Does Household Heterogeneity across Countries Matter for Optimal Monetary Policy within a Monetary Union?*

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Abstract

The financial situation of households differs substantially across countries, but the implications of this heterogeneity is still vastly understudied. We examine the implications of this asymmetry for optimal monetary policy in a currency union. We build a two-country monetary union model with heterogeneous households leading to inequality due to imperfect insurance. We introduce money through central bank digital currency (CBDC) as a liquid asset for self-insurance against idiosyncratic risk. CBDC is a new instrument which allows the central bank to target heterogeneity within a monetary union. We derive a welfare function with two additional objectives, consumption inequality within and across countries. The more heterogeneous households are, the less important inflation stabilization becomes in favor of stabilizing consumption inequality through providing money. We provide important policy implications as we show that it is beneficial for a monetary union to have a country-specific instrument to compensate for country differentials.

Keywords: Heterogeneous Households, Liquidity, Imperfect Insurance, Optimal Monetary Policy, CBDC, Monetary Union, Two-country model
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1 Introduction

The financial situation of households has been shown to differ substantially across countries, but the implications of this heterogeneity is still vastly understudied. Household heterogeneity matters for national monetary policy as financially constrained or so-called “hand-to-mouth” households affect the transmission of monetary policy on aggregate demand (see, e.g., Almgren et al. 2022, Kaplan et al. 2018 or Thiel 2024). However, empirical evidence also shows heterogeneity in terms of the share of constrained households *across* countries forming a monetary union. This asymmetry applies, for example, to the Euro area. The estimated share is around 0.3 on average, but it varies from 0.1 to 0.65 across Euro area countries (Almgren et al. 2022, Kaplan et al. 2014). Research on the implications of this heterogeneity for optimal monetary policy is still scarce.

In this paper, we show that cross-country heterogeneity within a monetary union has profound implications for monetary policy. We build a two-country monetary union model with heterogeneous shares of constrained households across countries. The model features household heterogeneity and imperfect insurance leading to inequality. We introduce money through central bank digital currency (CBDC) as a liquid asset to self-insure against idiosyncratic risk. In our framework, CBDC is a new country-specific instrument which allows the central bank to compensate for country differentials. We derive a welfare function and find that two additional stabilization objectives for the central bank arise: consumption inequality within and across countries. The more heterogeneous households are within and across countries, the less important inflation stabilization becomes in favor of providing consumption insurance through money. Our analysis highlights the welfare-enhancing potential of a country-specific monetary policy tool that can target even households that are non-participating in financial markets.

We find that the asymmetry across countries matters for optimal monetary policy. The share of constrained households determines how important inequality stabilization is relative to output and price stabilization. Heterogeneity creates new room for optimization of the central bank: to provide insurance against idiosyncratic and aggregate risk (Acharya et al. 2023). In face of household heterogeneity, it is optimal for the central bank to tolerate higher inflation in favor of stabilizing consumption inequality through providing money. The insurance motive is increasing in the share of constrained households as well as the consumption inequality, two inequality metrics in our framework. The higher the inequality (in terms of both metrics), the more money the central bank provides. The distribution of money between the countries depends on their asymmetry. The central bank redistributes in favor of the country with higher inequality. Additionally, we look at optimal monetary policy following union-wide and country-specific technology shocks. We find that the greater the heterogeneity across countries, the more important money as an instrument of redistribution becomes. With money, monetary policy becomes more efficient in closing the arising relative country gaps within a monetary union.

Our analysis is motivated by the large cross-national differentials in the Euro area.[1] Although the countries share a common monetary policy, their heterogeneity makes them differently vulnerable to shocks (Ampudia et al. 2016). Almgren et al. (2022) emphasize that monetary policy has heterogeneous effects on aggregate demand depending on the share of constrained households within a country. We focus on the asymmetry according to the share of constrained households as there is much literature underlining the importance of this household type for monetary policy. In addition, we implement imperfect insurance to capture inequality between the household types which is rising in the majority of advanced countries in the last decades (see, e.g., Dossche et al. 2021).[2]

To adequately account for the asymmetry across countries, we build an analytically tractable Heterogeneous Agent New Keynesian (HANK) model with two countries forming a monetary union, presented in section 2. We use the one-country model of Bilbiie and Ragot (2021) with two different household types (constrained and unconstrained), extend it to a currency union and implement heterogeneous shares of constrained households. As further agents, the model contains monopolistically competitive firms facing Rotemberg (1982) price adjustment cost, national governments redistributing firm profits and a central bank. There are two countries: Home and Foreign. Households in both countries face the idiosyncratic risk of switching household type in the next period. Unconstrained households participate in financial markets in contrast to constrained households. There are three assets: firm shares, bonds and money. Money is introduced through CBDC as the only liquid asset being accessible for both household types. In contrast to transfer payments, money is a self-insurance tool. Households optimally choose to save in CBDC to self-insure against idiosyncratic risk. The model features imperfect insurance, i.e. consumption inequality in equilibrium. The central bank conducts optimal monetary policy and has two instruments, the nominal interest rate as a union-wide and liquidity as a country-specific

[1] The countries differ, among others, in the share of constrained households, indebtedness of households and share of adjustable rate mortgages (Ampudia et al. 2016).

[2] The Gini coefficient of disposable income for the four largest Euro area countries (France, Germany, Italy and Spain) lies approximately between 0.33 (Italy) and 0.38 (Spain), whereby the values for consumption inequality are somewhat lower, except for France (Dossche et al. 2021, data refer to 2013 to 2018). However, consumption and income inequality can be seen as mirror images in the last decades (Aguiar and Bils 2015).

instrument.

CBDC is introduced as a non-interest bearing retail CBDC. This corresponds to the ECB's plans to issue the digital euro as an unrenumerated asset being accessible to all Euro area agents.[3] The central bank is able to use CBDC to transfer money directly to households in face of shocks to provide consumption insurance.[4] Hence, the central bank has a country-specific tool to target even households which are non-participating in financial markets.[5] CBDC is a national instrument to compensate for heterogeneity within and across countries.

In section 3, we derive a welfare function approximated around a zero-inflation steady state to provide some intuition about the insurance motive of the central bank arising from imperfect insurance. The motive depends, among others, on the share of constrained households and consumption inequality. The higher the inequality (in terms of both metrics), the more important becomes the distortion through inequality for optimal monetary policy, the larger the welfare gains from providing consumption insurance to constrained households. This logic applies to both countries.

Optimal inflation depends on the share of constrained households as it determines the steady state. The optimal long-run inflation lies between the Friedman rule and zero inflation based on the two distortions of the model, imperfect insurance and price adjustment costs. In contrast to Bilbiie and Ragot (2021), we assume different shares of constrained households. In our framework, it is decisive for optimal monetary policy how we adjust the share of constrained households, by adjusting the risk of becoming constrained or unconstrained in the next period. A change in the share of constrained households can push the optimal inflation rate toward either end of the spectrum (Friedman rule or zero inflation). It is welfare-enhancing to increase the consumption of constrained households as we start from an inefficient steady state.

Furthermore, we look at the short-run and long-run implications of different shares of constrained households in a symmetric union (union-wide share varies), section 4, and an asymmetric union (heterogeneous shares across countries), section 5. For the simulation, we target empirical counterparts for the share of constrained households. To analyze the short-run implications, we look at positive technology shocks affecting both countries (symmetric shock) or only one country (idiosyncratic shock) in the symmetric as well as asymmetric union case.

In a symmetric union, the share of constrained households determines long-run optimal

[3] See, for example, European Central Bank (2023) about the proposed digital euro access for all Euro area households, governments and firms.
[4] It could even withdraw money from them. Both would be technically feasible with CBDC.
[5] This is in line with Temperini et al. (2024) emphasizing the potential of CBDC to target specific agents or sectors.

inflation and liquidity provision. The larger the share, the more liquidity the central bank provides in equilibrium as the instrument becomes more important. The relation is non-linear. If the share falls by 40%, money demand falls by approx. 64% in equilibrium. The need for self-insurance is increasing in the idiosyncratic risk. In the short-run, facing a positive symmetric technology shock, the central bank tolerates inflation volatility in favor of providing insurance. Facing an idiosyncratic shock, the central bank additionally redistributes towards the affected country through liquidity injections. Through liquidity, the central bank manages to completely equalize consumption dynamics in both countries regardless of the symmetry of shock.

In an asymmetric union, the central bank faces an additional distortion through heterogeneity across countries. In the long run, optimal monetary policy changes due to the asymmetry. The greater the heterogeneity between countries, the higher optimal union-wide deflation as consumption insurance becomes more relevant. The union-wide money demand of unconstrained households is slightly increasing in the asymmetry. The distribution of money between countries depends on their asymmetry. At the country level, the central bank redistributes in favor of the country with a higher share of constrained households (higher average risk of being constrained) in equilibrium. If, for example, the share of constrained households is 40% higher than in the other country, the country with higher risk receives approx. 40% more money. As in a symmetric union, the central bank manages to equalize consumption across household types across countries regardless of the share of constrained households within a country.

In the short run as well as in the long run, there is redistribution in favor of the more vulnerable country. Even if both countries experience the same shock, the strength of its transmission differs due to their asymmetry. In the event of an idiosyncratic shock, it is no longer optimal for monetary policy to equalize consumption dynamics. It matters for optimal monetary policy which country experiences a shock in an asymmetric union. If the more distorted country is hit, monetary policy optimally reacts more expansively in the short run. We observe more volatility in macroeconomic dynamics.

Our analysis bears important policy implications. First, it is optimal to tolerate inflation in favor of providing consumption insurance in face of household heterogeneity. The greater the heterogeneity within and across countries, the more important money as an instrument of insurance becomes. Second, the more vulnerable a country is, the more the central bank should provide insurance to it. Third, our analysis shows the welfare-enhancing potential of a country-specific tool for central banks to reach even households that are non-participating in financial markets. For a monetary union, it is beneficial to introduce such a national instrument, for example through CBDC, to target heterogeneity across countries.

Related Literature. The main contribution of this paper is to provide first evidence on the implications of *cross-national* heterogeneity for optimal monetary policy in a monetary union HANK model. Our work complements the HANK literature in various ways. We add on work about monetary policy within a currency union (as Bayer et al. 2024), about optimal monetary policy in face of heterogeneous households (e.g., Acharya et al. 2023, Bhandari et al. 2021, Bilbiie 2024, Hansen et al. 2023, Ida 2023 or Nuño and Thomas 2022), as well as extend the HANK setup to an open economy (e.g., Auclert et al. 2021, focusing on exchange rates, Levine et al. 2023, focusing on trade openness). Additionally, we refrain from the assumption of full insurance. We also contribute on literature about optimal monetary policy within a currency union in face of heterogeneous countries as in Brissimis and Skotida (2008), but they assume a representative household.

In the HANK literature, there are already some contributions about the implications of inequality for optimal monetary policy. Most of them concentrate on a closed economy (as Bilbiie and Ragot 2021 or Bilbiie 2024), or refrain from idiosyncratic risk, assuming a Two Agent New Keynesian (TANK) model (as Areosa and Areosa 2016, Ascari et al. 2017, Bilbiie et al. 2024, Hansen et al. 2023 or Ida 2023). We go one step further and analyze the importance of constrained households and their asymmetric shares across countries for optimal monetary policy in a currency union.

In the one-country TANK model of Hansen et al. (2023), they focus on the implications of steady-state income inequality for optimal monetary policy and the design of Taylor rules. The authors assume a tech-bias in wage income to implement that rich households benefit more from positive productivity shocks leading to inequality. The central bank should also stabilize consumption inequality next to inflation and output. The larger the steady-state inequality, the less important becomes inflation stabilization, similar to our results. In case of steady-state equality, optimal monetary policy is the same as in the representative agent case. In contrast to our work, they look on the design of different Taylor rules, partly augmented by consumption inequality. The authors conclude that a Taylor rule including inequality is superior to a standard Taylor rule as there are large welfare gains. However, these welfare gains are only small in case of optimal monetary policy as there is not much left to be gained if the central bank includes inequality.

Ida (2023) implements different shares of constrained households and wage rigidity in a two-country TANK model, but refrains from idiosyncratic risk. With a focus on the transmission of an idiosyncratic positive productivity shock, Ida (2023) shows that the expectation channel of monetary policy is weakened through the existence of liquidity constrained households. The macroeconomic fluctuations become smaller because of the existence of this household type and wage rigidity. The share of constrained households affects the weights of the central bank on its objectives, as in our model. In the presence of household heterogeneity, strict inflation targeting is no longer optimal.

Acharya et al. (2023) and Bilbiie (2024) analyze optimal monetary policy in a tractable HANK framework with one country. Through heterogeneity, a new objective arises for monetary policy: insuring consumption risk by accounting for consumption inequality besides output and price stabilization. In Acharya et al. (2023), output stabilization becomes relatively more important than inflation stabilization to provide consumption insurance. In contrast to our work, Bilbiie (2024) refrains from liquidity (as Challe 2020) and assumes full insurance. In his model, the full insurance equilibrium is still the first-best allocation, the weight on output stabilization is the same as with a representative household. However, inequality arises as an additional target criteria, whereby its weight depends on, among others, the share of constrained households.

In our model, the central bank can use money to address household heterogeneity within and across countries. CBDC becomes part of optimal monetary policy. This is in contrast to Bilbiie et al. (2024) studying a TANK model in which fiscal and monetary policy are mixed. In their model framework, fiscal transfers and interest rate are both equally effective in managing the demand side. Similar to our analysis and Bilbiie and Ragot (2021), the central bank faces a trade-off between price and inequality distortions. However, in our paper, we study the additional distortion of heterogeneity across countries that is absent in one-country models as in Bilbiie and Ragot (2021) and Bilbiie et al. (2024).

Bayer et al. (2024) provide a tractable HANK as well as a more complex HANK model of two countries forming a monetary union to analyze the differences between a monetary union and national monetary policies as well as distributional effects in case of an idiosyncratic technology shock on the different household groups of the countries. Instead of optimal monetary policy, they implement a Taylor rule focusing on inflation and ignoring the output gap, reflecting the behavior of the European Central Bank. They find out that a common monetary policy leads to horizontal rather than vertical redistribution across countries as there is redistribution between the same wealth groups. The suffering of the poorest households of one country is mirrored by a similar gain of the poorest households of the other country. For the middle class, the comparison between a monetary union and national monetary policies does not matter as the emerging two effects (change in interest rate returns, change in tax burden) cancel out. Wealth determines the distributional effects.

In contrast to Debortoli and Galí (2022), Hedlund et al. (2017) and Thiel (2024), we assume exogenous shares of households. We let the extension by, e.g., endogenous transition probabilities between the household states as in Thiel (2024) for future work.

The HANK literature already delivered some answers to positive questions about the interaction between inequality and monetary policy (e.g., how does household heterogeneity change the transmission of monetary policy on aggregate demand?). We contribute to this literature by analyzing the implications of household heterogeneity within a currency union for optimal monetary policy. In addition, a normative question follows the positive questions: Should the central bank care about inequality? According to our analysis, the answer is yes. The central bank should take inequality into account since, as a first point, it is welfare-enhancing to provide consumption insurance to constrained households. The central bank acts as an "insurance-giver" (McKay and Wolf 2023), as shown by Acharya et al. (2023), Bilbiie and Ragot (2021) and Hansen et al. (2023). Ma and Park (2022) find that including a Gini coefficient for income into the Taylor rule can be welfare-improving as poorer households are benefiting the most from monetary policy targeting inequality. As a second point, inequality affects the design of optimal monetary policy, which is the focus of our paper.

It is noteworthy that one also has to consider if the central bank has the right instrument at all to tackle inequality. In general, redistribution is seen as the primary objective of fiscal policy. McKay and Wolf (2023) argue that monetary policy is not able to reduce inequality efficiently since it has only moderate distributional effects on consumption across households. Thus, a central bank has to react aggressively to have an effect on consumption distribution, but this comes at the (from their point of view too high) costs of more volatile output and inflation.

In our model framework, the central bank has an efficient instrument to provide consumption insurance. It is optimal for the central bank to use money to tackle imperfect insurance and to balance out asymmetry within a currency union, both at the expense of inflation stabilization. Through money, the central bank is able to react to country-specific dynamics and thus overcome the main disadvantage of a monetary union.[6]

2 Model

We build an analytically tractable HANK model with two countries forming a monetary union. To this end, we extend the one-country framework of Bilbiie and Ragot (2021) to a currency union, refrain from exogenous income of constrained households[7] and implement different shares of constrained households across countries. The model consists of different household types (constrained and unconstrained) which will lead to country-specific inequality. This friction motivates the introduction of a country-specific asset as an insurance tool against inequality. Households can self-insure against the idiosyncratic risk of changing types by holding a liquid asset that we will call money. We introduce money through CBDC. Equipped with CBDC as a new country-specific monetary policy instrument, the central bank is able to target heterogeneity within the monetary union.

The rest of the model ingredients are standard. Monopolistically competitive firms face price adjustment cost which leads to a New Keynesian Phillips curve. National governments collect taxes for redistribution purposes, while the central bank sets the nominal interest rate at the union-level.

We normalize the total population to one, where the mass on the segment $[0, \gamma)$ belongs to (H)ome, while the population on $[\gamma, 1]$ belongs to (F)oreign. Throughout the presentation of the model, we will focus on home country. We denote variables of the foreign counterpart by an asterisk $(*)$.

2.1 Households

Home country's private composite consumption index is defined as

$$C_t \equiv \frac{(C_{H,t})^{\gamma}(C_{F,t})^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}, \tag{1}$$

[6] Pallotti et al. (2024) underline the effectiveness of country-specific transfers as a national instrument to counteract the impact of the inflation rise on Euro area households in 2021 and 2022.

[7] Bilbiie and Ragot (2021) assume exogenous income for the constrained households in the model used in their main text. Their appendix B.6 contains the model extended by endogenous income for this household type.

where $C_{H,t}$ and $C_{F,t}$ are domestic bundles of H and F goods indexed by z, z^*, given by

$$C_{H,t} \equiv \left[\left(\frac{1}{\gamma}\right)^{\frac{1}{\epsilon}} \int_0^\gamma C_{H,t}(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}} \qquad C_{F,t} \equiv \left[\left(\frac{1}{1-\gamma}\right)^{\frac{1}{\epsilon}} \int_\gamma^1 C_{F,t}(z^*)^{\frac{\epsilon-1}{\epsilon}} dz^*\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

while $\epsilon > 1$ denotes the elasticity of substitution between any two varieties. There are no barriers to trade, so the law of one price holds for each good. Since preferences are assumed to be identical in the entire union, the consumer price index of the final good is identical across countries: $P_t = P_t^*$. This consumer price index reads $P_t = P_{H,t}^\gamma P_{F,t}^{1-\gamma}$, where the producer prices at home and abroad are given by $P_{H,t} = \left[\left(\frac{1}{\gamma}\right) \int_0^\gamma P_{H,t}(z)^{(1-\epsilon)} dz\right]^{\frac{1}{1-\epsilon}}$ and $P_{F,t} = \left[\left(\frac{1}{1-\gamma}\right) \int_\gamma^1 P_{F,t}(z^*)^{(1-\epsilon)} dz^*\right]^{\frac{1}{1-\epsilon}}$, respectively. It is useful to define the terms of trade as the relative price of the F bundle in terms of the H bundle, i.e. $ToT_t \equiv P_{F,t}/P_{H,t}$.

