcal{B}_t: \quad 0 = \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}(E_t\tilde{\mathcal{D}}_{t+1}-\tilde{\mathcal{D}}_t) -$$
$$+(1-a)\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}\kappa\left[\begin{array}{c}(E_t\gamma_{Lt+1}-\gamma_{Lt})+(E_t\gamma_{Lt+1}-\gamma_{Lt})\\ -(E_t\gamma^*_{Ut+1}-\gamma^*_{Ut})\end{array}\right]$$
$$-\left[(E_t\lambda_{t+1}-\beta^{-1}\lambda_t)-(\lambda_t-\beta^{-1}\lambda_{t-1})\right]$$
$$-\frac{2a-1}{4a(1-a)(\sigma\phi-1)+1}\left[(\beta E_t\gamma_{t+2}-E_t\gamma_{t+1})-(\beta E_t\gamma_{t+1}-\gamma_t)\right], \quad \text{(A38)}$$

$$\tilde{\Delta}_{Lt}: \quad 0 = -\frac{2a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t) + \kappa\frac{1}{4a(1-a)(\sigma\phi-1)+1}$$
$$\times\frac{1}{2}\left[\begin{array}{l}(4a(1-a)(\sigma\phi-1)+1)(\gamma_{Lt}-(\gamma^*_{Lt}+\gamma^*_{Ut}))\\ -\left\{(2a-1)-2(1-a)[2a(\phi-1)+1]\dfrac{2a(1-a)(\sigma\phi-1)+1-\phi}{2a(\phi-1)+1}\right\}\\ \times(\gamma_{Lt}+\gamma^*_{Lt}-\gamma^*_{Ut})\end{array}\right]$$

$$
\begin{aligned}
&- \frac{2a-1}{2a(\phi-1)+1}(\beta E_t \gamma_{t+1} - \gamma_t) \\
&- \frac{2a[2(1-a)(\sigma\phi-1)+1-\phi]}{2a(\phi-1)+1}(\lambda_t - \beta^{-1}\lambda_{t-1}). \qquad \text{(A39)}
\end{aligned}
$$

Under the assumption of $f_t = n_t^* = 0$, we have $E_t\tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t = 0$. The FOC for the net foreign asset implies:

$$
\begin{aligned}
\lambda_t - \beta^{-1}\lambda_{t-1} = &(1-a)\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}\theta(\gamma_{Lt} + \gamma_{Lt}^* - \gamma_{Ut}^*) \\
&- \frac{2a-1}{4a(1-a)(\sigma\phi-1)+1}(\beta E_t\gamma_{t+1} - \gamma_t). \qquad \text{(A40)}
\end{aligned}
$$

The sum rule for the output gaps is given by Equation (A17). Using the symmetry (Equation (A5)), the sum rule can be rewritten as:

$$
0 = \theta[a\pi_{Lt} + (1-a)\pi_{Lt}^* + \pi_{Ut}^*] + (\tilde{C}_t - \tilde{C}_{t-1}) + (\tilde{C}_t^* - \tilde{C}_{t-1}^*). \qquad \text{(A41)}
$$

To derive the difference rule, by taking the difference between Equations (A33) and (A34) and substituting Equation (A40), we obtain:

$$
\begin{aligned}
2\sigma(\beta E_t\gamma_{t+1} - \gamma_t) = &\sigma[(\tilde{Y}_{Lt} - \tilde{Y}_{Ut})] \\
&+ 4a(1-a)\phi\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t) \\
&- \sigma\kappa(\gamma_{Lt} + \gamma_{Lt}^* - \gamma_{Ut}^*). \qquad \text{(A42)}
\end{aligned}
$$

Moreover, substituting Equation (A40) into Equation (A39) yields:

$$
\begin{aligned}
&\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}(\tilde{Y}_{Lt} - \tilde{Y}_{Ut}) \\
&= -\frac{4a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t) + \kappa(\gamma_{Lt} - (\gamma_{Lt}^* + \gamma_{Ut}^*)) - (2a-1)\kappa\frac{\gamma_{Lt} + \gamma_{Lt}^* - \gamma_{Ut}^*}{4a(1-a)(\sigma\phi-1)+1}. \qquad \text{(A43)}
\end{aligned}
$$

