We are not in a Gaussian world anymore: Implications for the composition of official foreign assets[1]

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Motivation

- How do emerging markets (EM) protect themselves against risk?
  - Mostly through accumulation of international reserves as self-insurance... but little state-contingent assets.
- Here, we identify two related puzzles addressed by different strings of the literature
  - **Puzzle I**: Welfare gains of financial integration are surprisingly small. [Cole and Obstfeld, 1991; Gourinchas and Jeanne, 2006; and many others]
  - **Puzzle II**: Why not holding state-contingent assets instead of (or in addition to) international reserves (non-contingent)? [Caballero and Panageas, 2004, 2005]

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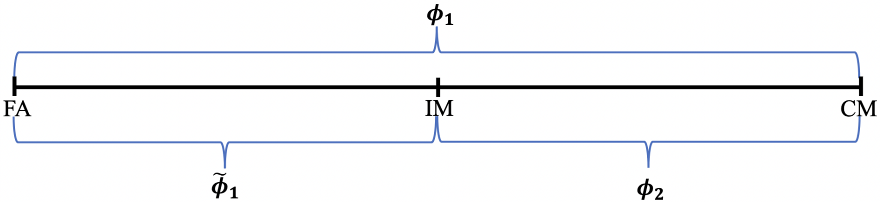
Decomposition of welfare gains

- Decompose the gain of financial integration in two segments: (i) from financial autarky to incomplete markets (only a risk-free asset); and (ii) from incomplete markets to complete markets (full set of Arrow-Debreu assets).

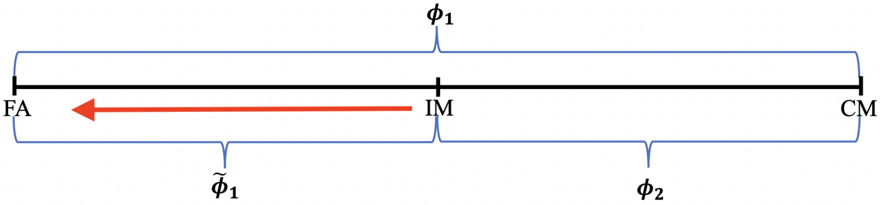


FA: Financial Autarky
IM: Incomplete Markets
CM: Complete Markets

- Size of $\widetilde{\phi}_1$ and ϕ_2 capture, respectively, the incentives to accumulate risk-free assets and state-contingent assets.

Decomposition of welfare gains II

- Adding our note to Puzzle II: There is a composition effect of welfare gains that has been overlooked when shocks are normally distributed:
  - **Most of the gains come from incomplete markets to complete markets** ($\phi_2 >> \widetilde{\phi}_1 \approx 0$)



FA: Financial Autarky
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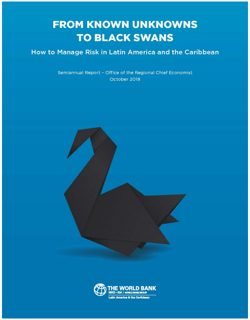
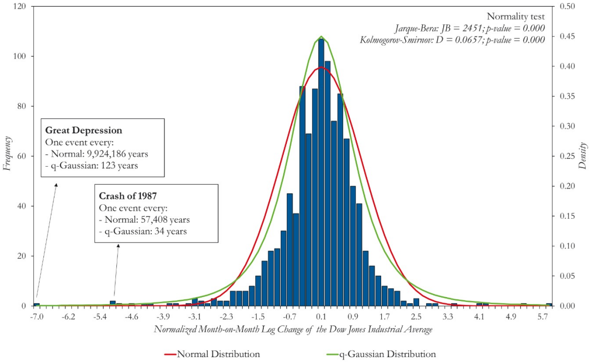
So..who is right? The shape of risk may hold the answer





Preview of main results

- We fit macroeconomic disaster governed by power-law distribution to the income risks and:
- We are able to increase substantially the size of welfare gains (ϕ_1) [**Puzzle I**]
- Not all gains come from incomplete to complete markets ($\widetilde{\phi}_1$ is not small relative to ϕ_2) to reconcile the observation of accumulation of risk-free asset in emerging economies [**Puzzle II**]

Related literature

- Gains of financial integration: Cole and Obstfeld (1991); Tesar (1995); Gourinchas and Jeanne (2003); Mendoza (1995); Martin (2010); Van Wincoop (1998)
- Accumulation of reserves as self-insurance to risks: Aizenman and Marion (2002); Jeanne and Ranciere (2006); Jeanne (2007); Bianchi et al (2018)
- Rare macroeconomic disasters and macro-finance implications: Rietz (1998); Barro (2006); Barro (2009); Barro and Urzua (2008); Barro and Jin (2011).
- Benefits of contingent asset holdings vs. FX reserves as precautionary savings: Caballero and Panageas (2004, 2005).

