Rank-Dependent Probability Weighting and the Macroeconomy: Insights from a Model with Incomplete Markets and Aggregate Shocks

Marco Cozzi - University of Victoria

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Motivation

- A vast experimental literature documents rank-dependent probability weighting in economic decisions characterized by risk.
- Individuals overestimate the likelihood of low-probability (bad) events, such as the probability of losing a job.
- It is not known whether this behavioral bias matters for macroeconomic outcomes, i.e. for business cycle analysis, and stabilization (fiscal) policies.
- I develop a model of Rank-Dependent Expected Utility (RDEU), with Incomplete Markets and Aggregate Shocks.
- Standard models with Incomplete Markets and Aggregate Shocks imply grossly unrealistic unemployment rates, I specify a better formulation of labor market dynamics.

Estimated Probability Weighting Functions

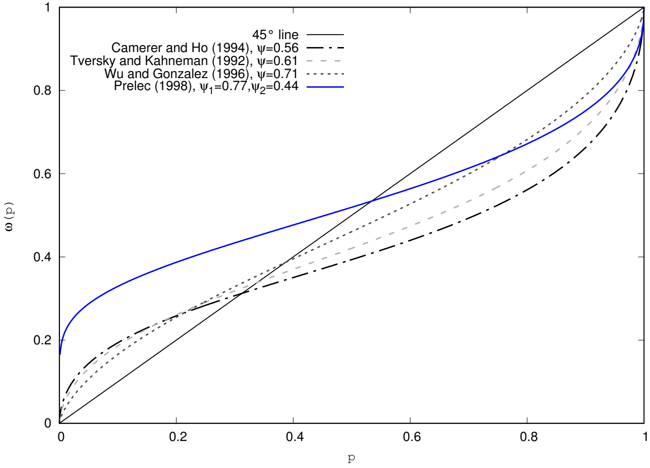

Figure: Probability weighting functions $\omega(p)$. One-parameter function (CH '94, TK '92, and WG '96): $\omega(p) = (p^{\psi}/(p^{\psi} + (1-p)^{\psi})^{1/\psi}$. Two-parameter function (Prelec '98): $\omega(p) = \exp(-\psi_1 * (-\ln(p))^{\psi_2})$.

Objective and Distorted Job Separations and Job Creations

Figure: Job separations (JSP, top panel) and Job creations (JFP, bottom panel): Times series of the data Vs. distorted (RDEUs) probabilities.

Contributions

- Implement a tractable –yet realistic– formulation of labor market dynamics in the processes driving the job finding and separation probabilities. $\rightarrow$ Individuals are facing an empirically relevant level of risk.
- Computationally quite challenging, but feasible and easier to work with than: a) prospect theory, and b) stochastic-volatility.
- Embed RDEU into a rich model with incomplete markets and aggregate shocks. $\rightarrow$ Explicit measurement of effects on business cycles.
- Apply the Krusell-Smith algorithm regarding household forecasting of future unemployment. $\rightarrow$ Implied bounded rationality allows not to rely on the objective transition probabilities for the aggregate dynamics.
- Quantify the importance of RDEU for household saving. $\rightarrow$ Explicit measurement of wealth inequality determinants.

Related Literature

- RDEU/Prospect Theory: Quiggin (JEBO 81), Segal, Spivak, and Zeira (EL 88), Prelec (ECTA 98), Tversky and Kahneman (JRU 92), Kahneman and Tversky (ECTA 79).
- "Exotic" Macroeconomics: Backus, Routledge, and Zin (NBER 05), Backus, Ferriere, and Zin (JME 15).
- Incomplete Markets and Aggregate Shocks: Krusell and Smith (JPE 98, RED 02), Krusell, Mukoyama, Sahin and Smith (RED 09), Castaneda, Diaz-Gimenez and Rios-Rull (JME 98), den Haan, Judd and Juillard (JEDC 10), and many more...

Ingredients of the Economic Model

- Incomplete markets, exogenous borrowing constraint and labor market risk (idiosyncratic and aggregate).
- Self-insurance through a (risky) asset to smooth income fluctuations.
- Ex-ante homogeneous infinitely-lived agents.
- Ex-post heterogeneity: employment status (ϵ) and assets (a) (high degree of wealth concentration can be obtained).
- Time varying labor market conditions and associated risk.
- Aggregate states: capital (K), unemployment rate (U), productivity (Z), job separation probability (s) and job finding probability (ϕ).
- Households maximize Expected Utility (EU), but they distort objective risks via Rank-Dependent Probability Weighting (RDEU).
- This formulation avoids: the need to estimate reference points, time-inconsistency (dynamic programming can be used).

