Accounting for the slowdown in output growth after the Great Recession: A wealth preference approach*

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Abstract

Previous studies have argued that US output growth declined persistently after the Great Recession. To explain the persistent slowdown in output growth, we develop a simple model that incorporates wealth preferences and downward nominal wage rigidity into a standard monetary growth model. Our model predicts that output initially grows at a constant steady rate and slows endogenously afterward. In the model, persistent stagnation occurs together with the declining real interest rate. Applying our model to the US data, we show that it successfully explains the slowdown in output growth along with the declines in the real interest rate. We also examine the model with the Japanese data. The model replicates the persistent stagnation that has been observed since the 1990s.

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Keywords: Secular stagnation, Wealth preferences, Liquidity preferences, the Great Recession, Downward nominal wage rigidity

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1 Introduction

The persistent decline in output growth in the United States (US) after the Great Recession has formed one of the most well-known debates in macroeconomics. In his "secular stagnation hypothesis," Summers (2014) argues that while long-run US output was expected to grow steadily before the Great Recession, the actual output after the Great Recession failed to catch up with the expected output trend. Figure 1 plots two estimated trends for log real gross domestic product (GDP) per capita along with the actual data. The upper panel shows the US output trends together with actual output. The linear trend (the dot-dashed line) is estimated using log real GDP data from 1990:Q1 to 2007:Q1 and extrapolated after 2007:Q1. While output was expected to grow based on the data up to 2007, actual real GDP after 2007:Q1 did not grow as fast as the expected output trend. Using the data that includes the period after 2007:Q1, we estimate the cubic trend of output (the dashed line). The cubic trend falls below the linear trend, exhibiting a slowdown in output growth.[1] A similar observation can be made for Japan in the 1990s, as shown in the lower panel of the figure. We estimate the linear trend of log real GDP from 1980:Q1 to 1991:Q1 and the cubic trend from 1980:Q1 to 2019:Q4.[2] Comparing the linear and cubic trends, we observe that Japan experienced a significant decline in output growth.

[Figure 1 about here.]

In this paper, we develop a model with wealth preferences and assess the model's ability to account for the slowdown in output growth. The literature on wealth preferences has found that strong wealth preferences could lead to an inherently stagnant economy in the steady state.[3] In the standard model without wealth preferences, households receive market interest from savings. In the model with wealth preferences, however, households receive additional benefits of savings, namely holding wealth. Under our preference assumptions, the additional benefits incentivize households to give up more consumption to enjoy holding more wealth. In turn, the permanent shortage in aggregate demand or the strong desire for savings leads to the permanently low real interest rate and low inflation, which well characterize the secular stagnation observed in advanced economies.

[1] We confirmed a similar pattern in the potential GDP series by the Congressional Budget Office.

[2] We use 1991:Q1 as the end of the sample period in calculating the linear trend of log GDP because a persistent decline in output growth started after this year.

[3] The earliest examples of these studies are Ono (1994) and Ono (2001), among others. In a recent paper, Michau (2018) also proposes a model with wealth preferences that leads to secular stagnation in the steady state.

We incorporate two components into the standard monetary growth model. As discussed, the first component is wealth preferences. Following the literature (e.g., Michau (2018), Michaillat and Saez (2021), Hashimoto, Ono, and Schlegl (2023)), we introduce wealth preferences with a strictly positive marginal utility in equilibrium. These preferences lead to a strong desire for savings compared with the case without wealth preferences. The second component is downward nominal wage rigidity (DNWR), which is widely discussed in recent studies.[4] Together with wealth preferences, DNWR plays an important role in generating secular stagnation in the monetary growth model.[5]

We demonstrate that our model endogenously generates a slowdown in output growth in the transition path to the steady state. In particular, we theoretically show that output initially equals the first-best allocation, but later falls below the first-best allocation. In our model, strong wealth preferences lead to disinflation, while DNWR is not binding. However, once DNWR binds, inflation no longer decreases and the household with wealth preferences accumulates wealth rather than consuming enough to reach the first-best allocation in output. Then, aggregate demand determines output, an aggregate demand shortage occurs, and the growth rate of output is lower than that under the first-best allocation.

In our numerical simulations, we focus on the slowdown in output growth in the US after the Great Recession. We measure the explanatory power of our model by the improvement in forecasts of slowdown in output growth compared to the forecasts of the linear trend. For the US data, our model explains slowdown in output growth 99.5 percent better than the forecasts of the linear trend. Our simple growth model explains the realized output trend after the Great Recession surprisingly better than the linear trend expected before the Great Recession. We also conduct numerical simulations for Japan. The simulation results also perform well for the Japanese data in the 1990s. In particular, the model explains slowdown in output growth 98.4 percent better than the forecasts of the linear trend.

Our model also predicts the permanent decline in the real interest rate, which is linked to the

[4] See Barattieri, Basu, and Gottschalk (2014), Sigurdsson and Sigurdardottir (2016), Schmitt-Grohé and Uribe (2016), Fallick, Lettau, and Wascher (2016), Hazell and Taska (2020), and Grigsby, Hurst, and Yildirmaz (2021), among others.

[5] We introduce the DNWR into the growth model. Recent studies emphasize that incorporating nominal rigidity into growth models yields important policy implications. See Oikawa and Ueda (2018) and Miyakawa, Oikawa, and Ueda (2022).

secular stagnation hypothesis.[6] In the standard consumption Euler equation, the real interest rate is determined by the household's subjective discount rate and the growth rate of consumption. However, as pointed out by Michaillat and Saez (2021), wealth preferences in the consumption Euler equation create discounting in the real interest rate. This discounting leads to a persistently low real interest rate, consistent with the data under secular stagnation.

In our model, inflation also declines persistently until the output trend deviates from the productivity trend. This observation is consistent with inflation in the US before and after the Great Recession. Hall (2011) points out that US inflation declined during the 1990s but became stable even in the presence of long-lasting slack in the economy from the Great Recession. Our model interprets the missing deflation as being a consequence of the binding DNWR, where inflation stops declining even if aggregate demand falls short of aggregate supply.

Previous studies have fallen into one of four groups in explaining secular stagnation. The first focuses on the productivity slowdown (e.g., Fernald (2015), Gordon (2015), Takahashi and Takayama (2022)). This group emphasizes the decline in productivity growth as a source of the secular stagnation. In our model, (labor) productivity growth also declines and the decline is driven by the decline in aggregate demand growth. Moreover, we intentionally remove exogenous declines in productivity growth from the model because we highlight the degree to which the model with wealth preferences alone explains the observed slowdown in output growth. The second group focuses on the impact of demographic changes on savings in explaining the declining real interest rate. (e.g., Carvalho, Ferrero, and Nechio (2016), Gagnon, Johannsen, and Lopez-Salido (2021), Jones (2022)). We exclude this potentially important factor from our model because we focus on the mechanism behind the impact of wealth preferences on savings. The third group relies on debt deleveraging (e.g., Hall (2011), Eggertsson and Krugman (2012), Mian and Sufi (2014), Guerrieri and Lorenzoni (2017), Eggertsson, Mehrotra, and Robbins (2019)). Among these studies, Eggertsson, Mehrotra, and Robbins (2019) introduce debt deleveraging into a model that incorporates declines in productivity and changes in demography. They numerically evaluate their model of the secular stagnation and discuss its policy implications.[7] Our paper is categorized into

[6] Characterizations of secular stagnation appear in Baldwin and Teulings (2014) and Krugman (2014).

[7] Ikeda and Kurozumi (2019) discuss monetary policy rules to prevent secular stagnation in a model with financial frictions and endogenous total factor productivity growth. Kobayashi and Ueda (2022) argue that a fear of large-scale taxation and capital misallocation arising from a debt crisis may be a driving force of the output slowdown in Japan. Mian, Straub, and Sufi (2021b) and Mian, Straub, and Sufi (2021a) point out that the combination of increases in

the fourth group, which introduces wealth preferences into a standard macroeconomic model. This group assumes a strong desire for liquidity or wealth (e.g., Michau (2018), Illing, Ono, and Schlegl (2018)).[8] The study closest to ours is Michau (2018), who incorporates wealth preferences and DNWR into the standard neoclassical growth model and shows the existence of both the neoclassical and stagnation steady states in his model. By contrast, our model has a unique steady state and endogenously generates a regime change from efficient allocation in the neoclassical economy to inefficient allocation in the stagnant economy.[9] Furthermore, we implement a quantitative assessment of whether the model with wealth preferences explains the slowdown in output growth together with permanent trend declines in the real interest rate and inflation.

The paper is organized as follows. Section 2 presents our simple growth model. Section 3 studies the model dynamics and presents the main analytical results. In Section 4, we simulate the model and show that its predictions are consistent with the data. Section 5 concludes.

2 The model

This section first presents the setup of the monetary growth model along with two important model assumptions: wealth preferences and DNWR. We then discuss the implications of DNWR for the goods market. Finally, we define the competitive equilibrium in this model.

top income share and financial deregulation raises the savings of the rich, which lowers interest rates and generates the secular stagnation.

[8] In recent studies, models with wealth preferences are analyzed using the New Keynesian framework (Michaillat and Saez (2021)) and search models (Michaillat and Saez (2022) and Hashimoto, Ono, and Schlegl (2023)).

[9] In this sense, our analysis also differs from Benigno and Fornaro (2018), who develop an endogenous growth model with DNWR. They show that weak growth depresses aggregate demand and that the resulting aggregate demand shortage may lead to the stagnation steady state. In contrast to our study, stagnation arises as a self-fulfilling equilibrium.

2.1 Setup

2.1.1 The household

The representative household solves the following maximization problem:

$$\max_{c_t, m_t, e_t, a_t, h_t} \quad \int_0^\infty \exp(-\rho t)\left[u(c_t) + v(m_t) - \phi(e_t) + \beta(a_t)\right] dt, \tag{1}$$

$$\text{s.t.} \quad \dot{a}_t = r_t(a_t - m_t) - \pi_t m_t + w_t e_t h_t - c_t + \tau_t, \tag{2}$$

$$h_t \leq 1, \tag{3}$$

where a_0 is given and the subjective discount rate ρ is strictly positive. Here c_t is consumption, m_t is real money balances ($m_t = M_t/P_t$, where M_t is nominal money balances and P_t is the price level), e_t is the labor effort (or the quality of labor), and a_t is total real asset holdings. In the budget constraint, r_t is the real interest rate ($a_t - m_t = b_t$ represents the bond holdings of the household), π_t is inflation or the opportunity cost of holding money, w_t is real wages, h_t is hours worked and τ_t is lump-sum transfers from government. Here, wage income is proportional to effective labor $n_t(= e_t h_t)$. The budget constraint (2) indicates that the sources of consumption and saving ($\dot{a}_t$) equal income from asset holdings ($r_t(a_t - m_t) - \pi_t m_t$), labor income ($w_t n_t$), and the lump-sum transfers from government (τ_t). In this maximization problem, the effective labor supply is decomposed into $h_t \leq 1$ and $0 \leq e_t < \infty$. For simplicity, we assume that the supply of bonds is zero (i.e., $b_t = 0$ in equilibrium); therefore, $a_t = m_t$ holds for all t.

