Selling Correlated Information Products

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Running Example: Consulting Services

- Consulting company (e.g. McKinsey) offering consulting services
- Clients differ in **type** (e.g. scale) of investment
- Client's value/size of the project $\equiv$ **willingness to pay** for consultants
- **Trade-off**:
  - high value clients can **downplay investment plan** to avoid higher fees,
  - might get hurt from **imperfect information**/expertise,
  - if **information spillovers** are strong then lying could be profitable

Questions

- How should a provider price these services given information spillovers?
- What are some environments in which common **fee structures** observed in practice are optimal contracts?
    - value-based fees (e.g. consultants charging a % of estimated profits)
    - flat/hourly rates (e.g. course fees)

MODEL

Model

- A monopolist seller (consultant) and a buyer (firm)
- Firm is privately informed about **type** (project's characteristics) $\theta \in \Theta$
  - θ distributed according to $F \in \Delta(\Theta)$
- Project **specific** state $\omega_\theta =$ G(ood) or B(ad)
  - E.g. $(\omega_{\theta_1}, \omega_{\theta_2}, \omega_{\theta_3}) = (B, B, G)$ for $\Theta = \{\theta_1, \theta_2, \theta_3\}$
  - Let $\Omega = \{G, B\}^\Theta$ be the set of states
- Common prior $\mu \in \triangle(\Omega)$
- (Marginal) probability that project θ is good (abusing notation):

$$\mu_\theta \equiv \mathbb{P}_\mu\Big(\omega_\theta = G\Big)$$

Decision Making Under Uncertainty

- Firm takes action a_g (invest) or a_b (don't invest)
- Ex-post payoff from taking $a \in A = \{a_g, a_b\}$:

$u(a, \omega_\theta)$	a_g	a_b
G	$u(\theta)$	0
B	0	$u(\theta)$

- Expected payoff under prior information:

$$\underline{U}(\theta) = \max_{a \in A}\{\mu_\theta, (1 - \mu_\theta)\}u(\theta) \qquad \text{(Outside Option)}$$

Information Design

- Seller can provide **additional** information at *zero* marginal cost
- An **information product** (Blackwell experiment) $E = (S, \pi)$, consists of a (possibly uncountable) set of signals S and signal function

$$\pi : \Omega \to \Delta(S)$$

- Let $U(E, \theta)$ be expected payoff of θ from E
- **Value** (WTP) for information product E is given by

$$V(E, \theta) = U(E, \theta) - \underline{U}(\theta)(\geq 0)$$

Seller's Problem

- Seller posts a **revenue maximizing** menu of $\mathcal{M} = \{\mathcal{E}, t\}$
  - $\mathcal{E}$ is a collection of experiments; tariff $t : \mathcal{E} \to \mathbb{R}_+$
- Seller **commits** ex-ante and state outcomes, actions and signal realizations are **not contractible**
- Simple Case 1: *correlation*$(\omega_\theta, \omega_{\theta'}) = 0 \Rightarrow$ **first degree** price discrimination
  - *horizontal differentiation* aspect of the model
- Simple Case 2: *correlation*$(\omega_\theta, \omega_{\theta'}) = 1 \Rightarrow$ standard **one** (information) good monopoly screening
  - *vertical differentiation* aspect of the model

Main Results

- **Revelation Principle and Simple Menus**
- Two Types (SKIP!)
- Continuum of Types

Revelation Principle

- By Revelation Principle, seller offers direct menu

$$\mathcal{M} = \{E(\theta), t(\theta)\}_\theta$$

- Seller's problem:

$$\max_{\{E(\theta),t(\theta)\}} \int_{\theta\in\theta} t(\theta)dF(\theta) \tag{Obj}$$

$$V(E(\theta),\theta) - t(\theta) \geq V(E(\theta'),\theta) - t(\theta') \tag{$\mathsf{IC}_{\theta,\theta'}$}$$

$$V(E(\theta),\theta) - t(\theta) \geq 0 \tag{IR_θ}$$

Simple Menus

Proposition 1. Seller can restrict without loss of generality to any IC and IR *simple* direct menu such that

i. *Customized* $E(\theta) = (\pi_\theta, S)$: Signal function $\pi_\theta : \Omega_\theta \to S$

ii. *Responsive* $E(\theta) = (\pi_\theta, \{s_g, s_b\})$: $a^*_{s_g,\theta} = a_g$ and $a^*_{s_b,\theta} = a_b$

