Endogenous Fiscal and Monetary Interactions:

The Curious Case of Brazil

Osvaldo Candido[*] Jose Angelo Divino[*]

Peter McAdam[**] Jaime Orrillo[*]

[*]Catholic University of Brasilia

[**]European Central Bank

Econometric Society European Meeting, 2024.
Rotterdam, Netherlands.

Outline

- Motivation
- Some Brazilian data
- Model economy
- Policy interactions
- Empirical evidence
- Conclusion

Motivation

- Government wants to stabilize the economy over the business cycle: on the fiscal side (τ, G) and/or on the monetary side (r).

Motivation

- Government wants to stabilize the economy over the business cycle: on the fiscal side (τ, G) and/or on the monetary side (r).
- Among developed countries, there is some tendency for co-ordination of domestic monetary and fiscal policies.
  - *Downturn:* High expenditure and accommodative monetary policy to stimulate economic activity.
  - *Upturn:* Tight fiscal policy to correct past deficits coupled with active monetary policy to fight inflation.

Motivation

- Government wants to stabilize the economy over the business cycle: on the fiscal side (τ, G) and/or on the monetary side (r).
- Among developed countries, there is some tendency for co-ordination of domestic monetary and fiscal policies.
  - *Downturn:* High expenditure and accommodative monetary policy to stimulate economic activity.
  - *Upturn:* Tight fiscal policy to correct past deficits coupled with active monetary policy to fight inflation.
- This behavior could have threshold characteristics that isn't normally part of the debate.

Motivation

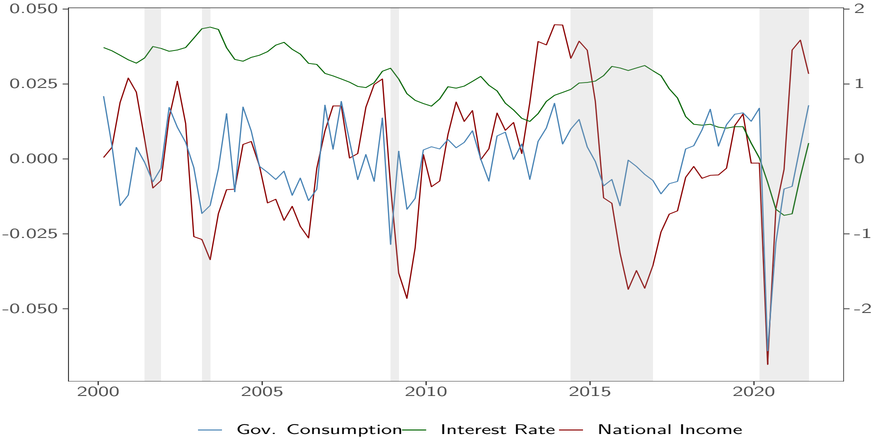
- Government wants to stabilize the economy over the business cycle: on the fiscal side (τ, G) and/or on the monetary side (r).
- Among developed countries, there is some tendency for co-ordination of domestic monetary and fiscal policies.
  - *Downturn:* High expenditure and accommodative monetary policy to stimulate economic activity.
  - *Upturn:* Tight fiscal policy to correct past deficits coupled with active monetary policy to fight inflation.
- This behavior could have threshold characteristics that isn't normally part of the debate.
- Emerging economies may not have such standard endogenous policy interactions, as policies are pro-cyclical and government spending is usually too high.

Motivation

- We explore this issue in the context of a simple General Equilibrium (GE) model and confront theoretical results with Brazilian data.
- What's so special about Brazil?
  - Highly volatile emerging economy;
  - Government consumption at extremes;
  - Stabilization difficulty;
  - Unclear fiscal and monetary policy coordination;
  - Local characteristics of stabilization patterns.

A look at the Brazilian data

Figure: Cyclical behavior of selected variables



What do the data suggest

- Practical coordination between fiscal and monetary policy during the downturn of the business cycle.

What do the data suggest

- Practical coordination between fiscal and monetary policy during the downturn of the business cycle.
- However, the same practice is not adopted during boom episodes, whereupon these policies seem to behave in an independent manner.