The functions $u(c_t)$, $v(m_t)$, and $\phi(e_t)$ represent the utility from consumption, the utility from real money balances, and the disutility from labor effort, respectively. They take a constant relative risk aversion form. These functions are expressed as follows:

$$u(c_t) = \ln c_t, \tag{4}$$

$$v(m_t) = v\frac{m_t^{1-\eta}}{1-\eta}, \qquad v > 0, \quad \eta > 0, \tag{5}$$

$$\phi(e_t) = \frac{(\phi_t e_t)^2}{2}, \qquad \phi_t > 0, \tag{6}$$

where, with a slight abuse of notations, $v > 0$ represents a parameter for the function $v(m_t)$. The time-varying parameter ϕ_t follows a deterministic process given by

$$\phi_t = \phi_0 \exp(-gt), \quad g > 0, \tag{7}$$

where $\phi_0 > 0$ denotes the initial value of ϕ_t. Here, ϕ_t decreases at the rate g. As we show below, ϕ_t enables us to consider endogenous variations in labor productivity through the household's effort.

Our preference assumptions on the utility from wealth are critical. We assume that the household has an insatiable desire for wealth. The utility from wealth satisfies $\beta'(a_t) > 0$, $\beta''(a_t) \leq 0$, and $\beta'(a_t)$ is strictly positive and constant in equilibrium. The simplest specification that satisfies these conditions for $\beta(a_t)$ is a linear function:

$$\beta(a_t) = \beta \times a_t, \qquad \beta > 0, \tag{8}$$

where β is a strictly positive parameter, again with a slight abuse of notations, for the function $\beta(a_t)$. This specification follows Michau (2018) and extends Ono (1994) and Ono (2001), in which the household has an insatiable desire for liquidity.

We assume (8) for simplicity, not for the necessity of our main results. A necessary condition for our main results is that marginal utility from wealth is strictly positive and constant in equilibrium. As argued by Michau (2018) and Michaillat and Saez (2021), there are a variety of alternative specifications for the utility from wealth that generate positive constant marginal utility. In these studies, while the concavity of the utility function is ensured, marginal utility from wealth is constant in equilibrium.[10] We employ linear utility (8) because these specifications lead to the same results in this paper.[11]

[10] Michau (2018) and Hashimoto, Ono, and Schlegl (2023) consider the preferences for wealth excluding money, $\beta(a_t - m_t^s)$, where m_t^s is the real money supply and the household takes it as given. They assume that $\beta'(a_t - m_t^s) > 0$, $\beta''(a_t - m_t^s) < 0$, and $\lim_{a_t \to \infty} \beta'(a_t - m_t^s) = 0$, but $\beta'(0) > 0$. Thus, marginal utility from wealth is constant in equilibrium, where $b_t = a_t - m_t^s = 0$. Ono and Yamada (2018) and Michaillat and Saez (2021) allow for utility from relative wealth $a_t(i) - \tilde{a}_t$. Here, $a_t(i)$ denotes wealth at the individual household level, and $\tilde{a}_t$ is average wealth in the economy, which the household takes as given. They assume that $\beta'(a_t(i) - \tilde{a}_t) > 0$, $\beta''(a_t(i) - \tilde{a}_t) < 0$ and $\lim_{a_t(i) \to \infty} \beta'(a_t(i) - \tilde{a}_t) = 0$, but again $\beta'(0) > 0$ where $a_t(i) = \tilde{a}_t$.

[11] Constant marginal utility results in equilibrium money holdings beyond the amount the consumers use for their transactions. Nevertheless, constant marginal utility is not necessarily inconsistent with the neuroscientific evidence. Based on lab experiments, Camerer, Loewenstein, and Prelec (2005) argue that "people value money without carefully computing what they plan to buy with it." (p. 35)

The first-order conditions are:

$$\frac{v'(m_t)}{u'(c_t)} = r_t + \pi_t, \tag{9}$$

$$\frac{\dot{c}_t}{c_t} = r_t - \rho + \frac{\beta'(a_t)}{u'(c_t)}, \tag{10}$$

$$\frac{\phi'(e_t)/h_t}{u'(c_t)} = w_t, \text{ if the DNWR is not binding,} \tag{11}$$

$$h_t = 1, \tag{12}$$

and the transversality condition is $\lim_{t\to\infty} \exp(-\rho t)u'(c_t)a_t = 0$. In (9), the household pays the opportunity cost of holding money, $r_t + \pi_t$, to receive the marginal benefits $v'(m_t)$ (or $v'(m_t)/u'(c_t)$ when measured in units of consumption goods). In (10), the household pays the marginal cost of savings, $\rho + \dot{c}_t/c_t$. This is the household's consumption discount rate, which allows for the household's risk aversion.[12] Regarding the marginal benefits of savings, the household receives market returns on bonds r_t and the marginal utility from wealth $\beta'(a_t)$ (or $\beta'(a_t)/u'(c_t)$ when measured in units of consumption goods). When the wealth preferences are absent, (10) reduces to the standard Euler equation $\dot{c}_t/c_t = r_t - \rho$ and savings only yield market returns r_t. When the wealth preferences are present, however, savings generate additional benefits of $\beta'(a_t)/u'(c_t)$. Thus, the household would give up more consumption and accept a lower interest to enjoy holding more wealth. Equation (11) has the standard interpretation that the marginal rate of substitution between effort and consumption (per unit of hours worked) equals the real wage. If we incorporate (12) into (11), the equation reduces to the standard first-order condition for labor supply. As we will discuss in Section 2.2, however, this equation only holds with equality when DNWR is not binding.

Eliminating r_t from (9) and (10) and allowing for $b_t = 0$ in equilibrium yields the condition for aggregate demand:

$$\Omega(m_t, c_t) = \rho + \frac{\dot{c}_t}{c_t} + \pi_t, \tag{13}$$

$$\text{where} \qquad \Omega(m_t, c_t) \equiv \frac{v'(m_t) + \beta'(m_t)}{u'(c_t)}.$$

[12]The consumption discount rate is generally represented by the sum of the steady-state discount rate and the growth rate of marginal utility. In the equation, it is given by $\rho - [du'(c_t)/dt][1/u'(c_t)] = \rho + \dot{c}_t/c_t$.

Here $\Omega(m_t, c_t)$ denotes the marginal benefits of holding wealth (measured in units of consumption goods). In $\Omega(m_t, c_t)$, $v'(m_t)$ is benefits of increasing money, and $\beta'(m_t) = \beta'(a_t)$ is the additional benefit of increasing wealth. The right-hand side of (13) represents the opportunity cost of holding wealth. Thus, (13) determines the rate at which the household substitutes wealth for consumption. To hold an additional unit of assets, the household must give up consumption goods by an amount equal to the household's consumption discount rate $(\rho + \dot{c}_t/c_t)$ and the inflation rate (π_t).

2.1.2 The firm

There is a representative firm in a competitive market in the economy. In our model, the firm's technology is linear in effective labor n_t:

$$y_t = n_t, \tag{14}$$

where y_t is output. With this production function, the firm's effective labor demand condition is:

$$w_t = 1. \tag{15}$$

2.1.3 The downward nominal wage rigidity

One of the most important assumptions in our model is the DNWR in the labor market. Following the literature (e.g., Schmitt-Grohé and Uribe (2016)), we assume that nominal wage inflation $\dot{W}_t/W_t$ cannot be lower than the lower bound γ:

$$\frac{\dot{W}_t}{W_t} \geq \gamma. \tag{16}$$

2.1.4 The government

The government has a budget constraint $\tau_t = \mu m_t^s$, where $m_t^s = M_t^s/P_t$ and M_t^s is the nominal money supply. Throughout this paper, we assume that the money growth rate is strictly positive $(\mu > 0)$ and sufficiently high:

$$\mu > \gamma, \tag{17}$$

which means that the money growth rate always exceeds the lowest level of inflation.

2.2 Implications of DNWR for the goods market

To explore the implications of DNWR for the goods market, we first rewrite the equation of DNWR as a complementary slackness condition in the labor market:

$$\left(\frac{\dot{W}_t}{W_t} - \gamma\right)(n_t^f - n_t) = 0,$$

where n_t^f is the first-best allocation of effective labor when DNWR is not binding. If the nominal wage inflation exceeds γ, the labor market achieves the first-best allocation of effective labor. If $\dot{W}_t/W_t$ is equal to γ, DNWR is binding. As a result, the demand for effective labor determines the allocation of n_t in the labor market.

We translate the complementary slackness condition in the labor market into that in the goods market. Note that DNWR equates to downward nominal price rigidity $\pi_t \geq \gamma$ because real wages $w_t = W_t/P_t$ are constant from (15). Furthermore, the production function is $y_t = n_t$. Combining these relations with the above condition yields:

$$(\pi_t - \gamma)(y_t^f - y_t) = 0, \tag{18}$$

where y_t^f is the first-best allocation of output.

The economy has two regimes. If inflation is high ($\pi_t > \gamma$), the goods market achieves the first-best allocation $y_t = y_t^f$. We refer to this regime as *the high-inflation regime.* Alternatively, if the goods market fails to achieve the first-best allocation, namely $y_t < y_t^f$, inflation hits the lower bound ($\pi_t = \gamma$).[13] We refer to this regime as *the low-inflation regime.*

When $\pi_t > \gamma$, output grows at an exogenous rate of $g > 0$. As ϕ_t decreases at the rate $g > 0$, marginal disutility of effort also decreases. This decline in ϕ_t raises the household's effort. Furthermore, (11) holds with equality. The assumptions in the model allow us to derive:

$$y_t^f = \phi_t^{-1} = \phi_0^{-1} \exp(gt), \tag{19}$$

which means that output grows at the exogenous rate $g > 0$.[14]

[13]We exclude the case of $y_t > y_t^f$ because it does not achieve feasibility of production.

