(Un-)Common Preferences, Ambiguity, and Coordination

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Motivation and goal

- The common prior assumption is pervasive in economic theory and helps us put restrictions on agents' beliefs
- Departures from Subjective Expected Utility (SEU) are both compelling (robustness as rationality) and present in experimental findings
- **The goal of this paper** is to:
  1. Propose notion of common priors for non-SEU preferences
     - Mutual dynamic consistency
  2. Characterize this notion in terms of interim preferences
     - Common limit of higher-order (nonlinear) expectations
  3. Study implications for coordination games on networks
     - Potential wedge between common ex-ante preference and (limit) coordination equilibrium

Setting

- Finite set of agents $I = \{1, ..., n\}$ and *finite* set of states Ω
- Each $i \in I$, endowed with a state partition (or information structure) Π_i
- If ω realizes, agent i knows that the state belongs to the cell $\Pi_i(\omega) \subseteq \Omega$
- Let Π_{meet} denote the meet (the public information) of the $\{\Pi_i\}_{i \in I}$
- Acts $f \in \mathbb{R}^\Omega$ represent state-contingent monetary consequences
- All agents have same risk preferences which we normalize wlog to $u = id$

The SEU case

- The interim belief (type) of every i is given by $P_i : \Omega \times 2^{\Omega} \to [0,1]$ such that
  1. For all $\omega \in \Omega$, $P_i(\omega, \cdot) \in \Delta(\Pi_i(\omega))$
  2. For all $E \subseteq \Omega$, the map $P_i(\cdot, E) : \Omega \to [0,1]$ is Π_i measurable

- $\bar{P} \in \Delta(\Omega)$ is a *prior* for i if

$$\bar{P}(E) = \sum_{\omega \in \Omega} P_i(\omega, E)\, \bar{P}(\omega) \qquad \forall E \in 2^{\Omega} \tag{1}$$

- $\bar{P} \in \Delta(\Omega)$ is a *common prior* if (1) holds for all $i \in I$

Ex-ante expectations

- A map $\bar{V} : \mathbb{R}^{\Omega} \to \mathbb{R}$ is an **ex-ante expectation** if it is normalized (i.e., $\bar{V}(1_{\Omega} k) = k$) and monotone.
- **Max-min preferences (Gilboa and Schmeidler, 1989)**: $\bar{V}(f) = \min_{p \in \bar{C}} \mathbb{E}_p[f]$ for some compact and convex set $\bar{C} \subseteq \Delta(\Omega)$
- **α-max-min preferences (Ghirardato Maccheroni Marinacci, 2004)**: $\bar{V}(f) = \alpha \min_{p \in \bar{C}} \mathbb{E}_p[f] + (1-\alpha) \max_{p \in \bar{C}} \mathbb{E}_p[f]$
- **Variational preferences (Maccheroni Marinacci Rustichini, 2006)**:

$$\bar{V}(f) = \min_{p \in \Delta(\Omega)} \{\mathbb{E}_p[f] + c(p)\}$$

  for some convex, lsc, grounded cost function $c : \Delta(\Omega) \to [0, \infty]$
  - Hansen and Sargent (2001) multiplier preferences: $c(p) = \lambda R(p || \bar{p})$

Interim expectations

- A map $V_i : \Omega \times \mathbb{R}^\Omega \to \mathbb{R}$ is an **interim expectation** for i if
  1. For all $\omega \in \Omega$, the function $V_i(\omega, \cdot) : \mathbb{R}^\Omega \to \mathbb{R}$ is normalized, monotone, *continuous*, and satisfies
  $$V_i\left(\omega, f1_{\Pi(\omega)} + h1_{\Pi(\omega)^c}\right) = V_i(\omega, f) \qquad \forall f, h \in \mathbb{R}^\Omega.$$
  2. For all $f \in \mathbb{R}^\Omega$ the function $V_i(\cdot, f) : \Omega \to \mathbb{R}$ is Π_i-measurable
- Say that $(\bar{V}, V_i, \Pi)$ is a **generalized conditional expectation** for i if they are *dynamically consistent*:
$$\bar{V}(f) = \bar{V}(V_i(\cdot, f)) \qquad \forall f \in \mathbb{R}^\Omega \tag{2}$$
- $\bar{V}$ is a **common ex-ante expectation** if (2) holds for all $i \in I$
- This definition captures both consistency among players and, for each player, consistency between periods

