THE NEW KEYNESIAN CLIMATE MODEL

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ABSTRACT. The accelerating pace of climate change confronts central banks with two significant obstacles to maintaining price stability. The first issue is *climateflation*, which refers to the inflationary effects resulting from a warmer planet and its adverse impact on the economy's productivity. The second challenge is *greenflation*, which pertains to inflationary pressures stemming from the implementation of climate-mitigation policies to achieve a low-carbon economy. This paper investigates these phenomena and their implications for monetary policy by developing and estimating a tractable New Keynesian Climate (NKC) model featuring climate change damages and mitigation policies for the global economy. We show that a central bank can decrease inflation associated with the green transition at the cost of reduced output expansion. However, the medium-term sacrifices are justified for long-term stability. Furthermore, monetary policy must consider the dependence of the natural interest rate on environmental development, as failing to do so will render it less effective.

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1 INTRODUCTION

As climate change accelerates, central banks worldwide are confronted with two emerging challenges to their price stability mandates. The first issue is *climateflation*, which refers to the inflationary effects resulting from climate-related events (extreme weather, natural disasters, resource scarcity) and their adverse effects on the economy's productivity. The second challenge is *greenflation*, which pertains to inflationary pressures stemming from the transition to a low-carbon economy, characterized by higher carbon taxes and abatement spending. While

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the first phenomenon can be characterized as an adverse supply shock, the second is a mixture of a positive cost-push shock and a positive demand shock. Thus, central banks face a delicate balancing act in addressing both climateflation and greenflation and maintaining price stability while supporting economic resilience.

In this paper, we investigate these two phenomena and their implications for central bank policy-making. To this end, we develop and estimate a tractable nonlinear New Keynesian Climate (NKC) model for the world economy featuring climate change damage and mitigation policies. As illustrated in Galí (2015)'s textbook, the New Keynesian framework captures the interplay between aggregate demand and supply, highlighting the role of inflation, output, and monetary policy in shaping the overall economic landscape. By augmenting the traditional 3-equation model with elements that capture climate externality and abatement costs, we aim to enrich our understanding of how climate change affects the economy. The tractability of this framework makes it possible to analytically decompose the effects of various economic mechanisms, including climateflation and greenflation.

Our first contribution is to bridge the gap between integrated assessment models (IAMs), developed to study carbon mitigation policies from a long-term perspective, and New Keynesian dynamic stochastic general equilibrium models (DSGEs), which are usually dedicated to the analysis of economic fluctuations. Our New Keynesian Climate (NKC) model keeps the elegance and tractability of the textbook model by incorporating *(i)* a single additional endogenous variable (the stock of carbon), *(ii)* four exogenous trends (population, carbon intensity, abatement efficiency and technological) and, *(iii)* one exogenous carbon tax. Consequently, the model is ultimately reduced to four equations. The IS curve incorporates green investment spending to reduce carbon emission. The Phillips curve consider the economic damage from rising carbon stocks and the production cost from abatement efforts. The monetary policy rule links the nominal interest rate to the deviation of inflation from its target and the output gap. Finally, the last equation is the law of motion that governs the accumulation of carbon dioxide emissions, which makes it depend on the current flow of production adjusted for abatement efforts. These equations form the basis for analyzing the impact of climate change on key macroeconomic variables and policy responses.

Our second contribution is to use a solution method that accounts for both *(i)* the structural change operating from rising carbon emissions and abatement spending and *(ii)* the stochastic fluctuations due to exogenous shocks around the evolving economy. Indeed, climate change and its associated mitigation policies have nonlinear and long-lasting effects on both the supply and demand side of the economy and the natural real interest rate. Therefore, the usual practice in the monetary policy literature for analyzing the propagation of small shocks around a balanced growth path is not appropriate. Consequently, we first use the extended path solution method from Fair and Taylor (1983) to numerically solve the stochastic path consistent with our model. The extended path approach uses a perfect foresight solver to obtain endogenous variables that are path-consistent with model equations. In each period, agents are surprised by the realization of shocks but still expect that in the future, shocks are zero on average, consistent with rational expectations. Second, an inversion filter is used to calculate the likelihood function. By extracting the sequence of innovations recursively through the inversion of observation equations for a given set of initial conditions, this filter has recently emerged as a computationally efficient method (Guerrieri and Iacoviello, 2017). Finally, using Bayesian techniques, as in Smets and Wouters (2003), we estimate the structural parameters using four World's macroeconomic and climate-related time series from 1985Q1 to 2023Q2.

Our third contribution is to use this estimated model to assess the importance of climateflation and greenflation and the associated monetary policy responses in different transition scenarios. The first scenario is a "laissez-faire" economy characterized by an increasing stock of carbon, that warms the planet and makes resources scarcer. The increasing damage to total factor productivity acts as a permanent negative supply shock that fuels inflation and drives output below its technological trend (Schnabel, 2022). The second scenario captures the "Paris-Agreement", which requires world governments to implement mitigation policies to reach net-zero carbon emissions by 2050. In our framework, this scenario takes the form of a linear increase in the carbon tax such that full abatement is reached in 2050. The rise in carbon tax forces firms to internalize the effects of their carbon emissions on aggregate productivity. In response, they reduce their emissions by increasing abatement expenditures, creating a demand-driven boom. We use these two scenarios *(i)* to explore the trade-offs between current abatement efforts and future damages, and *(ii)* to study their implications for natural output and real interest rate, inflation and monetary policy responses.

We find that....

Our paper is related to the burgeoning literature that focuses on climate issues using micro-founded structural models. Fischer and Springborn (2011), Heutel (2012), and Angelopoulos et al. (2013) were among the first to introduce carbon emissions in real business cycle models. They assumed that emissions stem from production and adversely impact utility or have a negative impact on productivity and production. Recent contributions have extended these models in several directions, including *(i)* multisector aspects (Golosov et al., 2014a; Dissou and Karnizova, 2016), *(ii)* labor market frictions (Gibson and Heutel, 2020; Finkelstein Shapiro and Metcalf, 2023), *(iii)* distortionary fiscal policy (Barrage, 2020), *(iv)* endogenous entry (Annicchiarico et al., 2018; Finkelstein Shapiro and Metcalf, 2023), *(v)* nominal rigidities and monetary policy (Annicchiarico and Di Dio, 2015, 2017; Carattini et al., 2021; Diluiso et al., 2021; Ferrari and Nispi Landi, 2022; Coenen et al., 2023; Del Negro et al., 2023; Ferrari and Nispi Landi, 2024), and *(vi)* distributional implications of energy price shocks similar to those of carbon price shocks (Auclert et al., 2023; Langot et al., 2023). While these studies provide interesting insights into the role of transition, they do not explicitly deal with the nonlinear nature of carbon accumulation and its permanent effects on the economy. By contrast, we consider long-run trends in carbon emissions and macroeconomic variables in the spirit of Jondeau et al. (2023). This makes our framework well-suited for studying the effects of environmental policies.

This is indeed the path followed by some recent papers. In a nonlinear New Keynesian model with climate change externalities, Nakov and Thomas (2023) analyze how optimal monetary policy trades off price stability and climate goals and how this trade-off depends on the (sub)optimality of green transition policies. In a nonlinear model with price and wage rigidities and two sectors, Olovsson and Vestin (2023) show that during the green transition, it is optimal for monetary policy to see through the increasing energy prices and focus on core inflation, resulting in a modest increase in inflation. Finally, using a small nonlinear open economy, Pappa et al. (2023) analyze energy efficiency and concentrate on the role played by fiscal policy during the transition.

The remainder of this paper is organized as follows. Section 2 presents the micro-foundations of the NKC and its summary into four core equations. Section 3 presents the data transformation, prior and posterior distributions, and assessment of the fit of the model. Section 4 presents the anatomy of the green transition and analyzes how the climateflation and greenflation phenomena may affect the world economy by 2100. Section 5 provides a sensitivity analysis. Section 6 studies the implications of the green transition for the central bank. Section 7 concludes.

2 The 4 equation New Keynesian climate (NKC) model

Our starting point is the textbook three equation New Keynesian model (Woodford, 2003, Galí, 2015), which includes an IS curve, a Phillips curve, and a Taylor-type rule. To this standard framework, we add climate dynamics by mixing Golosov et al. (2014a) and Nordhaus (1992).

2.1 Household sector. The economy is populated by a mass l_t of ex-ante atomistic, identical, and infinitely lived households. This mass is time-varying and captures the upward trend of the population observed over the last 60 years. Formally, it is assumed that the population asymptotically converges to a long-run level $l_T > 0$, such as $l_t = l_{t-1} (l_T / l_{t-1})^{\ell_g}$, with $\ell_g \in [0, 1]$ being the geometric rate of convergence to l_T. Each household indexed by $i \in [0, l_t]$ maximizes its sequence of present and future utility flows that depend positively on consumption $c_{i,t}$ and negatively on labor $n_{i,t}$:

$$\mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \tilde{\beta}_{t,t+s} \varepsilon_{b,t+s} \left(\frac{c_{i,t+s}^{1-\sigma_c} - 1}{1 - \sigma_c} - \psi_{t+s} \frac{n_{i,t+s}^{1+\sigma_n}}{1 + \sigma_n} \right) \right\}, \tag{1}$$

where $\mathbb{E}_t$ denotes the expectation conditional on the information available at t, $\tilde{\beta}_{t,t+s}$ is the technological-neutral discount factor,[1] $\sigma_c > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, $\sigma_n > 0$ is the inverse of the Frisch labor supply elasticity, and ψ_t is a scale variable pinning down hours worked in a balanced growth path.[2] In addition, $\varepsilon_{b,t}$ is a preference shock that captures the unexpected changes in aggregate demand. It follows an AR(1) process: $\varepsilon_{b,t} = (1 - \rho_b) + \rho_c \varepsilon_{b,t-1} + \eta_{b,t}$, with $\eta_{b,t} \sim \mathcal{N}(0, \sigma_b^2)$.

As in McKay et al. (2017), households are endowed with stochastic idiosyncratic employment status $\varsigma_{i,t} \in \{0, 1\}$, with 0 indicating low productivity (denominated "type L" worker) and 1 high productivity (denominated "type H" worker). The level of productivity is drawn i.i.d. with probabilities $\Pr(\varsigma_{i,t} = 0) = \omega$ and $\Pr(\varsigma_{i,t} = 1) = 1 - \omega$. The sequence of real

[1] The presence of a permanent increase in technology affects the Euler equation, and consequently the natural real interest rate and the monetary policy rule. To keep the framework tractable, we mute the effect of technology on the long-run equilibrium rate by imposing: $\tilde{\beta}_{t,t+s} = \beta (z_{t+s}/z_t)^{\sigma_c}$ with $\beta \in (0, 1)$. Note that this assumption is standard in models featuring recursive utility functions such as Epstein-Zin for instance.

[2] Note that ψ_t must grow proportionally with the flow of current consumption. Thus, if z_t denotes the trend in per capita consumption, $\psi_t = \psi z_t^{1-\sigma_c}$, with ψ as a scale parameter.

budget constraints for each type of households is as follows:

$$c_{i,t} + b_{i,t} + T^{s}_{i,t} = \frac{r_{t-1}}{\pi_t} b_{i,t-1} + \Pi_{i,t} + w_t n_{i,t} + T^{e}_{i,t}, \text{ if } \varsigma_{i,t} = 1, \tag{2}$$

$$c_{i,t} + b_{i,t} = \frac{r_{t-1}}{\pi_t} b_{i,t-1} + d_{i,t}, \text{ if } \varsigma_{i,t} = 0, \tag{3}$$

where variable $b_{i,t}$ is the one-period riskless bond, r_t is the gross nominal interest rate on bonds, $\pi_t = p_t / p_{t-1}$ is gross inflation with p_t being the price index, $\Pi_{i,t}$ are real dividend payments received from holding shares of firms, w_t is the aggregate real wage, and $T^{e}_{i,t}$ represents the revenues of the carbon tax redistributed through lum-sum transfers. Low-productivity households receive $d_{i,t}$ units of the consumption good as a transfer, and high-productivity households pay a tax of $T^{s}_{i,t} = \omega d_{i,t} / (1 - \omega)$ to finance the transfer. This transfer is assumed to be time-varying as it grows proportionally to productivity z_t.

The Euler equation associated with the problem of household i of productivity type $q \in \{H, L\}$ is thus given by:

$$\varepsilon_{b,t} c^{-\sigma_c}_{i,q,t} \geq \mathbb{E}_t \left\{ \frac{\tilde{\beta}_{t,t+1} \varepsilon_{b,t+1} r_t}{\pi_{t+1}} \left((1 - \omega)\, c^{-\sigma_c}_{i,H,t+1} + \omega c^{-\sigma_c}_{i,L,t+1} \right) \right\}, \tag{4}$$

where $c_{i,H,t}$ and $c_{i,L,t}$ denote consumption for high- and low-productive households, respectively.

2.2 Business sector. The business sector is characterized by final good producers who sell a homogeneous final good to households and the government. To produce, they buy and pack differentiated varieties produced by atomistic and infinitely lived intermediate goods firms that operate in a monopolistically competitive market. Intermediate goods firms contribute to climate change by emitting CO_2 as an unintended result of their production process.

2.2.1 Final good sector. At every point in time t, a perfectly competitive sector produces a final good Y_t by combining a continuum of intermediate goods $y_{j,t}$, $j \in [0, l_t]$, according to the technology $y_t = \left[l_t^{-1/\zeta} \int_0^{l_t} y_{j,t}^{\frac{\zeta-1}{\zeta}} \mathrm{d}j \right]^{\frac{\zeta}{\zeta-1}}$. The number of intermediate good firms owned by households is equal to the size of the population L_t. Parameter $\zeta > 1$ measures the substitutability across differentiated intermediate goods. Final good producing firms take their output price, p_t, and their input prices, $p_{i,t}$, as given and beyond their control. Profit maximization implies the demand curve $y_{j,t} = l_t^{-1} \left(p_{j,t} / p_t \right)^{-\zeta} y_t$, from which we deduce the relationship between the price of the final good and the prices of intermediate goods $p_t \equiv \left[l_t^{-1} \int_0^{L_t} p_{j,t}{}^{1-\zeta} \mathrm{d}j \right]^{\frac{1}{1-\zeta}}$.

2.2.2 *Intermediate goods sector.* Intermediate good j is produced by a monopolistic firm using the following production function:

$$y_{j,t} = \Gamma_t \left(n_{j,t}^d \right)^{\alpha}, \tag{5}$$

where Γ_t is the total factor productivity (TFP) that affects labor demand $n_{j,t}^d$, with intensity $\alpha \in [0,1]$.

The TFP is actually determined by two components:

$$\Gamma_t = z_t \Phi \left(m_t \right), \tag{6}$$

where z_t is the deterministic component of productivity and $\Phi(m_t)$ is a damage function that represents the impact of climate change on the production process. The deterministic component of TFP follows the process $z_t = z_{t-1}(1 + g_{z,t})$, where $g_{z,t} = g_{z,t-1} \left(1 - \delta_z\right)$ is the productivity growth rate and δ_z is the rate of decline in productivity. This formulation indicates that productivity growth decreases over time by a factor δ_z to match the observed slowdown in economic growth over the last 60 years.

Finally, following Golosov et al. (2014a), we assume an exponential damage function:[3]

$$\Phi(m_t) = \exp(-\gamma(m_t - m_{1750}))$$

where $m_t - m_{1750}$ is the excess carbon in the atmosphere net of its (natural) removal, with m_{1750} representing the stock of carbon in the preindustrial era, that is the steady-state level in the absence of anthropogenic emissions. Law of motion for atmospheric loading of CO_2 (in gigatons of CO_2) is given by:

$$m_t - m_{1750} = (1 - \delta_m)(m_{t-1} - m_{1750}) + \xi_m e_t, \tag{7}$$

where e_t denotes the anthropogenic carbon emissions in t, $\delta_M \in [0,1]$ represents the rate of transfer of atmospheric carbon to the deep ocean, and $\xi_m \geq 0$ is the atmospheric retention ratio.

