Supply Chain Frictions
Or: How to Iron if You Must

Ying-Ju Chen[1] Zhengqing Gui[2]
Elu von Thadden[3] Xiaojian Zhao[4]

[1]Hong Kong University of Science and Technology

[2]National University of Singapore

[3]University of Mannheim

[4]Monash University

EEA-ESEM, August 2024

Introduction

Consider a prototypical, simple supply chain:

Manufacturer (upstream) – Retailer (downstream) – Buyers

- Supply-demand mismatch: The manufacturer produces and delivers products to the retailer before demand is realized.
- Information asymmetry: The retailer is better informed about realized demand and his own actions than the manufacturer.
- Delay from Distance: Both these features are especially pronounced when both parties are geographically remote –> Ganapati-Wong (2023) on international supply chains.
- Interdependence: These frictions interact.

Introduction

- Many contracts are used in practice and have been studied in academic work, such as buyback, revenue sharing, wholesale price, etc.
- However, the conceptual question "what is the optimal contract" has received less attention (Cachon 2003).
- 16 years later: Chen (2019), still no satisfactory answer.

Introduction

- Many contracts are used in practice and have been studied in academic work, such as buyback, revenue sharing, wholesale price, etc.
- However, the conceptual question "what is the optimal contract" has received less attention (Cachon 2003).
- 16 years later: Chen (2019), still no satisfactory answer.

Introduction

- Many contracts are used in practice and have been studied in academic work, such as buyback, revenue sharing, wholesale price, etc.
- However, the conceptual question "what is the optimal contract" has received less attention (Cachon 2003).
- 16 years later: Chen (2019), still no satisfactory answer.

This Paper

- A model of an upstream manufacturer who produces a good and sells it to a downstream retailer who sells it on. Downstream demand is only observed by the retailer. Production precedes downstream sales, so the order quantity must be determined before demand is known.
- The contract between manufacturer and retailer specifies (at least) the wholesale price, quantity, the mismatch between delivery and payment, and how to handle unsold inventory. The optimal contract takes different forms, depending on the parameters. In particular, two commonly observed contracts can be optimal, wholesale contracts and buyback contracts.
- But there are twists.

This Paper

- A model of an upstream manufacturer who produces a good and sells it to a downstream retailer who sells it on. Downstream demand is only observed by the retailer. Production precedes downstream sales, so the order quantity must be determined before demand is known.
- The contract between manufacturer and retailer specifies (at least) the wholesale price, quantity, the mismatch between delivery and payment, and how to handle unsold inventory. The optimal contract takes different forms, depending on the parameters. In particular, two commonly observed contracts can be optimal, wholesale contracts and buyback contracts.
- But there are twists.

This Paper

- A model of an upstream manufacturer who produces a good and sells it to a downstream retailer who sells it on. Downstream demand is only observed by the retailer. Production precedes downstream sales, so the order quantity must be determined before demand is known.
- The contract between manufacturer and retailer specifies (at least) the wholesale price, quantity, the mismatch between delivery and payment, and how to handle unsold inventory. The optimal contract takes different forms, depending on the parameters. In particular, two commonly observed contracts can be optimal, wholesale contracts and buyback contracts.
- But there are twists.

Conceptual Contribution

- Standard screening theory: Incentive constraint implies a lot of structure. Ironing is just used to smooth out wrinkles.
- Our problem: Weak incentive constraint, implying less structure. Ironing used to create further local structure, global optimization still necessary.

Literature

- Vertical contracts: Winter (1993); Blair and Lewis (1994); Deneckere et al. (1996, 1997); Arya and Mittendorf (2004).
- Demand uncertainty and inventory decisions: Pasternack (1985); Marvel and Peck (1995); Montez and Schutz (2021).
- Buyback contracts: Yue and Raghunathan (2007), Hsieh et al. (2008), Babich et al. (2012).
- Mechanism design and ironing: Myerson (1981), Guesnerie and Laffont (1984), Loertscher and Muir (2022).
- Ex-post screening with hidden characteristics: Townsend (1979), Faure-Grimaud (2000), Hellwig (2010).

Model

- A manufacturer (m) sells a homogeneous product to a retailer (r) who sells it on to uncertain consumer demand.
- Constant marginal cost of production: c.
- The retailer has no initial funds: $W = 0$.
- The downstream market is sufficiently competitive, so retail price p is given and observable. Retail demand ω is stochastic with c.d.f. F and p.d.f. f.
- Realization of ω can be observed freely by the retailer, the manufacturer only knows F and f.
- Extension: Endogenous p.

Timeline

1. The manufacturer produces and delivers q to the retailer.
2. The retailer observes ω and decides his sales quantity $s(\omega) \leq q$.
   – If $s(\omega) = \omega$, retailer satisfies retail demand.
   – If $s(\omega) < \omega$, $\omega - s(\omega)$ is lost.
   – $q - s(\omega)$ is unsold inventory.
3. Revelation Principle: The retailer reports $\hat{\omega}$, repays $T(\hat{\omega})$ units of cash, and returns $R(\hat{\omega})$ units of unsold inventory to the manufacturer.

