Housing and Portfolio Choice over the Wealth Distribution

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ESEM, August 29, 2024

Asset choices vary significantly over wealth distribution

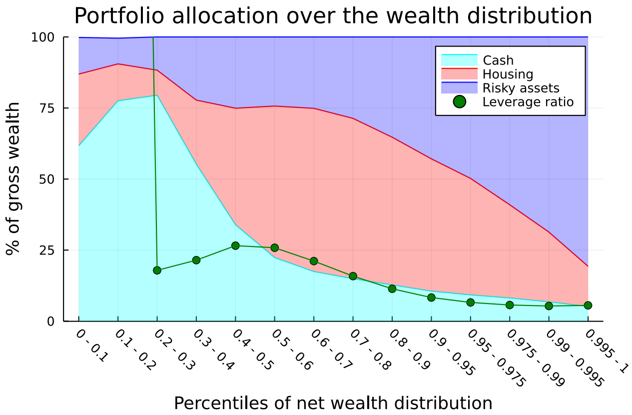

Figure: Composition of gross wealth in Sweden, 2000

- The poorest hold mostly cash
- Housing wealth is the dominant asset class for the middle class,
- ... largely financed by debt for the lower middle class
- For the richest other risky assets are the most important

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Need to look at portfolio choice over **wealth**, not only **age**!

This paper

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Result:

- Generate realistic portfolio choice patterns among home-owners

Contributions

- Optimal housing level increases in human capital, crowds out stocks: Yao and Zhang (2004), Cocco (2005) and Flavin and Yamashita (2011)

- Wedges between borrowing and lending rates affect risk premia and hence optimal portfolio choices: Davis et al. (2006), Willen and Kubler (2006).

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  - Formalize in a toy model how exactly this relationship works
  - Show suggestive evidence from data that this relationship exists
  - Show in a life-cycle model that this phenomenon is quantitatively large
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- Housing helps explain the risky share: Cioffi (2021).
  - Show that this is true even with homothetic preferences

First mechanism: Human capital & Housing choice

Higher ratio of human capital to finantial wealth $\Rightarrow$ Higher optimal share of housing to wealth

The aim of saving is smooth out net worth (wealth + human capital) to be consumed evenly over time.

- More wealth $\Rightarrow$ housing consumption $\uparrow$, and savings $\uparrow$
- More human capital $\Rightarrow$ housing consumption $\uparrow$, but savings $\downarrow$

Analytical solution in frictionless case Frictionless model of housing

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- Wealth is more inequally distributed than human capital
- Across individuals, wealth is negatively correlated with the ratio of human capital to wealth.

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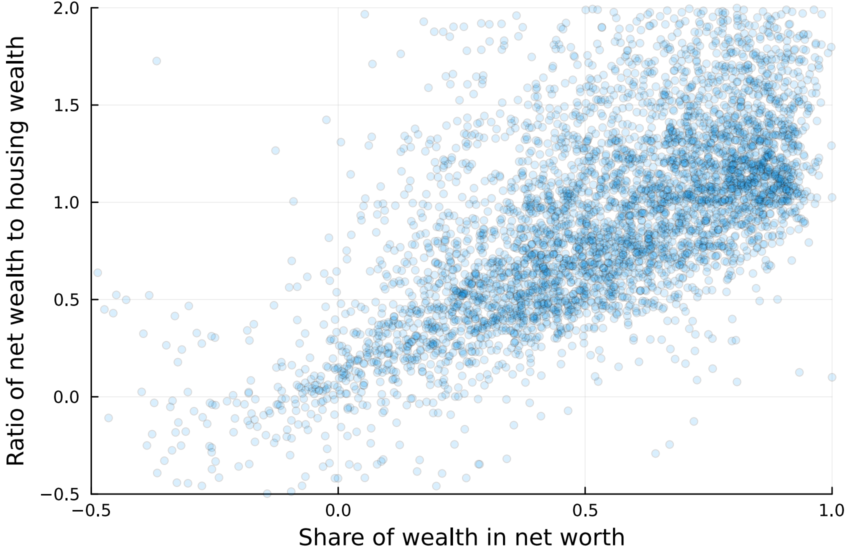
- Across individuals, wealth is negatively correlated with the ratio of human capital to wealth.