[3]This function approximates the damage function generally used in the DICE literature, which depends on atmospheric temperature.

A firm's CO_2 emissions stemming from its production process are denoted by $e_{i,t}$. As they are subject to carbon tax $\tau_{e,t}$, which aims to internalize the social cost of carbon emissions, the firm is incentivized to reduce its impact by investing in emission abatement technology. The abatement effort of the firm yields a reduction by $\mu_{i,t}$ (in %) in its CO_2 emissions. A firm's emissions take the following form:

$$e_{j,t} = \sigma_t \left(1 - \mu_{j,t}\right) y_{j,t} \varepsilon_{e,t},$$

where σ_t denotes aggregate carbon intensity in the production sector. Its law of motion is $\sigma_t = \sigma_{t-1}(1 - g_{\sigma,t})$, where $g_{\sigma,t}$ captures the possible changes in the decrease in the carbon decoupling rate. These changes follow $g_{\sigma,t} = (1 - \delta_\sigma) g_{\sigma,t-1}$, where $\delta_\sigma \in [0,1]$ is the rate of decline in the trend. This trend is set to match the decline in the emission-to-GDP ratio observed over the last 60 years. Finally, a firm's carbon intensity can be temporarily affected by an aggregate exogenous emissions shock, $\varepsilon_{e,t} = (1 - \rho_e) + \rho_e \varepsilon_{e,t-1} + \eta_{e,t}$, with $\eta_{e,t} \sim N(0, \sigma_e^2)$, which captures the cyclical changes in the emissions-to-output ratio. An increase in $\varepsilon_{e,t}$ induces a cyclical increase in the carbon intensity of the production sector.

Firms may substitute carbon-intensive technologies with low-carbon technologies; however this change in the existing lines of production is costly. We assume that the cost of abatement technology (in proportion to output) is given by:

$$\mathcal{C}^a_{j,t} = \theta_{1,t} \mu_{j,t}^{\theta_2} y_{j,t}, \tag{8}$$

where $\theta_{1,t} = (p_b/\theta_2)(1 - \delta_{pb})^{t-t_0}\sigma_t$ is the time-varying level of the abatement cost, $p_b > 0$ is a parameter determining the initial cost of abatement and $0 < \delta_{pb} < 1$ captures technological progress, which lowers the cost of abatement by a factor δ_{pb} each year. Finally, $\theta_2 > 0$ represents the curvature of the abatement cost function, which typically exhibits increasing returns in IAM literature.

Intermediate goods producers solve the typical two-stage problem. In the first stage, when input price w_t is taken as given, firms seek to maximize their one-period profits:

$$\max_{\{y_{j,t}, \mu_{j,t}\}} mc_{j,t} y_{j,t} - w_t \left(\frac{y_{j,t}}{\Gamma_t}\right)^{1/\alpha} - \mathcal{C}^a_{j,t} - \tau_{e,t}\sigma_t \left(1 - \mu_{j,t}\right) y_{j,t} \varepsilon_{e,t} \tag{9}$$

where $mc_{i,t}$ denotes the real marginal cost of producing one additional good.

In the second stage, firms decide their selling prices under the Rotemberg price setting. The Rotemberg price adjustment cost is given by:

$$\mathcal{C}^p_{j,t} = \frac{\kappa}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - \pi^\star_t\right)^2 \frac{y_t}{l_t} \tag{10}$$

where $\kappa > 0$ is the price stickiness parameter, y_t/l_t is the average market share per firm, and $\pi^\star_t$ is the gross inflation target, which follows a deterministic process (Ireland, 2007, Fève et al., 2010, Del Negro et al., 2015):

$$\pi^\star_t = (1 - \rho_{\pi^\star})\,\pi + \rho_{\pi^\star}\pi^\star_{t-1}, \tag{11}$$

where $\rho_{\pi^\star}$ is the autocorrelation coefficient that reflects the slow pace at which monetary authorities allegedly adjusted their inflation target, and π is the steady-state gross inflation.

To attenuate the expectation channel of inflation, an exogenous exit shock is introduced, consistent with empirical evidence on the survival rate of firms across time (OECD, 2017). As in Bilbiie et al. (2012), we assume a "death" shock, which occurs with probability $\vartheta \in (0, 1)$ in every period. This means that each firm's profit is subject to an idiosyncratic shock $\omega_{j,t}$ that takes the value of 0 for the fraction of firms exiting the market. Thus, the problem faced by firms can be expressed as follows:

$$\max_{\{p_{j,t}\}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \omega_{j,t+s} \left(y_{j,t+s}\frac{p_{j,t+s}}{p_{t+s}} - \varepsilon_{p,t+s} mc_{t+s} y_{j,t+s} - \mathcal{C}^p_{j,t+s} \right) \right\}, \tag{12}$$

subject to demand $y_{j,t} = l_t^{-1}\left(p_{j,t}/p_t\right)^{-\zeta} y_t$. $\varepsilon_{p,t}$ is a cost-push shock that follows an AR(1) process: $\varepsilon_{p,t} = (1-\rho_p) + \rho_p \varepsilon_{p,t-1} + \eta_{p,t}$, with $\eta_{p,t} \sim \mathcal{N}(0, \sigma^2_p)$.

Because all intermediate goods firms face an identical profit maximization problem, they choose the same price $p_{j,t} = p_t$. In a symmetric equilibrium, where $\int_0^{l_t} = \omega_{j,t+1}/\omega_{j,t}\mathrm{d}f(\omega_{j,t}) \simeq 1 - \vartheta$, the optimal pricing rule implies:

$$\kappa\left(\pi_t - \pi^\star_t\right)\pi_t = (1-\vartheta)\beta\kappa\mathbb{E}_t\left\{\left(\pi_{t+1} - \pi^\star_{t+1}\right)\pi_{t+1}\frac{y_{t+1}}{y_t}\frac{l_t}{l_{t+1}}\right\} + \zeta\varepsilon_{p,t} mc_t + (1-\zeta). \tag{13}$$

The above equation is the New Keynesian Phillips curve, which relates current inflation to the discounted sum of marginal costs.

2.3 Public sector. The government issues short-term bonds, collects revenue from the carbon tax and redistributes it entirely to households on a lump-sum basis:

$$\int_0^{l_t} b_{i,t}\mathrm{d}i + + \tau_{e,t} \int_0^{l_t} e_{j,t}\mathrm{d}j = \frac{r_{t-1}}{\pi_t} \int_0^{l_t} b_{i,t-1}\mathrm{d}i + \int_0^{l_t} \Pr\left(z_{i,t} = 1\right) T^e_{i,t}\mathrm{d}i. \tag{14}$$

The monetary policy authority follows a Taylor-type rule by gradually adjusting the nominal interest rate in response to *(i)* the inflation gap and *(ii)* the output gap:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho} \left[\left(\frac{\pi^\star_t}{\pi}\right)\left(\frac{\pi_t}{\pi^\star_t}\right)^{\phi_\pi}\left(\frac{y_t}{y^n_t}\right)^{\phi_y}\right]^{1-\rho} \varepsilon_{r,t}, \tag{15}$$

where r is the long-run nominal interest rate, y^n_t is the natural output and the parameters ρ_r, ϕ_π, ϕ_y capture the degree of interest-rate smoothing, and the responsiveness of the policy rate to the inflation and output gaps, respectively. Finally, $\varepsilon_{r,t}$ is a monetary policy shock that follows the process: $\varepsilon_{r,t} = (1-\rho_r) + \rho_r \varepsilon_{r,t-1} + \eta_{r,t}$, with $\eta_{r,t} \sim N(0, \sigma^2_r)$,

2.4 Aggregation. First, we aggregate consumption for the two types of households:

$$l_t c_t = \int_0^{l_t} \Pr\left(z_{i,t} = 0\right) c_{L,t}\mathrm{d}i + \int_0^{l_t} \Pr\left(z_{i,t} = 1\right) c_{H,t}\mathrm{d}i, \tag{16}$$

It leads to:

$$c_t = \omega c_{L,t} + \left(1-\omega\right) c_{H,t}. \tag{17}$$

It is assumed that bonds net supply is zero:

$$\int_0^{l_t} b_{i,t}\mathrm{d}i = 0. \tag{18}$$

As discussed by McKay et al. (2017), as long as $c_{L,t} < c_{H,t}$, the Euler equation for the low productive worker does not bind to equality as the right hand side will always be lower than the left hand side. Therefore, let $\lambda_t = \varepsilon_{b,t} c_{H,t}^{-\sigma_c} = \varepsilon_{b,t}\left(\frac{c_t - \omega d_t}{1-\omega}\right)^{-\sigma_c}$ denote the marginal utility of consumption of highly productive households, the *aggregate* Euler equation is as follows:

$$\lambda_t = \mathbb{E}_t \left\{ \frac{\tilde{\beta}_{t,t+1} r_t}{\pi_{t+1}} \left((1-\omega)\lambda_{t+1} + \omega \varepsilon_{b,t+1} d_t^{-\sigma_c} \right) \right\}. \tag{19}$$

In contrast, the general equilibrium for hours worked reads as:

$$\left(1-\omega\right) n_t = n^d_t. \tag{20}$$

Finally, the resource constraint is given by:

$$y_t = l_t c_t + \frac{\kappa}{2}\left(\pi_t - \pi_t^\star\right)^2 y_t + \theta_{1,t}\mu_t^{\theta_2} y_t + \vartheta \Pi_t. \tag{21}$$

2.5 Final System. The system can be summarized by the following set of four core equations that determine four endogenous variables $\{\tilde{y}_t, r_t, \pi_t, \tilde{m}_t\}$. These variables are respectively the detrended GDP ($\tilde{y}_t = y_t/(l_t z_t)$), the nominal interest rate, the inflation rate and the excess carbon in the atmosphere net of its (natural) removal ($\tilde{m}_t = m_t - m_{1750}$).

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(1) *IS curve:*

$$\left(\frac{x_t\tilde{y}_t - \omega d_t}{1-\omega}\right)^{-\sigma_c} = \beta\mathbb{E}_t\left\{\frac{\varepsilon_{b,t+1}}{\epsilon_{b,t}}\frac{r_t}{\pi_{t+1}}\left((1-\omega)\left(\frac{x_{t+1}\tilde{y}_{t+1} - \omega d_t}{1-\omega}\right)^{-\sigma_c} + \omega d_t^{-\sigma_c}\right)\right\},$$

where $x_t = 1 - (1-\vartheta)\frac{\kappa}{2}\left(\pi_t - \pi_t^\star\right)^2 - \theta_{1,t}\tilde{\tau}_{e,t}^{\theta_2/(\theta_2-1)} - \vartheta(1 - \epsilon_{p,t} mc_t)$.

(2) *Phillips curve:*

$$\left(\pi_t - \pi_t^\star\right)\pi_t = (1-\vartheta)\beta\mathbb{E}_t\left\{\left(1 + g_{z,t+1}\right)\frac{\tilde{y}_{t+1}}{\tilde{y}_t}\left(\pi_{t+1} - \pi_{t+1}^\star\right)\pi_{t+1}\right\} + \frac{\zeta}{\kappa}\varepsilon_{p,t} mc_t + \frac{1-\zeta}{\kappa}$$

where $mc_t = \frac{\psi}{\epsilon_{b,t}(1-\omega)^{\sigma_c+\sigma_n}}\frac{(x_t\tilde{y}_t - \omega d_t)^{\sigma_c}\tilde{y}_t^{\sigma_n}}{\Phi(\tilde{m}_t)^{1+\sigma_n}} + \theta_{1,t}\tilde{\tau}_{e,t}\left[\theta_2 + (1-\theta_2)\,\tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}\right]$

(3) *Monetary policy rule:*

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho}\left[\left(\frac{\pi_t^\star}{\pi}\right)\left(\frac{\pi_t}{\pi_t^\star}\right)^{\phi_\pi}\left(\frac{\tilde{y}_t}{\tilde{y}_t^n}\right)^{\phi_y}\right]^{1-\rho}\varepsilon_{r,t},$$

(4) *Pollution stock dynamics:*

$$\tilde{m}_t = (1-\delta_m)\tilde{m}_{t-1} + \xi_m\sigma_t\left(1 - \tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}\right) z_t l_t \tilde{y}_t \varepsilon_{e,t}.$$

3 Bayesian inference and model evaluation

In this section, we estimate the model using Bayesian methods (see An and Schorfheide, 2007, for an overview). The posterior distribution associated with the vector of observable variables is computed numerically using the Monte Carlo Markov Chain sampling approach.

Specifically, we rely on the Metropolis-Hastings algorithm to obtain a random draw of size 20,000 from the posterior distribution of the parameters (eight parallel chains simultaneously draw 2,500 iterations, with a common jump scale parameter to match an acceptance rate of approximately 30%). First, we describe how the non-linear model with trends is solved. We then discuss the retained data together with our choice of priors and comment on the posterior distribution of the structural parameters. Finally, we provide an assessment of the model's fit.

3.1 Numerical solution method with stochastic growth. We consider the *extended path solution method* from Fair and Taylor (1983) and Adjemian and Juillard (2014) to measure the non-linear effects of environmental constraints on growth accurately. Briefly, the extended path approach uses a perfect foresight solver to obtain endogenous variables that are path consistent with the model's equations. In each period, agents are surprised by the realization of shocks, but still expect that in the future, shocks are zero on average (consistent with rational expectations). The advantage of this method is that it provides an accurate and fast solution while considering all nonlinearities of the model. The drawback of this approach is that Jensen's inequality binds to equality, meaning that the non-linear uncertainty stemming from future shocks is neglected. Note that this drawback also applies to typical linearized DSGE models, such as Smets and Wouters (2007).

Taking nonlinear models to the data is a challenge, as nonlinear filters, which are required to form the likelihood function, are computationally expensive. An inversion filter has recently emerged as a computationally-inexpensive alternative (e.g., Guerrieri and Iacoviello, 2017, Atkinson et al., 2020). Initially pioneered by Fair and Taylor (1983), this filter recursively extracts the sequence of innovations by inverting the observation equation for a given set of initial conditions. Unlike other filters (e.g., Kalman or particle filters), the inversion filter relies on an analytic characterization of the likelihood function.[4]

The inversion performs using the perfect foresight solution proposed by Juillard et al. (1996). The standard approach is to compute the dynamics of the variables given the current and future shocks. In the extended path context, the inversion filter *(i)* substitutes current shocks and some endogenous variables when applying the perfect foresight solution and *(ii)* computes current shocks and nonobservable variable paths given the set of observable

[4]For a presentation of alternative filters to calculate the likelihood function, see Fernández-Villaverde et al. (2016). See also Cuba-Borda et al. (2019) and Atkinson et al. (2020) for details on the relative gains of the inversion filter.

variables. Finally, we use the Metropolis-Hastings algorithm as a sampler to draw from the parameter uncertainty.

