The Trilemma for Low Interest Rate Macroeconomics

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Secular stagnation

Japan since 1995:

- Nominal interest rate at the zero lower bound;
- Inflation near 0%;
- Weak GDP growth.

Despite:

- Money supply (M0): 100% of GDP;
- Public debt: 260% of GDP.

Secular Stagnation

Secular stagnation hypothesis:

- The economy fails to produce at full capacity due to a lack of demand;
- This is a permanent state of affairs.

Multiple equilibria problem:

- Keynesian **secular stagnation equilibrium** with:
  - Binding zero lower bound;
  - Low inflation;
  - Under-employment.
- **Neoclassical equilibrium** with:
  - Full employment;
  - Low (natural) real interest rate;
  - High inflation.
- **Ponzi equilibrium** with:
  - Full employment;
  - Low inflation;
  - Ponzi scheme.

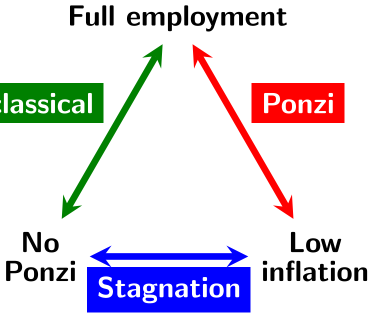
Secular Stagnation

Three desirable policy objectives:

- Full employment;
- Low inflation (on target);
- Low debt with no Ponzi scheme.

Trilemma:

- When the natural real interest rate is low, these three objectives are inconsistent.



Outline

1. **Model of secular stagnation**
2. Trilemma
3. Conclusion

Government

Evolution of nominal debt:

$$\dot{B}_t = i_t B_t - \tau_t P_t N_t.$$

Present value of real primary surpluses (per capita):

$$\Phi_t = \mathbb{E}_t \left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} \tau_s ds \right].$$

Real debt per capita:

$$b_t = \frac{B_t}{P_t N_t}.$$

Ponzi scheme:

$$\Delta_t = b_t - \Phi_t.$$

Government

The Ponzi scheme collapses at Poisson rate ε:

$$dP_t = \pi_t P_t dt + \frac{\Delta_t}{\Phi_t} P_t dJ_t,$$

where

$$dJ_t = \begin{cases} 1 & \text{with probability } \varepsilon dt \\ 0 & \text{with probability } (1 - \varepsilon dt) \end{cases} .$$

ε could be:

- A sunspost shock;
- A fundamental shock raising the natural real interest rate.

Model

Ramsey model:

- Preference for wealth:

$$\mathbb{E}_0 \left[\int_0^\infty e^{-(\rho - n)t} \left[u(c_t) + \gamma(a_t - b_t + \Delta_t) - \psi c \left(\frac{dP_t}{P_t} \right) \right] dt \right];$$

- Zero lower bound on the nominal interest rate;
- Downward nominal wage rigidity:
  - Nominal wage growth cannot fall below π^R,
  - Hence, $\pi_t \geq \pi^R$ and $L_t \leq 1$ with complementary slackness.

Equilibrium

Until the sunspot shock, the equilibrium $(c_t, \Delta_t, i_t, \pi_t, r_t)_{t=0}^{\infty}$ is given by:

$$\frac{\dot{c}_t}{c_t} = \left(\frac{u'(c_t)}{-c_t u''(c_t)}\right)\left[r_t - \rho + \frac{\gamma'(\Delta_t)}{u'(c_t)} + \varepsilon\left(\frac{u'(\bar{c}_t)}{u'(c_t)} - 1\right)\right];$$

$$\pi_t \geq \pi^R \text{ and } c_t \leq 1 \text{ with complementary slackness;}$$

$$(i_t - \pi_t) - r_t = \varepsilon \frac{\Delta_t}{b_t} \frac{u'(\bar{c}_t)}{u'(c_t)};$$

$$i_t = \max\{r^n + \pi^* + \phi[\pi_t - \pi^*], 0\};$$

$$\dot{\Delta}_t = \left[r_t - n + \varepsilon \frac{u'(\bar{c}_t)}{u'(c_t)}\right]\Delta_t;$$

$$\lim_{t\to\infty} e^{-(\rho - n + \varepsilon)t} u'(c_t)\Delta_t = 0.$$

Steady state equilibria

Neoclassical steady state:

- Full employment $c^n = 1$;
- No Ponzi scheme $\Delta^n = 0$;
- Real interest rate $r^n = \rho - \frac{\gamma'(0)}{u'(1)}$.

Secular stagnation steady state:

- Low inflation $\pi^{ss} = \pi^R$;
- Binding zero lower bound $i^{ss} = 0$;
- No Ponzi scheme $\Delta^{ss} = 0$;
- Underemployment $\frac{1}{u'(c^{ss})} = \frac{\rho+\pi^R}{\gamma'(0)}$.

