

# Uncertainty Shocks in An Intangible Economy

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## Abstract

This paper employs a micro-to-macro approach to study uncertainty shocks in the context of an intangible or knowledge-based economy. Based on quarterly firm level data, I find that intangible capital acts as a cushion to mitigate adverse effects of uncertainty shocks on investment. The study further develops a two-sector dynamic stochastic general equilibrium (DSGE) model, providing insights into the shifts in investment composition in an uncertainty-driven business cycle. Notably, the ascent of intangibles not only expands the size of the economy but also diminishes aggregate volatility in the uncertainty-driven business cycle. In essence, the economy becomes more knowledge-intensive in the wake of heightened uncertainty, indicating a dynamic interplay between intangibles and macroeconomic dynamics.

**JEL classification:** C33, E22, E32, O3

**Keywords:** Intangibles, Investment Compositions, Uncertainty Shocks, DSGE

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# 1 Introduction

Technology advance is transforming numerous countries toward a knowledge-based economy with increasing use of intangibles in production, such as knowledge derived from research and development (R&D), intellectual property, organizations, brands, and business strategy. The increasing importance of intangibles raises an essential question: what are the roles of intangibles in business cycles? While existing literature has made commendable strides in studying level (or first-moment) shocks, there is a lack of understanding the implications of intangibles for uncertainty shocks which are recognized as a pivotal source of macroeconomic fluctuations (Leduc & Liu 2016, Basu & Bundick 2017, Fernández-Villaverde & Guerrón-Quintana 2020).

To investigate macroeconomic implications of intangibles in a uncertainty-driven business cycle, I adopt a micro-to-macro approach by disciplining a macro model using micro empirical evidence. Employing a US firm-level dataset between 2000Q1 and 2023Q2, my first analysis delves into effects of uncertainty on investment and its components, and further explores roles of intangible capital in the relationship between uncertainty and investment. The empirical analysis exploits advantages of micro-level dataset in providing wider statistical coverage of intangible investment than officially published macro data. The empirical results document (1) negative relationships between uncertainty and both physical and intangible investment albeit the latter displaying less sensitivity to uncertainty, and (2) the observation that firms with more intangible capital exhibit a relative resilience in the face of uncertainty, evidenced by less pronounced impacts on their investment.

The analysis progresses to a structural interpretation of empirical findings using a theoretical framework, quantitatively evaluating the macroeconomic implications of the increasing importance of intangibles in business cycles. To this end, I build a two-sector Dynamic Stochastic General Equilibrium (DSGE) model with time-varying volatility in the preference shifter (i.e., the uncertainty shock)<sup>1</sup>. This model underscores the production of both tangible and intangible goods, and highlights two key differences between the two sectors. In line with existing literature (McGrattan & Prescott 2010, Mitra 2019), intangible capital is non-rival in that it can be used to produce two types of goods simultaneously while physical capital is rival and hence can only be used to produce one type of goods at a time. Departing from the literature, the model accounts for sophistication in producing intangible goods, such as technology innovation, which requires inputs from skilled labors. In equilibrium, firms determine optimal allocation of capital in the two sectors and demands for two types of labors.

Consistent with the empirical evidence, the model yields relatively muted responses of intangible investment to the uncertainty shock. This pattern is driven by two important forces, including capital reallocation and precautionary labor supply. Elevated uncertainty triggers precautionary saving effects, lowering demands for tangible goods and reducing marginal revenue product (MRPK) of the two types of capital. Given the

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<sup>1</sup>See Basu & Bundick (2017) among others. Bianchi et al. (2023) label time-varying volatility in the preference shifter as demand-sided or preference uncertainty shock.

long-term nature of intangible production, the effects of the uncertainty shock are partially absorbed by the tangible sector, resulting in a weakened transmission to the intangible sector. Consequently, MRPK of physical capital and wages in the intangible sector are less responsive than their counterparts in the tangible sector. The former leads to a gap in MRPK of physical capital, prompting capital reallocation towards the intangible sector. The latter widens the wage gap between skilled and unskilled labors, further strengthening precautionary labor supply motive (Basu & Bundick 2017) toward skilled labor.

In light of model mechanisms, this paper further assess the macroeconomic implications of the increasing importance of intangibles. The quantitative analysis suggests that intangibles act as a cushion to mitigate contractionary effects of heightened uncertainty—reducing investment volatility for the real side and curbing stock market volatility for the financial side. This macro theoretical finding echoes the micro empirical evidence, presenting a counterpoint to the amplification role traditionally ascribed to intangibles in the transmission of financial shocks (Lopez & Olivella 2018, Anzoategui et al. 2019, Ikeda & Kurozumi 2019)<sup>2</sup>.

Finally, this paper differentiates between “good uncertainty” and “bad uncertainty”. It shows that uncertainty in productivity of intangibles propagates as a form of good uncertainty, leading to expansionary effects on the economy in the medium-to-long run. However, the positive effects are quantitatively modest, incomparable with recessionary effects from the preference uncertainty. This finding provides insight into why scholars empirically observe negative effects of uncertainty based on aggregate measures (see Basu & Bundick (2017) among others).

This paper contributes to two major strands of emerging literature, i.e., macroeconomic consequences of uncertainty (Bloom et al. 2007, Fernández-Villaverde et al. 2015, Segal et al. 2015, Leduc & Liu 2016, Basu & Bundick 2017, Fasani et al. 2023, Alfaro et al. 2024) and intangibles or technology innovation in business cycles (Lopez & Olivella 2018, Ikeda & Kurozumi 2019, Mitra 2019, Döttling & Ratnovski 2023). This paper provides a crossroad between the two areas, being, to the best of my knowledge, the first paper to incorporate intangibles into a framework with uncertainty and examine their implications both empirically and theoretically. Specifically for the uncertainty literature, this paper aligns with Basu & Bundick (2017) by examining the transmission of the uncertainty shock in a sticky-price model and underscoring the importance of households’ precautionary motives. Departing from the common ingredients, I incorporate intangible production, a facet gaining increased prominence in advanced economies. Additionally, the model differentiates between skilled and unskilled labor, allowing for a nuanced exploration of precautionary labor motives specific to each type of labor. Thirdly, the inclusion of intangibles enables the study of intangible-specific uncertainty shocks, which propagate as good uncertainty.

Regarding the intangible literature, there is a debate whether the shift toward knowledge economy amplifies or dampens macroeconomic fluctuations. Previous focus on financial or liquidity shocks by Lopez & Olivella (2018), Anzoategui et al. (2019), and Ikeda & Kurozumi (2019) suggest an amplification effect

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<sup>2</sup>Anzoategui et al. (2019), Ikeda & Kurozumi (2019) focus on technology innovation which is a key component of intangibles.

of intangibles or technology innovation. Mitra (2019) shows that the rising importance of intangible could increase hours volatility. My paper provides a reconciliation that both the amplification and dampening effects are possible, but the latter is more likely to hold if the business cycle is dominated by the uncertainty shock. Furthermore, existing literature suggests that measurement issues are key concerns when studying intangibles (Corrado & Hulten 2010, McGrattan & Prescott 2010, Peters & Taylor 2017, Crouzet & Eberly 2021). In particular, intangible goods may not be (fully) included in national accounts. When measuring intangible investment, input-sided or cost approaches<sup>3</sup> are often used while output-sided measures<sup>4</sup> are difficult to observe or quantify. My theoretical analysis confirms adverse responses of intangible investment to uncertainty based on both output and input approaches, though the latter tends to be more responsive. This finding in turn validates my empirical results based on intangible investment measured by replacement costs, and on the other hand suggests potentially larger cushion effects provided by intangibles than observed.

This paper also connects to broader literature studying the secular change in corporate investment and its implications (Brown et al. 2009, Peters & Taylor 2017, Bianchi et al. 2019, Caggese & Pérez-Orive 2022, Döttling & Ratnovski 2023). In general, this paper complements the literature by investigating uncertainty as another important determinant of investment compositions, and developing a macroeconomic model to interpret the different impacts of uncertainty on tangible and intangible investment. Caggese & Pérez-Orive (2022) also build a general equilibrium model to study implications of rising importance of intangibles, but focusing on how differences in pledgeability affect sensitivity of the two types of investment to interest rates. In addition to differentiate different degree of pledgeability, my model also highlights non-rivalry and long-term features of intangibles. The latter two elements are more important to explain consequences of uncertainty shocks while limited pledgeability plays relatively a larger role in the propagation of financial shocks.

The rest of the paper is organized as follows. Section 2 empirically examines implications of intangibles for the effects of uncertainty. Section 3 presents the DSGE model with intangible production and the uncertainty shock, followed by calibration of parameters in Section 4. Sections 5-6 report main theoretical results. In Section 7, I investigate potential expansionary effects of uncertainty. Finally, Section 8 concludes with comments.

## 2 Empirical Evidence

This section provides empirical evidence to investigate the relationship between uncertainty and investment, while also exploring the role of intangibles in shaping this relationship. The empirical identification is divided into two stages. In the first stage, a structural VAR model is estimated to extract exogenous

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<sup>3</sup>A common measure of intangible investment is the replacement cost approach (Peters & Taylor 2017, Döttling & Ratnovski 2023).

<sup>4</sup>For example, R&D expenditure is an input-sided measure of intangibles while the knowledge derived from R&D activities, an outcome of intangible investment, would be difficult to measure and included in the national accounts.

components of an uncertainty measure<sup>5</sup>. Taking the adjusted uncertainty as a primary independent variable, I merge it with a firm-level panel dataset to study effects of uncertainty in the second stage.

In specific, I consider a six-variable VAR with the following variables: uncertainty measured by either the VIX index or Jurado et al. (2015)’s macroeconomic uncertainty (henceforth JLN index), real GDP, real private investment, CPI inflation, federal fund rate, and excess bond premium. Following existing literature, I order uncertainty as the first variable and identify contributions of each variables to the uncertainty according to a Cholesky decomposition. This identification scheme is also consistent with the theoretical implications which will be shown in Section 5. Then, contributions from non-uncertainty shocks to uncertainty are removed to obtain an adjusted uncertainty index  $u_t$ .

The firm-level variables used in the second-stage analysis are from Compustat based on quarterly financial statement of public firms between 2000Q1 and 2023Q2<sup>6</sup>. Following Peters & Taylor (2017) and Döttling & Ratnovski (2023), intangible investment is defined as the sum of research and development (R&D) expense and 30% of selling, general and administrative (SG&A) expense. This measure includes two important components of intangible investment—technology and organization capital. When constructing intangible capital, estimated off-balance sheet intangibles are added to the on-balance components (Intan). The off-balance sheet component is estimated using R&D and SG&A started from 1975Q1 based on a perpetual inventory method Peters & Taylor (2017).<sup>7</sup>

Following a commonly used sampling procedure (see Peters & Taylor (2017) and Döttling & Ratnovski (2023) among others), I exclude firms in utility (SIC 4900-4999), finance (SIC 6000-6999), and public service (SIC 9000 and above). Observations with missing or negative assets, sales, CAPX, R&D, or SD&A expenditure are also removed. Furthermore, very small firm with physical capital under \$5 million are excluded. In order to correctly match firm-level variables and the uncertainty index, fiscal quarters are mapped to calendar quarters using information on firms’ fiscal-year end. Finally, I manually check and drop duplicated observations<sup>8</sup>.

Figure 1 compares investment based on the aggregated Compustat series (Micro measures) and macroeconomic sources. While the former offers broader coverage of intangibles based on publicly listed firms, the latter is limited to technology activities but encompasses all establishments. In spite of different statistical coverage, the two total investment growth series display similar movement over time. The right panel depicts the increasing shares of intangible investment based on both measures, a trend that persists even during the COVID pandemic period.