A perfect foresight algorithm typically requires *(i)* a finite number of periods and *(ii)* a terminal period to compute each endogenous variable to realize economic surprises. To fix notation, this general representation in the presence of extended path takes the form:

$$\tilde{y}_t = g_\Theta \left(y_0, y, 0\right) \tag{22}$$

$$y_t = \mathbb{E}_{t,t+S} \left\{ g_\Theta \left(y_{t-1}, \tilde{y}_{t+S+1}, \varepsilon_t\right) \right\} \tag{23}$$

$$\mathcal{Y}_t = h_\Theta \left(y_t\right) \tag{24}$$

$$\varepsilon_t \sim \mathcal{N}\left(0, \Sigma_\varepsilon\right) \tag{25}$$

The first equation determines the deterministic evolution of the endogenous variables in the absence of shocks summarized in vector $\tilde{y}_t$ with initial conditions y_0 and terminal (asymptotic) state y for a given set of nonlinear equations $g_\Theta(\cdot)$. The second equation determines the path of endogenous variables y_t with economic surprise, ε_t is a vector of exogenous stochastic innovations that are normally distributed with mean zero and covariance Σ_ε; Θ is the vector of structural parameters; $h_\Theta(\cdot)$ and $g_\Theta(\cdot)$ are the set of nonlinear equations. $\mathbb{E}_{t,t+S}\{\cdot\}$ is the extended path-consistent expectation operator, which updates expectations over a specific time horizon of size S, and takes as given $\tilde{y}_{t+S+1}$ the terminal period of the expectation. Therefore, the size of the expectation window S must be sufficiently large to ensure that the value of $\tilde{y}_{t+S+1}$ does not affect the outcome.[5] The third equation relates the observations summarized in vector $\mathcal{Y}_t$ to the endogenous variables in y_t. The last equation concerns the distribution of exogenous innovations.

For each evaluation of the sample likelihood, we first compute the deterministic path providing the transition between the initial period $\{\tilde{y}_t\}_{t=1}^{T}$ and the terminal period. We select a value of $T = 1,000$ to allow convergence to the terminal state. Formally, we use Equation 22 assuming that (*i*) no shock with sequence $\{\varepsilon_t\}_{t=1}^{T}$ is all zeros, and (*ii*) a terminal condition that is the steady state of the model $\tilde{y}_{t+S+1} = y$, which can be written as $\tilde{y}_t = g_\Theta(y_{t_0}, y, 0)$. Next, we use the inversion filter to find the sequence of $\{\varepsilon_t\}_{t=1}^{T^*}$ that matches sample $\{\mathcal{Y}_t\}_{t=1}^{T^*}$

[5] One must strike a balance between the length of the expectation window to mimic infinite-horizon rational expectations, and the computational burden of updating the expectations. We select an expectation horizon of 40 years ($S = 160$). This length is sufficiently large to ensure that the terminal conditions do not quantitatively affect the numerical value of the likelihood function, but exhibits a moderate computational burden.

with T^* observations using $\{\tilde{y}_t\}_{t=1}^{T}$ as the terminal value of the expectation window. This implicitly assumes that agents expect the economy to return to its deterministic path $\tilde{y}$ after S periods. Based on the smoothed sequence $\{\varepsilon_t\}_{t=1}^{T^*}$, the likelihood function $\mathcal{L}(\theta, \mathcal{Y}_{1:T^*})$ of the model is obtained, conditional on the matrix of observations through time T^*.

3.2 Data description. The model is estimated using worldwide quarterly data from 1985Q1 to 2023Q2. As time series are not available on a quarterly basis, some transformations are necessary. First, the annual GDP in constant 2015 US$ is obtained from the *World Bank* (https://data.worldbank.org/indicator/NY.GDP.MKTP.KD), and is converted on a quarterly basis using the time disaggregation method of Chow and Lin (1971) using real quarterly GDP for total OECD countries from the *OECD Economic Outlook* database (https://data.oecd.org/gdp/quarterly-gdp.htm).[6] Quarterly headline inflation (https://db.nomics.world/OECD/EO?q=OECD%2FEO%2FOTO.CPI_YTYPCT.Q) and the nominal interest rates (https://db.nomics.world/OECD/EO?dimensions=%7B%22VARIABLE%22%3A%5B%22IRS%22%5D%7D&q=OECD%20economic%20outlook%20interest%20rate) are obtained from the *OECD Economic Outlook* database. The aggregate interest rate is the weighted average of the rates in the OECD countries. Annual CO_2 emissions, which correspond to the emissions from the burning of fossil fuels for energy and cement production, are from *Our World In Data* (https://ourworldindata.org/explorers/co2?facet=none&country=~OWID_WRL&Gas+or+Warming=CO2&Accounting=Territorial&Fuel+or+Land+Use+Change=All+fossil+emissions&Count=Per+country). We convert annual data into quarterly data using the same disaggregation approach as for GDP.

FIGURE 1. Observable variables

[6]This temporal disaggregation technique uses a statistical relationship between low-frequency data and higher-frequency indicator variables. First, regressions performed at the low-frequency level, at this level the target time series and the indicator time series are both available. Second, the resulting estimates are used to obtain the high-frequency target series.

Our solution method explicitly deals with trends and thus does not impose that variables must return to the steady state.[7] Consequently, we simply use the growth rate (i.e., the first difference of the logarithm) for GDP and CO_2 emissions and maintain the level of inflation and the interest rates. Figure 1 displays the temporal evolution of all observable variables of the model.

The measurement equations mapping our model to the four observable macroeconomic and climate-related time series are given by:

$$\begin{bmatrix} \text{Real output growth rate} \\ \text{Inflation rate} \\ \text{Short-term interest rate} \\ \text{CO}_2 \text{ emissions growth rate} \end{bmatrix} = \begin{bmatrix} \Delta \log (y_t) \\ \pi_t - 1 \\ r_t - 1 \\ \Delta \log (e_t) \end{bmatrix}. \tag{26}$$

3.3 Calibrated parameters. A first set of parameters is calibrated. These parameters can be divided into two groups: the structural parameters and initial conditions. We begin by discussing the calibration of the structural parameters reported in Panels A and B of Table 1.

These parameters are categorized into three panels. Panel A is related to the climate dynamics. These parameters are related to the carbon law of motion in Equation 7, and we assume that the carbon lifetime δ_m is set to 500 years (2,000 quarters). The first prototypes of DICE and IAM (e.g., Nordhaus 1992) as well as E-DSGE (e.g., Heutel 2012), typically assumed a short-lived cycle between 80 and 100 years. Recent advances in climate science, summarized by Dietz and Venmans (2019), highlight the presence of a much longer carbon lifetime than previously measured. Because this paper focuses on mitigation policies from a cost-efficiency perspective, the calibration of this parameter does not critically drive the quantitative outcome. Our assumption of a carbon lifetime of 2,000 quarters appears to be rather conservative with respect to the usual economic literature on climate dynamics. The next two parameters are also obtained from the DICE. Parameter ξ_m simply converts CO_2 units into carbon units as follows: GtC = ξ_m $GtCO_2$ (as damages are typically measured by the radiative forcing from carbon), while m_{1750} is the natural stock of carbon in the atmosphere back in 1750, while its value in 1984 was 719 GtC. The last parameter, γ, maps carbon stock to economic damage. Because temperatures and carbon stock are cointegrated variables, we

[7] Linearization methods impose to approximate any model's decision rules around a fixed point, and therefore, impose that the model is stationary in the neighborhood of the fixed point. Thus, the inference must be assessed based on stationary data, the latter implies a set of transformations (e.g., dividing by population, business cycle filters, etc.).

TABLE 1. Calibrated parameter values and initial conditions (quarterly basis)

NAME	PARAMETER	VALUE
Panel A: Climate Parameters		
CO_2 rate of transfer to deep oceans	δ_m	5.000e-04
Marginal atmospheric retention ratio	ξ_m	0.27273
Pre-industrial stock of carbon (GtC)	m_{1750}	545
Climate damage elasticity	γ	4.163e-05
Abatement cost curvature	θ_2	2.6
Decay abatement cost	δ_{pb}	0.004277
Panel B: Economic Parameters		
Risk aversion	σ_c	2.5
Firm exit shock	ν	0.03
Low productivity worker payoff-to-consumption	d/c	0.95
Share low productive workers	ω	0.02
Terminal population (billion)	L_∞	10.48
Population growth	l_g	0.00625
Goods substitution elasticity	ς	4
Decay TFP (annualized)	$\delta_z \times 400$	0.3
Decay rate emission intensity	δ_σ	
Labor intensity	α	0.7
Decline rate inflation trend (annualized)	$\rho_{\pi^\star} \times 400$	7.5
Long-term inflation target	$\pi^\star_\infty$	0.0025
Discount factor	β	0.9924
Panel C: Initial Conditions		
Initial GDP (trillion USD PPP)	Y_{t_0}	7.5
Initial inflation trend (annualized)	$\pi_{\star,t_0} \times 400$	10
Initial interest rate	$r_{t_0} \times 400$	6
Initial emissions ($GtCO_2$)	E_{t_0}	5.075
Initial abatement cost-to-gdp	θ_{1,t_0}	0.31454
Initial population (billion)	L_{t_0}	4.85
Initial stock of carbon (GtC)	m_{t_0}	719.94
Initial carbon price ($/ton)	τ_{t_0}	
Initial hours worked	h_{t_0}	1

follow Golosov et al. (2014b) and assume that damage directly emerges from atmospheric carbon concentration. Parameter γ is set to 4.13e-5 to entail a permanent 3% output loss in the business-as-usual scenario as in DICE. Note that this parameterization is higher than in Golosov et al. (2014b) to match the increased damage, by a factor of two, in DICE 2023 compared to its 2016R2 counterpart to account for possible tipping effects. Regarding the abatement sector, we build on Barrage and Nordhaus (2023). The abatement cost is meant to reach 10.9% of GDP in 2020 in the presence of full abatement ($\mu = 1$). As our first simulation date is much earlier, we extrapolate this value back to $t_0 = 1984Q4$ and find a value close to

0.18%. Technological progress through cost-efficient technologies causes this abatement cost to decrease by 1.7% per year, while the curvature remains fixed at 2.6.

Panel B concerns the calibration of the economic parameters. The risk aversion is set to 2.5 to match the relatively higher risk aversion parameter in emerging market economies (e.g., Aguiar and Gopinath 2007), thus attenuating the transmission channel of monetary policy. The discount factor is set to match the average real interest rate in the sample. Regarding the discounting of the Euler and Phillips curves, the exit rate ν is taken from the firm entry literature and assumes a 3%, consistent with OECD data documenting the death rates in the manufacturing and services sectors. In contrats, the Euler discount depends on ω and d; we impose a calibration such that we obtain a 3% percent discount as in McKay et al. (2017) by imposing a fraction of 2% of workers experiencing the income shock, while the insurance is set to 95% of consumption. Regarding the calibration for the exogenous process for the population, the population at the start of the sample (1984Q4) is set at 4.85 billion, but will converge at a 2% rate toward the long-term population $L_\infty = 10.48$ billion, consistently with United Nations forecasts for 2100. The substitutability of intermediate goods provides a 33% markup, which is typical for calibrating macroeconomic models with imperfect competition. The growth decay parameters δ_z and δ_σ are obtained from DICE 2023. The labor supply is normalized to one, while the labor intensity is set to 0.7, as in DICE. Finally, regarding the exogenous process for the inflation target, we run a residual minimization to match the inflation data. We obtain an initial inflation target of 10% annually, with a decay rate of 7.5% per year to reach a long-term inflation target of 1%. We set the annual central bank interest at 6%, which matches the average policy rate in the year preceding the sample. Finally, to capture realistic levels of GDP and CO_2 emissions, we set the level of GDP to 7.5 trillions USD and the level of emissions at 5 $GtCO_2$, as in 1984.

The last variable that needs to be discussed is the expected path of carbon tax $\tilde{\tau}_{e,t}$. The transition scenario that occurs out-of-sample affects expectations and therefore alters the representation of the data provided by the estimated model. Instead of imposing an arbitrary mitigation scenario (such as the Paris Agreement), we allow the data to be informative about the expected path of the carbon tax. Let $\{\tilde{\tau}_{e,t}\}_{1984Q4}^{T}$ denote the path of the carbon tax under temperature stabilization by 2050. We impose the following expectation scheme: $\mathbb{E}_{t,t+S}\{\tilde{\tau}_{e,t}\} = \varphi\tilde{\tau}_{e,t}$, where $\varphi \in [0,1]$ is the market belief about the realization of the Paris Agreement. Parameter φ can be interpreted as *(i)* the prior over the probability of realization of mitigation policy, *(ii)* the fraction of agents believing in a complete mitigation policy, or *(iii)* the belief about the policy's stringency.

3.4 Prior and posterior distributions. The remaining parameters are estimated. Their prior distributions are presented in Table 2. For exogenous disturbances, the standard deviations are imposed an inverse gamma "type 2" as Christiano et al. (2014) with a prior mean of 0.002 and a standard error of 0.0033. The AR of shocks follows a Beta distribution with a prior mean of 0.5 and a standard deviation of 0.1, which is a relatively more informative prior than in Smets and Wouters (2007). Regarding the structural parameters, we estimate the annualized slope of TFP growth and carbon decoupling. In the DICE models, these parameters typically lie between 1% and 1.5%. Therefore, we impose a diffuse Gamma distribution with a mean of 1.5 and a standard deviation of 0.5 in order to allow only values close to 1% up to 2%. Concerning the utility parameter determining labor supply, our prior is taken from Smets and Wouters (2007) with a Gamma distribution to impose a positive support for its posterior distribution. Concerning the Rotemberg adjustment cost, this parameter is typically between 20 and 200 in the literature. Therefore, we impose a Gamma distribution with a prior mean of 30 and a standard deviation of 5 to allow for possibly intense price stickiness or a quasi-flexible price scheme. Next, we comment on the parameters related to monetary policy. Regarding the stance of inflation, we mostly consider the prior distributions of Smets and Wouters (2007), but consider a Gamma shape for $\phi_\pi - 1$ and ensure that the Taylor principle holds for any posterior value. For ϕ_y, we employ the prior of Smets and Wouters (2007) but with a Gamma distribution. Finally, the mitigation belief probability φ follows a Beta distribution with a prior mean of 0.5 and a standard deviation of 0.1.