Outline

Evidence of macroeconomic disasters

Estimation of macroeconomic disasters

- Annual data for 156 countries, 1900-2018 (Bolt et al, 2018; WEO).
- Using real GDP per capital, we identify disaster observations following Barro and Jin (2011).
- For each disaster i, define x_i as the minimum real GDP per capita relative to the pre-disaster level.
- Reciprocal $z_i = 1/x_i$. Then, the pdf for z_i is [power-law distribution] $f(z_i) = (\alpha - 1)(z_{min})^{\alpha-1}(z_i)^{-\alpha}$, $\alpha > 1$
- MLE estimation for α conditional on z_{min} is:

$$\hat{\alpha} = 1 + n \left[\sum_{i=1}^{n} ln \left(\frac{z_i}{z_{min}} \right) \right]^{-1}$$

- We then follow Clauset et al (2009) to choose z_{min} that minimizes the distance between the cumulative density function of the data and of the fitted model for the observations for z.

Estimation of macroeconomic disasters

Table: Estimated parameters of the income distribution

Disaster regime (Power-law dist.)					*Tranquil times (Log-normal dist.)*	
$\hat{z}_{min}$		$\hat{\alpha}$		p	$\hat{\mu}$	$\hat{\sigma}$
Est.	95% Confidence Interval	Est.	95% Confidence Interval			
1.1899	(1.1158, 1.3917)	4.5980	(3.9846, 5.6343)	0.0274	0.0262	0.0450

Notes: The power-law and log-normal distributions are fitted through R implementation of Gillespie (2015) and Delignette-Muller and Dutang (2015), respectively. 95% confidence intervals of $\hat{z}_{min}$ and $\hat{\alpha}$ from 5,000 bootstrap simulations, following methods developed in Clauset *et al*. (2009). The point estimates for $\hat{\mu}$ and $\hat{\sigma}$ and the value of p are computed as explained in the text.

Basic Ideas

Prudent consumer

- Consider a two-period small open economy.
- The endowment in period 1, y_1, is deterministic.
- The endowment in period 2, y_2, is stochastic with $\mathbb{E}[y_2] = y_1$
- Lifetime expected utility (welfare) is given by: $W = u(C_1) + \beta\mathbb{E}[u(C_2)]$; $u(C) = (C^{1-\gamma} - 1)/(1-\gamma)$, and γ **is the coefficient of risk aversion**.
- 3 market arrangements:
  - Financial autarky (FA): $C_1^{FA} = y_1$, $C_2^{FA} = y_2$.
  - Incomplete markets (IM): $C_1^{IM} = y_1 - B_1$, $C_2^{IM} = y_2 + (1+r)B_1$
  - Complete markets (CM): $C_1^{CM} = y_1$, $C_2^{CM} = y_1$

Prudent consumer (cont.)

- Welfare is maximized under complete markets:

$$W_{CM} = u(y_1) + \beta u(y_1)$$

- The welfare gains of having complete markets relative to the case of financial autarky are the value of ϕ_1 that satisfies:

$$W_{FA}(\phi_1) = u((1+\phi_1)C_1^{FA}) + \beta \mathbb{E}\left[u((1+\phi_1)C_2^{FA})\right] = W_{CM}$$

- In the same vein, we define the relative gains from IM to CM:

$$W_{IM}(\phi_2) = u((1+\phi_2)C_1^{IM}) + \beta \mathbb{E}\left[u((1+\phi_2)C_2^{IM})\right] = W_{CM}$$

Relationship between welfare gains

- Let $\widetilde{\phi}_1$ denote the welfare gains from FA to IM.
- We can derive the following relationship:

$$\underbrace{\log\left(1+\phi_1\right)}_{\text{total gains}} = \underbrace{\log(1+\widetilde{\phi}_1)}_{\text{gains FA-IM}} + \underbrace{\log\left(1+\phi_2\right)}_{\text{gains IM-CM}}.$$

- Therefore, given ϕ_1 and ϕ_2, we can compute $\widetilde{\phi}_1$.