[14]To derive this equation, note that (11) and (15) imply that $\phi'(y_t) = u'(y_t)$ because $c_t = y_t = e_t h_t$ and $h_t = 1$

When $\pi_t = \gamma$, (11) no longer holds with equality. In particular, we obtain:

$$w_t > \frac{\phi'(e_t)/h_t}{u'(c_t)}, \text{ if the DNWR is binding.} \tag{20}$$

Here, while $h_t = 1$ continues to hold, DNWR introduces a wedge into the supply of effort. That is, the household cannot supply the optimal level of effort it desires at a given wage to the firm. We prove that (20) is equivalent to $y_t < y_t^f$.[15] Thus, (11) and (20) are consistent with the complementary slackness condition (18).[16]

Labor productivity varies endogenously depending on whether DNWR is binding. In this model, labor productivity is simply $y_t/h_t = e_t$ because of the production function $y_t = n_t = e_t h_t$. Increased effort improves the quality of labor, and the improved quality of labor enhances labor productivity. When DNWR is not binding, labor productivity $e_t = y_t^f/h_t$ grows at the rate g as seen from (19). When DNWR binds, labor productivity is no longer determined by y_t^f.

2.3 The competitive equilibrium

We are ready to discuss the competitive equilibrium. The market-clearing conditions are:

1. Goods market $c_t = y_t$,
2. Labor market $(\pi_t - \gamma)(y_t^f - y_t) = 0$,
3. Money market $m_t = m_t^s$,
4. Bond market $a_t - m_t = 0$.

A *competitive equilibrium* of the model is the set of allocations $\{c_t, y_t, m_t, a_t\}$ and prices $\{w_t, r_t, \pi_t\}$ that satisfy the following: (i) the representative household maximizes (1) subject to (2) and (3);

for all t. The utility functions (4) and (6) lead to $\phi_t^2 y_t = 1/y_t$. Given that the goods market achieves the first-best allocation (i.e., $y_t = y_t^f$), this condition leads to $y_t^f = 1/\phi_t = \phi_0^{-1}\exp(gt)$.

[15]Note that $w_t = 1$ and $h_t = 1$. Together with the goods market-clearing condition $c_t = y_t = e_t$, (20) becomes $(1 - y_t^2\phi_t^2) > 0$. This inequality is further rewritten as $(1 - y_t\phi_t)(1 + y_t\phi_t) > 0$. As y_t and ϕ_t are both strictly positive, this inequality is simplified to $1/\phi_t - y_t > 0$. Noting the definition of y_t^f given by (19), $y_t^f = 1/\phi_t$. Therefore, we have $y_t < y_t^f$.

[16]When DNWR is not binding, the household's decision of e_t influences its decision of c_t and m_t (see (9)–(11)). Once DNWR binds, however, (20) implies that the household's willingness to supply effort does not affect e_t, and thus the labor demand determines e_t in equilibrium. Note that the household's decisions of c_t and m_t are independent of e_t (See (9) and (10)). Consequently, the household does not need to reoptimize c_t and m_t after observing the difference between e_t and e_t^f in the labor market.

(ii) the representative firm maximizes profits; (iii) the government's transfers and money supply are specified as above; and (iv) all markets clear except for the labor market. The labor market-clearing condition depends on the complementary slackness condition (18).

3 The model dynamics

This section investigates the model dynamics from $t = 0$ to $t = \infty$, given the initial state at $t = 0$. We first show the system of equilibrium conditions under the two inflation regimes. Then, we demonstrate that a unique transition path to the "stagnation" steady state exists under wealth preferences. Along the transition path to the stagnation steady state, slowdown in output growth occurs endogenously. Finally, we show that the model without wealth preferences fails to generate a slowdown in output growth.

Throughout this section, we rely on the equation for aggregate demand, which is the key equation for understanding the model:

$$\Omega(m_t, y_t) \quad = \quad \rho + \frac{\dot{y}_t}{y_t} + \pi_t, \tag{21}$$

where we integrate the goods market-clearing condition with (13).

It is convenient to define the threshold value of $\Omega(m_t, y_t)$ for which output grows at the rate g (i.e., the first-best allocation) but inflation is as low as γ (i.e., $\pi_t = \gamma$). Substituting $\dot{y}_t/y_t = g$ and $\pi_t = \gamma$ into (13) gives the threshold value of $\Omega(m_t, y_t)$:

$$\Omega^* = \rho + g + \gamma. \tag{22}$$

We use the threshold Ω^* to evaluate the allocation and prices in the two inflation regimes.

3.1 High-inflation regime

The high-inflation regime is characterized by the first-best allocation $y_t = y_t^f$ and high inflation $\pi_t > \gamma$, where DNWR is not binding in (18). Using the goods and bond market-clearing conditions

$c_t = y_t$ and $b_t = 0$ (or $a_t = m_t$), we summarize the system of equilibrium conditions as follows:

$$\frac{\dot{y}_t}{y_t} = g, \tag{23}$$
$$\frac{\dot{m}_t}{m_t} = \mu - \gamma - [\Omega(m_t, y_t) - \Omega^*], \tag{24}$$
$$\pi_t = \gamma + [\Omega(m_t, y_t) - \Omega^*], \tag{25}$$
$$r_t = \rho + g - \frac{\beta'(m_t)}{u'(y_t)}. \tag{26}$$

Equation (23) immediately follows from (19) because $y_t = y_t^f$ holds under the first-best allocation. Next, we derive (26) from (10) using $\dot{y}_t/y_t = g$ in the high-inflation regime. Equation (25) is derived from (21) and (22). Finally, (24) is derived from the definition of $m_t = M_t/P_t$, $\dot{m}_t/m_t = \mu - \pi_t$. In solving the model, we compute output from $y_t = y_t^f = \phi_0^{-1}\exp(gt)$, given ϕ_0. Given $m_0 = a_0$, m_t can be solved numerically for m_t from (24).

3.1.1 Low-inflation regime

The low-inflation regime is characterized by inefficient allocation $y_t < y_t^f$ and the lower bound of inflation $\pi_t = \gamma$, where DNWR is binding in (18). Using the threshold value Ω^* given by (22), we summarize the equilibrium conditions as follows:

$$\frac{\dot{y}_t}{y_t} = g - [\Omega^* - \Omega(m_t, y_t)], \tag{27}$$
$$\frac{\dot{m}_t}{m_t} = \mu - \gamma, \tag{28}$$
$$\pi_t = \gamma, \tag{29}$$
$$r_t = \rho + g - \frac{\beta'(m_t)}{u'(y_t)} - [\Omega^* - \Omega(m_t, y_t)]. \tag{30}$$

Equation (29) shows that DNWR is binding. Equation (28) immediately follows from (29) because $\dot{m}_t/m_t = \mu - \pi_t$. To obtain (27), we use (21) and (22).[17] We derive (30) from (10) and (27).

Comparisons between the two regimes reveal that the difference between $\Omega(m_t, y_t)$ and Ω^* matters for the model dynamics. For example, (27) indicates that output growth is lower than g under the low-inflation regime if and only if $\Omega(m_t, y_t) < \Omega^*$. Furthermore, (25) suggests that by

[17] In particular, we use (21) to obtain $\dot{y}_t/y_t = \Omega(m_t, y_t) - \rho - \pi_t$. Because $\pi_t = \gamma$, $\dot{y}_t/y_t = \Omega(m_t, y_t) - \rho - \gamma$. By adding and subtracting g in this equation, we obtain $\dot{y}_t/y_t = g - \Omega^* + \Omega(m_t, y_t)$ from (22).

how much inflation under the high-inflation regime exceeds γ depends on the size of $\Omega(m_t, y_t) - \Omega^*$. The real interest rate under the low-inflation regime depends on the difference between $\Omega(m_t, y_t)$ and Ω^*. If $\Omega(m_t, y_t) < \Omega^*$, the difference generates downward pressure on the real interest rate as is seen from (30).

3.2 The stagnation steady state

We characterize the steady state before analyzing the model dynamics. The steady state in our model is the "stagnation" steady state where (i) DNWR binds and (ii) output converges to a constant value. We show that the stagnation steady state arises under our preference assumptions.

Let t^* be the period in which $\Omega(m_t, y_t)$ becomes equal to Ω^*. When $\Omega(m_t, y_t) = \Omega^*$, output growth is g and inflation is as low as γ. Equation (28) implies that m_t goes to ∞ as $t \to \infty$ because of (17). In particular, m_t can be solved as:

$$m_t = m_{t^*} \exp[(\mu - \gamma)(t - t^*)] \Rightarrow \lim_{t\to\infty} m_t = \infty \text{ for } t \geq t^*, \tag{31}$$

where m_{t^*} is the real money balances evaluated at $t = t^*$.

To prove the existence of the stagnation steady state, we use the transversality condition: $\lim_{t\to\infty} u'(y_t) a_t \exp(-\rho t) = \lim_{t\to\infty} m_t/y_t \exp(-\rho t) = 0$. Taking the time derivative of this condition translates the condition into $\dot{m}_t/m_t - \dot{y}_t/y_t - \rho < 0$. In the stagnation steady state, $\dot{m}_t/m_t = \mu - \pi_t = \mu - \gamma$ (because (i) DNWR binds) and $\dot{y}_t/y_t = 0$ (because (ii) output converges to a constant value). Thus, $\mu - \gamma < \rho$ ensures the transversality condition. Note that if the assumption $\mu > 0$ holds, the transversality condition implies that $\rho + \gamma > 0$.