Timeline

1. The manufacturer produces and delivers q to the retailer.
2. The retailer observes ω and decides his sales quantity $s(\omega) \leq q$.
   – If $s(\omega) = \omega$, retailer satisfies retail demand.
   – If $s(\omega) < \omega$, $\omega - s(\omega)$ is lost.
   – $q - s(\omega)$ is unsold inventory.
3. Revelation Principle: The retailer reports $\hat{\omega}$, repays $T(\hat{\omega})$ units of cash, and returns $R(\hat{\omega})$ units of unsold inventory to the manufacturer.

Timeline

1. The manufacturer produces and delivers q to the retailer.
2. The retailer observes ω and decides his sales quantity $s(\omega) \leq q$.
   – If $s(\omega) = \omega$, retailer satisfies retail demand.
   – If $s(\omega) < \omega$, $\omega - s(\omega)$ is lost.
   – $q - s(\omega)$ is unsold inventory.
3. Revelation Principle: The retailer reports $\hat{\omega}$, repays $T(\hat{\omega})$ units of cash, and returns $R(\hat{\omega})$ units of unsold inventory to the manufacturer.

Two Sources of Efficiency Loss

- First, the order quantity q is almost surely not correct.
- Second, return shipments are inefficient. One unit of unsold inventory generates value v_m to the manufacturer and v_r to the retailer.

$$v_m < v_r < c < p.$$

Contracts and Payoffs

- Denote a contract by $\Gamma = (q, s, T, R)$.
- Both parties are risk-neutral, so the retailer's payoff is

$$u_r(\omega, \hat{\omega}) = ps(\omega) - T(\hat{\omega}) + v_r[q - s(\omega) - R(\hat{\omega})],$$

and the manufacturer's payoff is

$$u_m(\hat{\omega}) = T(\hat{\omega}) + v_m R(\hat{\omega}) - cq.$$

Limited Liability and Feasibility

- Limited liability (LL): $T(\omega) \leq ps(\omega)$.
- Feasibility of sales (FS): $s(\omega) \leq \omega$.
- Feasibility of returns (FR): $R(\omega) \leq q - s(\omega)$.

Incentive-compatibility

- Incentive-compatibility:

$$ps(\omega) - T(\omega) + v_r[q - s(\omega) - R(\omega)] \geq p\hat{s} - T(\hat{\omega}) + v_r[q - \hat{s} - R(\hat{\omega})] \tag{IC}$$

for all ω, $\hat{\omega}$, and $\hat{s}$ such that

$$0 \leq \hat{s} \leq \min(\omega, q) \tag{IC-FS}$$
$$0 \leq R(\hat{\omega}) \leq q - \hat{s} \tag{IC-FR}$$
$$0 \leq T(\hat{\omega}) \leq p\hat{s} \tag{IC-LL}$$

- (FS), (FR), and (LL) restrict not only the retailer's optimal contractual choices, but also her set of possible deviations.

Incentive-compatibility

- Incentive-compatibility:

$$ps(\omega) - T(\omega) + v_r[q - s(\omega) - R(\omega)] \geq p\hat{s} - T(\hat{\omega}) + v_r[q - \hat{s} - R(\hat{\omega})] \tag{IC}$$

for all ω, $\hat{\omega}$, and $\hat{s}$ such that

$$0 \leq \hat{s} \leq \min(\omega, q) \tag{IC-FS}$$

$$0 \leq R(\hat{\omega}) \leq q - \hat{s} \tag{IC-FR}$$

$$0 \leq T(\hat{\omega}) \leq p\hat{s} \tag{IC-LL}$$

- (FS), (FR), and (LL) restrict not only the retailer's optimal contractual choices, but also her set of possible deviations.

The Contracting Problem

Assume that the manufacturer has full bargaining power. Thus, her problem is to find Γ that solves

$$\max_{\Gamma} \; E_\omega u_m(\omega),$$

subject to (LL), (FS), (FR), (IC),

$$E_\omega u_r(\omega, \omega) \geq \underline{u}_r \qquad (\text{IR}_r)$$

$$E_\omega u_m(\omega) \geq 0 \qquad (\text{IR}_m)$$

Note: $\underline{u}_r$ is a measure of the competitiveness of the supplier relationship.

First Best

- In the first-best environment, the manufacturer solves the same problem without (IC).
- Social surplus:

$$S(q) = \int_0^{+\infty} p\omega^+ + v_r(q - \omega^+)dF(\omega) - cq.$$

where $\omega^+ = \min(\omega, q)$.

- Denoting by $Q(q)$ the expected sales given q and price p:

$$Q(q) = \int_0^{+\infty} \omega^+ dF(\omega) = q - \int_0^q F(\omega)d\omega, \tag{1}$$

Then $S(q) = (p - v_r)Q(q) - (c - v_r)q$.