The Model - Production

- The supply side is kept as simple as possible.
- Aggregate Cobb-Douglas production function in aggregate capital K_t and labor L_t.
- Total labor is $L_t = l * N_t = l * (1 - U_t)$.
- For tractability, productivity is deterministically linked to the unemployment rate: $Z_t = 1 - \zeta * (U_t / \overline{U} - 1)$.
- Output is $Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$.
- Usual expressions for the net real return to capital $r_t = \alpha Z_t \left(\frac{L_t}{K_t} \right)^{1-\alpha} - \delta$ and the wage rate $w_t = (1 - \alpha) Z_t \left(\frac{K_t}{L_t} \right)$.

The Model - Households

$$V\left(a,\epsilon,s,\phi,K,U\right)=\max_{c\geq 0,a'\geq \underline{a}}\left\{\frac{c^{1-\gamma}-1}{1-\gamma}+\right.$$
$$\left.\beta\tilde{\mathbb{E}}_{\epsilon',s',\phi',K',U'|\epsilon,s,\phi,K,U}V\left(a',\epsilon',s',\phi',K',U'\right)\right\}$$
$$s.t.$$

$$c+a'=\left(1+r\right)a+\left(1-\tau\right)wl,\ \text{if } \epsilon=e$$
$$c+a'=\left(1+r\right)a+\chi wl,\ \text{if } \epsilon=u$$

$$\ln s'=(1-\rho_s)\mu_s+\rho_s\ln s+\varepsilon'_s,\varepsilon_s\sim N(0,\sigma_s^2)$$
$$\ln \phi'=(1-\rho_\phi)\mu_\phi+\rho_\phi\ln \phi+\varepsilon'_{\phi,t},\varepsilon_\phi\sim N(0,\sigma_\phi^2)$$

$$\ln K'=\theta_{K,0}+\theta_{K,1}\ln K+\theta_{K,2}U+\theta_{K,3}Z$$
$$U'=\theta_{U,0}+\theta_{U,1}U+\theta_{U,2}\pi(u,e')*U+\theta_{U,3}\pi(e,u')*N$$

The Model - Households

$$V(a,\epsilon,s,\phi,K,U) = \max_{c\geq 0, a'\geq \underline{a}} \left\{ \frac{c^{1-\gamma}-1}{1-\gamma} + \beta \tilde{\mathbb{E}}_{\epsilon',s',\phi',K',U'|\epsilon,s,\phi,K,U} V(a',\epsilon',s',\phi',K',U') \right\}$$

$s.t.$

$$c + a' = (1+r)a + (1-\tau)wl, \text{ if } \epsilon = e$$
$$c + a' = (1+r)a + \chi wl, \text{ if } \epsilon = u$$

$$\ln s' = (1-\rho_s)\mu_s + \rho_s \ln s + \varepsilon_s', \varepsilon_s \sim N(0,\sigma_s^2)$$
$$\ln \phi' = (1-\rho_\phi)\mu_\phi + \rho_\phi \ln \phi + \varepsilon_{\phi,t}', \varepsilon_\phi \sim N(0,\sigma_\phi^2)$$

$$\ln K' = \theta_{K,0} + \theta_{K,1} \ln K + \theta_{K,2} U + \theta_{K,3} Z$$
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$s.t.$

Budget Constraints

$$c+a'=(1+r)\,a+(1-\tau)\,wl,\ \text{if } \epsilon=e$$
$$c+a'=(1+r)\,a+\chi wl,\ \text{if } \epsilon=u$$

$$\ln s'=(1-\rho_s)\mu_s+\rho_s\ln s+\varepsilon_s',\varepsilon_s\sim N(0,\sigma_s^2)$$
$$\ln\phi'=(1-\rho_\phi)\mu_\phi+\rho_\phi\ln\phi+\varepsilon_{\phi,t}',\varepsilon_\phi\sim N(0,\sigma_\phi^2)$$

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Exogenous Stochastic Processes

$$\ln s'=(1-\rho_s)\mu_s+\rho_s\ln s+\varepsilon_s',\varepsilon_s\sim N(0,\sigma_s^2)$$
$$\ln\phi'=(1-\rho_\phi)\mu_\phi+\rho_\phi\ln\phi+\varepsilon_{\phi,t}',\varepsilon_\phi\sim N(0,\sigma_\phi^2)$$