Combining Equations (A42) and (A43) and rearranging the terms, we obtain:

$$
\begin{aligned}
&\frac{2a-1}{4a(1-a)(\sigma\phi-1)+1}\sigma(\tilde{Y}_{Lt} - \tilde{Y}_{Ut}) \\
&+ \left[(2a-1)\frac{2a(\sigma\phi-1)+1-\sigma}{4a(1-a)(\sigma\phi-1)+1} + \sigma\right]\frac{4a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t)
\end{aligned}
$$

$$=\sigma\kappa(y_{Lt}-(y_{Lt}+y^*_{Ut})).$$

Using the relationships (A4) and (A11), the left-hand side can be rewritten as:

$$\begin{aligned}
&(2a-1)\left[\tilde{\mathcal{T}}_t+\tilde{\Delta}_{Lt}+\frac{(2a-1)(\tilde{\mathcal{D}}_t+\tilde{\Delta}_{Lt})}{4a(1-a)(\sigma\phi-1)+1}\right]\\
&\quad+\left[(2a-1)\frac{2a(\sigma\phi-1)+1-\sigma}{4a(1-a)(\sigma\phi-1)+1}+\sigma\right]\frac{4a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\mathcal{D}}_t+\tilde{\Delta}_{Lt})\\
&=(\tilde{Q}_t-\tilde{\Delta}_{Lt})+(\tilde{\mathcal{D}}_t+\Delta_{Lt})-\frac{4a(1-a)\sigma\phi}{4a(1-a)(\sigma\phi-1)+1}(\tilde{\mathcal{D}}_t+\Delta_{Lt})\\
&\quad+\left[(2a-1)\frac{2a(\sigma\phi-1)+1-\sigma}{4a(1-a)(\sigma\phi-1)+1}+\sigma\right]\frac{4a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\mathcal{D}}_t+\tilde{\Delta}_{Lt})\\
&=\tilde{Q}_t+\tilde{\mathcal{D}}_t+\frac{4a(1-a)\phi(\sigma-1)}{2a(\phi-1)+1}(\tilde{\mathcal{D}}_t+\tilde{\Delta}_{Lt}).
\end{aligned}$$

Using the FOCs for the inflation rates and rearranging the terms, we obtain the difference rule:

$$\begin{aligned}
0=&\theta[a\pi_{Lt}-(1-a)\pi^*_{Lt}-\pi^*_{Ut}]+(\tilde{C}_t-\tilde{C}_{t-1})-(\tilde{C}^*_t-\tilde{C}^*_{t-1})\\
&+\frac{4a(1-a)\phi}{2a(\phi-1)+1}\frac{\sigma-1}{\sigma}(\tilde{\mathcal{D}}_t-\tilde{\mathcal{D}}_{t-1}+\tilde{\Delta}_{Lt}-\tilde{\Delta}_{Lt-1}).
\end{aligned}\tag{A44}$$

Combining the sum and difference rules (Equations (A41) and (A44)), the country-specific monetary policy rules are given by Equations (29) and (30). □

B.8 Optimal Monetary Policy and FXI under DCP

This section provides a full characterization of optimal monetary policy and FXI rules under DCP and provides proofs of Propositions 4 and 5. Let $y_{\Delta t}\equiv 2(\beta E_t y_{t+1}-y_t)$. First, combining the FOCs (A33) and (A34) for the output gap and (A31) for the FXI, the difference rule for the output gap can be written as:

$$\begin{aligned}
y_{\Delta t}=&\frac{\sigma}{4a(1-a)(\sigma\phi-1)+1}[2a(\phi-1)+1](\tilde{Y}_{Lt}-\tilde{Y}_{Ut})\\
&+\frac{4a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}[2a(\sigma\phi-1)+1-\sigma](\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t)\\
&+[\sigma(2a(\phi-1)+1)-2(1-a)(\sigma-1)]\theta[a\pi_{Lt}+(1-a)\pi^*_{Lt}-\pi^*_{Ut}].
\end{aligned}\tag{A45}$$