First Best

Proposition 1

The first-best quantity q^{FB} is unique and satisfies

$$F(q^{FB}) = \frac{p - c}{p - v_r}. \tag{2}$$

The manufacturer optimally chooses $R(\omega) = 0$ for all ω.

Second Best: Sales

First, sales are always maximal:

Lemma

If Γ is optimal, then $s(\omega) = \min(q, \omega)$ for all ω.

The Challenge of (IC)

- The combination of ex-post limited liability and asymmetric information yields a non-standard (IC).
- In a classical screening problem, the incentive constraint is

$$u_r(\omega, \omega) \geq u_r(\omega, \hat{\omega}) \text{ for any } \omega, \hat{\omega}. \qquad \text{(IC')}$$

- (IC') implies that the retailer's indirect utility $U_r(\omega)$ has

$$U_r'(\omega) = \frac{\partial u_r(\omega, \omega)}{\partial \omega}.$$

  Hence, U_r is absolutely continuous and increasing.
- This method relies on the fact that the retailer at state ω can misreport any $\hat{\omega}$ in its neighborhood $(\omega - \epsilon, \omega + \epsilon)$.

Second Best: The Challenge from (IC)

- However, when (IC) incorporates restrictions such as (LL) and (FC), which $\hat{\omega}$ can be misreported becomes endogenous.
- Recall that our (IC) is

$$u_r(\omega, \omega) \geq u_r(\omega, \hat{\omega}) \tag{IC}$$

  for any ω, $\hat{\omega}$ such that $T(\hat{\omega}) \leq p \min(q, \omega)$ and $R(\hat{\omega}) \leq q - \min(q, \omega)$.
- If the contract has $T(\hat{\omega}) > p \min(q, \omega)$ or $R(\hat{\omega}) > q - \min(q, \omega)$, then the retailer in state ω cannot misreport $\hat{\omega}$.
- Hence, (IC) is weaker than (IC').
  Consequence: The Envelope condition may no longer hold globally, U_r is no longer necessarily increasing.

Second Best: The Challenge from (IC)

- However, when (IC) incorporates restrictions such as (LL) and (FC), which $\hat{\omega}$ can be misreported becomes endogenous.
- Recall that our (IC) is

$$u_r(\omega, \omega) \geq u_r(\omega, \hat{\omega}) \tag{IC}$$

  for any ω, $\hat{\omega}$ such that $T(\hat{\omega}) \leq p \min(q, \omega)$ and $R(\hat{\omega}) \leq q - \min(q, \omega)$.
- If the contract has $T(\hat{\omega}) > p \min(q, \omega)$ or $R(\hat{\omega}) > q - \min(q, \omega)$, then the retailer in state ω cannot misreport $\hat{\omega}$.
- Hence, (IC) is weaker than (IC').
  Consequence: The Envelope condition may no longer hold globally, U_r is no longer necessarily increasing.

Second Best: The Challenge from (IC)

- However, when (IC) incorporates restrictions such as (LL) and (FC), which $\hat{\omega}$ can be misreported becomes endogenous.
- Recall that our (IC) is

$$u_r(\omega, \omega) \geq u_r(\omega, \hat{\omega}) \tag{IC}$$

  for any ω, $\hat{\omega}$ such that $T(\hat{\omega}) \leq p \min(q, \omega)$ and $R(\hat{\omega}) \leq q - \min(q, \omega)$.
- If the contract has $T(\hat{\omega}) > p \min(q, \omega)$ or $R(\hat{\omega}) > q - \min(q, \omega)$, then the retailer in state ω cannot misreport $\hat{\omega}$.
- Hence, (IC) is weaker than (IC').
  Consequence: The Envelope condition may no longer hold globally, U_r is no longer necessarily increasing.

Second Best: The Challenge from (IC)

- However, when (IC) incorporates restrictions such as (LL) and (FC), which $\hat{\omega}$ can be misreported becomes endogenous.
- Recall that our (IC) is

$$u_r(\omega, \omega) \geq u_r(\omega, \hat{\omega}) \tag{IC}$$

  for any ω, $\hat{\omega}$ such that $T(\hat{\omega}) \leq p \min(q, \omega)$ and $R(\hat{\omega}) \leq q - \min(q, \omega)$.
- If the contract has $T(\hat{\omega}) > p \min(q, \omega)$ or $R(\hat{\omega}) > q - \min(q, \omega)$, then the retailer in state ω cannot misreport $\hat{\omega}$.
- Hence, (IC) is weaker than (IC').
  Consequence: The Envelope condition may no longer hold globally, U_r is no longer necessarily increasing.

