Is macroprudential regulation desirable under endogenous capital formation?*

Fabian Knapp†

This version: August, 2024

Abstract

This paper is the first quantitative study on optimal macroprudential policy with both the price and the quantity of capital that serves as collateral being endogenous. I apply a small open economy DGSE with a collateral constraint to analyze time-consistent optimal policy and calibrate it to moments of the OECD member countries. I find that the optimal macroprudential debt tax is much lower than in comparable studies with a fixed amount of collateral and that the optimal policy reduces investment. The effectiveness of reducing crises is strongly diminished compared to the case with fixed capital and the welfare gain is small. The availability of an investment tax as a second instrument boosts welfare gains, turns crises into extremely rare events and implements optimal investment. The ex ante debt tax is much higher in this case.

Keywords: Macroprudential Policy, Investment, Optimal Policy, Financial Crises, Collateral Constraint

JEL Codes: D62, E32, E44, F32, F41

*I am grateful to Andreas Schabert for his support. Seminar participants at the Macro & Public Economics Seminar (University of Cologne) provided useful comments and suggestions.

†University of Cologne, Center for Macroeconomic Research, Albertus-Magnus-Platz, 50923 Cologne, Germany. Email: knapp@wiso.uni.koeln.de.

1 Introduction

Since the global financial crisis macroprudential regulation is intensively discussed and several studies emerged (Bianchi and Mendoza 2018, Bianchi 2016, Dávila and Korinek 2017, Benigno et al. 2013, Jeanne and Korinek 2019 among others). However, the impact of macroprudential regulation on capital formation is hardly analyzed. This paper is the first quantitative study on optimal macroprudential policy with both the price and the quantity of capital that serves as collateral being endogenous.

A significant part of research focuses on the question how collateral constraints amplify income-diminishing economic shocks and how macroprudential regulation can be implemented to reduce the adverse effects of this amplification mechanism. Models including an endogenous price of capital as well as an endogenous amount of capital have shown to fit the moments of macroeconomic time series quite well: Mendoza 2010 is able to reproduce the key features of sudden stops without analyzing policy interventions. Although there are several studies on optimal macroprudential regulation, there is no quantitative study with an endogenous price and an endogenous quantity of capital. Thus, the literature lacks so far an optimal policy analysis with both important elements of the collateral value being endogenous, although Mendoza 2010 has shown that exactly a specification with both elements being endogenous is in line with the data.

This paper will fill this gap in the literature on optimal macroprudential regulation by answering the following four questions:

1. Is macroprudential regulation desirable under endogenous capital formation?
2. If so, how does the effectiveness of an optimal debt tax change in relation to studies with fixed capital?
3. How does investment change through the introduction of the optimal tax?
4. How do the answers to the previous questions change if a second instrument (an investment tax/subsidy) is available?

To answer these questions theoretically and quantitatively, I use a real small open economy DSGE model which builds on Mendoza 2010 and calibrate it to match moments from OECD data. I show that this model is able to mimic key elements of financial crises. In order to analyze optimal policy, I follow Bianchi and Mendoza 2018 and solve for the Markov perfect equilibrium of my social planner problems to guarantee time consistency. I distinguish between two social planners: The first planner is able to influence the debt decision of households, but cannot affect the capital decisions of households and investment firms. His/her decisions can be decentralized by solving for the optimal state-contingent Pigouvian debt tax. The second social planner can additionally alter the investment decision of investment firms. The optimal choices can be decentralized by solving for the optimal state-contingent Pigouvian debt tax and investment tax. Both planners take the decision of future planners as given, but anticipate that their choices of the state variables will influence the future planner.

I find that that it is still optimal to implement an ex ante debt tax to reduce borrowing before crises and to reduce the frequency of crises in both cases. This is in line with the findings of Bianchi and Mendoza 2018 where capital is fixed. However, adding capital formation to the model substantially reduces the decrease of the crisis probability induced by a debt tax only. In this case the debt tax also reduces investment and the welfare gains as well as the size of the tax are small compared to other studies. When an investment tax in addition to the debt tax is available, the probability of crises falls very strongly and welfare gains multiply. Investment is subsidized when the constraint is not binding and taxed in case it is binding. Debt is taxed to a higher degree ex ante than in the case without an investment tax.

The key effect of models with a collateral constraint is a fire-sale mechanism: when the collateral constraint binds, agents cannot borrow more and therefore sell their collateral assets to repay debt. This, however, reduces the collateral price, which makes agents sell even more of their assets and so on. This vicious circle leads to a strong decrease of the collateral price and income. From the perspective of optimal regulation it is crucial that in economies with a collateral constraint there exists a pecuniary externality, which - according to the existing literature on macroprudential regulation - a social planner would like to correct by a Pigouvian ex ante tax on debt. The reason is that agents do not incorporate into their debt decision that the level of debt influences the price of collateral via consumption and therefore the borrowing capacity. In my model with capital accumulation there exists a second externality: agents do not take into account that the capital price is a function of the level of investment that emerges in equilibrium.

A macroprudential debt tax tends to reduce capital investment and thereby production capacities in crisis states. In an open economy 3-period model that builds on Dávila and Korinek 2017 I show that this is exactly the reason why a social planner chooses a comparably small Pigouvian debt tax (see Appendix). In my more complex infinite horizon model, too, it is analytically not clear whether the social planner wants to reduce borrowing ex ante. As opposed to the case with fixed collateral, the sign of the optimal ex ante tax can either be positive or negative. However, the calibrated version of the model shows that in both analyzed cases the sign of the mean ex ante tax is positive when capital is flexible, confirming what research has found out so far.

I find that in the 1-instrument case the optimal ex ante Pigouvian tax on debt is 0.52 per cent on average and that the optimal ex post Pigouvian tax on debt is -1.75 per cent on average. Thus, the social planner incentivizes households to reduce borrowing when the collateral constraint is not binding and supports borrowing when the constraint is binding. Both the ex-ante and the ex-post tax push up consumption and investment that increase the collateral price and the borrowing capacity.

The level of the ex ante tax is relatively low compared to the literature. The paper Bianchi and Mendoza 2018 that is model-wise and in terms of calibration very close to mine finds an optimal ex ante debt tax of 3.6%. This is the case because opposed to an economy with constant capital, the social planner now takes into account that a higher ex ante tax leads to higher investment tomorrow, since a higher collateral price emerges in the next period. However, a low level of investment would be desirable in a crisis situation. Thus, there is a novel cost channel of the macroprudential tax pushing its optimal level down.

The low optimal tax level leads to a reduction of the crisis probability of only 0.52 percentage points, which is much lower than the values in comparable papers. To summarize, macroprudential regulation under capital accumulation tends to be less effective with regard to preventing crises when there is a debt instrument only.

The availability of an investment tax as a second instrument boosts welfare gains, turns crises into extremely rare events and implements optimal investment. The capital price and the amount of investment are now detached and the ex ante debt tax can push up capital prices without the need for increased investment supporting these prices.

The remainder of the paper is structured as follows: Chapter 2 summarizes the related literature and chapter 3 describes the model and the competitive equilibrium. Chapter 4 analyzes the social planner problem and the optimal tax rates. Chapter 5 explains the calibration and summarizes the quantitatve results. Chapter 6 concludes.

2 Literature Review

There are two types of studies on optimal macroprudential policy. On the one hand, there are several analytical papers that use simple 3-period models. On the other hand, there are quantitative analyses. This paper places into this literature as follows: It is the first quantitative study with an optimal policy analysis of an economy with a collateral constraint in an infinite horizon framework with endogenous capital accumulation and endogenous capital pricing.

My paper is closest to Bianchi and Mendoza 2018. I use the same concept of time consistent optimal policy, a similar model as well as a comparable calibration, but I endogenize capital, whereas Bianchi and Mendoza simplify the analysis by fixing the amount of capital. In a calibrated version of their model Bianchi and Mendoza 2018 find a positive macroprudential debt tax, which decreases the probability of crises clearly. The welfare gain induced by the optimal policy is 0.3%.

Another quantitative paper on optimal macroprudential regulation is Ma 2020. In this article productivity is endogenized and growing. Thus, a collateral constraint in an economy with positive growth rates is analyzed. As in my paper, households can invest to increase future production, but the amount of collateral is fixed, however. In a calibrated version of the model Ma shows that the time consistent macroprudential debt tax is positive and that the occurrence of crises is strongly reduced under the optimal tax. The welfare gains sum up to 0.06%.

Bianchi 2016 is one of the few infinite horizon papers with an endogenous and time-varying amount of capital. However, the price of collateral is fixed to one, whereas my analysis features an endogenous price of capital. Moreover, Bianchi 2016 focuses merely on bailout policies.

A quantitative study that adds heterogeneity in terms of two agents (workers and entrepreneurs) is Biljanovska and Vardoulakis 2024. They fix capital and equip the social planner with a debt tax as well as a payroll tax. They find a positive debt tax and a state-dependent sign of the payroll tax.

Bianchi and Mendoza 2020 is - to my knowledge - the only quantitative analysis of macroprudential regulation that includes both an endogenous price of capital and an endogenous amount of capital serving as collateral. As opposed to my paper, Bianchi and Mendoza restrict the set of optimal policies to constant tax rates. They find that the optimal constant macroprudential debt tax is positive and compute welfare gains of 0.0179%. In their paper the macroprudential policy is very effective at reducing crisis events, too.

There are several papers using simplified 3-period models with flexible capital. Dávila and Korinek 2017 analyze collateral and distributive externalities in a 3-period model, where capital is only endogenous in the first period. The capital price which is relevant for the collateral constraint is only influenced by the capital decision in the previous period, whereas in my model it is the current capital choice that influences the capital price. They find that in the case of two instruments the social planner wants to tax debt and that the sign of the capital tax is not unambiguous. Lorenzoni 2008 analyzes collateral and distributive externalities in a 3-period model, where capital is endogenous in the first and the second period. Opposed to this paper, he does not solve for an optimal debt tax but discusses a capital requirement. Moreover, he focuses on distributive externalities. Lanteri and Rampini 2021 build a 3-period model with heterogeneous firms, old as well as new capital and distributive as well as collateral externalities. They focus on capital taxes and find that old capital should be taxed, whereas new capital should be subsidized.

3 Model

The model is a small open economy DSGE model with a collateral constraint. It resembles several elements of the models by Mendoza 2010 and Bianchi and Mendoza 2018, but differs in the way investment is modeled. Time is discrete and indexed by $t = 1, 2, ...$ All variables are in real terms.

I follow Bianchi and Mendoza and model production within a joint household-firm agent who also decides upon production factors[1]. There is a continuum of agents of measure one, but since they behave identically I will use the term “the (representative) household” from here on.

Furthermore, the model features an investment firm that buys depreciated capital from the households, invests and then sells the accumulated capital back to the households. Capital both serves as a production factor and as collateral. Therefore, from here on, I am going to use the terms capital and collateral interchangeably.

There are three shocks in the model: a real interest rate shock, a productivity shock and a financial shock. The shocks follow finite-state stationary Markov processes with compact support.

3.1 Set-up

Utility The representative household-firm derives utility from consumption c and disutility from supplying labor h. The utility function is a standard CRRA function. Thus, lifetime utility is given by the following expression:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{(c_t-\chi\frac{h_t^\omega}{\omega})^{1-\sigma}-1}{1-\sigma}\right] . \tag{1}$$

$\mathbb{E}$ denotes the expectations operator, β the subjective discount factor, σ the risk aversion coefficient, χ is a weighting parameter of the disutility of labor and $\frac{1}{\omega-1}$ the Frisch elasticity.

Production A final good is produced by combining capital k_H, labor h and an intermediate good v. The production function is a standard Cobb-Douglas production function with total factor productivity A:

$$F(k_{H,t},h_t,v_t)=A_t k_{H,t}^{\alpha_k}h_t^{\alpha_h}v_t^{\alpha_v} . \tag{2}$$

[1] Bianchi and Mendoza show that the same equilibrium conditions apply when there are two separate agents.

A_t is an exogenous productivity shock. Working time h increases production on the one hand, but decreases utility on the other hand. The intermediate good v also increases production, but has to be bought at the exogenous price p_v. Furthermore, a share θ of the costs of the intermediate good has to be paid before production and financed by foreign debt. This set-up of the production function, labor and the intermediate good is the same as in Bianchi and Mendoza 2018.

Capital The representative household starts the period with an amount of capital $k_{H,t}$ and uses it for production. Moreover, it serves as collateral in the current period. After production the household sells depreciated capital to an investment firm that can increase the capital stock at the cost of one unit of the final good and some investment adjustment costs which are specified as follows:

$$c(i_{F,t}) = \frac{a}{2}\left(i_{F,t} - \bar{i}\right)^2 \ . \tag{3}$$

Investment is given by

$$i_{F,t} = k_{F,t+1} - (1-\delta)k_{F,t} \ . \tag{4}$$

$\bar{i}$ is a parameter which serves as the investment benchmark. Later it will be calibrated as investment at the stochastic steady state. a is an adjustment cost parameter which later determines how strong the capital price q_t reacts to a deviation of investment from $\bar{i}$. δ denotes the depreciation rate of capital.

After the investment decision the investment firm sells its capital $k_{F,t+1}$ back to the households at price q_t. Thus, the static problem of the investment firm is:

$$\max_{i_{F,t}} \Pi_t \ = q_t i_{F,t} - i_{F,t} - \frac{a}{2}(i_{F,t} - \bar{i})^2 \ . \tag{5}$$

The optimal choice of the firm leads to the following pricing condition of capital

$$q_t = 1 + a\left(i_{F,t} - \bar{i}\right)$$

$$\Leftrightarrow q_t = 1 + a\left(k_{F,t+1} - (1-\delta)k_{F,t} - \bar{i}\right) \ , \tag{6}$$

which can either be written in terms of investment or in terms of capital. The investment firms are owned by the households so that profits/losses are part of the household's budget constraint.

Borrowing The household borrows in one-period foreign bonds b which yield a gross real interest of R. The interest rate R can be interpreted as the world interest rate which is taken as given by the domestic economy. It is therefore an exogenous shock and each level of bonds will be supplied[2]. Total borrowing in one period is the sum of (the negative amount of) bonds b_{t+1} divided by the gross interest rate and the working capital $\theta p_v v_t$.

Collateral constraint As the model's main financial friction the collateral constraint is at the heart of the model and generates the herein before mentioned fire sale mechanism. As it is usual in the literature, borrowing is limited by a fraction of the current value of collateral which is denoted by κ_t:

$$\frac{b_{t+1}}{R_t} - \theta p_v v_t \quad \geq \quad - \kappa_t q_t k_{H,t} \ . \tag{7}$$

This realistically reflects the fact that a significant part of firm borrowing is asset-based (Ivashina et al. 2022). Moreover, Bianchi and Mendoza 2018 show that this constraint can be microfounded by an incentive-compatibility constraint on borrowers. The intuition is that borrowers can divert funds after borrowing and because of limited enforcement the lender sells the collateral in that case at value $\kappa_t q_t k_{H,t}$.

As in Bianchi and Mendoza 2018 κ_t is a financial shock and can take two values. This mirrors that borrowing tends to be more restricted in crises. Opposed to the previous literature, there are two channels which influence the borrowing limit. On the one hand, there is the collateral price channel, which is standard. The current value of collateral obviously depends on the price which is paid when selling the collateral. On the other hand, there is the collateral quantity channel, which is novel. The current value of collateral depends on the choice of the amount of collateral which the households made

[2]Debt is, however, restricted by the collateral constraint below.

one period earlier.