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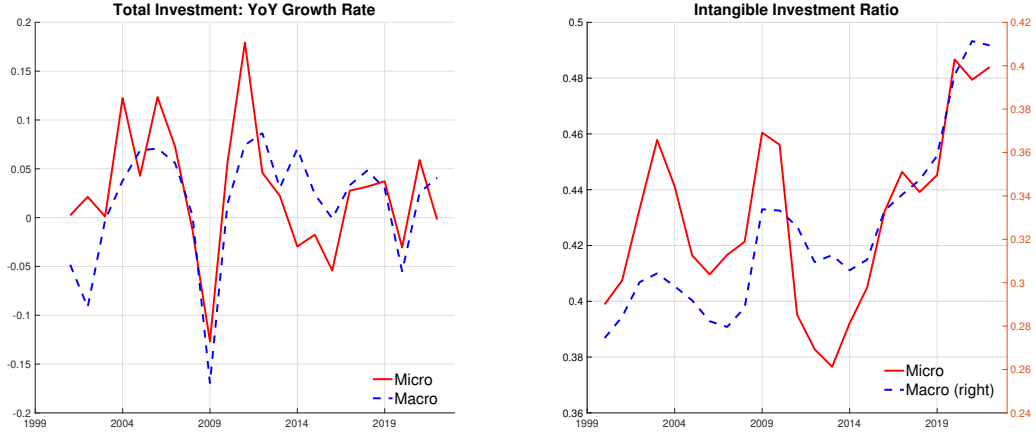
<sup>5</sup>See Bhattarai et al. (2020) among others using a VAR model to identify exogenous fluctuations of uncertainty.

<sup>6</sup>There are two important reasons for selecting the sample period. First, firms started to extensively report intangible investment since 2000. Second, when estimating intangible capital stock, the assumed initial value is unlikely to affect distant observations. To avoid potential bias due to the initial values, a late starting point of the sample is selected.

<sup>7</sup>As documented by Peters & Taylor (2017), the Federal Accounting Standards Board (FASB) requires firms to report R&D since 1975.

<sup>8</sup>Due to changes of the fiscal quarter, some observations may be counted twice.

Figure 1: Investment Comparison



*Note:* this figure compares total investment (in the left panel) and intangible investment ratio (in the right panel) based on Compustat (i.e., Micro) and Macro series. The total investment includes both intangible and tangible investment. The intangible investment ratio is defined as intangible investment over total investment. The Micro series define tangible investment as capital expenditure (CAPX), and intangible investment as that in R&D and organizational capital (measured as a portion of SG&A expenditures). The Macro series measure tangible investment as that in non-residential fixed investment, and intangible investment as that in intellectual property products.

## 2.1 Investment Regressions

To investigate overall effects of uncertainty on investment, I consider the following regression specifications:

$$I_{it} = \alpha_i + \beta u_t + \gamma' \mathbf{Z}_{it} + \psi_{fq} + \epsilon_{it} \quad (1)$$

where  $I_{it}$  is a measure of investment rate.  $\mathbf{Z}_{it}$  is a set of control variables, including Tobin's Q, leverage ratio, cash holding, cash flow, size, and age (see Döttling & Ratnovski (2023) among others).  $\alpha_i$  are firm fixed effects and  $\psi_{fq}$  are quarter fixed effects to control for seasonality. To account for potential implications of intangibles for the effects of uncertainty, Specification (1) is expanded with an interaction term between uncertainty and an intangible ratio, defined as intangible capital over total capital  $k_{it}^{int}$ . The main coefficients of interest are  $\beta_1$  and  $\beta_2$  which jointly capture the effect of uncertainty conditional on firm's intangible capital ratio.

$$I_{it} = \alpha_i + \beta_1 u_t + \beta_2 u_t \times k_{it}^{int} + \beta_3 k_{it}^{int} + \gamma' \mathbf{Z}_{it} + \psi_{fq} + \epsilon_{it} \quad (2)$$

To address the potential influence of outliers which are often presented in the firm-level studies, I adopt the Hampel Identifier (Wilcox 2011), an outlier detection approach based on median absolute deviation (MAD) statistics. MAD is a more robust statistic than standard deviation and hence the detection procedure is resilient to the presence of outliers. Detailed explanations of this method can be found from Appendix B.

Table 1: Investment Regression Results–Total Investment

	Baseline			No Outlier Detection		
	[1]	[2]	[3]	[4]	[5]	[6]
$u$	-0.0542*** (0.007)	-0.2431*** (0.032)		-0.0494*** (0.008)	-0.2081*** (0.042)	
$u \times k^{int}$		0.2361*** (0.039)	0.2350*** (0.039)		0.1984*** (0.051)	0.2049*** (0.051)
Observations	112167	103822	103747	116673	108516	108516
Adj. $R^2$	0.37	0.39	0.414	0.33	0.344	0.363
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	No	No	Yes	No	No	Yes

*Note:* the dependent variable–total investment rate is measured by the log of sum of tangible and intangible investment divided by lagged total assets.  $u$  is measured as the adjusted VIX index. Total assets are book assets plus off-balance intangible capital estimated based on the perpetual inventory method.  $k^{int}$  is the firm’s intangible-to-total capital ratio. Firm-level controls include Tobin’s Q, leverage ratio, cash holding, cash flow, size, and age. For regressions in columns [2], [3], [5], and [6],  $k^{int}$  is added as an additional control variable. Detailed descriptions about variables can be found in Appendix A. Robust standard errors are shown in parentheses. \*\*\*, \*\*, and \* represent significance level at 1%, 5%, and 10%, respectively.

Table 2: Investment Regression Results–Alternative Uncertainty Measure (JLN index)

	Baseline			No Outlier Detection		
	[1]	[2]	[3]	[4]	[5]	[6]
$u$	-0.1869*** (0.015)	-0.7023*** (0.070)		-0.1707*** (0.017)	-0.5936*** (0.094)	
$u \times k^{int}$		0.6310*** (0.085)	0.6572*** (0.085)		0.5171*** (0.114)	0.5581*** (0.113)
Observations	112177	103823	103750	116673	108516	108516
Adj. $R^2$	0.372	0.394	0.414	0.331	0.347	0.364
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	No	No	Yes	No	No	Yes

*Note:*  $u$  is measured as the adjusted JLN index. Others the same as above.

Table 1 report results based on overall investment rates, defined as the sum of tangible and intangible investment divided by firm’s assets in the last period. Uncertainty is measured as the adjusted VIX index. Column [1] shows a negative coefficient for  $u_t$  which is significant at 1% level, indicating an adverse effect of uncertainty on overall investment<sup>9</sup>. Column [2] confirms this result and further shows a positive and significant coefficients for the interaction term  $u_t \times k_{it}^{int}$ , implying that the negative effect of uncertainty can

<sup>9</sup>Focusing on firm-level uncertainty and annual data, Alfaro et al. (2024) also find negative effects of uncertainty on firm investment.

be weakened by a higher intangible ratio. Moreover, regressions without applying the Hampel Identifier not only confirm the overall effect but also the conditional effect of uncertainty.

$$I_{it} = \alpha_i + \beta_1 u_t \times k_{it}^{int} + \beta_2 k_{it}^{int} + \gamma' \mathbf{Z}_{it} + \eta_t + \psi_{fq} + \epsilon_{it} \quad (3)$$

To further confirm the implication of intangibles, I include time fixed effect  $\eta_t$  in Specification (3)<sup>10</sup>, and investigate if the weakened response of investment is driven by time-varying factors which cannot accounted in Specification (2). Results based on Specification (3) are reported in columns [3] and [6] in Table 1. I find that the presence of time fixed effects do not alter the mitigation role provided by intangibles. Table 2 reports invest regression results but uncertainty is measured as the adjusted JLN index. It confirms the negative relationship between total investment and uncertainty, and this adverse relationship could be mitigated by intangible capital ratio.

Table 3: Investment Regression Results–Investment Components

	VIX		JLN	
	Intangible Inv. Rate [1]	Tangible Inv. Rate [2]	Intangible Inv. Rate [3]	Tangible Inv. Rate [4]
<i>u</i>	-0.0322*** (0.005)	-0.0869*** (0.012)	-0.1418*** (0.012)	-0.2662*** (0.027)
Observations	111572	111520	111571	111481
Adj. $R^2$	0.214	0.544	0.216	0.545
Controls	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes	Yes

*Note:* the dependent variables are intangible investment rate in columns [1] and [3], and tangible investment rate in columns [2] and [4]. According to Peters & Taylor (2017) and Döttling & Ratnovski (2023), the intangible investment rate is measured as log of Compustat item *R&D* plus  $0.3 \times SG\&A$  scaled by lagged total assets. The tangible investment rate is measured as log of Compustat item *CAPX* scaled by lagged total asset. In columns [1] and [2], *u* is measured as the adjusted VIX index while the adjusted JLN index is used in columns [3] and [4]. The Hampel Identifier is applied. Others the same as above.

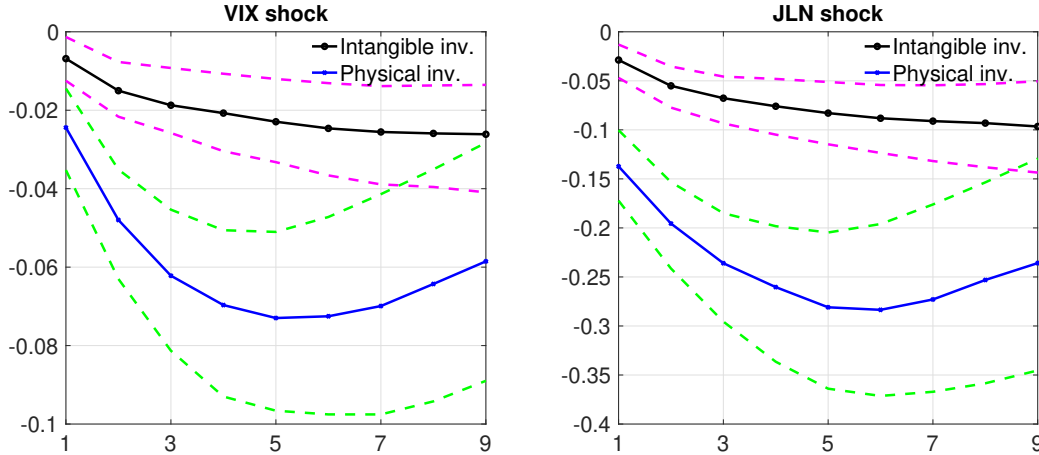
Next, I examine effects of uncertainty on different components of total investment, including tangible and intangible investment based on Specification (1). Both two investment variables are scaled by lagged asset as for total investment. Table 3 suggests that increased uncertainty reduces both types of investment, yet tangible investment tend to be more considerably affected compared to the intangible counterpart. For example, columns [1] and [2] suggest that 1% increase in the adjusted VIX index reduces tangible investment rate by 0.087% while the intangible investment is reduced by 0.032%. This finding implies that intangible investment is relatively less sensitive to the change of uncertainty level. The insensitivity of intangible investment to uncertainty can be further corroborated based on macro level evidence. Figure 2 plots impulse

<sup>10</sup>Note that it is unnecessary to include  $u_t$  in Specification (3) because  $\eta_t$  would absorb all time-series variations (Döttling & Ratnovski 2023).



responses of tangible and intangible investment measured at the macro level to uncertainty shocks<sup>11</sup>. Based on both two measures of uncertainty, Figure 2 suggests that the responses of intangible investment are relatively muted compared to tangible investment.

Figure 2: Responses of Investment based on the Macro Measure to Uncertainty Shocks



*Note:* this figure compares impulse responses of physical investment and intangible investment to 1% increase in uncertainty based on either VIX or JLN index. Physical investment is measured as non-residential fixed investment, and intangible investment is measured as investment in intellectual property products. Dashed lines show the 68% probability density intervals. Variables are expressed as percentage deviations.

## 2.2 Extension Analysis and Robustness Check

A set of extended estimation has been conducted to check robustness of main findings. First, in the stage one, I estimate the same VAR model but order uncertainty indices as the last variable to extract its exogenous components. Second, I run regressions using the original uncertainty indices in the regression analysis. Considering that investment may have slow movement, I further use lagged independent variables in the investment regressions.

In addition to studying the investment-uncertainty relationship, I also investigate stock price-uncertainty relationship. The regression equations are specified in a way similar to Specifications (1), (2), and (3), but replacing dependent variables with stock price and further including lagged stock price as an independent variable. The same set of firm-level control variables are included in the stock price regressions.

Table 12 in the Appendix C suggests a negative relationship between uncertainty and the stock price. However, the negative relationship can be mitigated by the presence of intangibles. Overall, I find that the negative effect of uncertainty and the mitigation effects of intangibles are robust to the alternative identification approaches. Extra empirical results are reported in the online Appendix C.

<sup>11</sup>In specific, I estimate the SVAR as in the first stage but separate the two types of investment as two variables.

### 3 The Model

This section outlines the theoretical framework which is used to analyse uncertainty shocks in an economy engaging in knowledge creation. Taking a medium-scale single-goods DSGE model as the benchmark, e.g., Basu & Bundick (2017), I augment it with production of intangible goods, incorporating skilled labor, and quantity-based financial frictions (i.e, a borrowing constraint).

