Robust Analysis of Short Panels

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Scope

- A new method for conducting econometric analysis using nonlinear **panel data** models.
- Example: **dynamic binary** response panel data model.

$$Y_{it} = 1\left[\gamma Y_{it-1} + Z_{it}\beta + C_i + U_{it} \geq 0\right], \quad t \in \mathcal{T} \equiv \{1, \ldots, T\}.$$

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- Each observational unit (OU) i – family, person, firm, etc. – delivers a value of Y_i and Z_i

$$Y_i \equiv (Y_{i1}, \ldots, Y_{iT}) \qquad Z_i \equiv (Z_{i1}, \ldots, Z_{iT})$$

given realizations of unobservable C_i and U_i

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given realizations of unobservable C_i and U_i

$$U_i \equiv (U_{i1}, \ldots, U_{iT}).$$

- C_i and initial condition Y_{i0} are OU-specific latent variables.

$$V_i = (C_i, Y_{i0}).$$

Henceforth V_i denotes the collection of OU-specific variables.

Short Panels

- Binary response example - now drop i subscripts

$$Y_t = 1\left[\gamma Y_{t-1} + Z_t\beta + C + U_t \geq 0\right], \quad t \in \mathcal{T} \equiv \{1, \ldots, T\}.$$

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- Study SMALL T – **short** panels. There is the **Incidental Parameter Problem**. Neyman and Scott (1948).

- By contrast with available approaches we proceed placing **no** additional restrictions on OU-specific $V = (C, Y_0)$.

Related Literature on Nonlinear Panel Data Models 1

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  - includes: *Rasch* (1960, 1961), *Andersen* (1973), *Honoré and Kyriazidou* (2000, 2019), Honoré and Tamer (2006), Honoré & de Paula (2021), *Davezies, D'Haultfoeuille & Laage* (2022), *Honoré & Weidner* (2022), Kitazawa (2022), Bonhomme, Dano & Graham (2023), Dano (2023), Davezies, D'Haultfoeuille & Mugnier (2023), *Honoré, Muris & Weidner* (2023).

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- $U \perp\!\!\!\perp (C, Z)$ is also imposed in likelihood based models including *Functional Differencing* – Bonhomme (2012).
- May not believe C and U independent if e.g. C captures OU-specific risk preference.

Related Literature on Nonlinear Panel Data Models 2

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- Models imposing conditional **stationarity** as in Manski (1987).

$$\forall (s,t) \in \mathcal{T} \qquad U_s | (C, Z) \sim U_t | (C, Z)$$

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- Requires scalar V, linear index, no life changing outcomes.

- Point identification requires rich support for Z.

Differencing and a new approach

- In linear models **differencing** removes individual effects.

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$$Y_t - Y_s = (Z_t - Z_s)\,\beta + (U_t - U_s)$$

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$$Y_{1t} = \max(Z_t\beta + C + U_t, Y_{2t})$$

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- Aristodemou (2021) (like Rasch (1960) etc) for particular models finds events whose probability of occurrence does not depend on OU-specific variables and so obtains some of the bounds we produce.

- We propose a new **universally applicable** approach to remove C when differencing **not feasible**.
  - leads to models imposing no restrictions on e.g. "fixed effects" C or unobserved initial conditions, Y_0 .

Structural functions

- Define a structural function, $h : \mathrm{Supp}(Y, Z, V, U) \to \mathbb{R}$, that embodies the structural relationships specified by a model via:

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- For example in the binary panel

$$h(Y, Z, V, U) = \sum_{t=1}^{T} \left(Y_t - 1\left[\gamma Y_{t-1} + Z_t\beta + C + U_t \geq 0\right]\right)^2$$

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where $V \equiv (C, Y_0)$.

- Model place restrictions on h and on the distribution of U given Z, $G_{U|Z=z}$ where

$$G_{U|Z=z}(\mathcal{S}) \equiv \mathbb{P}[U \in \mathcal{S}|Z = z]$$

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- Voila! **Moment inequalities**.

$$\mathbb{P}[Y = y|Z = z] \leq G_{U|Z=z}(\mathcal{U}^*(y, z; h)).$$

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  characterize the sharp identified set of functions h and distributions $G_{U|Z=z}$.

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  characterize the sharp identified set of functions h and distributions $G_{U|Z=z}$.