Welfare gains composition: simple binomial distribution for y_2

Table: Welfare gains as function of the size of the shocks

y^H	y^L	ϕ_1 (total)	ϕ_2 (segment 2)	ϕ_2/ϕ_1	$\tilde{\phi}_1$ (segment 1)
1.0	1.0	0	0	n/a	0
1.1	0.9	0.75	0.74	98.0%	0.01%
1.2	0.8	3.13	2.90	92.5%	0.2%
1.3	0.7	7.54	6.36	84.3%	1.11%
1.4	0.6	14.8	11.0	74.4%	3.41%
1.5	0.5	26.7	16.9	63.4%	8.35%
1.6	0.4	46.6	24.1	51.7%	18.2%

Note: Calculations assume $\gamma = 3$. Here, as the distance b/w y^H and y^L increases, the distribution of income in period 2 has fatter tails.

Model economy under normal and power-law distributions

Model setting

- More formal incorporation of income risk estimated from macroeconomic disasters
- Two-period model for a small open economy
- Y_1; $Y_2 = Yx$, x a continuous random variable with pdf given by $f_X(x)$. Assume $\mathbb{E}[x] = g_x$
- Rest of the world consists of a continuum of identical economies
- There is full risk sharing among the economies in the rest of world.

Complete markets

- Optimal conditions for the consumption are:

$$(C_1)^{-\gamma} = \lambda,$$

$$\beta f_X(x)(C_2(x))^{-\gamma} = \lambda p(x).$$

- The assumption of full risk sharing in the rest of the world implies:

$$p(x) = \beta f_X(x) \left(\frac{Y}{Yexp(g_x)} \right)^{\gamma} = \beta f_X(x) exp(-\gamma g_x).$$

- $\Rightarrow \quad C_1 = C_2(x) = Y.$
- Welfare under complete markets is given by:

$$W_{CM} = \frac{(Y)^{1-\gamma} - 1}{1-\gamma} + \beta \frac{(Y)^{1-\gamma} exp((1-\gamma)g_x) - 1}{1-\gamma} \tag{18}$$

Financial autarky

- Welfare under financial autarky, adding a potential compensation to consumption of a factor of $1+\phi_1$, is given by the following equation (19):

$$W_{FA}(\phi_1) = \frac{(Y(1+\phi_1))^{1-\gamma}-1}{1-\gamma} + \beta\mathbb{E}\left[\frac{(Yx(1+\phi_1))^{1-\gamma}-1}{1-\gamma}\right].$$

- Using (18) and (19), it follows that:

$$(1+\phi_1)^{1-\gamma} + (1+\phi_1)^{1-\gamma}\beta\mathbb{E}\left[(x)^{1-\gamma}\right] = (1+\beta exp((1-\gamma)g_x)).$$

- $\Rightarrow \quad \phi_1 = \left(\frac{1+\beta exp((1-\gamma)g_x)}{1+\beta\mathbb{E}\left[(x)^{1-\gamma}\right]}\right)^{\frac{1}{1-\gamma}} - 1.$

Incomplete markets

- Defining $\widetilde{b}_1 \equiv b_1 / Y$, then the optimal saving decision can be written as:

$$\left(1 - \widetilde{b}_1\right)^{-\gamma} = \beta R \mathbb{E}\left[\left(x + R\widetilde{b}_1\right)^{-\gamma}\right].$$

- R satisfies $1/R = \beta exp(-\gamma g_x)$. Hence, last equation becomes:

$$\left(1 - \widetilde{b}_1\right)^{-\gamma} = \exp(\gamma g_x)\mathbb{E}\left[(x + R\widetilde{b}_1)^{-\gamma}\right].$$

- $\Rightarrow \quad W_{IM}(\phi_2) = \frac{(Y(1+\phi_2)(1-\widetilde{b}_1))^{1-\gamma}-1}{1-\gamma} + \beta \mathbb{E}\left[\frac{(Y(1+\phi_2)(x+R\widetilde{b}_1))^{1-\gamma}-1}{1-\gamma}\right].$
- Using $W_{IM}(\phi_2)$ and (24), and solving for ϕ_2, we have:

$$\phi_2 = \left(\frac{1 + \beta exp((1-\gamma)g_x)}{(1-\widetilde{b}_1)^{1-\gamma} + \beta\mathbb{E}\left[(x + R\widetilde{b}_1)^{1-\gamma}\right]}\right)^{\frac{1}{1-\gamma}} - 1$$

Types of risk and welfare gains

- Income risk type I: Normal distribution
  - $\log(x) \sim N(\mu_x, \sigma_x^2)$.
  - μ_x and σ_x are computed indirectly based on the information in table with estimation of macroeconomic disasters

- Income risk type II: Combination of normal and power-law distributions
  - With probability $1 - p$: $\log(x) = \log(\tilde{x}) \sim N(\tilde{\mu}_x, \tilde{\sigma}_x^2)$.
    With probability p : $x = 1/z$.
  - z, the reciprocal of the contraction size in the case of disaster, follows a power law distribution with pdf:

$$f_Z(z) = (\alpha - 1)(z_{min})^{\alpha - 1} z^{-\alpha}, \alpha > 1$$

  - z_{min}, α, p, $\tilde{\mu}_x$, and $\tilde{\sigma}_x$ comes directly from table with estimation of macroeconomic disasters.