Now, the allocations and prices in the steady state are characterized as follows. Because (i) DNWR binds, inflation equals its lower bound: $\pi^{ss} = \gamma$. Output growth is zero in the stagnation steady state because (ii) output converges to a constant value. These results imply that the right-hand side of (21) is $\rho + \gamma$ in the stagnation steady state. Next, the left-hand side of (21) in the stagnation steady state is:

$$\begin{aligned}\Omega^{ss} &\equiv \lim_{t\to\infty} \Omega(m_t, y_t) \\ &= \lim_{t\to\infty} [v'(m_t) + \beta'(m_t)] y_t \\ &= \beta y^{ss}.\end{aligned} \tag{32}$$

Combining these results, we have the steady-state level of output y^{ss}:

$$y^{ss} = \frac{\rho + \gamma}{\beta}, \tag{33}$$

which is strictly positive because $\rho + \gamma > 0$. For the remaining variables, the steady-state growth rate of real money balances is $\mu - \gamma$ because (28) holds under binding DNWR. The nominal interest rate in the stagnation steady state is zero because of (9): $\lim_{t\to\infty}(r_t + \pi_t) = \lim_{t\to\infty}[v'(m_t)y_t] = 0$. Therefore, the real interest rate in the stagnation steady state is

$$r^{ss} = -\pi^{ss} = -\gamma. \tag{34}$$

3.3 Transition dynamics to the stagnation steady state

We are now ready to discuss the transition path to the stagnation steady state given the initial state at $t = 0$. We first present two lemmas. The first lemma describes the uniqueness of the transition path:

Lemma 1. *Suppose that $g > 0$, $\mu > 0$, $\mu > \gamma$, and $\mu - \gamma < \rho$. Under the preference assumptions specified by (4)–(8) with a strictly positive β, we have a unique dynamic path of output toward the stagnation steady state. Furthermore, assume that $\pi_0 > \gamma$ at the initial state. Then, the economy experiences a regime change from a high-inflation regime to a low-inflation regime at $t = t^*$.*

Proof. See Appendix A.1. □

The second lemma discusses the properties of $\Omega(m_t, y_t)$:

Lemma 2. *Under the assumptions in Lemma 1, the marginal benefits of holding wealth $\Omega(m_t, y_t)$*

satisfy the following:

$$\Omega(m_t, y_t) \;>\; \Omega^*, \quad \text{for} \quad 0 \leq t < t^*,$$
$$\Omega(m_t, y_t) \;<\; \Omega^*, \quad \text{for} \quad t > t^*,$$

Proof. See Appendix A.2. □

Using Lemmas 1 and 2, we demonstrate that the model exhibits an endogenous slowdown in output growth.

Proposition 1. *Under the assumptions in Lemma 1, the slowdown in output growth occurs at* $t = t^*$*. For* $0 \leq t \leq t^*$*, output growth is equal to* g*. For* $t^* < t < \infty$*, output growth is strictly positive but lower than* g*.*

Proof. It is obvious from (23) and (27). Equation (23) shows that output growth is g in the high-inflation regime. Equation (27) and Lemma 2 suggest that output growth is lower than g in the low-inflation regime. □

The slowdown in output growth results from a regime change from a high-inflation to low-inflation regime. In our model, steady growth of output persists as long as DNWR is not binding (i.e., $0 \leq t < t^*$). However, after DNWR binds at $t = t^*$, the growth rate becomes lower than g for $t > t^*$. As we suggested earlier, output growth in the low-inflation regime is lower than g if $\Omega(m_t, y_t) < \Omega^*$. Lemma 2 proves this inequality. Furthermore, output continues to slow until output growth converges to zero in the stagnation steady state at $t = \infty$:

$$\lim_{t\to\infty} \frac{\dot{y}_t}{y_t} = \lim_{t\to\infty} [g + \Omega(m_t, y_t) - \Omega^*] = \beta y^{ss} - \rho - \gamma = 0 \tag{35}$$

where the first equality is from (27) and the second equality is from (33).

The prediction in Proposition 1 is consistent with the fact emphasized in the literature on secular stagnation. That is, output growth in the US was high prior to the Great Recession but was low after the Great Recession. The slowdown in output growth is not transitory in the US data. Similar but more notable observations are confirmed for Japan after the early 1990s.

To better understand the model dynamics, it is useful to investigate the marginal benefits of holding wealth $\Omega(m_t, y_t)$ shown in (21). Lemma 2 means that $\Omega(m_t, y_t)$ declines between the high- and low-inflation regimes. While $\Omega(m_t, y_t)$ is always larger than Ω^* under the high-inflation regime, it is always smaller than Ω^* under the low-inflation regime. Thus, $\Omega(m_t, y_t)$ after t^* must be lower than $\Omega(m_t, y_t)$ before t^*. In simulations implemented in the subsequent section, we confirm that $\Omega(m_t, y_t)$ decreases smoothly over time. Under our specification of preferences, $\Omega(m_t, y_t) = (\beta + v m_t^{-\eta}) y_t$ is decreasing in m_t and increasing in y_t so that there are two offsetting effects on $\Omega(m_t, y_t)$. Numerically, $\Omega(m_t, y_t)$ tends to decrease over time because the effect of m_t on $\Omega(m_t, y_t)$ overwhelms the effect of y_t on $\Omega(m_t, y_t)$. Intuitively, this is because strong preferences for wealth make real money balances grow faster than aggregate demand.

The declining marginal benefits of holding wealth and DNWR give rise to the slowdown in output growth. Equation (21) shows that the marginal cost represented by the right-hand side of the equation must decline in equilibrium. Namely, either output growth ($\dot{y}_t/y_t$) or inflation (π_t) must decline, given a constant ρ. If the economy is initially in the high-inflation regime (i.e., $\pi_0 > \gamma$), output growth can be kept at $\dot{y}_t/y_t = g$. As long as DNWR is not binding, decreases in inflation make it possible to equalize marginal benefit and marginal cost of holding wealth. By contrast, if the economy turns to be in a low-inflation regime (i.e., $\pi_t = \gamma$), inflation no longer decreases. Only through the slowdown in output growth, the equalization of marginal benefit and marginal cost can be achieved.

We emphasize that the aggregate demand shortage drives this slowdown in output growth. In our model, the household's wealth preferences weaken aggregate demand growth by substituting wealth for consumption goods. Initially, because DNWR is not binding, the weakened aggregate demand leads to disinflation and the economy achieves the first-best allocation in output under flexible prices. However, when DNWR makes price adjustment rigid, the weakened aggregate demand determines equilibrium output. As a result, the growth rate of output is g for $t \leq t^*$ and becomes lower than g for $t > t^*$.

There are some remarks on the transition path. First, the initial state of the economy matters for the slowdown in output growth. In Lemma 1, we assume that inflation exceeds γ at $t = 0$ (i.e., $\pi_0 > \gamma$). This inequality is satisfied when wealth accumulation at $t = 0$ is low enough to ensure that the marginal benefits of holding wealth is high. When the marginal benefits of holding wealth is high, inflation is also high (see (21)). As wealth deepening proceeds, the marginal benefits of holding wealth decrease and thus disinflation occurs. When inflation hits the lower bound, output growth starts declining. The timing of the slowdown in output growth depends on preference parameters such as β and the degree of DNWR γ.

Second, the declining marginal benefits of holding wealth also account for the declining real interest rate over time. While Section 4 will demonstrate that the simulated real interest rate declines over time, the following proposition provides the analytical results of the declining real interest rate between the high- and the low-inflation regimes.

Proposition 2. *Let r_{t^*} be the real interest rate when the slowdown in output growth starts, where $r_{t^*} = \rho + g - \beta y_{t^*}$. Under the assumptions in Lemma 1, the real interest rate is strictly higher than r_{t^*} for $0 < t < t^*$, and the real interest rate is strictly lower than r_{t^*} for $t > t^*$.*

Proof. It immediately follows from (26) that $r_t = \rho + g - \beta y_t > r_{t^*} = \rho + g - \beta y_{t^*}$ for $t < t^*$. This is because Proposition 1 implies that output growth is positive; therefore, $y_t < y_{t^*}$ for $t < t^*$. It is straightforward to obtain $r_t = \rho + g - \beta y_t - [\Omega^* - \Omega(m_t, y_t)] < r_{t^*} = \rho + g - \beta y_{t^*}$ for $t > t^*$ from (30). This is because $\Omega^* > \Omega(m_t, y_t)$ from Lemma 2 and $y_t > y_{t^*}$ from Proposition 1. □

Finally, our model also predicts the slowdown in labor productivity growth as discussed by Fernald (2015). He argues that labor productivity slowed in the middle of the 2000s. Recall that labor productivity in our model is $e_t = y_t/h_t$ where $h_t = 1$. As suggested by Proposition 1, y_t initially grows at g and later slows at a rate lower than g. Therefore, our model also explains the observed slowdown in the labor productivity growth.

3.4 The role of wealth preferences

As we emphasized in the previous subsections, strong wealth preferences of $\beta > 0$ are a key assumption for generating the results of Proposition 1. To crystallize the role of wealth preferences, this subsection discusses the model's prediction under no wealth preferences (i.e., $\beta = 0$). The following proposition shows that, if the wealth preferences are absent, $\Omega(m_t, y_t)$ is constant over time.

Proposition 3. *Suppose that the assumptions in Lemma 1 hold except for $\beta > 0$. If $\beta = 0$, $\Omega(m_t, y_t)$ is constant over time. The regime is predetermined and regime change never occurs. There is no endogenous slowdown in output growth.*

Proof. See the Appendix A.3 □

The model without wealth preferences ($\beta = 0$) fails to generate an endogenous slowdown in output growth because $\Omega(m_t, y_t)$ is constant over time. The parameters in the model predetermine the regime in equilibrium. For example, if the economy is initially in a high-inflation regime, the economy is on the balanced growth path where the goods market achieves the first-best allocation with output growth of g. If the economy instead starts from a low-inflation regime, the economy experiences low economic growth given the rigid wage adjustment arising from DNWR. In this case, the allocation is inefficient because the aggregate demand shortage makes economic growth slower than g. In either case, however, we do not observe a slowdown in output growth, in contrast to the case of $\beta > 0$. Moreover, the growth rate of output with the binding DNWR does not converge to zero. Even if DNWR is binding, there is no stagnation steady state under $\beta = 0$. Therefore, the essential ingredient for generating a slowdown in output growth and the stagnation steady state is a strictly positive β in the wealth preferences.

3.5 The role of DNWR

The other key assumption in the model is DNWR. To understand the role of DNWR, we next consider what happens if DNWR is absent, but β is strictly positive. Using (21), Section 3.3 explained that, after π_t hits the lower bound, output growth must decline in response to decreases in $\Omega(m_t, y_t)$. However, a slowdown in output growth can occur even when π_t is decreasing over time in response to decreases in $\Omega(m_t, y_t)$ (See (13)). In this case, DNWR may not be necessary.

If DNWR is removed from the model, however, there is no monetary equilibrium with a strictly positive β. To see this, assume that $\gamma = -\infty$. This assumption ensures that DNWR never binds. In this case, the transversality condition $\mu - \gamma < \rho$ is not satisfied and steady-state output is negative from (33). As we discussed, when $\mu > 0$, the transversality condition becomes $\rho + \gamma > 0$. Thus, γ is at least strictly larger than $-\rho$ to guarantee the equilibrium in this model.

4 Simulating the model

While our model is qualitatively consistent with the observed slowdown in output growth, it is not necessarily clear whether the model numerically explains the data. It is also worth assessing macroeconomic variables other than output growth. In this section, we simulate the model for the US and Japan.[18] We will explore whether our model quantitatively explains the observed slowdown in output growth as well as the declining real interest rate and inflation.