For wealth-poor individuals, housing can crowd out other assets due to **optimal consumption decisions**.

Housing share and share of wealth in net worth are correlated in data



Second mechanism: Wedges between interest rates

Depressed risky share for the poor due to lower risk premia from debt

- Households with relatively more human capital are optimally more leveraged in average.
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Climbing out of debt $\Rightarrow$ Invest in risky assets

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More intuition: Optimal policies conditional on housing

Model

Starting point is a standard lifecycle model

Demographics | Income process | Bellman equation

Key to generate results:

- Housing choice
- A menu of different risk-free assets
- Calibration: target average holdings of different asset types over the life-cycle
- Accurate solution method: FOC-based, EGM + discrete choices Solution method

Housing

- Utility from non-durable consumption (c) and housing services (h):

$$U(c,h) = h^{\omega} c^{1-\omega}$$

  Housing services come either from renting or owning
- Owned house (H) provides services equal to its size ($h = H$ if $H > 0$)
- H has market value $P_t^h H$

$$P_t^h = G_h^t \exp(\tilde{p}_t^h)$$
$$\tilde{p}_t^h = \rho^h \tilde{p}_{t-1}^h + \varepsilon_t^h$$

- Costs for owners: maintenance and transaction costs (both when selling or buying)
- Rental cost is $\tau P_t^h h$, no frictions

Liquid assets

Bonds (B) offer a risk-free gross rate R_f; stocks (ξ) with risky gross rate R. Participating ($\xi > 0$) involves yearly participation cost F. Debt in the form of mortgage (M) and consumption loan (L) with constant rates R_m and R_l:

$$s_{it} = B_{it} + \xi_{it} + F\mathbb{1}_{\xi_{it}>0} + M_{it} + L_{it} \qquad R_f < R_m < R_l$$

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No short positions and borrowing limits (LTV and LTI):

$$\begin{aligned} 0 &\leq B_{it} \\ 0 &\leq \xi_{it} \\ 0 &\leq M_{it} \leq \min\left\{\eta_m HC(z_{it}, j), \delta P_t^h H_{it}\right\} \\ 0 &\leq L_{it} \leq \eta_c HC(z_{it}, j) \end{aligned}$$

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- Costless, but obligatory renegotiation in every period $\Rightarrow$ no extra state variable
- Bankruptcy

Calibration

- Most parameters are exogeneously set
- Rest (mostly preference parameters) are estimated through SMM

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Preference parameters | Return and income parameters | Housing parameters

Targeted moments:

- Age profiles of the means of
  net wealth, housing wealth, cash, risky assets, debt, participation rate, and home-ownership rate.
- Shape of wealth distribution

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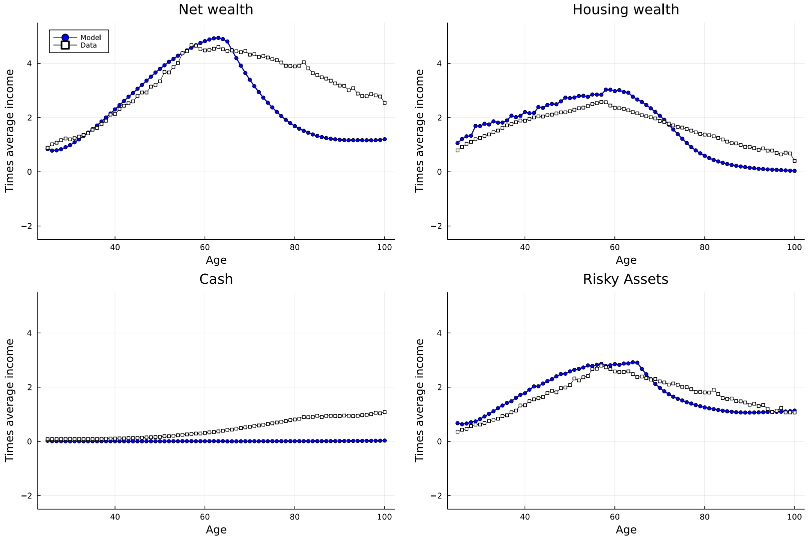
Validation: Portfolio allocation patterns over the wealth distribution.