Examples of DC non-SEU preferences

- $V_i(\omega, \cdot)$ *maxmin* wrt $C_i(\omega) \subseteq \Delta(\Pi_i(\omega))$ for all $i \in I$

- Then $\bar{V}$ is a **common ex-ante expectation** if and only if it is *maxmin* wrt a set $C \subseteq \Delta(\Omega)$ such that each $C_i(\omega)$ is obtained from C by Bayesian updating and C is *rectangular*

- $V_i(\omega, \cdot)$ *multiplier* wrt $p_{i,\omega} \in \Delta(\Pi_i(\omega))$ and $\lambda_i > 0$ for all $i \in I$

- Then $\bar{V}$ is a **common ex-ante expectation** if and only if it is multiplier wrt $\bar{p} \in \Delta(\Omega)$ and $\lambda > 0$ such that each $p_{i,\omega}$ is obtained from $\bar{p}$ by Bayesian updating and $\lambda_i = \lambda$ for all $i \in I$

Generalized iterated expectations

- For every $i \in I$, the interim expectation of $f \in \mathcal{F}$ is a Π_i-measurable act $V_i(\cdot, f) \in \mathbb{R}^\Omega$. Therefore, $V_i : \mathbb{R}^\Omega \to \mathbb{R}^\Omega$

- For every sequence $(i_k)_{k\in\mathbb{N}}$ of players in I, let $V_{1:k} : \mathbb{R}^\Omega \to \mathbb{R}^\Omega$ denote the operator

$$V_{1:k}(f) = V_{i_k} \circ V_{i_{k-1}} \circ ... \circ V_{i_1}(f)$$

- Under SEU, this is equivalent to

$$\mathbb{E}_{P_{i_3}}\left[\mathbb{E}_{P_{i_2}}\left[\mathbb{E}_{P_{i_1}}\left[f|\Pi_{i_1}(\omega)\right]|\Pi_{i_2}(\omega)\right]|\Pi_{i_3}(\omega)\right]$$

Existence of common ex-ante expectation

- We say that $(i_m)_{m\in\mathbb{N}} \in \mathcal{I} \subseteq I^{\mathbb{N}}$ is an I-**sequence** if each agent appears infinitely often

Theorem

Let $\{(V_i, \Pi_i)\}_{i\in I}$ be a collection of full support interim expectations such that $\Pi_{meet} = \{\Omega\}$. The following statements are equivalent:

(i) *There exists a common ex-ante preference $\bar{V}$ for $\{(V_i, \Pi_i)\}_{i\in I}$;*

(ii) *For each $f \in \mathbb{R}^{\Omega}$ there exists $k_f \in \mathbb{R}$ such that for each I-sequence $(i_t)_{t\in\mathbb{N}}$*

$$\lim_{t\to\infty} V_{i_t} \circ V_{i_{t-1}} \circ \ldots \circ V_{i_2} \circ V_{i_1}(f) = k_f 1_{\Omega}.$$

In this case, for each $f \in \mathbb{R}^{\Omega}$, we have $\bar{V}(f) = k_f$.