Next, from the FOC (A39) for the LOOP deviation and (A31) for the FXI:

$$\gamma_{\Delta t} = -\frac{4a(1-a)\phi}{2a-1}(\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t) + \theta\frac{1}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\phi-1)+1}{2a-1} \times \begin{bmatrix} (4a(1-a)(\sigma\phi-1)+1)[a\pi_{Lt} - ((1-a)\pi^*_{Lt} + \pi^*_{Ut})] \\ -\left\{(2a-1) - 2(1-a)[2a(\phi-1)+1]\frac{2a(1-a)(\sigma\phi-1)+1-\phi}{2a(\phi-1)+1}\right\} \\ \times[a\pi_{Lt} + (1-a)\pi^*_{Lt} + \pi^*_{Ut}] \end{bmatrix} \tag{A46}$$

The optimal monetary policy rules can be characterized by the sum rule (A17), the difference rule (A45), and the optimal LOOP deviation (A46). The general implication is that the monetary policy cannot close all gaps but instead, it faces a trade-off between stabilizing inflation, output, demand gap, and LOOP deviation.

To derive the optimal FXI rule, using the FOC (A38) and the UIP condition (12):

$$\begin{aligned} f_t = n^*_t &+ \frac{\theta}{2a\phi\chi_1}[2a(\sigma\phi-1)+1]E_t[a\pi_{Lt+1} + (1-a)\pi^*_{Lt+1} + \pi^*_{Ut+1}] \\ &+ \frac{2a-1}{4a(1-a)\phi\chi_1}(E_t\gamma_{\Delta t+t} - \gamma_{\Delta t}), \end{aligned} \tag{A47}$$

where $\gamma_{\Delta t}$ is given in Equation (A46).

Under Cole and Obstfeld (1991) case, the above conditions reduce to:

$$\begin{aligned} 0 &= (\tilde{Y}_{Lt} + \tilde{Y}_{Ut}) + \theta[a\pi_{Lt} + (1-a)\pi^*_{Lt} + \pi^*_{Ut}], \\ \gamma_{\Delta_{Lt}} - \gamma_{\Delta_{Lt-1}} &= (\tilde{Y}_{Lt} - \tilde{Y}_{Ut}) + \theta[a\pi_{Lt} + (1-a)\pi^*_{Lt} - \pi^*_{Ut}], \\ \gamma_{\Delta_{Lt}} &= -\frac{4a(1-a)}{2a-1}(\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t) + \theta\frac{1}{2a-1} \\ &\quad \times [a\pi_{Lt} - ((1-a)\pi^*_{Lt} + \pi^*_{Ut})] - (2a-1)[a\pi_{Lt} + (1-a)\pi^*_{Lt} + \pi^*_{Ut}] \\ f_t &= n^*_t + \frac{\theta}{2a\chi_1}E_t[a\pi_{Lt+1} + (1-a)\pi^*_{Lt+1} + \pi^*_{Ut+1}] \\ &\quad + \frac{2a-1}{2a(1-a)\chi_1}(E_t\gamma_{\Delta t+t} - \gamma_{\Delta t}). \end{aligned}$$

Combining these equations, optimal monetary policy and FXI rules are characterized by:

$$0 = \tilde{Y}_{Lt} + \theta[a\pi_{Lt} + (1-a)\pi^*_{Lt}], \tag{A48}$$

$$0 = \tilde{Y}_{Ut} + \theta\pi^*_{Ut} + \gamma_{\Delta_{Lt}} - \gamma_{\Delta_{Lt-1}}, \tag{A49}$$

$$\gamma_{\Delta_{Lt}} = -\frac{4a(1-a)}{2a-1}(\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t) + \theta\frac{1}{2a-1}$$
$$\times [a\pi_{Lt} - ((1-a)\pi^*_{Lt} + \pi^*_{Ut})] - (2a-1)[a\pi_{Lt} + (1-a)\pi^*_{Lt} + \pi^*_{Ut}] \quad \text{(A50)}$$
$$f_t = n^*_t + \frac{\theta}{2(1-a)\chi_1}E_t[a\pi_{Lt+1} + (1-a)\pi^*_{Lt+1} + \pi^*_{Ut+1}]$$
$$+ \frac{2a-1}{2a(1-a)\chi_1}[(E_t\tilde{Y}_{Lt+1} - \tilde{Y}_{Lt}) - (E_t\tilde{Y}_{Ut+1} - \tilde{Y}_{Ut})]. \quad \text{(A51)}$$

There are two key implications. First, the optimal monetary policy rule is asymmetric. The local central bank trades off inflation and output growth of locally produced goods. However, the US central bank trades off the US inflation and output growth, as well as LOOP deviation and demand gap. Second, and more importantly, the optimal FXI targets the LOOP deviation, as discussed in the next proposition.