Targeted moments



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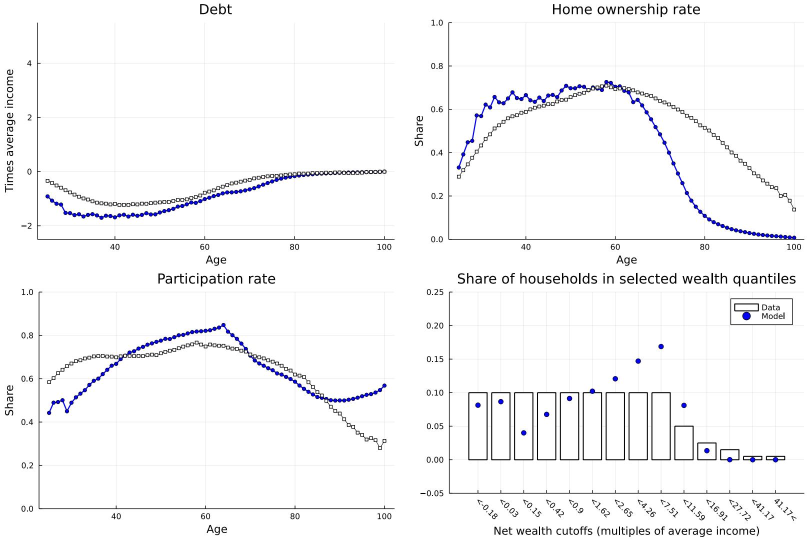
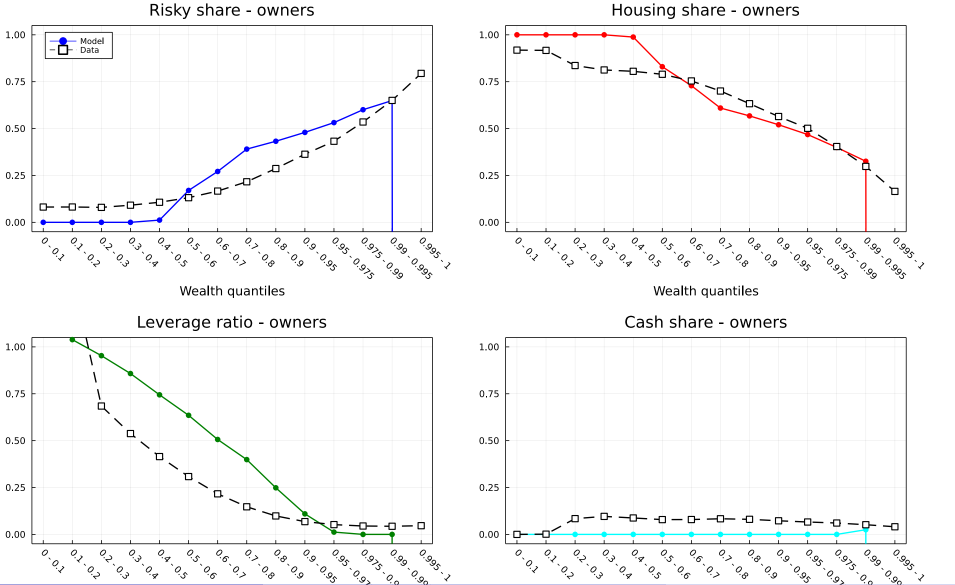
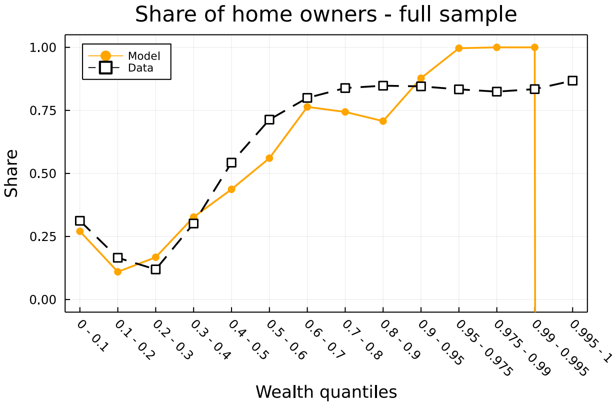