- The common prior, even beyond SEU (Samet, 1998), can be characterized through a condition that only involves the **interim preferences** of the agents

Asset pricing beauty contest Golub and Morris (2017)

- Assume that each $i \in I$ represents a continuum of agents with common information Π_i and *variational full support* interim preferences
- Single asset $\hat{f} \in \mathbb{R}^{\Omega}$, sequentially traded in discrete time $t \in \mathbb{N}$ with *random matching* described by a *strongly connected* $W = (w_{ij})_{i,j \in N}$
- If an agent i holds the asset, with probability $\beta \in (0, 1)$ they will *privately* sell the asset to an agent from a randomly selected class $j \in I$ (*no learning*)

- With probability $(1-\beta)$ they will have to liquidate the asset and obtain its fundamental (uncertain) value $\hat{f}$

- *Bertrand competition* among agents in class j matched with the asset holder i: the price for the trade is equal to the willingness to pay of agents in class j

- We focus on **Markov perfect equilibria** of this sequential game: the strategy $\sigma_i \in \mathbb{R}^{\Omega}$ of each i only depends on their own information set and specifies their bid price

Markov perfect equilibrium

- Given the assumptions, the unique Markov perfect equilibrium $\hat{\sigma}$ is the one satisfying the previous best-response map

$$\hat{\sigma}_i(\omega) = V_i\left(\omega, (1-\beta)\hat{f} + \beta \sum_{j \in I} w_{ij}\hat{\sigma}_j\right) \quad \forall \omega \in \Omega, \forall i \in I$$

- The RHS is the maximum willingness to pay of i given ω for the asset

- Taking the limit $\beta \to 1$ corresponds to the pure beauty contest limit

Multiple interaction structures

- Following Golub and Morris (2017), define

$$Q = \left\{ q \in \Delta(\Omega)^{I \times \Omega} : \forall (i, \omega) \in I \times \Omega, c_{i,\omega}(q_{i,\omega}) = 0 \right\}$$

- Each $q \in Q$ combined with network W gives *interaction structure* $W^q \in \mathbb{R}_+^{(I \times \Omega) \times (I \times \Omega)}$ that is strongly connected:

$$w^q_{(i,\omega)(j,\omega')} = w_{ij} q_{i,\omega}(\omega') \qquad \forall i, j \in I, \forall \omega, \omega' \in \Omega$$

- Denote left PF eigenvector $\gamma^q \in \Delta(I \times \Omega)$ for each q

Limit characterization

Theorem

For all $i \in I$ and $\omega \in \Omega$,

$$\lim_{\beta \to 1} \sigma_i^{\beta}(\omega) = \min_{q \in Q} \sum_{(j,\omega') \in I \times \Omega} \gamma_{j,\omega'}^{q} \mathbb{E}_{q_{j,\omega'}} \left[\hat{f}\right]$$

Moreover, if there exists a common ex-ante preference $\bar{V}$ for $\{(V_i, \Pi_i)\}_{i \in I}$, then, for all $i \in I$ and $\omega \in \Omega$,

$$\lim_{\beta \to 1} \sigma_i^{\beta}(\omega) \geq \bar{V}\left(\hat{f}\right)$$

Remarks on the limit equilibrium

1. Limit equilibrium price independent of state and agent: selects equilibrium of pure coordination game at $\beta = 1$

2. Strong coordination motives in the market attenuate the ambiguity concern exhibited by the equilibrium evaluation

$$\lim_{\beta \to 1} \sigma_i^\beta (\omega) \geq V_i \left(\omega, \hat{f}\right) \qquad \forall i \in I, \forall \omega \in \Omega,$$

3. Limit equilibrium price is higher than the *shared* ex-ante evaluation $\bar{V}\left(\hat{f}\right)$ when exists: sharp difference with respect to the SEU case

Example: Irrelevance of misspecification concern

- Common prior $\mu^* \in \Delta(\Omega)$, but heterogeneous aversion to misspecification: each $i \in I$ evaluates any $f \in \mathbb{R}^\Omega$

$$\min_{p \in \Delta} \left\{ \mathbb{E}_p \left[\hat{f} \right] + \lambda_i R(p || \mu^*) \right\}$$