Ponzi steady state:

- Full employment $c^p = 1$;
- Real interest rate $r^p = n - \varepsilon\frac{u'(\bar{c})}{u'(1)}$;
- Ponzi scheme $\gamma'(\Delta^p) = (\rho - n + \varepsilon)u'(1)$.

Steady state equilibria

Existence conditions:

- Secular stagnation steady state

$$r^n < -\pi^R;$$

- Neoclassical steady state

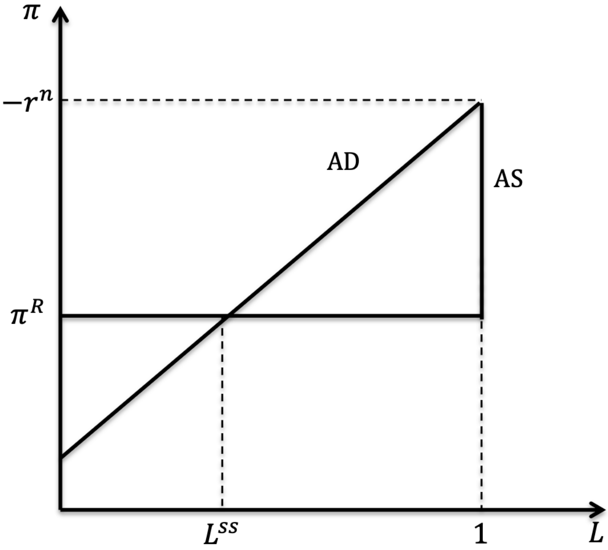
$$\pi^* \geq -r^n;$$

- Ponzi steady state

$$r^n < n - \varepsilon;$$

$$\pi^p \geq \max\left\{\varepsilon \frac{\Phi}{\Phi + \Delta^p} \frac{u'(\bar{c})}{u'(1)} - n, \pi^R\right\}.$$

Secular stagnation



Steady states with $i = 0$ and $\Delta = 0$

Secular stagnation

Paradox of flexibility:

- A rise in wage flexibility (lower π^R) reduces output!

Fundamental cause of secular stagnation:

- Existence of money!

The real interest rate is jointly determined by:

- Zero lower bound;
- Binding downward nominal wage rigidity.

Under-employment is a general equilibrium phenomenon:

- Excessive interest rate in financial markets $\Rightarrow$ Depressed demand for goods $\Rightarrow$ Insufficient demand for labor.

Outline

1. Model of secular stagnation
2. **Trilemma**
3. Conclusion

Trilemma

To reach the welfare maximizing steady state, the government chooses:

- the inflation target π^*;
- the magnitude of the Ponzi scheme Δ.

Welfare:

- Neoclassical steady state

$$u(1) + \gamma(0) + \psi r^n;$$

- Secular stagnation steady state

$$u(c^{ss}) + \gamma(0) - \psi \pi^R;$$

- Ponzi steady state

$$u(1) + \gamma(\Delta^p) - \psi \max\left\{\varepsilon \frac{\Phi}{\Phi + \Delta^p} \frac{u'(\bar{c})}{u'(1)} - n, \pi^R\right\} - \psi \varepsilon C\left(\frac{\Delta^p}{\Phi}\right).$$

Calibration

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta}$$

$$\gamma(a) = k \frac{(a - \underline{a})^{1-\sigma} - 1}{1 - \sigma}$$

Parameter	Calibrated value	Moment
Discount rate	$\rho = 5\%$	·
Population growth	$n = 0\%$	·
Reference rate of inflation for wage bargaining	$\pi^R = 0\%$	·
CRRA for consumption	$\theta = 4.46$	$r^n = -3\%$
CRRA for wealth (relative to reference level)	$\sigma = 1.16$	$\Delta^p = c^n$ when $\varepsilon = 0$
Scale parameter of preference for wealth	$k = 0.18$	$c^{ss} = (1 - 0.1)c^n$
Reference wealth level	$\underline{a} = -2$	$\underline{a} = -2c^n$
Present value of primary surpluses	$\Phi = 1$	$\Phi = c^n$