#### 3.1 Households

The representative household derives utility from consumption  $C_t$  and leisure, saves in the form of deposits  $D_t$  and equity shares  $S_t$ . It supplies two types of labor (Anzoategui et al. 2019), including unskilled labor which is used to produce tangible goods  $Y_t$  and skilled labor which is used in the production of intangibles  $X_t$ . The supply of two types of labor is measured by working hours  $H_t^u$  and  $H_t^s$ . Following Basu & Bundick (2017), the preference is given by a Epstein–Zin structure.

$$V_t = [\varepsilon_t^d U_t^{(1-\sigma)/\theta_v} + \beta(E_t V_{t+1}^{1-\sigma})^{1/\theta_v}]^{\theta_v/(1-\sigma)} \quad (4)$$

where the utility kernel  $U_t$  is

$$U_t = \log(C_t - h\bar{C}_{t-1}) e^{-\frac{\psi^u (H_t^u)^{1+\eta} + \psi^s (H_t^s)^{1+\eta}}{1+\eta}} \quad (5)$$

The household maximize lifetime utility (4) subject to the budget constraint

$$P_t C_t + D_t + P_t^E S_t = R_{t-1} D_{t-1} + (D_{t-1}^E + P_{t-1}^E) S_{t-1} + W_t^u H_t^u + W_t^s H_t^s + \Pi_t^f \quad (6)$$

where  $R_t$  denotes interest rate,  $P_t$  aggregate price level,  $W_t^u$  unskilled wages,  $W_t^s$  skilled wages, and  $\Pi_t^f$  profits due to owning the final goods firms. Equity shares have a price of  $P_t^E$  and pay dividends  $D_t^E$  for each share owned.  $h$  measures degree of external habits in consumption and  $\eta$  measures the elasticity of labour supply.  $\varepsilon_t^d$  is a preference shock following an AR(1) process:  $\varepsilon_t^d = (1 - \rho_d)\varepsilon^d + \rho_d\varepsilon_{t-1}^d + \sigma_t^d \epsilon_t^d$  and  $\epsilon_t^d$  follows an i.i.d  $N(0, 1)$ .  $\sigma_t^d$  is a uncertainty shock evolving as:  $\sigma_t^d = (1 - \rho_u)\sigma^d + \rho_u\sigma_{t-1}^d + \sigma^u \epsilon_t^u$  and  $\epsilon_t^u$  follows an i.i.d  $N(0, 1)$ .

Following Basu & Bundick (2017), the stochastic discount factor  $M_{t,t+1}$  is defined as

$$M_{t,t+1} = \frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t} = \beta \frac{\varepsilon_{t+1}^d}{\varepsilon_t^d} \left( \frac{U_{t+1}}{U_t} \right)^{(1-\sigma)/\theta_v} \left( \frac{C_t - h\bar{C}_{t-1}}{C_{t+1} - h\bar{C}_t} \right) \left( \frac{V_{t+1}^{1-\sigma}}{\mathbb{E}_t V_{t+1}^{1-\sigma}} \right)^{\theta_v/(1-\sigma)} \quad (7)$$

### 3.2 Intermediate Goods Producers

There is a continuum of monopolistic intermediate goods firms  $j$ , each of which produces tangible and intangible goods, accumulates physical capital and intangible capital, and finances business by debts and equity. The tangible goods  $Y_{jt}^m$  are sold to final goods producers, while the intangible goods  $X_{jt}$  are internally used for intangible investment.<sup>12</sup> Production functions of the two types goods for firm  $j$  are given by

$$Y_{jt}^m = A_t Z_{jt}^\zeta (u_{jt} K_{jt}^y)^\alpha (H_{jt}^u)^{1-\alpha-\zeta} \quad (8)$$

$$X_{jt} = \chi Z_{jt}^\zeta (u_{jt} K_{jt}^z)^\alpha (H_{jt}^s)^{1-\alpha-\zeta} \quad (9)$$

where  $Z_{jt}$  is knowledge or intangible capital,  $K_{jt}^y$  is physical or tangible capital used in producing  $Y_{jt}^m$ ,  $K_{jt}^z$  is physical capital used in producing  $X_{jt}$ , and  $u_{jt}$  is a utilization rate of physical capital.  $\chi$  is the intangible-specific productivity<sup>13</sup>,  $\zeta$  and  $\alpha$  are income shares of the two types of capital in the production functions.  $A_t$  is a productivity shock following an AR(1) process:  $A_t = (1 - \rho_a)A + \rho_a A_{t-1} + \sigma^a \epsilon_t^a$  and  $\epsilon_t^a$  follows an i.i.d  $N(0, 1)$ .

Equations (8) and (9) suggest two key differences in producing the two types of goods. In line with literature (McGrattan & Prescott 2010, Mitra 2019), I assume that physical capital, such as machines, can only be used to produce one type of goods at a time while intangible capital, such as patents and brands, is non-rival in that it can be used to produce two types of goods simultaneously. In other words,  $Z_{jt}$  provides a spillover effects in the production activities. Departing from literature, I also consider that production of intangible goods, such as technology innovation, is sophisticated which requires inputs from skilled labors.

The firm  $j$  accumulates physical and intangible capital according to the following laws of motions, respectively:

$$K_{j,t+1} = [1 - \delta(u_{jt})]K_{jt} + \Omega_{j,t}^k I_{jt} \quad (10)$$

$$Z_{j,t+1} = (1 - \delta_z)Z_{jt} + X_{jt} \quad (11)$$

where  $I_{jt}$  is physical investment and  $\delta_z$  is a depreciation rate of intangible capital. Denoting  $\delta_k$  as the depreciation rate of physical capital in the steady state, depreciation of physical capital  $\delta(u_{jt})$  depends on the utilization rate in the following functional form:

$$\delta(u_{jt}) = \delta_k + \delta_1(u_{jt} - 1) + \frac{\delta_2}{2}(u_{jt} - 1)^2 \quad (12)$$

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<sup>12</sup>The treatment of intangible investment is consistent with empirical literature (see Peters & Taylor (2017) among others) which suggests that intangible assets, such as R&D capital, are largely internally created.

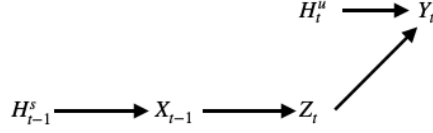
<sup>13</sup>In the extended analysis, I also incorporate a productivity shock in the intangible sector.

The physical investment adjustment cost  $\Omega_{j,t}^k$  is given by

$$\Omega_{j,t}^k = 1 - \frac{\phi_k}{2} \left( \frac{I_{jt}}{(1+g^y)I_{j,t-1}} - 1 \right)^2 \quad (13)$$

where  $g^y$  is the net growth rate of the economy in the steady state.

Figure 3: Contributions of the Two Types of Labors on  $Y_t$



The capital accumulation and productions functions (8), (9), (10) and (11) suggest different contributions of the two types of labors to tangible goods over different horizons. While unskilled labors contribute to tangible production contemporaneously, the contribution of skilled labor tends to have a lag (see Figure 3). Such a difference implies that skill labor tends to have a long-term nature. When the aggregate demand is subject to disturbances, the effect could be partially absorbed by the tangible sector before transmitted to the skilled labor market and hence intangible sector.

The firm  $j$  maximizes the expected present discounted value of the current and future dividends  $D_{jt}^E$ :

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} M_{t+s,t+s+1} D_{j,t+s}^E \quad (14)$$

subject to the two production functions (8) and (9), budget constraint (15), borrowing constraint (16), the demand scheme of tangible goods (17), and evolution of the two types of capital (10) and (11).

$$W_t^u H_{jt}^u + W_t^s H_{jt}^s + P_t I_{jt} + P_t \Phi(D_{jt}^E) + R_{t-1}^b B_{j,t-1} = P_{jt}^m Y_{jt} + B_{jt} \quad (15)$$

$$B_{jt} \leq \xi_t (P_t K_{jt} + \nu P_t Z_{jt}) \quad (16)$$

$$Y_{jt}^m = Y_t^m \left( \frac{P_{jt}^m}{P_t^m} \right)^{-\theta_m} \quad (17)$$

where  $P_{jt}^m$  is the price of intermediate tangible goods,  $\theta_m$  is elasticity of substitution for intermediate tangible goods.  $\nu \in (0, 1)$  captures limited pledgeability of intangible capital compared with tangible capital. Following Jermann & Quadrini (2012), each firm use debts and equity, and debts are preferred to equity due to its tax advantage though using debts is subject to the constraint. The gross lending rate is  $R_t^b = 1 + (1 - \tau)(R_t - 1)$ , where  $\tau \in (0, 1)$  denotes the tax rate. The dividends (or equity) and its associated payment (or equity raising) costs are subject to a quadratic adjustment style with  $\Phi(D_{jt}^E) = D_{jt}^E + \kappa/2(D_{jt}^E/(1+g_y)^t - d^E)^2$  where  $(1+g_y)^t$  is a scaling factor to ensure a balanced growth path,  $d^E = D^E/(1+g_y)$  is the detrended

dividend in the steady state, and  $\kappa$  is the elasticity of dividend payment costs.  $\xi_t$  is a financial shock following an AR(1) process:  $\xi_t = (1 - \rho_f)\xi + \rho_f\xi_{t-1} + \sigma^f\epsilon_t^f$  and  $\epsilon_t^f$  follows an i.i.d  $N(0, 1)$ .

### 3.3 Final Goods Producers

There are a continuum of monopolistic competitive final goods producers  $i$ , each of which is like a retailer, who buys intermediate goods  $Y_{it}^m$  and transfers them into differentiated final goods  $Y_{it}$  in a linear way, which is further used for consumption, physical investment, and government spending. The final goods firms face nominal price adjustment costs following Rotemberg's approach  $\frac{\phi_p}{2}(\frac{P_{it}}{\pi P_{i,t-1}} - 1)^2 Y_{it}$ .

The final goods producer  $i$  maximizes expected present discounted value of the current and future profit:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} M_{t+s,t+s+1} [(\frac{P_{i,t+s}}{P_{t+s}} - \frac{P_{t+s}^m}{P_{t+s}})Y_{i,t+s} - \frac{\phi_p}{2}(\frac{P_{i,t+s}}{\pi P_{i,t+s-1}} - 1)^2 Y_{t+s}] \quad (18)$$

subject to the demand of final goods:

$$Y_{it} = Y_t (\frac{P_{it}}{P_t})^{-\theta_f} \quad (19)$$

where  $\theta_f$  is elasticity of substitution for final goods. The maximization yields the following New-Keynesian Phillips Curve in the equilibrium:

$$p_t^m = \frac{\theta_f - 1}{\theta_f} + \frac{\phi_p}{\theta_f} (\frac{\pi_t}{\pi} - 1) \frac{\pi_t}{\pi} - \mathbb{E}_t M_{t,t+1} \frac{\phi_f}{\theta_f} (\frac{\pi_{t+1}}{\pi} - 1) \frac{\pi_{t+1}}{\pi} \frac{Y_{t+1}}{Y_t} \quad (20)$$

where  $p_t^m = P_t^m/P_t$  is the real price of intermediate goods (and also real marginal cost for final goods firms).

### 3.4 Equilibrium and Measure Issues

In the symmetric equilibrium, all intermediate goods firms and final goods firms have the same decisions, respectively. Hence,

$$Y_t = Y_t^m = A_t Z_t^\zeta (u_t K_t^y)^\alpha (H_t^u)^{1-\alpha-\zeta} \quad (21)$$

$$X_t = \chi Z_t^\zeta (u_t K_t^z)^\alpha (H_t^s)^{1-\alpha-\zeta} \quad (22)$$

The resource constraint is given by

$$Y_t = C_t + I_t + G_t + \frac{\phi_p}{2} (\frac{P_t}{\pi P_{t-1}} - 1)^2 Y_t + \frac{\kappa}{2} [\frac{D_t^E}{(1+g_y)^t} - d^E]^2 \quad (23)$$

where government spending  $G_t$  follows an AR(1) process:  $G_t/(1+g_y)^t = (1-\rho_g)g + \rho_g G_{t-1}/(1+g_y)^{t-1} + \sigma_g \epsilon_t^g$  and  $\epsilon_t^g$  follows i.i.d  $N(0, 1)$ .  $g = G/(1+g_y)$  is the detrended government spending in the steady state.