The derivation of total demand for goods consists of three steps. The family head's (intertemporal) optimization problem described below leads to a certain level of C_t. The second step implies solving for the optimal allocation between H and F goods:

$$\begin{aligned} C_{H,t} &= \gamma \left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\gamma} C_t = \gamma ToT_t^{1-\gamma} C_t \\ C_{F,t} &= (1-\gamma) \left(\frac{P_{F,t}}{P_{H,t}}\right)^{-\gamma} C_t = (1-\gamma) ToT_t^{-\gamma} C_t. \end{aligned} \tag{3}$$

Third, households decide on the optimal consumption choice between individual goods, which we will describe when analyzing the behavior of firms.

Households can switch between two states: unconstrained and constrained. Throughout the paper, we will denote the former by S as (financially) unconstrained households will end up being the savers in the economy. The latter state is indexed by N as (financially) constrained households are non-participating in financial markets, which means no access to credit markets, no or near zero liquid wealth.[8] The switching process is described by a Markov chain with exogenous transition probabilities. A saver household stays unconstrained with probability α and becomes constrained with probability $(1-\alpha)$. The corresponding transition probabilities for a constrained household are ρ (staying constrained) and $(1-\rho)$ (switching to unconstrained state). This switching process is helpful to introduce idiosyncratic risk in a tractable way. Here, $(1-\alpha)$ and ρ display the idiosyncratic risk. There is no migration between countries.[9] Given

[8] According to the categorization of Kaplan et al. (2014), most used in the literature, liquid reserves of constrained households are lower than two weeks' value of income or they only hold 1/2 of their monthly income as liquid reserves.

[9] This implies that the probability of switching between country H and F is 0.

these transition probabilities, the share of non-participating households reads:

$$\lambda = \frac{1-\alpha}{2-\alpha-\rho} \tag{4}$$

For F, it holds: $\lambda^* = (1-\alpha^*)(2-\alpha^*-\rho^*)^{-1}$.

We assume that households are part of a family. The family head weights all members equally when maximizing the utility of the family. However, there is a lack of risk sharing. Although the family head is able to pool all resources between households of the same state, this ability is limited as only some resources can be transferred between states. As the households are symmetric within a state, the family head distributes consumption and asset holdings equally across households of the same state.[10] Being in the unconstrained state means that households can adjust their bond and money holdings and receive dividends due to their firm shares. In the constrained state, households only have access to money. Hence, money is the only transferable asset between states. Thus, there is an incentive to hold money despite the lower return (zero) compared to bonds ($i_t > 0$): money can be used to self-insure against the idiosyncratic risk of becoming constrained in the next period.

Let m_t^N (m_t^S) be the real money balances per capita of non-participating (saver) households at the beginning of period t, with $m_t^N = M_t^N/P_{t-1}$ ($m_t^S = M_t^S/P_{t-1}$) in terms of the consumer price index. The corresponding balances at the end of period t, but before switching the states, are $\tilde{m}_{t+1}^N$ and $\tilde{m}_{t+1}^S$. After the switching process, households enter the next period ($t+1$) with m_{t+1}^N and m_{t+1}^S, respectively. Money flows are thus given by:

$$\begin{aligned} (1-\lambda)m_{t+1}^S &= \alpha(1-\lambda)\tilde{m}_{t+1}^S + (1-\rho)\lambda\tilde{m}_{t+1}^N \\ \lambda m_{t+1}^N &= (1-\alpha)(1-\lambda)\tilde{m}_{t+1}^S + \rho\lambda\tilde{m}_{t+1}^N \end{aligned} \tag{5}$$

Rearranging and using (4) leads to:

$$\begin{aligned} m_{t+1}^S &= \alpha\tilde{m}_{t+1}^S + (1-\alpha)\tilde{m}_{t+1}^N \\ m_{t+1}^N &= (1-\rho)\tilde{m}_{t+1}^S + \rho\tilde{m}_{t+1}^N \end{aligned} \tag{6}$$

Let β display the discount factor, χ a scaling parameter, σ as the inverse of the intertemporal elasticity of substitution, and φ the inverse Frisch elasticity. The family head maximizes the

[10] See Challe et al. (2017) for an in-depth discussion of the family metaphor in tractable New Keynesian models with heterogeneous agents.

following life-time utility, weighted by household shares:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\lambda) \frac{(C_t^S)^{1-\sigma}}{1-\sigma} + \lambda \frac{(C_t^N)^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right] \tag{7}$$

over consumption C_t^j, bonds $\tilde{b}_{t+1}^S$ and money holdings $\tilde{m}_{t+1}^j$ with $j \in \{S, N\}$ subject to the money flow conditions (6) and the following budget and credit constraints. Note that labor supply L_t will be determined by a union and not by this optimization choice.

The budget constraint for savers in real terms reads:

$$C_t^S + \tilde{b}_{t+1}^S + \tilde{m}_{t+1}^S = ToT_t^{\gamma-1} \left(w_t L_t + \frac{1}{1-\lambda} d_t - \tau_t^S \right) + \frac{1+i_{t-1}}{1+\pi_t} b_t^S + \frac{1}{1+\pi_t} m_t^S + x_t \tag{8}$$

and for constrained households:

$$C_t^N + \tilde{m}_{t+1}^N = ToT_t^{\gamma-1} \left(w_t L_t - \tau_t^N \right) + \frac{1}{1+\pi_t} m_t^N + x_t, \tag{9}$$

with $w_t = W_t / P_{H,t}$ as real wage and $d_t = D_t / P_{H,t}$ as real profits in terms of the H producer price index. The variables τ_t^S and τ_t^N are lump-sum taxes to finance the sales subsidy that corrects the markup distortion. Bonds pay out the net nominal return i_t from t to $t+1$. Note that $\tilde{b}_{t+1}^S$ and b_{t+1}^S are identical as bonds are only held in the unconstrained state. The net inflation rate is defined as $\pi_t \equiv \frac{P_t - P_{t-1}}{P_{t-1}}$.

We assume that liquidity is provided at a national level. The central bank injects ($x_t > 0$) or destroys ($x_t < 0$) money, whereby x_t denotes newly created or destroyed (real) liquidity at the beginning of period t, with $x_t = x_t^S = x_t^N$. Money balances must be non-negative:

$$\tilde{m}_{t+1}^S \geq 0, \qquad \tilde{m}_{t+1}^N \geq 0, \tag{10}$$

which can be interpreted as credit constraints.

After rearranging, the optimization process of the family head delivers:

$$1 \geq \beta E_t \left[\left(\frac{C_t^S}{C_{t+1}^S} \right)^{\sigma} \frac{1+i_t}{1+\pi_{t+1}} \right] \tag{11}$$

$$1 \geq \beta E_t \left[\left(\frac{C_t^S}{C_{t+1}^S} \right)^{\sigma} \frac{1}{1+\pi_{t+1}} (\alpha + (1-\alpha) q_{t+1}^{\sigma}) \right] \quad or \quad \tilde{m}_{t+1}^S = 0 \tag{12}$$

$$1 \geq \beta E_t \left[\left(\frac{C_t^N}{C_{t+1}^N} \right)^{\sigma} \frac{1}{1+\pi_{t+1}} (\rho + (1-\rho) q_{t+1}^{-\sigma}) \right] \quad or \quad \tilde{m}_{t+1}^N = 0, \tag{13}$$

with $q_t \equiv C_t^S / C_t^N$ as consumption inequality between S and N. While (11) describes the standard Euler equation for bond holdings, only unconstrained households are able to hold these assets and there is no insurance motive as bonds cannot be carried over to the constrained state. On the other hand, households save in money to self-insure. Eq. (12) and (13) correspond to the money choices of savers and non-participating households as they take into account the probability of switching states next period.

Following Bilbiie and Ragot (2021), we will focus on calibrations that allow for end-of-period money holdings of savers, i.e. $\tilde{m}_{t+1}^S > 0$, but constrained households choose not to do so, i.e. $\tilde{m}_{t+1}^N = 0$.[11]

For the sake of simplicity, we also follow Bilbiie and Ragot (2021) based on Galí et al. (2007) by assuming that labor is determined by firms' demand while a union pools hours worked. Hence, independently of the state, all households work the same amount: $L_t^S = L_t^N = L_t$. The aggregate amount is determined by

$$\chi (L_t)^{\varphi} ((1-\lambda)(C_t^S)^{-\sigma} + \lambda (C_t^N)^{-\sigma})^{-1} = w_t ToT_t^{\gamma-1}. \tag{14}$$

2.2 Firms

A continuum of monopolistically competitive firms with unit mass produce different goods indexed by z with labor L_t as sole input according to $Y_t(z) = A_t L_t(z)$, where A_t is an exogenous technology disturbance. The demand function for an individual intermediate good follows from the consumption structure described above and reads $Y_t(z) = \left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon} Y_t$. Following Rotemberg (1982), price adjustments are costly. Real profits of an individual firm z in terms of the producer price index are given by:

$$d_t(z) = (1+\tau) \frac{P_{H,t}(z)}{P_{H,t}} Y_t(z) - w_t L_t(z) - \frac{\nu}{2} \left(\frac{P_{H,t}(z)}{P_{H,t-1}(z)} - 1 \right)^2 Y_t, \tag{15}$$

where τ is a sales subsidy to correct distortions due to market power and $\frac{\nu}{2}\left(\frac{P_{H,t}(z)}{P_{H,t-1}(z)} - 1\right)^2 Y_t$ are the quadratic price-adjustment costs with $\nu \geq 0$ as the parameter that defines the degree of nominal price rigidity. For $\nu = 0$, prices are fully flexible.

[11] This applies for $1+\pi > \beta$. Constrained households then decide not to save, as their discounted utility of tomorrow's consumption is lower than their utility of today's consumption.

By maximizing the present value of life-time profits, discounted by the discount factor of saver households since they are the only shareholders, and assuming symmetry across firms, i.e. $P_{H,t}(z) = P_{H,t}$, we can derive the non-linear Phillips curve:

$$\pi_{H,t}(1+\pi_{H,t}) = \beta E_t \left[\left(\frac{C_t^S}{C_{t+1}^S} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \frac{1+\pi_{H,t+1}}{1+\pi_{t+1}} \pi_{H,t+1}(1+\pi_{H,t+1}) \right] + \frac{\epsilon}{\nu} \left(\frac{w_t}{A_t} - \frac{1}{\Phi} \right), \tag{16}$$

where $\pi_{H,t} \equiv (P_{H,t} - P_{H,t-1})/P_{H,t-1}$ denotes the producer price net inflation rate and the markup after the subsidy is given by $\Phi \equiv \epsilon/[(\epsilon-1)(1+\tau)]$.

Then, real profits in aggregate terms read

$$d_t = (1 + \tau - \frac{w_t}{A_t} - \frac{\nu}{2}\pi_{H,t}^2)Y_t. \tag{17}$$

2.3 Governmental and monetary authorities

Money creation. The central bank sets the nominal interest rate at the union-level. As mentioned above, CBDC allows for providing liquidity at the national level. Hereby x_t denotes newly created or destroyed money in period t in country H. Money in circulation at the end of each period evolves according to

$$m_{t+1} = \frac{1}{1+\pi_t} m_t + x_t \tag{18}$$

in real terms. An analogous equation holds abroad: $m_{t+1}^* = m_t^*/(1+\pi_t) + x_t^*$.

Government. Both countries have a government redistributing firm profits and subsidizing firms. We also assume that the sales subsidy is financed by lump-sum taxes levied on both household types uniformly according to

$$\tau_t^S = \tau_t^N = \tau Y_t \tag{19}$$

in terms of the producer price index. Recall that both household types receive the identical wage income due to $L_t = L_t^S = L_t^N$. Suppose an optimal subsidy so that the markup completely vanishes in steady state, i.e. $\Phi = 1$. The uniform taxation implies that unconstrained households make positive dividends net of taxes and therefore have an higher income stream than constrained households. We assume identical substitution elasticity between goods across countries, $\epsilon = \epsilon^*$, which implies the same uniform taxation and subsidy in both countries, $\tau = \tau^*$.

2.4 Market clearing and aggregation

Let us start with a simplifying notation. An aggregate (union) variable X_t^U is defined as the weighted average of country-specific (national) variables: $X_t^U = \gamma X_t + (1-\gamma)X_t^*$.

Recall that the labor market equilibrium reads $L_t^S = L_t^N = L_t$. The equilibrium in the money market is given by

$$m_{t+1} = (1-\lambda)\tilde{m}_{t+1}^S + \lambda \tilde{m}_{t+1}^N. \tag{20}$$

Goods market clearing in both countries implies:

$$\gamma(1 - \frac{\nu}{2}\pi_{H,t}^2)Y_t = \gamma C_{H,t} + (1-\gamma)C_{H,t}^* \tag{21}$$

and

$$(1-\gamma)(1 - \frac{\nu}{2}\pi_{F,t}^2)Y_t^* = \gamma C_{F,t} + (1-\gamma)C_{F,t}^*, \tag{22}$$

which can be rearranged in terms of union consumption:

$$(1-\pi_{H,t}^2)Y_t = ToT_t^{1-\gamma}C_t^U \qquad (1-\pi_{F,t}^2)Y_t^* = ToT_t^{-\gamma}C_t^U. \tag{23}$$

Aggregate consumption levels in both countries are given by $C_t = (1-\lambda)C_t^S + \lambda C_t^N$ and $C_t^* = (1-\lambda^*)C_t^{S^*} + \lambda^* C_t^{N*}$, respectively.

As saver households in both countries participate in financial markets and have access to riskless bonds, there is perfect risk sharing between them leading to $C_t^S = C_t^{S*}$. Thus, at the union level, bonds are in zero net supply: $\gamma(1-\lambda)b_t^S + (1-\gamma)(1-\lambda^*)b_t^{S*} = 0$.

It follows that the aggregate resource constraint can be stated as

$$C_t^U = \gamma(1 - \frac{\nu}{2}\pi_{H,t}^2)Y_t ToT_t^{\gamma-1} + (1-\gamma)(1 - \frac{\nu}{2}\pi_{F,t}^2)Y_t^* ToT_t^{\gamma}. \tag{24}$$

3 Optimal monetary policy: the role of imperfect insurance

The objective for the policy maker, i.e. the central bank, is to maximize the weighted aggregate of households' utility functions. Similar to Bilbiie and Ragot (2021), we want to give the reader a first glimpse behind the workings of the model by analyzing the steady state (3.1), in which nominal variables grow at a constant rate π while real variables stay constant, before analyzing the implications of stabilizing welfare-relevant variables (3.2).

3.1 Steady-state considerations

For analyzing the steady state, we focus here on country H. The same steady-state considerations apply to F.

Given a positive money demand by savers, we get from (11) and (12) for both assets: $1 = \beta(1+i)/(1+\pi)$ and $1 = \beta(\alpha + (1-\alpha)q^{\sigma})/(1+\pi)$. Rearranging results in the following steady-state consumption inequality:

$$q = \left(\frac{1+i-\alpha}{1-\alpha}\right)^{1/\sigma}. \tag{25}$$

At the Friedman rule, i.e. $i = 0$ and thus $1+\pi = \beta$, the difference between the household types vanishes, i.e. $q = 1$, as the returns on bonds and money are the same. In this situation, money is a "perfect" means for insurance. Despite $\lambda \neq 0$, there is no steady-state consumption inequality. However, this will not be an efficient steady state due to price adjustment costs. Alternatively, a zero-inflation steady state, i.e. $i = (1-\beta)/\beta$, eliminates the steady-state distortion of price adjustment costs. However, this leads to a lack of insurance as the return on money relative to the one on bonds shrinks. The optimal long-run inflation rate lies between these two cases, i.e $\beta - 1 \leq \pi^{optimal} \leq 0$, since lack of full insurance and price adjustment costs are the two distortions in the steady state.[12] As long as $i > 0$, steady-state inequality arises, i.e. $q > 1$, since the opportunity cost of insurance increases. In fact, q is increasing in i for given values of $\alpha \in (0,1)$ and $\sigma > 0$.

The total money evolution equation (18) at the steady state reads $\pi m/(1+\pi) = x$. Using this and the market clearing condition for money to evaluate the steady-state effects of money holdings on the budget constraints (8) and (9) reveals the insurance channel. The effect for savers is $-(2-\alpha-\rho+\pi)\lambda m/((1-\lambda)(1+\pi))$, which reflects the costs of self-insurance and is clearly negative for any reasonable calibration.[13] The opposite is true for the positive effect for constrained households that gain from self-insurance of switching S-households: $(2-\alpha-\rho+\pi)m/(1+\pi)$. Savers use part of their income (instead of consumption) to increase income in the case of becoming constrained and thus being able to consume more in that state. In the extreme case of $\alpha = \rho = 1$, we end up in a TANK version of this model. Then, there is no need to self-insure ($m = 0$) and the zero-inflation steady state is the optimal one.

Suppose a change in ρ, holding everything else constant. Lowering ρ leads to a reduction

[12] The same logic as in Bilbiie and Ragot (2021) applies here.
[13] Given $\pi^{optimal} \geq \beta - 1$ (the lower bound), $1 + \beta > \alpha + \rho$ must hold for this direct effect to be negative.

in the share of constrained households (see (4)) as it gets more likely to become unconstrained. This reduces the need for self-insurance. Hence, price distortions are more relevant and the optimal long-run inflation rate is closer to zero ($\pi^{optimal} \to 0$). However, this implies a higher i and, as shown above, an increase in q. These outcomes can also be seen by the effects on the budget constraints. While a lower ρ reduces the amount of the (negative) effect of money holdings on savers' budget constraint, the effect on the budget constraint of N-households[14] becomes stronger. In other words, less money is needed for the same insurance effect. But since money is less attractive due to the higher return on bonds (i), less money is used to self-insure which corresponds to an increase in q.

Now, suppose a change in α, holding everything else constant. Increasing α leads to a reduction in the share of constrained households (see (4)) as it gets more likely to stay unconstrained. Although this lower idiosyncratic risk decreases the need to self-insure through money, there is an opposing effect through higher steady-state consumption inequality at work. In contrast to varying ρ, a change in α has a direct effect on q. Raising α increases q. The steady-state inequality distortion worsens. When we concentrate on the effects of money holdings on the budget constraint, we observe that the effect on savers' budget constraint is also reduced. This is similar to a decrease in ρ. However, the effect of money on the budget constraint of N-households is also reduced. In other words, money is less effective and therefore less used for insurance purposes. To counteract these effects and the worsening steady-state inequality distortion, i.e. price distortions are less relevant, the central bank sets a stronger optimal deflation ($\pi^{optimal} \to \beta - 1$). This implies a lower i.

To sum up, the optimal long-run inflation rate lies between the Friedman rule and zero inflation as the central bank trades off consumption insurance through liquidity against price stabilization. A change in the share of constrained households (either via ρ or α) can push the optimal inflation rate towards either end of the spectrum.