The Challenge: Anything can happen

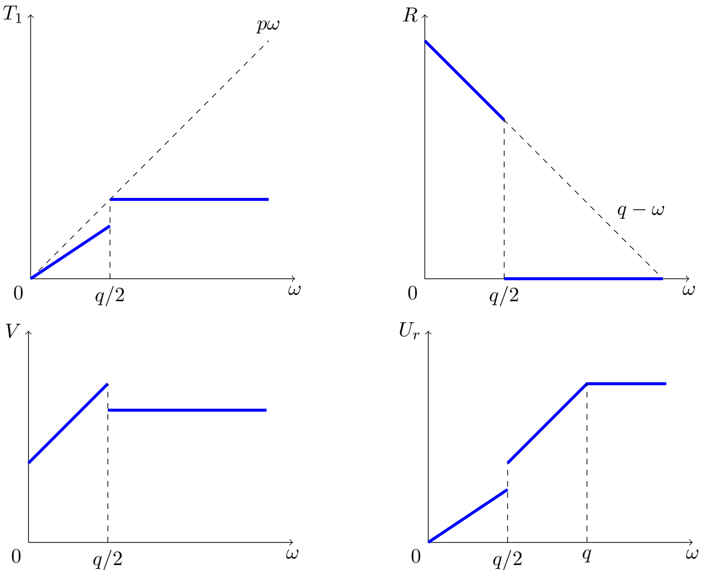
Consider the following contract for some given $q > \underline{q}$:

$$T_1(\omega) = \begin{cases} \alpha\omega & \omega < q/2, \\ pq/2 & \omega \geq q/2; \end{cases} \quad R(\omega) = \begin{cases} q - \omega & \omega < q/2, \\ 0 & \omega \geq q/2. \end{cases}$$

The retailer's indirect utility function is

$$U_r(\omega) = u_r(\omega, \omega, \min(\omega, q)) = \begin{cases} (p-\alpha)\omega & \omega < q/2, \\ p\omega + v_r(q-\omega) - pq/2 & q/2 \leq \omega < q, \\ pq/2 & \omega \geq q. \end{cases}$$

If $\alpha \in [p - v_r, p]$, then this contract is admissible.



T_1
$p\omega$
0
$q/2$
ω
R
$q-\omega$
0
$q/2$
ω
V
0
$q/2$
ω
U_r
0
$q/2$
q
ω

Our Approach: Ironing

- Remember:

$$u_r(\omega, \hat{\omega}, s) = ps - T_1(\hat{\omega}) + v_r[q - s - R(\hat{\omega})]$$
$$= (p - v_r)s - V(\hat{\omega}) + v_r q$$

where $V(\omega) = T(\omega) + v_r R(\omega)$ is the retailer's valuation of his total transfer: determines deviations in the (IC) constraint.
Neither is V necessarily increasing nor R decreasing.

- Map V into quantile space: For any $\phi \in [0, 1]$, let $\mathcal{V}(\phi)$ be the accumulated total transfer for all types below $F^{-1}(\phi)$:

$$\mathcal{V}(\phi) = \int_0^{F^{-1}(\phi)} V(\omega)\mathrm{d}F(\omega) = \int_0^{\phi} V(F^{-1}(\hat{\phi}))\mathrm{d}\hat{\phi}.$$

- Ironing: take the convex envelope of $\mathcal{V}$.

Our Approach: Ironing

- Remember:

$$u_r(\omega, \hat{\omega}, s) = ps - T_1(\hat{\omega}) + v_r[q - s - R(\hat{\omega})]$$
$$= (p - v_r)s - V(\hat{\omega}) + v_r q$$

where $V(\omega) = T(\omega) + v_r R(\omega)$ is the retailer's valuation of his total transfer: determines deviations in the (IC) constraint.
Neither is V necessarily increasing nor R decreasing.

- Map V into quantile space: For any $\phi \in [0, 1]$, let $\mathcal{V}(\phi)$ be the accumulated total transfer for all types below $F^{-1}(\phi)$:

$$\mathcal{V}(\phi) = \int_0^{F^{-1}(\phi)} V(\omega)\mathrm{d}F(\omega) = \int_0^{\phi} V(F^{-1}(\hat{\phi}))\mathrm{d}\hat{\phi}.$$

- Ironing: take the convex envelope of $\mathcal{V}$.

Our Approach: Ironing

- Remember:

$$u_r(\omega, \hat{\omega}, s) = ps - T_1(\hat{\omega}) + v_r[q - s - R(\hat{\omega})]$$
$$= (p - v_r)s - V(\hat{\omega}) + v_r q$$

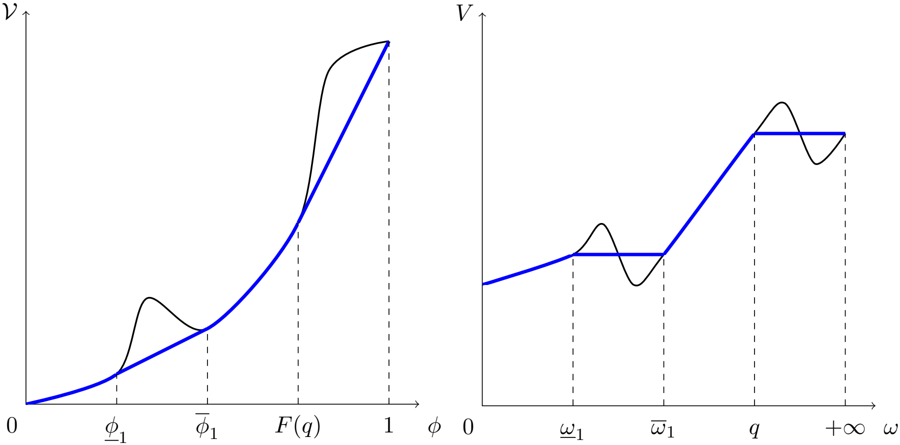
where $V(\omega) = T(\omega) + v_r R(\omega)$ is the retailer's valuation of his total transfer: determines deviations in the (IC) constraint.
Neither is V necessarily increasing nor R decreasing.

