# Kinks Know More:

## Policy Evaluation Beyond Bunching with an Application to Solar Subsidies

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# Imagine you are a policymaker...

- **Policy goal**: Aggregate capacity of solar panel installations in your country.
- **Policy tool**: Subsidy for rooftop solar panels to households and firms.
- **Problem**: $\approx$ 1% of German government spending; benefits the wealthy.
- **Attempted solution**: Nonlinear subsidy with several kink points.

# Research Questions and Challenge

## Research Questions

- *Is the nonlinear subsidy in Germany effective at reducing costs?*
- *What is the most cost-efficient nonlinear subsidy scheme?*

**Challenge:**

- Adopters react at the participation and the intensive margin simultaneously.
- The literature (Saez, 2010) exploits kinks to estimate intensive margin.

# Contributions of this Paper

**Methodological Contribution:**

- Exploit kinks to estimate intensive and participation margin simultaneously.
- Semi-nonparametric estimator with data-driven specification.
- Ignoring participation $\Rightarrow$ downward bias in intensive margin estimate.

**Applied Contribution:**

- Evaluation of German subsidy programme.

# Adopters' Behavior Details

- Heterogeneous, profit maximizing adopters $i$.

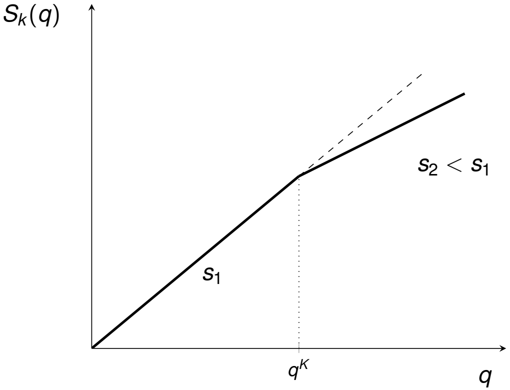
$$\pi^i = \max_q \quad S(q) - c_v^i(q) - c_f^i$$

$$\text{participate if } \pi^i \geq 0$$

- Choose capacity $q$ and participation.
- $S(q) :=$ subsidy
- $c_v^i(q) :=$ all variable economic and non-economic costs
- $c_f^i :=$ all fixed economic and non-economic costs

# Empirical Strategy: Exploit Kink

**The Kinked Subsidy-Function**

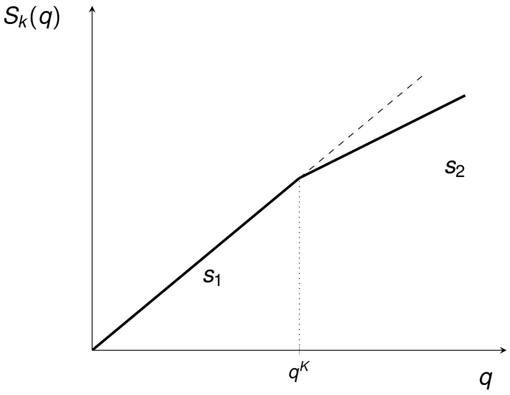


# Theoretical Effect of Kink: Intensive Margin

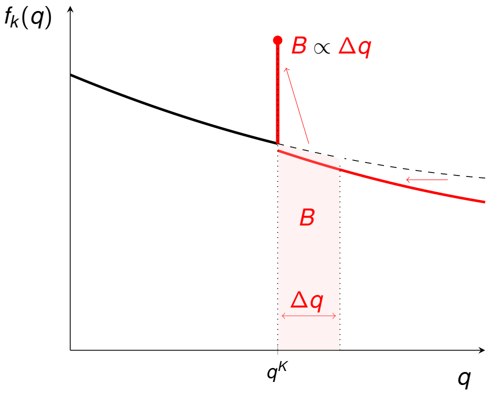
Details

$s_2 < s_1 \Rightarrow$ left shift $\Delta q$ above kink point $q^K$.

**The Kinked Subsidy**




**Distribution of Adoptions**

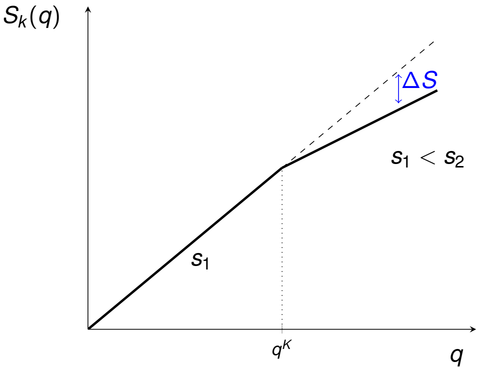


# Theoretical Effect of Kink: Intensive & Participation Margin

Details

Loss in subsidy $\Delta S \Rightarrow$ loss in profit $\Delta\pi \Rightarrow$ loss in participation $\Delta f$.

**The Kinked Subsidy**

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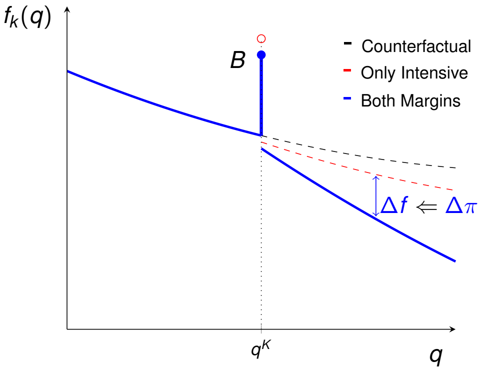
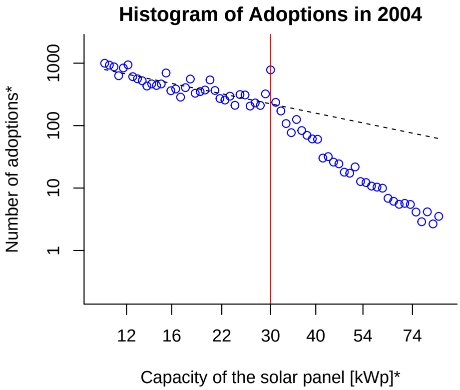