3.2 Welfare function

The following objective function, which the central bank minimizes, can be derived from a second-order Taylor expansion of the weighted aggregate of households' utility functions around

[14] This effect can also be described as indirect as it is based solely on the reaction of S-households, the only household type demanding money in equilibrium.

the zero-inflation steady state (see appendix A for details):

$$
\begin{aligned}
&-\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\Big[(\sigma+\varphi)(\tilde{C}_t^U)^2+\gamma\nu(\pi_{H,t})^2+(1-\gamma)\nu(\pi_{F,t})^2+\gamma(1-\gamma)(1+\varphi)(T\tilde{o}T_t)^2\\
&+\gamma(1-\gamma)\sigma\frac{CC^*}{(C^U)^2}(\hat{C}_t-\hat{C}_t^*)^2\\
&+\sigma\left(\gamma\lambda(1-\lambda)\frac{C^SC^N}{CC^U}(\hat{q}_t)^2+(1-\gamma)\lambda^*(1-\lambda^*)\frac{C^{S*}C^{N*}}{C^*C^U}(\hat{q}_t^*)^2\right)\\
&-2\gamma\lambda(q^\sigma-1)\left(\frac{C^N}{C^U}(\hat{C}_t^N+\frac{1-\sigma}{2}(\hat{C}_t^N)^2)-\hat{L}_t-\frac{1+\varphi}{2}(\hat{L}_t)^2\right)\\
&-2(1-\gamma)\lambda^*((q^*)^\sigma-1)\left(\frac{C^{N*}}{C^U}(\hat{C}_t^{N*}+\frac{1-\sigma}{2}(\hat{C}_t^{N*})^2)-\hat{L}_t^*-\frac{1+\varphi}{2}(\hat{L}_t^*)^2\right)\Big]
\end{aligned}
\tag{26}
$$

Variables without a time index stand for steady-state values. A "$\wedge$" is used to denote the log deviation of a variable from its steady-state value, while a "$\sim$" represents the gap between a variable and its efficient counterpart.[15]

The variables in the first line of (26) are the standard targets and weights for a two-country monetary union. The second line arises from a lack of full insurance between both countries as only saver households of both countries can participate in risk sharing. This can lead to welfare reducing differences in aggregate consumption at the country level. The inequality variables and weights in the third line are also common in the TANK literature[16] and therefore standard in a two-country monetary union with two different household types.

The last two lines arise due to a distorted steady state with inequality and can be interpreted as gains from consumption insurance ($\hat{C}_t^N$ and $\hat{C}_t^{N*}$). This insurance motive depends, among others, on two inequality metrics in this model: the share of constrained households and consumption inequality. The higher the inequality, the more important becomes the distortion through inequality for the central bank. Larger welfare gains from rising consumption of N arise. However, these gains via consumption have to be corrected by an increase in hours worked. The insurance motive vanishes for $q = 1$ (as the term $q^\sigma - 1$ disappears). The same applies to the foreign counterpart.

The role of λ (and thus α and ρ) can be better understood by using the welfare function in the case of a symmetric union (implying $q = q^*$, $\lambda = \lambda^*$ and $C = C^* = C^U$). A higher share of constrained households (λ) makes consumption insurance of N and inequality (for $\lambda < 0.5$) relatively more important compared to the other objectives. The optimal relative weight on

[15] The explanation for this gap and for the efficient steady state can be found in appendix A.
[16] See, among others, Ascari et al. (2017).

consumption insurance changes differently, depending on whether we vary α or ρ. If we adjust ρ, only λ changes, given a zero-inflation steady state.[17] If λ declines by lowering ρ, the welfare gains from consumption insurance decrease, thus the relative importance of consumption insurance declines. If we adjust α, there are effects on λ and q. If λ declines by increasing α, steady-state inequality q increases. While the former effect lowers the weight on the insurance motive, the latter increases the weight. Thus and as already discussed in the previous subsection, changing α leads to opposing effects on the insurance motive.

In the following sections, we want to emphasize the role of money as an insurance instrument by analyzing different values of λ in the cases of a symmetric and an asymmetric union.

4 Optimal monetary policy in a symmetric union

We look at the role of constrained households ($\lambda > 0$) for optimal monetary policy in a symmetric union ($\lambda = \lambda^* = \lambda^U$) and examine the implications of different shares of constrained households at the union-level. Section 4.1 provides the baseline calibration used for the simulation of the model. Section 4.2 examines the implications of the existence of constrained households for optimal monetary policy in the long run and therefore adresses the influence of λ on the trade-off faced by the central bank and on the model equilibrium. Section 4.3 examines how the central bank reacts optimally to a supply side shock affecting both countries (symmetric shock) or only one country (idiosyncratic shock).

4.1 Calibration

For simulating the model, we use the following calibration for both countries. For most of the parameters, we follow Bilbiie and Ragot (2021), summarized by Table 1.

Table 1: Baseline calibration based on Bilbiie and Ragot (2021)

Parameters	Values	Description
φ	0.25	Inverse Frisch elasticity
χ	1	Weight on disutility of labor
σ	1	Inverse intertemporal subsitution elasticity
β	0.98	Discount factor
ϵ	6	Substitution elasticity between goods
τ	$1/(\epsilon - 1)$	Optimal sales subsidy
ν	100	Rotemberg price adjustment cost
ρ_A	0.95	Persistence of technology shock

[17] In a zero-inflation steady state, there is no demand for money, which eliminates the effect of ρ on q.

The time interval is a quarter. We assume the countries to be of equal size, thus $\gamma = 0.5$.

Targeting the share of constrained households. We calibrate the union-wide share of constrained households (λ^U) to be 0.3 as it lies in the range for the estimates of the Euro area. According to Ampudia et al. (2018), the estimated share of constrained households is 0.24 and around 0.3 according to Almgren et al. (2022). In section 4.2, we additionally set λ^U equal to 0.5 and 0.2 to analyze the role of its share for optimal monetary policy.

For the symmetric union (section 4), we assume λ to be of equal size for both countries ($\lambda = \lambda^* = \lambda^U$). For the asymmetric union (section 5), we assume different shares of constrained households ($\lambda \neq \lambda^*$) as we observe large heterogeneity across Euro area countries (Almgren et al. 2022, Kaplan et al. 2014). Kaplan et al. (2014) deliver empirical evidence for a fraction of around 0.3 for Germany and around 0.2 for France and Spain, for example. According to Almgren et al. (2022), the share in France is around 0.2, in Italy, Spain and Germany above 20% in ascending order. For smaller Euro area countries, the shares vary from 10% in Malta to 65% in Latvia.

In the model, the share of constrained households in a country arises according to equation (4). Bilbiie and Ragot (2021) assume $\lambda = 0.5$ and $\alpha = \rho = 0.9$. As we want to target a lower (union-wide) share of $\lambda = 0.3$, we can let α or ρ adjust keeping the other parameter constant, as presented by Table 2, or adjust both parameters in combination. Furthermore, we concentrate on calibrations implying a positive money demand.

Table 2: Two approaches to target different λ values

λ	**(1) We let ρ vary**		**(2) We let α vary**	
	α	ρ	α	ρ
$\lambda = 0.5$	0.9	0.9	0.9	0.9
$\lambda = 0.3$	0.9	0.7667	0.9571	0.9
$\lambda = 0.2$	0.9	0.6	0.975	0.9

There are an infinite number of possible combinations between these two approaches. In the following analysis, we focus on letting ρ vary and holding α constant, see approach (1). We use this approach as we want to isolate the effect of different shares of constrained households ($\lambda = 0.2, 0.3$ and 0.5) on optimal monetary policy. In contrast to this approach, letting α vary, approach (2), also has an effect on other variables like steady-state inequality q, see for example equation (25). We discuss this further in section 4.2.

Choosing approach (1) results in $\rho = 0.7667$ for $\lambda = 0.3$ and α fixed to 0.9. In section 4.2, we present the long-run implications for both approaches as it is helpful to understand the model mechanisms. From section 4.3 onwards, we use approach (1) and provide approach (2) of varying α and holding ρ constant as a robustness check in appendix B.

4.2 Long-run implications

We analyze the long-run implications of the existence of constrained households for optimal monetary policy by simulating the model based on the calibration described in subsection 4.1. Table 3 presents steady-state values implied by Ramsey optimal policy for different values of λ. As we here look at a symmetric union, we refrain from using the country indices in this subsection. The case of $\lambda = 0.5$ is comparable to the model of Bilbiie and Ragot (2021) with an endogenous income of N.

Table 3: Implied steady-state values from Ramsey optimal policy in a symmetric union for different union-wide shares of constrained households (λ)

Model outcome					
	π	m	x	i	q
$\lambda = 0.5$	-0.366%	0.627	-0.002306	0.0167	1.167
$\lambda = 0.3$	-0.297%	0.278	-0.000828	0.0174	1.174
$\lambda = 0.2$	-0.226%	0.148	-0.000335	0.0181	1.181

Note: Since we look at a symmetric union (and same country size, $\gamma = 0.5$), the values for π, m, x and q are the same for both countries and for the union, e.g., $x = x^* = x^U = -0.002306$ in case of $\lambda = 0.5$.

Optimal monetary policy depends on λ as the optimal amount of money in circulation (m) and deflation (π) are increasing in λ. There is a non-linear relation between the share of constrained households and money demand. If the share falls by 40% (from $\lambda = 0.5$ to $\lambda = 0.3$), the demand for money falls by approx. 63.66% (from $m = 0.627$ to $m = 0.278$). As discussed in section 3, a lower ρ implying a lower λ implies less optimal deflation ($\pi^{optimal} \to 0^-$), at given α, as the need for self-insurance is lower. However, this implies a slightly higher i and thus a slight increase in q. An analogous disproportionately decrease in the usage of liquidity as insurance instrument can be seen by lowering the share of constrained households from $\lambda = 0.3$ to $\lambda = 0.2$. To put it differently, the insurance-through-liquidity motive of the central bank is increasing in λ. It vanishes for $\lambda \to 0$.

What if we let α vary instead of ρ? Furthermore, we check if the results survive if we let α vary instead of ρ. Table 4 reports the steady-state values implied by Ramsey optimal policy for different λ values for this approach:

Table 4: Implied steady-state values from Ramsey optimal policy in a symmetric union for different union-wide shares of constrained households (λ) if we vary α

Model outcome if we vary α ($\rho = 0.9$)					
	π	m	x	i	q
$\lambda = 0.5$	-0.366%	0.627	-0.002306	0.0167	1.167
$\lambda = 0.3$	-0.943%	0.369	-0.003511	0.0108	1.252
$\lambda = 0.2$	-1.303%	0.106	-0.001396	0.0071	1.285

The main message survives: The need for self-insurance increases with λ, the instrument liquidity (m) becomes more important with higher λ. A lower λ implies lower liquidity provision. As already discussed in the previous section, a higher α (lower idiosyncratic risk) implies higher consumption inequality in equilibrium (q)[18] increasing the relative optimal weight on consumption insurance and decreasing the importance of price stabilization. The central bank sets a stronger optimal deflation.

The decision on how we adjust λ determines the trade-off of the central bank between price stabilization and consumption insurance through liquidity. Both approaches change λ, but there are less implications for other variables (i and q) by adjusting ρ while holding α constant. Thus we get a more "pure" effect of a change in λ and can avoid mixed effects.

Later on, in case of an asymmetric union, we set different shares of constrained households for the countries, $\lambda \neq \lambda^*$.

4.3 Short-run implications

How does the central bank react optimally to a supply shock in face of heterogeneous households? To examine the short-run implications for optimal monetary policy in case of a symmetric union, we look at a positive technology shock.[19] We simulate a positive technology shock on both countries (symmetric shock, scenario 1) or on only one country (idiosyncratic shock, scenario 2) and compare both scenarios.

Technology follows an AR (1) process in each country: $\log A_t = \rho_A \log A_{t-1} + \epsilon_t$ with $0 \leq \rho_A < 1$ as persistence parameter and ϵ_t as shock term in the respective country. In case of a symmetric shock, H and F experience the same positive productivity shock of 1% increase ($\epsilon_1 = \epsilon_1^* = 0.01$). In case of an idiosyncratic shock, when only one country is hit, we double

[18] In contrast to approach (1), q increases considerably more.

[19] Most used shock in other models about optimal monetary policy and heterogeneous agents, see, for example, Bayer et al. 2024, Hansen et al. 2023 or Ida 2023.

the shock impulse strength ($\epsilon_1 = 0.02$ or $\epsilon_1^* = 0.02$) to have the same shock impulse strength at union level making the impulse response functions comparable.

Throughout the paper, we analyze a total of five different scenarios for working out the short-run implications of household heterogeneity for optimal monetary policy in a symmetric (section 4.3) and an asymmetric union (section 5.2) and compare them with each other. Table 5 provides an overview of the various scenarios.

Table 5: Five different scenarios to analyze the short-run implications in face of a technology shock (symmetric and idiosyncratic)

		Technology shock	
		Symmetric	Idiosyncratic
Union	Symmetric	1	2
	Asymmetric	3	4 and 5

Note: Scenario 1 and 2 refer to a symmetric union. Scenario 3, 4 and 5 refer to an asymmetric union, whereby in scenario 4 the country with lower λ (country F) and in scenario 5 with higher λ (country H) is hit by the shock.

In the RANK model with homogeneous households, a positive technology shock increases natural level of output through higher productivity and leads to deflation as marginal costs of firms decrease. The natural real interest rate declines, thus the central bank lowers the nominal interest rate. Technology shock has a deflationary effect in the adjustment process. No output gap arises.

In our model, households are heterogeneous according to asset holdings and income. S receives profit income in addition to labor income and receives higher returns on assets leading to consumption inequality. The heterogeneity across household types let a new motive arise for monetary policy: consumption insurance as already discussed in the previous chapter. It is welfare-enhancing to increase consumption of N as we start from an inefficient steady state.

Figure 1 and 2 show the impulse response functions (IRFs) after a positive productivity shock.[20] The IRFs depict absolute deviations from steady state. In figure 1, the IRFs relate to non-monetary variables, in figure 2, to monetary variables and terms of trade. In scenario 1, illustrated by the black line, both countries are hit by the same shock (symmetric shock).[21] In

[20] Appendix B.1 shows the corresponding IRFs for the second approach of targeting λ as a robustness check.

[21] Scenario 1 can be compared with the model of Bilbiie and Ragot (2021) to see the difference based on endogenous labor income of N. The productivity shock leads to higher wages for both household types and thus a higher demand of S and N. There is an upward pressure on prices, inflation rises (and thus profit income). The central bank provides liquidity in both countries, consumption of N rises more than of S, inequality decreases. In the model of Bilbiie and Ragot (2021), N does not benefit from higher productivity as N has fixed labor supply. The exogenous labor supply increases the inequality further in case of a positive productivity shock as only S benefits from it. We refrain from this assumption and still have a liquidity insurance motive of the central bank as $q > 1$.

scenario 2, blue dashed line, only country F is hit (idiosyncratic shock).[22] In both scenarios, we start from the same model equilibrium.

In scenario 1, the dynamics for both countries are the same. We therefore do not differentiate between countries in the description of scenario 1. N and S become more productive (panel A and B, fig 1), natural level of output increases. Wages (panel C and D, fig 1) increase for both households leading to higher demand and thus inflation. As marginal costs are decreasing in sum ($(w_1 - A_1) < 0$, panel A, B, C and D, fig 1), profits rise (panel E and F, fig 1), the firm owners S benefit more from the technology shock. Without liquidity, consumption inequality would rise (see Bilbiie and Ragot 2021, section 4.2). With liquidity, the central bank can balance the arising consumption inequality and even decrease it (q_t, panel G, fig 1). As we start from a distorted steady state, rising consumption of N is (always) welfare-enhancing. Thus the central bank tolerates inflation (π_t^U, panel A, fig 2) in favor of providing insurance.

It is optimal for the central bank to inject liquidity increasing income of N and S. N consumes all additional income, S saves some of it. The consumption of N rises relatively more than that of S ($C_1^N = 0.01046 > C_1^S = 0.00934$, panel H and I, fig 1), consumption inequality decreases (panel G, fig 1). Real money holdings increase (panel E and F, fig 2), too, as S households save some of the liquidity injections. The arising additional aggregate demand effect leads to inflation during the adjustment process. The welfare gains from tolerating inflation in favor of consumption insurance are larger.

The consumption inequality declines. However, income inequality rises as the disposable income of S increases more than that of N ($Y_1^S = 0.01379 > C_1^N = 0.01046$, panel K and L, fig 1). There are counteracting effects on income of N: N can consume the additional liquidity, but this leads to inflation decreasing the real value of money holdings.

Scenario 2 let us gain more insights into the mechanisms of our two-country model. In scenario 2, the central bank redistributes towards the country experiencing the shock. If the shock only affects country F, the terms of trade change (panel K, fig 2) to the disadvantage of country H. F experiences a productivity shock leading to union-wide inflation (CPI) through higher demand. This harms country H not experiencing higher productivity, but higher union-wide inflation and upward pressure on wages leading to higher marginal costs. In the short run, inflation (PPI) increases in H (panel B, fig 2), but decreases in F (panel C, fig 2) as F experiences lower marginal costs. However, the deflection of the curves is not very large ($\pi_1 = 0.00203$ and $\pi_1^* = -0.00201$).

[22] As we here assume a symmetric union, it does not matter which country experiences a productivity shock.