Household problem The household's maximization problem can be summarized as follows:

$$\max_{c_t, k_{H,t+1}, b_{t+1}, v_t, h_t} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{\left(c_t - \chi \frac{h_t^\omega}{\omega}\right)^{1-\sigma} - 1}{1-\sigma} \right] , \tag{8}$$

s.t.

$$(\lambda^{LF}) \qquad c_t + q_t k_{H,t+1} + \frac{b_{t+1}}{R_t} + p_v v_t = A_t k_{H,t}^{\alpha_k} h_t^{\alpha_h} v_t^{\alpha_v} + q_t (1-\delta) k_{H,t} + b_t + \Pi_t , \qquad \text{(Budget constraint)}$$

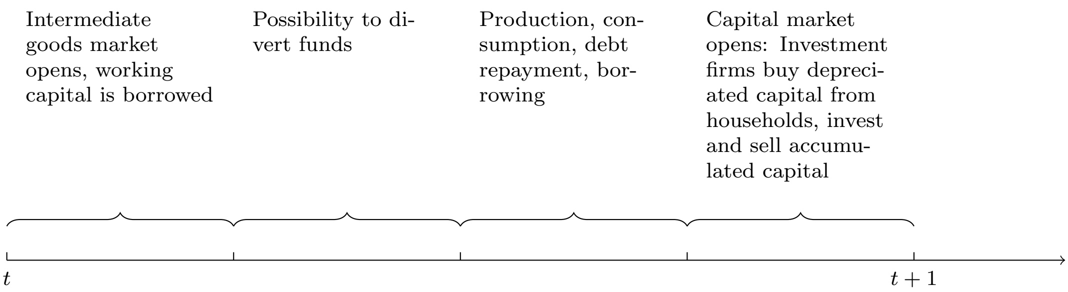
$$(\mu^{LF}) \qquad \frac{b_{t+1}}{R_t} - \theta p_v v_t \geq -\kappa_t q_t k_{H,t} . \qquad \text{(Collateral constraint)}$$

Market clearing In equilibrium, the capital market has to clear so that $k_{H,t} = k_{F,t} \; \forall \; t$ must hold. Furthermore, the final goods market must clear which leads to the following resource constraint:

$$A_t k_t^{\alpha_k} v_t^{\alpha_v} h_t^{\alpha_h} + b_t = \frac{b_{t+1}}{R_t} + c_t + k_{t+1} - (1-\delta) k_t + \frac{a}{2} (k_{t+1} - (1-\delta) k_t - \bar{i})^2 + p_v v_t . \tag{9}$$

Timing of events

The following picture summarizes the order of the events during one period.



3.2 Decentralized equilibrium

Definition 1 *A competitive equilibrium consists of a set of sequences* $\{c_t, h_t, v_t, k_{t+1}, b_{t+1}, q_t, \mu_t^{LF}\}_{t=0}^{\infty}$ *satisfying*

$$\left(c_t - \chi\frac{h_t^{\omega}}{\omega}\right)^{-\sigma} = \beta R_t \mathbb{E}_t\left[\left(c_{t+1} - \chi\frac{h_{t+1}^{\omega}}{\omega}\right)^{-\sigma}\right] + \mu_t^{LF} , \tag{10}$$

$$\left(c_t - \chi\frac{h_t^{\omega}}{\omega}\right)^{-\sigma} q_t = \beta \mathbb{E}_t\left[\left(c_{t+1} - \chi\frac{h_{t+1}^{\omega}}{\omega}\right)^{-\sigma}\left(q_{t+1}(1-\delta) + \ldots \right.\right.$$
$$\left.\left. \alpha_k A_{t+1} k_{t+1}^{\alpha_k - 1} v_{t+1}^{\alpha_v} h_{t+1}^{\alpha_h}\right) + \mu_{t+1}^{LF} q_{t+1} \kappa_{t+1}\right] , \tag{11}$$

$$\left(c_t - \chi\frac{h_t^{\omega}}{\omega}\right)^{-\sigma} p_v + \mu_t^{LF}\theta p_v = \left(c_t - \chi\frac{h_t^{\omega}}{\omega}\right)^{-\sigma} \alpha_v A_t k_t^{\alpha_k} v_t^{\alpha_v - 1} h_t^{\alpha_h} , \tag{12}$$

$$\chi h_t^{\omega - 1} = \alpha_h A_t k_t^{\alpha_k} h_t^{\alpha_h - 1} v_t^{\alpha_v} , \tag{13}$$

$$1 + a\left(k_{t+1} - (1-\delta)k_t - \bar{i}\right) = q_t , \tag{14}$$

$$A_t k_t^{\alpha_k} v_t^{\alpha_v} h_t^{\alpha_h} + b_t = c_t + p_v v_t + k_{t+1} - (1-\delta)k_t + \frac{b_{t+1}}{R_t} + \ldots$$
$$\frac{a}{2}(k_{t+1} - (1-\delta)k_t - \bar{i})^2 , \tag{15}$$

$$\frac{b_{t+1}}{R_t} - \theta p_v v_t \geq -\kappa_t q_t k_t , \tag{16}$$

$$\mu_t^{LF} \geq 0 , \tag{17}$$

$$\mu_t^{LF}\left(\frac{b_{t+1}}{R_t} + \kappa_t q_t k_t - \theta p_v v_t\right) = 0 , \tag{18}$$

given $\{A_t, R_t, \kappa_t\}_{t=0}^{\infty}$*,* b_0*,* k_0*, and the associated transversality conditions.*

I already used above that the capital market clears in equilibrium. Equations (10)-(13) are the household's optimality condition and equation (14) is the investment firm's optimality condition. The left hand side of these equations represents the costs from increasing the respective variable, whereas the right hand side depicts the gain of an increase. Equation (10) is the Euler equation where it is taken into account that more debt is costly when the collateral constraint is binding. Equation (11) describes the optimal household capital decision: the cost of one more unit of capital today is the price q_t, the gain of one more unit is that more depreciated capital can be sold tomorrow at price q_{t+1}, more is produced and more collateral is available if the constraint binds in the subsequent period. Equation (12) is the optimal decision on the intermediate good: on the one hand, it costs p_v and makes the collateral constraint tighter because of more working capital needed, on the other hand, there are gains in production. Equation (13) represents the optimal labor decision. One unit more labor means less leisure and therefore less utility, but also more production. Equation (14) is the optimal investment decision of investment firms where the definition of investment is already plugged in. From the firm's perspective more investment means on the one hand costs in terms of the final good as well as adjustment costs, but on the other hand it means increased revenues at price q_t. Equation (15) depicts the resource constraint, equation (16) the collateral constraint and equations (17) to (18) are the remaining Karush-Kuhn-Tucker conditions.

Pecuniary externalities There are two pecuniary externalities on the collateral price in the model: On the one hand, households do not take into account that in equilibrium the collateral price adapts to their choices affecting the capital demand (see equation (11)). This externality is the force which causes the Pigouvian tax in Bianchi and Mendoza 2018. On the other hand, households do not take into account that the capital they demand changes the firm's costs and therefore the capital price (see equation (14)). This additional channel is novel compared to Bianchi and Mendoza 2018.

Thus, if the collateral constraint is not binding, households do take into account that additional capital might be helpful for borrowing within the next period when the constraint may be binding. However, they do not take into account how their current and future choices of capital and consumption influence the price of capital. Therefore, their value of increasing capital differs from the one of the social planner.

4 Optimal time-consistent policy

Similar to Ma 2020 and Bianchi and Mendoza 2018 I solve for the optimal time-consistent discretionary policy. As Bianchi and Mendoza 2018 show, in models of the type used in this paper there is a time consistency issue under commitment, since the social planner tends to announce a low future consumption when the constraint is binding, which is no longer optimal ex post. Thus, it became standard in this literature to focus on optimal time-consistent policies. From here on, the notation changes to the recursive notation with $'$ denoting the subsequent state of the state variables.

I consider two variants of optimal policy: one in which the social planner is equipped with a debt tax only and one in which he/she also has an investment tax at his/her disposal. The time-consistent social planner with one instrument is able to influence the debt decision of households, but cannot affect the capital decisions of households and investment firms. The time-consistent social planner with two instruments is able to influence the debt decision of households and the capital decision of investment firms, but cannot affect the capital decisions of households. In both cases the planner takes the decision of future planners as given, but anticipates that his/her choices of the state variables will influence the future planner (see definition of Markov perfect equilibrium below).

4.1 Social planner problem (1 instrument)

The social planner who is equipped with a debt tax maximizes the household's utility subject to the resource constraint, the collateral constraint and two implementability constraints that stem from the laissez-faire choice of capital by the households and from the investment choice of the investment firm. In the Appendix I show that this reduced problem is equivalent to the maximization problem that also contains the optimality conditions with respect to the intermediate good v and labor h as well as the Karush-Kuhn-Tucker conditions of problem (8) as implementability constraints.

Time consistency is guaranteed by solving for the Markov perfect equilibrium. Given the policy functions of the future planner, the current planner chooses optimal values for all control variables and the subsequent states. He/she takes into account that the choice of the subsequent states, i.e. the choice of bonds and capital, will change the choice of control variables by the future planner. A Markov perfect equilibrium is given when the policy functions of the current and those of the future planner are identical. This means that the future and the current planner will take the same choices if the states they are in are the same.

Markov perfect equilibrium and first-order conditions The constrained social planner's optimization problem can be summarized as follows:

$$\mathcal{V}(b,k,s) = \max_{c,k',b',q,v} \frac{(c-\chi\frac{h^{\omega}}{\omega})^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_{s'|s}\mathcal{V}(b',k',s') \ , \tag{19}$$

s.t.

$$(\lambda^{SP}) \quad Ak^{\alpha_k}v^{\alpha_v}h^{\alpha_h} = \frac{b'}{R} - b + c + k' - (1-\delta)k + \frac{a}{2}(k'-(1-\delta)k-\bar{i})^2 + p_v v \ ,$$

(Resource constraint)

$$(\mu^{SP}) \quad \frac{b'}{R} - \theta p_v v \geq -\kappa q k \ , \quad \text{(Collateral constraint)}$$

$$(\xi) \quad q\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} = \beta\mathbb{E}_{s'|s}\Big[\left(\mathcal{C}_{fp}(b',k',s')-\chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma}\ldots$$
$$\Big((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\ldots$$
$$\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Big)\Big] + \beta\mathbb{E}_{s'|s}\left[\boldsymbol{\mu}_{fp}(b',k',s')\mathcal{Q}_{fp}(b',k',s')\kappa'\right] \ ,$$

(Household capital decision)

$$(\gamma) \quad q = 1 + a(k'-(1-\delta)k-\bar{i}) \ . \quad \text{(Firm investment decision)}$$

In order to facilitate notation, the following shortcuts of the policy functions of the current planner have been used above: $b' = \mathcal{B}(b,k,s)$, $k' = \mathcal{K}(b,k,s)$, $q = \mathcal{Q}(b,k,s)$, $c = \mathcal{C}(b,k,s)$, $v = \boldsymbol{v}(b,k,s)$ and $h = \boldsymbol{h}(b,k,s)$. There is an important difference between $\boldsymbol{\mu}/\boldsymbol{\mu}_{fp}$ and $\boldsymbol{\mu}^{SP}$: $\boldsymbol{\mu}/\boldsymbol{\mu}_{fp}$ denotes the private multiplier of the collateral constraint under the optimal choices of the current social planner/future social planner which is defined by (12). $\boldsymbol{\mu}^{SP}$, however, denotes the social multiplier of the collateral constraint, which reflects the social gain of marginally relaxing the collateral constraint.

Definition 2 *The Markov perfect constrained efficient equilibrium is defined by the policy functions* $\mathcal{B}(b,k,s)$, $\mathcal{K}(b,k,s)$, $\mathcal{Q}(b,k,s)$, $\mathcal{C}(b,k,s)$, $\boldsymbol{v}(b,k,s)$, $\boldsymbol{\mu}(b,k,s)$[3], $\boldsymbol{h}(b,k,s)$ *and*

[3] The household's multiplier on the collateral constraint is defined by equation (12) and is not binding for the social planner (see Appendix).

the value function $\mathcal{V}(b,k,s)$ that, first, solve the social planner optimization problem (19) and, second, are equal to the future planner's policy functions: $\mathcal{B}(b,k,s) = \mathcal{B}_{fp}(b,k,s)$, $\mathcal{K}(b,k,s) = \mathcal{K}_{fp}(b,k,s)$, $\mathcal{Q}(b,k,s) = \mathcal{Q}_{fp}(b,k,s)$, $\mathcal{C}(b,k,s) = \mathcal{C}_{fp}(b,k,s)$, $\boldsymbol{v}(b,k,s) = \boldsymbol{v}_{fp}(b,k,s)$, $\boldsymbol{\mu}(b,k,s) = \boldsymbol{\mu}_{fp}(b,k,s)$ and $\boldsymbol{v}(b,k,s) = \boldsymbol{v}_{fp}(b,k,s)$.

The optimization problem leads to the following first-order conditions:

$$\lambda^{SP} = \left(c - \chi\frac{h^\omega}{\omega}\right)^{-\sigma} + \xi\sigma\left(c - \chi\frac{h^\omega}{\omega}\right)^{-\sigma-1} q\,, \tag{20}$$

$$\lambda^{SP} = \beta R \mathbb{E}_{s'|s}\left[\lambda^{SP'}\right] + \mu^{SP} + \beta R \mathbb{E}_{s'|s}[\xi\Omega']\,, \tag{21}$$

$$\begin{aligned} \lambda^{SP}(1 + a(k' - (1-\delta)k - \bar{i})) &= \beta\mathbb{E}_{s'|s}\Big[\lambda^{SP'}\Big(\alpha_k A' k'^{\alpha_k - 1} v'^{\alpha_v} h'^{\alpha_h} \ldots \\ &\quad + (1-\delta)(1 + a(k'' - (1-\delta)k' - \bar{i}))\Big)\ldots \\ &\quad + \mu^{SP'} q' \kappa'\Big] + \gamma a + \beta\mathbb{E}_{s'|s}[\xi\Gamma' - \gamma'(1-\delta)a]\,, \end{aligned} \tag{22}$$

$$\begin{aligned} \chi h^{\omega-1}\left(c - \chi\frac{h^\omega}{\omega}\right)^{-\sigma} &= \lambda^{SP}\alpha_h A k^{\alpha_k} h^{\alpha_h - 1} v^{\alpha_v}\ldots \\ &\quad - \xi\chi h^{\omega-1}\sigma\left(c - \chi\frac{h^\omega}{\omega}\right)^{-\sigma-1} q\,, \end{aligned} \tag{23}$$

$$\mu^{SP}\kappa k = \xi\left(c - \chi\frac{h^\omega}{\omega}\right)^{-\sigma} + \gamma\,, \tag{24}$$

$$A k^{\alpha_k} v^{\alpha_v} h^{\alpha_h} - \frac{b'}{R} + b = c + k' - (1-\delta)k + \frac{a}{2}(k' - -(1-\delta)k - \bar{i})^2 - p_v v\,, \tag{25}$$

$$\lambda^{SP} p_v = \lambda^{SP}\alpha_v A k^{\alpha_k} v^{\alpha_v - 1} h^{\alpha_h} - \mu^{SP}\theta p_v\,, \tag{26}$$

$$\frac{b'}{R} - \theta p_v v \geq -\kappa q k\,, \tag{27}$$

$$\begin{aligned} q\left(c - \chi\frac{h^\omega}{\omega}\right)^{-\sigma} &= \beta\mathbb{E}_{s'|s}\Bigg[\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^\omega}{\omega}\right)^{-\sigma}\ldots \\ &\quad \Big((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k - 1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\ldots \\ &\quad \boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Big) + \boldsymbol{\mu}_{fp}(b',k',s')\mathcal{Q}_{fp}(b',k',s')\kappa'\Bigg]\,, \end{aligned} \tag{28}$$

$$q = 1 + a(k' - (1-\delta)k - \bar{i})\,, \tag{29}$$

$$\mu^{SP} \geq 0\,, \tag{30}$$

$$0 \quad = \quad \mu^{SP}\left(\frac{b'}{R} - \theta p_v v + \kappa q k\right) \ . \tag{31}$$

Ω captures the effects of the current planner's bond decision b' on the future planner's decisions, which is taken into account by the current planner:

$$\begin{aligned}
\Omega' = & -\sigma\left(\mathcal{C}_{fp,b}(b',k',s') - \chi \boldsymbol{h}_{fp,b}(b',k',s')\boldsymbol{h}_{fp}(b',k',s')^{\omega-1}\right)\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma-1}\dots \\
& \left((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\right)\dots \\
& + \left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma}\Bigg((1-\delta)\mathcal{Q}_{fp,b}(b',k',s')\dots \\
& + \alpha_k A' k'^{\alpha_k-1}\Big(\alpha_v \boldsymbol{v}_{fp,b}(b',k',s')\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v-1}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\dots \\
& + \alpha_h \boldsymbol{h}_{fp,b}(b',k',s')\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\Big)\Bigg)\dots \\
& + \mathcal{Q}_{fp,b}(b',k',s')\boldsymbol{\mu}(b',k',s')\kappa' + \mathcal{Q}_{fp}(b',k',s')\boldsymbol{\mu}_b(b',k',s')\kappa' \ .
\end{aligned} \tag{32}$$

Analogously, Γ captures the effect of the current capital decision k' on the future planner's decision on the choice variables:

$$\begin{aligned}
\Gamma' = & -\sigma\left(\mathcal{C}_{fp,k}(b',k',s') - \chi \boldsymbol{h}_{fp,k}(b',k',s')\boldsymbol{h}_{fp}(b',k',s')^{\omega-1}\right)\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma-1}\dots \\
& \left((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\right)\dots \\
& + \left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma}\dots \\
& \Bigg((1-\delta)\mathcal{Q}_{fp,k}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\Big(\alpha_v \boldsymbol{v}_{fp,k}(b',k',s')\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v-1}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\dots \\
& + \alpha_h \boldsymbol{h}_{fp,k}(b',k',s')\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\Big)\dots \\
& + \alpha_k(\alpha_k-1)A' k'^{\alpha_k-2}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Bigg)\dots \\
& + \mathcal{Q}_{fp,k}(b',k',s')\boldsymbol{\mu}_{fp}(b',k',s')\kappa' + \mathcal{Q}_{fp}(b',k',s')\boldsymbol{\mu}_{fp,k}(b',k',s')\kappa' \ .
\end{aligned} \tag{33}$$

Intuition The most important equations for the interpretation of the results in the next sections are equations (20)-(22) and equation (24). Equation (20) is the first-order condition on consumption. The second term on the right-hand side relates to the externality via consumption and indicates that the social planner takes the effect of higher consumption on the first implementability constraint (Household capital decision) into account: a higher level of consumption reduces the marginal utility of consumption and therefore reduces the cost of capital in utility terms for households.

