Dynamically Consistent Intergenerational Welfare

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Motivation

- How to evaluate intergenerational policies such as in climate change?
  - Disagreement about evaluation but not about how to measure the utilities.
- How to evaluate the value of an intergenerational utility stream $(u_0, u_1, u_2, \dots)$?
- Samuelson (1937), Koopman (1960). Discounted utility: $\delta \in (0,1)$

$$(1-\delta)\sum_{t=0}^{\infty} \delta^t u_t.$$

- Significant disagreement on the discount factor (Weitzman, 2001)
- Maxmin criterion: A set of discount factors $D \subset (0,1)$

$$\min_{\delta \in D}(1-\delta)\sum_{t=0}^{\infty} \delta^t u_t.$$

Motivation

- Maxmin criterion: A set of discount factors $D \subseteq (0,1)$

$$\min_{\delta \in D} (1-\delta) \sum_{t=0}^{\infty} \delta^t u_t.$$

- Intergenerational choice lacks commitment.
- Dynamic consistency crucial for a credible evaluation.
- Maxmin + dynamic consistency = dictatorship
- How to aggregate intergenerational welfare under dynamic consistency?

Summary

- Discounted utility recursively: $\delta \in (0,1)$

$$V(u_0, u_1, \dots) = u_0 + \delta\big(V(u_1, u_2, \dots) - u_0\big) = (1-\delta)\textstyle\sum_{t=0}^{\infty}\delta^t u_t$$

- General solution to aggregating intergenerational welfare under dynamic consistency:
- Under stationarity, for some $\delta^+, \delta^- \in (0,1)$, the utility stream evaluated recursively by

$$V(u_0, u_1, \dots) = u_0 + \delta^+ \max\left\{V(u_1, u_2, \dots) - u_0, 0\right\} + \delta^- \min\left\{V(u_1, u_2, \dots) - u_0, 0\right\}.$$

- Envy-guilt asymmetry for future generations utility as in Fern-Schmidt's other regarding preferences or loss aversion as in prospect theory. Tractable and simple model suitable for applications.
- If $\delta^- \geq \delta^+$, then

$$V(u_0, u_1, \dots) = \min_{\delta \in [\delta^+, \delta^-]} u_0 + \delta\big(V(u_1, u_2, \dots) - u_0\big)$$

- Without stationarity, δ_t^+, δ_t^- are time dependent e.g. (quasi-)hyperbolic discounting

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Outline

Setting

- The objects of choice are bounded sequences of real numbers $(u_t)_{t=0}^{\infty} \in \ell^{\infty}$.
  - The indices $t = 0, 1, 2, \ldots$ are generations and u_t is the utility of generation t from the utility stream $(u_t)_{t=0}^{\infty}$.
- The primitive is a binary relation $\succsim$ on ℓ^{∞}.
- $\succ$ and $\sim$ denote the asymmetric and symmetric parts of $\succsim$ respectively.

Axioms 1 & 2

The first axioms are standard axioms that $\succsim$ is complete, transitive, and continuous.

Axiom 1 (Complete & Transitive)

1. For each $(u_t)_{t=0}^{\infty}, (v_t)_{t=0}^{\infty} \in \ell^{\infty}$,

$$(u_0, u_1, \dots) \succsim (v_0, v_1, \dots) \quad \text{or} \quad (v_0, v_1, \dots) \succsim (u_0, u_1, \dots).$$

2. For each $(u_t)_{t=0}^{\infty}, (v_t)_{t=0}^{\infty}, (x_t)_{t=0}^{\infty} \in \ell^{\infty}$,

$$\text{if} \quad (u_0, u_1, \dots) \succsim (v_0, v_1, \dots) \quad \text{and} \quad (v_0, v_1, \dots) \succsim (x_0, x_1, \dots),$$

   then $(u_0, u_1, \dots) \succsim (x_0, x_1, \dots)$.

Axiom 2 (Continuity)

For each $(u_t)_{t=0}^{\infty}, (v_t)_{t=0}^{\infty} \in \ell^{\infty}$ such that $(u_0, u_1, \dots) \succ (v_0, v_1, \dots)$, there exists $\beta > 0$ such that

$$(u_0 - \beta, u_1 - \beta, \dots) \succ (v_0 + \beta, v_1 + \beta, \dots).$$

- Adding little to the utility sequence changes preferences only slightly.

Axiom 3

- The next axiom captures that choice alternatives are utility streams.
- Introduced in d'Aspremont and Gevers (1977) and Sen (1979).
- Positive affine transformation of the utility streams does not affect the comparisons.

