Optimal Testing in Disclosure Games

Avi Lichtig and Helene Mass (University of Bonn)

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Disclosure in settings such as regulation and investment

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- Banks disclose information to a financial regulator
- Factory owner self-reports pollution to an environmental regulator
- Investors consider financial statements provided by the company

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- Decision maker aims to align her action with the state; informed agent prefers extreme action
- Disclosure of hard evidence (fabrication of evidence impossible)

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The informed agent decides what to disclose strategically

Partially informed decision maker

Graham at al. (2005): "Many interviewed executives feel that the primary role of voluntary disclosure is to correct investors' perceptions about current or future performance, so that the stock is priced off company-provided information (...) may parlay into easier access to capital"

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Regulators and investors gather information

- Financial regulators run bank stress tests
- Environmental regulators collect air and water samples close to a factory
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How does the decision maker optimally obtain limited information when anticipating its impact on strategic interaction and disclosure?

Uncertain evaluation standards

Dye (1985)+ a limited access to information

- Decision maker has only limited access to information $\rightarrow$ disclosure incentives relevant
- If the agent is unable or unwilling to disclose $\rightarrow$ decision maker has to rely on her own information
- Trade-off between informativeness and disclosure incentives

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Probabilistic pooling resolves the trade-off

- Decision maker induces "intermediate" types to disclose by pooling them with lower types
- Probabilistic pooling is sufficient in order to induce disclosure
- Decision maker chooses the smallest probability sufficient to induce disclosure

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Interpretation: strategic use of uncertain evaluation standards

Literature

Standard voluntary disclosure models

Grossman (1981), Milgrom (1981), Dye (1985), and Jung and Kwon (1988)...

Voluntary disclosure with receiver's own information

Frenkel et al. (2020), Banerjee et al. (2024)...

Incentivizing disclosure in regulatiory settings

Lin (2010), Fukuyama et al. (2000), Evans et al. (2009), Harris and Raviv (2014)....

Communication games with receiver's information acquisition

Lai (2014), Dziuda & Salas (2018), Wei (2021), Matyskova and Montes (2023)...

Model

Disclosure model (based on Dye 1985)

Preliminaries

- Players: sender (informed agent) and receiver (decision maker)
- State X distributed on $[0, 1]$ with cont. diff. cdf F with $f > 0$

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Information

- Receiver chooses a test with at most k realizations $T : [0, 1] \to \Delta\{1, \ldots, k\}$
- Sender's type $(x, e) \in [0, 1] \times \{0, 1\}$, $e = 1$ with prob. $q \in (0, 1)$ (informed sender)
- If $e = 0$, sender sends $\emptyset$; if $e = 1$, sender decides whether to disclose x or send $\emptyset$

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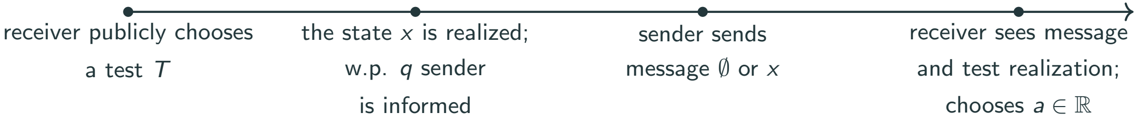
Payoffs

- The sender wants to maximize the receiver's action: $u_S(a,x) = a$ for all x and $a \in \mathbb{R}$
- The receiver wants to align her action with the state: $u_R(a,x) = -(x-a)^2$

Timing


receiver publicly chooses a test T
the state x is realized; w.p. q sender is informed
sender sends message $\emptyset$ or x
receiver sees message and test realization; chooses $a \in \mathbb{R}$

Timing

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receiver sees message and test realization; chooses $a \in \mathbb{R}$
(first: no commitment regarding the actions)

Reminder: equilibrium in Dye 1985

Sender plays a threshold strategy

$$\beta\left(x\right)=\begin{cases}0, & x<\underline{x}\\ 1, & x\geq\underline{x}\end{cases}$$