In equilibrium, capital markets and labor markets are clear.

$$K_t = K_t^y + K_t^z \quad (24)$$

$$H_t = H_t^u + H_t^s \quad (25)$$

The central bank sets nominal interest rate according to a Taylor rule:

$$R_t = R_{t-1}^{\rho_r} [R(\frac{\pi_t}{\pi})^{\rho_\pi} (\frac{Y_t}{(1+g^y)Y_{t-1}})^{\rho_y}]^{1-\rho_r} \quad (26)$$

Following Basu & Bundick (2017), the stock return  $R_t^E$  and model-implied stock market volatility in the annualized term  $V_t^M$  are defined as

$$R_t^E = \frac{P_t^E + D_t^E}{P_{t-1}^E} \quad (27)$$

$$V_t^M = 100\sqrt{4\text{VAR}(R_{t+1}^E)} \quad (28)$$

The presence of the intangible sector leads to measurement issues. First, we consider two measures of GDP, including the traditional GDP  $GDP$  which treat intangibles as intermediate costs and the actual GDP  $GDP^a$  which accounts intangibles.

$$GDP_t = C_t + I_t + G_t \quad (29)$$

$$GDP_t^a = C_t + I_t + X_t + G_t \quad (30)$$

When measuring intangible investment, we distinguish input-sided and output-sided measures. Taking technology creation as an example, R&D expenditures are inputs in the innovation process while patents are one outcome. The input-sided measure is based on the expenditure in creating intangibles, such as the replacement cost approach (Peters & Taylor 2017, Döttling & Ratnovski 2023). Given that  $K_t^z$  and  $H_t^s$  are exclusively used in the production of intangible goods, the measured intangible investment  $I_t^z$  is defined as the sum of capital costs  $R_t^{k,z}K_t^z$  and labor costs  $W_t^sH_t^s$  in producing  $X_t$ <sup>14</sup>

$$I_t^z = R_t^{k,z}K_t^z + W_t^sH_t^s \quad (31)$$

where  $R_t^{k,z} = \alpha\mu_t^z \frac{X_t}{K_t^z}$  is the return of physical capital used in  $X_t$  production, and  $\mu_t^z$  is the Lagrange multiplier associated with the intangible capital accumulation.

The measured intangible capital is given by

$$Z_{t+1}^m = (1 - \delta_z)Z_t^m + I_t^z \quad (32)$$

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<sup>14</sup>Alternatively, we can define  $I_t^z = R_t^{z,z}Z_t + R_t^{k,z}K_t^z + W_t^sH_t^s$  where  $R_t^{z,z} = \zeta\mu_t^z \frac{X_t}{Z_t}$ . I found that results based on the two definitions do not differ significantly.

which is consistent with the estimate based on perpetual inventory method. Online Appendix D lists all equilibrium conditions in the baseline model.

## 4 Solution and Calibration

Following the existing literature (see Born & Pfeifer (2014), Fernández-Villaverde et al. (2015), Basu & Bundick (2017) among others), the model is solved by a third-order perturbation around the balanced growth path. The third-order approximation is required to separate effects of the uncertainty shock from the corresponding level shock.

Table 4 presents calibrated parameters. The physical capital share  $\alpha$  is set as 0.3, in line with other US-based DSGE studies. The intangible capital share  $\zeta$  is calibrated as 0.15, which falls in the range suggested by the literature (Lopez & Olivella 2018, Mitra 2019). The discount factor  $\beta$  is calibrated as 0.995 to match quarterly interest rate. The habit parameter  $b$  is set as 0.75, a moderate value reported by the literature (see Born & Pfeifer (2014) and Bianchi et al. (2023) among others). The physical capital depreciation rate  $\delta_k$  is calibrated as 0.025, consistent with existing literature. Following Kung & Schmid (2015) and Jinnai (2015), the intangible capital depreciation rate  $\delta^a$  is set as 0.0375. The combination of  $\alpha$ ,  $\zeta$ ,  $\delta^k$  and  $\delta^a$  delivers the intangible-to-output ( $X/Y$ ) ratio as 11% and intangible investment share ( $X/(X + I)$ ) as 33%, consistent with my empirical evidence and literature (Aghion et al. 2010, Lopez & Olivella 2018).<sup>15</sup> The inverse labor elasticity  $\eta$  is calibrated as 2, consistent with literature (Smets & Wouters 2007, Basu & Bundick 2017). Following Born & Pfeifer (2014), the capital utilization cost  $\delta_2/\delta_1$  is calibrated as 0.1<sup>16</sup>. The physical capital adjustment cost  $\phi_k$  is calibrated as 1.6 which falls in the range suggested by the literature<sup>17</sup>. Following Basu & Bundick (2017), the price adjustment cost  $\phi_p$ , intertemporal elasticity of substitution  $\psi$ , and risk aversion  $\sigma$  are set as 100, 0.95, and 80, respectively. Following Jermann & Quadrini (2012), the equity adjustment cost  $\kappa$ , and tax rate  $\tau$  are set as 0.15 and 0.35, respectively. The two elasticity of substitution parameters  $\theta^m$  and  $\theta^f$  are calibrated as 10, implying markup as 1.1 in the final goods and intermediate goods sectors. Regarding the three Taylor parameters  $\rho$ ,  $\rho_\pi$ , and  $\rho_y$ , they are calibrated as commonly used values: 0.7, 1.85, and 0.25, respectively.

The middle part of Table 4 displays the calibrated values of steady-state parameters. The average per capita GDP growth rate is about 0.5%, implying  $g_y$  as 0.005. The government spending-to-output ratio is calibrated as 15%. The unskilled dis-utility parameter  $\psi^u$  is set to match 1/3 unskilled worked hours. The skilled dis-utility parameter  $\psi^s$  is set such that the productivity of intangibles  $\chi$  is normalized to unity.

<sup>15</sup>Lopez & Olivella (2018) find the  $X/Y$  ratio as 5% and the  $X$  investment share as 29%. Aghion et al. (2010) suggest that the  $X$  investment share is between 11% to 47%. Based on my micro and the macro datasets over 2000-2022, the averaged  $X$  investment shares are 43% and 25%, respectively.

<sup>16</sup>Note that  $\delta_1$  can be pinned down by return of physical capital in the steady state.

<sup>17</sup>Born & Pfeifer (2014) find a relative low value for  $\phi_k$  (1.6), while Bianchi et al. (2023) find a relatively high value (7.3). As will be shown shortly in Section 6, the inclusion of intangibles dampens investment volatility. In order to match data, a relatively low value of physical capital adjustment cost is chosen.

Table 4: Calibrated Parameters

Parameters	Description	Value
$\alpha$	physical capital share	0.3
$\zeta$	intangible capital share	0.15
$\beta$	discount factor	0.995
$b$	degree of habit formation	0.75
$\eta$	inverse labour elasticity	2
$\delta_k$	physical capital depreciation	0.025
$\delta_z$	intangible capital depreciation	0.0375
$\delta_2/\delta_1$	capital utilization cost	0.1
$\kappa$	equity adjustment cost	0.15
$\phi_k$	investment adjustment cost	1.6
$\phi_p$	price adjustment cost	100
$\psi$	intertemporal elasticity of substitution	0.95
$\sigma$	risk aversion	80
$\tau$	tax rate	0.35
$\theta_m$	IG elasticity of substitution	10
$\theta_f$	FG elasticity of substitution	10
$\rho$	taylor smoothing	0.7
$\rho_\pi$	taylor parameter	1.85
$\rho_y$	taylor parameter	0.25
$\nu$	intangible pledgeability	0.2
$1+g_y$	ss per capita GDP growth	1.005
$G/Y$	ss exo. demand share	0.15
$H^u$	ss unskilled hours worked	1/3
$H^s$	ss skilled hours worked	0.015
$\xi$	ss financial constraint	0.4
$\rho_a$	per. of tangible productivity	0.95
$\rho_d$	per. of preference	0.80
$\rho_g$	per. of government spending	0.98
$\rho_f$	per. of financial const.	0.98
$\rho_u$	per. of uncertainty	0.75
$\sigma_a$	std. of tangible productivity	0.007
$\sigma_d$	std. of preference	0.020
$\sigma_g$	std. of government spending	0.002
$\sigma_f$	std. of financial const.	0.009
$\sigma_u$	std. of uncertainty	0.009

Regarding the two financial constraint parameters, the intangible pledgeability  $\nu$  is set as 0.2, implying that 20% of intangible capital is pledgeable, in line with empirical observations (Mann 2018, OECD 2021, Caggese & Pérez-Orive 2022). Finally, the financial constraint  $\xi$  in the steady state is calibrated as 0.4, implying private debt-to-output ratio as 3.3 at quarterly frequency.

The lower part of Table 4 shows the calibrated values of shock processes. The persistence and standard deviation of the four level shocks are calibrated following estimated values in the literature (Jermann & Quadrini 2012, Christiano et al. 2014, Fernández-Villaverde et al. 2015, Bianchi et al. 2019). Regarding the



uncertainty shock, a moderately high persistence is chosen ( $\rho_u = 0.75$ ), in line with suggestions from existing literature (see Leduc & Liu (2016), Basu & Bundick (2017) among others). Following the approach adopted by Basu & Bundick (2017), I calibrate  $\sigma_u$  such that one standard deviation increase in the uncertainty shock increases stock market volatility by 15% percent. This implies  $\sigma_u$  as 0.009.

Table 5: Empirical and Model-implied Moments

Moment	Data	Model Baseline	Model w/o Un. shock
$\sigma(\Delta y)$	0.63	0.70	0.59
$\sigma(\Delta c)$	0.55	0.58	0.47
$\sigma(\Delta i)$	2.27	2.23	1.87
$\sigma(h)$	1.23	1.38	1.17

*Note:* the empirical sample period is 1986-2019 at quarterly frequency. The empirical counterpart of tangible output  $y$  is defined as GDP excluding intellectual property products (IPP). The empirical counterpart of tangible investment  $i$  is defined as fixed private investment excluding IPP.

In order to assess how the calibrated model fits data, a comparison is made between model-implied moments of key macroeconomic variables and their empirical counterparts. Table 5 suggests that the model closely matches the volatility of output growth, consumption growth, investment growth, and hours worked observed in data. Moreover, the last column in Table 5 also reports model-implied moments but shutting down the uncertainty shock. In this case, the four variables become substantially less volatile compared to the baseline case and data. For example, the standard deviation of investment growth is 83% as in the baseline case. This finding suggests important roles of the uncertainty shock in driving business cycles, consistent with the argument of Fernández-Villaverde & Guerrón-Quintana (2020).

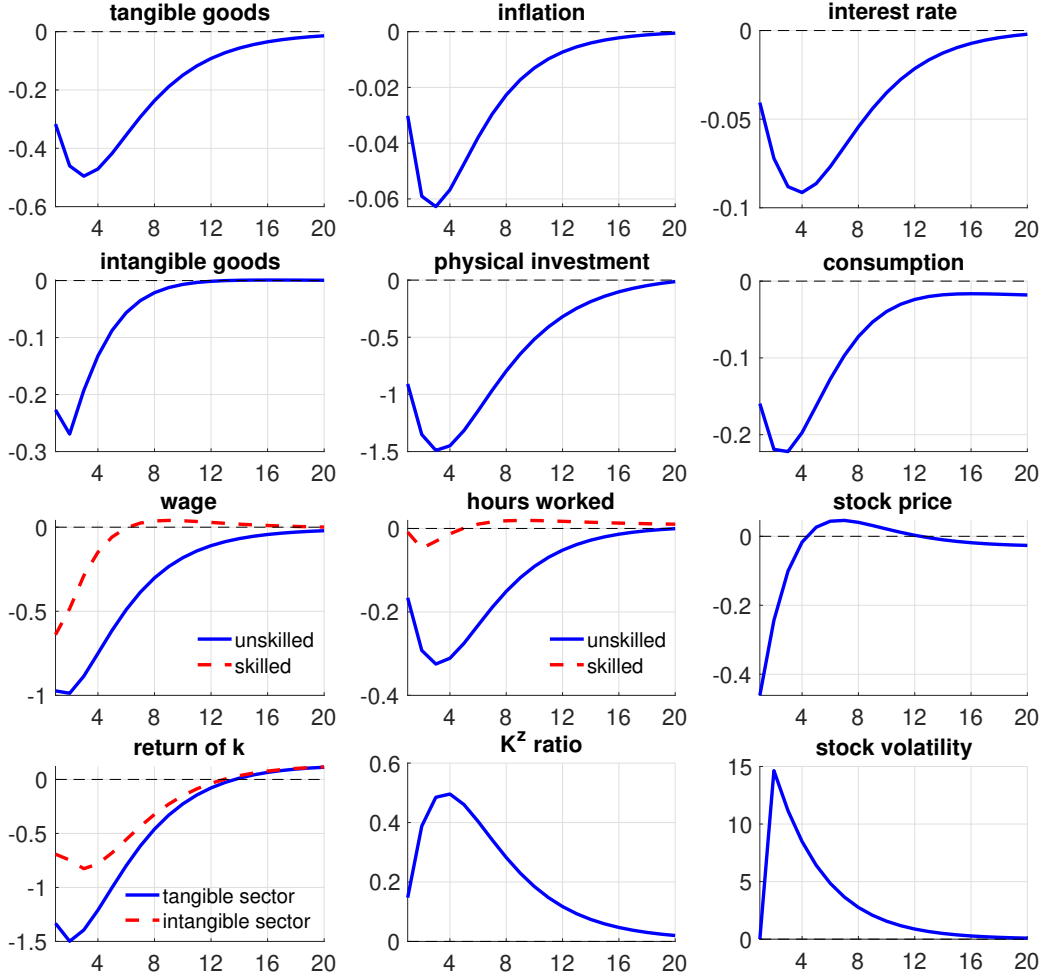
## 5 Uncertainty Shock in An Intangible Economy

This section investigates transmission of the uncertainty shock in a knowledge-intensive economy based on impulse response analysis.