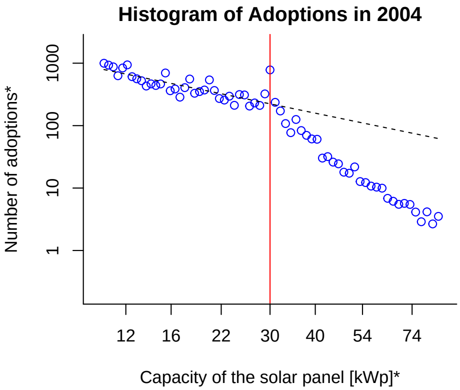
# The Effect in the Data

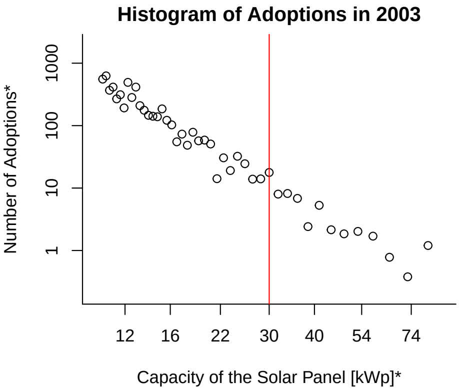



*Logarithmic Scales

# The Effect in the Data

Graphs




*Logarithmic Scales

# Assumptions

Details Intuition

## Assumptions

- *The intensive and participation margin elasticity are locally constant.*
- *The counterfactual distribution $f_l(.)$ is locally representable by a convergent power series (i.e., is real analytic):*

$$\ln f_l(q|\gamma) = \sum_{p=0}^{\infty} \gamma_p \left( \ln \frac{q}{q^K} \right)^p \quad \forall q \in (\underline{q}, \overline{q}).$$

Unknowns: $f_l(.|\gamma)$ := counterfactual; $\epsilon$ := intensive elasticity; $\eta$ := participation elasticity.

## Proposition

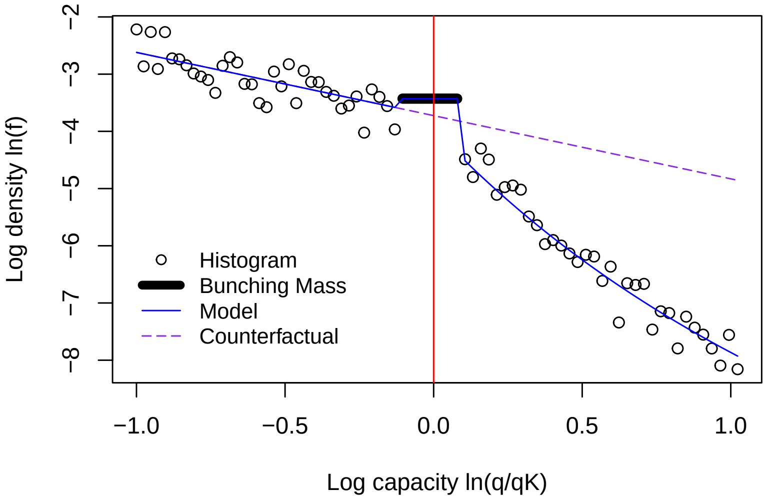
*The distribution $f_k$ and the bunching mass $B$ under the kinked subsidy $S_k$ is:*

$$f_k(q) = f_l(q|\gamma), \qquad \text{for } q < q^K; \tag{1}$$

$$B = \int_{q^K}^{q^K r^{-\epsilon}} R(q_l)^{\eta} f_l(q_l|\gamma) dq_l, \qquad \text{at } q = q^K; \tag{2}$$

$$f_k(q) = f_l(qr^{-\epsilon}|\gamma) r^{-\epsilon} R(qr^{-\epsilon})^{\eta}, \qquad \text{for } q > q^K. \tag{3}$$

*Known: $r$ := relative change in marginal subsidy; $R(.)$ := relative change in profit.*


−2
−3
−4
−5
−6
−7
−8
Log density ln(f)
−1.0
−0.5
0.0
0.5
1.0
Log capacity ln(q/qK)
Histogram
Bunching Mass
Model
Counterfactual

# Estimation



- Local nonlinear least square:

$$\min_{\widehat{\epsilon},\widehat{\eta},\widehat{\gamma_P}} \frac{1}{N}\sum_{j=1}^{N}\left(\widehat{\ln f(q_j)} - \ln f_k(q_j|\widehat{\epsilon},\widehat{\eta},\widehat{\gamma_P})\right)^2 .$$

- Semi-nonparametric Sieve Estimator (Chen 2007):

$$\ln f_l(q) = \sum_{p=0}^{P}\gamma_p\left(\ln\frac{q}{q^K}\right)^p$$

$$P \to \infty \text{ for sample size } \to \infty.$$

- Minimize estimate of mean squared error to select bandwidth and $P$.
- Standard errors: nonparametric bootstrap.

# Comparison to classic estimators

Details Biases Conclusion

Classic bunching estimator (Chetty et al., 2011):

- Ignores participation margin: 12 % downward bias in intensive margin.
- Implicitly relies on parametric functional form assumption on counterfactual distribution (Blomquist and Newey 2017): 11 % downward bias.
- Selection of specification is not based on MSE: 18 times larger standard error.

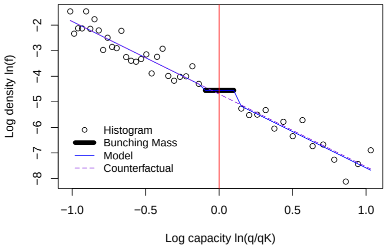
Regression kink design:

- Ignoring intensive margin: 5% upward bias in participation margin.
- $\Rightarrow$ RKD is not applicable. Simultaneous estimation is necessary.

# Robustness to Smoothness Assumption



- Placebo test on untreated data:




- Estimator of specification bias using untreated data. No evidence of specification bias.

# Results Years 2004-2008 Plots

| Capacity | Epsilon (SD) | Kappa (SD) |
|---|---|---|
| 30 kWp | 4.37 (0.13) | 2.31 (0.06) |
| 100 kWp | 4.63 (0.84) | 0.00 (0.02) |

Epsilon := intensive elasticity; Kappa:= participation semi-elasticity; SD:= standard errors.

- Isoelastic intensive margin response.
- Participation margin semi-elasticity decreases in capacity.

# Summary of the Empirical Approach 

**Advantages**

- Identification relies on quasi-experimental variation created by kink.
- No need for additional exogenous variation, instruments, control variables, panel data, covariates.
- Estimation only uses easily observable distribution of adopters.

**Potential Disadvantage**

- Local estimates. *Solution:* estimates from more than one kink point.

# Summary Counterfactual Details

**4 Counterfactual Exercises:**

1. Optimal linear subsidy: current subsidy is 0.14 % less costly.
2. Optimal nonlinear subsidy: saves 3 times more (0.45 % $\sim$ 45 mil. € per year).
3. Thought experiment, no participation: 8 % cost reduction.
4. Wrongly ignore participation: 3 % cost increase.

**Take away:**

- Government's strategy reduces costs, but can be improved.
- Due to participation, only moderate cost reduction; no sliver bullet.
- Considering both margins crucial when designing optimal policy.