What do the data suggest

- Practical coordination between fiscal and monetary policy during the downturn of the business cycle.
- However, the same practice is not adopted during boom episodes, whereupon these policies seem to behave in an independent manner.
- It is not clear the role each policy is playing in the different phases of the business cycle nor how they might eventually strength coordination to improve the country's macroeconomic performance.

Main Objectives

- To propose a simple two-period general equilibrium model with a representative lender, the government and no uncertainty.

Main Objectives

- To propose a simple two-period general equilibrium model with a representative lender, the government and no uncertainty.
- To construct a sequential game with perfect information played by the fiscal authority (the leader) and the monetary authority (the follower), and vice-versa to investigate (endogenous) policy interactions that might depend on a threshold.

Main Objectives

- To propose a simple two-period general equilibrium model with a representative lender, the government and no uncertainty.
- To construct a sequential game with perfect information played by the fiscal authority (the leader) and the monetary authority (the follower), and vice-versa to investigate (endogenous) policy interactions that might depend on a threshold.
- To estimate and test the relationships emerging from the theoretical model by using data for the Brazilian economy.

Related Literature

- Sargent and Wallace (1981)
- Leeper (1991)
- Sims (1994)
- Woodford (1994,1995, 2001, 2011),
- Cochrane (1998, 2001),
- Schmitt-Grohé and Uribe (2000),
- Bassetto (2002),
- Engwerda et al. (2002),
- Reis (2016),
- Bianchi and Melosi (2019) among others
- Elenev et al. (2021)

The major difference from our framework to this literature is the threshold-dependent endogenous interaction between policies.

The Model: Agents

- The economy comprises a representative taxpayer and a government.

The Model: Agents

- The economy comprises a representative taxpayer and a government.
- The government is collectively represented by the monetary and fiscal authorities that are assumed to be institutionally independent.

The Model: Agents

- The economy comprises a representative taxpayer and a government.
- The government is collectively represented by the monetary and fiscal authorities that are assumed to be institutionally independent.
- *Central Bank*: sets the policy interest rate.
- *Treasury*: decides the tax rate on income gains (financial and non-financial).

The Model: Resources and Constraints

- Two period perfect foresight endowment framework $\omega_1 \lesseqgtr \omega_2$
- $0 < G < \omega_1$.

The Model: Resources and Constraints

- Two period perfect foresight endowment framework $\omega_1 \lesseqgtr \omega_2$
- $0 < G < \omega_1$.
- Only one asset available for trade: risk-free government debt whose price, $q = (1 + r)^{-1}$, is determined in equilibrium with $r > -1$ being the risk-free interest rate ($r = (1 - q)/q$).

The Model: Resources and Constraints

- Two period perfect foresight endowment framework $\omega_1 \lesseqgtr \omega_2$
- $0 < G < \omega_1$.
- Only one asset available for trade: risk-free government debt whose price, $q = (1 + r)^{-1}$, is determined in equilibrium with $r > -1$ being the risk-free interest rate $(r = (1 - q)/q)$.
- Policy instruments are τ, r, and exogenous G.

The Model: Resources and Constraints

- Two period perfect foresight endowment framework $\omega_1 \lesseqgtr \omega_2$
- $0 < G < \omega_1$.
- Only one asset available for trade: risk-free government debt whose price, $q = (1+r)^{-1}$, is determined in equilibrium with $r > -1$ being the risk-free interest rate $(r = (1-q)/q)$.
- Policy instruments are τ, r, and exogenous G.
- The consumption good is the numeraire, so that all values are expressed in terms of this good.

The Model: The taxpayer's problem

Given $\tau \in (-1, 1)$ and q, the representative taxpayer chooses a consumption-investment plan (c_1, c_2, θ) to maximize utility s.t. the following budget constraints:

- In the first period:

$$c_1 = \omega_1 - q\theta, \tag{1}$$

- In the second period:

$$c_2 + \tau R = \omega_2 + \theta, \tag{2}$$

where R captures the total income gain (financial and non-financial) of the representative taxpayer:

$$R = (\omega_2 - \omega_1) + \theta(1 - q) \tag{3}$$

The Model: Government decision

Given the price q of the risk-free security, the government sells Θ units of the security to the representative taxpayer in the first period and levies an income-tax rate τ on any possible income gain in the second period.

