Limited Monotonicity and the Combined Compliers LATE

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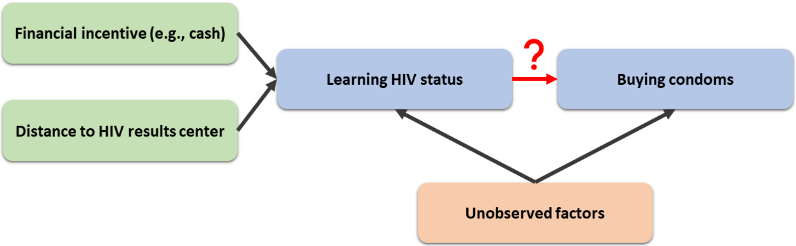

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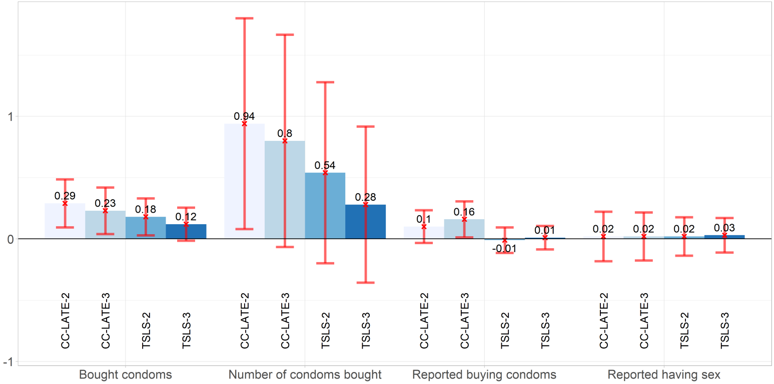
Our contribution

1. Limited monotonicity (**LiM**)
   - Generally less restrictive than other forms of monotonicity
2. Local Average Treatment Effect for Combined Compliers (**CC-LATE**)
   - Well-defined and intuitive causal parameter

Literature

	Assumption	**Interpretation of estimand**
Our project	Limited monotonicity (**LiM**)	**CC-LATE**: LATE for ”combined compliers”
Mogstad et al. (2021)	Partial monotonicity (**PM**)	**TSLS**: Weighted average of pairwise LATEs
Goff (2024)	Vector monotonicity	LATE for ”all compliers”
Sun & Wüthrich (2022)	Pairwise monotonicity	Weighted average of pairwise LATEs
Frölich (2007)	Imbens and Angrist monotonicity (IAM)	LATE for the largest complier group
Imbens & Angrist (1994)	Imbens and Angrist monotonicity (IAM)	TSLS: Weighted average of pairwise LATEs

Preview of estimates



Standard errors clustered at the village level. 95% confidence intervals in red.

Potential outcome framework

- Outcome Y
- Binary treatment D
- k binary instruments $Z_1, ..., Z_k$
- $D_i^{z_1 \ldots z_k} \in \{0, 1\}$ the potential treatment status of individual i
- $Y_i^{d, z_1 \ldots z_k}$ the potential outcome of individual i

Baseline assumptions

1. Random assignment and exclusion
2. SUTVA
3. Instrument relevance

Limited Monotonicity (LiM)

$$P(D^{1...1...1} \geq D^{0...0...0}) = 1 \text{ or } P(D^{1...1...1} \leq D^{0...0...0}) = 1.$$

Partial Monotonicity (PM)

$$P(D^{1...j...k} \geq D^{0...j...k}) = 1 \text{ or } P(D^{1...j...k} \leq D^{0...j...k}) = 1,$$

$$P(D^{i...1...k} \geq D^{i...0...k}) = 1 \text{ or } P(D^{i...1...k} \leq D^{i...0...k}) = 1, \text{ and}$$

$$P(D^{i...j...1} \geq D^{i...j...0}) = 1 \text{ or } P(D^{i...j...1} \leq D^{i...j...0}) = 1$$

$$\forall\ i \in \{0,1\}, ..., j \in \{0,1\}, ..., k \in \{0,1\}.$$

One instrument:

LiM and **PM**: $P(D^1 \geq D^0) = 1.$

Two instruments:

LiM: $P(D^{11} \geq D^{00}) = 1.$

PM:

$$P(D^{10} \geq D^{00}) = 1,$$
$$P(D^{01} \geq D^{00}) = 1,$$
$$P(D^{11} \geq D^{10}) = 1, \text{ and}$$
$$P(D^{11} \geq D^{01}) = 1.$$