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Aggregate Laws of Motion

$$\ln K'=\theta_{K,0}+\theta_{K,1}\ln K+\theta_{K,2}U+\theta_{K,3}Z$$
$$U'=\theta_{U,0}+\theta_{U,1}U+\theta_{U,2}\pi(u,e')*U+\theta_{U,3}\pi(e,u')*N$$

Labor Market Dynamics

- Markov Chains: the typical formulation à-la Krusell and Smith (JPE 98) has Markov Chains on s and ϕ such that the economy jumps randomly between $U_{low} = 4\%$ in booms and $U_{high} = 10\%$ in recessions.
- AR(1)'s: Using the Shimer (AER 05 and RED 12) data, I postulate and estimate with MLE two AR(1) processes for the job separation probability (JSP, s) and the job finding probability (JFP, ϕ):

$$\ln s' = (1-\rho_s)\mu_s + \rho_s \ln s + \varepsilon_s', \varepsilon_s \sim N(0, \sigma_s^2)$$
$$\ln \phi' = (1-\rho_\phi)\mu_\phi + \rho_\phi \ln \phi + \varepsilon_{\phi,t}', \varepsilon_\phi \sim N(0, \sigma_\phi^2)$$

- VAR(1): I tried it, but it is not needed (small parameter estimates, no Granger causality, and low correlation of residuals).

Probability Densities of JSP and JFP (quarterly)

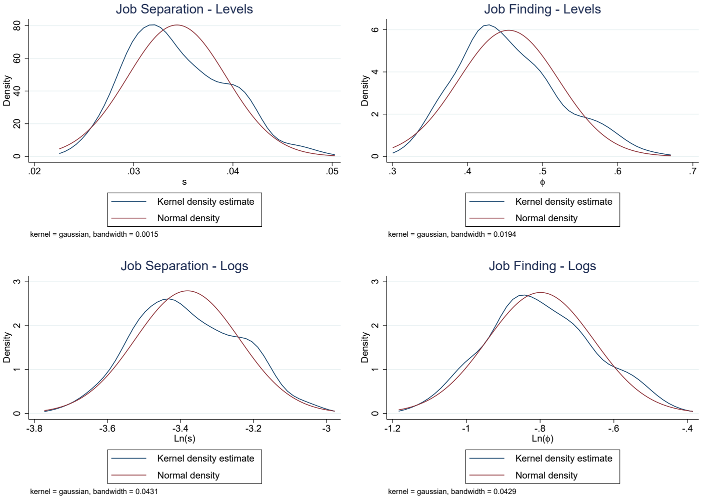

Figure: Limiting distributions of the two estimated AR(1) processes (in levels and logs) and kernel densities of the Shimer data.

Probability Densities of Quarterly Unemployment Rates

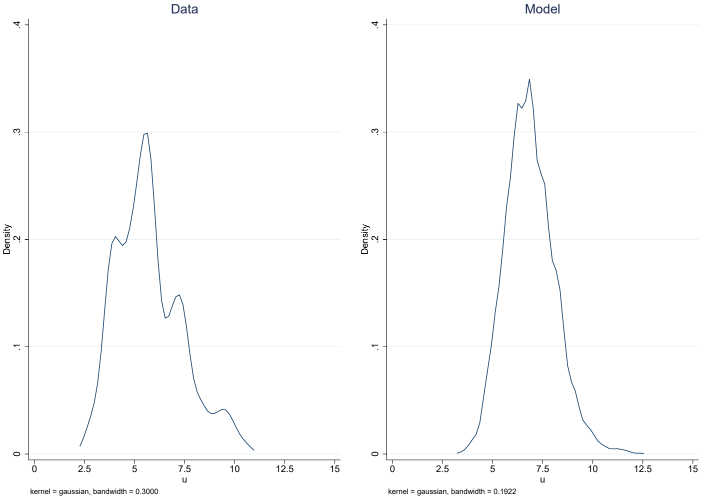

Figure: The data span the 1948Q1-2020Q1 period; the simulation draws long shock sequences from the estimated JFP and JSP AR(1) processes.