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The receiver's posterior-mean (nondisclosure)

$$\mathbb{E}[x|ND(\underline{x})] = \frac{qF(\underline{x})\mathbb{E}[x|x < \underline{x}] + (1-q)\mathbb{E}[x]}{qF(\underline{x}) + (1-q)}$$

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Equilibrium

$$\mathbb{E}[x|ND(\underline{x})] = \underline{x}$$

Solution concept: receiver-preferred PBE

Strategies in subgame induced by T

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Beliefs

$$f^T\left(x|\text{non-disclosure and signal realization is } i\right) = f^T\left(x|\text{nd} \wedge S = i\right) = \frac{\Pr\left(\text{nd} \wedge S = i|x\right) f(x)}{\Pr\left(\text{nd} \wedge S = i\right)}$$

▸ formal

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Sequential rationality

$$\text{(i)}\ \ \beta^T(x) = \begin{cases} 0, & x < \sum_{i=1}^{k} \overbrace{T_i(x)}^{\Pr_T(S=i|x)} a_i^T \\ 1, & x \geq \sum_{i=1}^{k} T_i(x) a_i^T \end{cases}$$

$$\text{(ii)}\ \ \text{For every } i \in \{1, \ldots, k\},\ a_i^T = \int_0^1 f^T\left(x \mid \text{nd} \wedge S = i\right) x\,dx$$

Receiver's optimal test choice

Receiver's loss

$$C(T) := \sum_{i=1}^{k} \int_0^1 \overbrace{T_i(x)}^{\Pr_T(S=i|x)} \underbrace{\left(x - a_i^T\right)^2}_{\text{quadratic loss } u_R(a_i^T, x)} \overbrace{\left[q\left(1 - \beta^T(x)\right) + (1-q)\right]}^{\Pr(\text{nd}|x)} f(x)dx$$

where $(\beta^T, \mathbf{a}^T)$ is the receiver-preferred equilibrium induced by T

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Receiver's optimization problem

$$\min_{T:[0,1]\to\Delta^k} C(T)$$

Analysis

Characterization of the optimal test

For this talk: focus on $k = 2$, binary tests

- Call realization with lower posterior mean "fail", with higher posterior mean "pass"
- Test is a function $T : [0, 1] \to \Delta\{\text{pass}, \text{fail}\}$
- Identify a test with function $T : [0, 1] \to [0, 1]$ assigning a passing probability to every state

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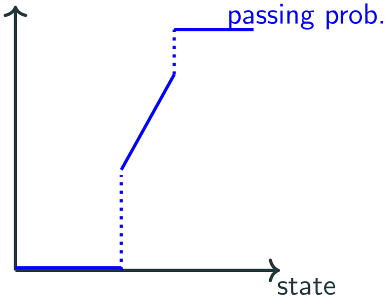
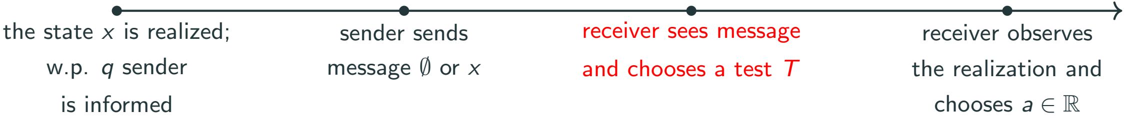
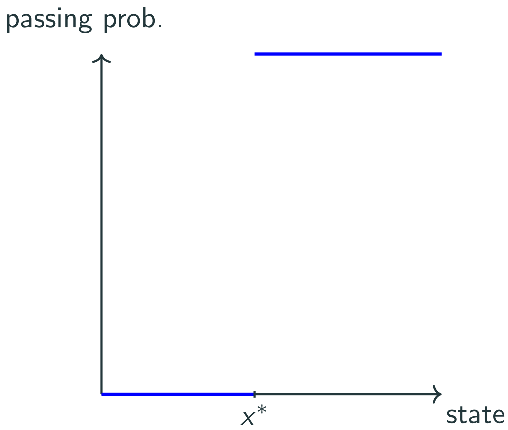