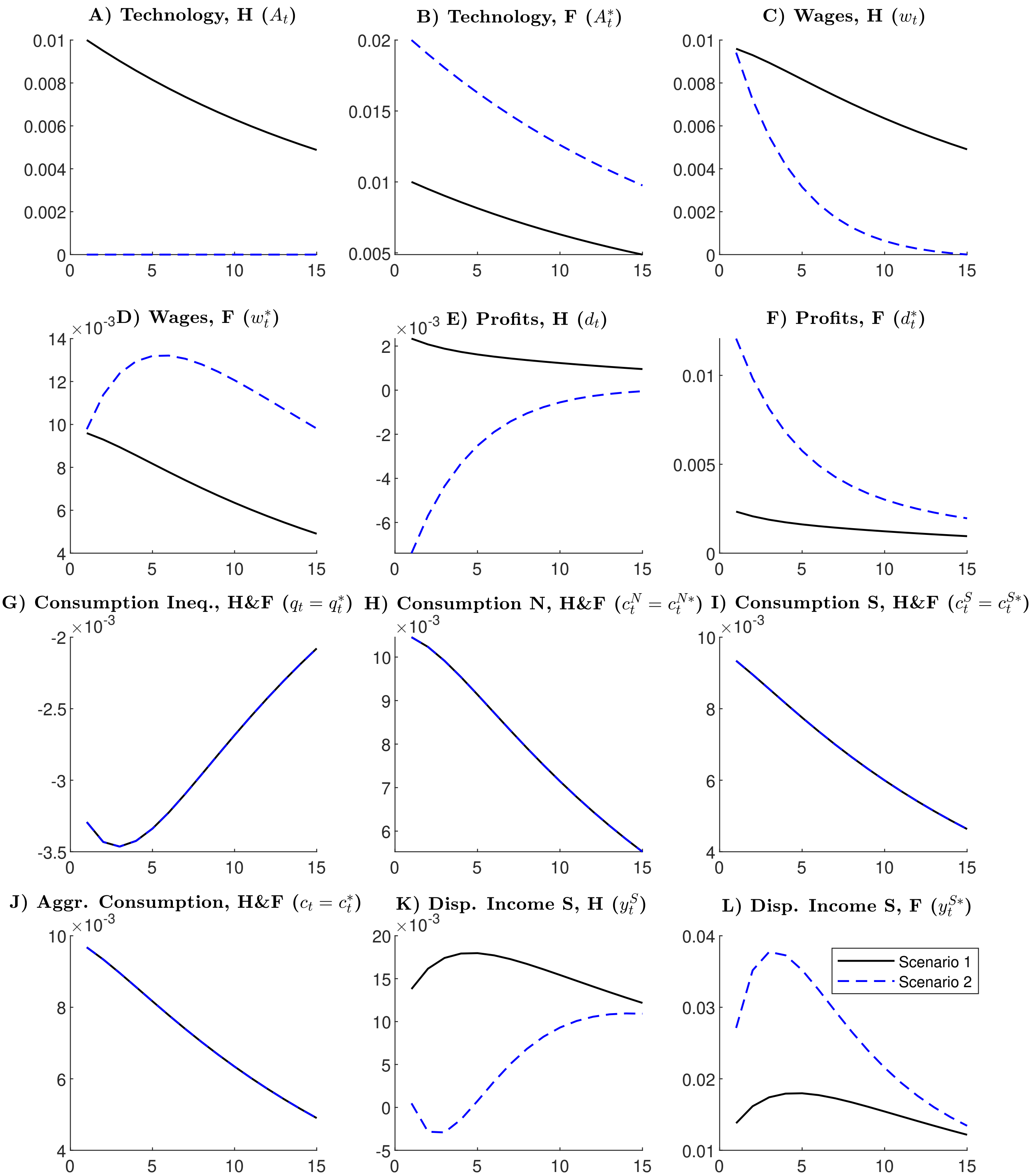

Figure 1: Impulse response functions (IRFs) of a symmetric (scenario 1) and idiosyncratic (scenario 2, shock in Foreign) positive productivity shock in a symmetric union with $\lambda = \lambda^* = \lambda^U = 0.3$ under Ramsey optimal monetary policy. The IRFs relate to non-monetary variables and depict absolute deviations from steady state. Variables with "H" refer to Home and "F" to Foreign. Part 1/2.

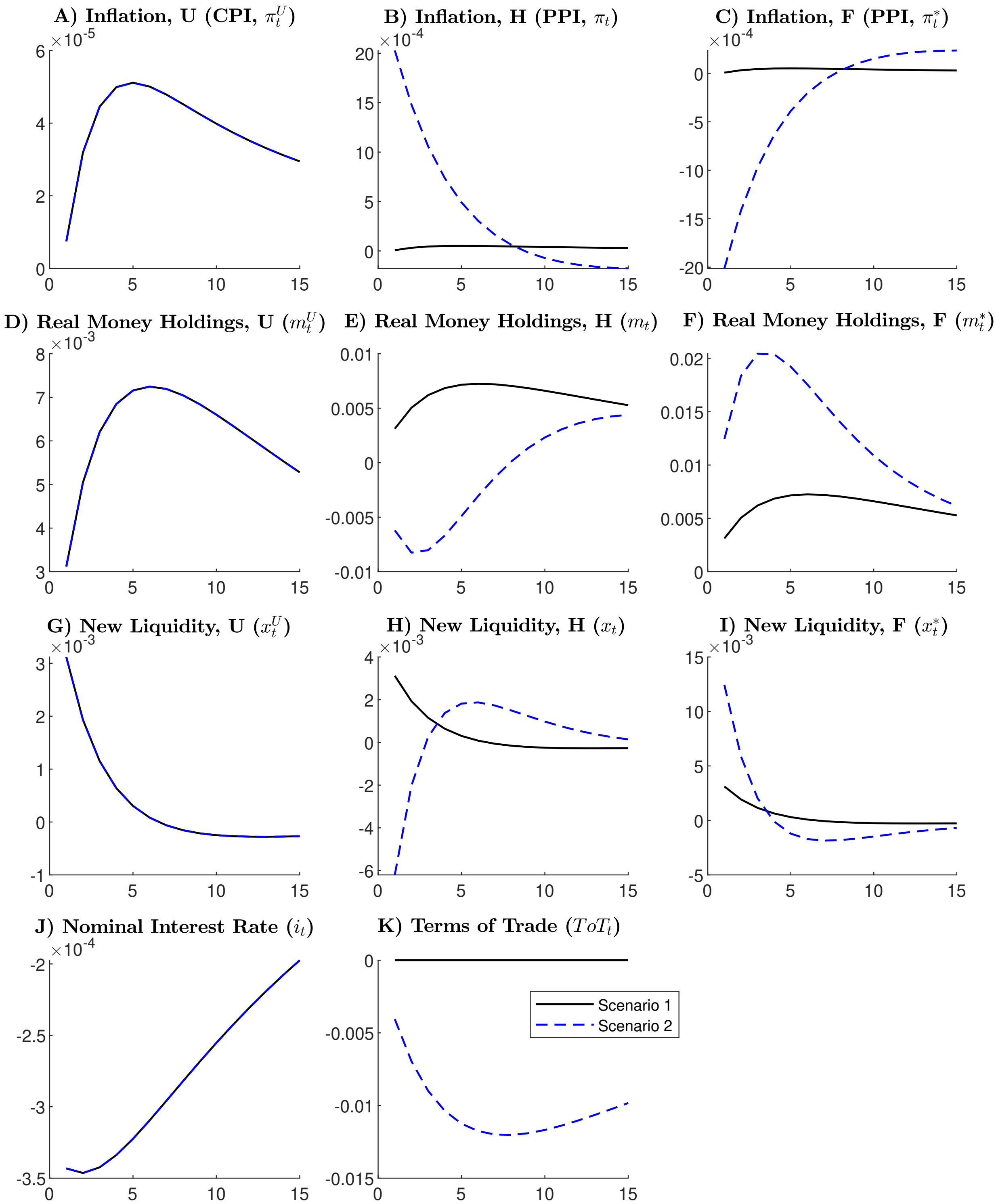

Figure 2: Impulse response functions (IRFs) of a symmetric (scenario 1) and idiosyncratic (scenario 2, shock in Foreign) positive productivity shock in a symmetric union with $\lambda = \lambda^* = \lambda^U = 0.3$ under Ramsey optimal monetary policy. The IRFs relate to monetary variables and terms of trade and depict absolute deviations from steady state. Variables with “H” refer to Home, “F” to Foreign and “U” to Union. Part 2/2.

Union-wide inflation (panel A, fig 2) reacts the same regardless of the symmetry of the shock, but also with a very small magnitude ($\pi_1^U = 0.000007$).

In both scenarios, the central bank tolerates inflation volatility in favor of consumption insurance. Total liquidity remains the same in both cases (m_t^U, panel D, fig 2). In case of an idiosyncratic shock, the central bank redistributes to the affected country through liquidity injections and withdraws money from the other one (panel H and I, fig 2) to mitigate consumption and inequality volatility. Wages in F increase more than in H. F receives positive liquidity ($x_1^* = 0.01244$), approximately four times the liquidity compared to the symmetric shock ($x_1^* = 0.00312$). H is withdrawn from liquidity (negative liquidity), but the extent is very small ($x_1 = -0.0062$). This changes during the adjustment process. From $t = 3$, H also receives positive liquidity. From $t = 5$, F is withdrawn from liquidity. At the union level, liquidity (x_t^U, panel G, fig 2) remains at the same level as in the symmetric shock case.

Through the instrument of liquidity, the central bank manages to completely equalize consumption reactions in both countries regardless of the type of the shock (symmetric vs. idiosyncratic). The IRFs are all stacked. Inequality falls in both countries by the same value ($q_1 = q_1^* = -0.00329$, panel G, fig 1) as the central bank equalizes consumption across countries and household types. Consumption reactions are the same across countries ($C_1 = C_1^* = 0.00968$, panel J, fig 1) and across households types N ($C_1^N = C_1^{N*} = 0.01046$, panel H, fig 1) and S ($C_1^S = C_1^{S*} = 0.00934$, panel I, fig 1). The central bank is able to compensate for the heterogeneity arising from productivity shock.

There are different effects on income inequality in the countries as the profit income rises in F (panel F, fig 1) and declines in H (panel E, fig 1) during the adjustment process. By income inequality we mean the difference between disposable income of S (Y_t^S) and that of N (Y_t^N), remember that N lives hand-to-mouth, thus $Y_t^N = C_t^N$. S in F (panel L, fig 1) experiences a higher income gain than in H (panel K, fig 1). Income inequality rises in F as disposable income of S exceeds that of N ($Y_t^{S*} > C_t^{N*}$ for all t, panel L and H, fig 1). Income and consumption inequality drift apart. In H, S even reduces money holdings during the adjustment process ($m_t < 0$ for $t < 8$, panel E, fig 2) in contrast to S in F (panel F, fig 2). Income inequality in H declines until $t = 12$ ($Y_t^S < C_t^N$ for $t < 12$, panel K and H, fig 1).

It is beneficial for the central bank to have an additional (country-specific) instrument through liquidity as it can redistribute between the countries through money. The extent to which the instruments are used at the union-level is the same in both scenarios (see union-wide variables

m_t^U, x_t^U and i_t, panel D, G and J, fig 2). The central bank sets the nominal interest rate i_t the same as in the symmetric shock case.

If the central bank pursues strict inflation targeting instead, it would harm the N-households resulting in welfare losses. Strict inflation targeting is not optimal. Divine coincidence in the sense of Blanchard and Galí (2007), i.e. stabilizing inflation and thereby closing the welfare-relevant output gap, does not hold due to the resulting inequality.

So far, this all refers to the case of a symmetric union (or one-country model). Things get even more interesting when we move on to the asymmetric union. Then, for example, ρ is lower in one of the countries, i.e. the optimal steady states of the countries would go in different directions. The country with lower ρ would prefer to approach more to zero inflation, the country with higher ρ to Friedman rule. However, this is not possible, as the central bank determines inflation throughout the union. In other words, the central bank is faced with an additional trade-off between the country heterogeneities, as the optimal country steady states (with country-specific monetary policy) drift apart. In the case of an asymmetric union, monetary policy must also compensate for country heterogeneities. In section 5 we discuss the implications of the existence of different λ levels across countries.

5 Optimal monetary policy in an asymmetric union

In this section, we focus on an asymmetric union with heterogeneous shares of constrained households across countries ($\lambda \neq \lambda^*$) as it is motivated by empirical evidence for the Euro area (Almgren et al. 2022, Kaplan et al. 2014). The central bank faces an additional distortion through heterogeneity across countries and needs to compensate for it. We analyze the long-run (5.1) and short-run (5.2) implications of heterogeneous shares of constrained households across countries in a currency union for optimal monetary policy.

For the asymmetric union, we assume the shares across countries to be $\lambda = 0.35$ and $\lambda^* = 0.25$. Additionally, we analyze a second combination of $\lambda \neq \lambda^*$ in section 5.1, assuming a larger distortion between the countries according to the share of constrained households, therefore $\lambda = 0.4$ and $\lambda^* = 0.2$. These values also lie in the range of implied values (from 0.21 to 0.41) for λ used in different HANK models summarized by Bilbiie (2020). For both combinations, we hold the union-wide share constant at $\lambda^U = 0.3$ to isolate the role of heterogeneous shares across countries.

Table 6 summarizes the calibration of α, α^*, ρ and ρ^*, implying λ, λ^* and λ^U, used for the analysis of an asymmetric union.

Table 6: Calibration of λ, λ^* and λ^U with fixed α and α^* and varying ρ and ρ^* in an asymmetric union

	Union	**Country H**			**Country F**			**Consumption inequality**
	λ^U	λ	α	ρ	λ^*	α^*	ρ^*	$q^U = q = q^*$
(1)	0.3	0.3	0.9	0.7667	0.3	0.9	0.7667	1.174
(2)	0.3	0.35	0.9	0.8143	0.25	0.9	0.7	1.174
(3)	0.3	0.4	0.9	0.85	0.2	0.9	0.6	1.173

Row (1) illustrates the case of a symmetric union, while rows (2) and (3) of an asymmetric union with increasing heterogeneity across countries in terms of $(\lambda - \lambda^*)$, which is largest in row (3).

As we assume α to be the same in both countries ($\alpha = \alpha^*$), there is no difference in consumption inequality q across countries for each combination of α (α^*) and ρ (ρ^*), respectively. Thus we can exclude that our results are driven by heterogeneous levels of steady-state inequality.

5.1 Long-run implications

To gain first insights into the implications of an asymmetric union for optimal monetary policy, Table 7 shows the model outcome under Ramsey optimal policy for the case of a symmetric, (1), and an asymmetric union, (2) and (3).[23]

Table 7: Implied steady-state values from Ramsey optimal policy in a currency union with symmetric and asymmetric shares of constrained households across countries

Model outcome	**Union-wide**				**Country-specific**				
	π^U	m^U	i	c^U		c	c^S	c^N	m
(1) Symmetric union	-0.297%	0.278	0.0174	0.999	Home	0.999	1.046	0.891	0.278
($\lambda = \lambda^* = 0.3$)					Foreign	0.999	1.046	0.891	0.278
(2) Asymmetric union	-0.299%	0.278	0.0174	0.999	Home	0.991	1.046	0.891	0.325
($\lambda = 0.35, \lambda^* = 0.25$)					Foreign	1.007	1.046	0.891	0.232
(3) Asymmetric union	-0.31%	0.280	0.0173	0.999	Home	0.984	1.045	0.891	0.374
($\lambda = 0.4, \lambda^* = 0.2$)					Foreign	1.015	1.045	0.891	0.186

In the long run, optimal steady-state values for inflation (π^U) and money holdings (m^U) change due to heterogeneity across countries. The greater the heterogeneity between the countries $(\lambda - \lambda^*)$, the higher optimal union-wide deflation as the objective consumption insurance becomes

[23] Appendix B.2.1 contains the corresponding model outcome for the second approach of targeting λ as a robustness check.

relatively more important. Remember that in equilibrium, $\pi^U = \pi = \pi^*$. The central bank sets the nominal interest rate i, as a union-wide instrument, nearly the same in all three cases.

Union-wide money demand (m^U) is slightly increasing in heterogeneity according to λ. At country level, the country with greater λ receives more money in case of an asymmetric union ($m > m^*$ for (2) and (3)). The greater the heterogeneity (the higher the idiosyncratic risk), the higher the money demand in a country.

The distribution of money between countries depends on their heterogeneity. In 2), H receives approximately 40% more money compared to F in equilibrium ($m = 0.325$ and $m^* = 0.232$). This corresponds to the increased share of constrained households of 40% in H ($\lambda = 0.35$) compared to F ($\lambda^* = 0.25$). In (3), H has twice the share of F, the money demand is about twice as high as in F ($m = 0.374$ and $m^* = 0.186$). Due to optimal deflation, the central bank needs to withdraw liquidity (negative x for each country; not shown) in equilibrium.

As λ illustrates average risk of being constrained, a higher λ leads to a higher need for self-insurance, thus higher money demand as a reflex of higher risk. It is optimal for monetary policy to redistribute between the countries in favor of the country with higher share of constrained households.

With liquidity as a country-specific instrument, the central bank is able to equalize consumption across N-households across countries ($C^N = C^{N*}$)[24] regardless of heterogeneity according to the share of constrained households. It follows that steady-state consumption inequality is nearly the same for all three cases. For (1) and (2), $q = q^* = 1.174$, for (3), $q = q^* = 1.173$, see also table 6.

These results underline the differences between the two monetary policy instruments. The central bank sets the nominal interest rate at the union level. As only S households hold bonds, the central bank can only affect their consumption with i. To target the consumption of N in a country, the central bank can use liquidity as an instrument to meet the demand for money and insure N. Liquidity is the appropriate instrument to address consumption inequality, nominal interest rate to address union-wide inflation rate.

5.2 Short-run implications

For the short-run implications of heterogeneous households for optimal monetary policy within a currency union, we simulate a positive technology shock as in section 4, but here we assume an

[24] Due to risk sharing between S-households across both countries, it also applies $C^S = C^{S*}$.

asymmetric union with different shares of constrained households across countries ($\lambda = 0.35$ and $\lambda^* = 0.25$).

In section 5.2.1, we compare the transmission of a symmetric shock in a symmetric union (scenario 1 from previous section) with the transmission in an asymmetric union (scenario 3), see table 5, to analyze if the asymmetry matters in face of a symmetric shock. In section 5.2.2, we examine the transmission of an idiosyncratic shock in an asymmetric union on the country with a lower share of constrained households (scenario 4) and on that with a higher share (scenario 5). We compare scenario 4 and 5 (idiosyncratic shock, respectively) to scenario 3 (symmetric shock) and analyze if it matters for optimal monetary policy which country is affected.[25] See table 5 for an overview of the different scenarios.

5.2.1 Does the asymmetry matter in face of a symmetric technology shock?

Figure 3 and 4 show the IRFs of a union-wide positive productivity shock for scenario 1 (black line) and 3 (blue dashed line). For the interpretation, we have to consider that the steady-state values implied from Ramsey optimal policy differ slightly between scenario 1 and 3 (see table 7, (1) and (2)). Consumption shares of S and N are equal across H and F for both scenarios ($C^S = C^{S*} = 1.046$ and $C^N = C^{N*} = 0.891$).

Although both countries experience the same technology shock in both scenarios, the strength of the transmission of the shock in the countries differs due to the heterogeneity. In an asymmetric union (scenario 3), wages increase more in H (panel B and C, fig 3) in response to the shock as in H are living more households hand-to-mouth thus consuming more out of the additional income gains, caused by the shock as described in the previous section in more detail. This leads to higher upward pressure on wages in H than in F. Marginal costs rise more ($w_t > w_t^*$), profits rise less (panel D and E, fig 3), terms of trade falls in disadvantage of H (panel K, fig 4). In the short run, this leads to heterogeneous inflation dynamics at country level. In $t = 1$, H faces positive inflation ($\pi_1 > 0$, panel B, fig 4) and F deflation ($\pi_1^* < 0$, panel C, fig 4) in contrast to a symmetric union (scenario 1) in which both countries face positive inflation ($\pi_1 = \pi_1^* > 0$) in the short run. It is noteworthy that the short-run inflation differences are minor in quantitative terms as $\pi_1 = 0.0000314$ and $\pi_1^* = -0.0000166$.

According to consumption, the consumption reactions differ only marginally in scenario 3. Aggregate demand in H increases slightly more than in F ($c_t > c_t^*$ for all t, panel I and J, fig

[25] Appendix B.2.2 and B.2.3 contain the corresponding robustness checks for the second approach of targeting λ.