Equation (21) displays the optimal decision on bonds. There are two differences compared to the laissez-faire optimality condition: First, the size of the social multipliers (in general) differs from the private ones. Second, the social planner incorporates into his/her decision that the choice of b' influences the future planner's choices of c, h, q, v and μ and therefore the first implementability constraint.

Equation (22) is the first-order condition on capital. Apart from the different multipliers compared to the laissez-faire, the equation displays that the current planner takes into account that k' influences the capital price via the second implementability constraint (Firm investment decision) both today and tomorrow. This relates to the externality via investment. Moreover, it is incorporated that the choice of k' influences the future planner's decisions regarding c, h, q, v and μ and therefore the first implementability constraint.

Finally, equation (24), which is the first-order condition on the capital price, shows that the multiplier of the collateral constraint and of the two implementability constraints are directly linked. This is the case since on the one hand, a higher q makes the collateral constraint less binding and on the other hand, the planner has to take into account the cost of reaching this price on the capital market.

4.2 Optimal debt tax (1-instrument case)

The social planner solution can be implemented through a tax on debt τ_b^{SP1}. A positive debt tax reduces the amount the household receives for a given level of newly issued debt. The revenues of the tax are remitted to the households via a lump-sum transfer T[4]. The household's budget constraint changes to:

$$c + qk'_H + \frac{b'}{(1+\tau_b^{SP1})R} + p_v v = Ak_H^{\alpha_k} h^{\alpha_h} v^{\alpha_v} + q(1-\delta)k_H + b + \Pi + T \; .$$

[4]In case of a subsidy the costs are paid by levying a lump sum tax.

The social planner solution can be decentralized through the following debt tax

$$\tau_b^{SP1} = \frac{\beta R \mathbb{E}_{s'|s}\left[\sigma\left(c' - \chi \frac{h'^{\omega}}{\omega}\right)^{-\sigma-1} \xi' q' + \xi \Omega'\right] - \sigma\left(c - \chi\frac{h^{\omega}}{\omega}\right)^{-\sigma-1} \xi q + \mu^{SP} - \mu^{LF}}{\beta R \mathbb{E}_{s'|s}\left[\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-\sigma}\right]} \tag{34}$$

with ξ being defined as

$$\xi = \left(c - \chi\frac{h^{\omega}}{\omega}\right)^{\sigma}\left(\mu^{SP}\kappa k - \gamma\right) \ . \tag{35}$$

Combining the two equations above gives an expression for the tax, which easier to interpret:

$$\begin{aligned}\tau_b^{SP1} = & \frac{\mathbb{E}_{s'|s}\left[\sigma\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-1}\left(\mu^{SP\prime}\kappa' k' - \gamma'\right) q' + \left(c - \chi\frac{h^{\omega}}{\omega}\right)^{\sigma}\left(\mu^{SP}\kappa k - \gamma\right)\Omega'\right]}{\mathbb{E}_{s'|s}\left[\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-\sigma}\right]} \dots \\ & + \frac{-\sigma\left(c - \chi\frac{h^{\omega}}{\omega}\right)^{-1}\left(\mu^{SP}\kappa k - \gamma\right) q + \mu^{SP} - \mu^{LF}}{\beta R \mathbb{E}_{s'|s}\left[\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-\sigma}\right]} \ .\end{aligned} \tag{36}$$

Interestingly, the expression of the optimal tax in equation (34) is the same as in Bianchi and Mendoza 2018, where capital is fixed. This is the case because the same externality of the consumption choice on the pricing condition (28) is internalized. The social planner takes the effect of consumption on the household pricing condition of bonds into account. If $\mu^{SP} - \mu^{LF}$ as well as Ω' were zero, the social planner would impose a tax in case the gains from marginally releasing the first implementability constraint tomorrow is larger than today.

However, opposed to Bianchi and Mendoza 2018 the multiplier of the household capital decision ξ does not solely depend on the collateral constraint multiplier μ^{SP} anymore: there is also a second relevant component measured by the multiplier on the optimality condition of the investment firm (γ). $\mu^{SP}\kappa k$ measures the gain of a higher capital price induced by a less binding first implementability constraint. A higher level of consumption decreases households' capital cost implying a less binding implementability constraint and a higher price of collateral. The higher price of collateral again makes the collateral constraint less binding. The multiplier γ, however, measures the cost of more consumption on the capital market. A higher level of consumption makes the first implementability condition less binding, which props up the capital price. This increase of the capital price is costly in terms of utility, since investment has to be raised so that this capital price emerges in equilibrium.

Analytically, the sign and size of γ is not clear. The quantitative analysis (see chapter 5) leads to the following conclusions: When the collateral constraint is binding, γ is negative in 7 states and positive in 9670 states. Moreover, it holds: the lower the capital level and the higher the initial debt, the higher is γ. This implies that the cost of relaxing the first implementability constraint (γ) is high in adverse states as is the gain (μ^{SP}). The average value under a binding collateral constraint is 0.158. When the constraint is not binding, γ is negative for most states but always close to zero.

Furthermore, the sign and size of $\mu^{SP} - \mu^{LF}$ as well as Ω' are also analytically not clear. Thus, without a quantitative analysis it is not possible to determine sign and size of the optimal tax.

4.3 Optimal taxes (2-instruments case)

The social planner that is equipped with two instruments has access to the same debt tax as the planner with one instrument. Additionally, he/she can set a tax on investment that influences the investment decision of an investment firm. The optimization problem can be found in the Appendix.

A higher investment tax τ_i^{SP2} leads to higher investment of the capital firm. The profit function of the investment firm looks as follows:

$$\Pi_t = q_t i_{F,t} - (1 + \tau_i^{SP2}) i_{F,t} - \frac{a}{2}(i_{F,t} - \bar{i})^2 \, .$$

A positive debt tax increases the amount of debt that has to be repayed in the next period. The household's budget constraint looks as follows:

$$c + qk'_H + \frac{b'}{(1+\tau_b^{SP2})R} + p_v v = Ak_H^{\alpha_k} h^{\alpha_h} v^{\alpha_v} + q(1-\delta)k_H + b + \Pi + T \ .$$

The optimal debt looks similar to the one in the 1-instrument case:

$$\tau_b^{SP2} = \frac{\beta R \mathbb{E}_{s'|s}\Big[\sigma\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-\sigma-1} \xi^{SP2'} q' + \xi^{SP2}\Omega'\Big] - \sigma\left(c - \chi\frac{h^{\omega}}{\omega}\right)^{-\sigma-1} \xi^{SP2} q + \mu^{SP2} - \mu^{LF}}{\beta R \mathbb{E}_{s'|s}\Big[\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-\sigma}\Big]} \ . \tag{37}$$

However, the size of the multipliers differ and owing to the investment tax there is no investment cost related to relaxing the first implementability constraint, i.e.

$$\xi^{SP2} = \left(c - \chi\frac{h^{\omega}}{\omega}\right)^{\sigma} \left(\mu^{SP2}\kappa k\right) \ . \tag{38}$$

Thus, the interpretation is the same as in Bianchi and Mendoza 2018: If the constraint is not binding today, i.e. $\mu^{SP2} = 0$, the ex ante tax is clearly positive making households borrow less and increasing the capital price and the collateral value tomorrow in an optimal way.
The costs of supporting a higher capital price through investment disappear in equation (37) because the investment tax assures that the optimal level of investment is met. The optimal investment tax is given by the following expression:

$$\tau_i^{SP2} = q - \frac{\beta \mathbb{E}_{s'|s}\Big[\lambda^{SP2'}\left(\alpha_k A' k'^{\alpha_k - 1} v'^{\alpha_v} h'^{\alpha_h} + (1-\delta)(1 + a(k'' - (1-\delta)k' - \bar{i}))\right) + \mu^{SP2'} q' \kappa' + \xi^{SP2}\Gamma'\Big]}{\lambda^{SP2}} \ . \tag{39}$$

If the optimal capital price is higher than the socially desirable level of investment, a tax is levied to create a wedge between the two values and to achieve the socially optimal level of both the capital price and investment.

5 Quantitative Analysis

In this chapter I first describe the calibration of the model and briefly summarize the algorithms for the decentralized equilibrium as well as for the optimal policies. Finally, I present the quantitative results. A detailed description of the data that has been used to compute the targets and how it was detrended can be found in the Appendix.

5.1 Calibration

In order to be comparable to the optimal macroprudential policy analysis by Bianchi and Mendoza 2018, the model is also calibrated to the OECD member countries between 1984 and 2012. As in their calibration, data of all 34 members of the OECD (as of 2012) were used and aggregated. So as to compute targets, the individual statistics are weighted by the 2012 real GDP in purchasing power standards.

The parameters are partly determined via simulation and partly determined ex ante. Mainly, those parameters were chosen to be determined by simulation which are hard to observe, but directly influence certain statistics that can be observed more easily.

Parameters determined ex ante I follow Bianchi and Mendoza 2018 and set the constant relative risk aversion parameter equal to one. Moreover, the shares of inputs and labor in gross output are also taken from Bianchi and Mendoza 2018[5]. The capital share is chosen in such a way that the exponents of the production sum up to one, i.e. we have constant returns to scale. The labor disutility coefficient is normalized so that labor equals one third in the deterministic steady state without collateral constraint. For the Frisch elasticity ω I picked a value that is standard in macroeconomics. Bianchi and Mendoza 2018 use data of the US flow of funds data set to compute the working capital coefficient which I also use. The logged interest rate follows the AR(1) process $\ln(R_t) = (1-\rho_R)\bar{R} + \rho_R \ln(R_{t-1}) + \varsigma_t$ with $\varsigma_t \sim N(0, \sigma_\varsigma)$. I again use the same parameters as Bianchi and Mendoza 2018 and set $\bar{R}$ to 1.1%, ρ_R to 0.68 and σ_ς to 1.38%.

The two outcomes of κ, which determine the maximum loan to value ratio in non-crisis and crisis times, are not easy to identify on a macroeconomic level. Capital in this paper

[5] For a detailed description of how these values are computed I refer to their paper.

is defined as total fixed capital. Since the loan-to-value ratios differ between debt contracts with different types of fixed capital as collateral, and since data on debt contracts are usually very selective, it is hard to pin down an average value for the whole economy. However, a recent study by Kermani and Ma 2022 uses data on the liquidation value of assets on firms' balance sheets in the US and find out that the average liquidation value of property, plant and equipment is 0.35. This is a good approximation for κ, since it is thought to reflect the expected selling price of collateral if the borrower is not able to repay. Thus, I set the loan to value ratio in normal times κ_H to 0.35[6]. To determine the value in crisis times, I apply the same relative reduction of κ as in Bianchi and Mendoza 2018.

There is no comparable data on the price of intermediate goods for the OECD countries. Moreover, the price of intermediate goods only changes the absolute size of v but neither the share of inputs to GDP nor the share of working capital to GDP. Thus, p_v is assumed to be equal to one.

Parameters determined by simulation There are eight parameters which are determined by simulation. The targets are mainly thought to represent several important moments of investment of the OECD member countries.

The discount factor is chosen so that the ratio of capital to GDP in the stochastic steady state matches the capital to GDP ratio of the OECD members of 2.89. Productivity A_t is defined as follows:

$$A_t = e^{z_t} , \tag{40}$$

$$z_t = \bar{z} + \rho_z z_{t-1} + \epsilon_t , \tag{41}$$

$$\epsilon_t \sim N(0, \sigma_\epsilon) . \tag{42}$$

The mean of $\bar{z}$ is set to zero so that the average of A is 1. In order to match the observed autocorrelation of GDP of 0.68, ρ_z equals 0.52. The standard deviation σ_ϵ is set to 2.25% to match a ratio of standard deviation of investment to the standard deviation of GDP

[6] Bianchi and Mendoza 2018 are able use higher values, since their capital price is lower than one in the steady state.

of 2.8.

The parameter $\bar{i}$, which determines at which investment level the price of capital is equal to 1, is set to the mean of investment in the stochastic steady state (0.0248). The capital price is also the investment price in my model. The sensitivity of the investment price to deviations of investment from $\bar{i}$ equals 6.2 in my calibration to match the standard deviation of the OECD investment price. Since the relative price of investment is not directly observable, I follow the method of the literature on the relative price of capital (e.g. Lian et al. 2020) and compute it as the ratio of the investment price level to the consumption price level. This parametrization leads to an average adjustment cost relative to GDP of 0.13% under laissez-faire.

The transition probabilities of the loan-to-value ratio are set so that a crisis probability of 4% and an average crisis duration of one year are matched. These targets are taken from Bianchi and Mendoza 2018 and the same definition of crises is applied: "A financial crisis is defined as an event in which the linearly detrended current account is above two standard deviations from its mean" (Bianchi and Mendoza 2018).