# Literature

**Methodology: Bunching Estimator, Regression Kink Design, Sieve Estimation**

- Ando (2017); Bachas, Soto (2018); Beffy, Blundell, Bozio, Laroque, To (2019); Bertanha, Caetano, Jales, Seegert (2023); Bertanha, McCallum, Seegert (2019); Blomquist, Newey, Kumar, Liang (2021 and 2024); Caetano, Caetano, Nielsen (2020); Card, Lee, Pei, Weber (2015); Chetty, Friedman, Olsen, Pistaferri (2011); Chen (2007); Cox, Liu, Morrison (2020); Ganong, Jaeger (2018); Gautier, Galliac (2021); Gelber, Jones, Sacks, Song (2017); Goff (2022); Iaria, Wang (2022); Kleven (2016); Kleven, Landais, Saez, Schultz (2013); Kleven, Waseem (2013); Kopczuk, Munroe (2015); Marx (2019); Moore (2021); Myhre (2022); Nielsen, Sorensen, Taber (2010); Ruh, Staubli (2019); Saez (2010), Slemrod, Weber, Shan (2017).
- Contribution: simultaneous estimation of both margins; semi-nonparametric estimator.

**Application: Solar Subsidies, 2nd degree price discrimination**

- Burr (2016); De Groote, Verboven (2019); Feger, Pavanini, Radulescu (2020); Gerarden (2018); Germeshausen (2018); Hughes, Podolefsky (2015); Jacquet, Lehmann, Van Der Linden (2013); Kraft, Bollinger, Gillingham, Lamp (2018); Rochet, Stole (2002); Saez (2002); Srivastav (2022).
- Contribution: evaluation of nonlinear solar subsidies.

# Conclusion

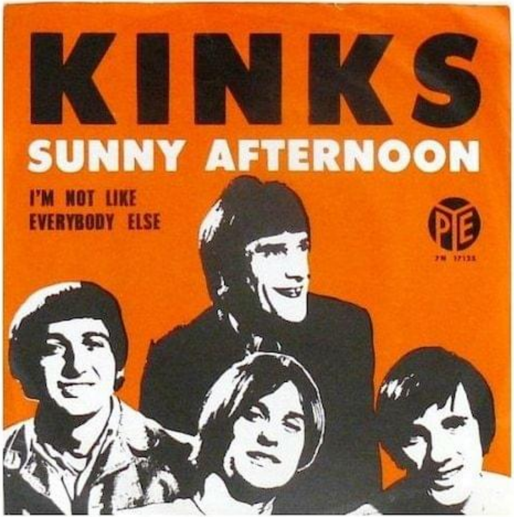
**Summary:**

- Methodology to simultaneously estimate intensive and participation margin using kinks in an incentive scheme.
- Evaluation of the German subsidy for solar panels.

**Methodology More Generally Applicable:**

- Generalizable to discontinuities.
- Kinks/discontinuities + intensive & participation margin are widespread.
- Similar problems: taxation, subsidies and transfers, product pricing.
- Costly deployment subsidies have moved to the forefront of climate action.

# Thank you!



# Appendix

# German Subsidy for Solar Panels

Assumptions

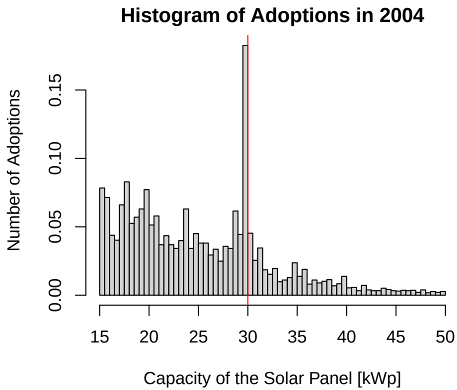
- Subsidy for rooftop solar panels for households and firms.
- From 2000 to 2003 net present value of subsidy linear in capacity.
- From 2004 net present value piecewise linear in capacity.
- Kink points: 30 kWp (5% drop in marginal rate) and 100 kWp (1% drop).

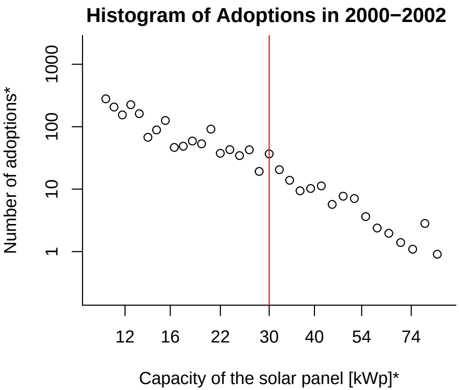
**30 kWp**

**100 kWp**

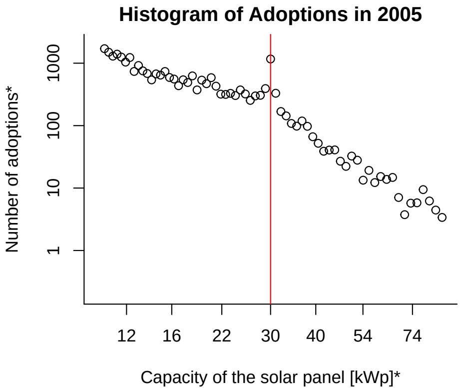
# Histogram of Adoptions

Assumptions Estimation



# Histograms of Adoptions 2005 Assumptions




*Logarithmic Scales

# Data Assumptions

- Administrative data from transmission system operators.
- Contains all solar panel installations in Germany.
- Installation date, capacity, subsidy payment.
- I use years 2000-2008.

# German Subsidy for Solar Panels

Assumptions

**Distribution Aggregate Capacity**

| Interval | Relative Capacity |
|---|---|
| $< 10$ kWp | 30 % |
| 10 to 30 kWp | 40 % |
| 30 to 100 kWp | 20 % |
| >100kWp | 10 % |

# Adopters' Behaviour: Extensions Back

- Heterogeneous, profit maximizing adopters $i$.

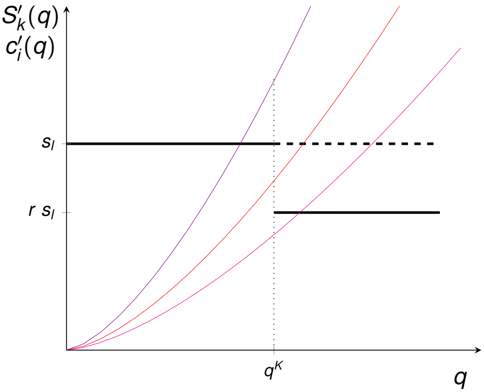
$$\pi^i = \max_q \beta^i S(q) - c_v^i(q)$$
$$\text{participate if } \pi^i \geq c_f^i$$

  $\beta^i :=$ individual specific discounting and productivity.
- The variable cost is convex because:
  - Opportunity and aesthetic cost of space on the roof are convex.
  - Price of solar panels is convex in their efficiency, i.e., their capacity per area.
- Fixed cost $c_f^i$ contains opportunity cost of adopting in a different period.