Types of risk and welfare gains - Risks type II (cont.)

- Income risk type II: Combination of normal and power-law distributions (cont.)
- From our estimation for α we consider three cases: (i) Power-law medium: $\alpha = 4.60$ (point estimate); (ii) Power-law low: $\alpha = 5.63$ (upper bound estimate); and (iii) Power-law high: $\alpha = 3.98$ (lower bound estimate)

Type I risks

Table: Welfare gains from financial autarky to complete markets (ϕ_1)

	γ						
	2.0	2.5	3.0	3.5	4.0	4.5	5.0
	0.177%	0.221%	0.264%	0.307%	0.350%	0.393%	0.435%

Table: Non-contingent bond holdings with type I risks ($\widetilde{b}_1$)

	γ						
	2.0	2.5	3.0	3.5	4.0	4.5	5.0
	0.26%	0.31%	0.35%	0.39%	0.43%	0.47%	0.51%

Table: Welfare gains from non-contingent bond to complete markets (ϕ_2)

	γ						
	2.0	2.5	3.0	3.5	4.0	4.5	5.0
	0.177%	0.220%	0.263%	0.305%	0.347%	0.387%	0.429%

Type II risks

Table: Welfare gains from financial autarky to complete markets (ϕ_1)

	γ						
α	2.0	2.5	3.0	3.5	4.0	4.5	5.0
3.98	0.69%	1.11%	1.91%	4.30%	∞	∞	∞
4.60	0.53%	0.80%	1.20%	1.95%	3.80%	20.50%	∞
5.63	0.40%	0.56%	0.77%	1.07%	1.52%	2.35%	4.34%

Type II risks (cont.)

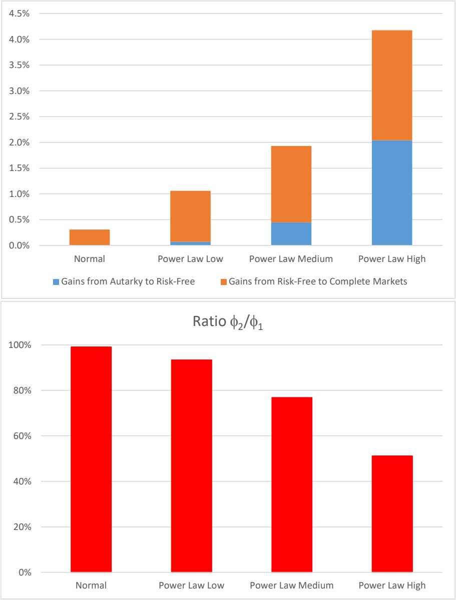
Table: Non-contingent bond holdings with type II risks ($\widetilde{b}_1$)

	γ						
α	2.0	2.5	3.0	3.5	4.0	4.5	5.0
3.98	1.58%	2.42%	3.46%	4.91%	6.06%	7.81%	9.52%
4.60	1.10%	1.53%	2.24%	3.12%	4.62%	5.72%	7.59%
5.63	0.74%	0.99%	1.32%	1.84%	2.39%	3.19%	4.33%

Table: Welfare gains from non-contingent bond to complete markets (ϕ_2)

	γ						
α	2.0	2.5	3.0	3.5	4.0	4.5	5.0
3.98	0.66%	1.01%	1.48%	2.16%	2.98%	4.16%	5.59%
4.60	0.52%	0.74%	1.06%	1.49%	2.19%	2.95%	4.14%
5.63	0.39%	0.54%	0.73%	1.00%	1.29%	1.74%	2.34%

Composition of welfare gains (illustration with $\gamma = 3.5$)



Conclusions

Policy conclusions

- Traditionally, welfare gains of financial integration are very small and almost all of them are accrued from incomplete (IM) to complete markets (CM).
- The policy implication in this Gaussian world would thus be to hold few non-contingent assets given that its pay-off is almost nil.
- In the presence of a power-law distribution, welfare gains are about the same in the first segment (from FA to IM) as in the second (from IM to CM).
- Our policy conclusion is thus that, in a world of fat tails, there is a clear theoretical case for self-insurance with non-contingent assets (FX reserves).