- Let $p_{\mu^*,i}(\omega, \cdot) = \mu^*(\cdot | \Pi_i(\omega))$. The interim evaluation of i at ω of any $f \in \mathbb{R}^\Omega$

$$V_i(\omega, f) = \min_{p \in \Delta} \left\{ \mathbb{E}_p[f] + \lambda_i R\left(p || p_{\mu^*,i}(\omega, \cdot)\right) \right\}$$

- Our theorem implies that

$$\lim_{\beta \to 1} \sigma_i^\beta(\omega) = \mathbb{E}_{\mu^*}\left[\hat{f}\right] \qquad \forall i \in I, \forall \omega \in \Omega$$

- When a common ex-ante preference $\bar{V}$ exists (i.e., $\lambda_i = \lambda$), we have $\lim_{\beta \to 1} \sigma_i^\beta(\omega) > \bar{V}\left[\hat{f}\right]$

Conclusion

- We have characterized the notion of common prior for a large class of preferences
- As in the SEU case, this characterization is expressed in terms of the agreement among infinite orders of iterated expectations
- These results allowed us to capture the effect of ambiguity attitudes in models of oligopolistic competition and strategic beauty contests
- In the paper, we provide sufficient and necessary conditions, both in terms of no trade, for the existence of a common rational preference

Weaker ex-ante expectations

- Often DC restrictive assumption for more general preferences than SEU with multiple info structures (Gumen and Savochkin, 2013, Ellis, 2018)

- Often weaker forms of ex-ante expectations are considered:

  1. We say that $V_{\circ}$ is a **lower common ex-ante expectation** for $(V_i, \Pi_i)_{i \in I}$ if

  $$V_{\circ}(f) \leq V_{\circ}(V_i(f)) \qquad \forall f \in \mathcal{F}, \forall i \in I$$

  2. We say that V° is a **upper common ex-ante expectation** for $(V_i, \Pi_i)_{i \in I}$ if

  $$V^{\circ}(f) \geq V^{\circ}(V_i(f)) \qquad \forall f \in \mathcal{F}, \forall i \in I$$

- Capture preference for gradual and one-shot resolution of uncertainty respectively

Extreme higher-order expectations

- Define $V_*, V^* : \mathbb{R}^\Omega \to \mathbb{R}$ by

$$V_*(f) = \inf_{\iota \in \mathcal{I}} \left\{ \lim_{m\to\infty} V_{i_m} \circ V_{i_{m-1}} \circ \ldots \circ V_{i_1}(f) \right\} = \inf_{\iota \in \mathcal{I}} \bar{V}_\iota(f)$$

and

$$V^*(f) = \sup_{\iota \in \mathcal{I}} \left\{ \lim_{m\to\infty} V_{i_m} \circ V_{i_{m-1}} \circ \ldots \circ V_{i_1}(f) \right\} = \sup_{\iota \in \mathcal{I}} \bar{V}_\iota(f)$$

- V_*, V^* are the lowest and the highest higher-order evaluations of act/bets

Extreme higher-order expectations

Theorem

Let $(V_i, \Pi_i)_{i \in I}$ *have full support and* $\Pi_{meet} = \{\Omega\}$. *The expectations* V_* *and* V^* *are respectively a lower and an upper common ex-ante expectation for* $(V_i, \Pi_i)_{i \in I}$. *Moreover, if* $V_\circ$ *and* V° *are a lower and an upper common ex-ante expectation for* $(V_i, \Pi_i)_{i \in I}$, *then*

$$V_*(f) \geq V_\circ(f) \quad \text{and} \quad V^*(f) \leq V^\circ \quad \forall f \in \mathcal{F}$$

- The extreme preferences constructed via higher-order expectations constitute tight bounds for the ex-ante preferences of the agents