Benchmark - no commitment

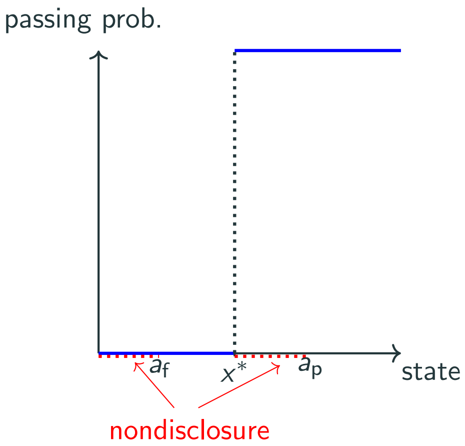


Benchmark: receiver cannot commit to a test



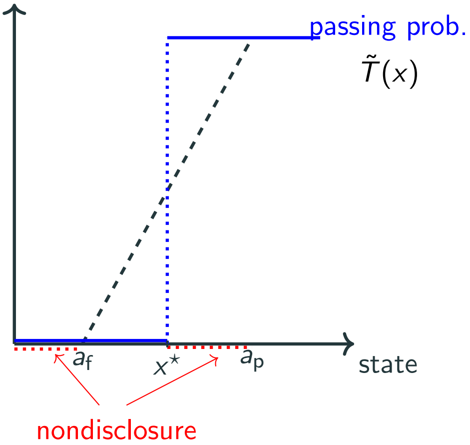
- First, the sender decides whether to disclose, then the receiver chooses a test
- The receiver's best reply solves a single-agent problem $\rightarrow$ deterministic test

Benchmark: receiver cannot commit to a test



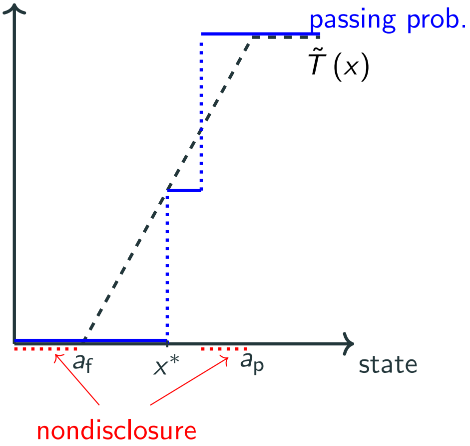
- Sender discloses if the state is higher than receiver's expectation in case of non-disclosure, given by a_f and a_p (x^* is equidistant point)

Benchmark: equilibrium without commitment



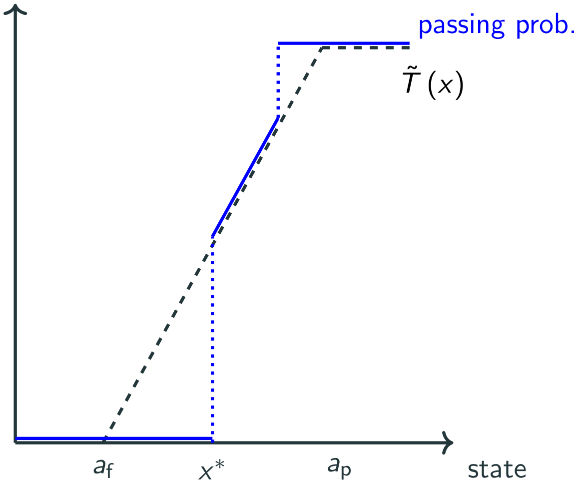
- Sender discloses iff $x \geq T(x)a_p + (1 - T(x))a_f \Leftrightarrow T(x) \leq \tilde{T}(x) := \frac{x - a_f}{a_p - a_f}$

Which improvement is possible with commitment?



- Decreasing the passing probability pools states with low states and induces disclosure
- Probabilistic pooling is sufficient and has no effect on loss at x^*

Which improvement is possible with commitment?