4.1 Calibration

We have eight parameters that need to be calibrated for simulation. Among these eight parameters, we set the subjective discount factor ρ at 0.04. This parameter is assumed to be common between the US and Japan. For the remaining parameters (g, μ, t^*, γ, β, v, η), we set the parameter values based on the data. In what follows, we explain the calibration, taking the US as an example.

The model has directly observable parameters (g, μ, t^*, and γ). Recall that we estimated the linear trend (dot-dashed line in the upper panel of Figure 1) from log real GDP over 1990:Q1 and 2007:Q1. The mean growth rate over the sample period (or the slope of the linear trend) is 0.022 at the annual rate. We take this value as the growth rate of output under the first-best allocation, $g = 0.022$. The money growth rate μ is 0.043, using the mean growth rate of M2 stock (per capita) over 1990:Q1–2019:Q4. Turning to the timing of the regime change from the high-inflation to the low-inflation regime t^*, we compare the linear trend and the cubic trend shown in Figure 1. We interpret that t^* is the period in which the cubic trend (dashed line in the figure) starts falling below the linear trend (dot-dashed line). The resulting t^* is 2001:Q3. Finally, to calibrate γ, we use $\pi_t = \gamma$ after t^* =2001:Q3 (see (29)). We extract trend inflation using a cubic trend over 1990:Q1–2019:Q4 and calculate the mean of the cubic trend after t^* =2001:Q3. The resulting mean trend inflation is 0.017. Here, the actual inflation is the year-on-year inflation calculated from the Personal Consumption Expenditure Price Index Excluding Food and Energy.

The remaining parameters are not directly observable (v, η, and β in the utility functions $v(m_t)$ and $\beta(a_t)$). These three parameters are calibrated from three target values from the data. The first target is the overall model performance in predicting output trend. More specifically, we compute

[18] The replication files are available at `https://sites.google.com/view/kazumainagaki/research`.

the mean squared error (MSE) for output trend between 2007:Q1 and 2019:Q4, where errors are the log-deviations of the model's forecast from the cubic trend in Figure 1. The second target for calibration is the cubic-trend value of the real interest rate in the beginning of 2001:Q3. With this target, we align the level of real interest rates with the data. For the data of the real interest rate, we use 1-year real Treasury yields from the database of the Cleveland Federal Reserve Bank and estimated by Haubrich, Pennacchi, and Ritchken (2012).[19] The real interest rate in the beginning of 2001:Q3 is 1.50 percent in the cubic trend. The third target for calibration is the cubic-trend value of the velocity of money in the beginning of 2001:Q3. The target ensures that the model can generate the observed ratio of consumption and real money balances because the velocity of money is defined as c_t/m_t. For the data of the velocity of money, we employ the velocity of M2. The velocity of money in the beginning of 2001:Q3 is 2.09 in the cubic trend. With these three targets, the resulting values of v, η, and β are $v = 0.072$, $\eta = 4.75$, and $\beta = 0.015$, respectively.

We turn to the parameters for the Japanese economy. We set $g = 0.039$ from the growth rate of real GDP per capita between 1980:Q1 and 1991:Q1 (corresponding to the linear trend in Figure 1). We parameterize $\mu = 0.041$ from per capita M2 growth averaged over 1980:Q2 –2019:Q4. Figure 1 suggests $t^* =$ 1989:Q1 for Japan. Finally, given t^* corresponds to 1989:Q1 for Japan, we calibrate γ at 0.003 based on trend inflation after 1989:Q1. Here, we calculate the trend inflation after 1989:Q1 from the Consumer Price Index for All items less fresh food.[20] To obtain preference parameters for Japan, the same targets are used. We compute the MSE for output trend between 1991:Q1 and 2019:Q4 as the first target. We also take the data of the real interest rate (3.67 percent in cubic trend) and the velocity (1.06 in cubic trend) in the beginning of 1989:Q1 as the second and third targets. We construct the real interest rate using the nominal interest rate and the actual inflation from 1986:Q3 to 2019:Q4.[21] We use the velocity of M2 as in the calibration for the US. After the calibration, we obtained $v = 0.048$, $\eta = 4.96$ and $\beta = 0.031$, respectively.

We solve the model using a method similar to the shooting algorithm. We calculate the transi-

[19] The most recent data are available at `https://www.clevelandfed.org/our-research/indicators-and-data/inflation-expectations/background-and-resources.aspx#research`.

[20] The price index is adjusted for consumption tax hikes in 1997 and 2014.

[21] To compute the 1-year real interest rate, we use the 12-month Japanese yen London Inter-Bank Offered Rate (JPY LIBOR) as an interest rate benchmark over 1986:Q3–2019:Q4. We combine core inflation from 1986:Q3 to 1989:Q4 and tax-adjusted core inflation from 1990:Q1 to 2019:Q4 because the consumption tax-adjusted core inflation is only available from 1990. Note that inflation is not adjusted for the introduction of the consumption tax in 1989:Q2 and the increase in 2019:Q4.

tion path forward from t^* to $\bar{t}$, where $\bar{t}$ is a sufficiently large number to approximate $t = \infty$. In this transition path, the economy moves toward the stagnation steady state. We guess output in the period of a regime change (denoted by y_{t^*}) and compute real money balances in the same period (denoted by m_{t^*}).[22] Together with y_{t^*} and m_{t^*}, we use (27) and (28) to obtain future variables $y_{t+\Delta}$ and $m_{t+\Delta}$, where Δ is a small increment of time. We iterate this calculation forward until we have $y_{\bar{t}} \simeq y_{\bar{t}+\Delta}$ and define $y_{\bar{t}}$ as a candidate of the steady-state output y^{ss}. If $y_{\bar{t}} \simeq y^{ss}$, we conclude that the transition path to the stagnation steady state is obtained. If not, we update the guess of the output y_{t^*} and iterate computations until we have $y_{\bar{t}} \simeq y^{ss}$. Regarding the transition path for $t < t^*$, output growth always equals g. We compute the transition path backward from t^* to $t = 0$, where $t = 0$ refers to the first period of the sample.[23]

4.2 Simulation results

Figures 2 and 3 report our simulation results for the US and Japan, respectively. The upper panel presents log real GDP per capita, the middle panel shows the real interest rate, and the lower panel is inflation. Each panel contains the simulated data (solid line), estimated cubic trend (dashed line), and actual data (dotted line). Our model has no stochastic shocks. Therefore, our model aims to explain the estimated cubic trend rather than the actual data.

4.2.1 The US

The upper panel of Figure 2 presents US output. In this panel, the simulated output at t^* is equalized to the linear trend in the same period. We investigate how closely the simulated output after t^* tracks the cubic trend. Note that, because the simulated output grows at g when $t \leq t^*$, the simulated output is designed to match perfectly with the linear trend, which grows at the same rate of g.

[Figure 2 about here.]

Comparing the solid and dashed lines in the upper panel of Figure 2 reveals that the model

[22] Here we use $\Omega(m_{t^*}, y_{t^*}) = \Omega^*$ to obtain m_{t^*}. The threshold value Ω^* is given by (22).

[23] All variables including $\dot{y}_t/y_t$ and $\dot{m}_t/m_t$ are smoothly connected between the high-inflation and the low-inflation regimes. We can confirm this smooth transition by evaluating $\Omega(m_t, y_t)$ in both systems of equations (23)–(26) and (27)–(30) at Ω^*. At $t = t^*$, the marginal benefits of holding wealth are equal to the threshold level, and (27)–(30) are equivalent to (23)–(26), respectively.

accounts for a substantial fraction of the slowdown in output growth relative to the linear trend (dotted line). In particular, our model explains the cubic trend much better than the linear trend does in terms of MSE. For example, when focusing on the initial 5 years following the onset of the Great Recession, we observe a 92.6 percent improvement in the MSE of our model from 2007Q1 to 2012Q1, compared to the MSE of the linear trend. Over the subsequent 5–10 years (from 2012Q1 to 2017Q1), the improvement in MSE further increases to 99.5 percent.

The middle panel shows the simulated real interest rate. Overall, the model successfully replicates the real interest rate that has decreased since the 1990s. The real interest rate in the cubic trend (dashed line) peaks at 2.43 percent in 1994:Q3 and reaches a minimum at -0.70 percent in 2014:Q1. The simulated real interest rate exhibits a similar pattern to the data. The simulated real interest rate was 2.20 percent in 1994:Q3 and -0.89 percent in 2014:Q1.

As discussed in the previous section, the decline in trend results from decreases in $\Omega(m_t, y_t)$ together with wealth preferences. Under the high-inflation regime, the additional benefit of holding wealth $\beta'(m_t)/u'(y_t) = \beta y_t$ in (30) lowers the real interest rate. Once DNWR binds and regime change occurs, the real interest rate further declines because of additional downward pressure on the real interest rate represented by $\Omega^* - \Omega(m_t, y_t)$ (see (26)).

The lower panel of Figure 2 plots inflation.[24] The simulated inflation (solid line) replicates the estimated cubic trend quite well, especially in replicating the decline in the 1990s. The figure indicates that the simulated inflation decreases steadily until $t = t^*$ (i.e., 2001:Q3) and becomes constant at γ (see (29)). In the data, while trend inflation (dashed line) decreases faster than the simulated inflation, it decreased steadily until 2003:Q2 and became stable at around 1.5–2 percent.[25]

4.2.2 Japan

Figure 3 conducts the same exercise with the Japanese data. As in the US case, the model performs well. Recall that we set the timing of the regime change to 1989:Q1. Thus, changes in model

[24] We use year-on-year inflation to remove noise in inflation.

[25] In our model, the regime change takes place when inflation hits the lower bound. As shown in the lower panel of Figure 2, the trend inflation decreases fast and nearly hits the lower bound prior to the calibrated t^*. To address the possibility that the regime change occurs earlier than t^*, we examine alternative t^*. Indeed, the model can better account for the slowdown in output growth when we set t^* at an earlier period. In particular, when t^* is 5 years earlier (i.e., 1996Q3), our model improves by 99.5 percent for the first 5 years and 99.9 percent for the next 5 years compared to the MSE of the linear trend.

dynamics become significant in the early 1990s. Once again, the model accounts for a significant portion of the slowdown in output growth compared to the linear trend. Shown in the upper panel of Figure 3, the model enhances the forecast by 90.5 percent over the five years following the peak of the business cycle (between 1996Q1 and 2001Q1). Furthermore, the improvement in MSE further increases to 98.4 percent over the subsequent 5–10 years following the business cycle's peak (between 2001Q1 and 2006Q1).[26] The simulated real interest rate closely keeps track of the cubic trend of the real interest rate that declined from about 4.94 percent in 1986:Q3 to a slightly negative value of around -0.53 percent in 2019:Q4. The simulated real interest rate is 4.82 percent in 1986:Q3 and -0.25 percent in 2019:Q4 before converging to the steady-state value of -0.27 percent.[27] Finally, simulated inflation also explains the observed decreases in the actual inflation in the early 1980s.