# Illustration, Optimization Problem

Back

**Marginal Subsidy and Marginal Cost Curves**

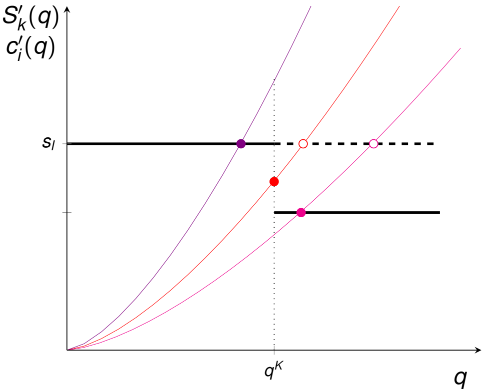



FOC: $\underbrace{S'(q)}_{\text{marginal subsidy}} - \underbrace{c_i'(q)}_{\text{marginal cost}} = 0$

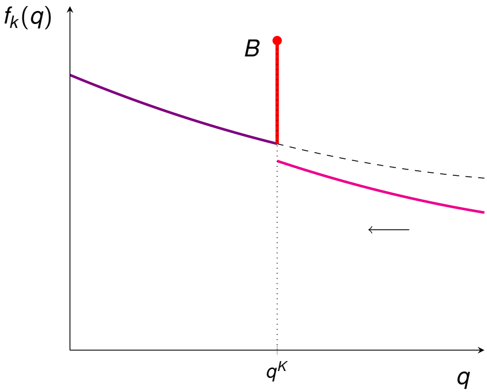
# Mechanism Only Intensive Margin Back

Marginal cost equal marginal subsidy (FOC) ⇒ bunching & left shift in distribution.

**Optimal Choices**




**Distribution, Only Intensive Margin**

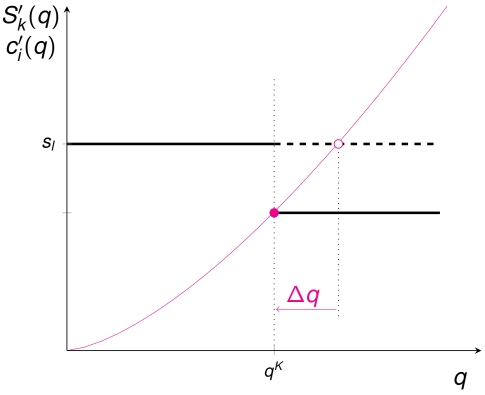


# Identification Only Intensive Margin

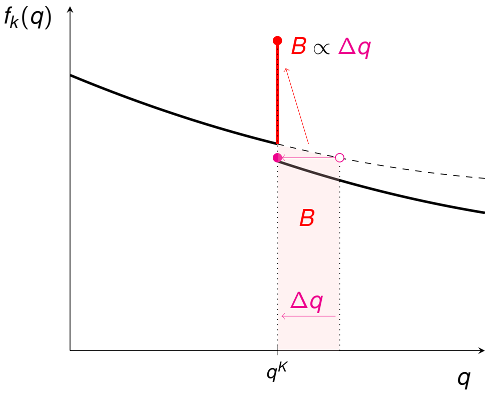
Back

Bunching mass $B$ proportional to left shift $\Delta q$ of marginal buncher.

**The Marginal Buncher**



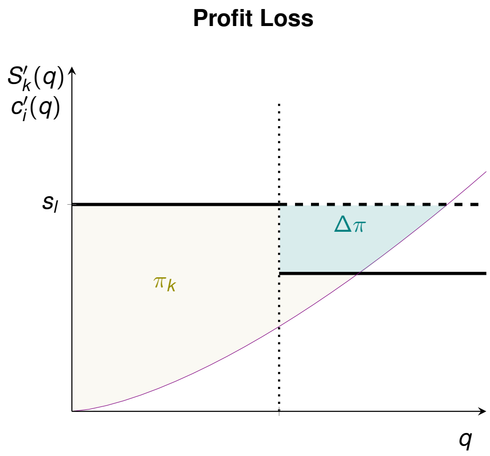

**Distribution, Only Intensive**

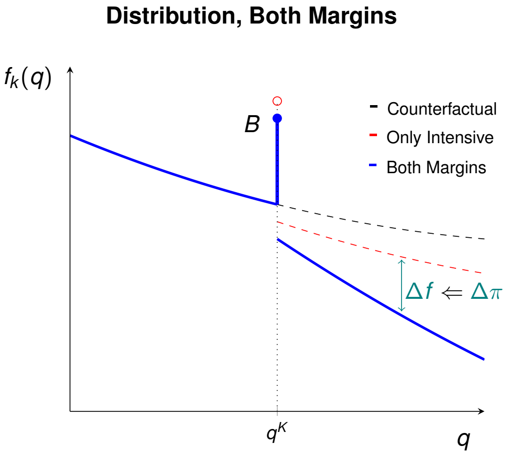


# Mechanism Both Margins

Back

Profit loss $\Delta\pi$ causes drop in participation $\Delta f$.



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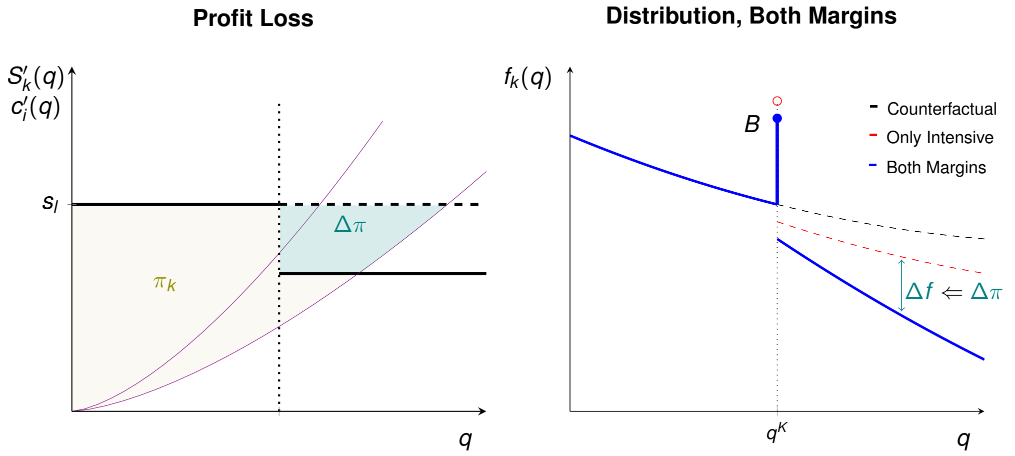
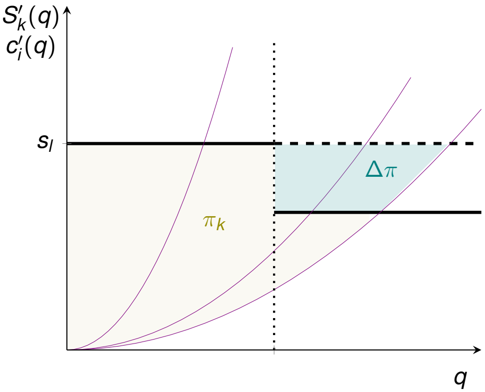


# Mechanism Both Margins

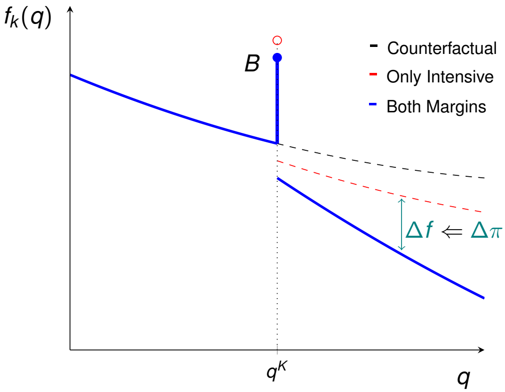
Back

Profit loss $\Delta\pi$ and participation loss $\Delta f$ increase in capacity.