We next turn to the posterior distribution generated by the Metropolis-Hasting sampler, expressed in 90% confidence intervals in Table 2 Regarding shocks, one can note that the main source of persistence is the nominal-related disturbances, as also found by Smets and Wouters (2007). In addition, the pollution shock also exhibits significant persistence, as obtained by Jondeau et al. (2023). The initial productivity growth is, on average, above the values found in DICE, but this is not surprising as our initial period of simulation (1985) exhibits more TFP Growth than the 2015-2020 period of DICE. Similar patterns are also observed for the decoupling rate. In comparison to Smets and Wouters (2007), we observe a much flatter labor supply equation, which causes the marginal cost and thus the inflation rate to be less responsive (than those in other papers) to a change in hours. Rotemberg price

TABLE 2. Prior and posterior distributions of structural parameters

		PRIOR DISTRIBUTION			POSTERIOR DISTRIBUTION		
		Shape	Mean	Std	Mode	Mean	[5%;95%]
Panel A: Shock processes							
Std demand	σ_b	$\mathcal{IG}_2$	0.002	0.0033	0.0546	0.0539	[0.0492;0.0591]
Std price	σ_p	$\mathcal{IG}_2$	0.002	0.0033	0.005	0.0051	[0.0046;0.0057]
Std MPR	σ_r	$\mathcal{IG}_2$	0.002	0.0033	0.0011	0.0011	[0.0010;0.0012]
Std emissions	σ_e	$\mathcal{IG}_2$	0.002	0.0033	0.0051	0.005	[0.0045;0.0055]
AR demand	ρ_b	$\mathcal{B}$	0.5	0.1	0.6407	0.6576	[0.6292;0.6895]
AR price	ρ_b	$\mathcal{B}$	0.5	0.1	0.982	0.9819	[0.9812;0.9822]
AR MPR	ρ_r	$\mathcal{B}$	0.5	0.1	0.2838	0.3051	[0.2269;0.3812]
AR emissions	ρ_e	$\mathcal{B}$	0.5	0.1	0.9579	0.9622	[0.9484;0.9746]
Panel B: Structural parameters							
Initial TFP growth	$g_{z,t_0} \times 400$	$\mathcal{G}$	1.5	0.5	1.7121	1.715	[1.6967;1.735]
Decay rate decoupling	g_{σ,t_0}	$\mathcal{G}$	1.5	0.5	1.3112	1.302	[1.2394;1.3643]
Labor disutility	σ_h	$\mathcal{G}$	2	0.75	0.1086	0.1307	[0.1018;0.184]
Rotemberg Cost	κ	$\mathcal{G}$	30	5	87.9231	86.3919	[82.9545;88.1328]
Inflation stance	$(\phi_\pi - 1)$	$\mathcal{G}$	0.5	0.05	0.3477	0.3783	[0.3147;0.4518]
MPR GDP stance	ϕ_y	$\mathcal{G}$	0.15	0.1	0.9088	0.9912	[0.8125;1.2632]
Mitigation policy belief	φ	$\mathcal{B}$	0.5	0.08	0.5364	0.4854	[0.3509;0.6406]
MPR smoothing	ρ	$\mathcal{B}$	0.5	0.1	0.9664	0.9679	[0.9616;0.9747]
Log marginal data density							-2901.86

Note: $\mathcal{B}$ denotes the Beta, $\mathcal{G}$ the Gamma, and $\mathcal{IG}_2$ the Inverse Gamma (type 2) distributions. 120,000 draws are used to compute the posterior mean and 90% confidence interval.

stickiness exhibits a large value, suggesting large nominal rigidities that are consistent with the Great Moderation period, characterized by a stable inflation rate. The smoothing of the interest rate is much larger than that reported by Smets and Wouters (2007), while the stance on inflation is relatively smaller. This can be explained by the presence of emerging countries that are relatively more hawkish than Western central banks. In addition, a marked contrast emerges in relation to the stance on the output gap parameter, where our estimates indicate a significantly larger weight for output stabilization. Finally, the model's estimation suggests that achieving full decarbonization policy is conceivable with a confidence level of approximately 53%.

3.5 Model evaluation. The quality of the model is first assessed by comparing the data and model-implied moments, as listed in Table 3. Data moments are computed for 1985Q1-2023Q2. Model-implied moments are reported with their 90% confidence intervals, based on 1000 random draws from the parameter distributions. Table 3 shows that the model reproduces the main data statistics relatively well, despite its small size.

Indeed, the model's performance is relatively good compared with the usual standards in the inference of real business cycle models. Specifically, the model accurately replicates the volatility of inflation and the nominal interest rate. However, it tends to underestimate the volatility of output and carbon emissions, suggesting a potential area for improvement. First-order autocorrelations are also successfully matched, with the exception of inflation which is found to be weaker in the model. Regarding the cross-correlation with output, the model performs well, accurately capturing the relationship between output and most variables, except for carbon emission growth. This suggests that further refinement or adjustments may be needed to better align the model's representation of carbon emission growth with the observed patterns.

TABLE 3. Empirical and model-implied moments

	DATA	MODEL [5%;95%]	DATA	MODEL [5%;95%]
	Mean		**Standard deviations**	
Output growth	0.007	[0.007;0.008]	0.012	[0.015;.0.034]
Inflation rate	0.011	[-0.007;0.015]	0.007	[0.007;0.023]
Nominal interest rate	0.008	[0.006;0.019]	0.006	[0.003;0.009]
Carbon emission growth	0.004	[0.003;0.004]	0.013	[0.016;0.034]
	Autocorrelation		**Correlation w/ output**	
Output growth	-0.199	[-0.348;-0.045]	1.000	[1.000;1.000]
Inflation rate	0.977	[0.700;0.916]	0.204	[-0.177;0.234]
Nominal interest rate	0.988	[0.919;0.990]	-0.038	[-0.175;0.236]
Carbon emission growth	-0.100	[-0.328;-0.021]	0.965	[0.877;0.945]

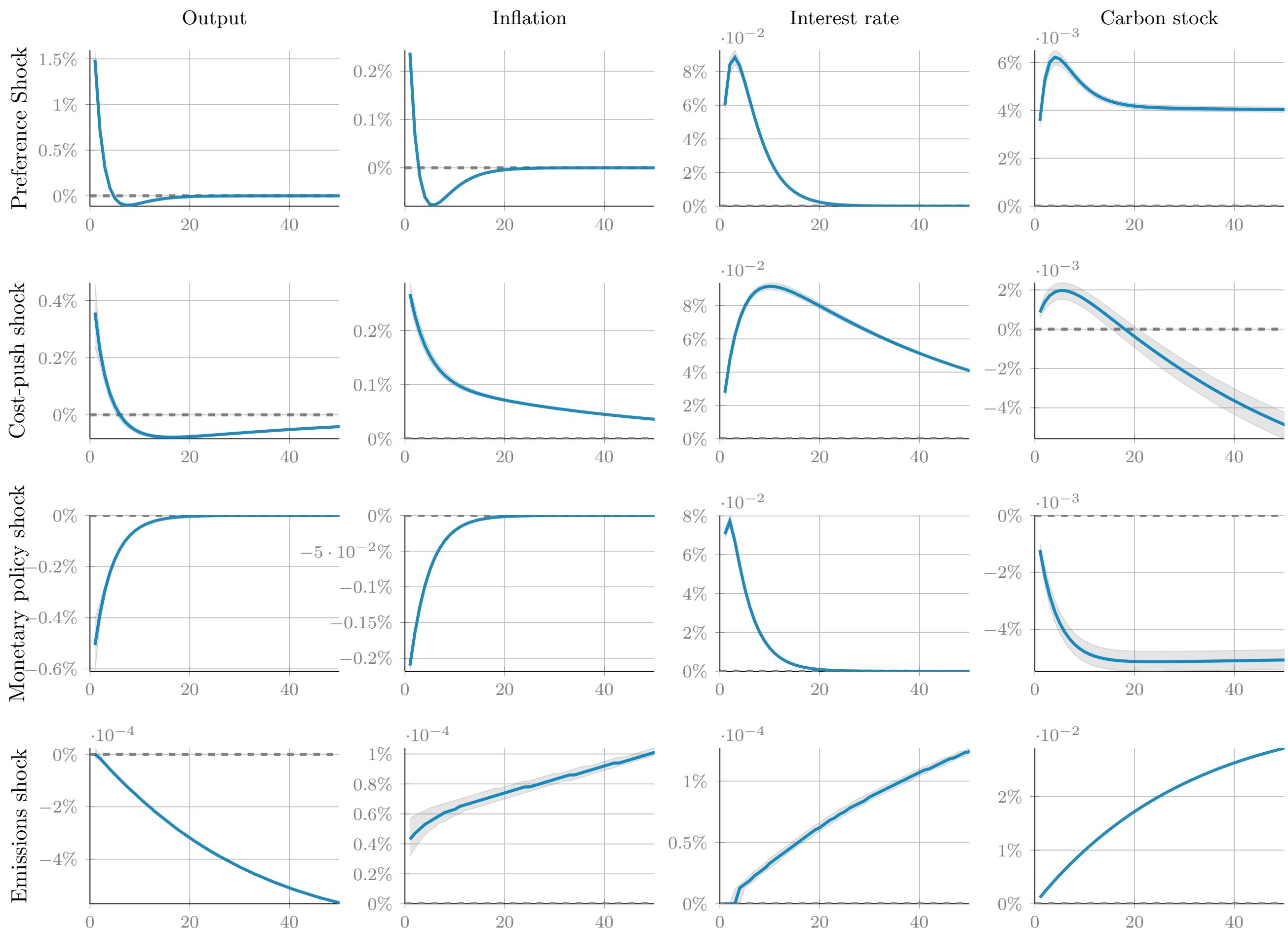
Note: Model-implied moments are computed across 1,000 random artificial series, each of the same size as the data sample.

Second, the model is evaluated based on impulse response functions (IRFs). IRFs are useful for assessing how shocks to economic variables reverberate through economic and climate systems. Figure 2 reports the generalized impulse response functions of the estimated model taking the parameters at their posterior mode among metropolis-hasting draws.

The first row of Figure 2 displays responses to a positive preference shock that boosts household consumption. In response, aggregate output increases, reflecting the positive impact of increased wealth on overall economic activity. This increase in output leads to a corresponding rise in both the inflation rate and the carbon stock. To counteract the inflationary effect of this positive demand shock, the interest rate increases, creating a modest recession when the shock process has decayed sufficiently.

The second row of Figure 2 reports the responses of the economy to the cost-push shock, similar to the markup shock of Smets and Wouters (2007). This supply shock increases firms'

FIGURE 2. Generalized impulse response functions of the estimated model



Note: The figure displays the generalized impulse response functions (GIRFs) of several variables to four shocks: preference, cost-push, monetary policy, and emissions in lines 1 to 4, respectively. GIRFs are computed using the value of the state variables in 2023Q2, and each GIRF is expressed as a percentage deviation from its initial value in 2023. GIRFs are averaged based on 500 exogenous draws.

selling price and is typically detrimental to the rest of the economy. The central bank must strike a balance between price and quantity stabilization, because the interest rate cannot stabilize when these two variables are in opposite directions. The real interest rate slightly increases following the realization of the shock which reduces output with a delay. Notably, the reduction in output also has a positive consequence in terms of emissions: there is a corresponding decrease in production and economic activity, resulting in reduced emissions.

The third row of Figure 2 shows the responses to a monetary policy shock. This shock is interpreted as a temporary deviation of the nominal rate from the systematic component of the policy rule. By boosting the return on safe assets, this shock reduces the willingness to consume and depresses aggregate demand. This decline in aggregate demand forces firms to reduce their hourly demand. The equilibrium wage clearing the labor market declines, thus creating a joint decline in the marginal cost and selling price of the goods. This decline in quantity also reduces emissions and makes the stock of carbon lower than expected.

The last shock is the emission intensity shock, which materializes as an exogenous increase in carbon emissions. This shock increases the carbon stock in the atmosphere and causes modest economic damage. However, its effects on inflation and interest rates are too small to measure.

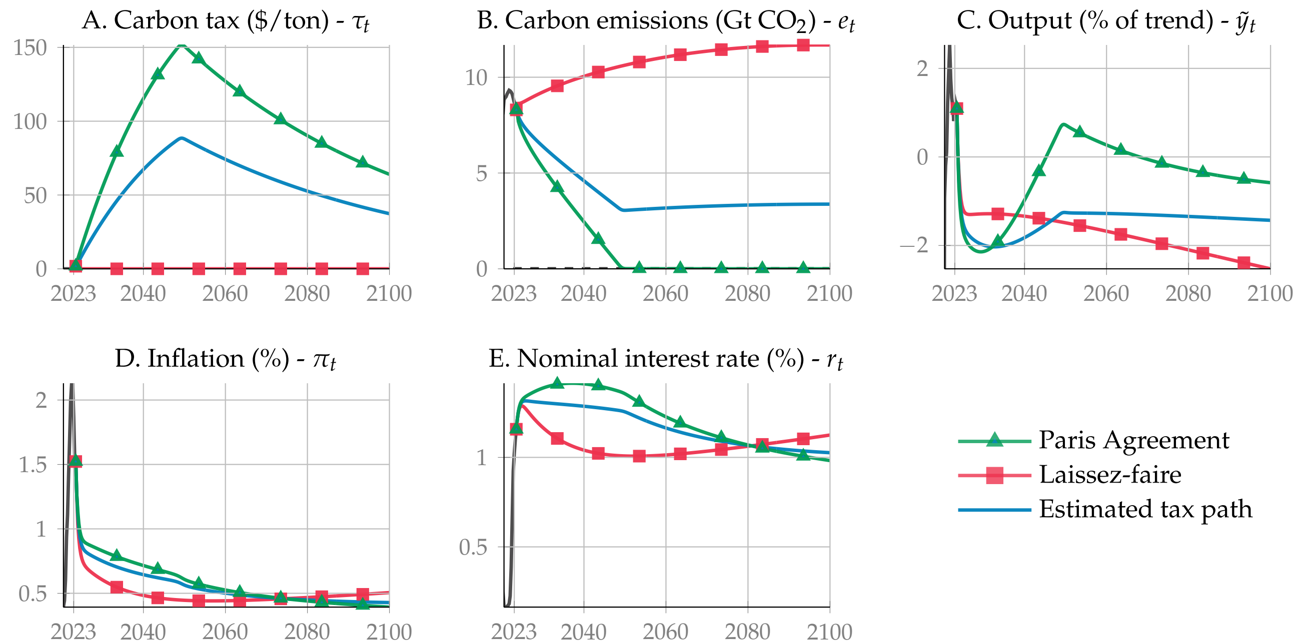
4 The anatomy of the green transition

In this section, we analyze how climateflation and greenflation phenomena may affect the world economy by 2100. The first stage of this analysis relies on long-term projections derived from the model under alternative scenarios. We then, decompose output and inflation into macroeconomic and environmental drivers.

4.1 Model-implied projections under CO2 emission scenarios. We first present long-term projections derived from the model to illustrate what can happens to the global economy by 2100. To this end, three alternative scenarios are implemented. The first scenario aligns with the so-called SSP1-1.9 pathway of the IPCC (2021) in terms of carbon emissions (*Paris Agreement*). It is assumed that carbon neutrality is reached in 2050 owing to the introduction of a carbon tax, with net emissions close to zero by 2050. The second scenario is equivalent to IPCC (2021)'s SSP3-7.0 pathway, which assumes that there are no environmental policies, resulting in a continuous increase in carbon emissions (*laissez-faire*). The third scenario relies directly on the projection of the estimated carbon tax process (*estimated tax path*). In our simulations, the value of the carbon tax is determined to match the desired control rate of emissions for each scenario, and the model endogenously generates out-of-sample forecasts based on the posterior distribution of both the parameters and shocks. The future path of the carbon tax rate was announced in 2023Q3 and expectations were adjusted in response to this new environment. It is important to note that our analysis focuses on climate change mitigation rather than on optimal tax per se.

Figure 3 shows the simulation results. The red line corresponds to the laissez-faire trajectory, which would result in a 4°C increase in the temperature. The green line is associated with a carbon trajectory that would be consistent with temperatures below 2°C above pre-industrial levels. The blue line represents the forecast of the model conditional on the

FIGURE 3. Model-implied projections based on alternative control rates of emissions



Note: This figure displays the projections of the main variables of the New-Keynesian climate model under three scenarios: *(i)* the Paris Agreement (carbon tax consistent with net zero in 2050), *(ii)* laissez faire (no carbon tax), and *(iii)* carbon tax consistent with the forecasts of the estimated model.