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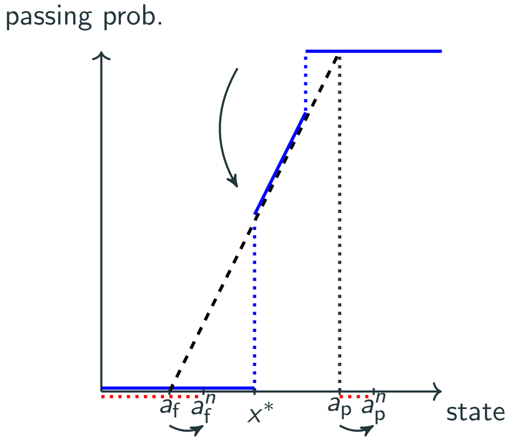

- a_f and a_p are weighted averages of some sets
- Decreasing the passing prob. of intermediate states increases the average of both sets

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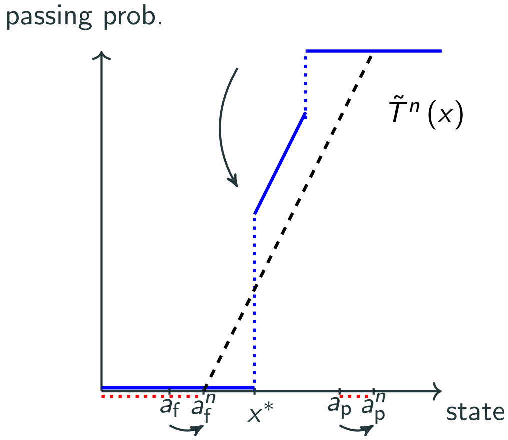

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Main Result

Let the vector $\mathbf{a}^* = (a_1^*, \ldots, a_k^*)$ be the receiver's actions after non-disclosure in the equilibrium induced by the optimal test T^*. The state space $[0,1]$ is partitioned into k intervals and for every interval $i \in \{1, \ldots, k\}$ there exist numbers x_i^- and x_i^+ s.t.

(i) The test assigns probability 1 to realization i for states below x_i^- and probability 1 to realization $i+1$ for states above x_i^+.

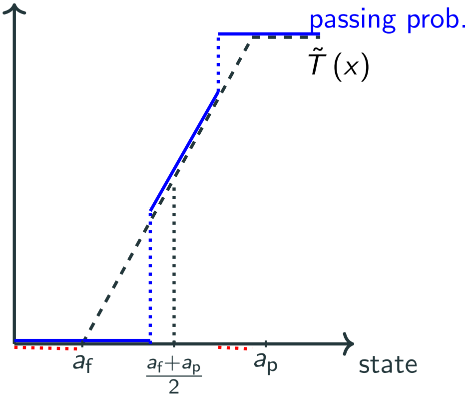
(ii) The disclosure condition is binding on the interval $\left[x_i^-, x_i^+\right]$, i.e. the test is equal to $\tilde{T}$.

(iii) The interval $\left[x_i^-, x_i^+\right]$ lies between the two actions conditional on non-disclosure, i.e.,

$$a_i^* < x_i^- < \frac{a_i^* + a_{i+1}^*}{2} < x_i^+ < a_{i+1}^*$$

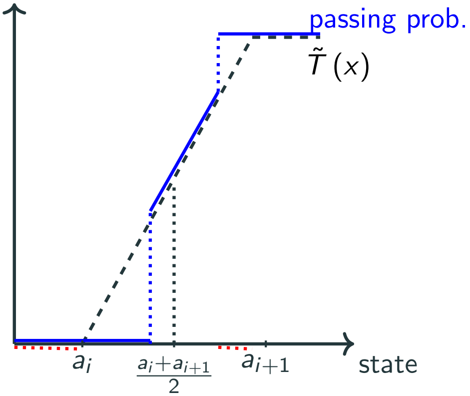
.