Simple Menus

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- Represent $E(\theta)$ as

$\Omega_\theta \backslash S$	s_g	s_b
G	$\pi_{g,\theta}$	$1 - \pi_{g,\theta}$
B	$1 - \pi_{b,\theta}$	$\pi_{b,\theta}$

  and impose (ii.) as additional constraint to seller's problem:

$$\underbrace{\mu_\theta \pi_{g,\theta} + (1 - \mu_\theta)\pi_{b,\theta}}_{\text{Probability of success (quality)}} \geq \max\{\mu_\theta, 1 - \mu_\theta\} \qquad (\text{Rsp}_\theta)$$

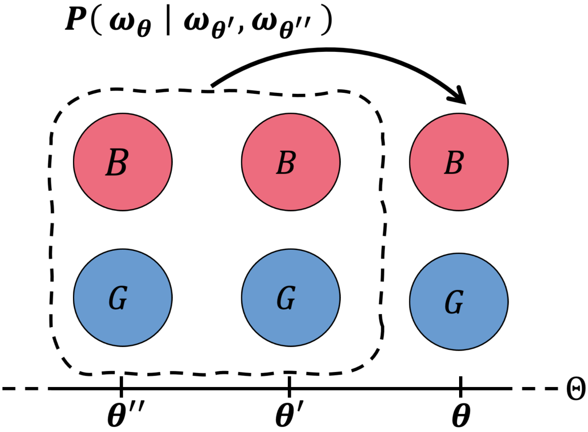
Main Results

- Revelation Principle and Simple Menus
- Two Types (SKIP!)
- **Continuum of Types**

Continuum of Types: Structure of μ

- Consider $\Theta = [0, \overline{\theta}]$ and non-decreasing $u(\cdot)$

(A1.) Markov Property(M)

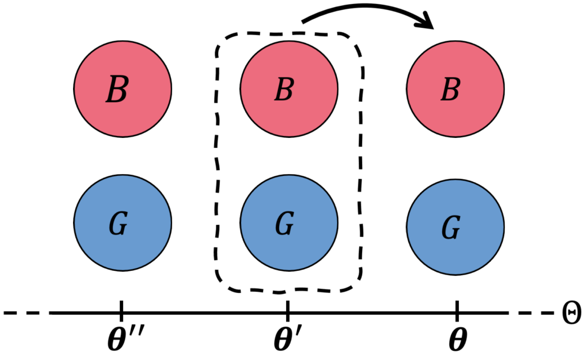


Continuum of Types: Structure of μ

- Consider $\Theta = [0, \overline{\theta}]$ and non-decreasing $u(\cdot)$

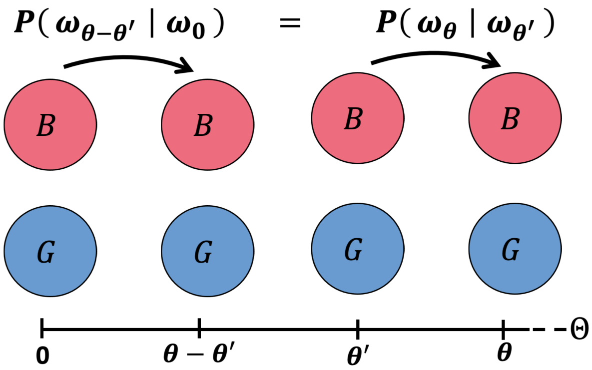
(A1.) Markov Property(M)

$$P(\omega_\theta \mid \omega_{\theta'}, \omega_{\theta''}) = P(\omega_\theta \mid \omega_{\theta'})$$



Continuum of Types: Structure of μ

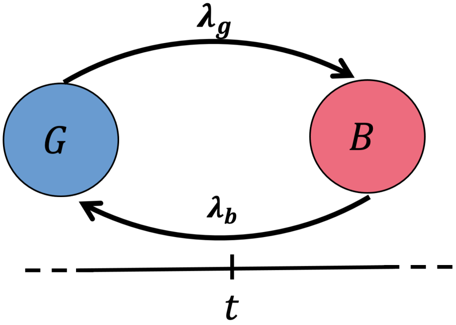
(A2.) Homogeneity(H)



Continuum of Types: Markov Chain

Lemma 1. If μ satisfies (A1) and (A2) $\Rightarrow$ 2-state Markov Chain:

Details



- $P(\Delta) = exp(Q \cdot \Delta)$
- $Q = \begin{pmatrix} -\lambda_g & \lambda_g \\ \lambda_b & -\lambda_b \end{pmatrix}$

- Solve for **steady state**: $\mu_\theta = \mu \in (0,1)$, where

$\mu = \lambda_b/(\lambda_g + \lambda_b)$, and let $\mu \geq 1/2$ w.l.o.g.