That is, the government chooses (Θ, τ) in order to balance its budget constraints in both periods:

$$G = q\Theta, \tag{4}$$

and

$$\Theta = \tau R, \tag{5}$$

where G is government consumption and R is given by (3).

The Model: Equilibrium

Definition

An equilibrium for the economy $\mathcal{E}$ consists of an allocation of consumption-investment plan $(\overline{c}_1, \overline{c}_2, \overline{\theta})$ and a fiscal policy for debt and income-tax rate, $(\overline{\Theta}, \overline{\tau})$, such that:

1. *The choices of the representative taxpayer are optimal. That is, the consumption-investment plan $(\overline{c}_1, \overline{c}_2, \overline{\theta})$ maximizes $U(c_1, c_2)$*
2. *Fiscal policy, $(\overline{\Theta}, \overline{\tau})$, satisfies the budget constraints (4) and (5).*
3. *Markets clear, meaning that $\overline{\Theta} = \overline{\theta}$ and $\overline{c}_1 + G = \omega_1$ in the first period and $c_2 = \omega_2$ in the second period.*

Equilibrium with log utility

The taxpayer's preferences are represented by a logarithmic utility function.

$$U(c_1, c_2) = \ln c_1 + \beta \ln c_2$$

where $\beta \in (0, 1)$ is the subjective discount factor.

Given q and τ, the taxpayer chooses $\theta \geq 0$ to maximize:

$$\ln(\omega_1 - q\theta) + \beta \ln\left(\omega_2 + \theta\Big(1 - \tau(1-q)\Big) - \tau(\omega_2 - \omega_1)\right). \quad (6)$$

Equilibrium with log utility

The FOC implies that:

$$\frac{q}{\omega_1 - q\theta} = \frac{\beta(1-\tau(1-q))}{\omega_2 + \theta\Big(1-\tau(1-q)\Big) - \tau\big(\omega_2 - \omega_1\big)}. \tag{7}$$

Given q, the government chooses (Θ, τ) to balance its budget constraints:

$$G = q\Theta, \tag{8}$$

$$\Theta = \tau R = \tau\Big(\omega_2 - \omega_1 + (1-q)\theta\Big). \tag{9}$$

Market clearing condition requires that:

$$\Theta = \theta. \tag{10}$$

Equations (7)-(10) characterize the equilibrium in this economy.

Computation of equilibrium

Solving for θ from (7) and Θ from (8), the demand and supply for bonds are, respectively:

$$\begin{aligned} \theta &= \frac{\beta\omega_1}{(1+\beta)q} - \frac{\omega_2 - \tau(\omega_2 - \omega_1)}{(1-\tau(1-q))(1+\beta)}, \\ \Theta &= \frac{G}{q}. \end{aligned} \quad (11)$$

Assuming market clearing, $\Theta = \theta$ one obtains the equilibrium price (or gross interest rate) as a function of τ:

$$(1/q \equiv 1 + r) = \frac{1}{1-\tau}\left(\frac{\omega_2 - \tau(\omega_2 - \omega_1)}{\beta\omega_1 - G(1+\beta)} - \tau\right). \quad (12)$$

Computation of equilibrium

Substituting this into (9), the income-tax rate levied by the government is:

$$\tau = \frac{G/q}{(\omega_2 - \omega_1) + \frac{G}{q} - G}. \tag{13}$$

Substituting the first relation of (12) in (13) and under the perfect foresight assumption, the equilibrium income-tax rate levied by the government is given by:

$$\overline{\tau} = \frac{G\omega_2}{\beta(\omega_1 - G)(\omega_2 - \omega_1)}. \tag{14}$$

Condition (14) demonstrates that fiscal policy is counter cyclical. That is, the government taxes in a "boom" (when $\omega_2 > \omega_1$) and subsidizes in a "bust" (when $\omega_2 < \omega_1$).