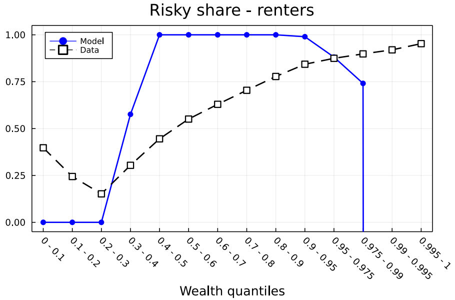
Portfolio choice over the wealth distribution - homeowners

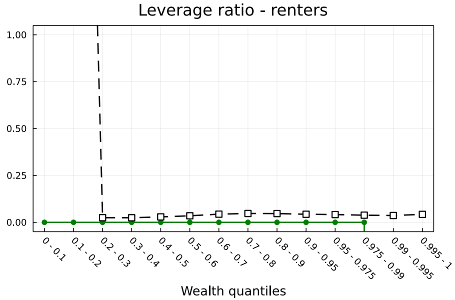


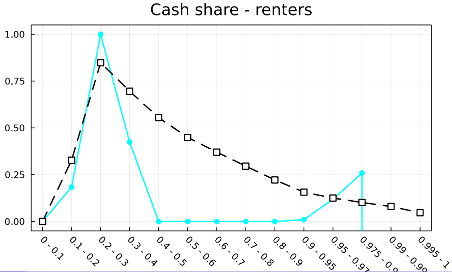
Home-ownership over the wealth distribution - whole economy



Portfolio choice over wealth distribution - renters







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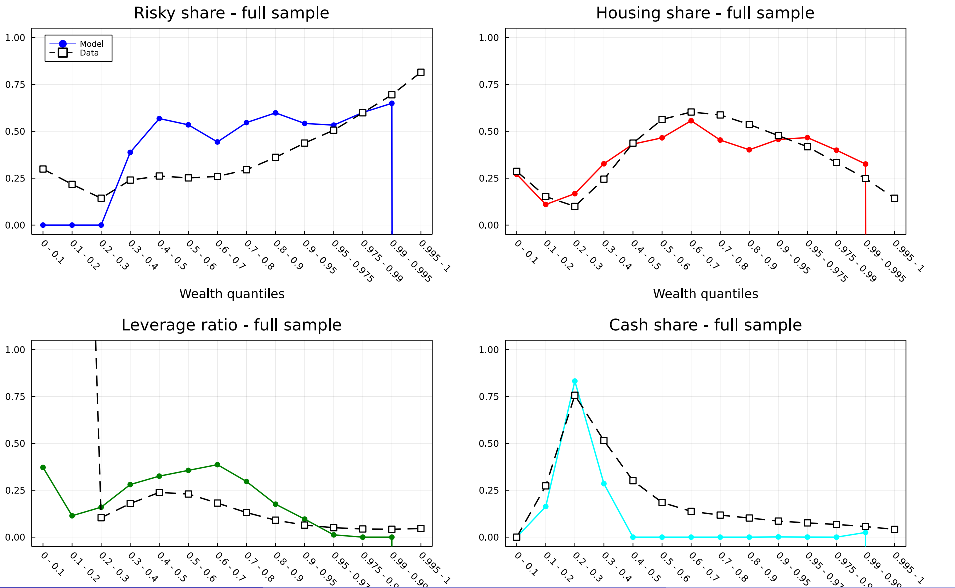

Counterfactuals

I claim two channels are important:

1. Optimal housing share varies over the wealth distribution, due to differences in human capital.
2. Wedges in interest rates

We want to see if these mechanisms indeed play a key role in generating the results.