### 5.1 Transmission of Uncertainty Shock with Intangibles

Figure 4 plots impulse responses to a positive uncertainty shock. Following the elevated uncertainty, the demand for consumption falls due to a precautionary saving effect, which further lowers tangible goods in the stick-price economy. The decline in tangible goods also decreases marginal revenue product of all types of capital (MRPK), reducing both physical and intangible investment. Consistent with the empirical findings as evidenced in Section 2, intangible investment shrinks with a smaller magnitude compared with physical investment. Moreover, Figure 4 also shows that skilled wages and hours tend to be less responsive than their unskilled counterparts. The last finding is consistent with empirical evidence found in Belianska (2023).

Figure 4: Uncertainty Shock

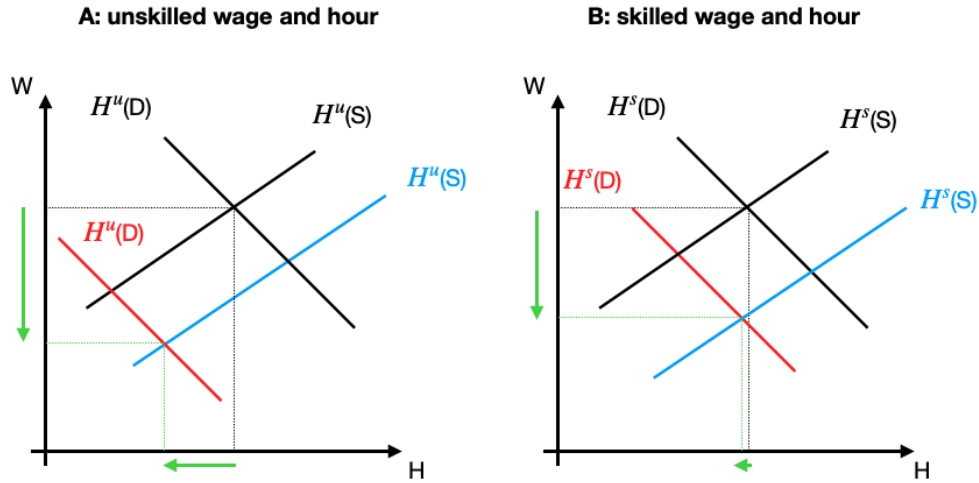


Note: variables are expressed as percentage deviations from the trend.

Two important forces—capital reallocation and precautionary labor supply—are at work to drive the relatively muted responses of intangible investment. Although MRPK of physical capital  $K_t^y$  and  $K_t^z$  both decline, the latter decreases less due to the long-term nature of intangible investment. Its return is less sensitive to the change of aggregate demand. Consequently, deploying physical capital in the intangible sector would be more attractive than in the tangible sector when facing increased uncertainty. The MRPK gap between  $K_t^y$  and  $K_t^z$  leads to a capital reallocation effect toward the intangible sector and hence  $K^z$  ratio rises.

Due to the complementarity between capital and labor, the capital reallocation effect also partially release downward pressure on skilled labor. The decreasing demand of skilled hours tends to be attenuated. As a result, skilled wages fall less significantly than unskilled wages. Moreover, the uncertainty shock induces precautionary labor supply, consistent with Basu & Bundick (2017). Given that skilled wages are relatively

Figure 5: Intuition: Equilibrium Wages and Hours Worked



Note: dark lines show pre-shock demand and supply of labor. The uncertainty shock reduces labor demand, shifting labor demand curves to the left (red lines). Due to the precautionary motive, households tend to increase labor supply, shifting labor supply curves to the right (blue lines).

higher than unskilled wages<sup>18</sup>, the precautionary labor motive for skilled work is stronger. Consequently, the skilled labor supply curve shift outward more significantly than the unskilled one, thereby dampening the decline in skilled hours worked. Figure 5 illustrates the determination of wages and hours worked for the two types of labor in response to the uncertainty shock.

Overall, elevated uncertainty channels resources toward the intangible sector, making the economy relatively more knowledge-intensive. A notable example can be found from the COVID pandemic period when uncertainty level substantially rises. OECD (2021) documents that firms tend to shift their investment composition toward intangibles, for example, adopting digital technology to weather the COVID-19 crisis. Given the sophistication of intangible production, such a shift of investment composition also changes firms' demand of labor to be more skilled-based.

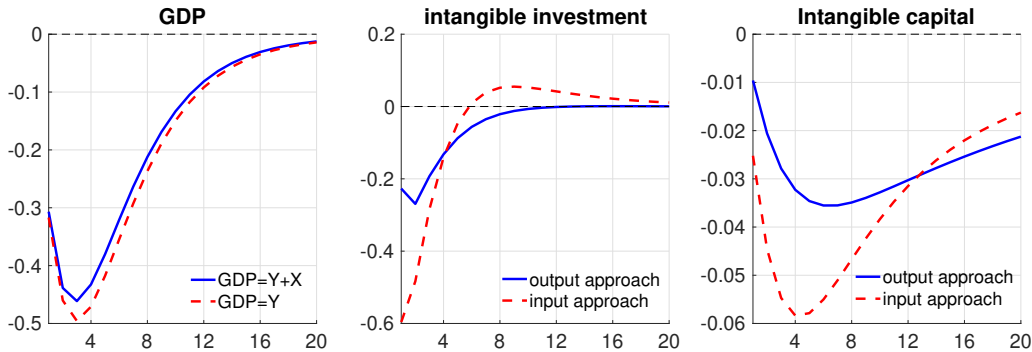
## 5.2 Measurement Issues

A potential concern in studying intangibles is measurement issues—intangibles tend to be measured by input or cost approaches and intangibles might not be fully included in the national account. The middle panel in Figure 6 displays responses of intangible investment measured by both output and input approaches. It shows that the input measure is more responsive than the output measure but both decline upon impact of the raised uncertainty. Hence, different measurement approaches would not fundamentally change the result.

<sup>18</sup>The model implies that skilled wages are higher than the unskilled wages in the steady state. This wage gap would enlarge when uncertainty increases.

Moreover, Figures 4 and 6 together suggests that the peak decline of physical investment is 2.5 times as large as that of measured intangible investment. The magnitude of this relative difference falls in the range suggested by empirical evidence in Table 3. Since intangible goods have a relative muted response, Figure 6 also suggests a smaller fall in the actual GDP than in the case that intangibles are omitted in the national account.

Figure 6: Uncertainty Shock: Measurement



*Note:* the output measure and the input measure of intangible investment are  $x_t$  and  $I_t^z = r_t^{k,z} K_t^z + w_t^s H_t^s$ , respectively. Variables are expressed as percentage deviations from the trend.

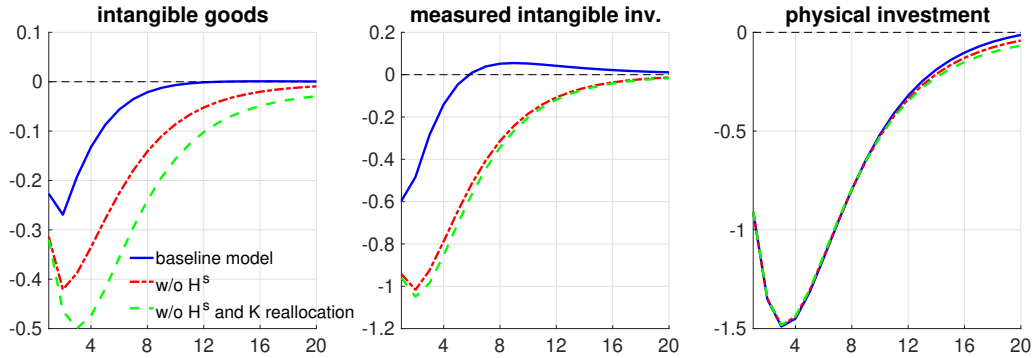
### 5.3 Further Discussions and Model Comparisons

The analysis highlights capital reallocation and precautionary labor motive in interpreting the results. It would be useful to quantitatively assess importance of the two factors in driving the muted response of intangibles to the uncertainty shock. To this end, I compare impulse responses based on the baseline model and counterfactual cases. One option is to remove skilled labor from the model and assume that unskilled labor is one of inputs used for intangible production. By doing so, the precautionary labor supply specific to skilled hours is shut down. Based on this counterfactual model, I further consider a case by assuming constant  $k^z$  ratio, which closes the capital reallocation channel. Figure 7a compares impulse responses of different types of investment based on the baseline model and the two counterfactual cases. When shutting down the skilled precautionary labor motive (red dash-dot line), the peak decline of  $x_t$  would be 0.4%, 60% larger than in the baseline model (0.25%). If the capital reallocation channel is further closed (green dash line), the peak decline of  $x_t$  could be doubled to 0.5%. Similarly, the measured intangible investment would also show more pronounced fall after removing the two elements.

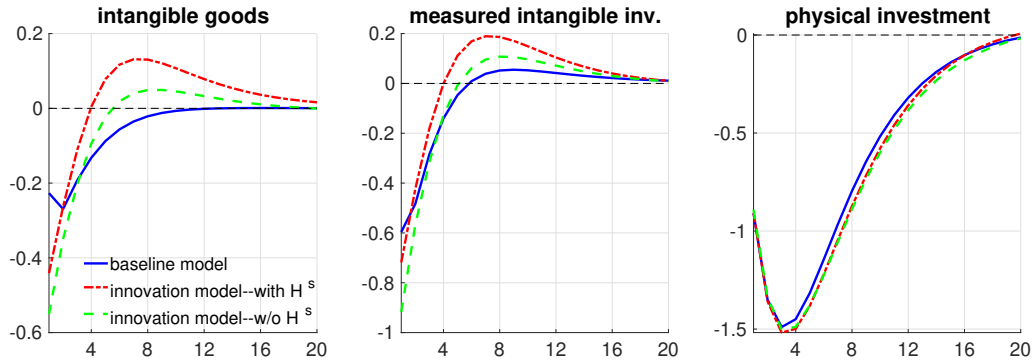
Focusing on technology components of intangibles–R&D, a body of literature incorporates endogenous growth in business cycle models (Comin & Gertler 2006, Anzoategui et al. 2019, Bianchi et al. 2019, Ikeda & Kurozumi 2019, Queralto 2020). I also consider modelling features as in these studies to examine the response of intangibles (or narrowly, technology innovation). In particular, I consider that intangible production (or technology creation) is conducted by using either tangible goods (see Comin & Gertler (2006) among others)

Figure 7: Uncertainty Shock: Model Comparisons

(a) Alternative Intangible Production Functions



(b) Baseline Model and Innovation Models



*Note:* variables are expressed as percentage deviations from the trend.