Computation of equilibrium

Inserting (14) into (12), yields the equilibrium bond price (equivalent to equilibrium gross interest rate):

$$1/\overline{q} = 1 + \overline{r} = \frac{\omega_2(\omega_2 - \omega_1) - G\omega_2}{\beta(\omega_1 - G)(\omega_2 - \omega_1) - G\omega_2}. \tag{15}$$

The amount of bonds in equilibrium is given by:

$$\overline{\theta} = \overline{\Theta} = (1 + \overline{r})G = \left(\frac{\omega_2(\omega_2 - \omega_1) - G\omega_2}{\beta(\omega_1 - G)(\omega_2 - \omega_1) - G\omega_2}\right)G \tag{16}$$

Finally, the levels of private consumption in equilibrium equal:

$$\begin{aligned} \overline{c}_1 &= \omega_1 - \overline{q}\overline{\theta} = \omega_1 - G \\ \overline{c}_2 &= \omega_2 \end{aligned} \tag{17}$$

Comparative statics

- Having computed the equilibrium, a natural question to ask is – how that equilibrium changes when some fundamentals change.
- We address this question by parameterizing the equilibrium monetary and fiscal policies in scenarios of a boom and a bust.
- The choice of G as a comparative static of interest is natural because it has a direct impact on consumption, see (17).
- Then, we analyze the behavior of both policies in equilibrium. We begin by setting the following curve $\alpha : \mathbb{R} \to \mathbb{R}^2$ as follows:

$$\alpha(G) = \left(1 + \bar{r}(G), \bar{\tau}(G)\right) \tag{18}$$

where $1 + \bar{r}$ and $\bar{\tau}$ are both defined as in (15) and (14), respectively.

Comparative statics

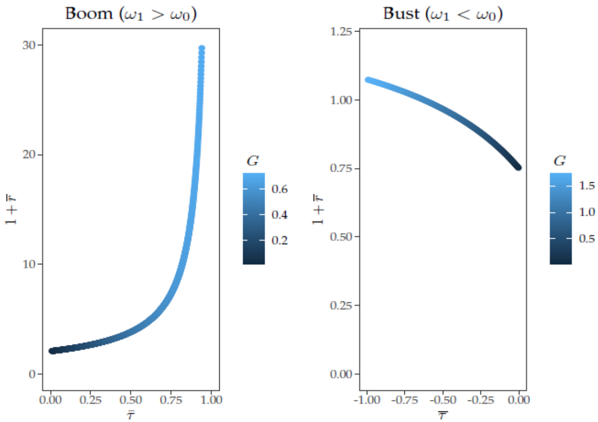
- Let's think about how $\alpha(\cdot)$ behaves over the business cycle:
  - A 'bust': $(\omega_2 - \omega_1) < 0$
    - here $\partial\bar{\tau}/\partial G < 0$
    - this implies an upper bound for public expenditure $(\tau(G) = -1)$:

$$0 < G^u_{bust} = \frac{\beta\omega_1(\omega_1 - \omega_2)}{\omega_2 + \beta(\omega_1 - \omega_2)} < \omega_1$$

  - A 'boom': $(\omega_2 - \omega_1) > 0$
    - here $\partial\bar{\tau}/\partial G > 0$
    - this implies an upper bound for public expenditure $(\tau(G) = 1)$:

$$0 < G^u_{boom} = \frac{\beta\omega_1(\omega_2 - \omega_1)}{\omega_2 + \beta(\omega_2 - \omega_1)} < \omega_1$$

Figure 2: Equilibrium path of monetary and fiscal policies parameterized by G



Notes: We plot the equilibrium path of the economy parameterized by the public expenditure, G. The left plot refers to a boom while the right one to a bust. The scale in blue represents an arbitrary increase in G, where the darker blue refers to a lower level of G. For the sake of the simulation, assuming $\beta = 0.8$ throughout, for a boom we choose

$$\omega_2 = 5 > \omega_1 = 3$$

while for a bust

$$\omega_2 = 3 < \omega_1 = 5$$

With these values, it is straightforward to see that the maximum G in a boom (given by (24)) is below 0.73 while the maximum G in a bust (given by (23)) is lower than 1.74.