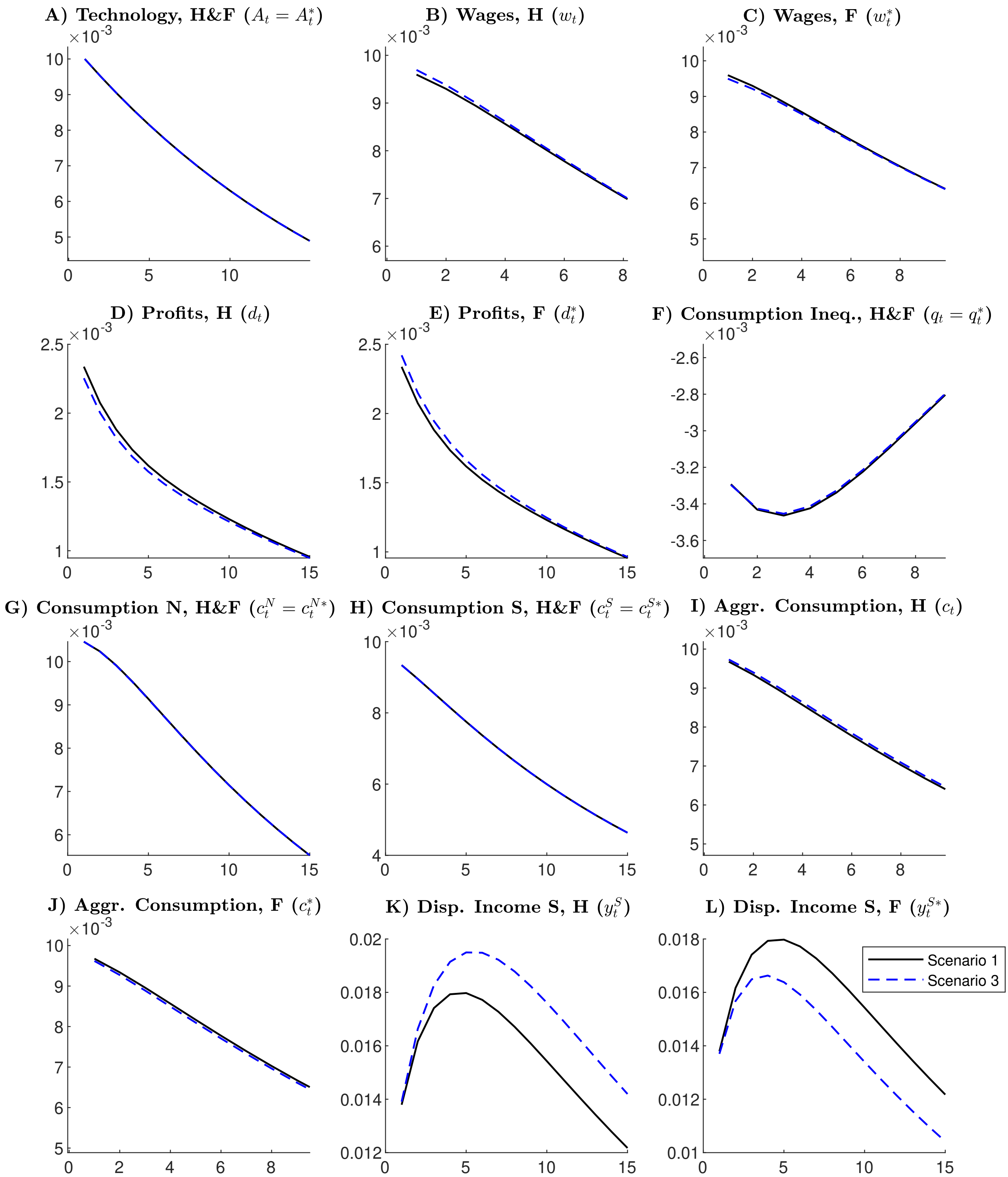

Figure 3: Impulse response functions (IRFs) of a symmetric positive productivity shock in a symmetric (scenario 1, with $\lambda = \lambda^* = \lambda^U = 0.3$) and an asymmetric union (scenario 3, with $\lambda = 0.35$, $\lambda^* = 0.25$ and $\lambda^U = 0.3$) under Ramsey optimal monetary policy. The IRFs relate to non-monetary variables and depict absolute deviations from steady state. Variables with "H" refer to Home and with "F" to Foreign. Part 1/2.

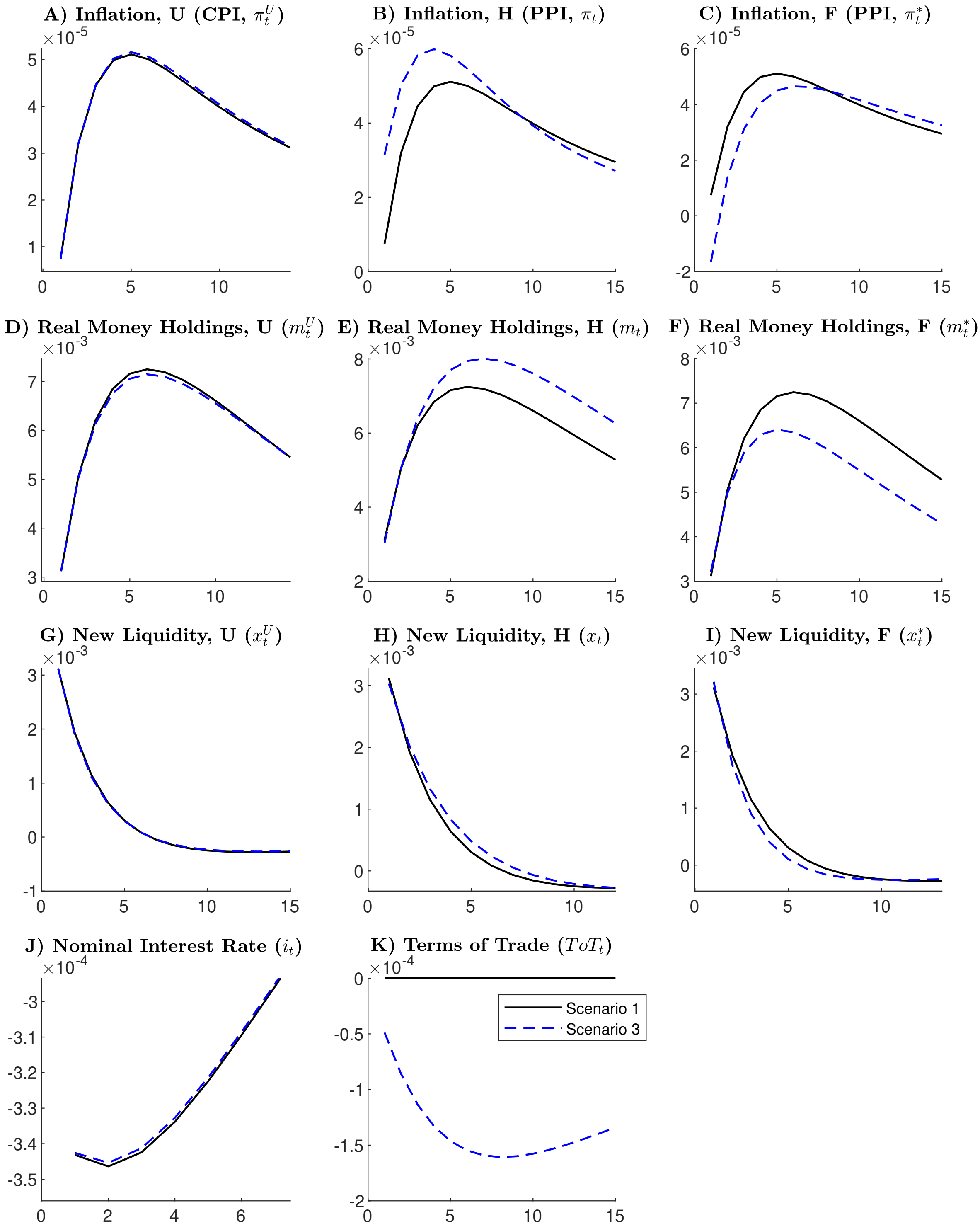

Figure 4: Impulse response functions (IRFs) of a symmetric positive productivity shock in a symmetric (scenario 1, with $\lambda = \lambda^* = \lambda^U = 0.3$) and an asymmetric union (scenario 3, with $\lambda = 0.35$, $\lambda^* = 0.25$ and $\lambda^U = 0.3$) under Ramsey optimal monetary policy. The IRFs relate to monetary variables and terms of trade and depict absolute deviations from steady state. Variables with "H" refer to Home, with "F" to Foreign and with "U" to Union. Part 2/2.

3). For the other consumption variables, the central bank manages to equalize their reactions. Consumption reactions of N across countries are equalized ($c_t^N = c_t^{N*}$ for all t, panel G, fig 3), hence consumption inequality drops by the same amount in both countries (panel F, fig 3) within a scenario. The consumption increase of N in scenario 3 is a little smaller compared to scenario 1, thus consumption inequality drops less. The central bank balances out the distortion between country H and F. Through optimal monetary policy, the central bank counteracts country heterogeneities within a currency union.

As in scenario 1, both countries receive liquidity to stabilize consumption inequality in each country. However, in scenario 3, the country with higher idiosyncratic risk (H), although it already keeps more money in equilibrium, builds up more money holdings (panel E and F, fig 4) during the adjustment process than F. There is a slight redistribution through liquidity injections in favor of H in the first periods after the shock (panel G and H, fig 4). Directly when the shock occurs ($t = 1$), F receives more liquidity ($x_1^* = 0.003213 > x_1 = 0.00303$), however, we have to consider that H already receives 40% more money in equilibrium compared to F.

The central bank sets the nominal interest rate slightly higher in scenario 3, but still expansive (panel J, fig 4).

In scenario 3, we observe a higher inflation compared to 1. In scenario 1, monetary policy is able to stabilize inflation faster (panel A, fig 4). The heterogeneity between countries leads to higher inflation volatility in the adjustment process. A higher inflation is tolerated in favor of providing consumption insurance.

In the long run, as well as in the short run, it is optimal for the central bank to redistribute through money in favor of the country with a higher share of constrained households in face of country heterogeneities. The asymmetry matters even in face of a symmetric technology shock.

5.2.2 Does it matter which country experience a shock?

Figure 5 and 6 show the IRFs after a positive productivity shock for three scenarios. In scenario 3 (black line), both countries experience the same productivity shock. In scenario 4 (blue dashed line) and 5 (red dashed-dotted line), only one country experiences the shock, in scenario 4 country F (lower share of constrained households), in scenario 5 country H (higher share of constrained households).[26] In all cases, monetary policy is conducting Ramsey optimal policy.[27]

[26] As in subsection 4.3, $\epsilon_1 = \epsilon_1^* = 0.01$ applies in case of a symmetric shock and $\epsilon_1 = 0.02$ or $\epsilon_1^* = 0.02$ in case of an idiosyncratic shock.

[27] In all three scenarios considered here, we start from the same steady state.

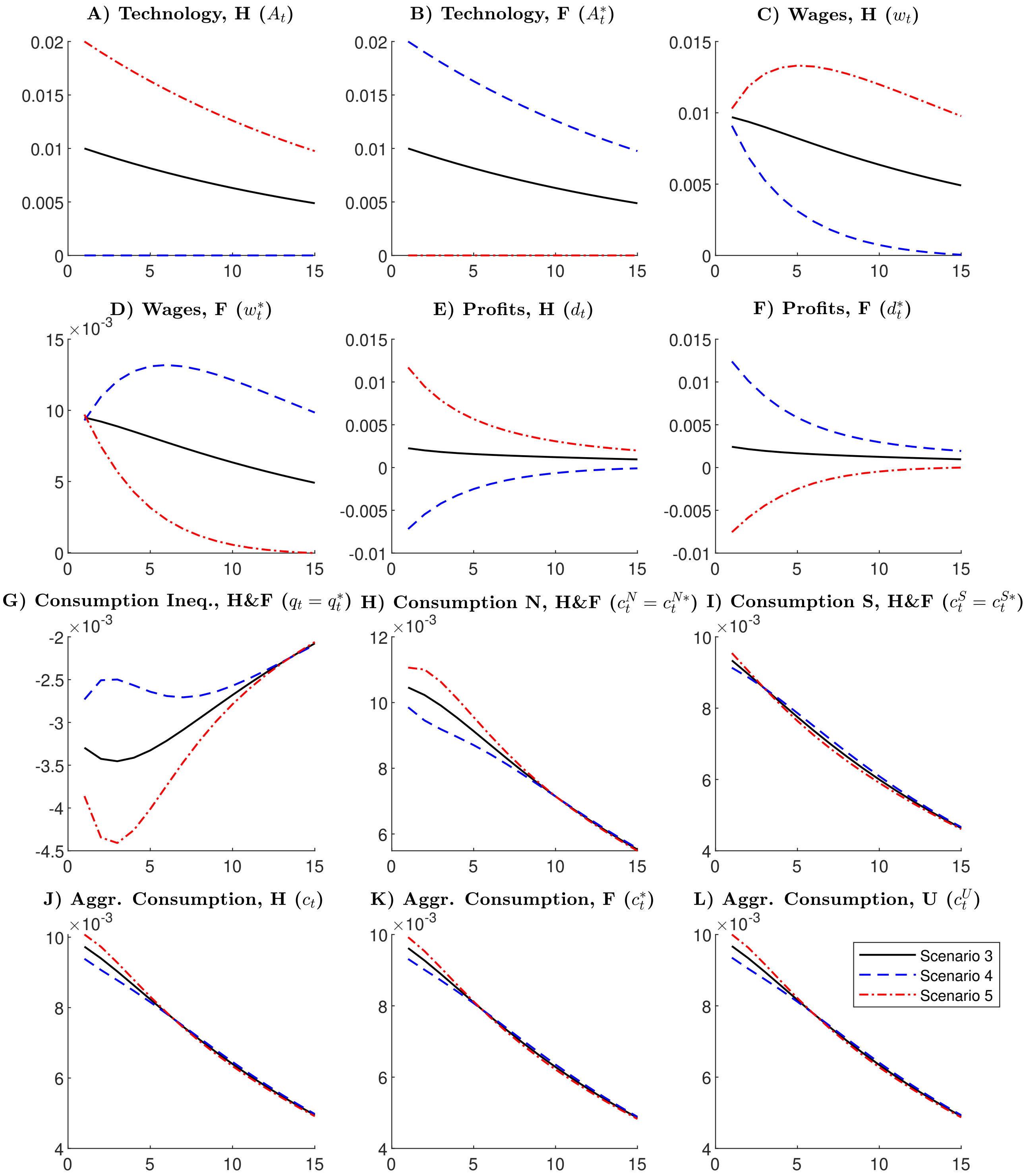

Figure 5: Impulse response functions (IRFs) of a symmetric (scenario 3) and an idiosyncratic positive productivity shock to F (scenario 4) and H (scenario 5) in an asymmetric union (with $\lambda = 0.35$, $\lambda^* = 0.25$ and $\lambda^U = 0.3$) under Ramsey optimal monetary policy. The IRFs relate to non-monetary variables and depict absolute deviations from steady state. Variables with “H” refer to Home, with “F” to Foreign and with “U” to Union. Part 1/2.

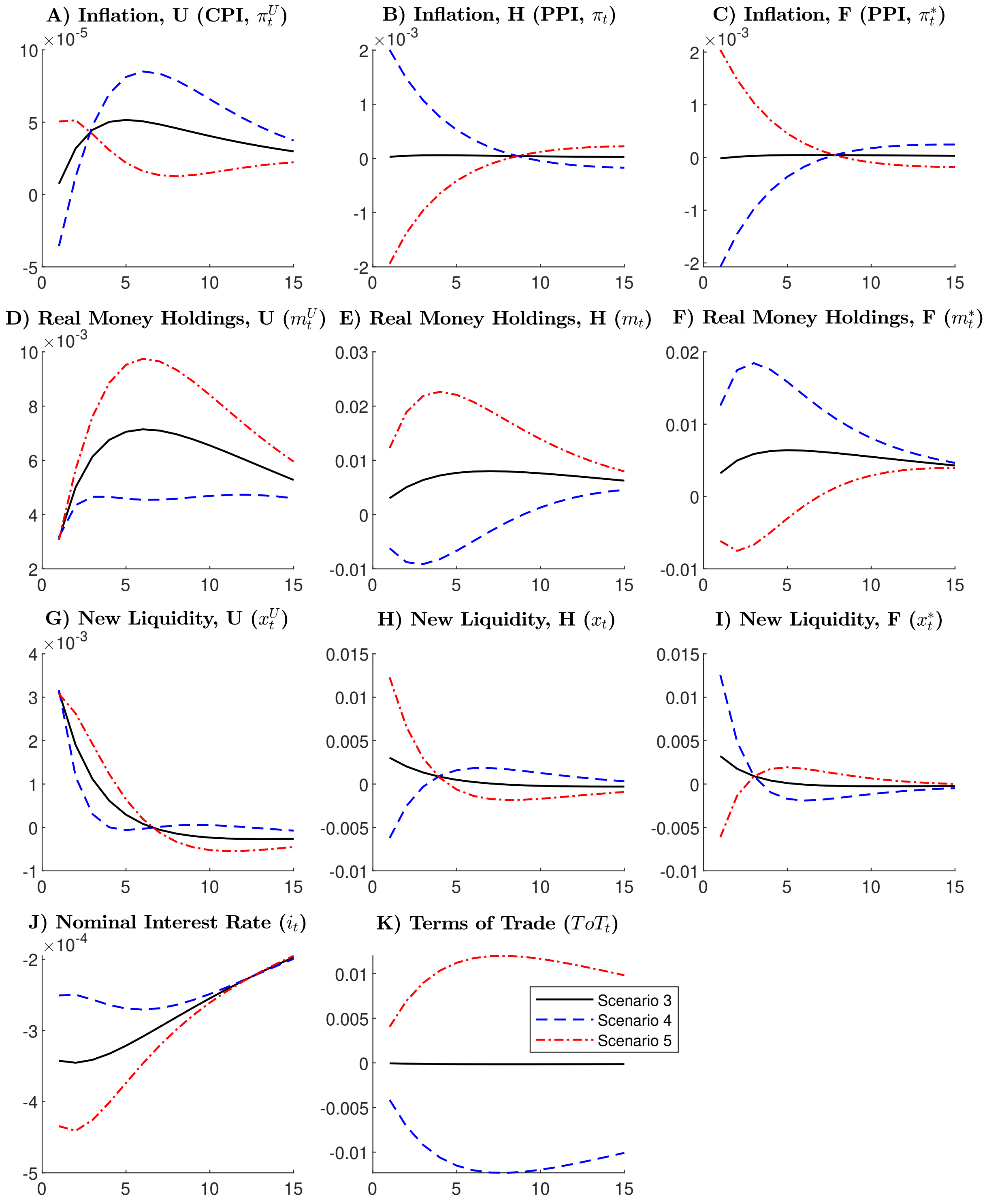

Figure 6: Impulse response functions (IRFs) of a symmetric (scenario 3) and an idiosyncratic positive productivity shock to F (scenario 4) and H (scenario 5) in an asymmetric union (with $\lambda = 0.35$, $\lambda^* = 0.25$ and $\lambda^U = 0.3$) under Ramsey optimal monetary policy. The IRFs relate to monetary variables and terms of trade and depict absolute deviations from steady state. Variables with "H" refer to Home, with "F" to Foreign and with "U" to Union. Part 2/2.