**Profit Loss**




**Distribution, Both Margins**



# Assumptions Details: Intensive Margin

Back

## Assumption (Locally isoelastic cost function)

*For agents and quantities close to the kink point the cost function is isoelastic:*

$$c_i(q) = \theta(i) q^{1+\frac{1}{\epsilon}} + c_f(i). \tag{4}$$

$(\theta, c_f) :=$ *(variable cost type, fixed cost type);* $\epsilon :=$ *intensive elasticity.*

For all firms $i$, define $q_l(i), c_t(i) :=$ choice and total cost under counterfactual.

$$\Rightarrow \ \theta(i) = \frac{s_l}{q_l(i)^{\frac{1}{\epsilon}}} \frac{\epsilon}{1+\epsilon} \text{ and } c_f(i) = c_t(i) - q_l(i) \frac{\epsilon \ s_l}{1+\epsilon}. \tag{5}$$

$\Rightarrow$ alternative type parameters $(q_l, c_t)$ with direct economic meaning.

# Assumptions Details: Participation Margin Back

## Assumption (Locally isoelastic and smooth type-distribution)

1. *The conditional CDF of the total cost $c_t$ is locally isoelastic:*

$$F_t(c_t|q_l) = (c_t)^\eta g(q_l). \tag{6}$$

$\eta :=$ *participation margin elasticity;* $g(q_l) :=$ *normalization term.*

2. *For an interval* $[\underline{q}, \overline{q}]$ *around the kink point the counterfactual density* $f_l(q_l)$ *is representable by a convergent power series (analytic):*

$$\ln f_l(q) = \sum_{p=0}^{\infty} \gamma_p \left(\ln \frac{q}{q^K}\right)^p.$$

*Definition* $f_l(q_l) := f(q_l|c_t \leq s_l q_l)$.

# Alternative Smoothness Assumption

Back

**Assumption (Smoothness)**

*The transformation of the counterfactual measure* $f_l(.)$ *is infinitely differentiable on* $(\underline{q}, \overline{q})$ *and the derivatives are bounded by*

$$\left| \frac{d^{(p)} \ln(f_l(q_l))}{d \ln(q_l)^{(p)}} \right| \leq M \frac{p!}{(\ln(\overline{q}) - \ln(q^K))^p}, \tag{7}$$

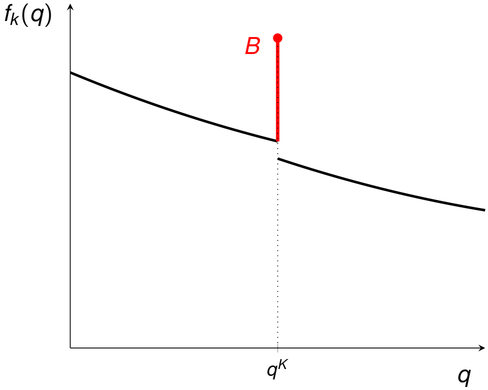
*where the bound* $M > 0$ *denotes a large real number.*

# Intuition Assumptions

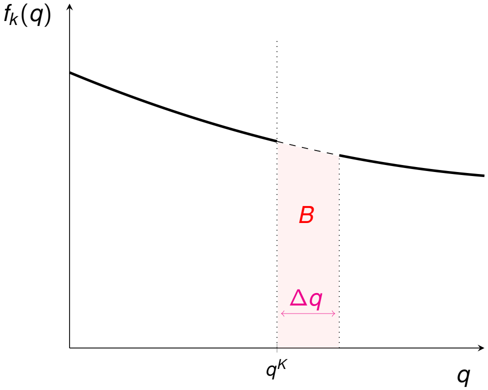
Back

Smoothness assumption identifies counterfactual distribution of bunching mass.

**Observable Distribution**



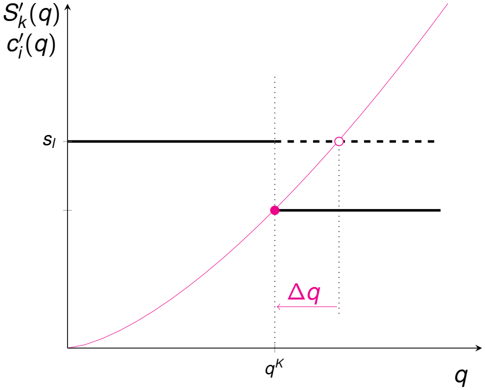

**Identification $\Delta q$**



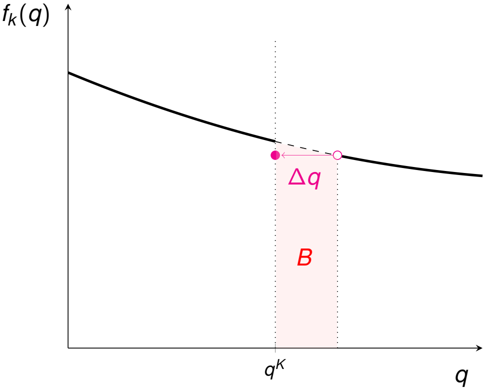
# Intuition Assumptions Back

The shift $\Delta q$ identifies the elasticity of the marginal buncher.

**The Marginal Buncher**




**Identification $\Delta q$**



# Identification Back

Population Criterion

$$Q(\epsilon,\eta,\gamma) = \int_{\underline{b}}^{\overline{b}} \left(\ln f_k^o(q) - \ln f_k\left(q \mid \epsilon,\eta,\gamma\right)\right)^2 \, dF^w. \tag{8}$$

- $f_l(.)$ real analyticity $\Rightarrow$ parameter space is $(\epsilon,\eta,\gamma) \in \mathbb{R}^\infty$.
- True $(\epsilon^o,\eta^o,\gamma^o)$ is the unique minimum of $Q(.)$.
- Parameter space is compact; $Q(.)$ is continuous.