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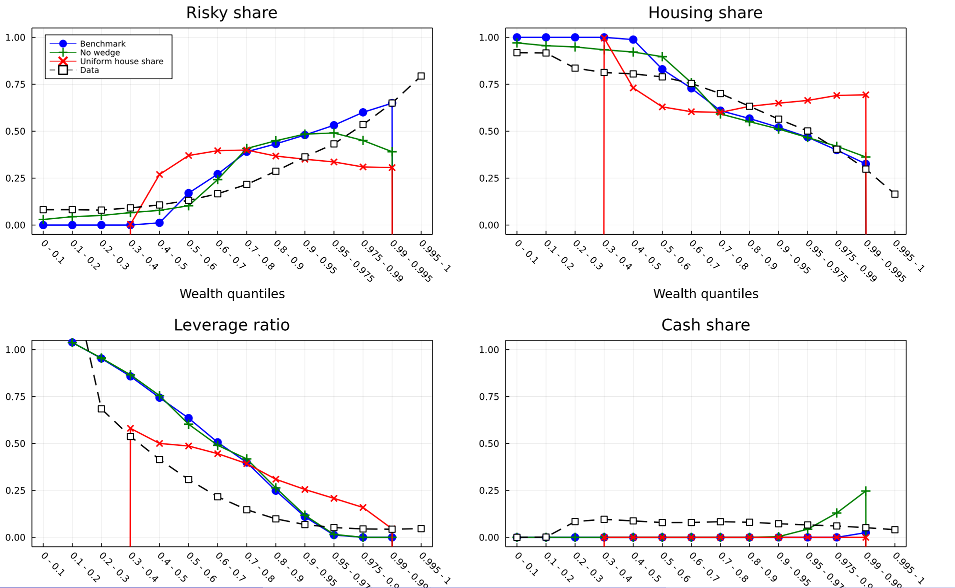
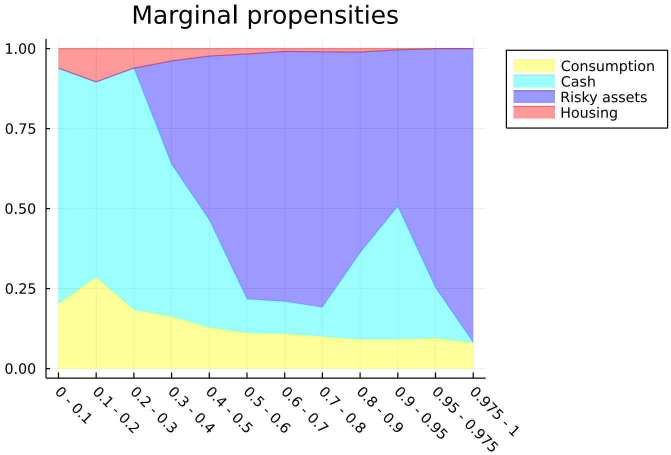
Two corresponding counterfactuals:

1. All homeowners are forced to have the same (the average) ratio of housing to net wealth
2. Replace the three risk-free assets with one (with interest rate R^m)

Counterfactuals - homeowners



An Implication: What do people do with an extra cent?



Optimal policies

Figure: Marginal propensities of different means of saving and expenditure.

Conclusion

- Housing wealth crowds out risky investment for households with low wealth-to-income ratios
- Wedge between borrowing and lending rates imply lower risk premium for the leveraged $\Rightarrow$ lower risky share for the poor
- These effects survive in a standard life-cycle model and can explain the increasing risky share in household wealth among homeowners.
- In this model, helicopter money for the rich ends up in the stock market.

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Optimal housing choice, no frictions

- Finite horizon, deterministic
- Utility from non-durable consumption and housing
- Saving into bonds and housing

The household maximizes

$$\sum_{t=0}^{T} \beta^t U(c_t, h_t)$$

such that

$$b_t + h_t = Rb_{t-1} + R^h h_{t-1} + y_t - c_t \qquad \forall t$$

Assumption: $R^h < R$, ignoring the services it provides, housing is a bad investment.

Optimal housing choice, no frictions

$$b_t + h_t - \frac{R - R^h}{R} h_t = R b_{t-1} + R^h h_{t-1} + y_t - c_t - \frac{R - R^h}{R} h_t$$

$\frac{R-R^h}{R} h_t$ is the foregone capital income from consuming housing instead of saving in bonds.

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$\frac{R-R^h}{R} h_t$ is the foregone capital income from consuming housing instead of saving in bonds.
Define:

$$a_t = b_t + \frac{R^h}{R} h_t$$
$$x_t = c_t + \frac{R - R^h}{R} h_t$$

Problem is equivalent to

$$\sum_{t=0}^{T} \beta^t u(x_t)$$
$$\text{s.t.} \quad a_t = R a_{t-1} + y_t - x_t \qquad \forall t$$

Housing is like consumption

Assume:

- $R\beta = 1$ and
- $u(c_t, h_t) = v\big(c_t^{1-\omega} \cdot h_t^{\omega}\big)$.