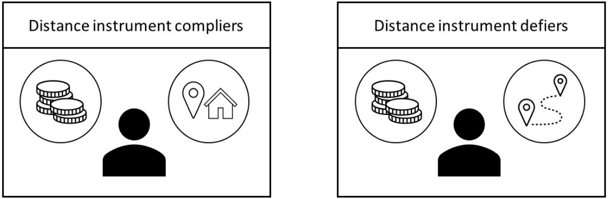
Four initial response types with one binary instrument

D^1	D^0	Notion	LiM	PM
1	1	Always-taker	✓	✓
1	0	Complier	✓	✓
0	1	Defier		
0	0	Never-taker	✓	✓

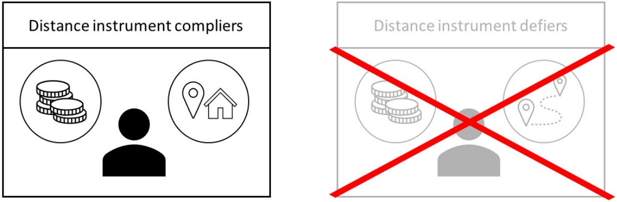
16 initial response types with two binary instruments

D^{11}	D^{10}	D^{01}	D^{00}	Notion	LiM	PM
1	1	1	1	Always-taker	✓	✓
1	1	1	0	Eager complier	✓	✓
1	0	0	0	Reluctant complier	✓	✓
1	1	0	0	First instrument complier	✓	✓
1	0	1	0	Second instrument complier	✓	✓
1	0	1	1	First instrument defier	✓	
1	1	0	1	Second instrument defier	✓	
1	0	0	1	Eager defier	✓	
0	1	1	0	Reluctant defier	✓	
0	1	0	0	Defier type 1	✓	
0	0	1	0	Defier type 2	✓	
0	1	1	1	Defier type 3		
0	1	0	1	Defier type 4		
0	0	1	1	Defier type 5		
0	0	0	1	Defier type 6		
0	0	0	0	Never-taker	✓	✓

LiM allows for:



PM rules out one:



Example 1 - Returns to education

Treatment: College

Instruments: Financial incentive & distance to college

LiM still holds if the following two types co-exist:

- Individuals who attend college only when it is cheap and far
- Individuals who attend college only when it is cheap and close

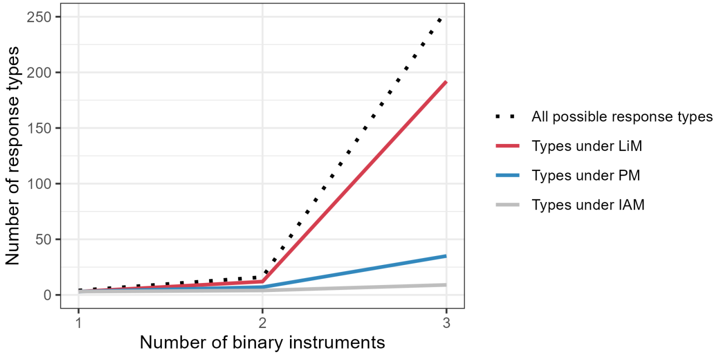
Example 2 - Twinning and same-sex instrument

Treatment: Having a third child

Instruments: Twinning & same-sex instruments

LiM still holds if the following two types exist:

- Parents who have a preference for the two firstborn being of opposite sex
- Parents who have a preference for the two firstborn being of the same sex



250
200
150
100
50
0
Number of response types
1
2
3
Number of binary instruments
All possible response types
Types under LiM
Types under PM
Types under IAM

D^{11}	D^{10}	D^{01}	D^{00}	Notion	LiM	PM
1	1	1	1	Always-taker	✓	✓
1	1	1	0	Eager **complier**	✓	✓
1	0	0	0	Reluctant **complier**	✓	✓
1	1	0	0	First instrument **complier**	✓	✓
1	0	1	0	Second instrument **complier**	✓	✓
1	0	1	1	First instrument defier	✓	
1	1	0	1	Second instrument defier	✓	
1	0	0	1	Eager defier	✓	
0	1	1	0	Reluctant defier	✓	
0	1	0	0	Defier type 1	✓	
0	0	1	0	Defier type 2	✓	
0	1	1	1	Defier type 3		
0	1	0	1	Defier type 4		
0	0	1	1	Defier type 5		
0	0	0	1	Defier type 6		
0	0	0	0	Never-taker	✓	✓

Theorem 1

Let Assumptions 1, 2, 3, and LiM hold.