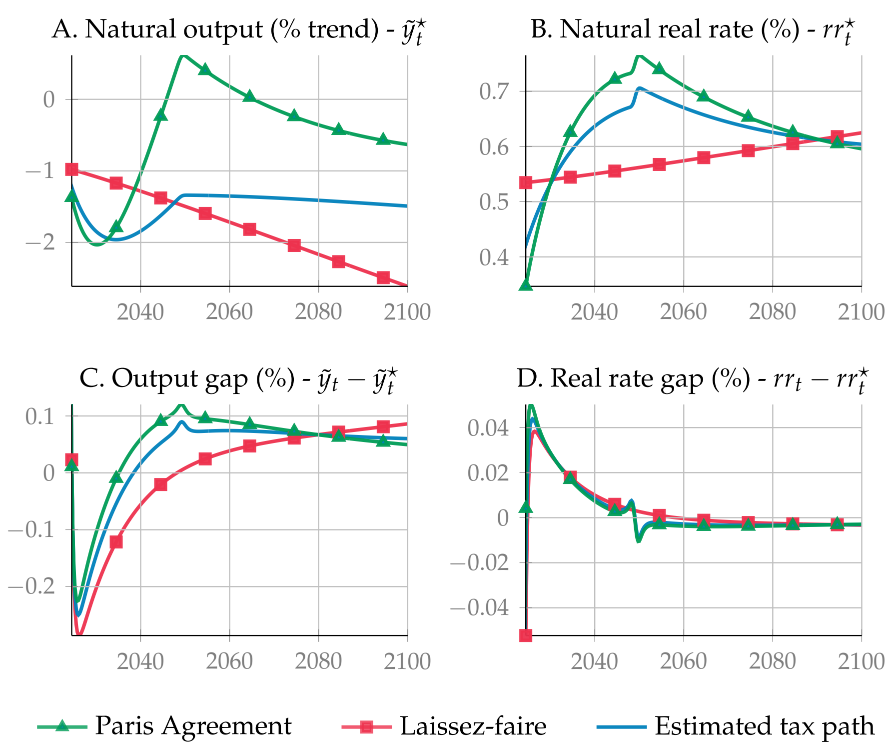
estimated carbon tax process. Under the laissez-faire scenario, in which no emission control measures are implemented, our simulations reveal a significant increase in climate damages over time. Consequently, total factor productivity (TFP) experiences a downward trajectory, reflecting the detrimental impact of climate change on productive resources. This negative effect on productivity leads to a drop in output. This reduction in demand pushes inflation down. Given these negative developments, the central bank reduces interest rates.

Under the Paris Agreement scenario, our simulations demonstrate a markedly different economic landscape. In this scenario, a carbon tax is implemented, which gradually increases and eventually reaches a level of $150 per ton of emissions, which proves to be sufficient for achieving the transition to a net-zero carbon economy. This significant policy intervention boosts the aggregate demand. However, investment-led expansion brings about certain challenges. One notable consequence is the emergence of heightened inflationary pressures. The joint combination of the rising carbon tax and output expansion contributes to a substantial surge in inflation, which we term as "greenflation." This phenomenon reflects a price increase stemming from the cost of transitioning for a sustainable low-carbon economy.

In response to an elevated inflationary environment, monetary policy takes a more restrictive stance. The central bank implements measures to tighten monetary conditions, aiming to temper the investment boom and alleviates inflationary pressures associated with the transition process. By stabilizing the climate, economic damages are stabilized by approximately 1% below the technological trend, while they grow in the laissez-faire economy, continuously depressing output.

4.2 Implications for the natural rate of interest. The natural rate of interest, $rr_t^\star$, is defined here as the real interest rate that would prevail in imperfectly competitive markets but with flexible prices. It serves as a benchmark for central banks to set policy rates and stabilize the economy. Estimates of $rr_t^\star$ guide monetary policy decisions, balancing economic growth and inflation controls under varying economic conditions. Consequently, we assess how the natural interest rate modifies in the wake of green transition scenarios.

FIGURE 4. The natural economy under alternative transition scenarios



Note: This figure displays the projections of the model when prices are flexible under two scenarios: *(i)* the Paris Agreement (carbon tax consistent with net zero in 2050) and *(ii)* laissez faire (no carbon tax). The paths are shown as deviations from the estimated tax scenario.

Figure 4 reports the natural economy (Panels A and B) under the two scenarios (Paris Agreement and laissez faire), together with the gaps between the nominal friction and natural economies. These gaps highlight either an overheated economy or underutilization of resources. A positive gap indicates that the economy is operating above its potential, leading to inflationary pressures, while a negative gap suggests that the economy is below potential, resulting in unemployment and low level of inflation.

We observe that the natural economy is affected by climate change and environmental policies. Thus, monetary policy decisions must be adapted. Indeed, a higher $rr_t^\star$ implies a higher terminal policy rate. Under the Paris Agreement scenario, natural output grows as demand increases, following the increase in abatement expenditures. Because the natural economy is frictionless, natural output responds much faster than in the nominal friction economy, initially creating a negative output gap, followed by a small positive one after 2035. The natural rate responds to the initial tax shift by increasing. The demand effects from abatement expenditures push $rr_t^\star$ upward, and the real rate adjusts so that the gap between the two converges to zero. The economy converges quite quickly towards natural levels because the model does not incorporate endogenous mechanisms, such as habit formation and price indexation, which could slow down the propagation of the effects associated with carbon tax and the flow of carbon emissions.

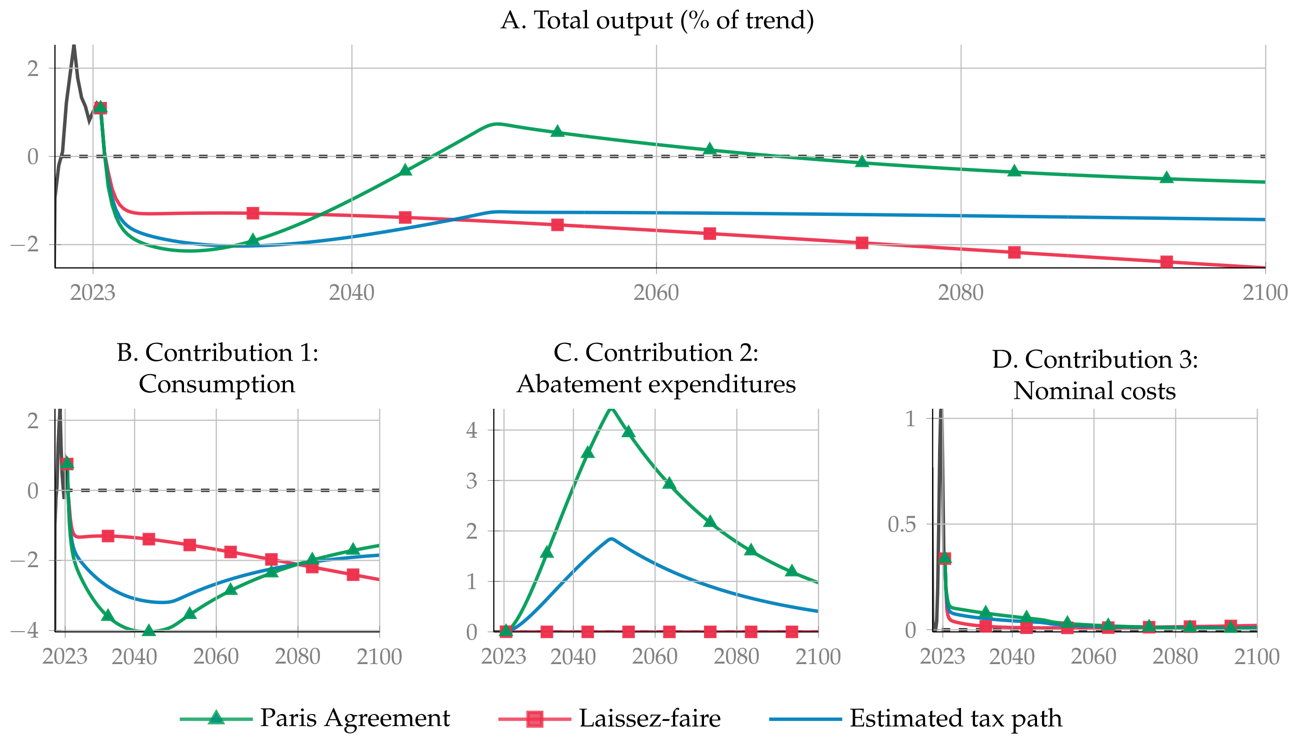
4.3 Decomposing output. In this section we examine the various forces that influence total output. The logarithm of the detrended output ($\hat{y}_t = \log(\tilde{y}_t/\tilde{y})$) can be approximated as follows:

$$\hat{y}_t \simeq \underbrace{\widehat{IS}_t}_{\text{consumption}} + \underbrace{\theta_{1,t}\tilde{\tau}_{e,t}^{\theta_2/(\theta_2-1)}}_{\text{abatement expenditures}} + \underbrace{(1-\vartheta)\frac{\kappa}{2}(\pi_t - \pi_t^\star)^2 + \vartheta\,(1-mc_t)}_{\text{nominal costs}}. \tag{27}$$

In this expression, three main forces can be disinguished. The term $\widehat{IS}_t$ captures the role of the future product of real interest rates in current **consumption** spending.[8] This is the main channel of the transmission of monetary policy, which is attenuated by discounting within the Euler equation. The second term, **abatement expenditures**, captures the extra spending in abatement equipment that is necessary to reach net zero. The last term, named **nominal costs**, captures how changes in menu cost/price rigidity diverts a fraction of resources by reducing current consumption. In the literature on first-best allocations in New Keynesian models, monetary policy is typically committed to reducing this wedge.

[8] $\widehat{IS}_t = \log(IS_t/IS)$ where $IS_t = \omega d_t + (1-\omega)\left[\omega\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\beta\,(1-\omega)^s\,\varepsilon_{b,t+s}d_{t+s}^{-\sigma_c}\prod_{j=0}^{s}\frac{r_{t+j}}{\pi_{t+1+j}}\right\}\right]^{-1/\sigma_c}$.

FIGURE 5. Decomposition of detrended output during the transition



Note: This figure displays the projections of the detrended output under three scenarios: *(i)* the Paris Agreement (carbon tax consistent with net zero in 2050), *(ii)* laissez faire (no carbon tax), and *(iii)* carbon tax consistent with the forecasts of the estimated model.

Figure 5 displays the decomposition of detrended output into these three complementary components under three scenarios: *(i)* the carbon tax is adjusted to achieve a net-zero emissions target by 2050 (green line), *(ii)* the carbon tax is consistent with the forecasts of the estimated model (blue line), and *(iii)* no carbon tax (red line). In the Paris Agreement scenario (green line), the rise in carbon tax creates a recession leading to output levels approximately 2% below the technological-neutral average (Panel A). This recessionary effect is primarily driven by the real interest rate channel (Panel B). This outcome is attributable to inflation pressures stemming from the implementation of the carbon tax, which leads to greenflation. In response to these inflationary pressures, the central bank strongly increases its policy rate, which in turn reduces demand. By contrast, achieving net zero requires a transformation of production lines and massive investment abatement equipment (Panel C). These expenditures increase proportionally to the carbon tax, eventually reaching a level equivalent to 4.2% of output at the peak of the transition, before gradually decreasing as a result of technological efficiency. Finally, the contribution of nominal rigidities arising from the Rotemberg pricing and exit shock (Panel D) is relatively small compared with the other two forces. We also note that the share is similar across the scenarios. The estimated tax path scenario (blue line) resembles that of the Paris Agreement albeit with a lower amplitude. This is because the carbon tax increases less sharply, as Panel A of Figure 3 shows. Finally, the path under laissez-faire (red line) is characterized by decreasing GDP, which is driven by economic damage from a warming planet. Much of this decline is explained by a continued increase in the real interest rate, as the central bank seeks to contain climateflation. Because no carbon tax is implemented, firms do not abate carbon emissions, making abatement expenditure zero over the century. This is discussed in the following subsection.

4.4 Decomposing inflation. We proceed in the same way to analyze the various forces that influence inflation. The Phillips curve is a highly nonlinear equation. In order to keep track of most nonlinearities in the decomposition, we propose a semi-linearization approach, which allows us to decompose the "inflation gap" ($\hat{\pi}_t = \pi_t - \pi_t^{\star}$) into four drivers:

$$\hat{\pi}_t \simeq \underbrace{\hat{\pi}_t^{w}}_{\text{real wage}} + \underbrace{\hat{\pi}_t^{c}}_{\text{climateflation}} + \underbrace{\hat{\pi}_t^{g}}_{\text{greenflation}} + \underbrace{\hat{\pi}_t^{x}}_{\text{exogenous shocks}}, \tag{28}$$

with

$$\begin{aligned}
\hat{\pi}_t^{j} &= \frac{\xi - 1}{\kappa}\widehat{mc}_t^{j} + \mathbb{E}_t\{\beta_{t+1}^{\pi}\hat{\pi}_{t+1}^{j}\} \quad \text{for } j = \{w, c, g\}, \\
\hat{\pi}_t^{x} &= \frac{\xi - 1}{\kappa}\left(\varepsilon_{p,t} - 1\right) mc_t + \mathbb{E}_t\{\beta_{t+1}^{\pi}\hat{\pi}_{t+1}^{x}\},
\end{aligned}$$

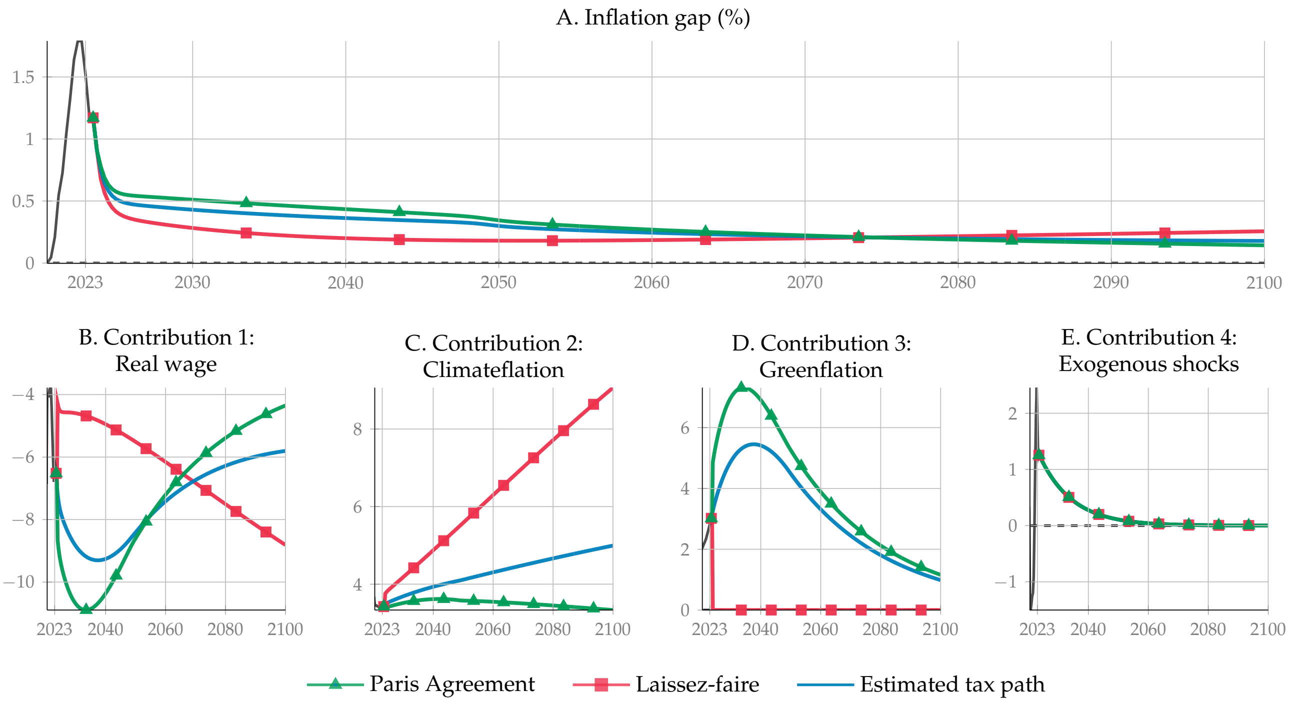
where $\beta_{t+1}^{\pi} = (1 + g_{z,t+1})\,\tilde{y}_{t+1}/\tilde{y}_t$ is the discount factor adjusted by GDP growth, and $\widehat{mc}_t^{j}$ are the elements obtained from the linear approximation of the marginal cost expression:

$$mc_t = \underbrace{\frac{\psi\left(x_t\tilde{y}_t - \omega d_t\right)^{\sigma_c}\tilde{y}_t^{\frac{\sigma_n}{\alpha}}}{\varepsilon_{b,t}\left(1-\omega\right)^{\sigma_c+\sigma_n}}}_{mc_t^{w}}\underbrace{\frac{1}{\Phi\left(\tilde{m}_t\right)^{1+\frac{\sigma_n}{\alpha}}}}_{mc_t^{c}} + \underbrace{\tilde{\tau}_{e,t}\theta_{1,t}\left(\theta_2 + \tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}\left(1-\theta_2\right)\right)}_{mc_t^{g}}$$

The term mc_t^{w} represents the standard component of marginal cost, which is influenced by real wages. The term mc_t^{c} arises from the damage function, which lowers the TFP and raises the marginal cost as carbon emissions intensify. The final term mc_t^{g} pertains to the implementation of a carbon tax, which imposes financial burdens on a firm.