- Map V into quantile space: For any $\phi \in [0, 1]$, let $\mathcal{V}(\phi)$ be the accumulated total transfer for all types below $F^{-1}(\phi)$:

$$\mathcal{V}(\phi) = \int_0^{F^{-1}(\phi)} V(\omega) \mathrm{d}F(\omega) = \int_0^{\phi} V(F^{-1}(\hat{\phi})) \mathrm{d}\hat{\phi}.$$

- Ironing: take the convex envelope of $\mathcal{V}$.

Ironing: An Example



Ironed: New Structure

- Result: V becomes nondecreasing on the state space.
- Next step: restructuring the returns function R to make it non-increasing.
- The ironed and restructured contract has an important property: $p\omega \geq T_1(\hat{\omega})$ implies $V(\omega) \leq V(\hat{\omega})$.
  We therefore can use cash transfers to replace return shipments on those ironed intervals.
- After ironing, the resulting contract is defined piecewise, and on each piece it resembles a buyback contract.
- Hence, these pieces are "local buyback contracts".
- Intuitively, they require the retailer to make some targeted total transfer on each piece.

Ironed: New Structure

- Result: V becomes nondecreasing on the state space.
- Next step: restructuring the returns function R to make it non-increasing.
- The ironed and restructured contract has an important property: $p\omega \geq T_1(\hat{\omega})$ implies $V(\omega) \leq V(\hat{\omega})$.
  We therefore can use cash transfers to replace return shipments on those ironed intervals.
- After ironing, the resulting contract is defined piecewise, and on each piece it resembles a buyback contract.
- Hence, these pieces are "local buyback contracts".
- Intuitively, they require the retailer to make some targeted total transfer on each piece.

Ironed: New Structure

- Result: V becomes nondecreasing on the state space.
- Next step: restructuring the returns function R to make it non-increasing.
- The ironed and restructured contract has an important property: $p\omega \geq T_1(\hat{\omega})$ implies $V(\omega) \leq V(\hat{\omega})$.
  We therefore can use cash transfers to replace return shipments on those ironed intervals.
- After ironing, the resulting contract is defined piecewise, and on each piece it resembles a buyback contract.
- Hence, these pieces are "local buyback contracts".
- Intuitively, they require the retailer to make some targeted total transfer on each piece.

Ironed: New Structure

- Result: V becomes nondecreasing on the state space.
- Next step: restructuring the returns function R to make it non-increasing.
- The ironed and restructured contract has an important property: $p\omega \geq T_1(\hat{\omega})$ implies $V(\omega) \leq V(\hat{\omega})$.
  We therefore can use cash transfers to replace return shipments on those ironed intervals.
- After ironing, the resulting contract is defined piecewise, and on each piece it resembles a buyback contract.
- Hence, these pieces are "local buyback contracts".
- Intuitively, they require the retailer to make some targeted total transfer on each piece.

Ironed: New Structure

- Result: V becomes nondecreasing on the state space.
- Next step: restructuring the returns function R to make it non-increasing.
- The ironed and restructured contract has an important property: $p\omega \geq T_1(\hat{\omega})$ implies $V(\omega) \leq V(\hat{\omega})$.
  We therefore can use cash transfers to replace return shipments on those ironed intervals.
- After ironing, the resulting contract is defined piecewise, and on each piece it resembles a buyback contract.
- Hence, these pieces are ”local buyback contracts".
- Intuitively, they require the retailer to make some targeted total transfer on each piece.

Ironed: New Structure

- Result: V becomes nondecreasing on the state space.
- Next step: restructuring the returns function R to make it non-increasing.
- The ironed and restructured contract has an important property: $p\omega \geq T_1(\hat{\omega})$ implies $V(\omega) \leq V(\hat{\omega})$.
  We therefore can use cash transfers to replace return shipments on those ironed intervals.
- After ironing, the resulting contract is defined piecewise, and on each piece it resembles a buyback contract.
- Hence, these pieces are "local buyback contracts".
- Intuitively, they require the retailer to make some targeted total transfer on each piece.