The **Combined Compliers Local Average Treatment Effect** (CC-LATE) is identified as

$$E(Y^1 - Y^0 | T \in cc) = \frac{E(Y|Z_1 = 1, \ldots, Z_k = 1) - E(Y|Z_1 = 0, \ldots, Z_k = 0)}{E(D|Z_1 = 1, \ldots, Z_k = 1) - E(D|Z_1 = 0, \ldots, Z_k = 0)}$$

where T denotes type and cc the set of combined compliers.

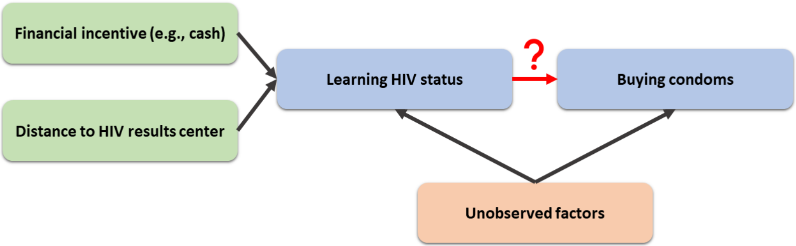
Note that the CC-LATE under LiM simply is:

$$\beta_{\textbf{CC-LATE}} = \sum_{g \in cc} P(G_i = g) \cdot E[Y_i^1 - Y_i^0 | G_i = g]$$

Assuming PM, Mogstad et al. (2021) show:

$$\beta_{\textbf{TSLS}} = \sum_{g \in \mathcal{G} : \mathcal{C}_g \neq \emptyset} \omega_g \cdot E[Y_i^1 - Y_i^0 | G_i = g],$$

$$\omega_g = P(G_i = g) \sum_{k=2}^{K} (1[k \in \mathcal{C}_g] - 1[k \in \mathcal{D}_g]) \left(\frac{\text{Cov}(D_i, 1[p(Z_i) \geq p(z^k)])}{\text{Var}(p(Z_i))} \right)$$



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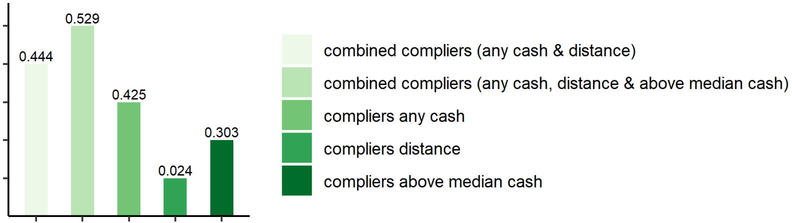
Motivation

1. **LiM** allows for:

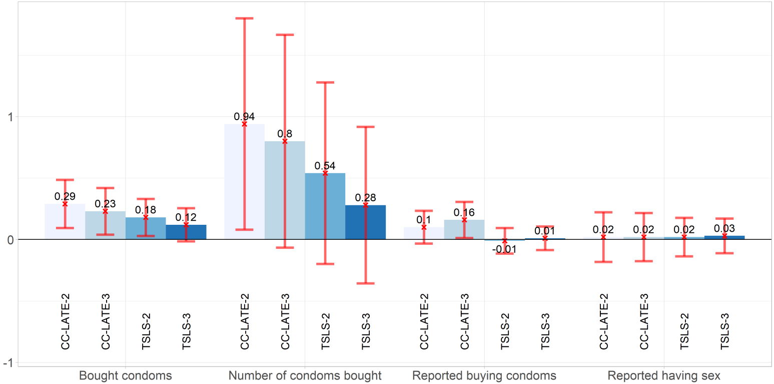
D^{11}	D^{10}	D^{01}	D^{00}	Notion	LiM	PM
1	0	1	0	Distance instrument complier	✓	✓
1	1	0	1	Distance instrument defier	✓	

2. **CC-LATE** estimand has intuitive interpretation



Trade-off when adding instruments:

	nr. observations	% observations
two instruments (*CC-LATE-2*)	432	43%
three instruments (*CC-LATE-3*)	278	28%
total	1008	100%



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Contribution

1. We introduce **LiM**
   - a more plausible monotonicity assumption
2. We show that the **CC-LATE** estimand is identified
   - parameter with an intuitive interpretation
3. CC-LATE estimates provide slightly more evidence for protective behavior when learning HIV status

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