# The Relative Net Change in Subsidy Payment $R(.)$ Back

Define the function $R(q_l)$ as the net subsidy of firm $q_l$ under the kinked scheme as a fraction of the subsidy under the linear scheme:

$$R(q_l) = \frac{S_k(q_k(q_l)) - \Delta c(q_l)}{S_l(q_l)}. \tag{9}$$

The function $R(q_l)$ is:

$$R(q_l) = 1, \qquad \text{for } q_l < q^K; \tag{10}$$

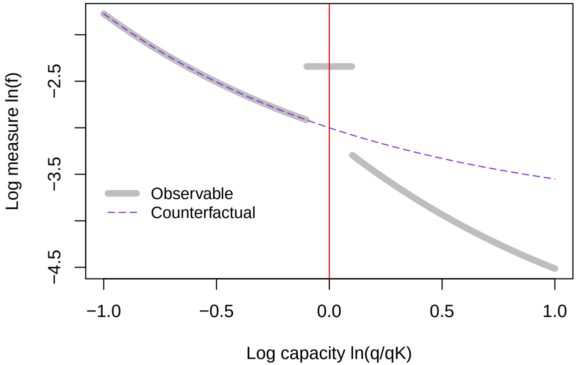
$$R(q_l) = \frac{q^K}{q_l} + \frac{\epsilon}{1+\epsilon}\left(1 - \left(\frac{q^K}{q_l}\right)^{\frac{1+\epsilon}{\epsilon}}\right), \qquad \text{for } q_l \in [q^K, q^K r^{-\epsilon}]; \tag{11}$$

$$R(q_l) = (1-r)\frac{q^K}{q_l} + \frac{\epsilon}{1+\epsilon}\left(1 + \frac{r^{\epsilon+1}}{\epsilon}\right), \qquad \text{for } q_l > q^K r^{-\epsilon}. \tag{12}$$

# Simulation Exponential Counterfactual

Back

$$\ln f_l^o(q) = \lambda_0 + e^{-\lambda_1 \ln\left(q/q^K\right)}.$$



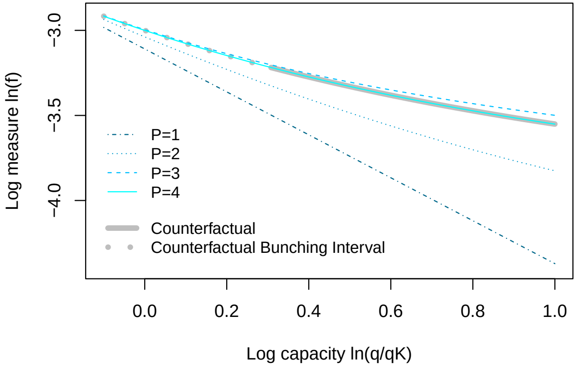
# Simulation Exponential Counterfactual Back

**Table:** Inferred parameters exponential $\ln f_l^o(.)$

| | Epsilon | Bias Epsilon [%] | Eta | Bias Eta [%] |
|---|---|---|---|---|
| True Value | 0.30000 | 0.000 | 3.0000 | 0.000 |
| P=1 | 0.36239 | 20.795 | 0.3976 | -86.747 |
| P=2 | 0.32369 | 7.895 | 2.1423 | -28.591 |
| P=3 | 0.29792 | -0.694 | 3.1557 | 5.190 |
| P=4 | 0.30000 | 0.000 | 2.9996 | -0.012 |

# Simulation Exponential Counterfactual Back

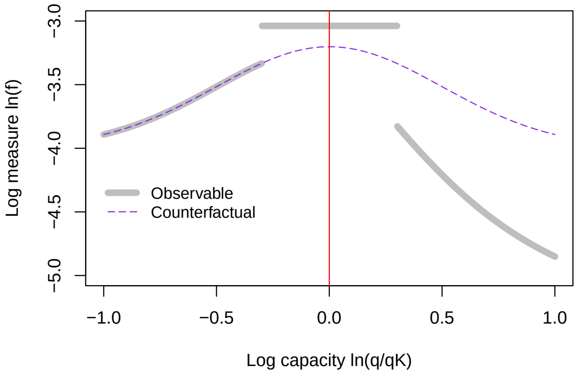
**Figure:** True and inferred counterfactual, exponential $\ln f_l(.)$.



# Simulation Normal Counterfactual

Back

$$\ln f_l^o(q) = \lambda_0 + \frac{1}{\sqrt{2\pi}\lambda_1} e^{-\frac{1}{2}\left(\frac{\ln\left(q/q^K\right)}{\lambda_1}\right)^2}.$$



# Simulation Normal Counterfactual

Back

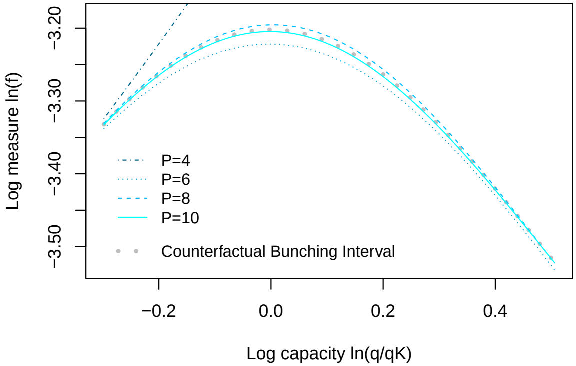
**Table:** Inferred parameters, normal $\ln f_l$

| | Epsilon | Bias Epsilon [%] | Eta | Bias Eta [%] |
|---|---|---|---|---|
| True Value | 0.30000 | 0.000 | 3.0000 | 0.000 |
| P=4 | 0.19170 | -36.101 | 9.1228 | 204.093 |
| P=6 | 0.31168 | 3.893 | 2.9096 | -3.013 |
| P=8 | 0.29439 | -1.869 | 2.9933 | -0.223 |
| P=10 | 0.30248 | 0.827 | 3.0015 | 0.050 |

# Simulation Normal Counterfactual

Back

**Figure:** True and inferred counterfactual, normal ln $f_l$.



# Rank Condition

Back

**Rank Condition:** Equation (4) and (5) have a unique intercept in $(\epsilon, \eta)$.

Holds generically. Moreover, each of the following is sufficient:

1. If $\frac{d \ln f_l(q)}{d \ln q} < -1$.
2. If $f_l(q)$ or $\ln f_l(q)$ is real analytic on the interval $[0, \overline{q})$
3. If order of series expansion $P$ is finite.
4. If $f_l(.)$ is real analytic on $(0, \overline{q})$ and there exists a P such that $\lim_{q \downarrow 0} \frac{d^P f_l^o(q)}{dq^P} \neq 0$ or $\pm \infty$.
5. If $\eta^o$ is known; in particular $\eta^o = 0$.

# Nonparametric Specification Back

- Mean squared error: $\mathbb{E}(\hat{\eta}(P,n)-\eta)^2 = \underbrace{\mathbb{E}(\hat{\eta}(P,n)-\tilde{\eta}(P))^2}_{\text{Variance}} + \underbrace{(\tilde{\eta}(P)-\eta)^2}_{\text{Bias}^2}.$

  $\eta$ :=parameter; $n :=$ sample size; $P :=$ order of series; $\tilde{\eta}(P) :=$ biased value.