Local Buyback Contracts

Definition

Γ is a local buyback contract if there exists disjoint intervals $[\underline{\omega}_n, \overline{\omega}_n)$ with $\overline{\omega}_n \leq q$, $n = 1, ..., , N$, and an equal number of constants t_n, such that:

1. for any $\omega \leq q$,
   1. if $\omega \in [\underline{\omega}_n, \overline{\omega}_n)$, then $T(\omega) = \min(p\omega, t_n)$, $V(\omega) = t_n + v_r(q - \overline{\omega}_n)$;
   2. if $\omega \notin \bigcup[\underline{\omega}_n, \overline{\omega}_n)$, then $T'(\omega) = p$, $R(\omega) = \max(q - \omega, 0)$;
2. for any $\omega > q$, $T_1(\omega) = T_1(q)$ and $R(\omega) = R(q) = 0$;
3. $V(\omega)$ is nondecreasing and continuous.

- $[\underline{\omega}_n, \overline{\omega}_n)$ is the "ironed" interval, on which V is constant.
- Otherwise, (FR) binds.

Local Buyback Contracts

Proposition 2

If Γ is optimal, it is a local buyback contract.

- Cash transfer is efficient, returning unsold inventory is costly.
- A local buyback contract uses cash as much as possible whenever the "ironed" V function is constant.

But: The converse is not true.

Local Buyback Contracts

Proposition 2

If Γ is optimal, it is a local buyback contract.

- Cash transfer is efficient, returning unsold inventory is costly.
- A local buyback contract uses cash as much as possible whenever the "ironed" V function is constant.

But: The converse is not true.

From Local to Global Optimization: Buyback Contracts

A local buyback contract with $n = 1$ is called a buyback contract:

Definition

A contract is a **buyback contract** if there exist a constant $t \in [0, pq]$, such that for any ω,

$$T_1(\omega) = \min(p\omega, t);$$

$$R(\omega) = \begin{cases} q - \omega & \omega < \underline{\omega}, \\ \max((t - p\omega)/v_r, 0) & \omega \geq \underline{\omega}; \end{cases}$$

where $\underline{\omega} = \max\left(0, \frac{t - v_r q}{p - v_r}\right)$.

Buyback Contract

- Ex ante, the manufacturer provides the retailer with a fixed quantity q at the wholesale price $p_w < p$.
- Ex post, the retailer has to repay $t = p_w q$ to the manufacturer after the selling season. For this, he must sell at least

$$\bar{q} = (p_w/p)q.$$

  - If $\omega \geq \bar{q}$, then t is repaid in cash, and the retailer salvages unsold inventories.
  - If $\omega < \bar{q}$, then some of unsold inventories must be returned to the manufacturer so as to make the value of the total transfer equal to t.
- In the latter case, it is as if the manufacturer buys back unsold inventories at a price p_b.

Buyback Contract

- Ex ante, the manufacturer provides the retailer with a fixed quantity q at the wholesale price $p_w < p$.
- Ex post, the retailer has to repay $t = p_w q$ to the manufacturer after the selling season. For this, he must sell at least

$$\bar{q} = (p_w/p)q.$$

  - If $\omega \geq \bar{q}$, then t is repaid in cash, and the retailer salvages unsold inventories.
  - If $\omega < \bar{q}$, then some of unsold inventories must be returned to the manufacturer so as to make the value of the total transfer equal to t.
- In the latter case, it is as if the manufacturer buys back unsold inventories at a price p_b.

Buyback Contract

- Ex ante, the manufacturer provides the retailer with a fixed quantity q at the wholesale price $p_w < p$.
- Ex post, the retailer has to repay $t = p_w q$ to the manufacturer after the selling season. For this, he must sell at least

$$\bar{q} = (p_w/p)q.$$

  - If $\omega \geq \bar{q}$, then t is repaid in cash, and the retailer salvages unsold inventories.
  - If $\omega < \bar{q}$, then some of unsold inventories must be returned to the manufacturer so as to make the value of the total transfer equal to t.
- In the latter case, it is as if the manufacturer buys back unsold inventories at a price p_b.

Buyback Contract

- Ex ante, the manufacturer provides the retailer with a fixed quantity q at the wholesale price $p_w < p$.
- Ex post, the retailer has to repay $t = p_w q$ to the manufacturer after the selling season. For this, he must sell at least

$$\bar{q} = (p_w/p)q.$$

  - If $\omega \geq \bar{q}$, then t is repaid in cash, and the retailer salvages unsold inventories.
  - If $\omega < \bar{q}$, then some of unsold inventories must be returned to the manufacturer so as to make the value of the total transfer equal to t.
- In the latter case, it is as if the manufacturer buys back unsold inventories at a price p_b.

Buyback Contract

- Ex ante, the manufacturer provides the retailer with a fixed quantity q at the wholesale price $p_w < p$.
- Ex post, the retailer has to repay $t = p_w q$ to the manufacturer after the selling season. For this, he must sell at least

$$\bar{q} = (p_w/p)q.$$

  - If $\omega \geq \bar{q}$, then t is repaid in cash, and the retailer salvages unsold inventories.
  - If $\omega < \bar{q}$, then some of unsold inventories must be returned to the manufacturer so as to make the value of the total transfer equal to t.
- In the latter case, it is as if the manufacturer buys back unsold inventories at a price p_b.