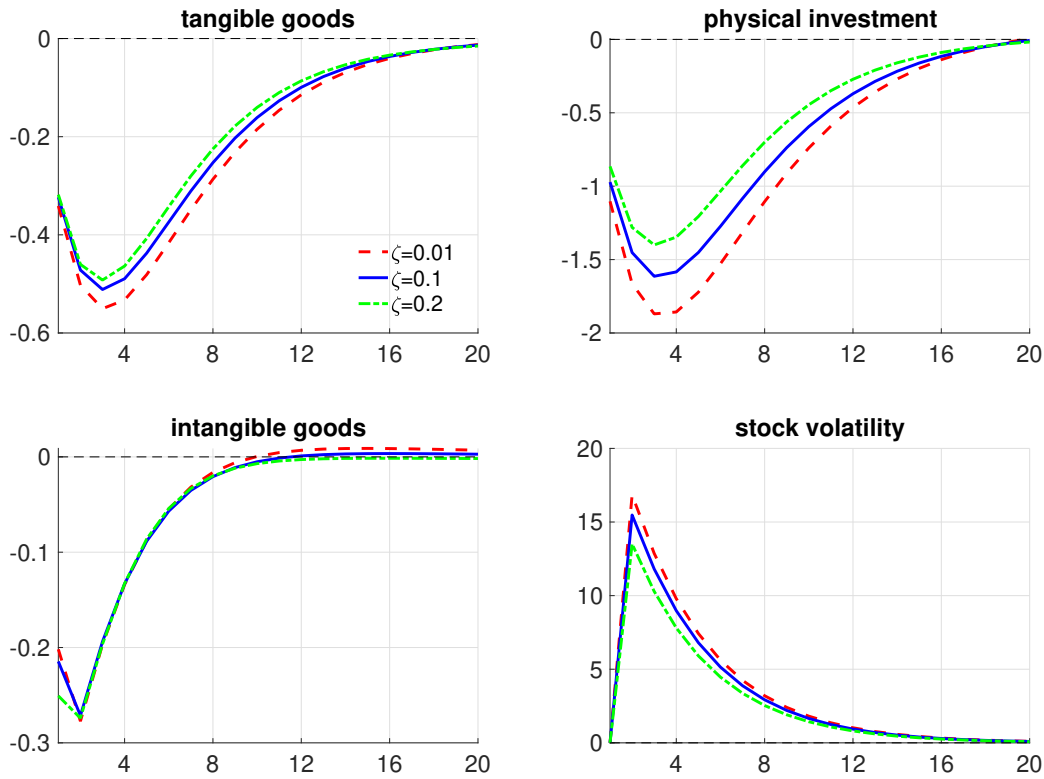
or both tangible goods and skilled labor (see Queralto (2020) among others). Figure 7b suggests more significant responses of intangible investment based on the two innovation models<sup>19</sup> compared to the baseline model. One may interpret that technology components tend to be more responsive to uncertainty than other intangible components. Such an implication is consistent with data patterns—R&D expenditure is more volatile than organization capital investment. Alternatively, by focusing on different modelling features, we may take these two models as extra exercises to assess importance of capital reallocation and precautionary labor motive in contributing to the response of intangible investment. In the two innovation models, the capital reallocation channel and/or the skilled precautionary labor motive are absent. Consistent with the implications from Figure 7a, Figure 7b also confirms the importance of the two channels in driving the muted response of intangible investment.

<sup>19</sup>Technical details of the two innovation models are reported in online Appendix DII.

## 6 The Rise of Intangibles

Given the importance and transmission of the uncertainty shock established in Sections 4 and 5, this section proceeds to study the implications of rising intangibles in the model with the uncertainty shock.

Figure 8: Uncertainty Shock: Alternative Intangible Ratios

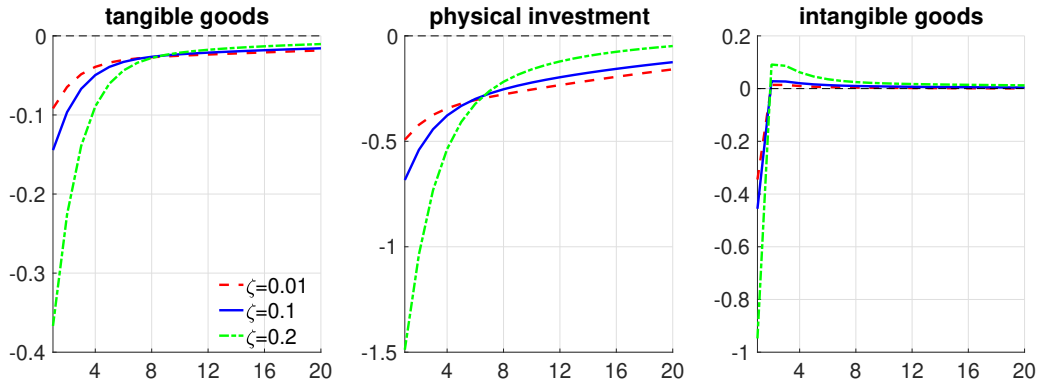


*Note:* this figure compares IRFs to the uncertainty shock at different values of the intangible share  $\zeta$ . Variables are expressed as percentage deviations from the trend.

Figure 8 compares responses of output, investment, and stock market volatility to the uncertainty shock while setting the intangible share  $\zeta$  at alternative values. When there is almost no intangible in the economy, the red dash lines show the largest decline of tangible goods, physical investment, and the largest increase of stock market volatility in response to the uncertainty shock. If we increase the intangible share to 0.1, the responses of output, investment, and stock market volatility would all be dampened. Such a dampening effect could be further strengthened if the intangible share becomes 0.2. In particular, the dampening effect on physical investment is the most pronounced; increasing  $\zeta$  from 0.01 to 0.2 reduces the peak decline of physical investment by around 0.5%. The results based on Figure 8 suggest some volatility reduction effects of intangibles on both the real side (e.g., output and investment) and the financial side (e.g., stock market). These results are consistent with the empirical findings as found in Section 2.

Figure 8 implies two important roles of rising intangibles in propagations of the uncertainty shock. On one hand, higher values in  $\zeta$  lead to larger weights on intangible sectors in the economy. Given that intangible goods and investment are insensitive to uncertainty levels, the increased shares of intangibles tends to reduce aggregate responses directly. On the other hand, due to the complementarity between the physical and intangible capital in production, the inertia response of intangible capital also mitigates the adverse effects of uncertainty on tangible sectors. Hence, increased  $\zeta$  also leads to a spillover effect which attenuates responses of tangible variables.

Figure 9: Financial Shock: Alternative Intangible Ratios



*Note:* this figure compares IRFs to the financial shock at different intangible ratios. Variables are expressed as percentage deviations from the trend.

Some literature suggests that technology innovation (see Ikeda & Kurozumi (2019) among others) or more broadly intangibles (see Lopez & Olivella (2018) among others) may play amplification roles in the business cycles focusing on financial shocks. Since intangibles dampen the transmission of the uncertainty shock, it remains unclear what the aggregate role of intangibles is in business cycles. This question is investigated here using the model with both first-moment and second-moment shocks. To this end, I first revisit effects of intangibles on transmissions of a financial shock, and then explore implications of intangibles for macroeconomic volatility.

Figure 9 confirms findings in the existing literature—the rising shares of intangibles magnify effects of an adverse financial shock. Upon impact, the borrowing constraint of firms would be tightened. Given that physical capital is more pledgeable than intangible capital, firms tend to substitute the latter with the former, leading to substantial decline of intangible investment. Due to the complementarity of the two types of capital in production, the slow accumulation of intangible capital would reduce marginal efficiency of physical capital, further leading to larger decreases in physical investment and hence tangible goods.

To investigate business cycle implications of intangibles, I compare model-implied moments of key macroeconomic aggregates at alternative values of intangible shares. Table 6 reports relative volatility between the benchmark case—an intangible economy ( $\zeta = 0.15$ ) and a tangible economy with very low share of intangibles ( $\zeta = 0.01$ ). When the financial shock alone is switched on, Table 6 suggests amplification effects of

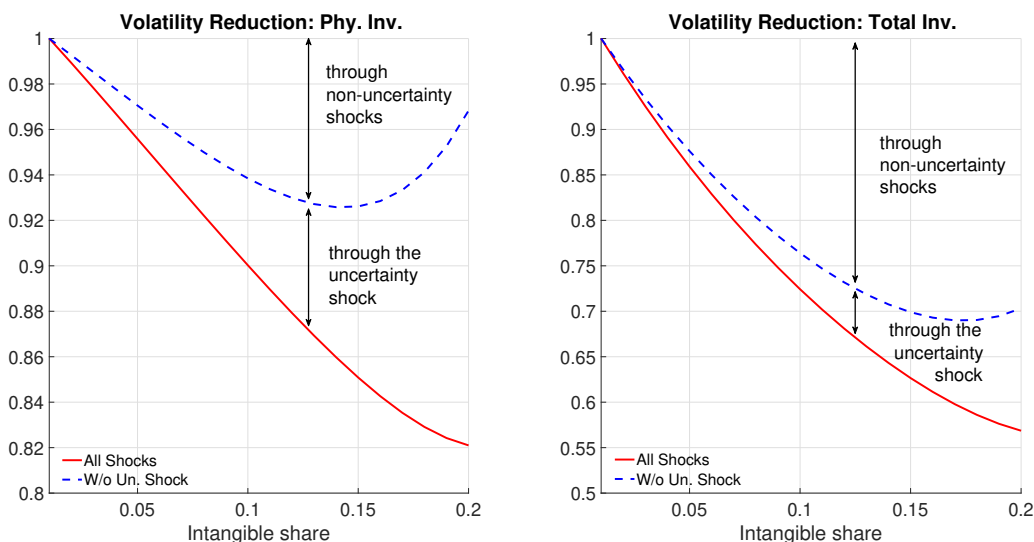
Table 6: Effects of Intangibles on Aggregate Volatility

Variable	Fin. Shock Only	All Level Shocks	All Shocks
$y$	2.42	1.06	0.98
$c$	1.35	1.01	0.99
$i$	1.78	0.93	0.85
$x$	1.73	1.06	0.96
$h$	1.93	1.13	0.96

*Note:* this table contains relative volatility of key macroeconomic aggregates between an intangible economy ( $\zeta = 0.15$ ) and a tangible economy ( $\zeta = 0.01$ ). Volatility is measured with model-implied standard deviation. An entry below (above) 1 implies that intangibles dampens (amplifies) volatility of a variable.

intangibles on output, investment, and hours worked. After including all level shocks, the relative volatility of tangible investment becomes less than one, indicating some dampening effects provided by intangibles. When we further include the uncertainty shock, the relative volatility of all variables is smaller than unity. The last finding indicates important interactions between intangibles and uncertainty—intangibles could play as cushions in an uncertainty-driven business cycle.

Figure 10: Volatility Reduction Effects of Intangibles on Investment



*Note:* this figure shows relative volatility of physical investment (left panel) and total investment  $i+x$  (right panel) conditional on different intangible ratios. Volatility is measured with model-implied standard deviation. Investment volatility is indexed as unity when  $\zeta=0.01$ .

Focusing on investment, I further explore the interaction between intangibles and uncertainty, and assess its importance in contributing to the volatility reduction process. Figure 10 displays volatility of physical investment and total investment (i.e., physical plus intangible investment) at different intangible shares relative to the tangible economy. To distinguish the contribution specifically through uncertainty, Figure 10 plots the relative volatility with and without the uncertainty shock; differences between the two cases



suggest how largely the volatility reduction process comes through the uncertainty shock. Figure 10 shows substantial fall of both investment volatility owing to the interaction between intangibles and the uncertainty shock. Moreover, this contribution increases with the rise of intangible shares, indicating that the interaction between intangibles and uncertainty is important to maintain the volatility reduction effect. Figure 10 also suggests that total investment volatility tends to decline faster than physical investment volatility. This is not a surprising result since the former includes the inertia component of investment–intangible investment.

Table 7: Effects of Intangibles on Steady State

	DSS			SSS		
	Tangible Economy	Intangible Economy	Intangible/Tangible	Tangible Economy	Intangible Economy	Intangible/Tangible
$y$	0.765	1.312	1.715	0.762	1.308	1.717
$gdp^a$	0.771	1.477	1.916	0.768	1.475	1.921
$i$	0.166	0.326	1.964	0.160	0.317	1.981
$x$	0.006	0.165	27.500	0.006	0.167	27.833
$c$	0.484	0.789	1.630	0.487	0.794	1.630

*Note:* this table compares steady-state value of key macroeconomic aggregates between the intangible economy ( $\zeta = 0.15$ ) and the tangible economy ( $\zeta = 0.01$ ). Columns Intangible/Tangible show relative value of a variable between the two cases in either deterministic steady state or stochastic steady state.