[Figure 3 about here.]

5 Conclusion

Output growth in the US was persistently low after the Great Recession. In this paper, we explained this slowdown in output growth by introducing wealth preferences and DNWR into a standard monetary growth model. We theoretically showed that our model generates a slowdown in output growth in the transition path to the stagnation steady state. Consistent with the literature on secular stagnation, the model also explains the declining real interest rate over time. Using numerical simulations, we found that our model explains slowdown in output growth 99.5 percent better than the forecasts of the linear trend. We also implemented simulations for Japan and confirmed that the model also explains the stagnation in Japan.

It is quite surprising that a simple model can account for the long-run patterns in the data. However, we are not arguing that wealth preference and DNWR dominate alternative explanations for secular stagnation in the literature. In particular, demographic and financial factors could be important drivers of secular stagnation. Further research that incorporates these factors into our model would enrich our understanding of secular stagnation. Especially for its policy prescriptions, we need a fully-fledged model to answer many remaining questions: What are the implications of

[26] We also confirm that the forecasting power of the model is robust to alternative t^*.

[27] Note that we plot real interest rates from 1986:Q3 because of data availability of the nominal interest rate.

growth-enhancing policy on the model dynamics? What happens to output growth and the real interest rate if we prompt nominal wage adjustment by removing institutional frictions in the labor market? What are the impacts of forward guidance on output growth? Exploring these questions would be important for future research.

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A Proofs of lemmas and propositions

A.1 Proof of Lemma 1

To prove Lemma 1, the phase diagram in the (m_t, y_t) plane is convenient. See Figure 4. We derive the loci in each inflation regime.

[Figure 4 about here.]

We first consider the high-inflation regime. In this regime, (23) implies that $\dot{y}_t > 0$. We thus focus on the $\dot{m}_t = 0$ locus drawn as the red solid line in Figure 4. The $\dot{m}_t = 0$ locus is obtained from (24) and given by $\mu - \gamma - \Omega(m_t, y_t) + \Omega^* = 0$. Under our preference assumptions, $\Omega(m_t, y_t) = (\beta + v m_t^{-\eta}) y_t$. Therefore, the $\dot{m}_t = 0$ locus is:

$$\begin{aligned} y_t &= \frac{\Omega^* + \mu - \gamma}{\beta + v m_t^{-\eta}} \\ &= \frac{\rho + g + \gamma + (\mu - \gamma)}{\beta + v m_t^{-\eta}} \\ &= f_H(m_t), \end{aligned} \tag{36}$$

where the second equality comes from (22) and we define the $\dot{m}_t = 0$ locus as $y_t = f_H(m_t)$. Here m_t increases with time whenever (m_t, y_t) lies to the right of the $\dot{m}_t = 0$ locus. Together with $\dot{y}_t > 0$ under the high-inflation regime, the directions of the changes are indicated by red arrows in the figure.

Next, consider the low-inflation regime. Under the assumption of $\mu > \gamma$ (see (17)), (28) implies that $\dot{m}_t > 0$. We thus focus on the $\dot{y}_t = 0$ locus drawn as the blue solid line in Figure 4. The $\dot{y}_t = 0$ locus is obtained from (27) and given by $g - \Omega^* + \Omega(m_t, y_t) = 0$. Again, noting that $\Omega(m_t, y_t) = (\beta + vm_t^{-\eta})y_t$ and $\Omega^* = \rho + g + \gamma$, we have the $\dot{y}_t = 0$ locus as follows:

$$\begin{aligned} y_t &= \frac{\Omega^* - g}{\beta + vm_t^{-\eta}} \\ &= \frac{\rho + \gamma}{\beta + vm_t^{-\eta}} \\ &= f_L(m_t), \end{aligned} \tag{37}$$

where we define the $\dot{y}_t = 0$ locus as $y_t = f_L(m_t)$. Note that $f_L(m_t) < f_H(m_t)$ holds for any $m_t > 0$ because $\mu - \gamma > 0$ and $g > 0$. Here y_t increases with time whenever (m_t, y_t) lies to the left of the $\dot{y}_t = 0$ locus. Together with $\dot{m}_t > 0$ under the low-inflation regime, the directions of the changes are indicated by blue arrows in the figure.

Let us introduce another locus that determines the regime change. This locus is drawn as the black solid line in Figure 4 and defined as a set of (m_t, y_t) in which output growth is g and inflation is γ. In other words, the locus specifies a set of (m_t, y_t) that satisfies $\Omega(m_t, y_t) = \Omega^*$. Using $\Omega(m_t, y_t) = (\beta + vm_t^{-\eta})y_t$, we rewrite $\Omega(m_t, y_t) = \Omega^*$ as:

$$\begin{aligned} y_t &= \frac{\Omega^*}{\beta + vm_t^{-\eta}} \\ &= \frac{\rho + g + \gamma}{\beta + vm_t^{-\eta}} \\ &= f_T(m_t), \end{aligned} \tag{38}$$

where we define the locus as $y_t = f_T(m_t)$. As shown in Figure 4, we have $f_L(m_t) < f_T(m_t) < f_H(m_t)$ given m_t because of $\mu > \gamma$ and $g > 0$. It is easy to show that the economy is in the high-inflation regime if (m_t, y_t) lies above the locus.[28] When (m_t, y_t) lies below the locus, DNWR is binding and the economy is in the low-inflation regime.

Figure 4 also draws the optimal transition path (curve with arrows) starting from the initial

[28] We prove this claim by contradiction. Suppose that $\pi_t = \gamma$ and $\dot{y}_t/y_t < g$. Then, (21) implies that $\Omega(m_t, y_t) = \rho + \dot{y}_t/y_t + \pi_t < \rho + g + \gamma = \Omega^*$. That is, $\Omega(m_t, y_t) < \Omega^*$. It immediately follows from $\Omega(m_t, y_t) = (\beta + vm_t^{-\eta})y_t$ that $y_t = \Omega(m_t, y_t)/(\beta + vm_t^{-\eta}) < \Omega^*/(\beta + vm_t^{-\eta}) = f_T(m_t)$, which contradicts the supposition. Note also that we do not need to consider $\pi_t < \gamma$ and $\dot{y}_t/y_t > g$ because they are infeasible.

state of the economy. In the figure, $\pi_0 > \gamma$ is assumed. Thus, the economy is in the high-inflation regime, and (m_0, y_0) is located above the locus of $f_T(m_t)$. Once (m_t, y_t) moves to the right of the $f_T(m_t)$ locus, the regime in the economy turns to the low-inflation regime. Note that, as $t \to \infty$, $m_t \to \infty$ from (31) and $\dot{y}_t/y_t \to 0$ from (35). Thus, (m_t, y_t) asymptotically converges to the dotted line located at the bottom where $y_t = y^{ss} = (\rho + \gamma)/\beta$. In the transition, $\dot{y}_t/y_t$ is always positive because (m_t, y_t) is located above the blue solid line (see Figure 4).

We prove two claims to show the uniqueness of the transition path to the stagnation steady state. First, we show that the transition path is saddle-path stable around the stagnation steady state. Second, we prove that the regime change from the high-inflation to the low-inflation regime occurs at a unique pair of $(m_t, y_t) = (m_{t^*}, y_{t^*})$. The first claim ensures that the transition path in the low-inflation regime is unique. The second claim implies that the transition path in the high-inflation regime is unique because (m_t, y_t) is smoothly and uniquely connected to the transition path in the low-inflation regime. With these claims, we prove that the transition path to the stagnation steady state is unique as a whole. It is necessary to prove the second claim because, if (m_t, y_t) moves along the curve represented by $y_t = f_T(m_t)$ in which $\dot{y}_t/y_t = g$ and $\pi_t = \gamma$, multiple transition paths connected to the curve may exist in the high-inflation regime.

Let us prove the first claim. Define $z_t = 1/m_t$ and consider the system of the equations under the low-inflation regime:

$$\begin{aligned} \frac{\dot{y}_t}{y_t} &= g - \left[\Omega^* - \Omega(1/z_t, y_t)\right], \\ \frac{\dot{z}_t}{z_t} &= -(\mu - \gamma). \end{aligned}$$

By linearizing the above two equations around the stagnation steady state $(z_t, y_t) = (0, y^{ss})$, we have the eigenvalues ζ such that:

$$\begin{vmatrix} \rho + \gamma - \zeta & 0 \\ 0 & -(\mu - \gamma) - \zeta \end{vmatrix} = 0,$$

where we use (33) to replace βy^{ss} by $\rho + \gamma$. As mentioned in the main text, we assume that $\rho + \gamma > 0$. The condition $\rho + \gamma > 0$ then ensures that one eigenvalue is positive. In addition,

(17) implies that the other eigenvalue is negative. Because the system of the equations under the low-inflation regime includes one jump variable y_t and one predetermined variable z_t, the transition path is saddle-path stable and unique under this regime. Along any path located above the saddle path, m_t eventually becomes negative so that it is infeasible. Along any path located below the saddle path, y_t eventually becomes zero so that the transversality condition does not hold.

Next, we prove the second claim by contradiction. Suppose that there exist other pairs of (m_t, y_t) in which output grows at a rate of g and inflation is equal to γ. In this case, the transition path of (m_t, y_t) must be along the curve represented by $y_t = f_T(m_t)$ in Figure 4. Note also that m_t and y_t cannot jump between the high- and the low-inflation regimes and (m_t, y_t) must be connected to the unique transition path in the low-inflation regime that goes through (m_{t^*}, y_{t^*}). In other words, if (m_t, y_t) moves along the curve without a jump, then there exists a pair of $(m_t, y_t) = (m_{\tilde{t}}, y_{\tilde{t}})$ that is not only along the curve but is also close to (m_{t^*}, y_{t^*}) in time. Because $(m_{\tilde{t}}, y_{\tilde{t}})$ exists in the neighborhood of (m_{t^*}, y_{t^*}), $\tilde{t} = t^* + \Delta$ where Δ is an infinitesimally small increment of time. In this case, the growth rate of $\Omega_{\tilde{t}}$ must be zero around $t = t^*$ because $\Omega(m_{t^*}, y_{t^*}) = \Omega^* = \rho + g + \gamma$ along the curve.