Axiom 3 (Co-Cardinality)

For each $(u_t)_{t=0}^{\infty}, (v_t)_{t=0}^{\infty} \in \ell^{\infty}$, $\alpha > 0$ and $\beta \in \mathbb{R}$,

$$(u_0, u_1, \ldots) \succsim (v_0, v_1, \ldots) \Longrightarrow (\alpha u_0 + \beta, \alpha u_1 + \beta, \ldots) \succsim (\alpha v_0 + \beta, \alpha v_1 + \beta, \ldots).$$

Axiom 4

- The next axiom assumes that the preferences respect unanimous improvements for every generation.

Axiom 4 (Generation-wise Unanimity)

For all $(u_t)_{t=0}^{\infty}, (v_t)_{t=0}^{\infty} \in \ell^{\infty}$, if for all $t \in \mathbb{N}$, $u_t \geq v_t$ and for some $t' \in \mathbb{N}$, $u_{t'} > v_{t'}$, then $(u_0, u_1, \dots) \succ (v_0, v_1, \dots)$.

Previous Literature

Axiom 5

- The last axiom is Koopman's (1960) Stationarity.
- The passage of time does not affect the preferences.
- Every generation has the same time preferences.
- Especially gives dynamic consistency.

Axiom 5 (Stationarity)

Let $(u_t)_{t=0}^{\infty}, (v_t)_{t=0}^{\infty} \in \ell^{\infty}$ and $\theta \in \mathbb{R}$. Then

$$(u_0, u_1, \dots) \succsim (v_0, v_1, \dots) \iff (\theta, u_0, u_1, \dots) \succsim (\theta, v_0, v_1, \dots).$$

Characterization

- General characterization for intergenerational welfare aggregation under stationarity by the envy-guilt asymmetry.

Theorem 1 (Stationary Intergenerational Welfare)

$\succsim$ satisfies Axioms 1 to 5 iff. there exist unique $\delta^+, \delta^- \in (0,1)$ such that there exists a recursive function $V : \ell^\infty \to \mathbb{R}$ defined by for each $(u_t)_{t=0}^\infty \in \ell^\infty$,

$$V(u_0, u_1, ...) = u_0 + \delta^+ \max\{V(u_1, u_2, ...) - u_0, 0\} + \delta^- \min\{V(u_1, u_2, ...) - u_0, 0\}$$

with $\limsup_{t\to\infty} |V(u_t, u_{t+1}, \ldots)| < \infty$ and V represents $\succsim$.

- $\limsup_{t\to\infty} |V(u_t, u_{t+1}, \ldots)| < \infty$ gives the convergence of the recursive formula.
- If $\delta^- \geq \delta^+$, then

$$V(u_0, u_1, \ldots) = \min_{\delta\in[\delta^+,\delta^-]} u_0 + \delta\big(V(u_1, u_2, \ldots) - u_0\big)$$

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Axiom 6

- Relax stationarity for general dynamically consistent preferences.
- History independence is dynamic consistency from Epstein (2003) in the current setting.
- Guarantees time-consistent choices.

Axiom 6 (History Independence)

For all $t \in \mathbb{N}, (a_t)_{t=0}^{\infty}, (b_t)_{t=0}^{\infty}, (u_t)_{t=0}^{\infty}, (v_t)_{t=0}^{\infty} \in \ell^{\infty}$,

$$(a_0, \ldots, a_{t-1}, u_t, u_{t+1}, \ldots) \succsim (a_0, \ldots, a_{t-1}, v_t, v_{t+1}, \ldots)$$
$$\iff (b_0, \ldots, b_{t-1}, u_t, u_{t+1}, \ldots) \succsim (b_0, \ldots, b_{t-1}, v_t, v_{t+1}, \ldots).$$

- Each generation has preferences for consumption streams starting at their generation that are history independent

Axiom 7

- Monotone continuity from Villegas (1964) and Arrow (1966).
- The limit of the utility stream is not given a positive weight.

Axiom 7 (Monotone Continuity)

For all $(u_t)_{t=0}^{\infty}, (v_t)_{t=0}^{\infty}, (x_t)_{t=0}^{\infty} \in \ell^{\infty}$ such that $(u_0, u_1, \dots) \succ (v_0, v_1, \dots)$, then there exists $t \in \mathbb{N}$ such that

$$(u_0, \dots, u_{t-1}, x_t, x_{t+1}, \dots) \succ (v_0, v_1, \dots) \text{ and}$$

$$(u_0, u_1, \dots) \succ (v_0, \dots, v_{t-1}, x_t, x_{t+1}, \dots).$$

Characterization

- Replacing stationarity with history independence and monotone continuity gives time-dependent discount factors.