Optimal test: characterization



- Between $\frac{a_f+a_p}{2}$ and second jump: decrease passing prob. to induce disclosure
- Between first jump and $\frac{a_f+a_p}{2}$: increase passing prob. to decrease a_f and a_p

Optimal test: characterization



- Between $\frac{a_i+a_{i+1}}{2}$ and second jump: decrease passing prob. to induce disclosure
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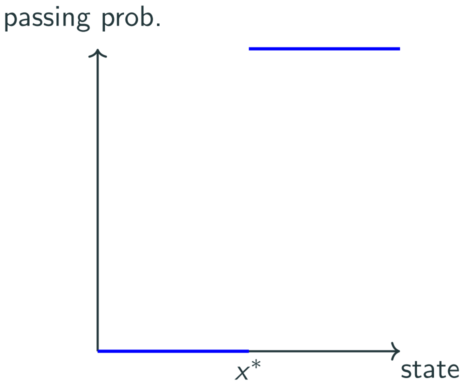
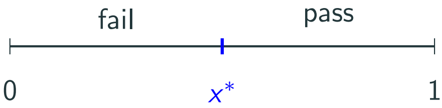
Implementation and Interpretation

Monotone binary tests are simple pass/fail evaluation

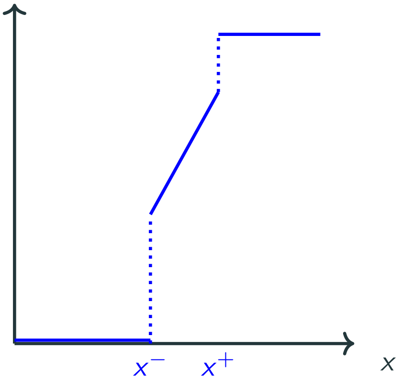


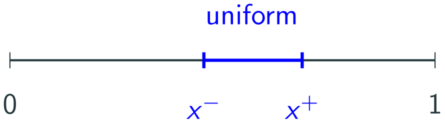


- Implement any monotone binary test as a pass/fail evaluation
- Outcome is pass when the state is above threshold

The optimal binary test is an evaluation with a random threshold

passing prob.





- The passing probability at x corresponds to x being above the random threshold
- $T : [0,1] \to [0,1]$ corresponds to the cdf of the random threshold

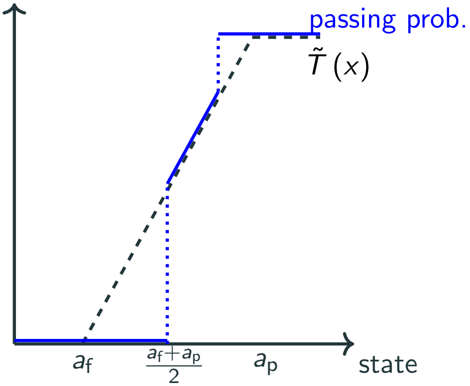
Interpretation

Uncertain evaluation standards

- The optimal test is monotone $\rightarrow$ can be implemented as a simple pass/fail test with a random threshold
- A bank with a low risk level is certain to pass, a bank with a high risk level is certain to fail
- A bank with an intermediate risk level is uncertain about the evaluation outcome
- Empirical observation: financial regulators use opaque evaluation standards in bank stress testing

Commitment to Actions

Optimal test with commitment to actions



- Due to the lack of the equilibrium effect, the first jumping point equals to the equidistant point
- The second jumping point balances informativeness and disclosure

Commitment to actions

Commitment to actions

- If decision maker can commit to actions, the structure of the optimal information gathering process remains (almost) the same
- Decision maker uses commitment power to reward disclosure and punish non-disclosure

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Value of commitment - partially informed receiver

- In general disclosure models, commitment to actions does not change the outcome (Glazer and Rubinstein 2004, Hart et al. 2017)
- If the receiver has access to additional information, commitment plays a role in determining the outcome (actions are not ex-post optimal)
- Empirical evidence for commitment to actions by regulators (e.g. reduced fines in case of self-reporting)

Conclusion

Insights

- The decision maker can leverage the possibility to obtain own information and influence voluntary disclosure
- The optimal test pools intermediate states with low states to incentivize them to disclose
- In order to resolve the trade-off with informativeness, probabilistic pooling is optimal
- Optimal test can be implemented as an evaluation with random thresholds