Continuum of Types: Full Surplus Extraction

- Suppose seller offers $\bar{E}(\theta)$ and sets price at highest WTP

$$\begin{aligned}\bar{t}(\theta) &= V(\bar{E}(\theta), \theta) \\ &= u(\theta) - \max\{\mu_\theta, 1-\mu_\theta\}u(\theta) \\ &= (1-\mu)u(\theta)\end{aligned}$$

- *Necessary condition*: Local downward deviations $\theta - \Delta$ are not profitable:

$$\underbrace{(1-\mu)\Big(u(\theta) - u(\theta-\Delta)\Big)}_{\bar{t}(\theta)-\bar{t}(\theta-\Delta)\ \textbf{(Marginal gain)}} \quad (?) \quad \underbrace{\left(\mu P_{gb}(\Delta) + (1-\mu)P_{bg}(\Delta)\right)u(\theta)}_{\textbf{(Marginal cost)}}$$

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Continuum of Types: Full Surplus Extraction

Proposition 2. Seller extracts full surplus if and only if

$$(1-\mu)u'(\theta) \leq (\mu\lambda_g + (1-\mu)\lambda_b)u(\theta), \quad \forall\theta \in \Theta. \tag{C1}$$

- Writing $\lambda_b = \mu\lambda_g/(1-\mu)$, re-arrange (C1):

$$u'(\theta) \leq 2\lambda_b u(\theta), \quad \forall\theta \in \Theta$$

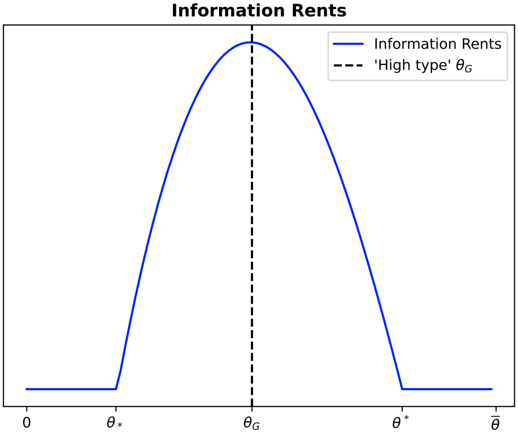
- Suppose (C1) doesn't hold
- Let $u(\cdot)$ be concave and suppose there exists $\theta_G \in (0, \overline{\theta})$ such that

$$u'(\theta_G) = 2\lambda_b u(\theta_G),$$

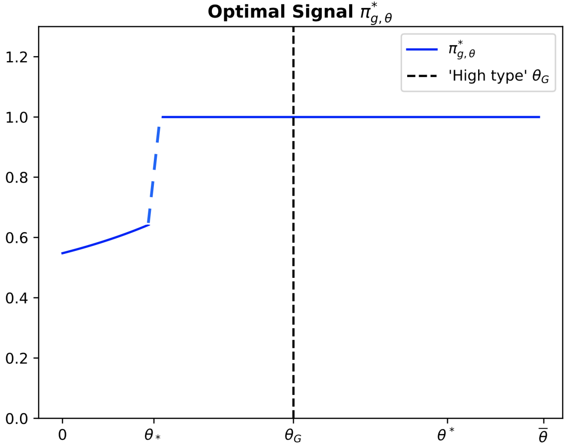
  - Note: $u'(\theta) > 2\lambda_b u(\theta)$ for $\theta < \theta_G$ and $u'(\theta) < 2\lambda_b u(\theta)$ for $\theta > \theta_G$

Continuum of Types: Optimal Menu (Graphs)

- Buyer's surplus $V(E^*(\theta), \theta) - t^*(\theta)$

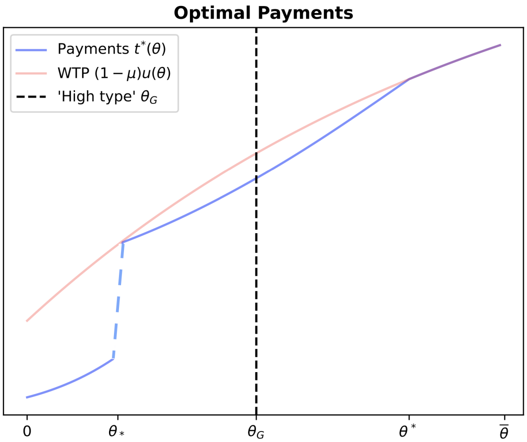


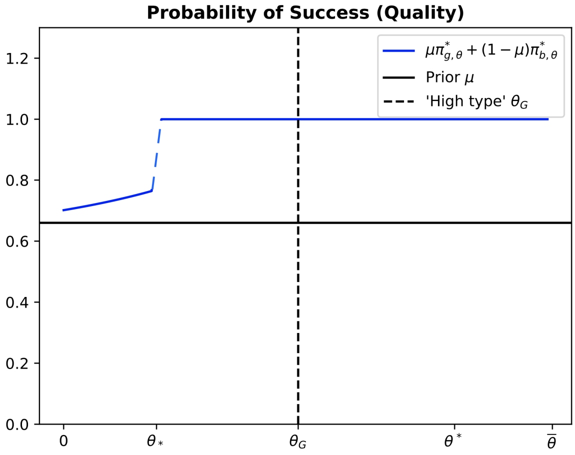
- Distort only one signal: $\pi^*_{b,\theta} = 1$