Theorem 2 (Dynamically Consistent Intergenerational Welfare)

$\succsim$ satisfies Axioms 1 to 4, 6 and 7 iff. there exist for each $t \in \mathbb{N}$, unique $\delta_t^+, \delta_t^- \in (0,1)$ such that for each $t \in \mathbb{N}$ and $(u_l)_{l=0}^{\infty} \in \ell^\infty$, we have a recursive function

$$\begin{aligned} & V_t(u_t, u_{t+1}, \ldots) \\ & = u_t + \delta_t^+ \max\left\{V_{t+1}(u_{t+1}, u_{t+2}, \ldots) - u_t, 0\right\} + \delta_t^- \min\left\{V_{t+1}(u_{t+1}, u_{t+2}, \ldots) - u_t, 0\right\} \end{aligned}$$

with $\prod_{t=0}^{\infty} \max\{\delta_t^+, \delta_t^-\} = 0$, $\limsup_{t\to\infty} |V_t(u_t, u_{t+1}, \ldots)| < \infty$, and the recursive solution V_0 represents $\succsim$.

- $\prod_{t=1}^{\infty} \max\{\delta_t^+, \delta_t^-\} = 0$ and $\limsup_{t\to\infty} |V_t(f_t, f_{t+1}, \ldots)| < \infty$ give the convergence of the recursive solution.
- Allows for (quasi-)hyperbolic discounting.

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Open Questions

- How to estimate δ^+ and δ^-?
  - How are they related to individual's discount factors
- Does this model give the same predictions as exponential discounting.
  - Does the differences matter for applications.
- Is the model computationally tractable and how to solve for the optimal policy.
- The role of co-cardinality.

Conclusion

- Dynamic consistency crucial for credible intergenerational choice plans.
- Offered a general characterization for intergenerational welfare aggregation under stationarity or dynamic consistency.
- A simple and tractable model that is suitable for applications.

Appendix

Axiom 3

- The next axiom captures that choice alternatives are utility streams.
- Introduced in d'Aspremont and Gevers (1977) and Sen (1979).
- Positive affine transformation of the utility streams does not affect the comparisons.

Axiom 3 (Co-Cardinality)

For each $(u_t)_{t=0}^{\infty}, (v_t)_{t=0}^{\infty} \in \ell^{\infty}$, $\alpha > 0$ and $\beta \in \mathbb{R}$,

$$(u_0, u_1, \dots) \succsim (v_0, v_1, \dots) \Longrightarrow (\alpha u_0 + \beta, \alpha u_1 + \beta, \dots) \succsim (\alpha v_0 + \beta, \alpha v_1 + \beta, \dots).$$

- Discounted utility assumes for all $(u_t)_{t=0}^{\infty}, (v_t)_{t=0}^{\infty}, (x_t)_{t=0}^{\infty} \in \ell^{\infty}$

$$(u_0, u_1, \dots) \succsim (v_0, v_1, \dots) \Longrightarrow (u_0 + x_0, u_1 + x_1, \dots) \succsim (v_0 + x_0, v_1 + x_1, \dots).$$

- Assumes that utility levels are not comparable across generations

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Axiom 4

- The next axiom assumes that the preferences respect unanimous improvements for every generation.

Axiom 4 (Generation-wise Unanimity)

For all $(u_t)_{t=0}^{\infty}, (v_t)_{t=0}^{\infty} \in \ell^{\infty}$, if for all $t \in \mathbb{N}$, $u_t \geq v_t$ and for some $t' \in \mathbb{N}$, $u_{t'} > v_{t'}$, then $(u_0, u_1, \dots) \succ (v_0, v_1, \dots)$.

- The previous literature assumed unanimity for exponential discounters:
  - There exists a finite set $D \subset (0, 1)$ such that
    - if for all $\delta \in D$, $\sum_{t=0}^{\infty} \delta^t u_t > \sum_{t=0}^{\infty} \delta^t v_t$, then $(u_0, u_1, \dots) \succ (v_0, v_1, \dots)$.
  - Incompatible with dynamic consistency unless there is a dictator (Zuber, 2011; Jackson & Yariv, 2015)

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