The Optimal Contract

Proposition

If Γ is optimal and piecewise continuous, then it is a buyback contract.

- The wholesale price is $p_w = t/q$.
- The buyback price is $p_b = (t - p\omega)/R(\omega)$. If we write p_b as a function of R ("Taxation Principle"), then

$$p_b(R) = \begin{cases} v_r & t < v_r q, \\ p - \frac{pq-t}{R} & t \geq v_r q. \end{cases}$$

- Hence, the buyback price can be state-dependent.

The Optimal Contract

Proposition

If Γ is optimal and piecewise continuous, then it is a buyback contract.

- The wholesale price is $p_w = t/q$.
- The buyback price is $p_b = (t - p\omega)/R(\omega)$. If we write p_b as a function of R ("Taxation Principle"), then

$$p_b(R) = \begin{cases} v_r & t < v_r q, \\ p - \frac{pq-t}{R} & t \geq v_r q. \end{cases}$$

- Hence, the buyback price can be state-dependent.

The Optimal Contract

Proposition

If Γ is optimal and piecewise continuous, then it is a buyback contract.

- The wholesale price is $p_w = t/q$.
- The buyback price is $p_b = (t - p\omega)/R(\omega)$. If we write p_b as a function of R (”Taxation Principle"), then

$$p_b(R) = \begin{cases} v_r & t < v_r q, \\ p - \frac{pq-t}{R} & t \geq v_r q. \end{cases}$$

- Hence, the buyback price can be state-dependent.

Constant vs. State-dependent Buyback Prices

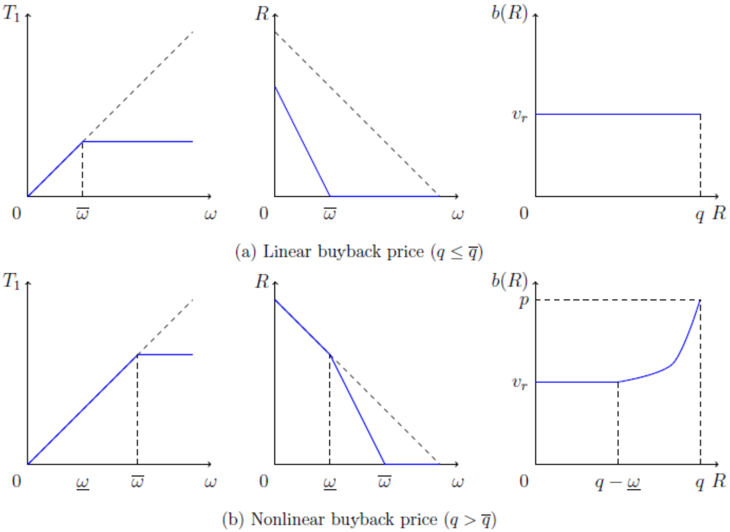


Figure 5: Two types of buyback contracts

Optimal Order Quantity

Proposition 4

1. The optimal contract is a buyback contract and implements a quantity $q^* < q^{FB}$.
2. There is a $\overline{q} < q^{FB}$ such that, if $q^* > \overline{q}$, the buyback price is variable; if $q^* \leq \overline{q}$, the buyback price is constant.

Proposition 5

There is a threshold $U^{VB} > 0$ such that

1. If $\underline{u}_r < U^{VB}$, the optimal contract implements $q^* > \overline{q}$ and is buyback with variable buyback pricing.
2. If $\underline{u}_r > U^{VB}$, the optimal contract implements $q^* < \overline{q}$ and is buyback with constant buyback pricing.

Discussion and Extensions

- What if the demand function $F(\omega; p)$ is price-dependent?
  - The optimal contract is still buyback, but the price is lower than the first-best level.
- What if the retailer has initial liquidity, $W > 0$?
  - The optimal contract may shift from buyback to wholesale, depending on relative bargaining powers.
- What if the retailer can use firesales to generate emergency cash at price v_r?
  - Returns are still optimal, but for a smaller range of ω, if $\underline{v}$ is small. If $\underline{v}$ is larger, the optimal contract uses no returns and essentially mimics the efficiency properties of a wholesale contract by means of retail firesales.
- What if there are multiple retailers?
  - The first best price and order quantity can be implemented when there are sufficiently many retailers.

Discussion and Extensions

- What if the demand function $F(\omega; p)$ is price-dependent?
  - The optimal contract is still buyback, but the price is lower than the first-best level.
- What if the retailer has initial liquidity, $W > 0$?
  - The optimal contract may shift from buyback to wholesale, depending on relative bargaining powers.
- What if the retailer can use firesales to generate emergency cash at price v_r?
  - Returns are still optimal, but for a smaller range of ω, if $\underline{u}$ is small. If $\underline{u}$ is larger, the optimal contract uses no returns and essentially mimics the efficiency properties of a wholesale contract by means of retail firesales.
- What if there are multiple retailers?
  - The first best price and order quantity can be implemented when there are sufficiently many retailers.