Calibration

Welfare cost of inflation under a Ponzi scheme:

$$\psi\left[\pi^p + \varepsilon C\left(\frac{\Delta^p}{\Phi}\right)\right].$$

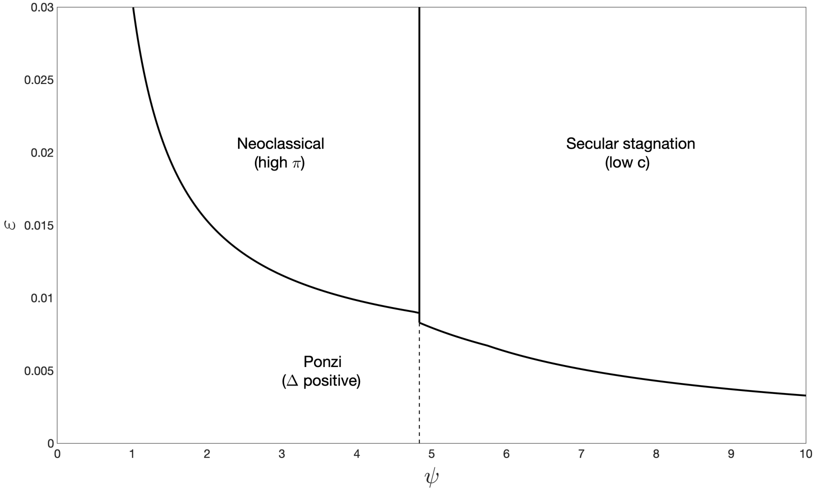
The convex welfare cost of a price level jump:

$$C(x) = \alpha\frac{(x+1)^\beta - 1}{\beta}$$

Cases:

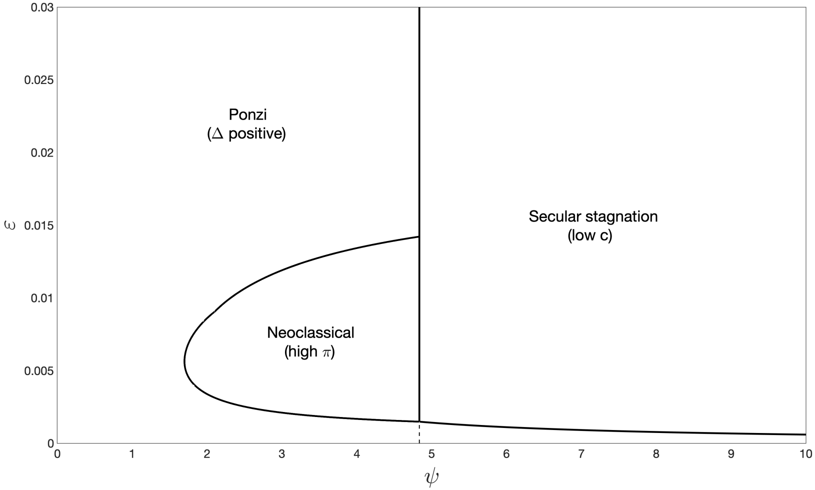
- $\alpha = 5$ and $\beta = 1$;
- $\alpha = 1$ and $\beta = 6$;
- $\alpha = 1$ and $\beta = 1$.

Trilemma



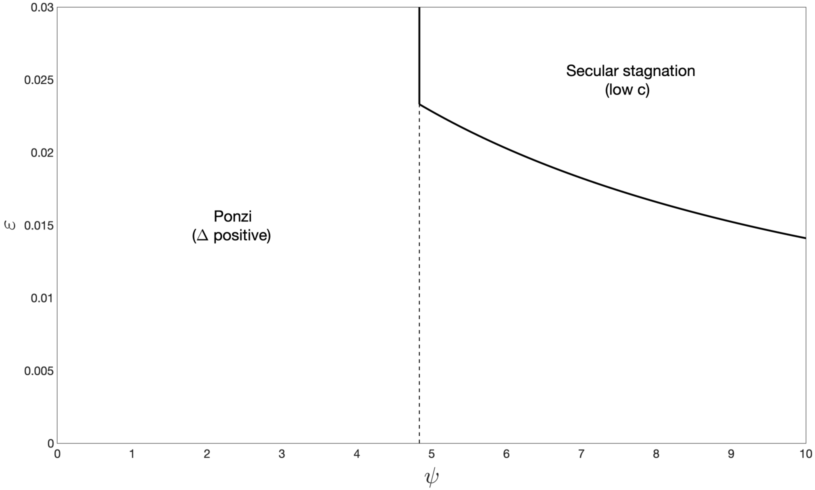
Trilemma for $\alpha = 5$ and $\beta = 1$

Trilemma



Trilemma for $\alpha = 1$ and $\beta = 6$

Trilemma



Trilemma for $\alpha = 1$ and $\beta = 1$

Breaking through the trilemma

How can we break through the trilemma (such as to have full employment, low inflation, and no Ponzi scheme)?

- Electronic money (abolish cash);
- Tax wealth or set an increasing consumption tax;
- Government spending;
- Redistribute across heterogeneous households.

Conclusion

If the 2% inflation target is too low, we must either have:

- Secular stagnation
  - Inflation is below target;
- Ponzi scheme
  - Inflation is much above target when the Ponzi scheme collapses.

In both cases, the central bank is powerless!

The trilemma is a fundamental challenge to the inflation targeting framework (with a low inflation target).