B.9 Proof of Proposition 4

From Equation (A50), $\partial\gamma_{\Delta t}/\partial\tilde{\Delta}_{Lt} < 0$. Hence,

$$\frac{\partial f_t}{\partial\tilde{\Delta}_{Lt}} = \frac{\partial f_t}{\partial\gamma_{\Delta t}}\frac{\partial\gamma_{\Delta t}}{\partial\tilde{\Delta}_{Lt}} > 0.$$

Thus, the optimal FXI is increasing in Δ_{Lt}. Next, similarly to the PCP case,

$$\frac{\partial\tilde{Q}_0}{\partial\mu^*_0} > \frac{\partial\tilde{Q}_1}{\partial\mu^*_0} > \cdots > 0, \quad \frac{\partial\tilde{Q}_t}{\partial\mu^*_0} > \frac{\partial\tilde{Q}^{FXI}_t}{\partial\mu^*_0}.$$

Since $\tilde{\mathcal{E}}_t$ is close to $\tilde{Q}_t$ when the price stickiness is sufficiently high,

$$\frac{\partial\tilde{\Delta}_{L0}}{\partial\mu^*_0} > \frac{\partial\tilde{\Delta}_{L1}}{\partial\mu^*_0} > \cdots > 0, \quad \frac{\partial\tilde{\Delta}_{Lt}}{\partial\mu^*_0} > \frac{\partial\tilde{\Delta}^{FXI}_{Lt}}{\partial\mu^*_0}.$$

Hence, the FXI reduces the LOOP deviation. Finally, to show that the optimal FXI is larger under DCP than PCP, the optimal FXI rule under PCP and $\sigma = \phi = 1$ is characterized by:

$$f_t = n^*_t + \frac{\theta}{2a\chi_1}E_t(\pi_{Lt+1} - \pi^*_{Ut+1}). \quad \text{(A52)}$$

I compare the optimal FXI rules (A52) under PCP and (A51) under DCP. First, for the output gap term in Equation (A51), when $\sigma = \phi = 1$, since $\partial\tilde{\mathcal{D}}_t/\partial\tilde{\Delta}_{Lt} = 0$, $\partial\tilde{Y}_{Lt}/\partial\tilde{\Delta}_{Lt} = (\partial\tilde{Y}_{Lt}/\partial\tilde{\mathcal{D}}_t)(\partial\tilde{\mathcal{D}}_t/\partial\tilde{\Delta}_{Lt}) = 0$. Similarly, $\partial\tilde{Y}_{Ut}/\partial\tilde{\Delta}_{Lt} = 0$. Next, for the local inflation, from the

NKPCs (26) and (27), $\partial\pi_{Lt}/\partial\tilde{\Delta}_{Lt} = 0$ under PCP and $\partial(a\pi_{Lt} + (1-a)\pi^*_{Ut})/\partial\tilde{\Delta}_{Lt} = -2\kappa(1-a)$, which is second-order when the home bias is large enough (large a). For the US inflation, $\partial\pi^*_{Ut}/\partial\tilde{\Delta}_{Lt} = 0$ under both PCP and DCP. Hence, up to the first order and without FXI,

$$\left(\frac{\partial E_t(\pi_{Lt+1} - \pi^*_{Ut+1})}{\partial\mu^*_0}\right)^{PCP} \fallingdotseq \left(\frac{\partial E_t(a\pi_{Lt+1} + (1-a)\pi^*_{Lt+1} - \pi^*_{Ut+1})}{\partial\mu^*_0}\right)^{DCP} > 0.$$

The reaction coefficient to the inflation differential is larger under DCP than PCP: [24]

$$\frac{\theta}{2(1-a)\chi_1} > \frac{\theta}{2a\chi_1}.$$

Hence,

$$\left(\frac{\partial f_t}{\partial\mu^*_t}\right)^{DCP} > \left(\frac{\partial f_t}{\partial\mu^*_t}\right)^{PCP}.$$

□

B.10 Proof of Proposition 5

First, I consider the PCP case. Since $\partial f_t/\partial\mu^*_t > 0$ and $\partial\tilde{\mathcal{D}}_t/\partial f_t > 0$, I consider the elasticity of inflation to the demand gap. From the NKPCs for the domestic good inflation in the two countries,

$$\frac{\partial\pi_{Lt}}{\partial\tilde{\mathcal{D}}_t} = -\frac{\partial\pi^*_{Ut}}{\partial\tilde{\mathcal{D}}_t} = \kappa(1-a).$$

Hence, $\partial\pi_{Lt}/\partial\mu^*_t = -\partial\pi^*_{Ut}/\partial\mu^*_t$.