Table 1: Calibration

Parameter	Description	Value	Source/Target
σ	Risk aversion	1	Standard
α_v	Share of inputs in gross output	0.45	Bianchi and Mendoza 2018
α_h	Share of labor in gross output	0.352	Bianchi and Mendoza 2018
α_k	Share of capital in gross output	0.198	Constant returns to scale
χ	Labor disutility coefficient	0.49	Normalization so that $h = \frac{1}{3}$ in SS without CC
$\frac{1}{\omega-1}$	Frisch elasticity	1	Standard
θ	Working capital coefficient	0.16	Bianchi and Mendoza 2018
κ_H	Normal credit regime	0.35	Average liquidation value of fixed assets (Kermani and Ma 2022)
κ_L	Tight credit regime	0.29	Procentual reduction of LTV observed for housing
$\bar{R}$	Mean of interest rate process	1.1%	Bianchi and Mendoza 2018
ρ_R	Autocorrelation of interest rate process	0.68	Bianchi and Mendoza 2018
σ_ς	Conditional SD of interest rate process	1.38 %	Bianchi and Mendoza 2018
Parameters determined by simulation			
β	Discount factor	0.97	Ratio of capital to GDP = 2.89
a	Adjustment cost parameter I	6.2	SD of investment price = 0.04
$\bar{i}$	Adjustment cost parameter II	0.0248	Mean of investment (stochastic SS)
$\bar{z}$	Mean of TFP process	0	Normalization
ρ_z	Autocorrelation of TFP process	0.52	Autocorrelation of GDP of 0.68
σ_ϵ	Conditional SD of TFP process	2.25%	Ratio of SD of investment and SD of GDP = 2.8
$P_{H,L}$	Transition probability κ_H to κ_L	0.06	Crises probability of 4 %
$P_{L,L}$	Transition probability κ_L to κ_L	0	Average crises duration of 1 year

5.2 Algorithms

Since there is evidence that local solution methods are particularly imprecise for the type of model used in this paper (Groot et al. 2023), I use a global solution method for both laissez-faire and the social planner problems. In particular, I use fixed point iteration on the model's Euler equations to compute policy functions on a discrete grid $b \times k \times z \times R \times \kappa$[7]. The grid has 32400 elements. For values that do not lie on the grid I

[7] z denotes the productivity shock, see next subsection.

use bilinear interpolation.

In case of laissez-faire, the algorithm is a modified version of the "FiPit-algorithm" by Mendoza and Villalvazo 2020, which was created to solve the model by Mendoza 2010 very fast. I adapt the algorithm to my model's equations and shocks. For the two optimal policy problems I use two nested fixed point algorithms that differ from other macroprudential policy papers in the solution method I use in the inner loop. In the inner loop I solve for the policy functions given the policy functions of the future planner by using a fixed point algorithm. In the outer loop I update the policy functions of the future planner. A detailed description of the three algorithms can be found in the Appendix.

5.3 Crisis events under laissez-faire

The calibrated version of the model is able to produce crisis events which are in line with the study by Mendoza 2010, who focuses on replicating the dynamics around sudden stop events, and with Bianchi and Mendoza 2018. To summarize the dynamics around financial crises, I first simulate the laissez-faire economy for 100,000 periods and identify all events as a financial crisis where the current account is two standard deviations above its mean. Afterwards, for each event I create time series from 5 years before to four years after the crisis and compute the mean of all events for each period.

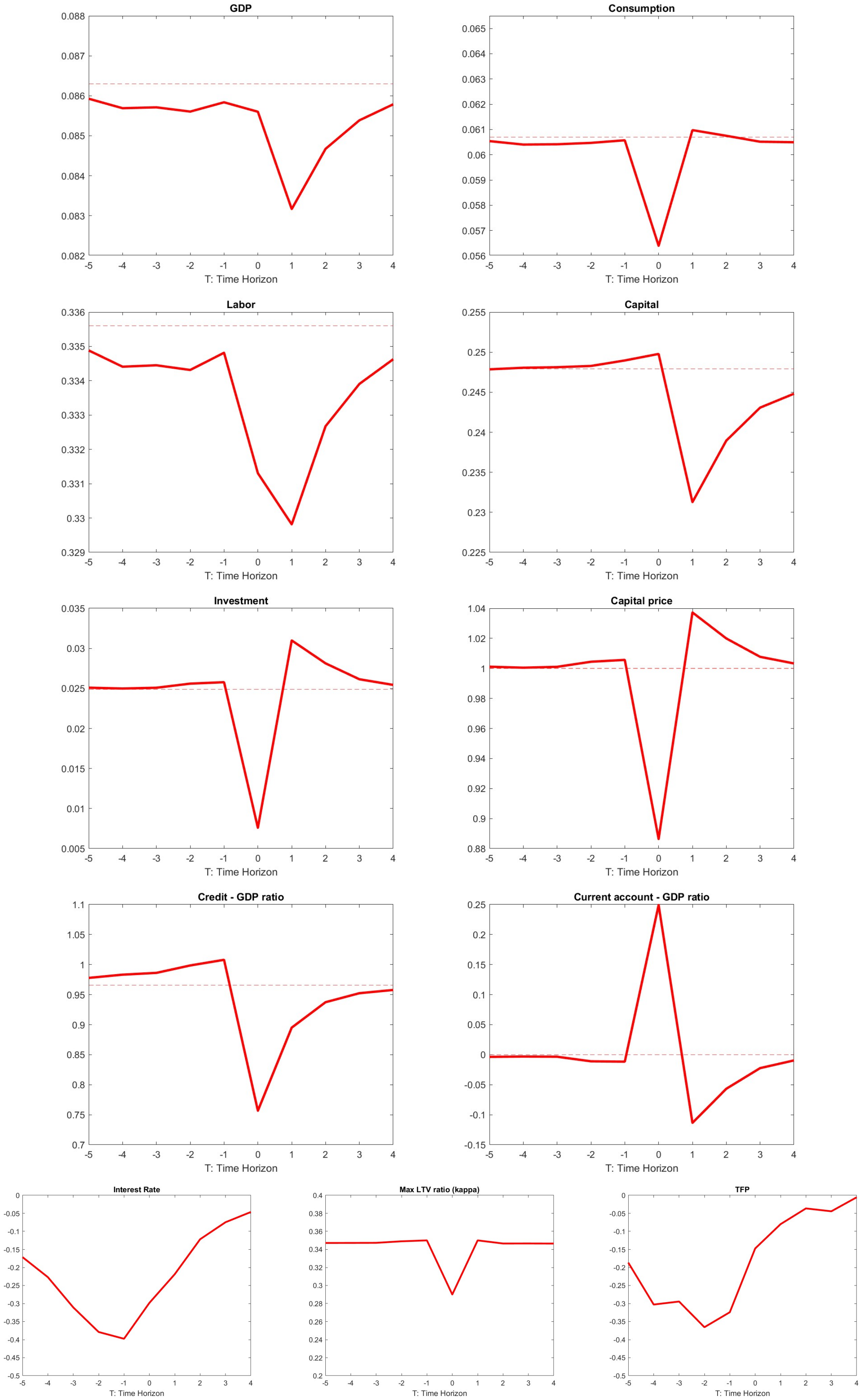


Figure 1: *Dynamics around crises (dotted line = long-run average)*

The figure summarizes the dynamics around crises which occur in $T = 0$. All values are in levels - except for R and TFP which are in percentage deviations from the long run mean. The dotted line in each subfigure displays the long-run mean of the respective variable.

The credit to GDP ratio drops in a crisis, since the collateral constraint is binding, the maximum loan-to-value ratio is lower than usual and collateral prices are down due to the Fisherian deflation mechanism. The reaction of investment in a crisis is - as expected - much stronger than the one of consumption. Since the loan-to-value ratio will increase again in the next period and more can be borrowed and consumed, the future marginal utility is quite low compared to today. Therefore, the current cost of capital is quite high and households decide to choose a low level of capital tomorrow. This leads to a low investment level. Moreover, the sum of investment and consumption has to decrease as lower GDP and borrowing lead to a decrease of available funds. This is in line with Bianchi and Mendoza 2018. Per definition the current account increases strongly during a crisis. After the crisis the credit to GDP ratio recovers (current account is negative), since financial conditions improve (κ rises). Capital and GDP are declining one period after the financial crisis due to the reduction of investment one period earlier. In Mendoza 2010 there is a reduction of GDP during the crisis because he uses a different crisis definition and in Bianchi and Mendoza 2018 GDP decreases during the crisis since capital is fixed. Labor shrinks in a crisis, since the reduced level of intermediate goods due to tight financing conditions reduce the return to labor. In the period after the crisis the reduced level of capital further diminishes the return to labor and therefore working time. Investment increases strongly in the first period after the crisis because TFP has risen and the current level of capital is low.

Similar to Mendoza 2010 crises occur in my model when the already low interest rate has decreased ex ante, stimulating overborrowing, and then rises again. Moreover, crises occur when there is a financial shock, i.e. κ is at the lower level, and there were no financial shocks in the periods before. The capital price decreases up to a level of 0.9, which is perfectly in line with the reduction observed in the data by Mendoza 2010. The capital price is low because of Fisherian deflation: since investment and consumption are low, both first-order conditions related to capital imply a lower capital price. The lower price reduces the collateral value implying less debt and a further pronounced reduction in consumption and investment. To sum up, the model produces strong recessions under laissez-faire that are in line with the literature.

5.4 Results

Optimal taxes The following figures show the optimal debt tax in the 1-instrument case when the collateral constraint is not binding (ex ante tax) and the optimal tax when the collateral constraint is binding (ex post tax). I display the ex ante tax for intermediate values of the interest rate and the productivity shock and a high maximum loan to value ratio (κ). For the ex post tax I choose the same values of the interest rate and the productivity shock but a low κ[8].

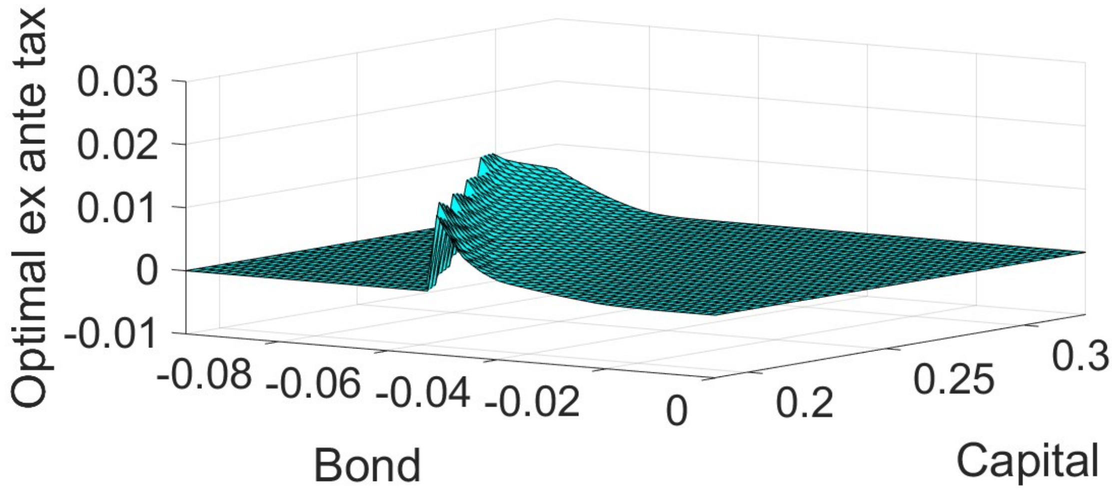


Figure 2: *Optimal Pigouvian ex ante debt tax (1-instrument case)*

[8]The figure only changes slightly when considering the same financial shock, but the probability that the constraint is binding, i.e. an ex post tax has to be imposed, is much higher when the financial shock is a the low realization.

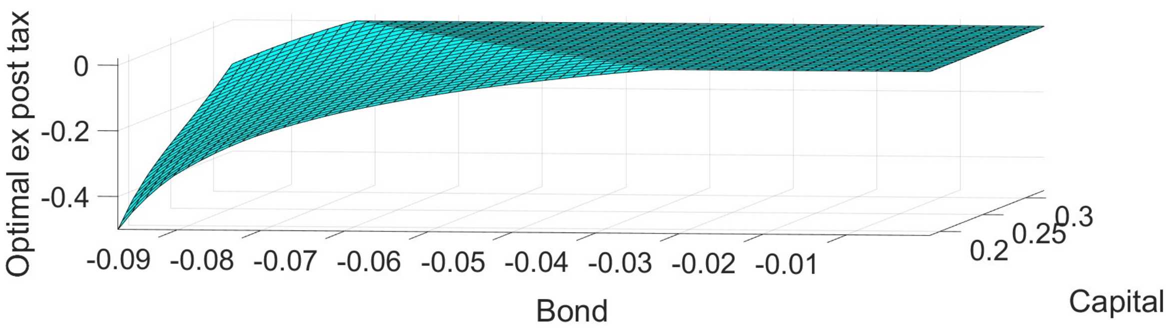


Figure 3: *Optimal Pigouvian ex post debt tax (1-instrument case)*

The optimal ex ante tax is almost zero if debt is low and capital is high, since the probability of being constrained within the next period is very low. The closer the state is to the binding region (left part of the figure), the higher is the macroprudential tax. This is due to the high probability that the collateral constraint will be binding in the next period with the pecuniary externality having strong adverse effects. In the binding region I have set the ex ante tax to zero, since the ex post tax is operating there.

The optimal ex post tax is slightly positive when the constraint is hardly binding and becomes more negative the higher the debt and the lower the capital. Since the capital price is lower in that region, the amplification effects are strong and it is optimal to increase q by reducing the amount of debt that has to be repayed and therefore increasing investment and consumption. Households do not take into account that their consumption level influences the capital price, which means that they consume to less implying a decreased capital value and collateral value. This, however, has adverse consequences, since the lower collateral level would induce an even lower consumption level and an even lower capital price. The social planner is aware of the Fisherian debt deflation mechanism and subsidizes borrowing to increase consumption. The lower the debt and capital levels, the stronger is the reduction in q, which makes a very high debt subsidy optimal.

The next two figures display the optimal ex ante and ex post tax in the 2-instruments case:

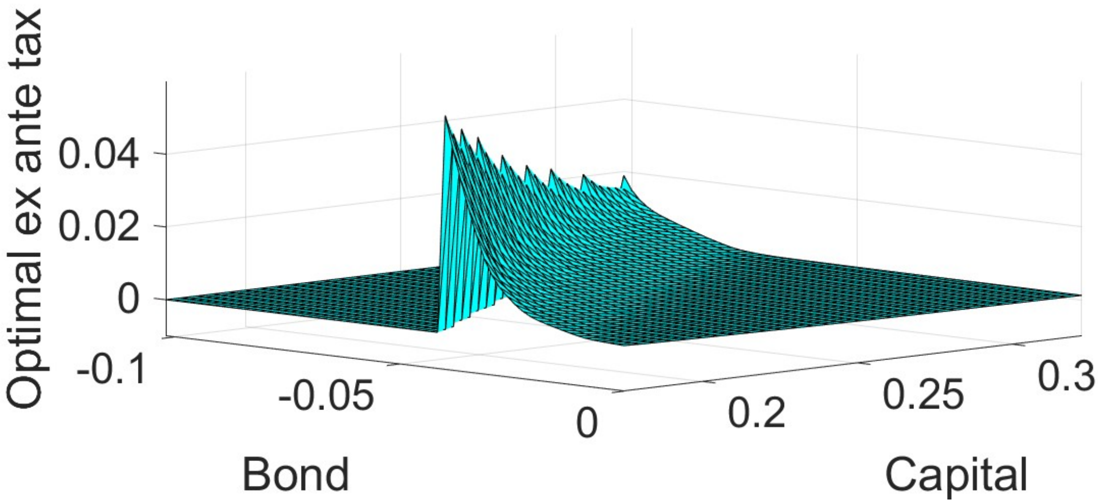


Figure 4: *Optimal Pigouvian ex ante debt tax (2-instruments case)*

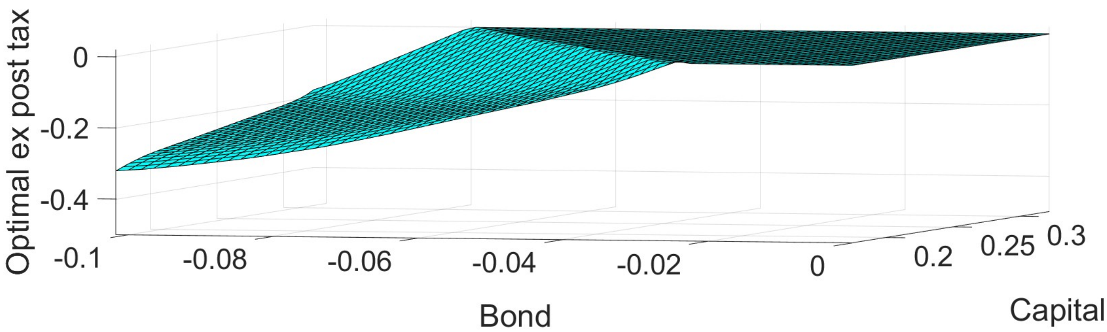


Figure 5: *Optimal Pigouvian ex post debt tax (2-instruments case)*

The sign of the taxes are the same as in the 1-instrument case: the ex ante tax is positive and the ex post tax is negative, i.e. a subsidy is paid. However, the ex ante tax is higher than in the 1-instrument case. This is due to the fact that a higher level of consumption in the next period still props up the capital price, but does not have to be accompanied by increased investment. Thus, it is even more desirable to decrease debt today in order to stabilize the capital price tomorrow. Still, it is optimal to have the highest debt tax close to the binding region because there the probability of being in a constrained state tomorrow is highest. The ex post tax falls less with a higher debt and a lower capital because the ex ante tax has been more effective compared to the 1-instrument case.