Discussion and Extensions

- What if the demand function $F(\omega; p)$ is price-dependent?
  - The optimal contract is still buyback, but the price is lower than the first-best level.
- What if the retailer has initial liquidity, $W > 0$?
  - The optimal contract may shift from buyback to wholesale, depending on relative bargaining powers.
- What if the retailer can use firesales to generate emergency cash at price v_r?
  - Returns are still optimal, but for a smaller range of ω, if $\underline{u}$ is small. If $\underline{u}$ is larger, the optimal contract uses no returns and essentially mimics the efficiency properties of a wholesale contract by means of retail firesales.
- What if there are multiple retailers?
  - The first best price and order quantity can be implemented when there are sufficiently many retailers.

Discussion and Extensions

- What if the demand function $F(\omega; p)$ is price-dependent?
  - The optimal contract is still buyback, but the price is lower than the first-best level.
- What if the retailer has initial liquidity, $W > 0$?
  - The optimal contract may shift from buyback to wholesale, depending on relative bargaining powers.
- What if the retailer can use firesales to generate emergency cash at price v_r?
  - Returns are still optimal, but for a smaller range of ω, if $\underline{u}$ is small. If $\underline{u}$ is larger, the optimal contract uses no returns and essentially mimics the efficiency properties of a wholesale contract by means of retail firesales.
- What if there are multiple retailers?
  - The first best price and order quantity can be implemented when there are sufficiently many retailers.

Discussion and Extensions

- What if the demand function $F(\omega; p)$ is price-dependent?
  - The optimal contract is still buyback, but the price is lower than the first-best level.
- What if the retailer has initial liquidity, $W > 0$?
  - The optimal contract may shift from buyback to wholesale, depending on relative bargaining powers.
- What if the retailer can use firesales to generate emergency cash at price v_r?
  - Returns are still optimal, but for a smaller range of ω, if $\underline{u}$ is small. If $\underline{u}$ is larger, the optimal contract uses no returns and essentially mimics the efficiency properties of a wholesale contract by means of retail firesales.
- What if there are multiple retailers?
  - The first best price and order quantity can be implemented when there are sufficiently many retailers.

Discussion and Extensions

- What if the demand function $F(\omega; p)$ is price-dependent?
  - The optimal contract is still buyback, but the price is lower than the first-best level.
- What if the retailer has initial liquidity, $W > 0$?
  - The optimal contract may shift from buyback to wholesale, depending on relative bargaining powers.
- What if the retailer can use firesales to generate emergency cash at price v_r?
  - Returns are still optimal, but for a smaller range of ω, if $\underline{u}$ is small. If $\underline{u}$ is larger, the optimal contract uses no returns and essentially mimics the efficiency properties of a wholesale contract by means of retail firesales.
- What if there are multiple retailers?
  - The first best price and order quantity can be implemented when there are sufficiently many retailers.

Discussion and Extensions

- What if the demand function $F(\omega; p)$ is price-dependent?
  - The optimal contract is still buyback, but the price is lower than the first-best level.
- What if the retailer has initial liquidity, $W > 0$?
  - The optimal contract may shift from buyback to wholesale, depending on relative bargaining powers.
- What if the retailer can use firesales to generate emergency cash at price v_r?
  - Returns are still optimal, but for a smaller range of ω, if $\underline{u}$ is small. If $\underline{u}$ is larger, the optimal contract uses no returns and essentially mimics the efficiency properties of a wholesale contract by means of retail firesales.
- What if there are multiple retailers?
  - The first best price and order quantity can be implemented when there are sufficiently many retailers.

Discussion and Extensions

- What if the demand function $F(\omega; p)$ is price-dependent?
  - The optimal contract is still buyback, but the price is lower than the first-best level.
- What if the retailer has initial liquidity, $W > 0$?
  - The optimal contract may shift from buyback to wholesale, depending on relative bargaining powers.
- What if the retailer can use firesales to generate emergency cash at price v_r?
  - Returns are still optimal, but for a smaller range of ω, if $\underline{u}$ is small. If $\underline{u}$ is larger, the optimal contract uses no returns and essentially mimics the efficiency properties of a wholesale contract by means of retail firesales.
- What if there are multiple retailers?
  - The first best price and order quantity can be implemented when there are sufficiently many retailers.

Conclusion

- This paper studies how supply chains can react optimally to informational frictions between upstream and downstream firms when production precedes sales.
- The nature of this problem gives rise to a special incentive constraint that is weaker than those of standard screening problems.
- The contract analysis generalizes the ironing approach by Myerson, Guesnerie, and Laffont to combine local and global optimization techniques.
- We show that the optimal contract is a wholesale or a buyback contract, where the optimal buyback price can be state-dependent. This depends, in particular, on the competitiveness of the supply relationship and the distribution of bargaining power along the chain.
- The result is robust to a number of variations, such as price-dependent demand and competition.