Then

$$h_t = A_t\Big(w_t + \sum_{s=0}^{T-t} \frac{y_{t+1}}{R^s}\Big)$$

- Agents want to consume housing against their future labor income $\leftrightarrow$ opposite direction to total savings. People with more future income should save less!

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- Agents want to consume housing against their future labor income $\leftrightarrow$ opposite direction to total savings. People with more future income should save less!
- Optimal housing/wealth ratio is increasing in the ratio of human capital to wealth

$$\frac{h_t}{w_t} = A_t \frac{w_t + HC_t}{w_t}$$

Back

Demographics

- Partial equilibrium overlapping generation economy
- Each period a measure one of 25 years old households are born
- Survival is stochastic, ...
- until certain death at age 100
- Bequests:
  - a fixed fraction of wealth is given to a random newborn
  - the rest is distributed evenly

Back

Income process

Log labor income is composed of a deterministic secular growth term (gt), a deterministic age term (f), a permanent (z) part following an AR(1) process and a transitory (ν) stochastic part.

$$y_{ij} = gt + f_j + z_{ij} + \nu_{ij}$$
$$z_{ij} = \rho z_{ij-1} + \varepsilon_{ij}$$

iid shocks

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$
$$\nu_{ij} \sim N(0, \sigma_\nu^2)$$

for ages $j > 65$ we have

$$y_{ij} = f_j + z_{i,65} + \nu_{ij}^r$$
$$\nu_{ij}^r \sim N(0, \sigma_{\nu^r}^2)$$

Back

Solution method

Combining EGM Carroll (2006) with discrete choices as in Fella (2014) and Iskhakov et al. (2017)

- All risky share and consumption decisions are based on first order conditions - higher precision than VFI
- Difficulties arise as the value function is only piecewise concave
  - global optimization is needed to solve for ξ
  - piecewise integration
  - developed a substitution method to ensure accuracy even close to bankruptcy
  - optimal saving policy is still increasing but can have jumps -> check several candidates and find jumping points
- Comparing values is used only for discrete decisions (participation and housing)

Back

Bankruptcy

- Due to bad income draws or
- tightening borrowing constraints,

bankruptcy can occur.

- House is lost;
- asset level is set to borrowing limit;
- expenditure is set to a consumption floor ζ.

ζ determines how hard households try to avoid being close to their borrowing limit Back

Bellman-equation

$$V_j(P_t^h, a_{it}, z_{it}, H_{it-1}) = \max_{\{c,B,L,M,\xi,H,h\}} \Big\{ (1-\beta) U(c_{it}, h_{it})^{1-\psi} + \\ + \beta \Big(q_{j+1} \mathbb{E}_t \Big[V(P_{t+1}^H, a_{it+1}, z_{it+1}, H_{it})^{1-\gamma} \Big] + \\ + (1-q_{j+1}) \mathbb{E}_t \Big[B(P_{t+1}^H, a_{it+1}, H_{it})^{1-\gamma} \Big] \Big)^{\frac{1-\psi}{1-\gamma}} \Big\}^{\frac{1}{1-\psi}}$$

subject to the budget constraints

$$a_{it} = c_{it} + s_{it} + \tau h_{it} P_t^h \mathbb{1}_{h_{it}=0} + D(H_{it-1}, H_{it}, P_t^h)$$