Figure 6 shows the share of each component in the effects on total inflation for the three alternative scenarios (Paris Agreement, laissez faire, and estimated tax path). We observe that the dynamics of the inflation gap are similar between the scenarios, even if the level is lower in the laissez-faire scenario with inflation at 1% annualized by 2050. However, different sources led to this result.

FIGURE 6. Decomposition of inflation during the green transition



Note: This figure displays the projections of the inflation gap (inflation relative to its target) under three scenarios: *(i)* the Paris Agreement (carbon tax consistent with net zero in 2050), *(ii)* laissez faire (no carbon tax), and *(iii)* carbon tax consistent with the forecasts of the estimated model..

Under the laissez-faire scenario, the presence of climate change-related factors significantly affects inflation dynamics. The climateflation term, $\hat{\pi}_t^c$, representing the impact of climate change on inflation through resource scarcity, pushes the inflation gap up to 4% in 2023 and is expected to rise further in the future as resources become increasingly scarce. This scarcity emerges from the damage function, which leads to a decline in total factor productivity (TFP) over time as carbon stock increases. At the same time, this inflationary effect of climate is counterbalanced by a decrease in the standard term of the New Keynesian Phillips curve $\hat{\pi}_t^s$. Indeed, the indirect effect of climate change on the wealth effect of labor supply causes marginal costs and, consequently, inflation to mechanically decrease over time.[9] Note also that the contribution of exogenous cost-push shocks remains small out-of-sample.

In contrast, under the Paris Agreement scenario, inflation is relatively higher than that in the laissez-faire scenario. This surge in inflation is primarily driven by the greenflation term,

[9]Our decomposition exercise relies on a nonlinear model in which cross-products are not washed out by the linearization phase. Therefore, it should be noted that the factors are not orthogonal to $cov\left(\hat{\pi}_t^g, \hat{\pi}_t^w\right) \neq 0$.

which represents the effect of the carbon tax on firms' pricing and increases their production costs. In particular, aggressive carbon taxation increases inflation by 7 pp above its average by 2030. The implementation of a carbon tax leads to a relatively larger cost of production for firms, contributing to an overall increase in inflation. The gain of such a policy is to stabilize the climate, which stabilizes the climateflation term to remain approximately 3%.

5 Sensitivity analysis

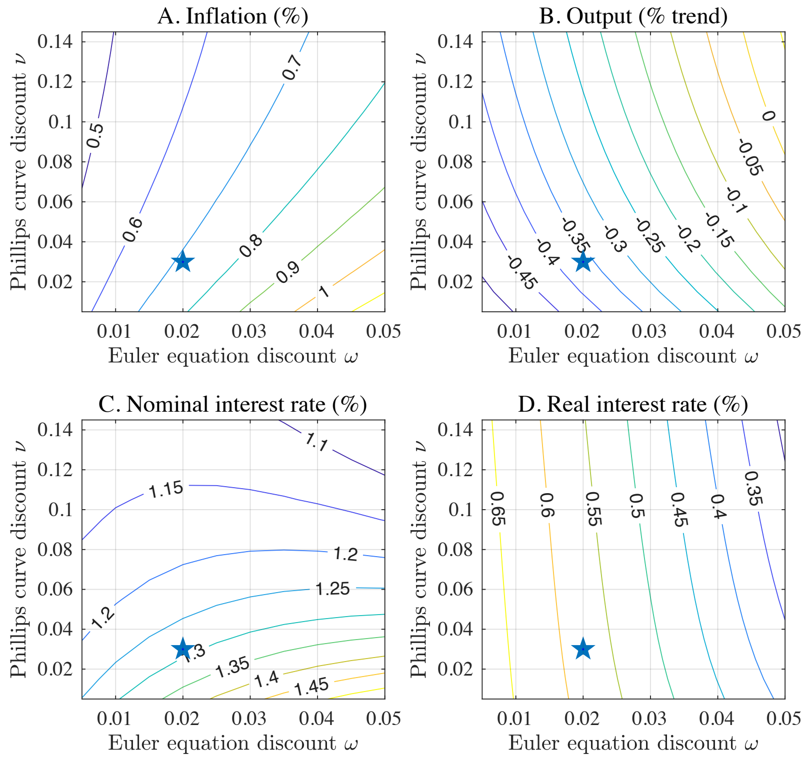
In this section, we conduct a sensitivity analysis to examine the effects of various structural parameters on inflation and output dynamics during the green transition. To this end, we vary key factors such as discounting, monetary policy coefficients, climate-related parameters, and the slopes of the aggregate demand and supply curves.

5.1 The role of attenuated expectations. The forward guidance puzzle found in New Keynesian models highlights the issue that the model predicts unrealistically large effects of future policy announcements on current economic outcomes, leading to implausibly strong responses in output and inflation. An announced carbon tax creates a similar effect because it leads to disproportionately large anticipatory changes in current economic behavior, such as investment and consumption, based on the expectation of future policy impacts. Therefore, we examine the sensitivity of our model to this puzzle. Our model comprises two main parameters on the Euler equation and Phillips curve that attenuate the effect of forward real rates and marginal costs on current outcomes. Figure 7 shows the average values of inflation, output, interest rate, and real rate under the alternative attenuation levels.

Attenuation by more intense discounting of the future marginal utilities of consumption tends to increase inflation. By weighting future high real interest rates relatively less, households tend to consume more during the transition, which translates into higher inflation through the demand effect. In contrast, greater discounting makes the Phillips curve less sensitive to future increases in the carbon tax, resulting in relatively lower inflation with the degree of attenuation. Because the economy is less inflationary, it yields a higher output on average during the transition.

5.2 The Taylor rule. What are the main implication of dovish against hawkish central bank for the transition? To answer this question, we vary the coefficients of the Taylor rule by

FIGURE 7. Sensitivity to discount parameters

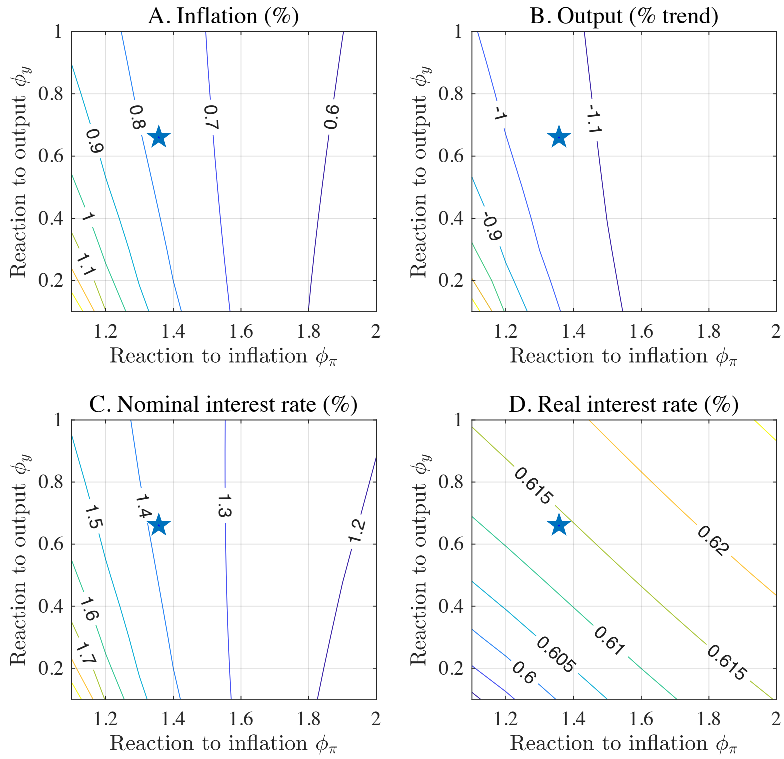


Note: This figure displays the average value between 2023Q4 and 2050Q1 of inflation, output, and the nominal and real interest rates under the Paris Agreement scenario. The star represents the outcome when the parameters are set to their estimated values.

exploring a relatively higher coefficient on inflation $\phi_\pi \in [1.15, 2]$ and output $\phi_y \in [0.1, 1]$. Figure 8 reports the outcome.

We obtain the typical stabilization mechanisms: an increase in inflation (resp. output gap) coefficient reduces average inflation (resp. output gap) during the transition. However, the effects are limited, as an increase in the coefficient does not necessarily yield a substantial decrease in the average value of the targeted variable. This suggests that the usual trade-off between output and inflation, as discussed by Clarida et al. (1999) and Woodford (2003), does not emerge strongly. A subsequent section further discusses this aspect. It is also noteworthy that the divine coincidence principle, characterized by a situation in which stabilizing inflation also naturally stabilizes the output gap, does not hold. Indeed, an increase in the output gap coefficient does not reduce inflation, suggesting that the transition is similar to a supply-side phenomenon.

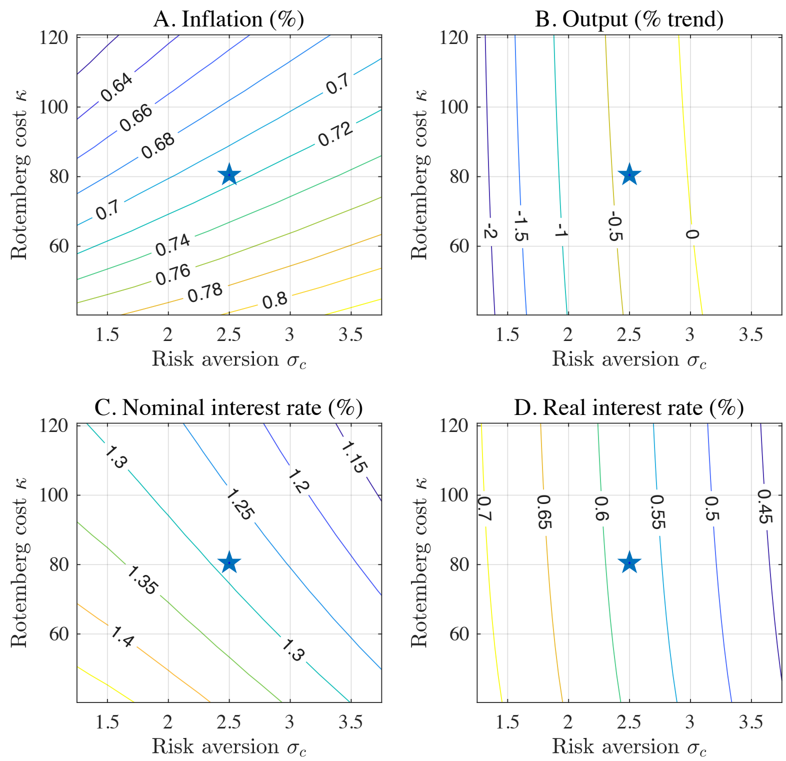
FIGURE 8. Sensitivity to the parameters of the monetary policy rule



Note: This figure displays the average value between 2023Q4 and 2050Q1 for inflation, output, and the nominal and real interest rates under the Paris Agreement scenario. The star represents the outcome when the parameters are set to their estimated values.

5.3 Slopes of aggregate demand and supply curves. Recent literature, such as Hazell et al. (2022), has shown that the New Keynesian Phillips curve has been relatively flat since the 1980s. However, the recent surge in inflation has led to a substantial revision of the price-setting mechanism to accommodate the observed increase in inflation following the Ukrainian war, resulting in a much steeper New Keynesian Phillips curve (e.g., Harding et al., 2023, Benigno and Eggertsson, 2023). Thus, we assess the sensitivity of our results to adjustments in the slope of the Phillips curve, by altering the Rotemberg coefficient from 50 to 120. Similarly, we investigate the role of the risk aversion coefficient in the household utility function. In a New Keynesian framework, higher risk aversion dampens the transmission mechanism of monetary policy by making households less responsive to changes in interest rates, thereby reducing their impact on consumption and savings decisions. We explore parameter values starting from 1.4, as used in Smets and Wouters (2007) and Nordhaus (2017), up to higher values common in asset pricing models. The results are shown in Figure 9.

FIGURE 9. Sensitivity to the slopes of demand and supply curves

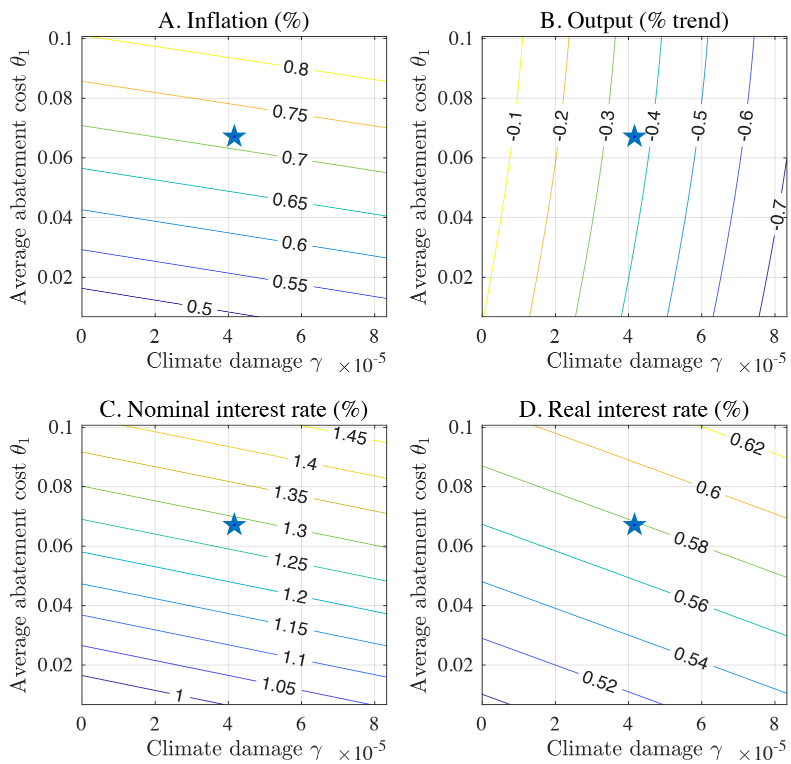


Note: This figure displays the average value between 2023Q4 and 2050Q1 for inflation, output, and the nominal and real interest rates under the Paris Agreement scenario. The star represents the outcome when the parameters are set to their estimated values.

We find that the degree of nominal rigidities reduces the response to inflation during the transition, but is not significant enough to change the overall outcome. This finding suggests that the specific degree of nominal rigidities in the model does not play a critical role in driving the inflation dynamics associated with the transition. In contrast, the risk aversion parameter, which governs the responsiveness of aggregate demand to future interest rates, plays a much more important role in driving inflation and output during the transition. By increasing the risk aversion coefficient, the desire for consumption smoothing increases, as households prefer a more stable consumption path over time to avoid the uncertainty associated with fluctuating consumption levels. Consequently, consumption is less sensitive to real interest rates. This parameter is particularly critical for determining output during the transition, as relatively high risk aversion reduces the contractionary IS effect of monetary policy. An expansion is feasible if output is relatively inelastic to the real rate, allowing abatement expenditure to dominate.

