Tensor-Train Decomposition for High-Dimensional Economic Models

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Introduction

Motivation

Quantitative Macro Models

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   realistically large shocks.

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High-Accuracy Solutions

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High-Accuracy Solutions

- **Global Solution Techniques**: when shocks are large.

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High-Accuracy Solutions

- **Global Solution Techniques**: when shocks are large.
- **Curse of Dimensionality**: infeasible in high dimensions!

Related Literature

Standard Methods

grid size grows exponentially in dimensions. Infeasible for $d > 3$.

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1. Sparse Grids: Brumm and Scheidegger (2017), Krueger and Kubler (2006), ...
    - reduce grid sizes, focus on important points
    - good in medium dimensions
    - grids still grow to an overwhelming size for $d > 20$

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$\rightarrow$ Still room for improvement!

This Paper

Novel **global solution technique** based on the **Tensor-Train Decomposition**.

Method

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   $\Rightarrow$ Tensor-Train Decomposition

Approximation

$$\hat{f}(\vec{x}) = \sum_{i_1=1}^{m} \cdots \sum_{i_d=1}^{m} A[i_1, ..., i_d] \cdot \left(\phi_{i_1}(x_1) \cdot \ldots \cdot \phi_{i_d}(x_d) \right)$$

Approximation

- Products of one-dimensional basis-functions ϕ

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$\rightarrow$ A is an order-d tensor with m^d entries!

Handling the exponential size of A

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2. **Decomposition**:
   Decompose A into a sequence of smaller objects with lower rank.
3. **Oseledets (2011)**:
   Decompose order-d tensor of size m^d into a sequence of d order-3 tensors with size mr^2.

   $\rightarrow$ **Tensor-Train Decomposition**

Tensor-Train Decomposition – 2D Special Case

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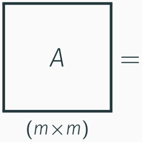
- Approximate $(m \times m)$-matrix A



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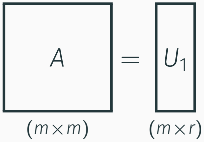

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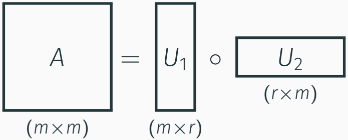

Tensor-Train Decomposition – 2D Special Case

- Approximate $(m \times m)$-matrix A
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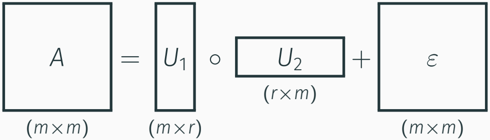
- error ε low if A has low rank



Matrix Completion

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$$(m \times r) \circ (r \times m)$$

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$$\overbrace{(m \times r)}^{U_1} \circ \overbrace{(r \times m)}^{U_2}$$

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same principle for $d > 3$!

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Tensor-Train Decomposition

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Scaling Example

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Scaling Example

d	#A	$\sum$ #U
2	100	100
5	10^5	850
10	10^{10}	2100
100	10^{100}	24600

Table 1: Object size for different d with $m = 10$ and $r = 5$.

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Computing the Decomposition

- Non-linear approximation problem
- Decomposed in a sequence of linear problems, Holtz et al. (2012)

Economic Application

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1. Impact of large shocks on debt, inflation and taxation?

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3. Intergenerational impact of different policy?

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- 60 Generations
- two types of assets
- three aggregate shocks
- five policy parameters

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$\rightarrow$ The proposed method solves the problem in 2h on a PC!

Results

- Average relative error in consumption 0.014%
- Average market clearing error 0.036%
- Q99 consumption error $< 0.08\%$

	Q99	AVG	Q50
Euler-Equation 1	7.40E-04	1.31E-04	8.03E-05
Euler-Equation 2	7.97E-04	1.40E-04	8.40E-05
Labor Supply	1.13E-03	3.43E-04	2.57E-04
Phillips-Curve	1.03E-03	2.08E-04	1.63E-04
Market Clearing	1.29E-03	3.61E-04	2.35E-04

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2. We exploit the **Tensor-Train Decomposition** for efficient approximations.

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2. We exploit the **Tensor-Train Decomposition** for efficient approximations.
3. We solve a high-dimensional **economic application** with **high accuracy**, at low **computational cost**.

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