In addition to investigate how intangibles affect macroeconomic fluctuations, this section also studies joint implications of intangibles for both long-run equilibrium and short-run fluctuations. Table 7 reports and compares steady-state values of key macroeconomic aggregates in the tangible and intangible economies based on both deterministic steady state (DSS) and stochastic steady state (SSS). The latter includes not only effects of intangibles on DSS but also interactions between intangibles and stochastic shocks, hence joint effects of intangibles on long-run equilibrium and short-run fluctuations. Overall, Table 7 suggests that the presence of intangibles expands the size of the economy, leading to higher values of output, investment, and consumption. Two interesting observations are worthy of particular highlight. First, the presence of intangible not only boosts actual GDP ( $gdp^a$ ) but also the size of tangible sector (e.g.,  $y$ ), implying both direct effect ( $x \rightarrow gdp^a$ ) and indirect effect ( $x \rightarrow y \rightarrow gdp^a$ ) of intangibles on the economy. Second, effects of intangibles based on SSS are slightly larger than those based on DSS. For example, the effect of intangibles on actual GDP can be 0.5% larger based on SSS than DSS. This is due to the cushion effect on macroeconomic fluctuations as established in Table 6; a more resilient macroeconomic environment enables agents to smooth influences of stochastic disturbances and hence facilitates expansion of business. This finding further implies a connection between the size and volatility of the economy through intangibles.

## 7 Good v.s. Bad Uncertainty Shock

The majority of literature suggests recessionary effects of uncertainty. However, it is also possible for an uncertainty shock to deliver expansionary effects which is often interpreted as good uncertainty (Bloom 2014, Segal et al. 2015). Some types of uncertainty may bring promising investment opportunities, leading to potentially high expected return in the future. In such a case, firms are encouraged to expand, and hence uncertainty results in expansionary effects. One typical example used to explain this intuition can be found from the hi-tech boom in the 1990s.

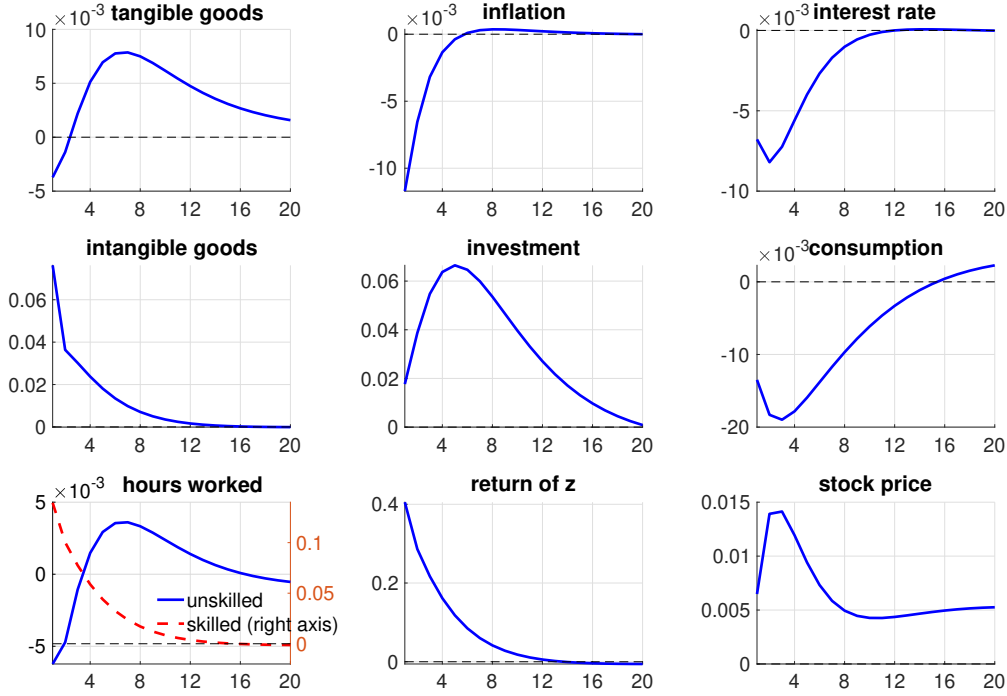
Motivated by implications from the literature and historical experiences, this section explores potential expansionary effects of uncertainty as an extended analysis. To this end, I incorporate an intangible-specific uncertainty shock and investigate its transmission. In particular, I allow the intangible productivity  $\chi$  in equation (9) to follow an AR(1) process:  $\chi_t = (1 - \rho_x)\chi + \rho_x\chi_{t-1} + \sigma_{t-1}^x\epsilon_t^x$ .  $\sigma_t^x$  is the intangible-specific uncertainty shock evolving as:  $\sigma_t^x = (1 - \rho_{ux})\sigma^x + \rho_{ux}\sigma_{t-1}^x + \sigma^x\epsilon_t^{ux}$  and  $\epsilon_t^{ux}$  follows an i.i.d  $N(0, 1)$ . Increased volatility in  $\chi_t$  suggests more dispersed productivity in producing intangible goods, which leads to uncertain outcomes in the intangible production.

The information about intangible productivity and related uncertainty shocks is limited in the literature, particularly for the latter. I set the persistence of intangible productivity shock  $\rho_x$  as 0.95 and that of the intangible uncertainty shocks  $\rho_{ux}$  as 0.75 based on values of  $\rho_a$  and  $\rho_u$ . In the online Appendix, a range of value for  $\rho_x$  and  $\rho_{ux}$  are used to conduct a sensitivity analysis. Following the approach used by Mitra (2019), the volatility of intangible productivity shock  $\sigma^x$  in the steady state is calibrated to match the volatility of intangible investment rate ( $x_t/z_t$ ).

Figure 11 displays impulse responses to the positive intangible uncertainty shock (rise of  $\sigma_t^x$ ). In contrast to the recessionary effects as shown in Section 5 and 6, the intangible-specific uncertainty shock leads to expansionary effects on investment, output, and stock prices. In the short run, the intangible uncertainty shock triggers the precautionary saving effect which lowers consumption and hence imposes downward pressure on tangible goods. However, the rise of intangible uncertainty also increase the return of intangible capital, implying potential business opportunities which could bear fruit in the future. The increased return triggers a growth-option effect which encourages intangible investment and stimulates skilled hours worked, resulting in an expansion in the intangible sector. Owing to the complementarity between the two types of capital, the accumulation of intangible capital also provides a spillover effect on the tangible sector, which further releases the downward pressure on the return of physical capital, gradually increasing physical investment and unskilled hours worked. Overall, the economy enters a expansion in the mid-to-long run.

In explaining the expansionary effect of the intangible uncertainty shock, the growth-option effect plays important role in driving the results. Essentially, growth options rely on time-to-build/develop features of a project (Bar-Ilan & Strange 1996) which is the case for intangible production. To further corroborate the role of growth options or long-term feature of intangibles in explaining the results, I compare the baseline results

Figure 11: Intangible Uncertainty Shock



*Note:* the size of the intangible uncertainty shock is set as five times large as the preference uncertainty shock. Variables are expressed as percentage deviations from the trend.

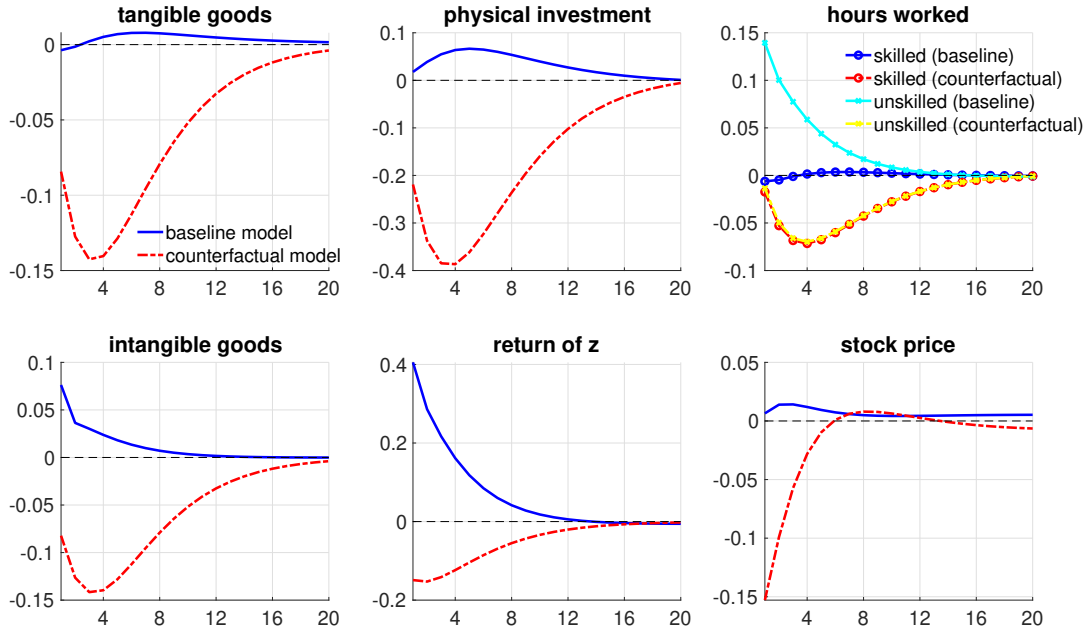
with a counterfactual case that intangibles does not have the long-term feature. In details, I consider that intangible goods contribute to tangible production contemporaneously. The tangible production function becomes

$$Y_t^m = A_t X_t^\zeta (u_t K_t^y)^\alpha (H_t^u)^{1-\alpha-\zeta} \quad (33)$$

Equation (33) also implies that output is a function of contemporaneous term of the intangible-specific uncertainty shock  $Y_t = f(\sigma_t^x, H_t^s, \dots)$ , and hence  $\sigma_t^x$  immediately affects output. This contrasts to the baseline model where  $\sigma_t^x$  primarily affects output in the future. Figure 12 compares responses of some key macroeconomic aggregates in the baseline model with the counterfactual model. In the latter case, the return of intangible capital declines, discouraging intangible investment. Hence, the growth-option effect is unlikely to occur. Since skill labor in the counterfactual model contributes to tangible production contemporaneously, responses of the two types of labor (the red-dash-dot line and the yellow-cross line) become almost the same. Overall, figure 12 shows that the expansionary effects disappear once the long-term feature of intangibles is removed.

If an uncertainty shock could potentially lead to expansionary effects, it is questionable why such effects

Figure 12: Intangible Uncertainty Shock, Compared to A Counterfactual Case



*Note:* this figure compares responses of some key macroeconomic aggregates in the baseline case with the counterfactual case that intangible production does not have the long-term feature. Variables are expressed as percentage deviations from the trend.

are hardly observed from empirical evidence. The analysis in this section provides two explanations. First, the expansionary effects could be quantitatively incomparable to the recessionary effects as shown in Section 5. Although the volatility of intangible productivity and the corresponding uncertainty shocks are calibrated at a fairly large size, the magnitude of responses is small as suggested by Figure 11. This is due to the two competing effects (i.e., precautionary saving and growth options) which tend to counteract each other. Thus, when considering multiple sources of uncertainty at the aggregate level, the expansionary effects would be dominated by the recessionary effects. Second, the expansionary effects tend to appear in the mid-to-long run. However, short-run responses of variables might dominate results under business cycle frequency. Both factors lead to recessionary effects of uncertainty, as found by existing empirical studies.

## 8 Conclusion

The rapid growth of intangible investment, which may exceed tangible investment in the recent two decades (Corrado & Hulten 2010), raise an important question—what implications do intangibles hold for business cycle fluctuations? Focusing on uncertainty shocks, this paper adds to the literature another set of macroeconomic consequences of rising intangibles. An essential finding is that intangibles dampen the transmission of the uncertainty shock, contrast to the amplification role in the financial shock. My results

imply that intangibles play as a cushion in the uncertainty-driven business cycles.

To study the macroeconomic consequences of intangibles, I empirically test effects of uncertainty on corporate investment, and further develop a two-sector DSGE model with productions of tangible and intangible goods. The empirical evidence suggests that intangible investment is less sensitive to uncertainty compared to tangible investment. Moreover, firms with more intangible capital tend to experience less pronounced effects of uncertainty on their investment decisions.