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Future steps

- Informational budget for receiver
- Complex evidence structures

Proof Sketch

Proof Idea

Challenge

- The sender's disclosure strategy (and the receiver's utility) is not continuous in the test
- The receiver's utility in a given state depends on the whole test and not just on the distribution over realizations in the given state

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General approach

- Assume, by contradiction, that the optimal test does not fulfill one of the characteristics
- Find a directional derivative of the receiver's loss that is strictly negative
- Thus, there exists a nearby test that leads to a strictly lower loss

Monotonicity

- For a given test T and state x, consider the following change:
  - Increasing the probability of success at x by δ
  - Insuring that disclosure behavior does not change (globally); e.g. by decreasing the probability of success for binding states.

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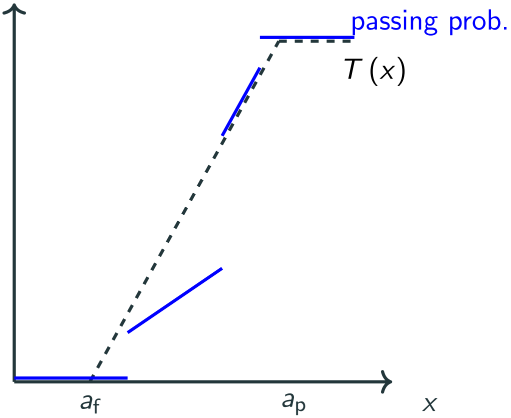
  - Increasing the probability of success at x by δ
  - Insuring that disclosure behavior does not change (globally); e.g. by decreasing the probability of success for binding states.
- Informativeness: $\delta f(x)\left((x-a_f)^2-(x-a_p)^2\right)=(a_p-a_f)(2x-a_p-a_f)$
- Equilibrium effects: $\delta f(x)(a_p-x)C_p$; $\delta f(x)(x-a_f)C_f$

Example: either the test is deterministic or the disclosure condition is binding



- Find a (small) adjustment that decreases the receiver's loss
- Small adjustments do not change disclosure locally
- Choose adjustment that (weakly) decreases a_f and a_p and improves informativeness

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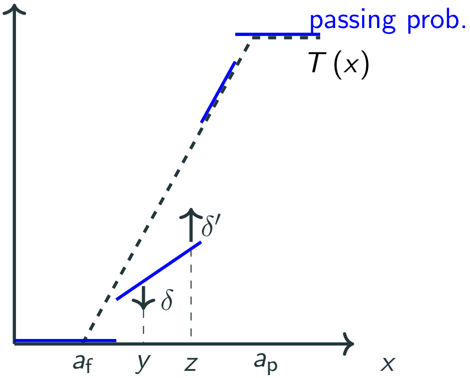
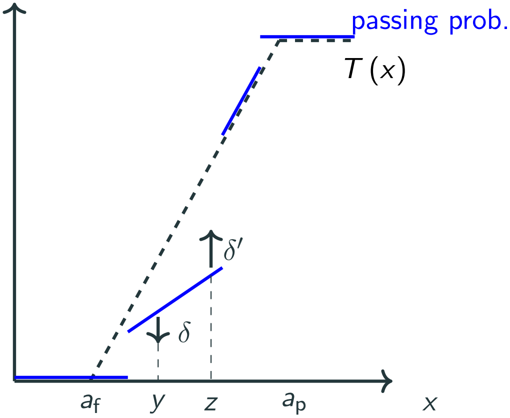


- Decrease the passing probability on $(y-\epsilon, y)$ by δ
- Increase the passing probability on $(z-\epsilon, z)$ by δ'

Show that adjustment induces a lower loss



Equilibrium effect

- Effect of adjustment at y on a_p: take away states from a_p-set that are below a_p (bad)
- Magnitude depends on $a_p - y$ and $f(y)$