Source: Authors' numerical simulations.

Interaction between policies

- We've seen how both policies respond to exogenous changes in G. However, how do these policies interact with each other in equilibrium?
- We address this by investigating how the equilibrium τ responds to changes in the equilibrium r.
- Specifically, we ask what happens with the tax rate if the interest rate is not in equilibrium and vice-versa.
- To do this, we build a sequential game with complete information played by the monetary and the fiscal authority.

Interaction of fiscal and monetary policy

- Our main objective is to assess the effects of changes in the base interest rate on the tax rate in equilibrium.

Interaction of fiscal and monetary policy

- Our main objective is to assess the effects of changes in the base interest rate on the tax rate in equilibrium.
- We consider either (12) or (13). Depending on the equation, a different interaction might arise.

Interaction of fiscal and monetary policy

- Our main objective is to assess the effects of changes in the base interest rate on the tax rate in equilibrium.
- We consider either (12) or (13). Depending on the equation, a different interaction might arise.
- The interaction depends on the level of government consumption relative to private income, which rests on the assumption of what is considered a ‘low’ or ‘high’ public consumption.

Interaction of fiscal and monetary policy

- Our main objective is to assess the effects of changes in the base interest rate on the tax rate in equilibrium.
- We consider either (12) or (13). Depending on the equation, a different interaction might arise.
- The interaction depends on the level of government consumption relative to private income, which rests on the assumption of what is considered a 'low' or 'high' public consumption.
- We adopt two criteria:
  - *Prop 1*: a low or a high public consumption if $G < (\omega_2 - \omega_1)$ or $G > (\omega_2 - \omega_1)$, respectively, since $(\omega_2 - \omega_1) > 0$.
  - *Prop 2*: depends on how close is G to ω_1.

Monetary authority as Leader

We consider (13) to be the best response of fiscal policy to monetary policy setting, which sets $r = (1-q)/q$ according to the market equilibrium. Then, we can define a sequential game with perfect information played between the monetary authority (Leader) and the fiscal authority (Follower).

The perfect sub-game equilibrium is given by:

$$\begin{cases} \overline{r} & = \dfrac{(\omega_2 - \omega_1)\Big(\omega_2 - \beta(\omega_1 - G)\Big)}{\beta(\omega_2 - \omega_1)(\omega_1 - G) - G\omega_2} \\ \\ \tau(r) & = \dfrac{G(1+r)}{\omega_2 - \omega_1 + rG} \end{cases} \tag{19}$$

By substituting $\overline{r}$ in $\tau(r)$, we find that the backward-induction equilibrium $(\overline{r}, \overline{\tau})$ coincides with (14) and (15), which are part of the equilibrium.

Monetary authority as Leader

Proposition

Assume that $0 < G < \omega_1$. *Then, the following holds:*

1. *If* $G < (\omega_2 - \omega_1)$, *then* $\frac{d\tau}{dr} > 0$.
2. *If* $G > (\omega_2 - \omega_1) > 0$, *then* $\frac{d\tau}{dr} < 0$.
3. *If* $(\omega_2 - \omega_1) < 0$, *then* $\frac{d\tau}{dr} < 0$.

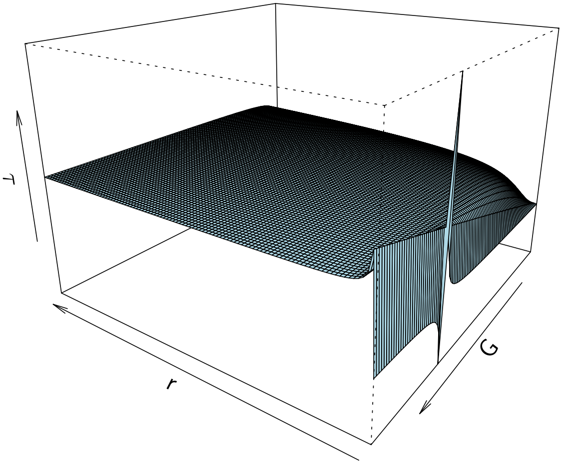
Proof.