The IRFs for scenario 4 and 5 are not mirror images. Due to the heterogeneity across countries, it matters for optimal monetary policy which country is hit. The central bank behaves asymmetrically. The country experiencing a positive productivity shock is better off, as it experiences decreasing marginal costs. This is reflected by the reactions of, for example, the terms of trades (panel K, fig 6) and producer price index (panel B and C, fig 6) in each country. When, for example, H is hit, marginal costs are decreasing in H, thus π_t decreases in the short run. The terms of trades develop positively for H harming country F experiencing positive inflation.

The reactions of wages (panel C and D, fig 5) are also not mirror images comparing scenario 4 and 5. In scenario 5, the wage response is 10.8% higher in the country affected by the shock in $t = 1$ than in scenario 4 ($w_1 = 0.010297$ in scenario 5 compared to $w_1^* = 0.009293$ in scenario 4) driven by higher aggregate demand in H.

In case of an idiosyncratic shock in an asymmetric union, the central bank is not able to fully equalize aggregate consumption across countries in comparison to scenarios 1, 2 and 3 discussed in the previous sections. Aggregate demand rises more in H than in F ($c_t > c_t^*$ for all t, panel J and K, fig 5) in all three scenarios due to the higher share of constrained households consuming more out of the additional income gained by the positive productivity shock, regardless of the symmetry of the shock. All consumption variables ($c_t^N, c_t^{N*}, c_t^S, c_t, c_t^*$, panel H, I, J and K, fig 5) show the highest increase when country H is hit, even union-wide consumption (c_t^U, panel L, fig 5) is higher compared to the other scenarios. As in all three cases the consumption of N increases more than that of S in both countries, inequality drops during the adjustment process (panel G, fig 5), strongest in scenario 5. The central bank manages to equalize the drops in inequality across countries for all cases ($q_t = q_t^*$ for all t), it harmonizes the deviations across countries. For the central bank, it is optimal to minimize the difference in consumption volatility across countries.

The central bank provides insurance in all three cases, but the extent is highest when country H is hit. As the idiosyncratic risk is higher in H than in F ($\lambda > \lambda^*$), consumption insurance in H is more important and has a relatively higher optimal weight as the distortion is higher in H than in F. Providing insurance through liquidity is more important for country H than F, thus the central bank provides more union-wide liquidity during the first periods after the shock occurs (x_t^U, panel G, fig 6) in scenario 5. In scenario 4, it provides the lowest additional liquidity compared to scenario 3 and 5. Union-wide money (m_t^U, panel D, fig 6) rises in all three scenarios, but the most in scenario 5, the least in scenario 4. The central bank makes greater use of money as an instrument in scenario 5.

At country level, monetary policy redistributes newly created money (panel G and H, fig 6) in favor of the affected country in the short run. Country F receives slightly higher liquidity injections in scenario 4 ($x_1^* = 0.0125648$) than H in scenario 5 ($x_1 = 0.0122897$). However, we have to bear in mind that H has higher money demand in equilibrium ($m = 0.325 > m^* = 0.232$, Table 7).

If the more vulnerable country is hit, monetary policy tolerates higher union-wide inflation (π_t^U, panel A, fig 6) in the first two periods compared to scenarios 3 and 4. In scenario 3, the central bank also tolerates positive inflation, but to a lesser extent. In the first period, the inflation rate in scenario 5 ($\pi_1^U = 0.00005045$) corresponds to approximately 6.8 times the one in scenario 3 ($\pi_1^U = 0.00000741$). In scenario 4, the central bank even tolerates deflation in the very short run. Monetary policy sacrifices inflation stabilization in favor of consumption stabilization across countries (c_t, c_t^*) and household types (c_t^S, c_t^N, c_t^{N*}).

In scenario 5, monetary policy optimally reacts more expansively in the short run. This applies to both instruments of monetary policy, m_t^U (panel D, fig 6) and i_t (panel J, fig 6). The central bank sets the nominal interest rate the lowest in scenario 5 during the adjustment process. Thus there are different short-run implications for optimal monetary policy in face of asymmetric countries and idiosyncratic shocks.

In the event of an idiosyncratic shock and an asymmetric union, it is no longer optimal for monetary policy to equalize consumption reactions as in the previous sections. In general, we observe more volatility in macroeconomic dynamics when the more distorted country is affected.

Due to the heterogeneity of countries within the monetary union, it matters for optimal monetary policy which country experiences a shock. If the country with a higher proportion of constrained households is hit by a positive technology shock, monetary policy optimally reacts more expansively in the short run. This applies to both instruments of monetary policy, money and nominal interest rate.

6 Conclusion

This paper builds on the empirical evidence of heterogeneous shares of financially constrained households across countries forming a monetary union, as it is the case for the Euro area (Almgren et al. 2022, Kaplan et al. 2014). We analyze the implications of this asymmetry for optimal monetary policy in an analytically tractable HANK model for a currency union.

We find that the asymmetry matters for optimal monetary policy as the trade-off of the central bank changes in face of household heterogeneity. It is optimal to tolerate inflation in favor of providing consumption insurance for the constrained households through liquidity. Strict inflation targeting is not optimal in face of technology shocks. This applies in the case of a symmetric and an asymmetric union. In an asymmetric union, the central bank optimally redistributes through liquidity in favor of the country with a higher share of constrained households. The greater the heterogeneity between countries, the more important liquidity as an instrument of redistribution becomes. The higher the share of constrained households, the more liquidity the central bank provides, the better the central bank can close the arising relative country gaps. With an additional country-specific instrument, the central bank can balance country heterogeneities. Additionally, it matters which country experiences a shock. If the more distorted country is affected, optimal monetary policy reacts more expansively, liquidity becomes even more important. In face of an asymmetric union, the central bank shows a liquidity-insurance motive to deal with household heterogeneity across countries as it is welfare-enhancing.

We further develop the HANK literature by extending the HANK framework to a monetary union and implementing this asymmetry. We focus on this asymmetry as the HANK literature (e.g., Kaplan et al. 2018) already showed the importance of constrained households for the transmission of monetary policy on aggregate demand. Other promising future developments could include the extension by endogenous household shares (as in Debortoli and Galí 2022 or Thiel 2024), by implementing poor and wealthy hand-to-mouth households, Kaplan et al. (2014) provide empirical evidence for this distinction, or by implementing capital (Bilbiie et al. 2022). In our model, CBDC becomes an integral part of optimal monetary policy to address household heterogeneity (within and across countries), as one dimension of heterogeneity within a monetary union. Another promising research direction is to apply CBDC as a country-specific tool to address various dimensions of heterogeneity within a monetary union.

Our analysis provides important implications for monetary unions. It shows the welfare-enhancing potential of a country-specific tool for central banks to reach even households that are non-participating in financial markets. For a monetary union, it is beneficial to introduce such a national instrument, for example through CBDC, to target heterogeneity across countries.

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Appendix A: Derivation of the welfare function

Let X_t be a generic variable and X its steady state. Then, we define $\hat{X}_t$ as the log deviation of X_t around X, $\hat{X}_t \equiv \log(X_t/X)$. Hence, using a second-order approximation yields

$$\frac{X_t - X}{X} = \exp(\hat{X}_t) - 1 \simeq \hat{X}_t + \frac{1}{2}\hat{X}_t^2. \tag{A.1}$$

Let $U(C_t^S, C_t^N, L_t)$ and $U(C_t^{S*}, C_t^{N*}, L_t^*)$ be the period utility function of H and F households. Then the central bank's period loss function is given by a weighted sum of these utility functions:

$$U_t^U = \gamma U(C_t^S, C_t^N, L_t) + (1-\gamma)U(C_t^{S*}, C_t^{N*}, L_t^*). \tag{A.2}$$

We take a second-order approximation around the zero-inflation steady state and drop terms of third or higher order:

$$\begin{aligned} U_t^U - U^U =& \gamma(1-\lambda)(C^S)^{1-\sigma}(\hat{C}_t^S + \frac{1-\sigma}{2}(\hat{C}_t^S)^2) \\ &+ \gamma\lambda(C^N)^{1-\sigma}(\hat{C}_t^N + \frac{1-\sigma}{2}(\hat{C}_t^N)^2) \\ &- \gamma\chi L^{1+\varphi}(\hat{L}_t + \frac{1+\varphi}{2}(\hat{L}_t)^2) \\ &+ (1-\gamma)(1-\lambda^*)(C^{S*})^{1-\sigma}(\hat{C}_t^{S*} + \frac{1-\sigma}{2}(\hat{C}_t^{S*})^2) \\ &+ (1-\gamma)\lambda^*(C^{S*})^{1-\sigma}(\hat{C}_t^{N*} + \frac{1-\sigma}{2}(\hat{C}_t^{N*})^2) \\ &- (1-\gamma)\chi(L^*)^{1+\varphi}(\hat{L}_t^* + \frac{1+\varphi}{2}(\hat{L}_t^*)^2). \end{aligned} \tag{A.3}$$

Assuming optimal sales subsidies at the zero-inflation steady state implies $w = w^* = 1$ and evaluating (14) and (23) at the steady state leads to

$$\chi L^{1+\varphi} = ToT^{\gamma-1}L((1-\lambda)(C^S)^{-\sigma} + \lambda(C^N)^{-\sigma}) = C^U(C^S)^{-\sigma}((1-\lambda) + \lambda q^\sigma), \tag{A.4}$$

and an analogous equation for the foreign counterpart.

Using this and due to risk sharing between S-households in both countries we can rewrite

(A.3):

$$
\begin{aligned}
\frac{U_t^U - U^U}{(C^S)^{-\sigma} C^U} =& \gamma(1-\lambda)\left(\frac{C^S}{C^U}(\hat{C}_t^S + \frac{1-\sigma}{2}(\hat{C}_t^S)^2) - \hat{L}_t - \frac{1+\varphi}{2}(\hat{L}_t)^2\right) \\
&+ \gamma\lambda q^\sigma \left(\frac{C^N}{C^U}(\hat{C}_t^N + \frac{1-\sigma}{2}(\hat{C}_t^N)^2) - \hat{L}_t - \frac{1+\varphi}{2}(\hat{L}_t)^2\right) \\
&+ (1-\gamma)(1-\lambda^*)\left(\frac{C^{S*}}{C^U}(\hat{C}_t^{S*} + \frac{1-\sigma}{2}(\hat{C}_t^{S*})^2) - \hat{L}_t^* - \frac{1+\varphi}{2}(\hat{L}_t^*)^2\right) \\
&+ (1-\gamma)\lambda^*(q^*)^\sigma \left(\frac{C^{N*}}{C^U}(\hat{C}_t^{N*} + \frac{1-\sigma}{2}(\hat{C}_t^{N*})^2) - \hat{L}_t^* - \frac{1+\varphi}{2}(\hat{L}_t^*)^2\right)
\end{aligned}
\tag{A.5}
$$

Now, take a second-order approximation of (23) around the zero-inflation steady state:

$$
\begin{aligned}
\hat{L}_t + \frac{1}{2}(\hat{L}_t)^2 + \hat{A}_t\hat{L}_t + \hat{A}_t + \frac{1}{2}(\hat{A}_t)^2 =& (1-\gamma)\left(T\hat{o}T_t + \frac{1-\gamma}{2}T\hat{o}T_t^2 + T\hat{o}T_t\hat{C}_t^U\right) \\
&+ \hat{C}_t^U + \frac{1}{2}(\hat{C}_t^U)^2 + \frac{\nu}{2}(\pi_{H,t})^2, \\
\hat{L}_t^* + \frac{1}{2}(\hat{L}_t^*)^2 + \hat{A}_t^*\hat{L}_t^* + \hat{A}_t^* + \frac{1}{2}(\hat{A}_t^*)^2 =& -\gamma\left(T\hat{o}T_t - \frac{\gamma}{2}T\hat{o}T_t^2 + T\hat{o}T_t\hat{C}_t^U\right) \\
&+ \hat{C}_t^U + \frac{1}{2}(\hat{C}_t^U)^2 + \frac{\nu}{2}(\pi_{F,t})^2.
\end{aligned}
\tag{A.6}
$$

Combining the second-order approximations of the aggregate consumption equations $C_t = (1-\lambda)C_t^S + \lambda C_t^N$, $C_t^* = (1-\lambda^*)C_t^{S^*} + \lambda^* C_t^{N^*}$, and $C_t^U = \gamma C_t + (1-\gamma)C_t^*$ leads to the following expression:

$$
\begin{aligned}
\hat{C}_t^U + \frac{1}{2}(\hat{C}_t^U)^2 = \gamma(1-\lambda)\left(\frac{C^S}{C^U}(\hat{C}_t^S + \frac{1}{2}(\hat{C}_t^S)^2)\right) + \gamma\lambda\left(\frac{C^N}{C^U}(\hat{C}_t^N + \frac{1}{2}(\hat{C}_t^N)^2)\right) \\
+ (1-\gamma)(1-\lambda^*)\left(\frac{C^{S*}}{C^U}(\hat{C}_t^{S*} + \frac{1}{2}(\hat{C}_t^{S*})^2)\right) + (1-\gamma)\lambda^*\left(\frac{C^{N*}}{C^U}(\hat{C}_t^{N*} + \frac{1}{2}(\hat{C}_t^{N*})^2)\right)
\end{aligned}
\tag{A.7}
$$

Using (A.6) and (A.7) to rewrite and rearrange (A.5) yields

$$
\begin{aligned}
\frac{U_t^U - U^U}{(C^S)^{-\sigma} C^U} =& \frac{\gamma}{2}(\hat{A}_t)^2 + \frac{1-\gamma}{2}(\hat{A}_t^*)^2 - \gamma\frac{\nu}{2}(\pi_{H,t})^2 - (1-\gamma)\frac{\nu}{2}(\pi_{F,t})^2 - \frac{1}{2}\gamma(1-\gamma)\hat{ToT}_t^2 \\
&+ \gamma \hat{A}_t \hat{L}_t + (1-\gamma)\hat{A}_t^* \hat{L}_t^* - \gamma\frac{\varphi}{2}(\hat{L}_t)^2 - (1-\gamma)\frac{\varphi}{2}(\hat{L}_t^*)^2 \\
&- \gamma\frac{\sigma}{2}\left((1-\lambda)\frac{C^S}{C^U}(\hat{C}_t^S)^2 + \lambda\frac{C^N}{C^U}(\hat{C}_t^N)^2\right) \\
&- (1-\gamma)\frac{\sigma}{2}\left((1-\lambda^*)\frac{C^{S*}}{C^U}(\hat{C}_t^{S*})^2 + \lambda^*\frac{C^{N*}}{C^U}(\hat{C}_t^{N*})^2\right) \\
&+ \gamma\lambda(q^\sigma - 1)\left(\frac{C^N}{C^U}(\hat{C}_t^N + \frac{1-\sigma}{2}(\hat{C}_t^N)^2) - \hat{L}_t - \frac{1+\varphi}{2}(\hat{L}_t)^2\right) \\
&+ (1-\gamma)\lambda^*((q^*)^\sigma - 1)\left(\frac{C^{N*}}{C^U}(\hat{C}_t^{N*} + \frac{1-\sigma}{2}(\hat{C}_t^{N*})^2) - \hat{L}_t^* - \frac{1+\varphi}{2}(\hat{L}_t^*)^2\right)
\end{aligned}
\tag{A.8}
$$

The labor terms in the second line can be replaced by a first-order version of (A.6) and then rearranged, while the next two lines can be rewritten by using $\hat{C}_t^S = \hat{C}_t + \lambda\frac{C^N}{C}\hat{q}_t$ and $\hat{C}_t^N = \hat{C}_t - (1-\lambda)\frac{C^S}{C}\hat{q}_t$ and the foreign counterparts which can be obtained by combining the aggregate consumption equations for $\hat{C}_t$ and $\hat{C}_t^*$ and the definitions of $\hat{q}_t$ and $\hat{q}_t^*$. This leads to

$$
\begin{aligned}
\frac{U_t^U - U^U}{(C^S)^{-\sigma} C^U} =& -\frac{\varphi}{2}(\hat{C}_t^U)^2 - \frac{1+\varphi}{2}\left((\hat{A}_t^U)^2 - 2\hat{C}_t^U\hat{A}_t^U\right) - \gamma\frac{\nu}{2}(\pi_{H,t})^2 - (1-\gamma)\frac{\nu}{2}(\pi_{F,t})^2 \\
&- \frac{1}{2}\gamma(1-\gamma)(1+\varphi)\left(\hat{ToT}_t^2 - 2\hat{ToT}_t(\hat{A}_t - \hat{A}_t^*) + (\hat{A}_t - \hat{A}_t^*)^2\right) \\
&- \gamma\frac{\sigma}{2}\left(\frac{C}{C^U}(\hat{C}_t)^2 + \lambda(1-\lambda)\frac{C^S C^N}{C C^U}(\hat{q}_t)^2\right) \\
&- (1-\gamma)\frac{\sigma}{2}\left(\frac{C^*}{C^U}(\hat{C}_t^*)^2 + \lambda^*(1-\lambda^*)\frac{C^{S*}C^{N*}}{C^* C^U}(\hat{q}_t^*)^2\right) \\
&+ \gamma\lambda(q^\sigma - 1)\left(\frac{C^N}{C^U}(\hat{C}_t^N + \frac{1-\sigma}{2}(\hat{C}_t^N)^2) - \hat{L}_t - \frac{1+\varphi}{2}(\hat{L}_t)^2\right) \\
&+ (1-\gamma)\lambda^*((q^*)^\sigma - 1)\left(\frac{C^{N*}}{C^U}(\hat{C}_t^{N*} + \frac{1-\sigma}{2}(\hat{C}_t^{N*})^2) - \hat{L}_t^* - \frac{1+\varphi}{2}(\hat{L}_t^*)^2\right)
\end{aligned}
\tag{A.9}
$$

Using $\hat{C}_t = \hat{C}_t^U + (1-\gamma)\frac{C^*}{C^U}(\hat{C}_t - \hat{C}_t^*)$ and $\hat{C}_t^* = \hat{C}_t^U - \gamma\frac{C}{C^U}(\hat{C}_t - \hat{C}_t^*)$, the aggregate consumption terms can be rewritten. Collecting terms yields

$$\begin{aligned}
\frac{U_t^U - U^U}{(C^S)^{-\sigma} C^U} = &- \frac{\sigma+\varphi}{2}(\hat{C}_t^U)^2 - \frac{1+\varphi}{2}\left((\hat{A}_t^U)^2 - 2\hat{C}_t^U \hat{A}_t^U\right) - \gamma\frac{\nu}{2}(\pi_{H,t})^2 - (1-\gamma)\frac{\nu}{2}(\pi_{F,t})^2 \\
&- \frac{1}{2}\gamma(1-\gamma)(1+\varphi)\left(T\hat{o}T_t - (\hat{A}_t - \hat{A}_t^*)\right)^2 - \frac{1}{2}\gamma(1-\gamma)\sigma\frac{CC^*}{(C^U)^2}(\hat{C}_t - \hat{C}_t^*)^2 \\
&- \frac{\sigma}{2}\left(\gamma\lambda(1-\lambda)\frac{C^S C^N}{CC^U}(\hat{q}_t)^2 + (1-\gamma)\lambda^*(1-\lambda^*)\frac{C^{S*}C^{N*}}{C^* C^U}(\hat{q}_t^*)^2\right) \\
&+ \gamma\lambda(q^\sigma - 1)\left(\frac{C^N}{C^U}(\hat{C}_t^N + \frac{1-\sigma}{2}(\hat{C}_t^N)^2) - \hat{L}_t - \frac{1+\varphi}{2}(\hat{L}_t)^2\right) \\
&+ (1-\gamma)\lambda^*((q^*)^\sigma - 1)\left(\frac{C^{N*}}{C^U}(\hat{C}_t^{N*} + \frac{1-\sigma}{2}(\hat{C}_t^{N*})^2) - \hat{L}_t^* - \frac{1+\varphi}{2}(\hat{L}_t^*)^2\right)
\end{aligned} \tag{A.10}$$

Now, we will use the shock variables $\hat{A}_t$, $\hat{A}_t^*$, and $\hat{A}_t^U$ to introduce gaps between variables and their efficient counterpart. The efficient steady state can be implemented by changing the financing of the sales subsidy from our assumed uniform taxation form to a distribution in which saver households pay for the subsidy exclusively. Consider a more general financing framework:

$$\tau_t^S = \frac{1-\theta}{1-\lambda}\tau Y_t, \qquad \tau_t^N = \frac{\theta}{\lambda}\tau Y_t \tag{A.11}$$

in terms of the producer price index. The parameter θ captures the possibility of redistribution in steady state. Recall that both household types receive the identical wage income due to $L_t = L_t^S = L_t^N$. Due to an optimal subsidy, the markup completely vanishes in steady state, i.e. $\Phi = 1$. In the case of a uniform taxation, i.e. $\theta = \lambda$, saver households make positive dividends net of taxes and therefore have an higher income stream than constrained households despite zero inflation. This implies imperfect insurance in steady state. In the case of $\theta = 0$, savers receive no dividends. The resulting income structure implies perfect steady-state insurance, i.e. $q = 1$, and there is no need to hold money. This results in the efficient steady state with zero inflation and no inequality.