Assumption on the preferences

- We say that a preference is **rational** if the function $\bar{V}_i$ is
  1. **Normalized**, that is, $\bar{V}_i(1_\Omega k) = k$ for all $k \in X$
  2. **Monotone**, that is, $f \geq g \implies \bar{V}_i(f) \geq \bar{V}_i(g)$
- These properties are equivalent to the following axioms:
  1. **Weak order**: the preference $\succsim_i$ is complete and transitive
  2. **Monotonicity**: $f \geq g \implies f \succsim_i g$ and $x > y \implies x1_\Omega \succ_i y1_\Omega$
  3. **Certainty Equivalent**: For each $f \in \mathcal{F}$ there exists $k \in X$ such that $f \sim_i k1_\Omega$

“Full-support” assumption

- Let e_ω denote the ω-element of the basis of $\mathbb{R}^\Omega$
- We say that the interim expectations $(V_i, \Pi_i)_{i\in I}$ have **full support** if there exists $\varepsilon > 0$ such that

$$V_i(\omega, f + \delta e_{\omega'}) - V_i(\omega, f) \geq \delta\varepsilon$$

for all $i \in I$, $\omega \in \Omega$, $\omega' \in \Pi_i(\omega)$, $f \in \mathbb{R}^\Omega$, $\delta > 0$ with $f + \delta e_{\omega'} \in \mathbb{R}^\Omega$

- Interpretation: $V_i(\omega, \cdot)$ is responsive (“derivative” bdd away from 0) to changes in consequences at all states $\omega' \in \Pi_i(\omega)$
- If $V_i(\omega, \cdot)$ is SEU or multiplier, then it has full support* if $p_{i,\omega}(\omega') > 0$ for all $\omega' \in \Pi_i(\omega)$
- If $V_i(\omega, \cdot)$ is maxmin, then it has full support if $p(\omega') > 0$ for all $\omega' \in \Pi_i(\omega)$ and $p \in C_i(\omega)$

Sketch of the proof

- Fix $j \in I$, $\omega \in \Omega$ and define the "unambiguous" preference relation $\succsim^*_{j,\omega}$ of agent j at state ω on $\mathcal{F}$ by

$$f \succsim^*_{j,\omega} g \iff V_j(\omega, \lambda f + (1-\lambda) h) \geq V_j(\omega, \lambda g + (1-\lambda) h) \quad \forall \lambda \in (0,1]$$

- $V_j(\omega, \cdot)$ is normalized, monotone, and continuous $\Longrightarrow$ there exists a compact and convex $C_j(\omega) \subseteq \Delta(\Omega)$ such that

$$f \succsim^*_{j.\omega} g \iff \mathbb{E}_p[f] \geq \mathbb{E}_p[g] \qquad \forall p \in C_j(\omega)$$

and

$$V_j(\omega, f) = \alpha_j(f) \min_{p \in C_j(\omega)} \mathbb{E}_p[f] + (1 - \alpha_j(f)) \max_{p \in C_j(\omega)} \mathbb{E}_p[f] \quad \forall f \in \mathcal{F}$$

where $\alpha_j : \mathcal{F} \to [0,1]$

- Full support implies that $p(\omega') > 0$ for all $\omega' \in \Delta(\Pi_j(\omega))$ and $p \in C_j(\omega)$

- Therefore, for each $j \in I$ and $f \in \mathcal{F}$ we can build a Markov transition (stochastic matrix) M_j such that each row $M_j(\omega) \in C_j(\omega) \subseteq \Delta(\Pi_j(\omega))$ and

$$V_j(\omega, f) = M_j(\omega) f.$$

- Since $\Pi_{meet} = \{\Omega\}$, for every finite sequence $\{i_1, ..., i_m\}$ such that each $i \in I$ appears at least once, the matrix

$$M_{i_m} \cdot ... \cdot M_{i_1}$$

has all strictly positive entries, so it is irreducible

- Finally, adapt techniques for irreducible Markov chains to get that

$$\lim_{m \to \infty} V_{i_m} \circ V_{i_{m-1}} \circ ... \circ V_{i_1}(f) = k1_\Omega$$