5.4 Macro-climate parameters Finally, we investigate the sensitivity of the main variables of interest to the macro-climate parameters. Specifically, Figure 10 reports the implications of varying damage parameter γ and abatement cost θ_1 on the outcome.

FIGURE 10. Sensitivity to macro-climate parameters



Note: This figure displays the average value between 2023Q4 and 2050Q1 for inflation, output, and the nominal and real interest rates under the Paris Agreement scenario. The star represents the outcome when the parameters are set to their estimated values.

To interpret γ, in the baseline scenario with no mitigation policy, a carbon stock of 1,700 gigatons generates an approximately 6.83 percent TFP loss. We explore damage parameters ranging from no climate damage to 13 percent TFP loss. For the abatement parameter, this can be interpreted as the percentage of GDP spent on average to reach net zero. In our baseline simulation, it requires about 6.5 percent of GDP to decarbonize the economy, but we explore higher abatement costs of up to 10 percent of GDP.

We find that inflation tends to increase in response to climate damage and abatement costs. This observation aligns with the earlier discussion on the effects of greenflation and climateflation on inflation. The implementation of carbon taxes and the impact of climate change on production costs contribute to inflationary pressures. Abatement costs are relatively more important in driving the cost of the transition, because higher abatement costs increase the marginal cost of production for firms, which translates into more inflation through the greenflation channel discussed earlier. Similarly, an increased damage parameter boosts the climateflation terms, which deteriorates output as the real interest rates increase. The damage parameter is essential for driving output during the transition.

5.5 Carbon tax path

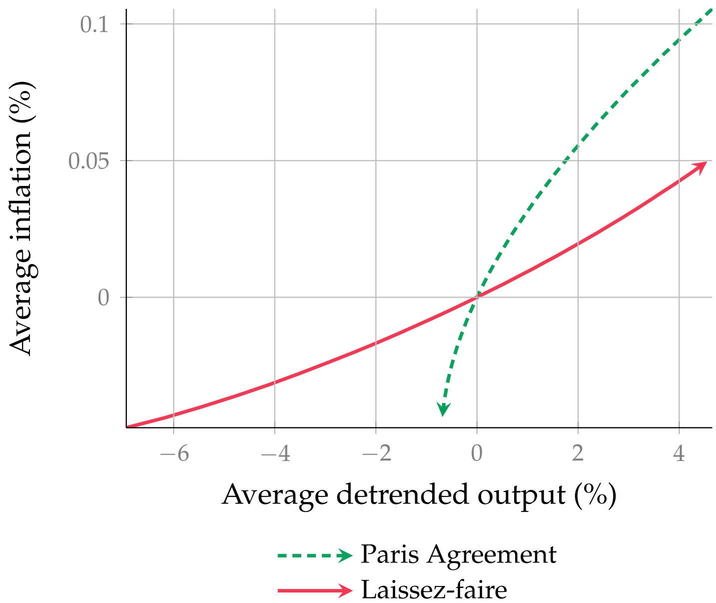
6 IMPLICATIONS FOR THE CENTRAL BANK

6.1 Managing the structural changes associated with climate change and environmental policies. We approximate the set of central bank strategies by exploring alternative values of $\phi_y \in [0, 1]$. This parameter modifies the relative weight of the output gap versus inflation in the operational conduct of monetary policy.

Figure 11 displays the average inflation and output gap gains for 2024-2050 for alternative parameter values of ϕ_y. The arrows indicate the direction of movement as the relative weight of the output gap in the policy rule increases. We observe that the two scenarios (Paris Agreement and laissez faire) exhibit substitutions between output and inflation gains that can be expected along the two transition paths. However, an increase in the output gap stance leads to the opposite effects under these scenarios. Under the Paris Agreement scenario, an increase in abatement spending fuels an inflationary boom. A central bank aiming to reduce inflation related to the structural transition over this period will achieve this at the cost of reduced output expansion. Cooling the boom leads to reduced inflation, similar to the response in the textbook New Keynesian model, which faces a persistent demand shock.

In contrast, under the laissez-faire scenario, climateflation generates a persistent source of inflation that forces the central bank to strike a balance between output and inflation gains. Damage represents a 2% permanent total factor productivity (TFP) loss, and the central bank decides whether these damages materialize in real or nominal terms. An increase in damage creates climateflation. Under a low stance on the output gap, the central bank focuses on inflation stabilization, particularly climateflation, and increases the real interest rate through the Taylor principle, thereby worsening the recession as the planet warms. Conversely, a dovish central bank that does not commit to reducing inflation will see a relatively small increase in the real rate, leading to a reduced cost in terms of output, albeit with more inflation.

FIGURE 11. Average macroeconomic gains under alternative output gap weight in the Taylor rule (*relative to the estimated weight*)

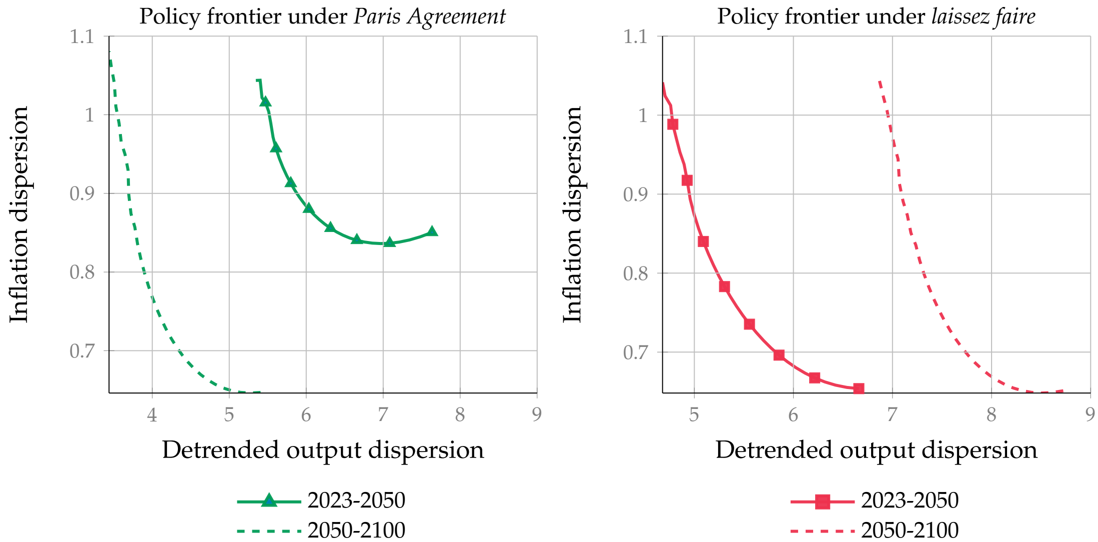


Note: This figure displays the macroeconomic effects in terms of average inflation (y-axis) and detrended output (x-axis) over 2024-2050 for alternative parameter values ϕ_y under two scenarios: *(i)* Paris Agreement (green) and *(ii)* laissez-faire (red). The gains are shown relative to those obtained with the estimated weight on the output gap in the Taylor rule. The arrows indicate the direction of movement as the relative weight of the output gap increases. These effects are obtained in the model without shocks.

6.2 Managing structural changes and the business cycle. The central bank would like to address both the structural change and usual business cycle component (emanating from shocks). As illustrated in Figure 11, the usual tradeoff between inflation and output stabilization emerges. We now analyze the inflation-output variability frontier according to the horizon: *(i)* the medium run (i.e., during the transition, from 2024 to 2050) and *(ii)* the long run (i.e., after the transition, between 2050 and 2100). Figure 12 shows the two frontiers for the two transition scenarios (*Paris Agreement* and *laissez faire*).

In the medium run, the Paris Agreement scenario presents a less favorable trade-off compared to the laissez-faire scenario, which is primarily characterized by relatively higher inflation. The increase in abatement spending required to mitigate climate change in accordance with the Paris Agreement results in an inflationary boom. In this context, the central bank faces a challenging decision: reducing inflation related to structural transition but only at the cost of limiting output expansion. This cooling down of the economy mirrors the New Keynesian textbook's response to a persistent demand shock. Essentially, saving the planet

FIGURE 12. Inflation-output variability frontier under alternative output gap weight in the Taylor rule



Note: This figure displays the dispersion of inflation (y-axis) and the dispersion of detrended output (x-axis) for alternative parameter values ϕ_y under two scenarios: *(i)* the Paris Agreement (green) and *(ii)* laissez-faire (red). The inflation and output dispersions are defined as $\mathbb{E}\{(\pi_t - \pi_t^\star)^2\})$ and $\mathbb{E}\{(\tilde{y}_t - \tilde{y})^2\}$, respectively.

induces an immediate economic cost that manifests as higher inflation and constrained output growth.

Despite the medium-run costs, the Paris Agreement scenario offers significant long-term benefits. By addressing climate change proactively, this approach aims to stabilize the economy with lower dispersion in inflation and output fluctuations in the future. The structural changes implemented during the transition period lay the groundwork for a more stable economic environment post-2050. The initial investment in abatement spending and the corresponding inflationary pressures are counterbalanced by reduced volatility in the long run, making medium-term sacrifices worthwhile for long-term stability.

By contrast, the laissez-faire scenario postpones climate-mitigation efforts, which initially results in relatively lower inflation and fewer constraints on output growth. However, this approach generates higher economic dispersion in the future. By delaying necessary climate actions, the economy faces persistent sources of inflation, known as climateflation, driven by increasing damage from climate change. These damages, representing a 2% permanent loss in total factor productivity (TFP), force the central bank to balance between stabilizing inflation and maintaining output levels. In the long run, the central bank's efforts to mitigate climateflation result in increased volatility in both inflation and output, creating a more unstable economic environment post-2050.

6.3 On the design of the monetary policy rule. The New Keynesian framework and its policy measures are designed based on the assumption that the economy will revert to a steady state. Hence, monetary policy rules are typically built in deviations from long-term fundamentals, such as output and interest rates. The presence of climate and the structural transformation involved in the transition shake the usual conception of monetary policy built into deviation from the long-term observable average. As highlighted by Orphanides (2002), the misperception of long-term outcome can have detrimental implication for the effectiveness of monetary policy. To gauge the importance of structural factors in the monetary policy rule for the determination of equilibrium, the monetary policy rule can be expressed as follows:

$$\varsigma_{r,t} = \varsigma_{r,t-1}^{\rho} \left[\frac{\pi_t^\star}{\pi} \left(\frac{\pi_t}{\pi_t^\star} \right)^{\phi_\pi} \varsigma_{y,t}^{\phi_y} \right]^{1-\rho} \varepsilon_{r,t}. \tag{29}$$

Notice that the estimated rule is the one where:

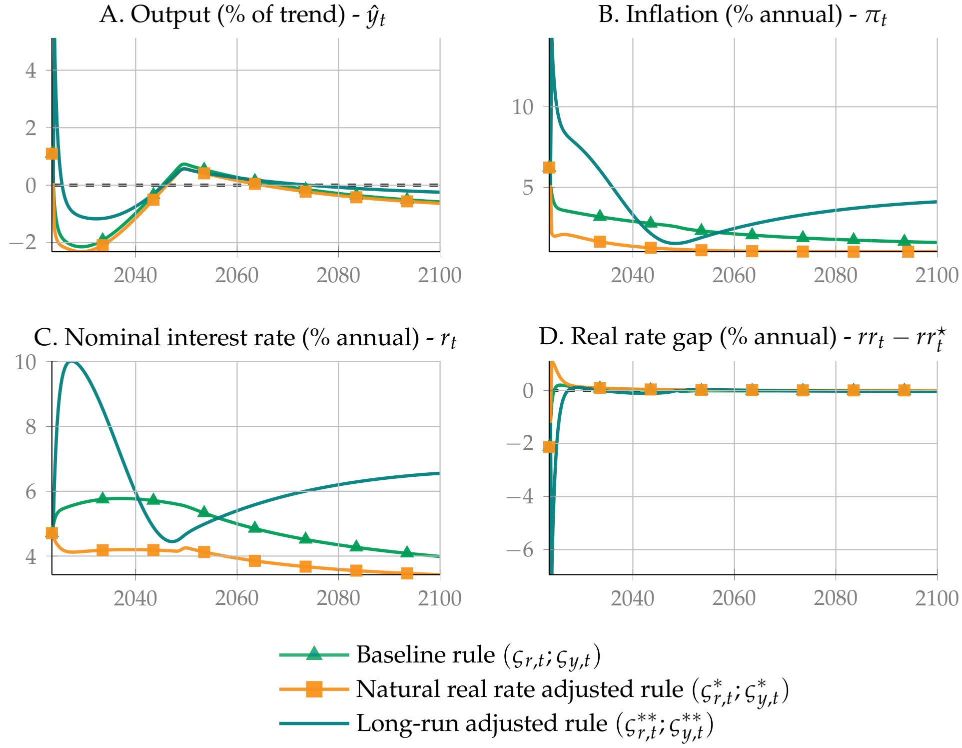
$$\varsigma_{r,t} = \frac{r_t}{r} \quad \text{and} \quad \varsigma_{y,t} = \frac{y_t}{y_t^\star}.$$

This monetary policy rule implies that the long-term value of the nominal interest rate is fixed. However, the long-term determinants of the economy, such as productivity growth and population, may affect the natural rate of interest (Laubach and Williams, 2003, Holston et al., 2017). Consequently, using a steady-state interest rate as a reference to set the current policy rate may be misleading, leading more inflation in a manner similar to that in the 1970s (Orphanides, 2002).

An alternative approach would be to follow a monetary policy rule that adjusts for deviations from the natural rate of interest. This approach allows the central bank to set the policy rate based on a reference level consistent with the economy's structural drivers, which comprise both long-term changes from damages and green transition. This operational rule can be represented by:

$$\varsigma_{r,t}^* = \frac{r_t}{r_t^\star \pi_t^\star} \quad \text{and} \quad \varsigma_{y,t}^* = \frac{\tilde{y}_t}{\tilde{y}_t^\star}.$$

FIGURE 13. A transition under alternative monetary policy rules



Note: This figure displays the path of the key variables consistent with the Paris Agreement scenario under *(i)* the estimated policy rule (green), *(ii)* a policy rule adjusted with the natural real rate (orange), and *(iii)* a policy rule in which the variables deviate from their steady-state levels.

Another option is to use the Taylor rule, which considers the steady state of the economy as a reference, as in Woodford (2003)'s textbook. This rule imposes that:

$$\varsigma_{r,t}^{**} = \frac{r_t}{r} \quad \text{and} \quad \varsigma_{y,t}^{**} = \frac{\tilde{y}_t}{\tilde{y}}.$$

Among these three rules, the most efficient in stabilizing output and inflation is adjusted by the natural level of the economy. Indeed, this rule is more relevant because the natural rate of interest and the natural rate of output provide benchmarks for assessing the stance of monetary policy. By anchoring policies to these natural rates, a central bank can better stabilize the economy by aligning the actual output and inflation with their natural levels. This approach helps to avoid systematic policy errors that can arise from relying on real-time estimates that might be inaccurate, thus improving the effectiveness and credibility of monetary policy. By contrast, if the central bank does not adjust to the structural changes in the economy and relies on the classic steady-state adjusted rule, it constantly overestimates the potential of the economy. The output gap is systematically negative, leading to an accommodative monetary policy, which, in turn, causes a dramatic rise in inflation during the transition.

6.4 The effects of forgetting climate aspects in the monetary policy rule

7 Conclusion

This paper has developed and analyzed a New Keynesian model that considers the trade-off between the cost of mitigation and the cost of climate change. By extending the traditional New Keynesian framework to include abatement costs, climate externalities, and carbon stock dynamics, this paper provides a comprehensive framework to understand the interactions between environmental policies and macroeconomic outcomes. Our empirical analysis, based on Bayesian techniques and fully nonlinear methods provides a data grounded quantitative analysis of the transition to a net-zero carbon economy.