We follow Levine et al. (2023) and characterize the efficient allocation of our model by flexible prices and eliminating the distortion from limited asset market participation which results in perfect insurance, i.e. $C_t^{Se} = C_t^{Ne} = C_t^e = C_t^{Se*} = C_t^{Ne*} = C_t^{e*} = C_t^{Ue}$.[1] Marginal cost pricing implies $w_t/A_t = 1$. Using this and applying a first-order approximation to (14) around the aforementioned steady state leads to $\hat{A}_t = \varphi\hat{L}_t^e + \sigma\hat{C}_t^{Ue} + (1-\gamma)T\hat{o}T_t^e$. An analogous equation

[1] Levine et al. (2023) name this the equitable allocation.

holds abroad: $\hat{A}_t^* = \varphi\hat{L}_t^{e*} + \sigma\hat{C}_t^{Ue} - \gamma T\hat{o}T_t^e$. The labor terms in these equations can be eliminated by evaluating (A.6) up to first-order for the efficient allocation. Rearranging yields

$$\hat{A}_t^U = \frac{\sigma+\varphi}{1+\varphi}\hat{C}^{Ue}, \qquad \hat{A}_t - \hat{A}_t^* = T\hat{o}T_t^e. \tag{A.12}$$

Now, we can use these expressions to introduce gaps between variables and their efficient counterpart. Rewriting (A.10) and dropping terms independent of policy leads to

$$\begin{aligned}\frac{U_t^U - U^U}{(C^S)^{-\sigma}C^U} =& -\frac{\sigma+\varphi}{2}(\tilde{C}_t^U)^2 - \gamma\frac{\nu}{2}(\pi_{H,t})^2 - (1-\gamma)\frac{\nu}{2}(\pi_{F,t})^2 \\ & - \frac{1}{2}\gamma(1-\gamma)(1+\varphi)(T\tilde{o}T_t)^2 - \frac{1}{2}\gamma(1-\gamma)\sigma\frac{CC^*}{(C^U)^2}(\hat{C}_t - \hat{C}_t^*)^2 \\ & - \frac{\sigma}{2}\left(\gamma\lambda(1-\lambda)\frac{C^SC^N}{CC^U}(\hat{q}_t)^2 + (1-\gamma)\lambda^*(1-\lambda^*)\frac{C^{S*}C^{N*}}{C^*C^U}(\hat{q}_t^*)^2\right) \\ & + \gamma\lambda(q^\sigma - 1)\left(\frac{C^N}{C^U}(\hat{C}_t^N + \frac{1-\sigma}{2}(\hat{C}_t^N)^2) - \hat{L}_t - \frac{1+\varphi}{2}(\hat{L}_t)^2\right) \\ & + (1-\gamma)\lambda^*((q^*)^\sigma - 1)\left(\frac{C^{N*}}{C^U}(\hat{C}_t^{N*} + \frac{1-\sigma}{2}(\hat{C}_t^{N*})^2) - \hat{L}_t^* - \frac{1+\varphi}{2}(\hat{L}_t^*)^2\right)\end{aligned} \tag{A.13}$$

where $\tilde{C}_t^U \equiv \hat{C}_t^U - \hat{C}_t^{Ue}$ and $T\tilde{o}T_t \equiv T\hat{o}T_t - T\hat{o}T_t^e$.

The Welfare function can now be written as

$$\begin{aligned}& E_0\sum_{t=0}^{\infty}\beta^t\frac{U_t^U - U^U}{(C^S)^{-\sigma}C^U} = \\ & -\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\Big[(\sigma+\varphi)(\tilde{C}_t^U)^2 + \gamma\nu(\pi_{H,t})^2 + (1-\gamma)\nu(\pi_{F,t})^2 + \gamma(1-\gamma)(1+\varphi)(T\tilde{o}T_t)^2 \\ & + \gamma(1-\gamma)\sigma\frac{CC^*}{(C^U)^2}(\hat{C}_t - \hat{C}_t^*)^2 \\ & + \sigma\left(\gamma\lambda(1-\lambda)\frac{C^SC^N}{CC^U}(\hat{q}_t)^2 + (1-\gamma)\lambda^*(1-\lambda^*)\frac{C^{S*}C^{N*}}{C^*C^U}(\hat{q}_t^*)^2\right) \\ & - 2\gamma\lambda(q^\sigma - 1)\left(\frac{C^N}{C^U}(\hat{C}_t^N + \frac{1-\sigma}{2}(\hat{C}_t^N)^2) - \hat{L}_t - \frac{1+\varphi}{2}(\hat{L}_t)^2\right) \\ & -2(1-\gamma)\lambda^*((q^*)^\sigma - 1)\left(\frac{C^{N*}}{C^U}(\hat{C}_t^{N*} + \frac{1-\sigma}{2}(\hat{C}_t^{N*})^2) - \hat{L}_t^* - \frac{1+\varphi}{2}(\hat{L}_t^*)^2\right)\Big]\end{aligned} \tag{A.14}$$

Appendix B: Robustness checks

Appendix B contains the model equilibria and dynamics in face of a positive technology shock for the second approach of targeting λ^U as a robustness check (see table 2). Appendix B.1 shows the IRFs of a symmetric (scenario 1) and idiosyncratic shock (scenario 2) in case of a symmetric union. Appendix B.2 covers the case of an asymmetric union. It contains the corresponding model equilibria in case of a symmetric and asymmetric union with different levels of heterogeneity (B.2.1) as well as the model dynamics in case of an asymmetric union (B.2.2 and B.2.3). Appendix B.2.2 contains the IRFs of a symmetric shock in a symmetric union (scenario 1) and an asymmetric union (scenario 3). Appendix B.2.3 compares the IRFs of a symmetric (scenario 3) with an idiosyncratic shock to country F (scenario 4) or to H (scenario 5) in an asymmetric union. Table 5 summarizes the different scenarios. As in the main text, the central bank conducts Ramsey optimal monetary policy.

B.1 Symmetric union

Scenario 1 vs 2. We hold ρ constant and let α vary to target $\lambda^U = 0.3$. This leads to another steady state (less optimal deflation) than in the main text. The IRFs look quite similar. In case of an idiosyncratic shock (scenario 2), the central bank redistributes towards the affected country F through liquidity injections (panel H and I, fig B.2) and equalizes consumption reactions across countries and N-households (panel G, H and J, fig B.1). It tolerates inflation volatility (panel A, fig B.2) in favor of providing consumption insurance.

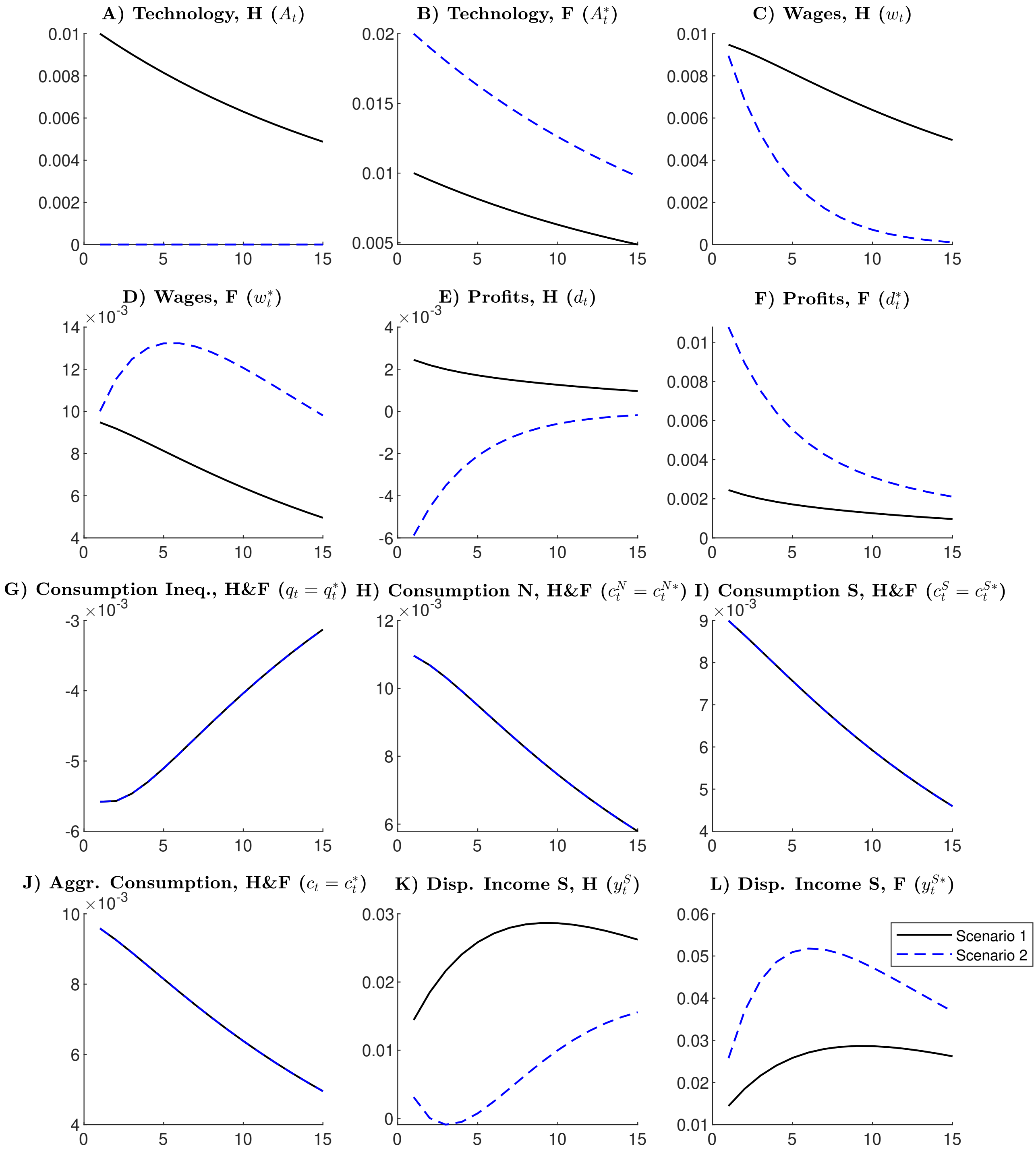

Figure B.1: Impulse response functions (IRFs) of a symmetric (scenario 1) and idiosyncratic (scenario 2, shock in Foreign) positive productivity shock in a symmetric union with $\lambda = \lambda^* = \lambda^U = 0.3$ under Ramsey optimal monetary policy. The IRFs relate to non-monetary variables and depict absolute deviations from steady state. Variables with "H" refer to Home and "F" to Foreign. Part 1/2.

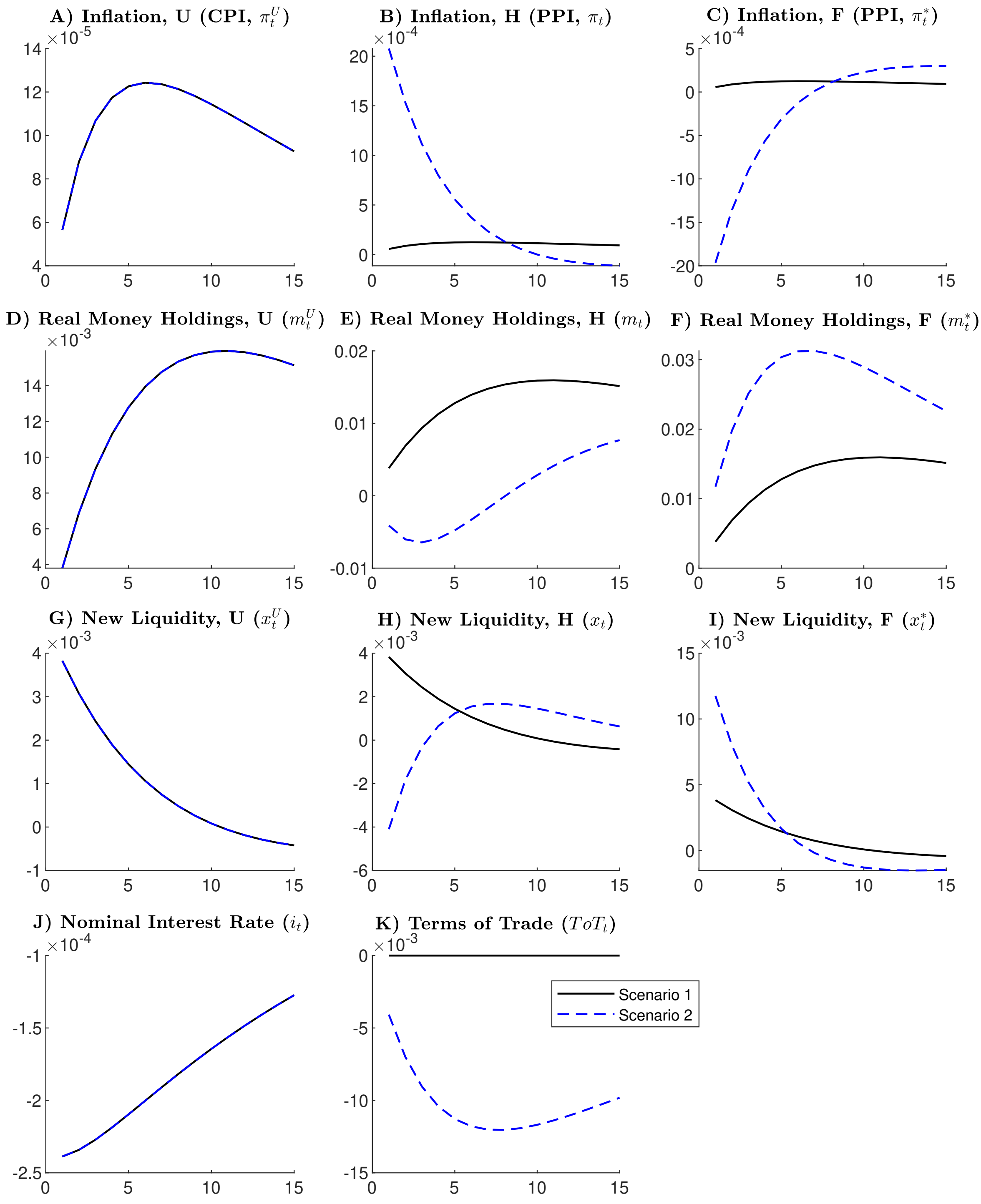

Figure B.2: Impulse response functions (IRFs) of a symmetric (scenario 1) and idiosyncratic (scenario 2, shock in Foreign) positive productivity shock in a symmetric union with $\lambda = \lambda^* = \lambda^U = 0.3$ under Ramsey optimal monetary policy. The IRFs relate to monetary variables and terms of trade and depict absolute deviations from steady state. Variables with "H" refer to Home, with "F" to Foreign and with "U" to Union. Part 2/2.

B.2 Asymmetric union

We hold ρ and ρ^* constant and let α and α^* vary to target $\lambda^U = 0.3$. Table B.1 summarizes the calibration of α, α^*, ρ and ρ^*, implying λ, λ^* and λ^U, used for the analysis of an asymmetric union for the robustness check provided in this appendix.

Table B.1: Calibration of λ, λ^* and λ^U with varying α and α^* and fixed ρ and ρ^* in an asymmetric union

	Union	**Country H**			**Country F**		
	λ^U	λ	α	ρ	λ^*	α^*	ρ^*
(1)	0.3	0.3	0.9571	0.9	0.3	0.9571	0.9
(2)	0.3	0.35	0.9462	0.9	0.25	0.9667	0.9
(3)	0.3	0.4	0.9333	0.9	0.2	0.975	0.9

B.2.1 Model equilibria

Table **??** shows the implied steady-state values from Ramsey optimal policy for a symmetric, (1), and an asymmetric union, (2) and (3):

Table B.2: Implied steady-state values from Ramsey optimal policy in a currency union with symmetric and asymmetric shares of constrained households across countries

Model outcome	**Union-wide**				**Country-specific**					
	π^U	m^U	i	c^U		c	c^S	c^N	q	m
(1) Symmetric union	-0.94%	0.369	0.0108	0.997	Home	0.997	1.061	0.847	1.252	0.369
$(\lambda = \lambda^* = 0.3)$					Foreign	0.997	1.061	0.847	1.252	0.369
(2) Asymmetric union	-0.96%	0.322	0.0106	0.997	Home	0.998	1.059	0.885	1.197	0.598
$(\lambda = 0.35, \lambda^* = 0.25)$					Foreign	0.995	1.059	0.804	1.318	0.047
(3) Asymmetric union	-1.02%	0.178	0.01	0.996	Home	0.999	1.054	0.916	1.15	0.749
$(\lambda = 0.4, \lambda^* = 0.2)$					Foreign	0.994	1.054	0.752	1.401	-0.392

In case of an asymmetric union, N in F is suffering the most. The consumption of N is much lower, lowest in F in (3), the inequality rises and differs between the countries. Due to the higher inequality, optimal deflation is larger in case of asymmetric union compared to symmetric union as the central bank wants to compensate for. There are mixed effects. A lower share of constrained households through higher α implies a higher consumption inequality in equilibrium. The higher α, the less S households demand money, harming H. On the one hand, heterogeneity according to share of constrained households across countries $(\lambda - \lambda^*)$ rises from (1) to (2) to (3), but on the other hand, inequality across countries increases, too $(q - q^*)$.