By differentiating τ in (19) wrt r, we obtain:

$$\frac{d\tau}{dr} = \frac{G\Big((\omega_2 - \omega_1) - G\Big)}{\Big((\omega_2 - \omega_1) + rG\Big)^2}$$

Then, the proposition follows.

Figure: Graphical representation of system (19)



Optimal monetary policy

- The choice of the monetary authority in the perfect sub-game equilibrium given by (19) represents an optimal interest rate rule in the following sense:
  1. In this sequential game, it seems that the fiscal authority has an advantage because its choice comes after the Leader.
  2. However, since this is a complete information game, the Leader can solve the Follower's game in such a way that the optimal interest rate is essentially an optimal response to the choice of the fiscal authority.
  3. Nonetheless, if the authorities' roles were reversed, a similar argument could be applied to deliver an optimal fiscal rule.

Fiscal authority as Leader

Now, consider (12) as the reaction function of the monetary authority to the fiscal policy. More precisely, (12) defines r as being a function of τ, not necessarily the equilibrium one that is given by (14). In this case, we have that:

$$\begin{cases} \overline{\tau} & = \dfrac{G\omega_2}{\beta(\omega_1 - G)(\omega_2 - \omega_1)} \\ r(\tau) & = \dfrac{1}{1-\tau}\left(\dfrac{\omega_2 - \tau(\omega_2 - \omega_1)}{\beta\omega_1 - G(1+\beta)} - \tau\right) - 1 \end{cases} \tag{20}$$

Equations (20) can be thought of as being a perfect sub-game equilibrium of a sequential game with perfect information played by the fiscal authority (Leader), and the monetary authority (Follower).When $\overline{\tau}$ is replaced in $q(\tau)$, we obtain a backward-induction equilibrium $(\overline{\tau}, \overline{q})$ that coincides with the equilibrium price of this finance-fiscal economy.

Fiscal authority as Leader

If we allow for the best response $q(\tau)$ to depend also on government consumption, G, we have a hyperbole with vertical asymptote at

$$\overline{G} = \frac{\beta}{1+\beta}\omega_1.$$

and summarized by the following proposition:

Fiscal authority as Leader

Proposition

Let $0 < G < \omega_1$*. Then the following results hold:*

1. *if* $G < \frac{\beta}{1+\beta}\omega_1$*, then* $\frac{dr}{d\tau} > 0$.
2. *if* $G > \frac{\beta}{1+\beta}\omega_1$*, then* $\frac{dr}{d\tau} < 0$.

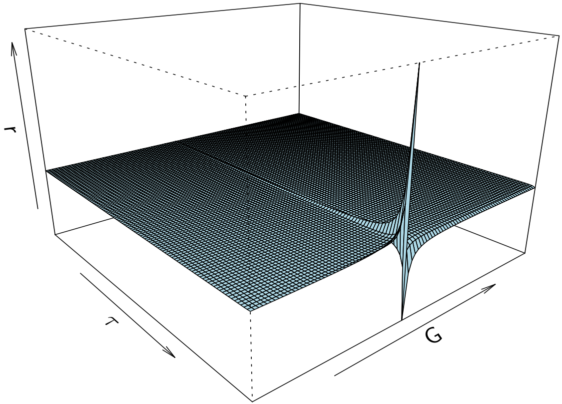
Proof.

By differentiating r in (20) wrt τ, we obtain

$$\frac{dr}{d\tau} = \frac{(1-\beta)\omega_1 + G(1+\beta)}{(1-\tau)^2\Big(\beta\omega_1 - G(1+\beta)\Big)}$$

From this, the proposition follows immediately. □

Figure: Graphical representation of system (20)



Empirical evidence: Threshold regression

We investigate the empirical relationship between income-tax rate and interest rate for Brazil in a threshold model that depends on the level of government consumption, G, as described in *Proposition* 1.