Continuum of Types: Optimal Menu (Graphs)

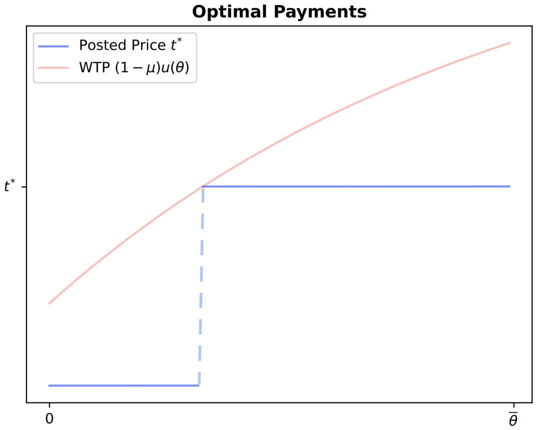
- Payments $t^*(\theta)$ vs. WTP $V(\bar{E}(\theta), \theta)$



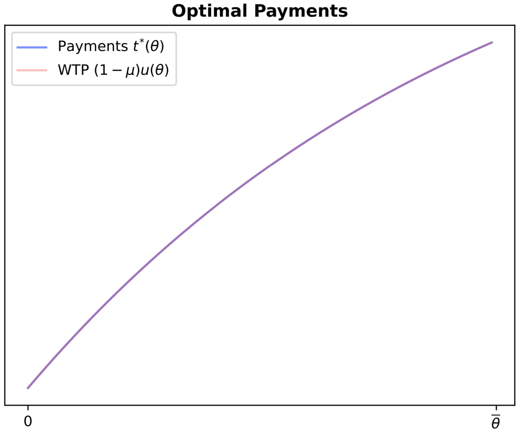


Continuum of Types: Fee Structures in Practice

- As $\lambda_g, \lambda_b \to 0$ (correlation ≈ 1)
  $\Rightarrow$ **flat rate fees**



- As $\lambda_g, \lambda_b \to \infty$ (correlation ≈ 0)
  $\Rightarrow$ **project-based fees**



Conclusion

- Take aways:
  - **imitation costs** when buying information goods
  - unlike typical results in mechanism design, monopolist can **extract full surplus** or otherwise leave **highest rents to 'middle' types**

THANK YOU!

Appendix

Related Literature

- **Design and Price of Information:** Bergemann et al. (2018), Admati and Pfleiderer (1986, 1990), Babaioff et al. (2012), Liu et al. (2021),Eső and Szentes (2007), etc.
- **Screening and Product Differentiation:** Mussa and Rosen (1978), Maskin and Riley (1984),Perloff and Salop (1985), Spulber (1989),Rochet and Stole (2002), etc.
- **Complex Environments:** Jovanovic and Rob (1990), Callander (2008)

Related Literature

	Prior μ_θ	**Utility**	**Correlation**
Bergemann et al. (2018)	private	$u(\theta) = u(\theta')$	corr$(\theta, \theta') = 1$
this project	common	$u(\theta) \neq u(\theta')$	corr$(\theta, \theta') \in [0, 1]$

- Bergemann et al. (2018) considers a common state ω (as if corr$(\theta, \theta') = 1$), but types differ in private interim beliefs μ_θ
- This project considers many states (ω_θ), but common prior beliefs, and different ex-post payoffs

$\Rightarrow$ Switch off screening over differences in prior beliefs

Continuum of Types: Structure of μ

- Transition matrix function

$$P(\Delta) = \exp(Q\Delta) = \frac{1}{\lambda_g + \lambda_b} \begin{pmatrix} \lambda_b + \lambda_g e^{-\Delta(\lambda_g+\lambda_b)} & \lambda_g - \lambda_g e^{-\Delta(\lambda_g+\lambda_b)} \\ \lambda_b - \lambda_b e^{-\Delta(\lambda_g+\lambda_b)} & \lambda_g + \lambda_b e^{-\Delta(\lambda_g+\lambda_b)} \end{pmatrix}$$

satisfying *forward equation* $P'(\theta) = P(\theta)Q$ and *backward equation* $P'(\theta) = QP(\theta)$.

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