- Estimate of Variance$\Rightarrow$ bootstrap
- Estimate of Bias $\Rightarrow$ untreated data.
  -
    $$\tilde{\eta}-\eta = \frac{\sum_{p=P}^{\infty} \gamma_p \frac{1}{p!}\left(\ln\left(\rho^{-\epsilon}\right)\right)^p}{\ln R(q^K \rho^{-\epsilon}, \epsilon)} \tag{13}$$
  - Intuition: On untreated data, any effect $\widehat{\eta}_{nt}(P)$ estimates bias from specification.
  - The bias in $\eta$ depends only on $\epsilon$ and on the un-estimated rest of the parameter $(\gamma_{P+1}, \gamma_{P+2}, \ldots)$.

# Specification Bias Back

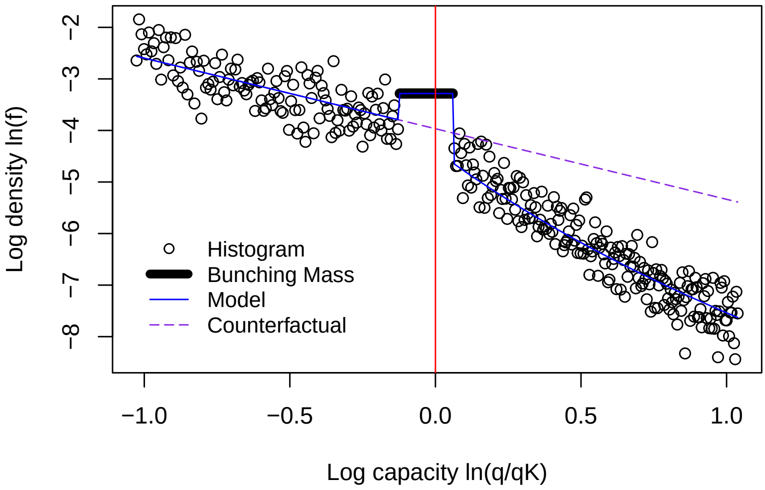
- Assume $\hat{\epsilon} \approx \epsilon^o$ and $\ln f_l(.)$ is $P$ times differentiable.
- Analytic expression of specification bias:

$$\mathbb{E}\left[\hat{\eta} - \eta^o\right] = \frac{h(q^K \rho^{-\epsilon^o}) \left(\ln\left(\frac{q^K \rho^{-\epsilon^o}}{q^K}\right)\right)^P}{\ln R(q^K \rho^{-\epsilon^o}, \epsilon^o)}, \tag{14}$$

- Numerator is the rest of P-th order Taylor approximation.
- Assume it is the same in the treated and untreated data.
- Simulate intensive margin in untreated data using $\hat{\epsilon}$. $\hat{\eta}$ is estimate of bias.
- Small and statistically insignificant specification bias.

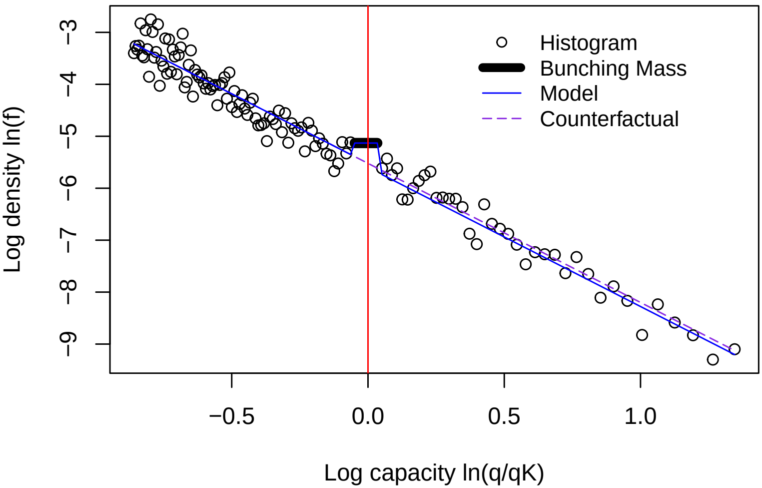
# Estimation 2004-08 at 30 kWp

Back



# Estimation 2004-08 at 100 kWp

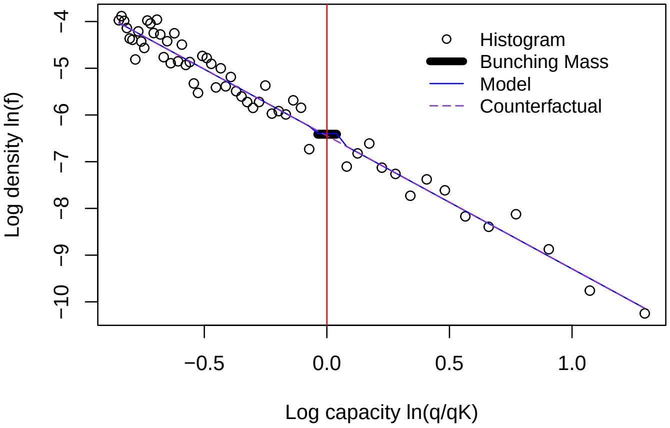
Back



# Robustness 100 kWp

Back

Both parameters are insignificant.



# Comparison classic estimates: details

Summary

| Parameter | Unbiased Estimate | Biased Estimate | Relative Difference in % |
|---|---|---|---|
| $\tilde{\epsilon}_1$ | 4.37 | 3.39 | -23 |
| $\tilde{\epsilon}_2$ | 4.37 | 3.87 | -12 |
| $\tilde{\kappa}$ | 2.31 | 2.43 | 5 |

| Parameter | Optimal Estimate | P=7 |
|---|---|---|
| $\epsilon$ | 4.37 (0.13) | 3.78 (0.39) |
| $\kappa$ | 2.31 (0.06) | 3.25 (1.02) |

# The Government's Objective Back

Achieve a certain capacity goal at minimal public costs:

$$\min_S \int S(q)f(q)dq \quad \text{such that} \quad Q \geq Q^T;$$

$Q :=$ aggregate capacity, $Q^T :=$ capacity goal.

- Zero welfare weight for rents paid to adopters.
- Special case of more general welfare criterion.

# Details Welfare Back

**Utility function of adopter:**

$$U\left(S(q) - c(q,\theta_q) + y - T(y) - e(y,\theta_y)\right) \tag{15}$$

$c(.) :=$ cost of producing capacity $q$, $y :=$ other income, $T(.) :=$ income tax, $e(.) :=$ effort cost to produce other income, $\theta = (\theta_q, \theta_y)$ type parameters.