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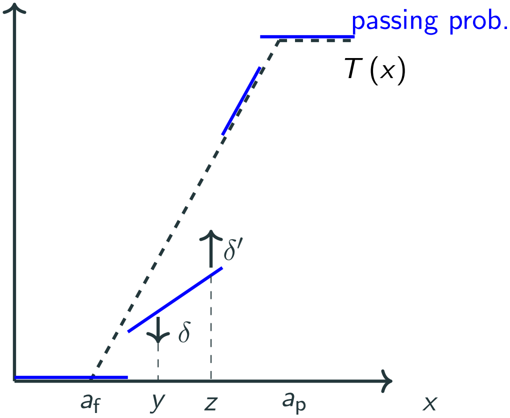

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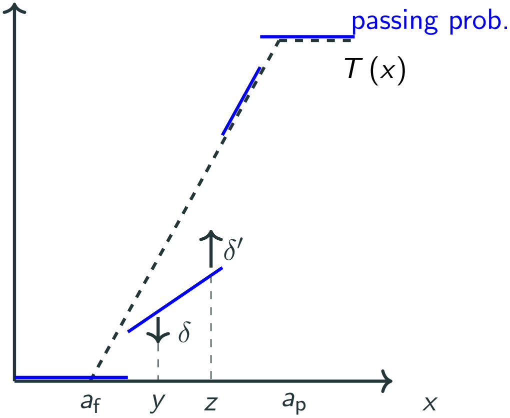

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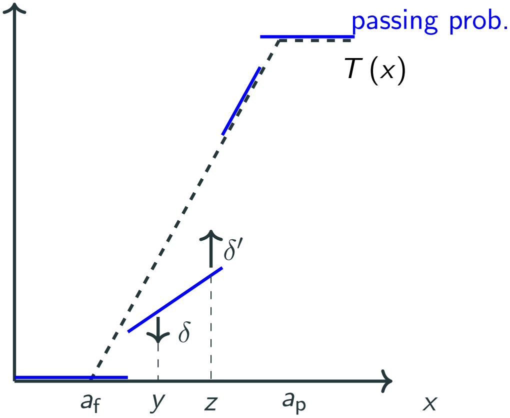

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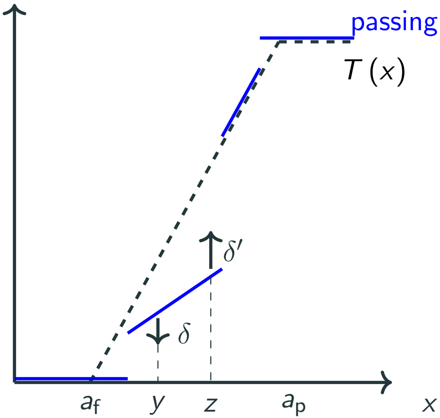
- Choose $\delta f(y)(a_p - y) = \delta' f(z)(a_p - z)$ to keep a_p fixed
- For a_f good effect is stronger (relatively higher state taken away) and bad effect is weaker (relatively closer state added)

Show that adjustment induces a lower cost



Overall effect

- Locally, there is no effect on disclosure
- For the given δ and δ', the informativeness increases
- Non-disclosure is punished harder and sender discloses more
- For a sufficiently small ϵ the cost decreases and we obtain a contradiction

Conditional Distributions

Strategies in subgame induced by T

sender's strategy $\beta^T : [0,1] \to \{0,1\}$, receiver's strategy $\mathbf{a}^T = \left(a_{\mathrm{f}}^T, \ldots, a_k^T\right)$

Beliefs

$$f^T\left(x|S=i\wedge \text{nd}\right) = \frac{\Pr\left(S=i\wedge \text{nd}|x\right)f(x)}{\Pr\left(S=i\wedge \text{nd}\right)} = \frac{\overbrace{\left[q\left(1-\beta^T\left(x\right)\right)+\left(1-q\right)\right]}^{\Pr(\text{nd})}\ \overbrace{T_i(x)}^{\Pr_T(S=i|x)}\ f\left(x\right)}{\int\limits_0^1\left[q\left(1-\beta^T\left(z\right)\right)+\left(1-q\right)\right]T_i\left(z\right)f\left(z\right)dz}$$

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