For the imported inflation, from the law of one price,

$$\pi^*_{Ut} = \tilde{\mathcal{E}}_t - \tilde{\mathcal{E}}_{t-1} + \pi^*_{Ut}, \quad \pi^*_{Lt} = -(\tilde{\mathcal{E}}_t - \tilde{\mathcal{E}}_{t-1}) + \pi_{Lt}. \tag{A53}$$

Hence,

$$\frac{\partial\pi_{Ut}}{\partial\tilde{\mathcal{D}}_t} = -\frac{\partial\pi^*_{Lt}}{\partial\tilde{\mathcal{D}}_t}, \quad \frac{\partial\pi_{Ut}}{\partial\mu^*_t} = -\frac{\partial\pi^*_{Lt}}{\partial\mu^*_t}$$

[24]The difference between inflation differential terms under PCP and DCP is quantitatively at most second-order. The difference in the optimal FXI volumes under PCP and DCP is mainly because the reaction coefficient to the inflation is larger under DCP, which is due to the deviation from the LOOP.

Hence, the response of CPI inflation is symmetric.

$$\left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{PCP} = -\left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{PCP} \quad (> 0).$$

Next, I consider the DCP case. Since the optimal FXI is larger under DCP than PCP (Equation (31)),

$$\left(\frac{\partial \tilde{Q}_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \tilde{Q}_t}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \tilde{Q}_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \tilde{Q}_t}{\partial \mu_t^*}\right)^{PCP} \quad (< 0),$$

For the local imports of US goods, since the LOOP holds,

$$\left(\frac{\partial \pi_{Ut}^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_{Ut}}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_{Ut}^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_{Ut}}{\partial \mu_t^*}\right)^{PCP} \quad (< 0),$$
$$\left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{PCP} \quad (< 0).$$

Next, consider the US imports of local goods. Combining Equations (A4) and (A11) and using $\partial \tilde{\Delta}_{Lt} / \partial \tilde{\mathcal{D}}_t = 0$ when $\sigma = \phi = 1$,

$$\frac{\partial \tilde{Q}_t}{\partial \tilde{\mathcal{D}}_t} = -(2a-1)^2 < 0.$$

When the price stickiness is sufficiently high, $\partial \tilde{\mathcal{E}}_t / \partial \tilde{\mathcal{D}}_t \fallingdotseq (2a-1)^2 < 0$.

Under PCP, since π_{Lt}^* is determined by the LOOP condition (A53),

$$\left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mathcal{D}}_t}\right)^{PCP} = -\frac{\partial \tilde{\mathcal{E}}_t}{\partial \tilde{\mathcal{D}}_t} + \frac{\partial \tilde{\pi}_{Lt}^*}{\partial \tilde{\mathcal{D}}_t} \fallingdotseq (2a-1)^2 + \kappa(1-a).$$

Under DCP, since the LOOP does not hold and π_{Lt}^* is determined by the NKPC (27),

$$\left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mathcal{D}}_t}\right)^{DCP} = \kappa(1-a).$$

Comparing the PCP and DCP cases,

$$\left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mathcal{D}}_t}\right)^{DCP} - \left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mathcal{D}}_t}\right)^{PCP} = -(2a-1)^2,$$

$$\left(\frac{\partial \pi^*_{Lt}}{\partial \tilde{\mu}^*_t}\right)^{DCP} - \left(\frac{\partial \pi^*_{Lt}}{\partial \tilde{\mu}^*_t}\right)^{PCP} < 0.$$

The difference is first order when a is sufficiently large. Hence,

$$\left(\frac{\partial \pi^{*FXI}_t}{\partial \mu^*_t} - \frac{\partial \pi^*_t}{\partial \mu^*_t}\right)^{DCP} < \left(\frac{\partial \pi^{*FXI}_t}{\partial \mu^*_t} - \frac{\partial \pi^*_t}{\partial \mu^*_t}\right)^{PCP} (> 0).$$

□