The two figures below display the optimal ex ante and ex post investment tax:

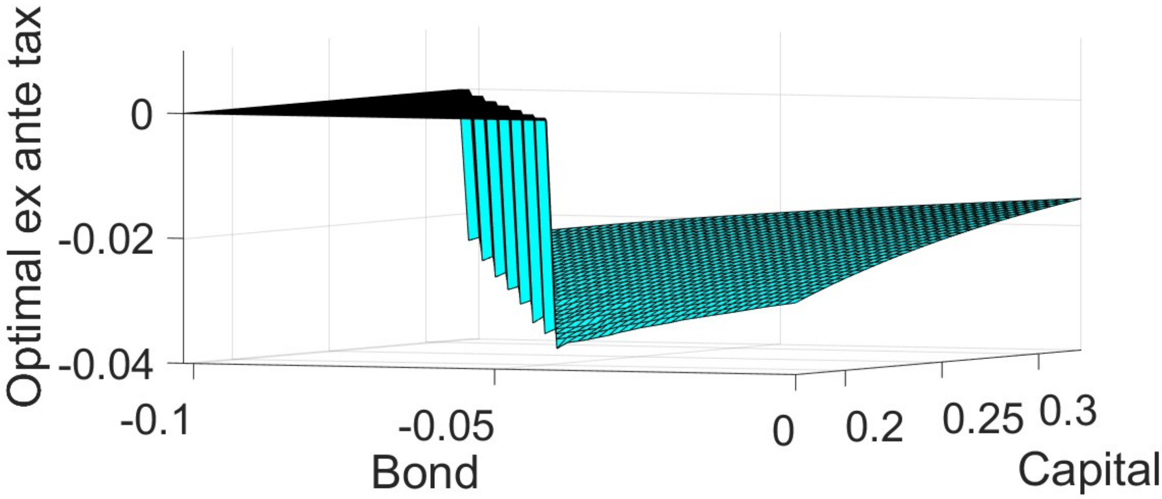


Figure 6: *Optimal Pigouvian ex ante investment tax*

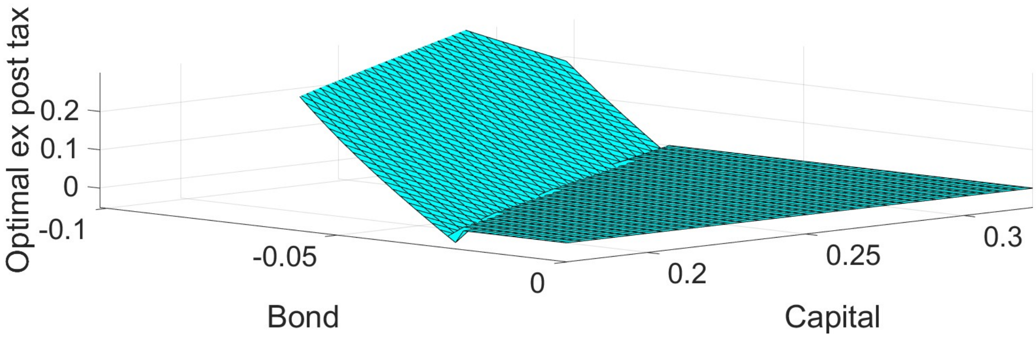


Figure 7: *Optimal Pigouvian ex post investment tax*

The ex post investment tax is merely positive, i.e. investment firms are taxed. Since a high capital price to increase the collateral value as well as a low investment are desirable, the tax creates a wedge that achieves the split of both variables. The ex ante investment tax is negative, i.e. a subsidy is paid. Thus, the optimal capital price that internalizes the externality of consumption on asset prices is lower than the price that would emerge with optimal investment. The subsidy makes it possible to both achieve the optimal level of investment as well as the optimal capital price level.

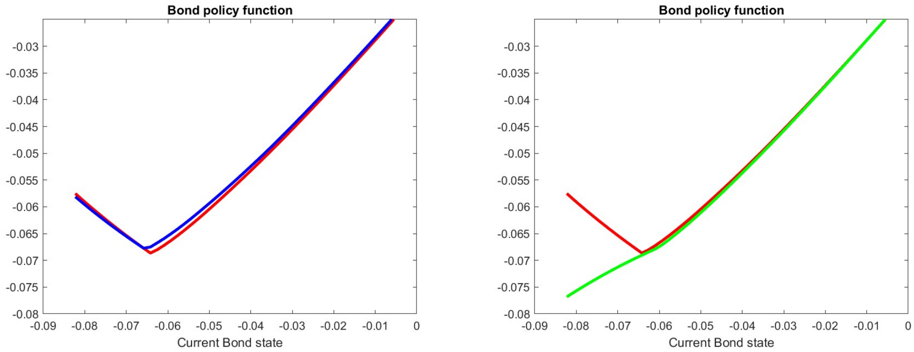


Figure 8: *Bond policy functions for a given level of capital and exogenous states: Laissez-faire (red) vs. 1-instrument case (blue) / 2-instruments case (green)*

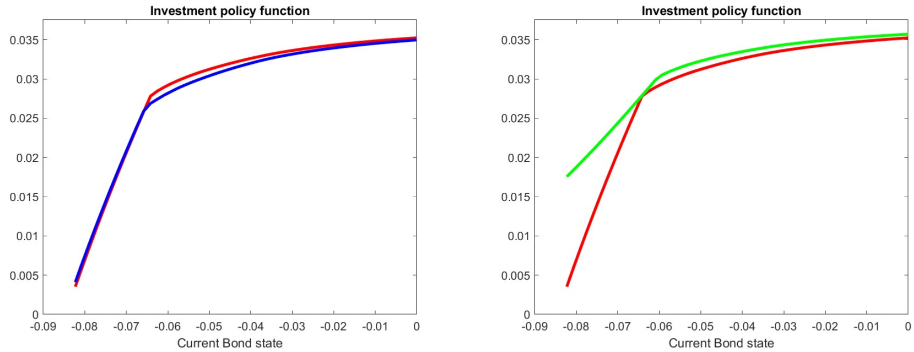


Figure 9: *Investment policy functions for a given level of capital and exogenous states: Laissez-faire (red) vs. 1-instrument case (blue) / 2-instruments case (green)*

Differences in borrowing and investment The figures above respectively depict the bond and investment policy functions in the 1-instrument case and the 2-instruments case compared to laissez-faire. For the sake of more visible differences, capital is fixed at an intermediate level. The interest rate shock as well as the productivity shock are also at an intermediate level. The maximum loan-to-value ratio (κ) is high.

In the 1-instrument case the ex ante debt tax reduces debt before the constraint binds, which is the point of the kink of the bond policy function. This lower debt also leads to lower values of the investment policy function compared to laissez-faire. In the 2- instruments case there is slightly more borrowing until the constraint binds under laissez-faire and much more borrowing when the constraint is binding in an unregulated economy. Thus, the optimal policy is very effective at increasing the borrowing capacity. The higher level of borrowing despite of an ex ante debt tax is caused by general equilibrium forces. Investment is higher at any level of current bonds.

Comparison of welfare and averages across economies The following table summarizes the main results of the quantitative analysis. Welfare is computed as the standard compensating consumption variations for each initial state that equates current expected utility of laissez-faire and the economies of interest (with consumption c^* and labor h^*). In particular, welfare is computed as $\vartheta(b, k, s)$ that solves

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t^{LF}(1+\vartheta), h_t^{LF}) \right] = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t^*, h_t^*) \right] .$$

Table 2: Main results

	Laissez-faire	**1 instrument: debt tax** (ex ante & ex post)	**1 instrument: debt tax** (ex ante only)	**2 instruments: debt and capital tax** (both ex ante & ex post)
Average ex ante debt tax (in %)	–	0.52	0.52	1.4
Average ex post debt tax (in %)	–	-1.75	–	-1.21
Average ex ante investment tax (in %)	–	–	–	-2.64
Average ex post investment tax (in %)	–	–	–	7.55
Probability of crises (in %)	4.47	3.95	4.06	0.0321
Binding collateral constraint (in %)	39.7	25.67	24.29	80.76
Change of investment compared to laissez-faire (in %)	–	-0.23	-0.35	-4.08
Average debt to GDP	0.836	0.82	0.815	0.899
Average capital to GDP	2.884	2.882	2.879	2.831
Average capital	0.248	0.248	0.247	0.238
Average welfare gain (in %)	–	0.0022	0.0018	0.0289

I find that in the 1-instrument case the optimal ex ante Pigouvian tax on debt is 0.52 percent on average, which is much lower than what other studies find (see below). This is due to the cost of macroprudential policy in terms of high investment in crises as argued above. The optimal ex post Pigouvian tax on debt is -1.75 percent on average. The optimal policy decreases the probability of crises by 0.52 percentage points[9]. Thus, the availability of a debt tax alone is not enough to prevent crises. Still, the debt tax is very effective at reducing the probability of being in a constrained state. This suggests that the tax is effective to make the laissez-faire states non-binding where the constraint is hardly binding, but not effective at improving conditions when the constraint is strongly binding under laissez-faire. Average investment is reduced by 0.23% due to reduced borrowing. Welfare increases by 0.0022 percent. This value is very low compared to the optimal macroprudential policy literature where capital is fixed. In order that my research findings are comparable to the other studies I also compute policy functions and the ergodic distribution when only the optimal ex ante tax is applied[10]. This experiment shows that the major part of the welfare gain is due to the ex ante tax. However, the probability of crises is higher without an ex post tax.

Table 3: Optimal ex ante tax and its effects across the literature

	Bianchi and Mendoza 2018	Bianchi and Mendoza 2020	Ma 2020
Optimal ex ante debt tax (in %)	3.6	1.31	1.28
Welfare gain (in %)	0.3	0.0179	0.06
Reduction of crisis probability (in %)	4 to 0.02	3.15 to 1.06	6.23 to 1.89

In the 2-instruments case there are almost no crises anymore. Thus, the availability of an investment tax additional to a debt tax is crucial for preventing crises. The ex ante debt tax is 1.4 percent on average, which is more than twice as high as in the 1-instrument case. As discussed above the cost of an ex ante tax in terms of higher investment in the binding state tomorrow does not exist owing to the investment tax. The ex post debt subsidy is 1.21 percent on average. It is lower than in the case without an investment tax since the ex ante debt policy is more aggressive and successful. The ex ante investment tax is -2.64 percent on average and a level of 7.55 percent regarding the

[9]The current account values that define a crisis are taken from laissez-faire.

[10]Opposed to Bianchi and Mendoza 2018 the ex ante tax is not necessarily optimal without the ex post part due to the optimality condition of investment firms. However, it is a good reference point

ex post investment tax is optimal. Since the instruments are very successful at making binding states less harmful, the probability of being in a state with a binding constraint is roughly 80 percent. This shift in the ergodic distribution is also the reason why there is less investment on average, although the policy functions indicate higher investment in the same states. Moreover, the debt level is higher owing to the fact that agents are impatient and a binding constraint is much less harmful. The welfare gain in the 2-instruments case is more than 13 times larger than the welfare gain when only the debt tax is available. That means that the availability of an investment tax boosts the welfare gain.

Crises situations under laissez-faire: what would a social planner do? The following figures again show the average crises dynamics of several variables under laissez faire, but now there is also the comparison to how the economies with 1 respectively 2 instruments would have evolved: Given the bonds and capital states of five periods before the laissez-faire crisis as well as all shocks until four periods after the crisis, I computed the optimal choice of the social planners by simulating their economies. It is important to underline that the graphs cannot be interpreted as a time series, since they do not depict an example crisis but the average of all laissez-faire crisis situations. However, if the averages diverge, the differences point at systematical deviations of the social planner economies under unfavorable economic situations.

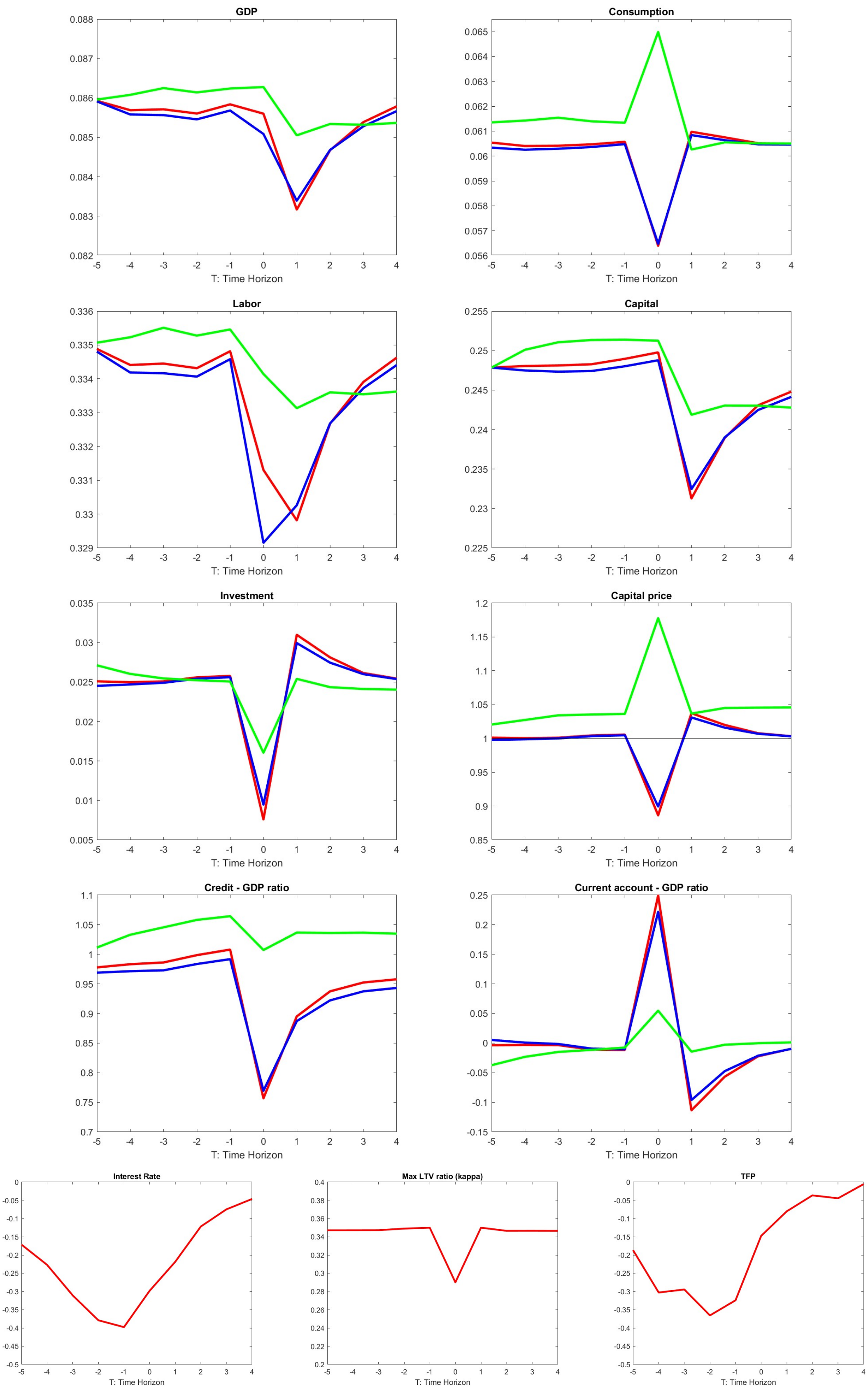


Figure 10: *Crises situations under laissez-faire: what would a social planner do? Red line = Laissez-faire, blue line = 1 instrument, green line = 2 instruments*

Before the crisis In the 1-instrument case the ex ante borrowing tax reduces the average credit to GDP ratio since borrowing becomes more expensive. Lower borrowing reduces investment and therefore capital before the crisis compared to laissez-faire. The lower level of capital implies a lower return to labor and therefore less labor. Consequently, GDP is also lower in the 1-instrument case compared to laissez-faire. Moreover, less borrowing implies a slightly positive/less negative current account to GDP ratio. The capital price is almost the same before the crisis at $T = 0$, since the difference in investment is small. There is also a slightly lower level of consumption in the 1-instrument case compared to laissez-faire due to reduced borrowing. The social planner imposes a debt tax before a crisis to reach a higher consumption level in the crisis period propping up the capital price and increasing the collateral value. The debt tax and the implied differences in consumption and investment are quite small as the social planner takes into account that an induced higher collateral price in the crisis can only be achieved by higher investment, although a low investment is desirable.