Our framework can rationalize either green or climateflation mechanisms by adjusting the carbon tax. The tractability of the NKC model allows the decomposition of the relative forces by which climate change and climate mitigation policies affect the demand and supply sides of the economy. In a laissez-faire economy, the increasing damage to TFP creates a long and lasting recession, similar to a permanent supply shock that fuels inflation and makes output below its technological trend. In contrast, in the wake of a net zero transition, the rise in the production cost of firms boosts inflation, which combined with an increase in green investment, creates an economic expansion. This alternative macroeconomic era is usually referred to as *greenflation* and exhibits features similar to a persistent demand shock. From a central bank policy making perspective, this shock is much easier to manage than climateflation as it does not involve the usual sacrifice between price and quantity stabilization.

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INTERNET APPENDIX

(not for publication)

A FULL MODEL

Our model includes four core equations and variables $\{\tilde{y}_t, \pi_t, r_t, \tilde{m}_t\}$:

$$\left(\frac{x_t\tilde{y}_t - \omega d_t}{1-\omega}\right)^{-\sigma_c} = \beta\mathbb{E}_t\left\{\frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}}\frac{r_t}{\pi_{t+1}}\left((1-\omega)\left(\frac{x_{t+1}\tilde{y}_{t+1} - \omega d_t}{1-\omega}\right)^{-\sigma_c} + \omega d_t^{-\sigma_c}\right)\right\} \tag{A.1}$$

$$(\pi_t - \pi_t^\star)\,\pi_t = (1-\vartheta)\beta\mathbb{E}_t\left\{(1+g_{z,t+1})\frac{\tilde{y}_{t+1}}{\tilde{y}_t}\left(\pi_{t+1} - \pi_{t+1}^\star\right)\pi_{t+1}\right\} + \frac{\zeta}{\kappa}\varepsilon_{p,t}mc_t + \frac{1-\zeta}{\kappa} \tag{A.2}$$

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^\rho\left[\left(\frac{\pi_t^\star}{\pi}\right)\left(\frac{\pi_t}{\pi_t^\star}\right)^{\phi_\pi}\left(\frac{\tilde{y}_t}{\tilde{y}_t^n}\right)^{\phi_y}\right]^{1-\rho}\varepsilon_t^r \tag{A.3}$$

$$\tilde{m}_t = (1-\delta_m)\tilde{m}_{t-1} + \xi_m\sigma_t\left(1 - \tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}\right)z_t l_t \tilde{y}_t \varepsilon_{e,t} \tag{A.4}$$

Our model also includes auxiliary variables:

$$x_t = 1 - (1-\vartheta)\frac{\kappa}{2}\left(\pi_t - \pi_t^\star\right)^2 - \theta_{1,t}\tilde{\tau}_{e,t}^{\theta_2/(\theta_2-1)} - \vartheta(1-\epsilon_{p,t}mc_t) \tag{A.5}$$

$$mc_t = \frac{\psi}{\epsilon_{b,t}\,(1-\omega)^{\sigma_c+\sigma_n}}\frac{(x_t\tilde{y}_t - \omega d_t)^{\sigma_c}\,\tilde{y}_t^{\sigma_n}}{\Phi\left(\tilde{m}_t\right)^{1+\sigma_n}} + \theta_{1,t}\tilde{\tau}_{e,t}\left[\theta_2 + (1-\theta_2)\,\tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}\right] \tag{A.6}$$

where $\tilde{y}_t = y_t/(z_t l_t)$, $\tilde{\tau}_{e,t} = \tau_{e,t}\sigma_t\varepsilon_{e,t}/(\theta_2\theta_{1,t})$, and $\tilde{m}_t = m_t - m_{1750}$.

It also comprises five trend related deterministic processes:

$$\sigma_t = \sigma_{t-1}(1 - g_{\sigma,t}) \tag{A.7}$$

$$g_{\sigma,t} = (1-\delta_\sigma)\,g_{\sigma,t-1} \tag{A.8}$$

$$z_t = z_{t-1}(1 + g_{z,t}) \tag{A.9}$$

$$g_{z,t} = g_{z,t-1}\,(1-\delta_z) \tag{A.10}$$

$$\theta_{1,t} = (p_b/\theta_2)(1-\delta_{pb})^{t-t_0}\sigma_t \tag{A.11}$$

$$l_t = l_{t-1}\,(l_T/l_{t-1})^{\ell_g} \tag{A.12}$$

And four stochastic processes:

$$\varepsilon_{b,t} = (1-\rho_b) + \rho_b\varepsilon_{b,t-1} + \eta_{b,t}$$

$$\varepsilon_{p,t} = (1-\rho_p) + \rho_p\varepsilon_{p,t-1} + \eta_{p,t}$$

$$\varepsilon_{e,t} = (1-\rho_e) + \rho_e\varepsilon_{e,t-1} + \eta_{e,t}$$

$$\pi_t^\star = (1-\rho_{\pi^\star})\,\pi + \rho_{\pi^\star}\pi_{t-1}^\star + \eta_{\pi^\star,t}$$

B Math derivations

Demand part. Detrended Euler equation reads as:

$$\tilde{\lambda}_t = \mathbb{E}_t\left\{\beta\frac{r_t}{\pi_{t+1}}\left((1-\omega)\tilde{\lambda}_{t+1}+\omega\varepsilon_{b,t+1}d_t^{-\sigma_c}\right)\right\}. \tag{B.13}$$

It can be rewritten as:

$$\begin{aligned}\tilde{\lambda}_t &= \mathbb{E}_t\left\{R_t\left((1-\omega)\tilde{\lambda}_{t+1}+\omega\varepsilon_{b,t+1}d_t^{-\sigma_c}\right)\right\}\\ &= \omega\mathbb{E}_t\left\{\textstyle\sum_{s=0}^{\infty}(1-\omega)^s\varepsilon_{b,t+s}d_{t+s}^{-\sigma_c}\prod_{j=0}^{s}R_{t+j}\right\},\end{aligned}$$

where $R_t = \beta r_t/\pi_{t+1}$.

Recall that: $\tilde{\lambda}_t = \varepsilon_{b,t}\left(\frac{\tilde{c}_t-\omega d_t}{1-\omega}\right)^{-\sigma_c}$, the Euler equation becomes:

$$\left(\frac{c_t/z_t-\omega d_t}{(1-\omega)}\right)^{-\sigma_c} = \omega\mathbb{E}_t\left\{\textstyle\sum_{s=0}^{\infty}(1-\omega)^s\varepsilon_{b,t+s}d_{t+s}^{-\sigma_c}\prod_{j=0}^{s}R_{t+j}\right\}$$

which can be rewritten as:

$$c_t/z_t = IS_t,$$

where $IS_t = \omega d_t + (1-\omega)\left[\omega\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\beta(1-\omega)^s\varepsilon_{b,t+s}d_{t+s}^{-\sigma_c}\prod_{j=0}^{s}\frac{r_{t+j}}{\pi_{t+1+j}}\right\}\right]^{-1/\sigma_c}$.

In addition, we know that:

$$IS_t = c_t/z_t = x_t y_t/(z_t l_t),$$

where $x_t = 1-(1-\vartheta)\frac{\kappa}{2}(\pi_t-\pi_t^{\star})^2 - \theta_{1,t}\tilde{\tau}_{e,t}^{\theta_2/(\theta_2-1)} - \vartheta(1-\epsilon_{p,t}mc_t)$, with $\mu_t = \tilde{\tau}_{e,t}^{1/(\theta_2-1)}$.

As $c_t = x_t y_t/l_t$, it comes:

$$IS_t = x_t\tilde{y}_t.$$

Therefore, applying the logarithm yields:

$$\hat{y}_t \simeq \widehat{IS}_t + \theta_{1,t}\tilde{\tau}_{e,t}^{\theta_2/(\theta_2-1)} + (1-\vartheta)\frac{\kappa}{2}(\pi_t-\pi_t^{*})^2 + \vartheta(1-\epsilon_{p,t}mc_t),$$

with $\hat{y}_t = \log(\tilde{y}_t/\tilde{y})$ and $\widehat{IS}_t = \log(IS_t/IS)$.

Marginal cost. The marginal cost is given by:

$$mc_t = \frac{w_t}{\Gamma_t} + \theta_{1,t}\mu_t^{\theta_2} + \tau_{e,t}\sigma_t(1-\mu_t)\varepsilon_{e,t}$$

Let us consider the real wage of the high productive worker $w_t = \psi_t n_t^{\sigma_n}/\lambda_t$, the general equilibrium condition $(1-\omega)n_t = n_t^d = N_t$ and the production function $y_t = l_t\Gamma_t N_t^{\alpha}$, we get:

$$mc_t = \frac{1}{\lambda_t\Gamma_t}\psi_t\left(\left(\frac{y_t}{l_t\Gamma_t}\right)^{\frac{1}{\alpha}}\frac{1}{1-\omega}\right)^{\sigma_n} + \theta_{1,t}\mu_t^{\theta_2} + \tau_{e,t}\sigma_t(1-\mu_t)\varepsilon_{e,t}$$

Recall that $\Gamma_t = \Phi(\tilde{m}_t)z_t$ and $\tilde{y}_t = y_t/(l_t z_t)$, thus:

$$mc_t = \frac{1}{\lambda_t\Phi(\tilde{m}_t)}\psi z_t^{-\sigma_c}\left(\left(\frac{\tilde{y}_t}{\Phi(\tilde{m}_t)}\right)^{\frac{1}{\alpha}}\frac{1}{1-\omega}\right)^{\sigma_n} + \theta_{1,t}\mu_t^{\theta_2} + \tau_{e,t}\sigma_t(1-\mu_t)\varepsilon_{e,t}$$

Next, replacing λ_t by its expression in function of $\tilde{c}_t$ gives:

$$mc_t = \frac{\psi}{\varepsilon_{b,t}\left(\frac{\tilde{c}_t-\omega d_t}{1-\omega}\right)^{-\sigma_c}\Phi\left(\tilde{m}_t\right)}\left(\left(\frac{\tilde{y}_t}{\Phi\left(\tilde{m}_t\right)}\right)^{\frac{1}{\alpha}}\frac{1}{1-\omega}\right)^{\sigma_n}+\tilde{\tau}_{e,t}\theta_{1,t}\left(\theta_2+\tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}\left(1-\theta_2\right)\right).$$

Finally,

$$mc_t = \frac{\psi}{\left(1-\omega\right)^{\sigma_c+\sigma_n}}\frac{\left(x_t\tilde{y}_t-\omega d_t\right)^{\sigma_c}\tilde{y}_t^{\frac{\sigma_n}{\alpha}}}{\varepsilon_{b,t}\Phi\left(\tilde{m}_t\right)^{1+\frac{\sigma_n}{\alpha}}}+\tilde{\tau}_{e,t}\theta_{1,t}\left(\theta_2+\tilde{\tau}_{e,t}^{\frac{1}{\theta_2-1}}\left(1-\theta_2\right)\right).$$

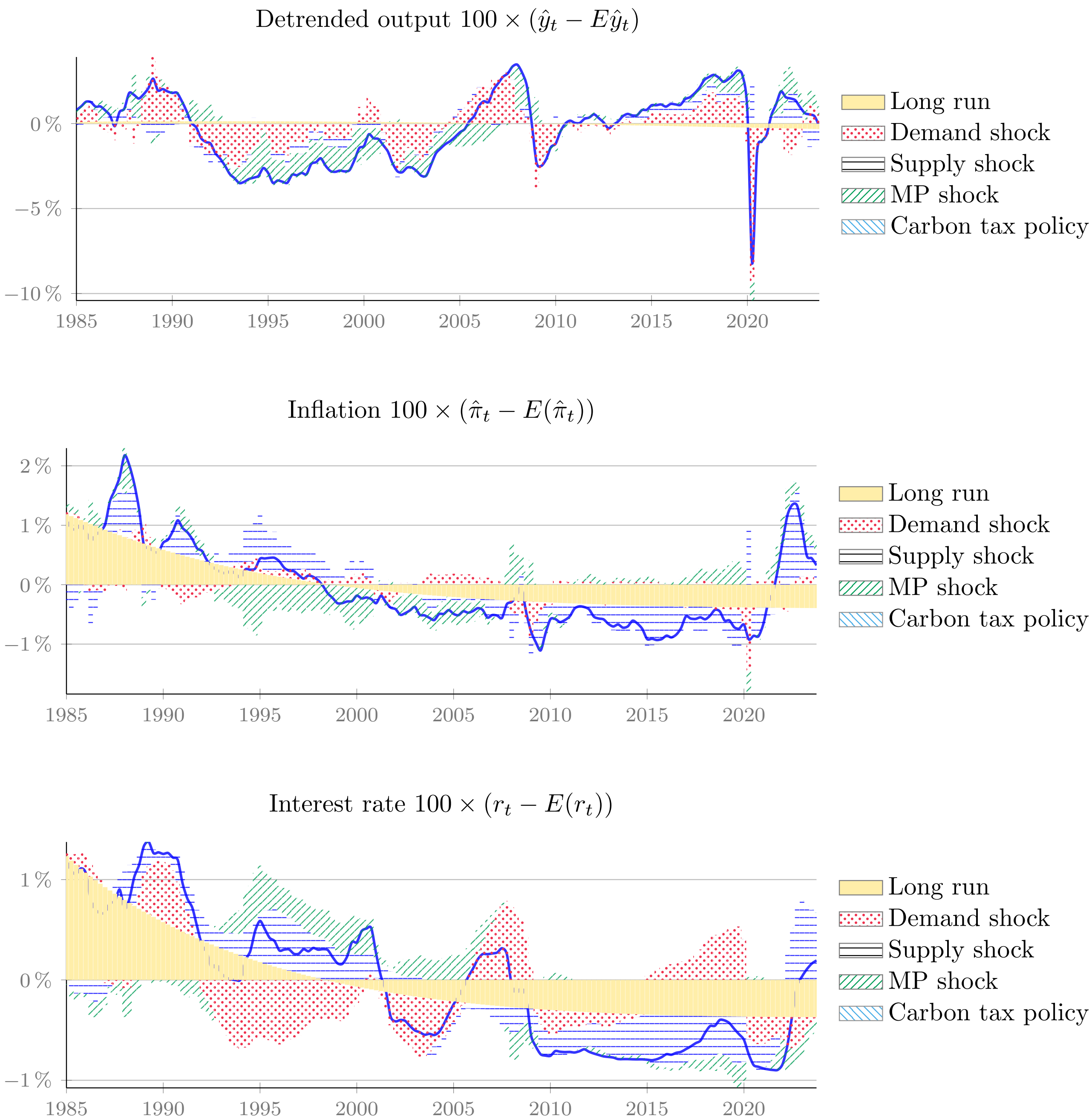
Phillips curve. The Phillips curve is a discounted sum of future marginal costs

$$\pi_t = \frac{\zeta}{\kappa}\mathbb{E}_t\sum_{s=0}^{\infty}\hat{\beta}_{t,t+s}\left[\varepsilon_{p,t+s}mc_{t+s}+\frac{1-\zeta}{\zeta}\right],$$

with $\hat{\beta}_{t,t+s} = \beta^s\frac{y_{t+s}l_t}{y_t l_{t+s}}\frac{1}{\pi_t-\ \pi_t^{\star}}$.

C Historical decomposition

FIGURE 14. Historical decomposition of detrended output, inflation and the nominal interest rate on the sample period



Note: This figure displays the approximate contribution of each shock to the determination of the variable of interest. The cross-products across the contribution of shocks were neglected.