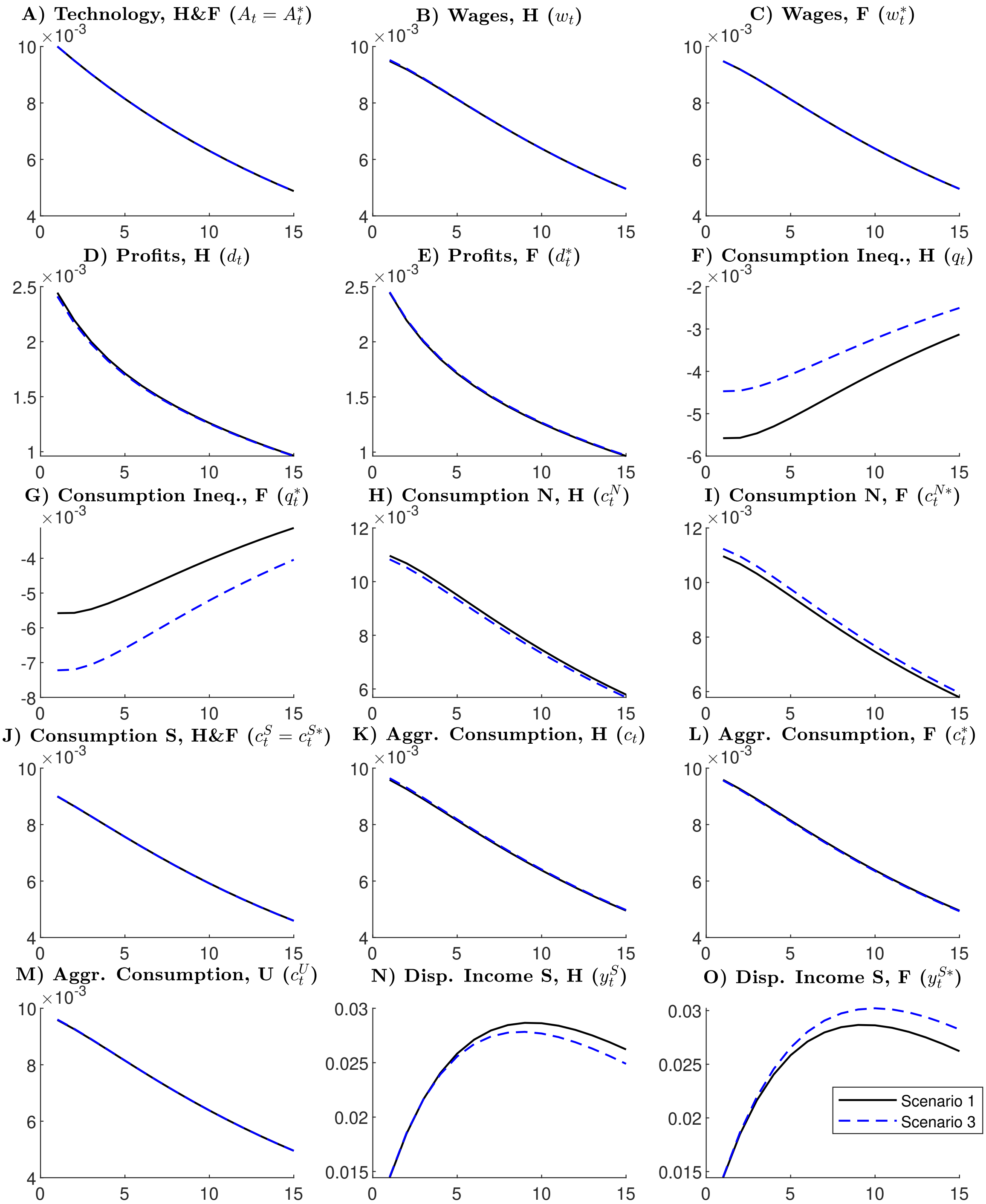

Figure B.3: Impulse response functions (IRFs) of a symmetric positive productivity shock in a symmetric (scenario 1, with $\lambda = \lambda^* = \lambda^U = 0.3$) and an asymmetric union (scenario 3, with $\lambda = 0.35$, $\lambda^* = 0.25$ and $\lambda^U = 0.3$) under Ramsey optimal monetary policy. The IRFs relate to non-monetary variables and depict absolute deviations from steady state. Variables with "H" refer to Home, with "F" to Foreign and with "U" to Union. Part 1/2.

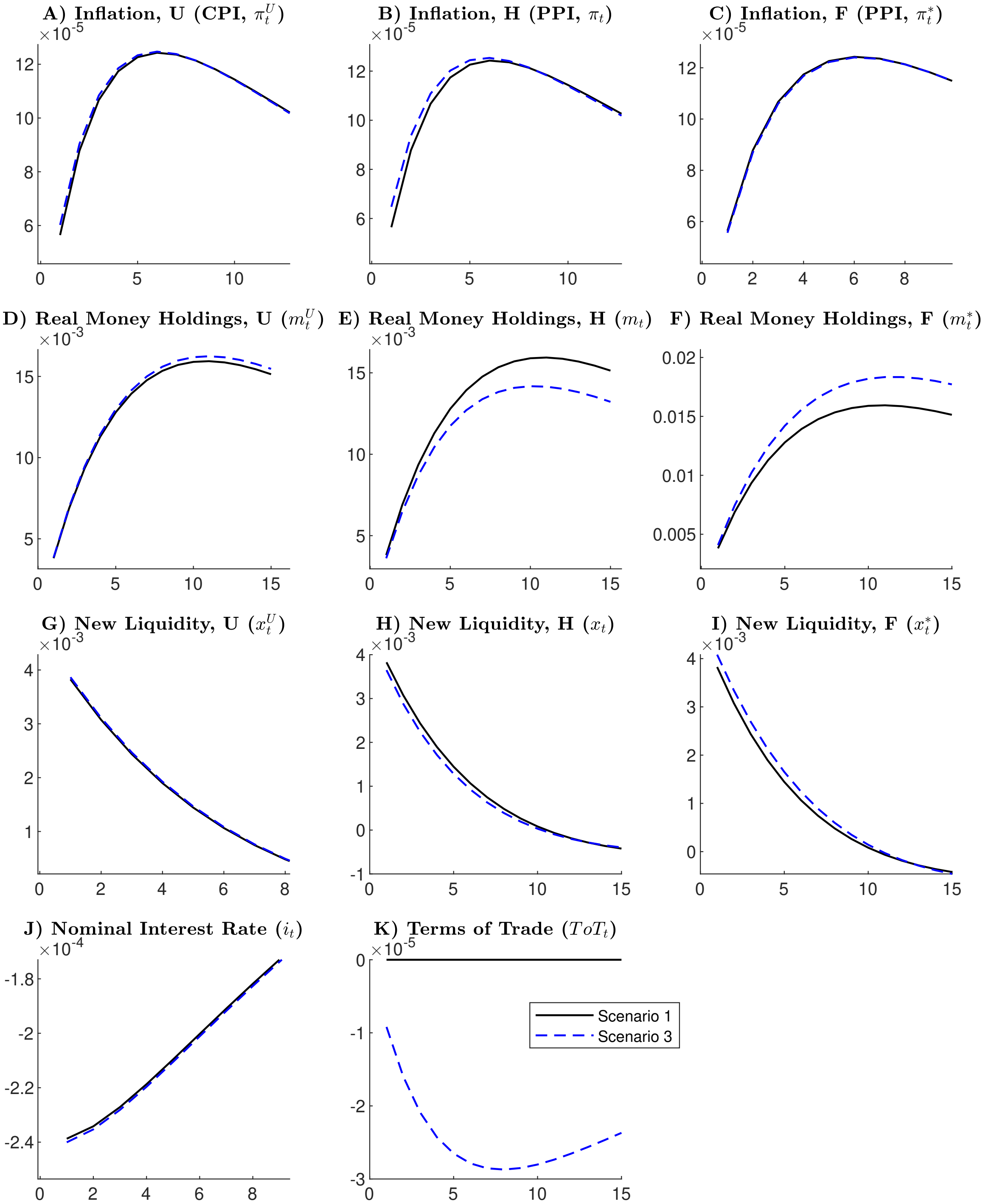

Figure B.4: Impulse response functions (IRFs) of a symmetric positive productivity shock in a symmetric (scenario 1, with $\lambda = \lambda^* = \lambda^U = 0.3$) and an asymmetric union (scenario 3, with $\lambda = 0.35$, $\lambda^* = 0.25$ and $\lambda^U = 0.3$) under Ramsey optimal monetary policy. The IRFs relate to monetary variables and terms of trade and depict absolute deviations from steady state. Variables with "H" refer to Home, with "F" to Foreign and with "U" to Union. Part 2/2.

B.2.2 Scenario 1 vs 3

The effect of steady-state inequality on optimal monetary policy overshadows the effect of different λ values across countries. The central bank compensates for higher q^* in F, although H has a higher value of λ. The central bank redistributes to F (panel E, F, H and I, fig B.4). This is in contrast to optimal monetary policy if only λ is heterogeneous across countries (main text) and not additionally q. Here, there are two increasing distortions: distortion through λ heterogeneity and through q heterogeneity.

There are opposite effects for redistribution between the countries. A higher λ leads to more redistribution to H, but F has a higher q^*, which leads to more redistribution to F. Thus, the effects are not clearly separable.

B.2.3 Scenario 3 vs 4 vs 5

Does it matter which country experience a shock? Yes, it does as the non-participating households are gaining the most when the country with the higher share of constrained households (H) is hit by a productivity shock (scenario 5, panel I and J, fig B.5). Optimally, the central bank sets its interest rate even more expansively (panel J, fig B.6). Both dynamics are in line with the optimal monetary policy described in the main text.

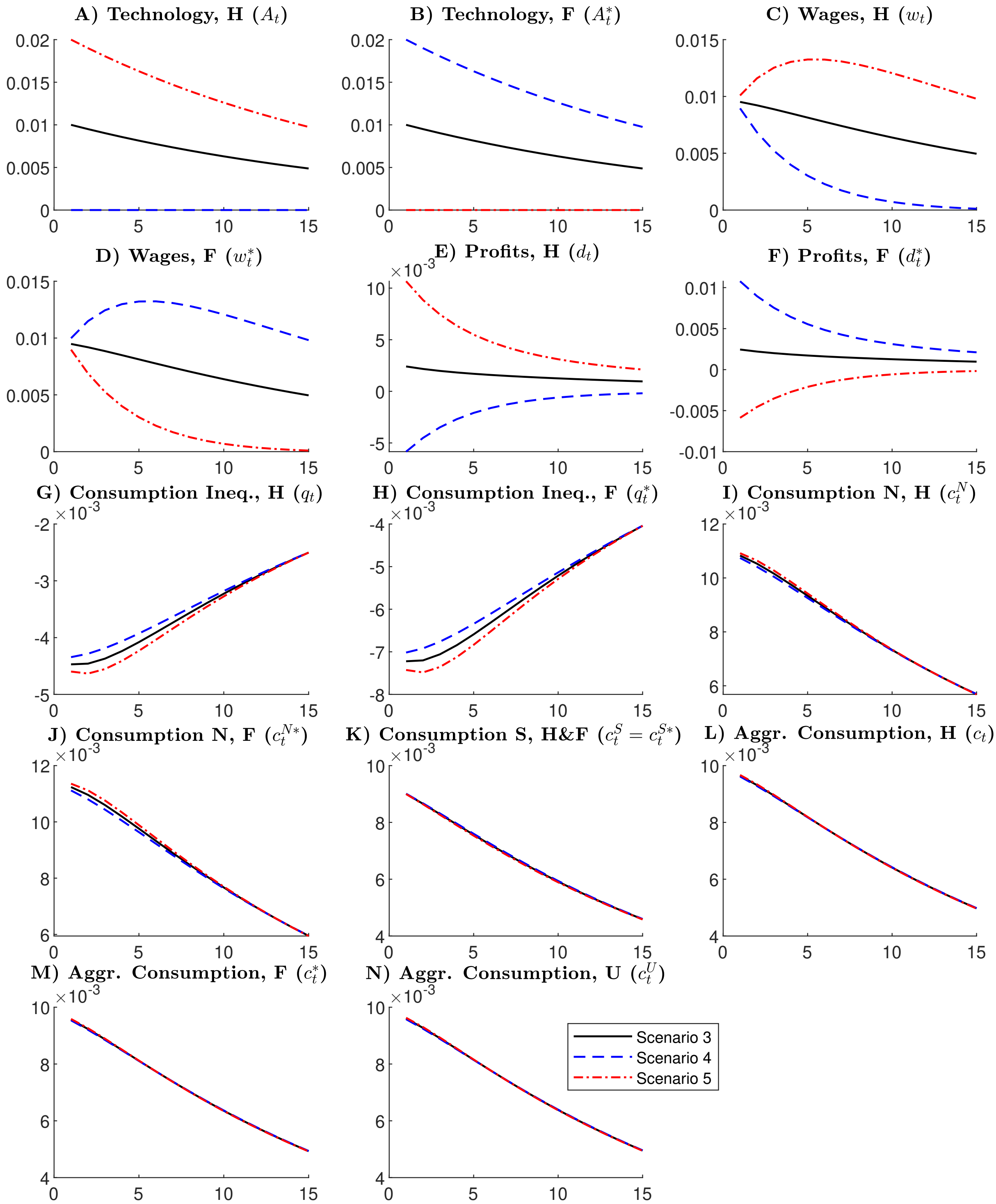

Figure B.5: Impulse response functions (IRFs) of a symmetric (scenario 3) and an idiosyncratic positive productivity shock to F (scenario 4) and H (scenario 5) in an asymmetric union (with $\lambda = 0.35$, $\lambda^* = 0.25$ and $\lambda^U = 0.3$) under Ramsey optimal monetary policy. The IRFs relate to non-monetary variables and depict absolute deviations from steady state. Variables with "H" refer to Home, with "F" to Foreign and with "U" to Union. Part 1/2.

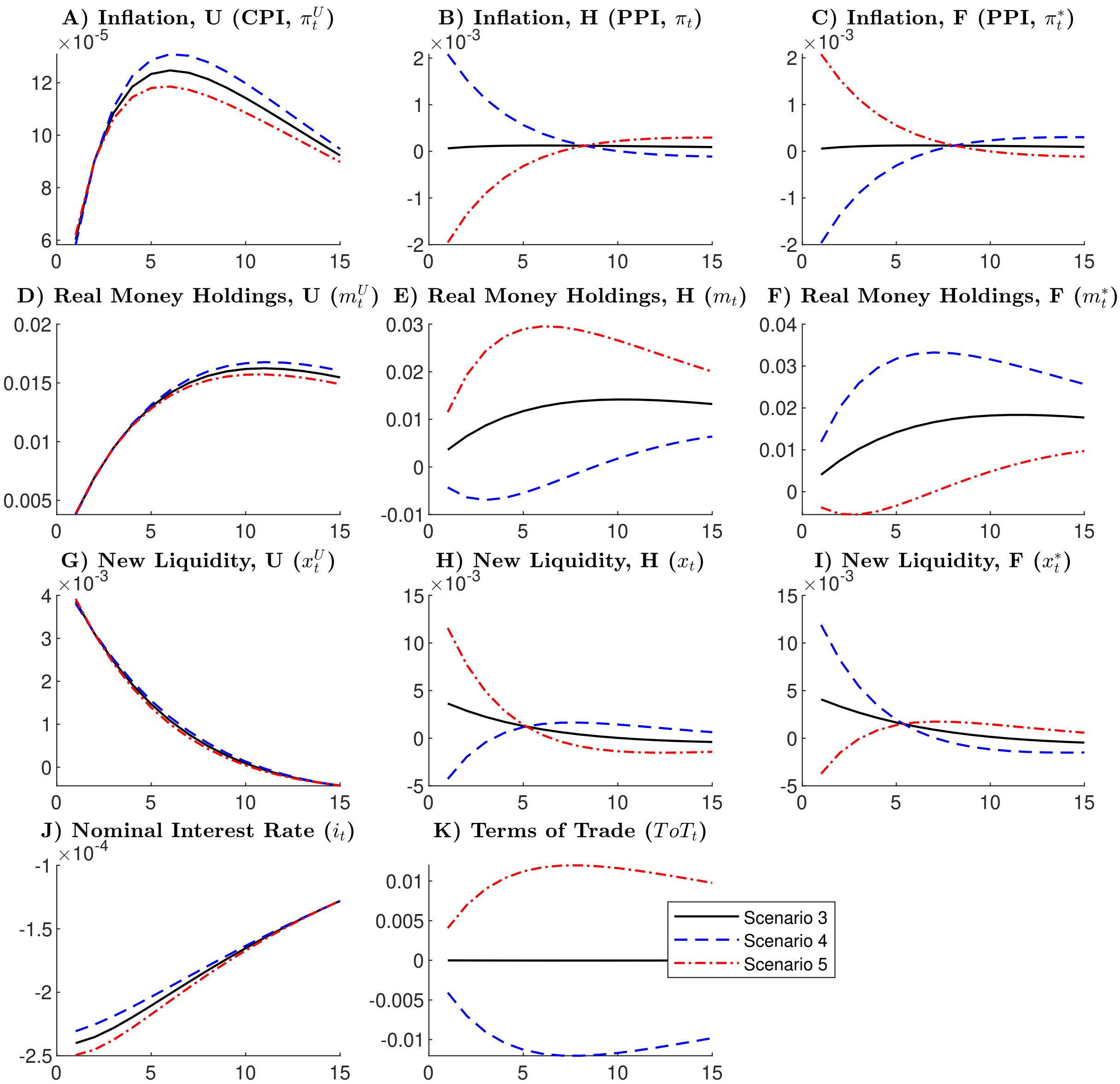

Figure B.6: Impulse response functions (IRFs) of a symmetric (scenario 3) and an idiosyncratic positive productivity shock to F (scenario 4) and H (scenario 5) in an asymmetric union (with $\lambda = 0.35$, $\lambda^* = 0.25$ and $\lambda^U = 0.3$) under Ramsey optimal monetary policy. The IRFs relate to monetary variables and terms of trade and depict absolute deviations from steady state. Variables with "H" refer to Home, with "F" to Foreign and with "U" to Union. Part 2/2.