Let us denote $\Omega(m_t, y_t)$ by Ω_t. The growth rate of Ω_t is given by:

$$\begin{aligned}\frac{\dot{\Omega}_{\tilde{t}}}{\Omega_{\tilde{t}}} &= \frac{\dot{y}_{\tilde{t}}}{y_{\tilde{t}}} - \eta \frac{v}{v + \beta m_{\tilde{t}}^{\eta}} \frac{\dot{m}_{\tilde{t}}}{m_{\tilde{t}}} \\ &= g - \eta \frac{v}{v + \beta m_{\tilde{t}}^{\eta}} (\mu - \gamma) \\ &= 0.\end{aligned} \tag{39}$$

The first equality is derived from the total derivative of $\Omega_t = \Omega(m_t, y_t) = (\beta + v m_t^{-\eta}) y_t$. In the second equality, $\dot{y}_t / y_t = g$ and $\dot{m}_t / m_t = \mu - \gamma$ because $\Omega(m_{\tilde{t}}, y_{\tilde{t}}) = \Omega^*$. The third equality shows that the growth rate of $\Omega_{\tilde{t}}$ must be zero. The above equation suggests that $m_{\tilde{t}} = \{v[\eta(\mu - \gamma) - g]/\beta\}^{1/\eta}$ is constant. However, this contradicts the assumption that $\dot{m}_{\tilde{t}} / m_{\tilde{t}} = \mu - \gamma > 0$ if $\Omega(m_t, y_t) = \Omega^*$ holds.

We conclude that the transition path to the stagnation steady state is unique as a whole. The regime change occurs only at $t = t^*$ and the transition path under the high-inflation regime is smoothly and uniquely connected to the unique transition path under the low-inflation regime.

A.2 Proof of Lemma 2

We first prove that $\Omega(m_t, y_t) > \Omega^*$ for $0 \leq t < t^*$. The economy is in the high-inflation regime in $0 \leq t < t^*$. As shown in Figure 4, y_t is located above the locus of $f_T(m_t)$. Thus, we have $y_t > f_T(m_t)$ for $0 \leq t < t^*$. Then, the definition of $f_T(m_t)$ shown in (38) leads to $\Omega(m_t, y_t) > \Omega^*$ for $0 \leq t < t^*$.

We also prove that $\Omega(m_t, y_t) < \Omega^*$ for $t > t^*$ from Figure 4. The figure indicates that $y_t < f_T(m_t)$ for $t > t^*$ because the economy is in the low-inflation regime. Again, the definition of $f_T(m_t)$ shown in (38) leads to $\Omega(m_t, y_t) < \Omega^*$ for $t > t^*$.

A.3 Proof of Proposition 3

To prove the proposition, we focus on the growth rate of $\Omega_t = \Omega(m_t, y_t) = (\beta + vm_t^{-\eta})y_t$. When $\beta = 0$, the growth rate of $\Omega(m_t, y_t)$ is written as:

$$\frac{\dot{\Omega}_t}{\Omega_t} = \frac{\dot{y}_t}{y_t} - \eta\frac{\dot{m}_t}{m_t}. \tag{40}$$

In the general case of $\beta \neq 0$, $\dot{\Omega}_t/\Omega_t = \dot{y}_t/y_t - \eta v/(v + \beta m_t^{\eta})\dot{m}_t/m_t$. In the case of $\beta = 0$, however, the coefficient on $\dot{m}_t/m_t$ is a constant η. We will show below that Ω_t is constant in equilibrium when $\beta = 0$. The constant Ω_t rules out the possibility of an endogenous slowdown in output growth.

Output growth under the high-inflation regime Suppose that the economy is initially in the high-inflation regime. Substituting (23) and (24) into (40) yields the differential equation for Ω_t under the high-inflation regime:

$$\begin{aligned}\frac{\dot{\Omega}_t}{\Omega_t} &= g - \eta(\mu - \gamma - \Omega_t + \Omega^*) \\ &= \eta(\Omega_t - \Omega_H),\end{aligned} \tag{41}$$

where

$$\Omega_H = \Omega^* + (\mu - \gamma) - \frac{g}{\eta}. \tag{42}$$

Equation (41) is the differential equation for Ω_t with a positive coefficient on Ω_t. If there is a deviation of Ω_t from Ω_H, Ω_t would explode to either ∞ or $-\infty$. Therefore, when $\beta = 0$, only

$\dot{\Omega}_t = 0$ is feasible in equilibrium. Therefore, Ω_t in equilibrium is constant at $\Omega_t = \Omega_H$.

The high-inflation regime under $\beta = 0$ is feasible only when $g < \eta(\mu - \gamma)$. To see this, suppose that $g \geq \eta(\mu - \gamma)$ in the high-inflation regime. In this case, (42) implies that $\Omega_H \leq \Omega^*$. Given $\dot{y}_t/y_t = g$ in the high-inflation regime, (21) and (22) imply that $\Omega_H \leq \Omega^*$ is rewritten as $\Omega_H = \rho + \dot{y}_t/y_t + \pi_t = \rho + g + \pi_t < \Omega^* = \rho + g + \gamma$ and thus $\pi_t < \gamma$. However, it violates the assumption of DNWR, $\pi_t > \gamma$.

If Ω_t is constant at Ω_H, $\dot{y}_t/y_t$, $\dot{m}_t/m_t$, π_t, and r_t are all constant over time. In particular, (23)–(26) reduce to $\dot{y}_t/y_t = g$, $\dot{m}_t/m_t = g/\eta$, $\pi_t = \mu - g/\eta$, and $r_t = \rho + g$. The economy achieves the first-best allocation on the balanced growth path.

Output growth under the low-inflation regime Substituting (27) and (28) into (40) yields:

$$\begin{aligned} \frac{\dot{\Omega}_t}{\Omega_t} &= g + (\Omega_t - \Omega^*) - \eta(\mu - \gamma) \\ &= (\Omega_t - \Omega_L), \end{aligned} \tag{43}$$

where

$$\Omega_L = \Omega^* + \eta(\mu - \gamma) - g. \tag{44}$$

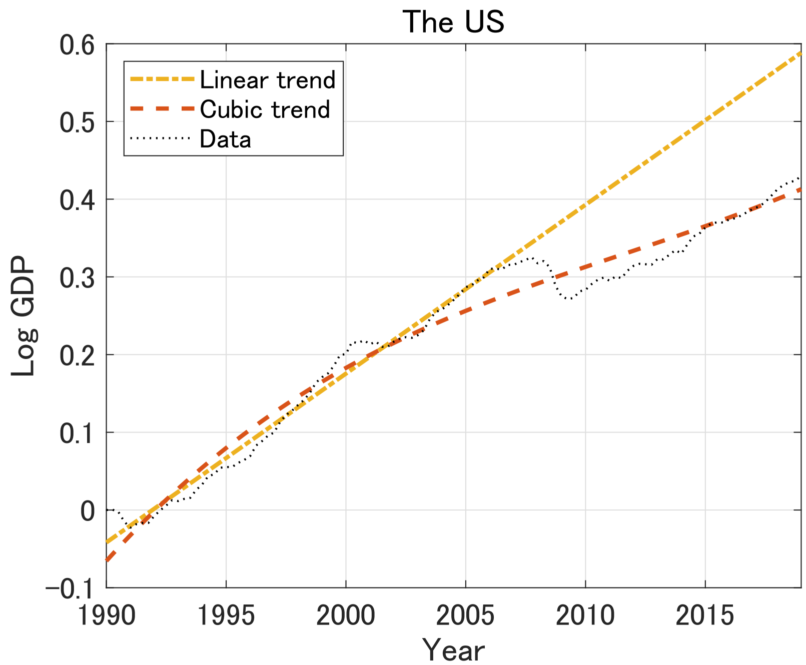
As in (41), (43) is the differential equation for Ω_t with a positive coefficient on Ω_t. Once again, only $\dot{\Omega}_t = 0$ is feasible in equilibrium. Therefore, Ω_t in equilibrium is constant at $\Omega_t = \Omega_L$.

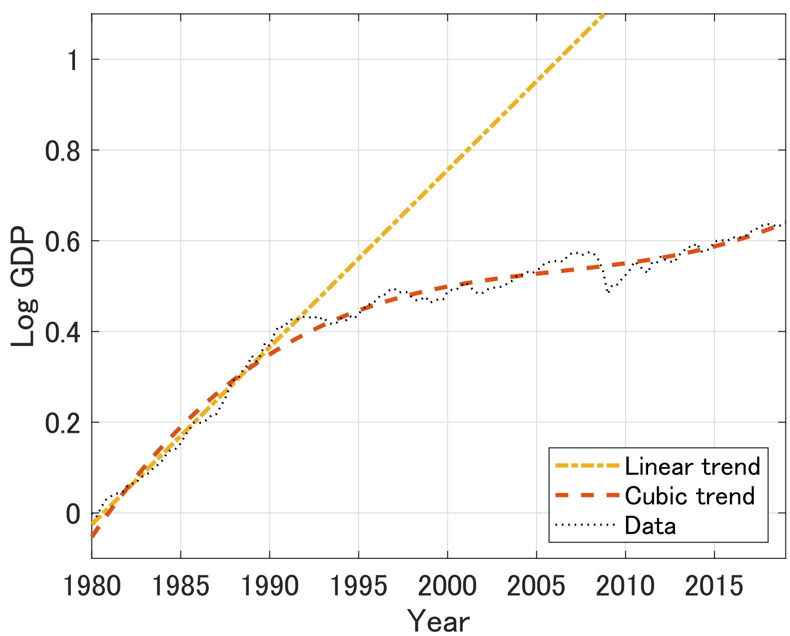
The low-inflation regime under $\beta = 0$ is feasible only when $g \geq \eta(\mu - \gamma)$. To prove this, suppose that $g < \eta(\mu - \gamma)$ in the low-inflation regime. In this case, (44) implies that $\Omega_L > \Omega^*$. Given $\pi_t = \gamma$ in the low-inflation regime, (21) and (22) imply that the condition $\Omega_L > \Omega^*$ is rewritten as $\Omega_L = \rho + \dot{y}_t/y_t + \pi_t = \rho + \dot{y}_t/y_t + \gamma > \Omega^* = \rho + g + \gamma$ and thus $\dot{y}_t/y_t > g$. However, it is infeasible because output growth cannot exceed g.