We focus on *Prop*.1 because the Central Bank acts as a Leader in a consolidated inflation-targeting monetary policy regime.

We apply a threshold regression framework that captures the dependence of τ on the level of G:

$$\tau_t = \sum_{j=1}^{m+1} \boldsymbol{\alpha}_j \mathbf{x}_t \mathbb{I}_{(G_t, \gamma_j)} + \varepsilon_t$$

where $\mathbf{x_t} = (\mathbf{1}, \omega_t, r_t)$, and $\mathbb{I}_{(G_t, \gamma_j)} = \mathbb{I}(\gamma_{j-1} \leq G_t < \gamma_j)$ is an indicator function for the j^{th} threshold (or break), m is the number of ordered threshold(s) with $\gamma_0 = -\infty$ and $\gamma_{m+1} = +\infty$, and $(\boldsymbol{\alpha}_j, \gamma_j)$ $\forall$ i, j are parameters to be estimated.

Data

The variables are quarterly from 2000q1 to 2021q4 and described as:

- τ_t : logarithm of the total income-tax revenue as a ratio of gross national income (SA). This give us the effective income-tax rate, i.e., the fraction of income that was effectively paid as income-tax by the private sector:
- ω_t : logarithm of gross national income at constant prices, SA;
- r_t : logarithm of the Over Selic interest rate, which is monetary policy instrument (the Brazilian equivalent of the Federal Funds Rate);
- G_t : logarithm of total government consumption index in real terms (SA).

Threshold regression estimates

Variables	$m = 1$		
	$G_t < \gamma = 4.9351$	$G_t \geq \gamma = 4.9351$	
ω_t	0.6568***	-0.0083	
	(0.1234)	(0.2640)	
r_t	0.1828***	-0.0628***	
	(0.0415)	(0.0193)	
Constant	-6.8830***	1.8970	
	(1.6256)	(3.4831)	
% obs.	47%	53%	
$R^2 = 0.45$	AIC = -2.7245	BIC = -2.5545	HQ = -2.6561
	$m = 2$		
	$G_t < \gamma_1 = 4.9351$	$\gamma_1 \leq G_t < \gamma_2$	$G_t \geq \gamma_2 = 4.9858$
ω_t	0.6568***	1.1190***	-0.7414***
	(0.1258)	(0.1574)	(0.2684)
r_t	0.1828***	0.0328**	-0.0662**
	(0.0423)	(0.0137)	(0.0323)
Constant	-6.8830***	-12.9659***	11.5746***
	(1.6565)	(2.0762)	(3.5395)
% obs.	47%	15%	38%
Threshold test	0 vs. 1	1 vs. 2	2 vs. 3
F-stat	12.5624***	26.2378***	4.5426
$R^2 = 0.57$	AIC = -2.9017	BIC = -2.6466	HQ = -2.7990

Conclusions

- This paper investigated theoretically and empirically the endogenous interaction of fiscal and monetary policy in a two-period GE model with a representative lender, the government and no uncertainty.
- We compute the equilibrium with logarithmic preferences and derive endogenously a sequential game with perfect information played by the fiscal (the Leader) and the monetary authority (the Follower) (and vice-versa).
- The best response of the fiscal authority depends on government consumption, which defines a threshold for the relationship between the effective income tax rate and the policy interest rate.

Conclusions

- The threshold regression estimated for recent Brazilian data closely resembled the theoretical findings.
- The policy interest rate has positive or negative effects on the effective income tax rate depending on whether public consumption is below or above the estimated threshold, respectively.
- When government consumption is "too high" relative to private income, the relationship between interest rate and effective income tax rate is negative. However, if public consumption is "low", the relationship is positive.
- These theoretical and empirical findings suggest a threshold-dependent interaction between the fiscal and the monetary policies that departures from the conventional wisdom.

Thanks!

jangelo@p.ucb.br