**General Welfare Function:**

$$\max \int_{\Theta} G\left[S(q) - c(q,\theta_q) + y - T(y) - e(y,\theta_y)\right] f(\theta)d\theta + V(Q) \tag{16}$$

$$\text{s.t.} \int_{\Theta} T(y) - S(q)f(\theta)d\theta = R \text{ and } Q = \int_{\Theta} q\, f(\theta)d\theta \tag{17}$$

$G(.) :=$ redistributive preferences; $V(.) :=$ valuation of aggregate capacity.

# Assumptions and Solution Method Back

**Assumption (Global Assumptions)**

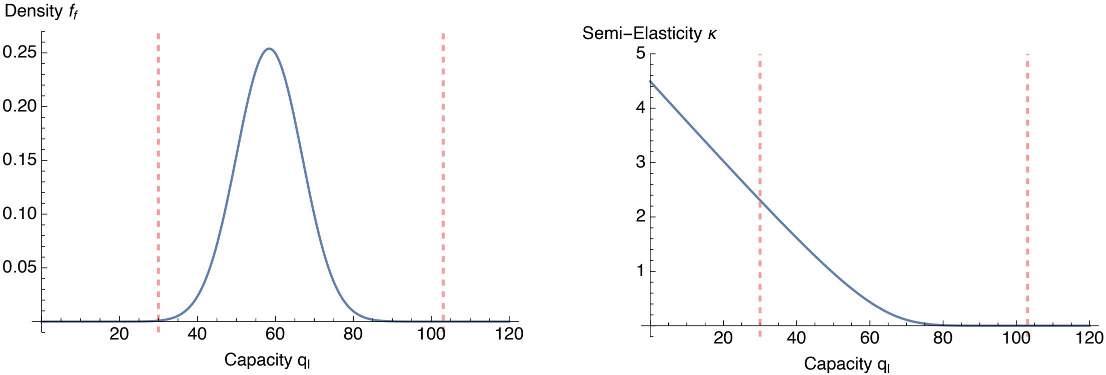
1. *The cost function is isoelastic.*
2. *The fixed cost follows an independent, Normal distribution.*

**Solution Method:**

- Mechanism design approach following Rochet and Stole (2002).

# Normal Distribution of Fixed Costs & Implied Semi-Elasticity

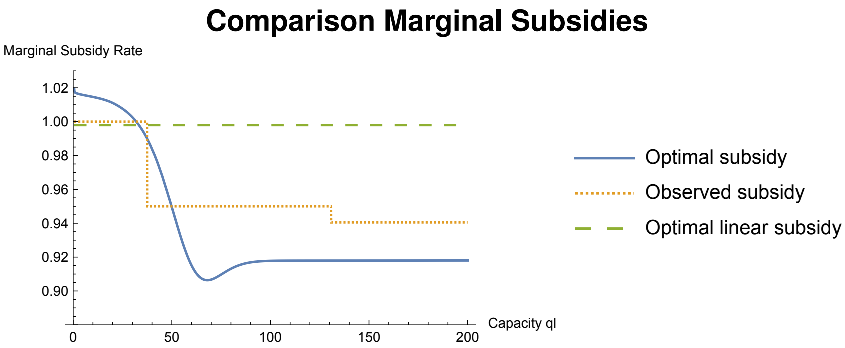
Back




The x-axis shows variable profit under the counterfactual subsidy as a function of capacity $q_l$.

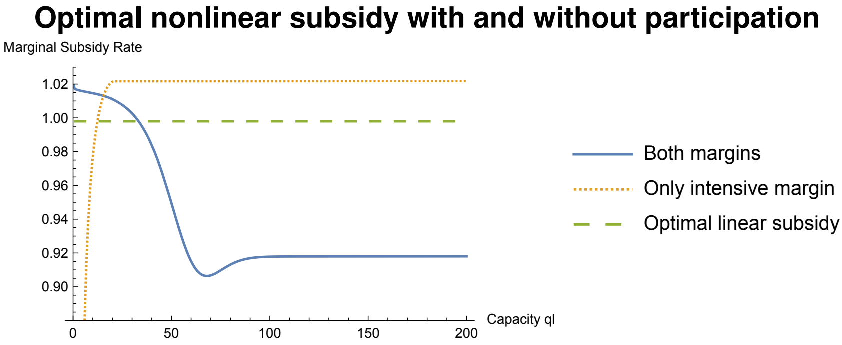
# Counterfactual: The Optimal Marginal Subsidy

Back




- Actual subsidy is 0.14 % less costly than optimal linear subsidy.
- Optimal nonlinear saves 3 times more (0.45 % $\sim$ 45 mil. € per year).

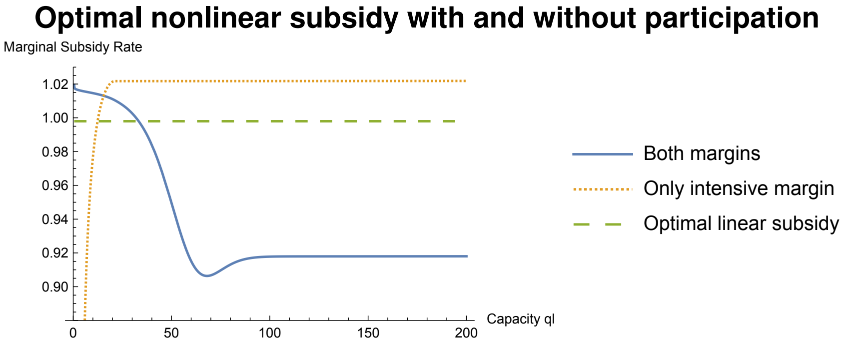
# Thought Experiment: No Participation Back




- Without participation margin ⇒ Cost savings are 9% (900 mil. € per year).

# Counterfactual: Wrongly Ignoring Participation

Back




Suppose policymaker wrongly ignores participation:

- Cost increase of 3 % instead of cost decrease.
- Taking participation into account is key.

# Details Optimal Subsidies Back

- Denote a subsidy in place by $S(q)$.
- Change the marginal subsidy at point $q$ by $\mathrm{d}S'(q)$: $\hat{S}'(q) = S'(q) + \mathrm{d}S'(q)$.

Change in principal's payoff, only intensive margin:

$$(V'(Q) - S'(q))\frac{\mathrm{d}q(\epsilon)}{\mathrm{d}S'(q)}f(q) \qquad - \int_q^\infty f(\tilde{q})d\tilde{q}$$

Both margins:

$$(V'(Q) - S'(q))\frac{\mathrm{d}q(\epsilon)}{\mathrm{d}S'(q)}f(q) + \int_q^\infty (V'(Q)\tilde{q} - S(\tilde{q}))\frac{\eta(\tilde{q})}{S(\tilde{q})}f(\tilde{q})d\tilde{q} - \int_q^\infty f(\tilde{q})d\tilde{q}$$

In the optimum the above expressions are zero for all $q$.