$$a_{it} = \hat{s}_{it} + \exp(y_{it})$$

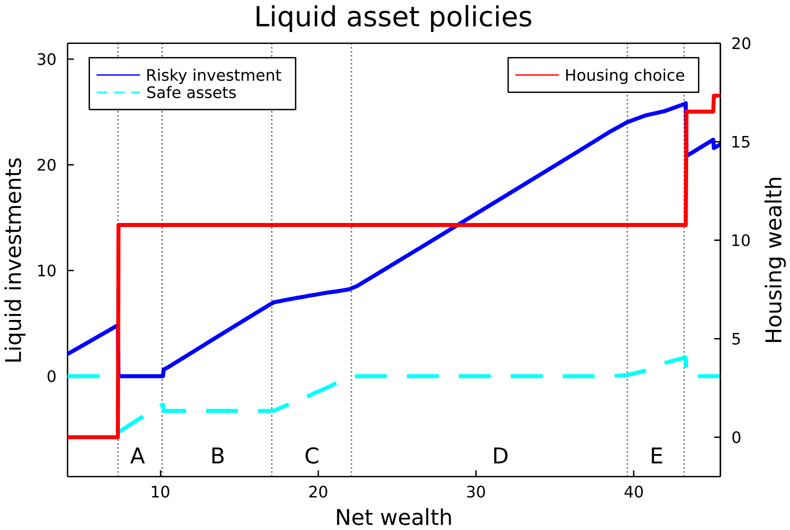
$$\hat{s}_{it} = \xi_{i,t-1} R_t + R^f B_{i,t-1} + R^m M_{i,t-1} + R^l L_{i,t-1}$$

$$h_{it} = H_{it} \quad \text{when } H_{it} > 0$$

$$H_t \in \{0, H_1, \ldots H_l\}$$

Back

Why wedges between interest rates matter?



- A: consumption loan, non-participant
- B: maximal mortgage, participant
- C: trade-off between stocks and mortgage
- D: no risk-free assets
- E: trade-off between stocks and bonds

Back

Parameters

Preference parameters			
β	time preference rate	0.938	estimated
κ	Bequest strength	0.932	estimated
θ	Bequest share to offspring	0.473	estimated
γ	risk aversion	8.81	estimated
ψ	inverse EIS	0.761	estimated
ω	housing share	0.276	SCB - renters
ζ	consumption insurance	0.045%*	estimated

Table: Calibrated values for model parameters. Quantities marked with an asterisk * are expressed relative to average yearly income.

Back

Parameters

Returns and participation cost			
R^f	deposit rate	1.013	SCB
μ_M	expected log stock market return	0.0646	SIXRX
σ_M	s.d. of log stock market return	0.14	SIXRX
R^m	interest rate - mortgage	1.04	SCB
R^c	interest rate - consumption loan	1.075	SCB
F	fixed participation cost	1.8%*	estimated
Income			
g	drift of aggregate wage growth	0.0213	data
ρ	auto-correlation of persistent component	0.924	data
σ_ε	s.d. of shocks to persistent income	0.171	data
σ_ν	s.d. of shocks to transitory income	0.356	data
$\sigma_{\nu pen}$	transitory pension	0.094	data

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Parameters

Housing			
ρ_h	autocorrelation of housing prices	0.9334	data
σ_h	s.d. of housing price shocks	0.0836	data
$\min_h$	minimal housing size	1*	preset
Φ	buying costs	1.035	preset
α	selling costs	0.96	preset
τ	rental costs to price ratio	0.071	estimated
η_m	PTI mortgage	0.18	preset
η_c	PTI consumption loan	0.2	FI
$\overline{L}$	maximal consumption loan	2*	FI
δ	mortgage max LTV	0.85	preset
χ	maintenance cost	0.04	Svensson (2023)

Table: Calibrated values for model parameters. Quantities marked with an asterisk * are expressed relative to average yearly income.

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Demographics

- Partial equilibrium overlapping generation economy
- Each period a measure one of 25 years old households are born
- Survival is stochastic, ...
- until certain death at age 100
- Bequests:
  - a fixed fraction of wealth is given to a random newborn
  - the rest is distributed evenly

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Income process

Log labor income is composed of a deterministic secular growth term (gt), a deterministic age term (f), a permanent (z) part following an AR(1) process and a transitory (ν) stochastic part.

$$y_{ij} = gt + f_j + z_{ij} + \nu_{ij}$$
$$z_{ij} = \rho z_{ij-1} + \varepsilon_{ij}$$

iid shocks

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$
$$\nu_{ij} \sim N(0, \sigma_\nu^2)$$

for ages $j > 65$ we have

$$y_{ij} = f_j + z_{i,65} + \nu_{ij}^r$$
$$\nu_{ij}^r \sim N(0, \sigma_{\nu^r}^2)$$

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