If Ω_t is constant at Ω_L, then $\dot{y}_t/y_t$, $\dot{m}_t/m_t$, π_t, and r_t are all constant over time. In particular, (27)–(30) reduce to $\dot{y}_t/y_t = \eta(\mu - \gamma)$, $\dot{m}_t/m_t = \mu - \gamma$, $\pi_t = \gamma$, and $r_t = \rho + \eta(\mu - \gamma)$. The economy achieves the inefficient allocation on the balanced growth path because output growth is $\eta(\mu - \gamma)$, which is lower than g.

To summarize, there is no endogenous slowdown in output growth when $\beta = 0$. The parameters in the model fully determine the regime. If $g < \eta(\mu - \gamma)$, the growth rate of output is g and the economy remains in the high-inflation regime. Alternatively, if $g \geq \eta(\mu - \gamma)$, the growth rate of output is $\eta(\mu - \gamma)$ and the economy is always in the low-inflation regime starting from the initial period.

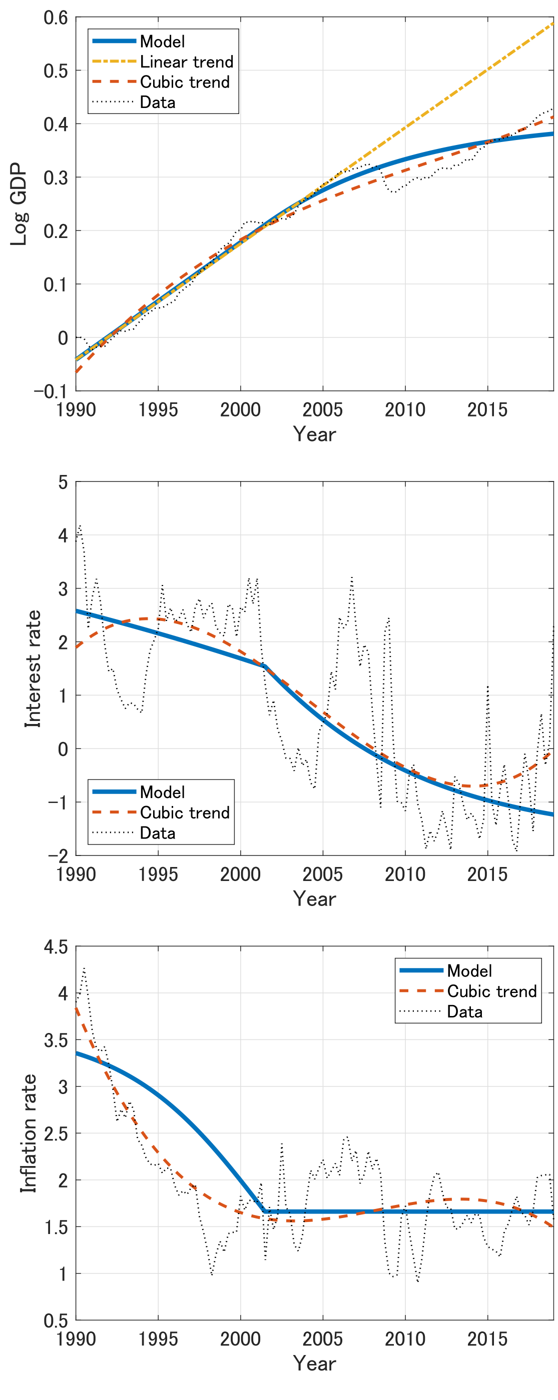
Figure 1: Real GDP per capita and trend





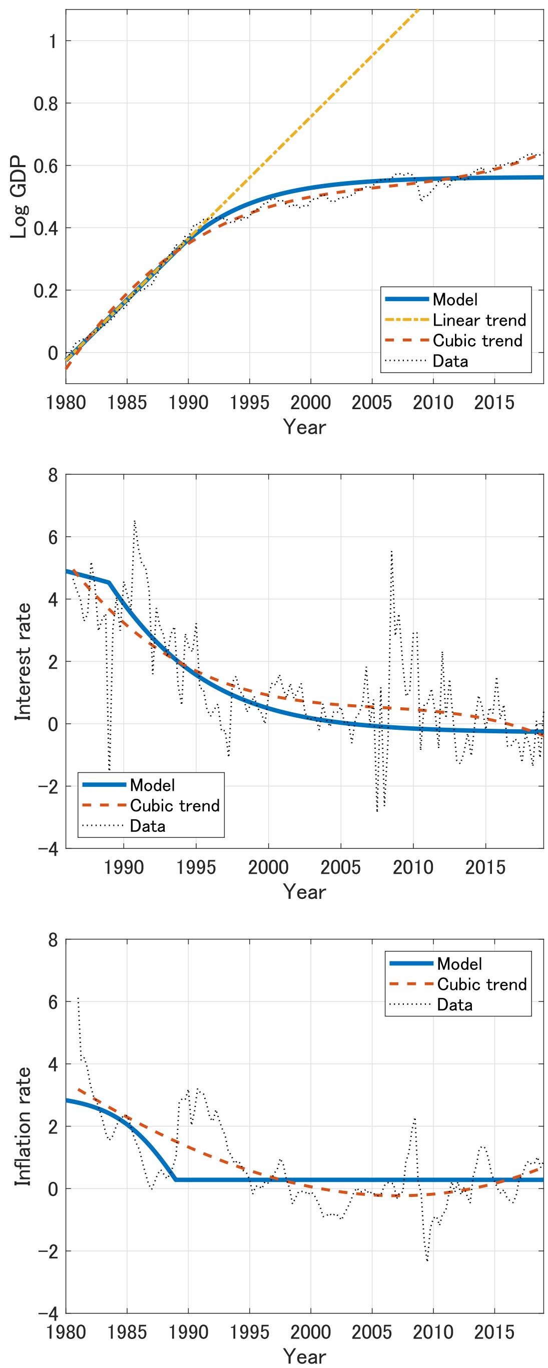
Notes: Each panel of the figure plots the estimated output trend for log real GDP per capita, along with the actual data. The upper panel plots US data for 1990:Q1–2019:Q4, while the lower panel shows Japanese data for 1980:Q1–2019:Q4. In each panel, the dotted line represents actual GDP. The dashed line is the cubic trend of output. The dot-dashed line is the linear trend estimated from the subsample. We use the period over 1990:Q1–2007:Q1 for the US and the period over 1980:Q1–1991:Q1 for Japan as the subsample for their linear trend. The linear trend after the last period of the subsample represents the projected values.

Figure 2: Simulation results for the US: Output, real interest rate, and inflation



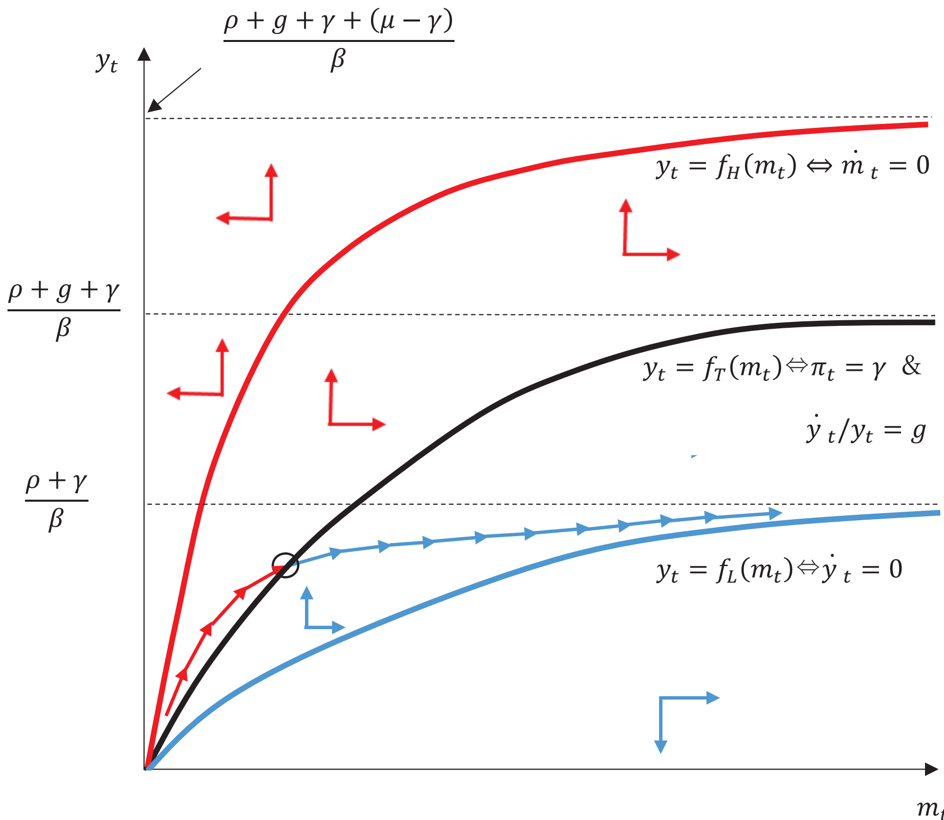
Notes: Each panel of the figure compares the simulated data, estimated cubic trend, and actual data in the US. The solid line represents the simulated data, the dashed line represents the estimated cubic trend, and the dotted line represents the actual data. The upper panel is log real GDP, the middle panel is the real interest rate, and the lower panel is (year-on-year) inflation. The dot-dashed line in the upper panel is the linear trend estimated from the data over 1990:Q1–2007:Q1. For details of the data, see the main text.

Figure 3: Simulation results for Japan: Output, real interest rate, and inflation



Notes: Each panel of the figure compares the simulated data, estimated cubic trend, and actual data in Japan. The dot-dashed line in the upper panel is the linear trend estimated from the data over 1980:Q1–1991:Q1. For other details, see the notes in Figure 2.

Figure 4: Phase diagram



Notes: The red solid line denoted by $y_t = f_H(m_t)$ represents the locus that achieves $\dot{m}_t = 0$ when the economy is in the high-inflation regime. This $\dot{m}_t = 0$ locus determines the direction of change in m_t in the high-inflation regime. In this regime, $\dot{y}_t > 0$ always holds. The blue solid line denoted by $y_t = f_L(m_t)$ is the locus that achieves $\dot{y}_t = 0$ when the economy is in the low-inflation regime. This $\dot{y}_t = 0$ locus determines the direction of change in y_t in the low-inflation regime. In this regime, $\dot{m}_t > 0$ always holds. The black solid line denoted by $y_t = f_T(m_t)$ points to the locus that achieves $\dot{y}_t/y_t = g$ at the lowest level of inflation $\pi_t = \gamma$. If (m_t, y_t) is located above (below) the locus, the economy is in the high- (low-)inflation regime.