- Models will restrict $G_{U|Z}$ e.g. $U \perp\!\!\!\perp Z$, parametrically, or nonparametrically.

Example - Dynamic Binary Panel Model

- 2 period dynamic binary panel model with Y_0 **observed** so $V = C$.

$$Y_t = 1[Z_t\beta + \gamma Y_{t-1} + C + U_t \geq 0], \quad t \in \{1, 2\}, \quad \text{Supp}(C) = \mathbb{R}$$

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- What values of (u_1, u_2) are in the set $\mathcal{U}^*(y,(z,y_0);\theta)$?

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- Consider $(y_1, y_2) = (0,1)$. There are inequalities:

$$u_1 \leq -z_1\beta - \gamma y_0 - c, \quad u_2 \geq -z_2\beta - c$$

so (u_1, u_2) can deliver $(y_1, y_2) = (0,1)$ if there exists c such that

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- Define $\Delta_{21} u \equiv u_2 - u_1$ and $\Delta_{21} z \equiv z_2 - z_1$. With y_0 **observed**:

$$\mathcal{U}^*((0, 1), (z, y_0); \theta) = \{u : \Delta_{21} u \geq -\Delta_{21} z\beta + \gamma y_0\}.$$

Example: Dynamic Binary Panel Model: Summary

$$Y_t = 1[Z_t\beta + \gamma Y_{t-1} + C + U_t \geq 0], \quad t \in \{1,2\}, \quad \text{Supp}(C) = \mathbb{R}$$

- The **sharp** (see Chesher and Rosen (2017)**)**.identified set for $(\beta, \gamma, G_{U|Z=z})$ comprises values satisfying, for all $z \in \text{Supp}(Z)$

$$\mathbb{P}[Y = (0,1)|Z = z] \leq G_{U|Z=z}\left(\{u : \Delta_{21}u \geq -\Delta_{21}z\beta + \gamma y_0\}\right)$$

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- Can impose e.g. U and Z independent and calculate with U_1, U_2 IID e.g. logistic. But we can drop logistic!

Dynamic Binary Panel Model: Initial Y NOT observed

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- With y_0 **not** observed $\mathcal{U}^*(\,(0,1), z; \theta)$ is the **UNION** of the sets (**) with $y_0 = 0$ and $y_0 = 1$.

$$\mathcal{U}^*(\,(0,1), z; \theta) = \{u : \Delta_{21} u \geq -\Delta_{21} z\beta\} \bigcup \{u : \Delta_{21} u \geq -\Delta_{21} z\beta + \gamma\}$$

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- Can impose e.g. U and Z independent and calculate with U_1, U_2 IID e.g. logistic. But we can drop logistic!

How to drop the logistic restriction - nonparametrics

- Restrict $U \perp\!\!\!\perp Z$. The sharp identified set of $(\beta, \gamma, G_{U|Z=z})$: values satisfying, $\forall z \in \mathrm{Supp}(Z)$:

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- Specify N intervals $(-\infty, \tau_1], (\tau_1, \tau_2], \ldots, (\tau_{N-1}, \infty)$. Define probabilities:

$$p_i \equiv \mathbb{P}[\Delta_{21} U \in [\tau_{i-1}, \tau_i]]$$

$$\forall z \in \mathcal{R}_Z, \qquad \begin{cases} \mathbb{P}[Y = (0,1)|Z = z] \leq \sum_{i:\tau_i \geq -\Delta z\beta + \min(\gamma,0)} p_i \\ \mathbb{P}[Y = (1,0)|Z = z] \leq \sum_{i:\tau_i \leq -\Delta z\beta - \min(\gamma,0)} p_i \end{cases}$$

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- **Linear programme**. Inference using e.g. Kline and Tamer (2016).

Scope

- The paper gives many examples of applications, e.g. to multiple discrete, and ordered, choice panels.
  - **Endogenous** explanatory variables are permitted.
  - Easy to allow for unobserved **initial conditions** in dynamic models.
  - With linear indexes U^* sets obtained using Fourier-Motzkin elimination.
  - Can **dispense** with linear index restrictions.
  - There can be **multiple** OU-specific effects.
  - A **nonparametric** specification of the distribution of U can be accommodated.
  - In **likelihood** based models incidental parameters can be removed in the same way.