Quarterly Unemployment Rates, Data Vs. Simulated path

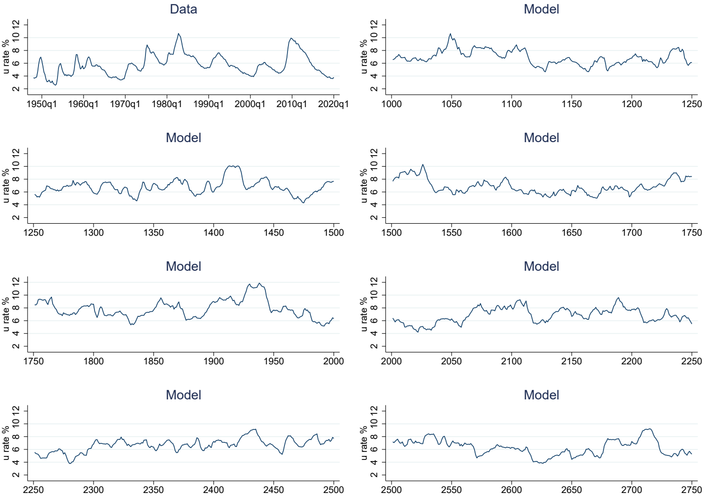

Figure: The simulation draws long shock sequences from the estimated JFP and JSP AR(1) processes, and the law of motion for the unemployment rate.

The Model - Quasi-Aggregation

- Also this model features Quasi-Aggregation: first moments deliver an almost perfect fit of the forecasting equations.
- Both the Capital and Unemployment Aggregate Laws Motion (ALM) have $R^2 > 99.6\%$.
- The Unemployment ALMs based on the objective transition probabilities have a marginally higher $R^2 = 99.9\%$.

Calibration - RDEU Model

Parameter	*Value*	*Target*
α - *Capital share*	0.36	*Labor share of output* $= 64\%$
δ - *Capital depreciation rate*	0.025	*Avg. inv. share of output* $= 25\%$
θ - *Risk Aversion*	2.0	*EIS* $= 0.5$
β - *Discount factor*	0.988	*Avg. quarterly interest rate* $= 1\%$
l - *Hours worked*	0.327	*Time share of work* $= 33\%$
ζ - *Productivity sens. to u.*	0.02	*S.d. of aggregate income* $= 1.9\%$
χ - *UI Replacement Rate*	0.4	*Benefits are 40% of labor earnings*
$\underline{a}$ - *Borrowing limit*	-0.8	*Households can borrow up to 20%*
ψ - *Curv. of prob. weighting*	$-$	*Estimates in the literature*
ρ_ϕ - *Pers. of the AR(1) JFP*	0.936	*MLE estimates on CPS data*
σ_ϕ - *S.d. of the JFP shocks*	0.051	*MLE estimates on CPS data*
μ_ϕ - *Uncond. avg. of the JFP*	-0.8	*MLE estimates on CPS data*
ρ_s - *Pers. of the AR(1) JSP*	0.923	*MLE estimates on CPS data*
σ_s - *S.d. of the JSP shocks*	0.056	*MLE estimates on CPS data*
μ_s - *Uncond. avg. of the JSP*	-3.4	*MLE estimates on CPS data*

Results - Business Cycle Statistics

Variable j	σ_j	σ_j / σ_Y	ρ_j	$\rho_{j,Y}$
HP-filtered Data				
Y	0.0189	1.000	0.892	1.000
C	0.0111	0.586	0.850	0.901
I	0.0658	3.487	0.847	0.937
L	0.0173	0.916	0.915	0.821
EU Model				
Y	0.0162	1.000	0.975	1.000
C	0.0138	0.850	0.929	0.911
I	0.0315	1.948	0.841	0.852
L	0.0138	0.855	0.910	0.944
RDEU M1 (CH)				
Y	0.0195	1.000	0.982	1.000
C	0.0137	0.702	0.953	0.837
I	0.0504	2.577	0.901	0.886
L	0.0138	0.709	0.910	0.900

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C	0.0111	0.586	0.850	0.901
I	0.0658	3.487	0.847	0.937
L	0.0173	0.916	0.915	0.821
RDEU M2 (WG)				
Y	0.0188	1.000	0.981	1.000
C	0.0138	0.732	0.949	0.859
I	0.0453	2.408	0.895	0.883
L	0.0138	0.735	0.910	0.907
RDEU M3 (P)				
Y	0.0192	1.000	0.982	1.000
C	0.0131	0.681	0.949	0.837
I	0.0477	2.488	0.901	0.902
L	0.0138	0.722	0.910	0.902

Conclusions

- In terms of business-cycle statistics, the RDEU models attain lower root mean squared errors than the EU one.
- RDEU helps improving the fit of the volatilities of the macro aggregates.
- RDEU does not help improving the fit of the autocorrelations of some macro aggregates (C and Y).
- In terms of long-run outcomes (e.g., capital level and wealth inequality), the RDEU models display mixed results: quantitatively important, but can go in either direction.
- Future work: consider the welfare costs of business cycles, exploiting a counterfactual that shuts down only the aggregate unemployment risk. $\rightarrow$ Develop a recursive methodology to address the so-called integration principle.