The 2-instrument case is not easy to interpret since the less severe crisis period has an effect on all other periods as well. There is a higher level of investment in periods -5 to -3, which leads to a higher capital stock compared to the two other cases before the crisis. More capital has the effect that the return to labor rises and therefore working hours are higher. The development of the capital price is - due to the availability of the investment tax - separated from the investment level. The social planner chooses on average a higher consumption level than in the 1-instrument case and under laissez-faire. The higher consumption level increases the capital price and borrowing capacity. Thus, the credit to GDP ratio is higher than in the other cases and the current account is slightly negative until period -1.

The crisis In the crisis, i.e. $T = 0$, the lower initial level of debt in the 1-instrument case compared to laissez-faire and a debt subsidy lead to a lower debt service and to decreased credit costs. Thus, there are more funds available, which leads to a slightly less pronounced decrease in investment and capital as well as a tinily higher consumption level than under laissez-faire, which stabilizes the capital price and dampens the Fisherian deflation mechanism. This is exactly the externality the social planner internalizes: the debt subsidy props up consumption increasing the collateral price and alleviating the debt deflation. The borrowing capacity does not drop as strongly as under laissez-faire, which is reflected by the less positive current account and the higher credit to GDP ratio. Labor falls more strongly compared to laissez-faire, since the intermediate good also falls more than in the 1-instrument case. In the crisis, the increase in households' utility merely comes from reduced working hours.

The picture changes clearly in the 2-instruments case. Now, only consumption and the capital price are closely related: a high capital price can only be generated if consumption is high (via the household pricing condition on bonds). The movements of investment and capital prices are now detached. Thus, compared to the social planner with 1 instrument it is now possible to generate both a high capital price due to high consumption and a low level of investment. As in the 1-instrument case the debt subsidy increases the funds available. But now the debt subsidy is even bigger and the increased amount of funds is merely used for higher consumption propping up the capital price and increasing the borrowing capacity. Since the borrowing capacity and therefore the funds available increase strongly, investment falls less than in the other two cases. Again, the social planner internalizes that a higher level of consumption is making the collateral constraint less binding, but he/she does no longer have to assign big shares of the increased borrowed funds to investment.

The credit-GDP ratio drops only a bit in the 2-instruments case because the high capital price increases the collateral value and therefore the borrowing capacity. This is also the reason why the reaction of the current account is much less pronounced than under laissez-faire / in the 1-instrument case. Labor falls as the binding constraint increases the cost of v, reducing the return to labor. In the 2-instruments case the (much bigger) utility gain in the average crisis comes from consumption, not from labor.

After the crisis After the crisis the average of most variables align, since the economies return to "normal times". What strikes the eye is that the capital price and the credit to GDP ratio are higher in the 2-instruments economy. This is because on average the collateral constraint is more binding than in the other two economies.

6 Conclusion

This paper has provided evidence that the integration of capital formation in models used to analyze macroprudential regulation makes a less severe intervention - in form of a very small debt tax - optimal before crises. Moreover, under the optimal policy crises occur less often than without any intervention, but more often than in other studies under the optimal policy. The welfare gains are also small compared to studies with fixed capital. The availability of an investment tax in addition to a debt tax boosts welfare and makes crises extremely rare events. Thus, from a policy perspective it is important to think of macroprudential policy as debt **and** investment regulation.

There are two interesting questions which arise from my analysis and which have not been answered up to this point: First, how would the optimal tax change if the effects of lower investment on growth were also taken into account? Second, this analysis aggregates over different types of capital. How would the results change if different types of capital with different prices and loan-to-value-ratios were analyzed? These questions are left for future research.

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Appendix

Contents

A 3-period model

As stated above it is possible to show in a 3-period model that the optimal borrowing tax in a 1-instrument case takes into account that the imposed tax reduces capital, which again reduces the optimal tax. The following model is based on Dávila and Korinek 2017 but builds on an open economy setting instead of a two-agent setting..

A.1 Set-up

Agents/households live for three periods $t = 1, 2, 3$. There is a continuum of agents of mass one. They derive utility from consumption c_t of a non-durable good. Agents maximize their lifetime utility $\sum_{t=1}^{3} \beta^t u(c_t)$ and their preferences satisfy $u_t = \log c_t$ for $t = 1, 2$ and $u_t = c_t$ for $t = 3$. There is no uncertainty.
In the first period, the household receives an exogenous income y and can borrow $-b_2$ at the interest rate R_1 in that period. Income and borrowed funds are spent on consumption c_1 and on investment. The investment technology is given by a quadratic cost function as in chapter 4 of Dávila and Korinek 2017. Thus, higher investment increases the stock of capital in the next period k_2, but also increases investment cost in the current period. The income in period 2 is then given by $A_2 k_2$ (A_2 denotes productivity). In this period households are subject to a collateral constraint and can only borrow up to a fraction ϕ of their collateral value which is given by end-of-period capital holding times the price of capital q_2. Income plus borrowing $-b_3$ at rate R_2 can be spent on consumption c_2 and on capital accumulation: households can trade non-depreciating capital among each other at price q_2. In the last period the household receives income $A_3 k_3$, repays its debt b_3 and consumes c_3.

Table 4: Summary

	Period I	**Period II**	**Period III**
Income	y	$A_2 k_2$	$A_3 k_3$
Budget constraint	$y - \frac{b_2}{R_1} - c_1 - \frac{a}{2} k_2^2$	$A_2 k_2 - \frac{b_3}{R_2} - c_2 - q_2(k_3 - k_2) + b_2$	$A_3 k_3 + b_3 - c_3$
Collateral constraint	–	$\frac{b_3}{R_2} \geq -\phi q_2 k_3$	–

Borrowed funds are assumed to be supplied in any amount by a foreign country. Aggregate capital in period $t = 2$ is fixed so that the capital market clearing condition is $k = k_2 = k_3$.

A.2 Laissez-faire

The Lagrangian of the household's maximization problem looks as follows:

$$
\begin{aligned}
L = & \log c_1 + \beta \log c_2 + \beta^2 [A_3 k_3 + b_3] \\
& + \lambda_1 [y - \frac{b_2}{R_1} - c_1 - \frac{a}{2} k_2^2] \\
& + \beta \lambda_2 [A_2 k_2 - \frac{b_3}{R_2} - c_2 - q_2 (k_3 - k_2) + b_2] \\
& + \beta \mu [\frac{b_3}{R_2} + \phi q_2 k_3] \ . \qquad \text{(App.1)}
\end{aligned}
$$

Parameters are restricted to the subspace that leads to a binding collateral constraint under laissez-faire. Thus, the problem above leads to the following first-order conditions:

$$
\begin{aligned}
c_1: & \quad \frac{1}{c_1} - \lambda_1 = 0 && \text{(App.2)} \\
c_2: & \quad \frac{1}{c_2} - \lambda_2 = 0 && \text{(App.3)} \\
b_2: & \quad -\frac{\lambda_1}{R_1} + \beta \lambda_2 = 0 && \text{(App.4)} \\
b_3: & \quad \beta - \frac{\lambda_2}{R_2} + \frac{\mu}{R_2} = 0 && \text{(App.5)} \\
k_2: & \quad -a k_2 \lambda_1 + \beta (q_2 + A_2) \lambda_2 = 0 && \text{(App.6)} \\
k_3: & \quad \beta A_3 + \mu \phi q_2 - q_2 \lambda_2 = 0 \ . && \text{(App.7)}
\end{aligned}
$$

Equilibrium:
Capital is assumed to be fixed so that capital market clearing implies

$$
k := k_2 = k_3 \ . \qquad \text{(App.8)}
$$

Plugging in of the capital market clearing condition into the budget constraints yields the following resource constraints:

$$
\begin{aligned}
t = 1: & \ y - \frac{b_2}{R_1} - c_1 - \frac{a}{2} k^2 && = 0 && \text{(App.9)} \\
t = 2: & \ A_2 k - \frac{b_3}{R_2} - c_2 + b_2 && = 0 && \text{(App.10)} \\
t = 3: & \ A_3 k + b_3 - c_3 && = 0 \ . && \text{(App.11)}
\end{aligned}
$$

Furthermore, the collateral constraint is binding:

$$
\frac{b_3}{R_2} = -\phi q_2 k \ . \qquad \text{(App.12)}
$$

The household's optimality conditions can be rearranged in the following way:

$$c_2 = c_1 \beta R_1 \tag{App.13}$$

$$\mu = \frac{1}{c_2} - \beta R_2 \tag{App.14}$$

$$k = \beta \frac{(A_2 + q_2) c_1}{a c_2} \tag{App.15}$$

$$q_2 = \frac{\beta A_3 c_2}{1 - \phi + \beta \phi R_2 c_2} \ . \tag{App.16}$$

Equations (App.15) and (App.16) will be taken into account by the social planner who is equipped with a debt tax and an investment subsidy. The social planner who is equipped with a debt tax only just takes equation (App.16) into account.

As can be seen in equation (App.16), the price of capital crucially depends on the loan to value ratio ϕ. If the borrowing limit is equal to the entire collateral value, i.e. $\phi = 1$, the price of collateral no longer depends on the level of consumption and no externality exists. I define this price as q^*:

$$q^* = \frac{A_3}{R_2} \ . \tag{App.17}$$

It is now possible to split the price of collateral into the price without externality q^* and the distorting externality $\Lambda(\underset{+}{c_2})$:

$$q_2^{LF} = \Lambda(\underset{+}{c_2}) q^* \ , \tag{App.18}$$

$$\text{with} \quad \Lambda(\underset{+}{c_2}) = \frac{\beta R_2 c_2}{1 - \phi + \beta \phi R_2 c_2} \ . \tag{App.19}$$

A.3 Optimal policy (2 instruments)

Let us now assume that the planner is equipped with a Pigouvian debt tax and a Pigouvian investment tax. A positive debt tax τ_b^{SP2} reduces the amount the household receives for a given level of newly issued debt in period 1. A positive investment tax τ_i^{SP2} increases the quadratic investment cost in the first period. Thus, the household's budget constraint in period 1 changes to:

$$y - \frac{(1 - \tau_b^{SP2}) b_2}{R_1} - c_1 - (1 + \tau_i^{SP2}) \frac{a}{2} k^2 \ . \tag{App.20}$$

Moreover, the household's decisions on borrowing and investment (App.13) and (App.15) change to:

$$c_2 (1 - \tau_b^{SP2}) = c_1 \beta R_1 \tag{App.21}$$

$$k = \beta \frac{(A_2 + q_2)c_1}{(1 + \tau_i^{SP2})ac_2} \; . \tag{App.22}$$

A social planner who is equipped with these two instruments maximizes households' consumption subject to the resource constraints as well as to the collateral constraint. He/she takes into account that the price of collateral in the collateral constraint is directly influenced by the level of consumption in the second period. Thus, the Lagrangian of the planner's maximization problem looks as follows:

$$\begin{aligned} L = & \log c_1 + \beta \log c_2 + \beta^2[A_3 k + b_3] \\ & + \lambda_1^{SP2}[y - \frac{b_2}{R_1} - c_1 - \frac{a}{2}k^2] \\ & + \beta\lambda_2^{SP2}[A_2 k - \frac{b_3}{R_2} - c_2 + b_2] \\ & + \beta\mu^{SP2}[\frac{b_3}{R_2} + \phi q^* \Lambda(\underset{+}{c_2})k] \; . \end{aligned} \tag{App.23}$$

The problem above leads to the following first-order conditions:

$$\begin{aligned} c_1 : & \quad \frac{1}{c_1} - \lambda_1^{SP2} & = 0 \; , \quad & \text{(App.24)} \\ c_2 : & \quad \frac{1}{c_2} - \lambda_2^{SP2} + \phi\mu^{SP2} q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k & = 0 \; , \quad & \text{(App.25)} \\ b_1 : & \quad -\frac{\lambda_1^{SP2}}{R_1} + \beta\lambda_2^{SP2} & = 0 \; , \quad & \text{(App.26)} \\ b_2 : & \quad -\frac{\lambda_2^{SP2}}{R_2} + \beta + \frac{\mu^{SP2}}{R_2} & = 0 \; , \quad & \text{(App.27)} \\ k : & \quad -ak\lambda_1^{SP2} + \beta\lambda_2^{SP2} A_2 + \beta^2 A_3 + \beta\mu^{SP2}\phi q^* \Lambda(\underset{+}{c_2}) & = 0 \; . \quad & \text{(App.28)} \end{aligned}$$

It is possible to write down all multipliers as functions of c_1:

$$\begin{aligned} \lambda_1^{SP2} &= \frac{1}{c_1} & \text{(App.29)} \\ \lambda_2^{SP2} &= \frac{1}{\beta c_1 R_1} & \text{(App.30)} \\ \mu^{SP2} &= \frac{1}{\beta c_1 R_1} - \beta R_2 \; . & \text{(App.31)} \end{aligned}$$

The combination of equations (App.25) and (App.30) leads to the following equation:

$$\frac{1}{c_2} - \frac{1}{\beta c_1 R_1} + \phi\mu^{SP2} q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k = 0 \; . \tag{App.32}$$

Furthermore, the investment decision can be rewritten by using equations (App.28), (App.29), (App.30) and (App.31):

$$-ak\frac{1}{c_1} + \beta\frac{1}{\beta c_1 R_1}A_2 + \beta^2 A_3 + \beta\left(\frac{1}{\beta c_1 R_1} - \beta R_2\right)\phi q^* \Lambda(\underset{+}{c_2}) = 0\ . \tag{App.33}$$

It is now possible to use the equations above as well as equations (App.21) and (App.22) to solve for the optimal debt and investment tax:

$$\begin{aligned} \frac{1-\tau_b^{SP2}}{\beta c_1 R_1} - \frac{1}{\beta c_1 R_1} + \phi\mu^{SP2} q^* \frac{\partial\Lambda}{\underset{+}{\partial c_2}} k = 0 \\ \Leftrightarrow \beta c_1 R_1 \phi \mu^{SP2} q^* \frac{\partial \Lambda}{\underset{+}{\partial c_2}} k = \tau_b^{SP2} \end{aligned} \tag{App.34}$$

$$\begin{aligned} \left(\tau_i^{SP2} ak\frac{1}{c_1} - \beta^2 A_3 - \beta\phi(\frac{1}{c_2} - \beta R_2) q^* \Lambda(\underset{+}{c_2}) - \beta\frac{A_2}{c_2}\right) + ... \\ \frac{1}{c_1 R_1}A_2 + \beta^2 A_3 + \beta\left(\frac{1}{\beta c_1 R_1} - \beta R_2\right)\phi q^*\Lambda(\underset{+}{c_2}) = 0 \\ \Leftrightarrow \beta\left(\frac{1}{c_2} - \frac{1}{\beta c_1 R_1}\right)\left(\phi q^* \Lambda(\underset{+}{c_2}) + A_2\right) = \tau_i^{SP2} ak\frac{1}{c_1} \\ \Leftrightarrow -\frac{\beta\frac{\tau_b}{\beta c_1 R_1}\left(\phi q^*\Lambda(\underset{+}{c_2}) + A_2\right)}{ak\frac{1}{c_1}} = \tau_i^{SP2}\ . \end{aligned} \tag{App.35}$$

The optimal debt tax τ_b^{SP2} has a positive sign, which means that under laissez-faire there is overborrowing and the social planner incentivizes the household to issue less debt. This is due to the fact that less debt to be repaid in period 2 leads to a higher level of consumption, which in turn increases the price of capital and loosens the collateral constraint. Furthermore, it can be seen that the tax is a function of the externality $\frac{\partial\Lambda}{\partial c_2}$ and is zero if there is no externality, i.e. $\frac{\partial\Lambda}{\partial c_2} = 0$.

The optimal investment tax has a negative sign which means that a subsidy is paid. Thus, the social planner incentivizes the household to invest more. However, this under-investment stems from the presence of the debt tax. If the debt tax τ_b^{SP2} was zero, the optimal investment tax would also be zero. The role of the investment tax is to undo the distortion in the investment decision which is caused by the debt tax. The debt tax leads to a wedge in the Euler equation, which makes the laissez-faire capital choice by the household no longer optimal.

A.4 Optimal policy (1 instrument)

Let us now assume that the planner is equipped with a Pigouvian debt tax only. A positive debt tax τ_b^{SP1} reduces the amount the household receives for a given level of newly issued debt in period 1. Thus, the household's budget constraint in period 1 changes to:

$$y - \frac{(1-\tau_b^{SP1})b_2}{R_1} - c_1 - \frac{a}{2}k_2^2 \, . \tag{App.36}$$

Moreover, the household's decision on borrowing (App.13) changes to:

$$c_2(1-\tau_b^{SP1}) = c_1 \beta R_1 \, . \tag{App.37}$$

A social planner that is equipped with this instrument maximizes household's consumption subject to the resource constraints as well as to the collateral constraint. He/she takes into account that the price of collateral in the collateral constraint is directly influenced by the level of consumption in the second period and that capital is a function of the level of consumption in period 1 and 2. Thus, the Lagrangian of the planner's maximization problem looks as follows:

$$\begin{aligned}
L = & \log c_1 + \beta \log c_2 + \beta^2 \left[y_3 + b_3 + A_3 k(\underset{+}{c_1}, \underset{-}{c_2}) \right] \\
& + \lambda_1^{SP1} \left[y_1 - c_1 - a \frac{k(\underset{+}{c_1}, \underset{-}{c_2})^2}{2} - \frac{b_2}{R_1} \right] \\
& + \beta \lambda_2^{SP1} \left[y_2 + b_2 + A_2 k(\underset{+}{c_1}, \underset{-}{c_2}) - c_2 - \frac{b_3}{R_2} \right] \\
& + \beta \mu_2^{SP1} \left[\frac{b_3}{R_2} + \phi q^* \Lambda(\underset{+}{c_2}) k(\underset{+}{c_1}, \underset{-}{c_2}) \right] \, .
\end{aligned}$$

The problem above leads to the following first-order conditions:

$$b_1: \; -\frac{\lambda_1^{SP1}}{R_1} + \beta \lambda_2^{SP1} = 0 \, , \tag{App.38}$$

$$b_2: \; \frac{\mu_2^{SP1}}{R_2} + \beta - \frac{\lambda_2^{SP1}}{R_2} = 0 \, , \tag{App.39}$$

$$\begin{aligned}
c_1: \; & \frac{1}{c_1} + \beta^2 A_3 \underset{+}{\frac{\partial k}{\partial c_1}} - \lambda_1^{SP1} \left(a \underset{+}{\frac{\partial k}{\partial c_1}} k(\underset{+}{c_1}, \underset{-}{c_2}) + 1 \right) + \beta \lambda_2^{SP1} A_2 \underset{+}{\frac{\partial k}{\partial c_1}} \ldots \\
& + \beta \mu_2^{SP1} \phi q^* \Lambda(\underset{+}{c_2}) \underset{+}{\frac{\partial k}{\partial c_1}} = 0 \, ,
\end{aligned} \tag{App.40}$$

$$
\begin{aligned}
c_2 :\ & \frac{1}{c_2} + \beta A_3 \underset{-}{\frac{\partial k}{\partial c_2}} - \frac{1}{\beta}\lambda_1^{SP1}\left(a \underset{-}{\frac{\partial k}{\partial c_2}} k(\underset{+}{c_1}, \underset{-}{c_2})\right) + \lambda_2^{SP1}\left(A_2 \underset{-}{\frac{\partial k}{\partial c_2}} - 1\right) \ldots \\
& + \mu_2^{SP1}\left(\phi q^* \Lambda(\underset{+}{c_2}) \underset{-}{\frac{\partial k}{\partial c_2}} + \phi q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k(\underset{+}{c_1}, \underset{-}{c_2})\right) = 0\ . \qquad \text{(App.41)}
\end{aligned}
$$

Equation (App.38) can be used to eliminate λ_2^{SP1} in equations (App.40) and (App.41):

$$
\begin{aligned}
& \frac{1}{c_1} + \beta^2 A_3 \underset{+}{\frac{\partial k}{\partial c_1}} - \lambda_1^{SP1}\left(a \underset{+}{\frac{\partial k}{\partial c_1}} k(\underset{+}{c_1}, \underset{-}{c_2}) + 1\right) + \frac{\lambda_1^{SP1}}{R_1} A_2 \underset{+}{\frac{\partial k}{\partial c_1}} \\
& + \beta \mu_2^{SP1} \phi q^* \Lambda(\underset{+}{c_2}) \underset{+}{\frac{\partial k}{\partial c_1}} = 0 \qquad \text{(App.42)} \\
& \frac{1}{c_2} + \beta A_3 \underset{-}{\frac{\partial k}{\partial c_2}} - \frac{1}{\beta}\lambda_1^{SP1}\left(a \underset{-}{\frac{\partial k}{\partial c_2}} k(\underset{+}{c_1}, \underset{-}{c_2})\right) + \frac{\lambda_1^{SP1}}{\beta R_1}\left(A_2 \underset{-}{\frac{\partial k}{\partial c_2}} - 1\right) \ldots \\
& + \mu_2^{SP1}\left(\phi q^* \Lambda(\underset{+}{c_2}) \underset{-}{\frac{\partial k}{\partial c_2}} + \phi q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k(\underset{+}{c_1}, \underset{-}{c_2})\right) = 0\ . \qquad \text{(App.43)}
\end{aligned}
$$

The two equations above and equation (App.37) make it possible to derive a first expression of the optimal tax:

$$
\begin{aligned}
\frac{\tau_b^{SP1}}{c_1} = & \left(R_1 \underset{-}{\frac{\partial k}{\partial c_2}} - \underset{+}{\frac{\partial k}{\partial c_1}}\right)\left(\beta^2 A_3 - \lambda_1^{SP1} a k(\underset{+}{c_1}, \underset{-}{c_2}) + \frac{\lambda_1^{SP1}}{R_1} A_2 + \beta \mu_2^{SP1} \phi q^* \Lambda(\underset{+}{c_2})\right) \ldots \\
& + \beta R_1 \mu_2^{SP1} \phi q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k(\underset{+}{c_1}, \underset{-}{c_2})\ . \qquad \text{(App.44)}
\end{aligned}
$$

The forces that determine the optimal tax can be interpreted in an intuitive way. The second line is - despite the size of the collateral constraint multiplier - exactly the same expression as the optimal debt tax in the 2-instruments case. Thus, this part is a point in favor of a positive debt tax, since a higher level of consumption in period 2 increases the collateral price, which is not taken into account by the agents.
The first line incorporates the effects that arise because of the distorting effect of the debt tax on the level of capital. The expression inside the second bracket reflects the marginal social gain of having an additional unit of capital. Since a higher tax reduces capital, i.e. $R_1 \frac{\partial k}{\partial c_2} - \frac{\partial k}{\partial c_1} < 0$, a positive marginal gain of increasing capital means that a positive tax foregoes gains from a higher stock capital, pushing for a smaller debt tax. The social gain of increasing capital is not positive per construction, however, the 2-instruments case has shown that this value is positive for the optimal debt tax τ_b^{SP2} (otherwise an investment

subsidy would not be optimal).
To further evaluate the potential sign of the optimal tax, I use the fact that $\lambda_1^{SP1} = \beta R_1(\beta R_2 + \mu_2^{SP1})$ and substitute the $\frac{c_1}{c_2}$ in equation (App.15) by $\frac{1-\tau_b^{SP1}}{\beta R_1}$ to take into account that the marginal costs of capital, $ak(c_1, c_2)$ are directly influenced by the tax:

$$\tau_b^{SP1} = \frac{\beta c_1 \left(R_1 \underset{-}{\frac{\partial k}{\partial c_2}} - \underset{+}{\frac{\partial k}{\partial c_1}} \right) \left(\beta R_2 \left(1 - \underset{+}{\Lambda(c_2)} \right) q^* + \mu_2^{SP1} q^* \left(\phi - 1\right) \underset{+}{\Lambda(c_2)} \right) + \beta c_1 R_1 \phi \mu_2^{SP1} q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k(\underset{+}{c_1}, \underset{-}{c_2})}{\left(1 - c_1 \left(R_1 \underset{-}{\frac{\partial k}{\partial c_2}} - \underset{+}{\frac{\partial k}{\partial c_1}} \right) \beta \left(\beta R_2 + \mu_2^{SP1} \right) \right) \left(A_2 + \underset{+}{\Lambda(c_2)} q^* \right)}. \tag{App.45}$$

As one would have expected, the first thing to notice is that if there is no externality, i.e. $\phi = 1$, $\Lambda(c_2) = 1$ and $\frac{\partial \Lambda}{\partial c_2} = 0$, the optimal tax is zero. If there is no externality, the social planner cannot improve the allocation since the household's consumption choice is optimal given the presence of the collateral constraint, which cannot be abolished by the planner. Second, given that the denominator is strictly positive, there are effects working in opposite directions in the numerator: two parts that are positive and one part which is negative. The third part of the numerator, as already discussed, is positive. The expression $\mu_2^{SP1} q^* \left(\phi - 1\right) \Lambda(c_2)$ is negative since $\phi < 1$ but is multiplied with a negative term, $R_1 \frac{\partial k}{\partial c_2} - \frac{\partial k}{\partial c_1}$, and the sign of the product is therefore positive. The expression $\beta R_2(1 - \Lambda(c_2))q^*$ is positive and also multiplied with the negative term above so that the product is negative. Thus, there are opposing effects and in contrast to the 2-instruments case there are also opposing forces with a negative sign that reduce the size of the optimal debt tax.
It is important to underline that in this simplified 3-period model there is no further capital accumulation in period 2, since capital is assumed to be fixed in this period and there is no depreciation. Furthermore, the stock of capital is only productive for 3 periods. Thus, some important effects influencing the allocation as well as the sign and size of the debt tax are missing. Still, this simple model makes it easier to understand some of the effects which are incorporated in a more complex model and it already shows that even without the effects outlined above there are forces leading to a small(er) debt tax.
A more realistic model containing all these complex capital effects is needed to fully evaluate the interaction of a debt tax and capital accumulation and is analyzed in the main part of this paper.

B Proofs

The following proof is analogous to a proof in the appendix of Bianchi and Mendoza 2018. The reduced social planner problem is equivalent to the social planner problem incorporating all first-order conditions of the decentralized equilibrium (except for the first-order condition on bonds):

$$\begin{aligned}
\mathcal{V}(b,k,s) &= \max_{c,k',b',q,v} \frac{(c-\chi\frac{h^\omega}{\omega})^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_{s'|s}\mathcal{V}(b',k',s') \ , \\
&\text{s.t.} \\
Ak^{\alpha_k}v^{\alpha_v}h^{\alpha_h} &= \frac{b'}{R} - b + c + k' - (1-\delta)k + \frac{a}{2}(k'-(1-\delta)k-\bar{i})^2 + p_v v \ , \\
&\qquad\qquad\qquad\qquad\qquad\qquad \text{(Resource constraint)} \\
\frac{b'}{R} - \theta p_v v &\geq -\kappa q k \ , \qquad\qquad \text{(Collateral constraint)} \\
q\left(c-\chi\frac{h^\omega}{\omega}\right)^{-\sigma} &= \beta\mathbb{E}_{s'|s}\Big[\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^\omega}{\omega}\right)^{-\sigma} \dots \\
&\Big((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\dots \\
&\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Big)\Big] + \beta\mathbb{E}_{s'|s}\left[\boldsymbol{\mu}_{fp}(b',k',s')\mathcal{Q}_{fp}(b',k',s')\kappa'\right] \ , \\
&\qquad\qquad\qquad\qquad \text{(Household capital decision)} \\
q &= 1 + a(k'-(1-\delta)k-\bar{i}) \ , \\
&\qquad\qquad\qquad\qquad \text{(Firm investment decision)} \\
\left(c-\chi\frac{h^\omega}{\omega}\right)^{-\sigma} p_v + \mu^{LF}\theta p_v &= \left(c-\chi\frac{h^\omega}{\omega}\right)^{-\sigma}\alpha_v A k^{\alpha_k} v^{\alpha_v-1} h^{\alpha_h} \ , \qquad \text{(App.46)} \\
\chi h^{\omega-1} &= \alpha_h A k^{\alpha_k} h^{\alpha_h-1} v^{\alpha_v} \ , \qquad \text{(App.47)} \\
\mu^{LF} &\geq 0 \ , \qquad \text{(App.48)} \\
\mu^{LF}\left(\frac{b'}{R} - \theta p_v v + \kappa q k\right) &= 0 \ . \qquad \text{(App.49)}
\end{aligned}$$

μ^{LF} is defined via equation (App.46). Combining equations (26) and (App.46) leads to the following relation between μ^{LF} and μ^{SP}:

$$\mu^{LF} = \left(c-\chi\frac{h^\omega}{\omega}\right)^{-\sigma}\frac{\mu^{SP}}{\lambda^{SP}} \ .$$

Equation (App.48) is not binding since the equation above shows that it is positively related to μ^{SP}, which is either positive or zero. If the collateral constraint is not binding, μ^{SP} and - because of the equation above - also μ^{LF} are equal to zero. Thus, equation (App.49) is not binding. Equation (App.47) is not binding because the combination of equations (20) and (23) of the reduced planner problem yield the same condition. Equation (App.46) is not binding because it defines μ^{LF}, which does not influence any other equations.

C Investment tax as second instrument

The constrained social planner's optimization problem can be summarized as follows:

$$\mathcal{V}(b,k,s) = \max_{c,k',b',q,v} \frac{(c-\chi\frac{h^{\omega}}{\omega})^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_{s'|s}\mathcal{V}(b',k',s') \;, \tag{App.50}$$

s.t.

$$(\lambda^{SP2}) \quad Ak^{\alpha_k}v^{\alpha_v}h^{\alpha_h} = \frac{b'}{R} - b + c + k' - (1-\delta)k + \frac{a}{2}(k'-(1-\delta)k-\bar{i})^2 + p_v v \;, \quad \text{(Resource constraint)}$$

$$(\mu^{SP2}) \quad \frac{b'}{R} - \theta p_v v \geq -\kappa q k \;, \quad \text{(Collateral constraint)}$$

$$(\xi^{SP2}) \quad q\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} = \beta\mathbb{E}_{s'|s}\Big[\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma} \ldots$$
$$\Big((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\ldots$$
$$\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Big)\Big] + \beta\mathbb{E}_{s'|s}\left[\boldsymbol{\mu}_{fp}(b',k',s')\mathcal{Q}_{fp}(b',k',s')\kappa'\right] . \quad \text{(Household capital decision)}$$

The first-order conditions of the optimal policy problem are as follows:

$$\lambda^{SP2} = \left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} + \xi^{SP2}\sigma\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma-1} q \;, \tag{App.51}$$

$$\lambda^{SP2} = \beta R\mathbb{E}_{s'|s}\left[\lambda^{SP2'}\right] + \mu^{SP2} + \beta R\mathbb{E}_{s'|s}[\xi^{SP2}\Omega'] \;, \tag{App.52}$$

$$\lambda^{SP2}(1+a(k'-(1-\delta)k-\bar{i})) = \beta\mathbb{E}_{s'|s}\Big[\lambda^{SP2'}\Big(\alpha_k A' k'^{\alpha_k-1}v'^{\alpha_v}h'^{\alpha_h}\ldots$$
$$+(1-\delta)(1+a(k''-(1-\delta)k'-\bar{i}))\Big)\ldots$$
$$+\mu^{SP2'}q'\kappa'\Big] + \beta\mathbb{E}_{s'|s}[\xi^{SP2}\Gamma'] \;, \tag{App.53}$$

$$\chi h^{\omega-1}\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} = \lambda^{SP2}\alpha_h A k^{\alpha_k}h^{\alpha_h-1}v^{\alpha_v}\ldots$$
$$-\xi^{SP2}\chi h^{\omega-1}\sigma\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma-1} q \;, \tag{App.54}$$

$$\mu^{SP2}\kappa k = \xi^{SP2}\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} \;, \tag{App.55}$$

$$Ak^{\alpha_k}v^{\alpha_v}h^{\alpha_h} - \frac{b'}{R} + b = c + k' - (1-\delta)k + \frac{a}{2}(k'-(1-\delta)k-\bar{i})^2 - p_v v \;, \tag{App.56}$$

$$\lambda^{SP2}p_v = \lambda^{SP2}\alpha_v A k^{\alpha_k}v^{\alpha_v-1}h^{\alpha_h} - \mu^{SP}\theta p_v \;, \tag{App.57}$$

$$\frac{b'}{R} - \theta p_v v \geq -\kappa q k \,, \tag{App.58}$$

$$q\left(c - \chi\frac{h^\omega}{\omega}\right)^{-\sigma} = \beta\mathbb{E}_{s'|s}\Bigg[\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^\omega}{\omega}\right)^{-\sigma}\ldots$$
$$\Big((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\ldots$$
$$\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Big) + \boldsymbol{\mu}_{fp}(b',k',s')\mathcal{Q}_{fp}(b',k',s')\kappa'\Bigg] \,, \tag{App.59}$$

$$\mu^{SP2} \geq 0 \,, \tag{App.60}$$

$$0 = \mu^{SP2}\left(\frac{b'}{R} - \theta p_v v + \kappa q k\right) \,. \tag{App.61}$$

Ω captures the effects of the current planner's bond decision b' on the future planner's decisions, which is taken into account by the current planner:

$$\Omega' = -\sigma\left(\mathcal{C}_{fp,b}(b',k',s') - \chi\boldsymbol{h}_{fp,b}(b',k',s')\boldsymbol{h}_{fp}(b',k',s')^{\omega-1}\right)\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^\omega}{\omega}\right)^{-\sigma-1}\ldots$$
$$\Big((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Big)\ldots$$
$$+\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^\omega}{\omega}\right)^{-\sigma}\Bigg((1-\delta)\mathcal{Q}_{fp,b}(b',k',s')\ldots$$
$$+\alpha_k A' k'^{\alpha_k-1}\Big(\alpha_v\boldsymbol{v}_{fp,b}(b',k',s')\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v-1}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\ldots$$
$$+\alpha_h\boldsymbol{h}_{fp,b}(b',k',s')\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\Big)\Bigg)\ldots$$
$$+\mathcal{Q}_{fp,b}(b',k',s')\boldsymbol{\mu}(b',k',s')\kappa' + \mathcal{Q}_{fp}(b',k',s')\boldsymbol{\mu}_b(b',k',s')\kappa' \,.$$

Definition 3 *The Markov perfect constrained efficient equilibrium is defined by the policy functions* $\mathcal{B}(b,k,s)$, $\mathcal{K}(b,k,s)$, $\mathcal{Q}(b,k,s)$, $\mathcal{C}(b,k,s)$, $\boldsymbol{v}(b,k,s)$, $\boldsymbol{\mu}(b,k,s)$[11], $\boldsymbol{h}(b,k,s)$ *and the value function* $\mathcal{V}(b,k,s)$ *that, first, solve the social planner optimization problem (App.50) and, second, are equal to the future planner's policy functions:* $\mathcal{B}(b,k,s) = \mathcal{B}_{fp}(b,k,s)$, $\mathcal{K}(b,k,s) = \mathcal{K}_{fp}(b,k,s)$, $\mathcal{Q}(b,k,s) = \mathcal{Q}_{fp}(b,k,s)$, $\mathcal{C}(b,k,s) = \mathcal{C}_{fp}(b,k,s)$, $\boldsymbol{v}(b,k,s) = \boldsymbol{v}_{fp}(b,k,s)$, $\boldsymbol{\mu}(b,k,s) = \boldsymbol{\mu}_{fp}(b,k,s)$ *and* $\boldsymbol{v}(b,k,s) = \boldsymbol{v}_{fp}(b,k,s)$.

[11]The household's multiplier on the collateral constraint is defined by equation (12) and is not binding for the social planner (see Appendix).

D Data

Table 5: Data Sources

Data	Measure	Unit	Source	URL / Reference	Usage
Annual National Accounts, 9B. Balance sheets for non-financial assets: Fixed assets & GDP (expenditure approach)	Current prices	Local currency, Millions	OECD	https://stats.oecd.org/	Computation of capital-to-GDP ratio
Annual National Accounts, 1. Gross domestic product (GDP): Gross fixed capital formation & GDP (expenditure approach)	VOB: Constant Prices: OECD base year	Local currency, Millions	OECD	https://stats.oecd.org/	Logarithmized and linearly detrended time series of investment and GDP per worker used to compute standard deviation (sd) of investment relative to sd of to GDP and autocorrelation of GDP
Annual National Accounts, 1. Gross domestic product (GDP): GDP (expenditure approach)	Constant Prices, Constant PPPs, OECD base year	US Dollar Millions, 2015	OECD	https://stats.oecd.org/	Computation of country weights (2012 share of summed real GDP)
Annual Labor Force: Statistics: ALFS Summary tables	–	Persons, Thousands	OECD	https://stats.oecd.org/	Computation of investment and GDP per worker
Penn World Table, version 10.01, Price level of capital formation & of consumption	Price level relative to price level of US GDP in 2017	–	University of Groningen	https://dataverse.nl/api/access/datafile/354095 Feenstra et al. 2015	Logarithmized and HP-filter detrended time series of relative price of capital used to compute sd

E Algorithms

Laissez-faire (for an even more detailed description of the updating steps see Mendoza and Villalvazo 2020[12])

1. Uniformly spaced discrete grids for the state variables bond b (60 nodes) and capital k (30 nodes) as well as a grid for the shock state space are created. The interest rate shock and the productivity shock are discretized by Tauchen's method with 3 realizations each. The financial shock has two realizations so that there are 18 different possible combinations of shocks. Thus, the state space has 60 × 30 × 18 elements. The interpolation scheme is a bilinear interpolation.
2. Guess the ratio of the collateral constraint multiplier μ to λ which is denoted by $\hat{\boldsymbol{\mu}}_0^{Guess}$, the bond policy function $\mathcal{B}_0^{Guess}$ and the capital price policy function $\mathcal{Q}_0^{Guess}$. I used $\mathcal{Q}_0^{Guess} = \text{ones}(b,k,s)$, $\mathcal{B}_0^{Guess} = b(b,k,s)$ and $\hat{\boldsymbol{\mu}}_0^{Guess} = \text{zeros}(b,k,s)$ as initial guesses.
3. Use guesses $\hat{\boldsymbol{\mu}}_j^{Guess}$, $\mathcal{B}_j^{Guess}$, $\mathcal{Q}_j^{Guess}$ to compute guesses $\mathcal{K}_j^{Guess}$, $\mathcal{C}_j^{Guess}$, $\boldsymbol{h}_j^{Guess}$ and $\boldsymbol{v}_j^{Guess}$.
4. Assume that the collateral constraint is not binding and use the equilibrium conditions to update all policy functions except $\mathcal{Q}$.
5. Check whether the collateral constraint is binding.
6. Solve for $\hat{\mu}_j$ in the binding states. Update all other policy functions except for $\mathcal{Q}$.
7. Use all updated policy functions and equation (11) to compute $\mathcal{Q}_j$.
8. Check convergence. If $sup_{B,K,S}\|x_j(b,k,s) - x_{j-1}(b,k,s)\| \geq \epsilon$ for $x = \mathcal{Q}$, $\mathcal{B}$, $\hat{\boldsymbol{\mu}}$, compute $\mathcal{Q}_{j+1}^{Guess}$, $\mathcal{B}_{j+1}^{Guess}$ and $\hat{\boldsymbol{\mu}}_{j+1}^{Guess}$ as weighted sums of $\mathcal{Q}_j^{Guess}$, $\mathcal{B}_j^{Guess}$, $\hat{\boldsymbol{\mu}}_j^{Guess}$ and $\mathcal{Q}_j$, $\mathcal{B}_j$ and $\hat{\boldsymbol{\mu}}_j$. Then go to step 3. Else stop.

Optimal policy: 1 instrument

Outer loop

1. Equivalent to the decentralized equilibrium, uniformly spaced discrete grids for the state variables bond (60 nodes) and capital (30 nodes) as well as a grid for the shock state space are created. The interest rate shock and the productivity shock are discretized by Tauchen's method with 3 realizations each. The financial shock has two realizations so that there are 18 different possible combinations of shocks. Thus, the state space has 60 × 30 × 18 elements. The interpolation scheme is bilinear interpolation.
2. Guess the policy functions of the future planner $\mathcal{Q}_{fp}^{Guess}$, $\boldsymbol{\mu}_{fp}^{Guess}$, $\boldsymbol{h}_{fp}^{Guess}$, $\boldsymbol{v}_{fp}^{Guess}$ and $\mathcal{C}_{fp}^{Guess}$. Compute derivatives of these policy functions with respect to b and k.

[12] Please note that I use a different notation.

Inner Loop:

(a) Guess the ratio of the collateral constraint multiplier μ to λ, which is denoted by $\hat{\boldsymbol{\mu}}_0^{Guess}$, the bond policy function $\mathcal{B}_0^{Guess}$, the capital price policy function $\mathcal{Q}_0^{Guess}$ and the investment implementability constraint multiplier $\boldsymbol{\gamma}_0^{Guess}$.

(b) Use guesses $\hat{\boldsymbol{\mu}}_j^{Guess}$, $\mathcal{B}_j^{Guess}$, $\mathcal{Q}_j^{Guess}$ to compute guesses $\mathcal{K}_j^{Guess}$, $\mathcal{C}_j^{Guess}$, $\boldsymbol{h}_j^{Guess}$ and $\boldsymbol{v}_j^{Guess}$.

(c) Eliminate ξ in all equations by using equation (25). Assume that the collateral constraint is not binding and update all policy functions except $\mathcal{Q}$.

(d) Check whether the collateral constraint is binding.

(e) Solve for $\hat{\boldsymbol{\mu}}_j$ in the binding states. Update all other policy functions except for $\mathcal{Q}$.

(f) Use updated policy functions and equation (28) to compute $\mathcal{Q}_j$.

(g) Check convergence. If $sup_{B,K,S}\|x_j(b,k,s) - x_{j-1}(b,k,s)\| \geq \epsilon$ for $x = \mathcal{Q}$, $\mathcal{B}$, $\hat{\boldsymbol{\mu}}$, $\boldsymbol{\gamma}$, compute $\mathcal{Q}_{j+1}^{Guess}$, $\mathcal{B}_{j+1}^{Guess}$, $\hat{\boldsymbol{\mu}}_{j+1}^{Guess}$ and $\boldsymbol{\gamma}_{j+1}^{Guess}$ as weighted sums of $\mathcal{Q}_j^{Guess}$, $\mathcal{B}_j^{Guess}$, $\hat{\boldsymbol{\mu}}_j^{Guess}$, $\boldsymbol{\gamma}_j^{Guess}$ and $\mathcal{Q}_j$, $\mathcal{B}_j$, $\hat{\boldsymbol{\mu}}_j$, $\boldsymbol{\gamma}_j$. Then go to step b. Else stop.

3. Check convergence. If $sup_{B,K,S}\|x_j(b,k,s) - x_{j-1}(b,k,s)\| \geq \epsilon^*$ for $x = \mathcal{Q}_{fp}$, $\boldsymbol{\mu}_{fp}$, $\boldsymbol{h}_{fp}$, $\boldsymbol{v}_{fp}$, $\mathcal{C}_{fp}$, compute new guesses as weighted sums of old guesses and converged policy functions. Compute derivatives of these policy functions with respect to b and k. Then go to the inner loop. Else stop.

Optimal policy: 2 instruments

Outer loop

1. Equivalent to the decentralized equilibrium, uniformly spaced discrete grids for the state variables bond (60 nodes) and capital (30 nodes) as well as a grid for the shock state space are created. The interest rate shock and the productivity shock are discretized by Tauchen's method with 3 realizations each. The financial shock has two realizations so that there are 18 different possible combinations of shocks. Thus, the state space has 60 × 30 × 18 elements. The interpolation scheme is bilinear interpolation.
2. Guess the policy functions of the future planner $\mathcal{Q}_{fp}^{Guess}$, $\boldsymbol{\mu}_{fp}^{Guess}$, $\boldsymbol{h}_{fp}^{Guess}$, $\boldsymbol{v}_{fp}^{Guess}$ and $\mathcal{C}_{fp}^{Guess}$. Compute derivatives of these policy functions with respect to b and k.

Inner Loop:

(a) Guess the ratio of the collateral constraint multiplier μ to λ, which is denoted by $\hat{\boldsymbol{\mu}}_0^{Guess}$, the bond policy function $\mathcal{B}_0^{Guess}$ and the capital policy function $\mathcal{K}_0^{Guess}$.

(b) Use guesses $\hat{\boldsymbol{\mu}}_j^{Guess}$, $\mathcal{B}_j^{Guess}$, $\mathcal{K}_j^{Guess}$ to compute guesses $\mathcal{C}_j^{Guess}$, $\boldsymbol{h}_j^{Guess}$, $\boldsymbol{v}_j^{Guess}$ and $\hat{\boldsymbol{q}}_j^{Guess}$. $\hat{q}$ is an auxiliary variable defined as follows: $\hat{q} := q\left(c - \chi\frac{h^\omega}{\omega}\right)^{-\sigma}$.

(c) Eliminate ξ in all equations by using equation (App.55). Assume that the collateral constraint is not binding and update all policy functions.

(d) Check whether collateral constraint is binding.

(e) Solve for $\hat{\boldsymbol{\mu}}_j$ in the binding states. Update all other policy functions.

(f) Check convergence. If $sup_{B,K,S}\|x_j(b,k,s) - x_{j-1}(b,k,s)\| \geq \epsilon$ for $x = \mathcal{K}$, $\mathcal{B}$, $\hat{\boldsymbol{\mu}}$, compute $\mathcal{K}_{j+1}^{Guess}$, $\mathcal{B}_{j+1}^{Guess}$ and $\hat{\boldsymbol{\mu}}_{j+1}^{Guess}$ as weighted sums of $\mathcal{K}_j^{Guess}$, $\mathcal{B}_j^{Guess}$, $\hat{\boldsymbol{\mu}}_j^{Guess}$ and $\mathcal{K}_j$, $\mathcal{B}_j$ and $\hat{\boldsymbol{\mu}}_j$. Then go to step b. Else stop.

3. Check convergence. If $sup_{B,K,S}\|x_j(b,k,s) - x_{j-1}(b,k,s)\| \geq \epsilon^*$ for $x = \mathcal{Q}_{fp}$, $\boldsymbol{\mu}_{fp}$, $\boldsymbol{h}_{fp}$, $\boldsymbol{v}_{fp}$, $\mathcal{C}_{fp}$, compute new guesses as weighted sums of old guesses and converged policy functions. Compute derivatives of these new guesses with respect to b and k. Then go to the inner loop. Else stop.