The quantitative analysis based on the DSGE model suggests that precautionary labor supply and capital reallocation effects are two important forces shaping the response of intangibles to the uncertainty shocks. Alongside the rise of the intangible share in the production, aggregate volatility would be dampened. Finally, this paper investigates effects of a intangible-specific uncertainty shock, i.e., uncertainty surrounding the productivity in the intangible sector. I show that intangible-specific uncertainty shock could trigger growth option effects, leading to expansion in the medium-to-long run. The last finding suggests that intangible-specific uncertainty can be a source of good uncertainty.

In conclusion, this paper contributes insights into the intricate interplay between intangibles and business cycle dynamics. The evidence presented not only highlights the stabilizing role of intangibles in uncertainty-driven business cycles but also sheds light on the potential positive impacts stemming from intangible-specific uncertainty. These findings extend our understanding of the nuanced relationship between intangibles and macroeconomic fluctuations, offering a comprehensive perspective on their multifaceted implications for economic stability.

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## Appendix A Data

The data used for VAR are from the FRED and Gilchrist & Zakrajšek (2012), including VIX index, real GDP, real fixed private investment, CPI inflation, effective federal funds rate, and excess bond premium. VIX index, real GDP, and real fixed private investment are expressed as logarithm.

Table 8 lists definitions of firm-level variables use in Section 2.1 and 2.2, where capitalized abbreviations refer to the item names in Compustat. In Section 2.1, the dependent variables—investment and components, are measured by log intangible investment rate  $I_{it}^{int}$ , log total investment rate  $I_{it}^{tan}$ , and log tangible investment rate  $I_{it}^{tot}$  as follows:

$$I_{it}^{int} = \log\left(\frac{XRD_{it} + 0.3 \times XSGA_{it}}{AT_{it}}\right)$$

$$I_{it}^{tan} = \log\left(\frac{CAPX_{it}}{AT_{it}}\right)$$

$$I_{it}^{tot} = \log\left(\frac{XRD_{it} + 0.3 \times XSGA_{it} + CAPX_{it}}{AT_{it}}\right)$$

Regarding control variables, I follow Döttling & Ratnovski (2023) to construct measures. In particular, firm size is measured by log of total assets which is equals to the sum of book assets and (off-balance sheet) intangible capital. The latter component is estimated based on the perpetual inventory method. Total q is an extension of Tobin'q which uses total assets in construction.

Table 8: Definition of Firm-level Variables

Variable	Definition
Intangible investment	XRD + 0.3 × XSGA
Tangible investment	CAPX
Total investment	Intangible investment + Tangible investment
Intangible capital	INTAN + Off-balance sheet intangible capital estimated by the author using perpetual inventory method
Tangible capital	PPENT
Total capital	Intangible capital + Tangible capital
Intangible ratio	Intangible capital / Total capital
Total assets	Book assets (AT) + Off-balance sheet intangible capital
Total q	(CSHO × PRCC + Total Assets - CE) / Total Assets
Cash	CHE/AT
Leverage	(DLTT + DLC) / AT
Cashflow	OIBDP / lagged AT
Size	log of total assets
Age	log of quarters since first observation in Compustat
Stock price	log of PRCC

The off-balance sheet intangible capital is estimated based on two components—technology and organization capital. To estimate the former component, I capitalize the stock of R&D spending based on

replacement costs using the perpetual inventory approach Peters & Taylor (2017) as follows

$$Z_{i,t}^{tech} = (1 - \delta_{rd})Z_{i,t-1}^{tech} + R\&D_{it}$$

Following Li & Hall (2020), BEA’s industry-specific R&D depreciation rates are used to measure  $\delta_{rd}$ . The initial technology stock  $G_{i,0}$  is estimated using first recorded R&D spending in Compustat and averaged R&D growth rate.

To estimate the organization capital, a fraction of SG&A spending is capitalized based on the perpetual inventory method.

$$Z_{i,t}^{org} = (1 - \delta_{sga})Z_{i,t-1}^{org} + SG\&A_{it}$$

Existing literature (Hulten & Hao 2008, Eisfeldt & Papanikolaou 2014, Zhang et al. 2014) suggests that 30% of SG&A spending represents investment in organization capital through advertising, spending on distribution systems, employee training, and payments to strategy consultants. Following Falato et al. (2022), organization capital depreciation rate  $\delta_{sga}$  is set as 20% on the annual basis. Finally, summing up  $Z_{i,t}^{tech}$  and  $Z_{i,t}^{org}$  yields the off-balance sheet intangible capital.

## Appendix B Hampel Identifier

To remove outliers, I first run a regression and then apply the Hampel Identifier (HI) as suggested by Wilcox (2011) to residuals stacked over time and individual firms ( $R_i$ ). In details, observations are treated as outliers for which the following is true:

$$HI = \frac{|R_i - M|}{MAD/z_{0.75}} > c$$

where  $M$  is the median of residuals  $R_1, R_2, \dots, R_n$ ,  $MAD$  is the median of the centred absolute values  $|R_i - M|$ ,  $z_{0.75}=0.6745$  is the 75th quantile of the standard normal distribution, and  $c$  is the critical value or cut-off.  $MAD/0.6745$  is a consistent estimator for standard deviation<sup>20</sup>. Following Wilcox (2011), the cut-off is set as 2.24. Observations with HI greater than the cut-off will be treated as outliers and hence removed.

## Appendix C Additional Empirical Evidence

Appendix C presents extended empirical results for robustness check. In the first robustness check, I estimate the same VAR models as in the mainly analysis but order uncertainty indices as the last variable to extract its exogenous components in the stage one. The second-stage results for investment regressions based on the alternative adjusted uncertainty index is reported in Table 9. Second, I run regressions using

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<sup>20</sup>MAD is a more robust statistic compared with standard deviation and hence is more resilient to outliers.

Table 9: Investment Regression Results–Alternative Order of Uncertainty Indices

	Intangible Inv. Rate		Tangible Inv. Rate		Total Inv. Rate						
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	
$u$	-0.0390*** (0.006)	-0.1736*** (0.016)	-0.0725*** (0.013)	-0.3085*** (0.035)	-0.0532*** (0.008)	-0.2234*** (0.019)	-0.2628*** (0.036)	-0.8934*** (0.090)			
$u \times k^{int}$							0.2415*** (0.044)	0.7805*** (0.111)	0.2451*** (0.043)	0.8247*** (0.110)	
Observations	111568	111547	111515	111496	112165	112173	103820	103820	103745	103736	
Adj. $R^2$	0.214	0.217	0.544	0.545	0.370	0.372	0.391	0.396	0.413	0.415	
Constant	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	
Quarter FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	
Time FE	No	No	No	No	No	No	No	No	Yes	Yes	

*Note:* uncertainty  $u$  is measured as the adjusted VIX in columns [1], [3], [5], [7], [9], and the adjusted JLN index in other columns. The VIX index or the JLN index is order as the last variable in the first-step estimation. For regressions in columns [7]–[10],  $k^{int}$  is added as an additional control variable. Robust standard errors are shown in parentheses. \*\*\*, \*\*, and \* represent significance level at 1%, 5%, and 10%, respectively.

Table 10: Investment Regression Results–Original VIX Index

	Intangible Inv. Rate		Tangible Inv. Rate		Total Inv. Rate					
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
$u$	-0.0452*** (0.005)		-0.0602*** (0.010)		-0.0501*** (0.006)		-0.1937*** (0.027)			
$L.u$		-0.0338*** (0.005)		-0.1081*** (0.010)		-0.0650*** (0.006)		-0.3317*** (0.027)		
$u \times k^{int}$							0.1651*** (0.033)		0.1689*** (0.032)	
$L.u \times L.k^{int}$								0.3312*** (0.034)		0.3320*** (0.034)
Observations	111565	105766	111514	105229	112156	105522	103768	97908	103738	98004
Adj. $R^2$	0.216	0.258	0.544	0.569	0.371	0.398	0.392	0.413	0.413	0.432
Constant	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	No	No	No	No	No	No	No	No	Yes	Yes

*Note:* uncertainty  $u$  is measured as the original VIX index. For regressions in columns [7] and [9],  $k^{int}$  is added as an additional control variable. For regressions in columns [8] and [10], lagged  $k^{int}$  is added as an additional control variable. Robust standard errors are shown in parentheses. \*\*\*, \*\*, and \* represent significance level at 1%, 5%, and 10%, respectively.

the original VIX index in the regression analysis. Considering that investment may have slow movement, I further use lagged independent variables in the investment regressions. Such a treatment is also useful to address potential reverse causality issues. Results based on the original VIX index is reported in Table 10. Similar to the treatment of original VIX index, the original JLN index is used in the analysis, and the investment regression results are presented in Table 11. Results based on stock price regressions are presented in Table 12. Overall, I find that the negative effect of uncertainty and the mitigation effects of intangibles are robust to the alternative identification approaches.

Table 11: Investment Regression Results–Original JLN Index

	Intangible Inv. Rate		Tangible Inv. Rate		Total Inv. Rate					
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
u	-0.1504*** (0.011)		-0.1874*** (0.024)		-0.1543*** (0.013)		-0.5441*** (0.061)			
L.u		-0.1257*** (0.011)		-0.2857*** (0.024)		-0.1904*** (0.013)		-0.7133*** (0.062)		
u $\times$ $k^{int}$							0.4540*** (0.076)		0.4730*** (0.075)	
L.u $\times$ $L.k^{int}$								0.6409*** (0.076)		0.6513*** (0.077)
Observations	110253	105790	110264	105225	110842	105575	102462	98027	102410	97973
Adj. $R^2$	0.219	0.261	0.543	0.571	0.373	0.401	0.396	0.417	0.414	0.432
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	No	No	No	No	No	No	No	No	Yes	Yes

*Note:* uncertainty  $u$  is measured as one-period ahead macroeconomic uncertainty index from (Jurado et al. 2015). For regressions in columns [7] and [9],  $k^{int}$  is added as an additional control variable. For regressions in columns [8] and [10], lagged  $k^{int}$  is added as an additional control variable. Robust standard errors are shown in parentheses. \*\*\*, \*\*, and \* represent significance level at 1%, 5%, and 10%, respectively.

Table 12: Stock Price Regression Results

	VIX			JLN		
	[1]	[2]	[3]	[4]	[5]	[6]
u	-0.1215*** (0.003)	-0.1536*** (0.014)		-0.0273*** (0.007)	-0.0417 (0.030)	
u $\times$ $k^{int}$		0.0549*** (0.017)	0.0523*** (0.014)		0.0287 (0.037)	0.0839*** (0.030)
Observations	103770	97352	96751	103489	97341	96758
Adj. $R^2$	0.891	0.891	0.912	0.891	0.891	0.912
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	No	No	Yes	No	No	Yes

*Note:* the dependent variables is a firm's stock price  $s\_price$ . For regressions in columns [2], [3], [5], and [6],  $k^{int}$  is added as an additional control variable. Others the same as above.

## Appendix D Model Details

### Appendix DI Households

The representative household maximize the utility subject to the budget constraint, yielding following first order conditions.

$$1 = \mathbb{E}_t M_{t,t+1} \frac{R_t}{\pi_{t+1}}$$

$$\psi^u(h_t^u)^\eta = \frac{W_t^u}{C_t - h\bar{C}_{t-1}}$$

$$\psi^s(h_t^s)^n = \frac{W_t^s}{C_t - h\bar{C}_{t-1}}$$

$$\frac{P_t^E}{P_t} = \mathbb{E}_t M_{t,t+1} \frac{D_{t+1}^E + P_{t+1}^E}{P_{t+1}}$$

Following Basu & Bundick (2017), the stochastic discount factor  $M_{t,t+1}$  is defined as

$$M_{t,t+1} = \frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t} = \beta \frac{\varepsilon_{t+1}^d}{\varepsilon_t^d} \left( \frac{U_{t+1}}{U_t} \right)^{(1-\sigma)/\theta_v} \left( \frac{C_t - h\bar{C}_{t-1}}{C_{t+1} - h\bar{C}_t} \right) \left( \frac{V_{t+1}^{1-\sigma}}{\mathbb{E}_t V_{t+1}^{1-\sigma}} \right)^{\theta_v/(1-\sigma)}$$

## Appendix DII Intermediate Goods Producers

The firm  $j$  maximizes the expected present discounted value of the current and future dividends  $D_{jt}^E$ :

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} M_{t+s,t+s+1} D_{j,t+s}^E$$

subject to

$$W_t^u H_{jt}^u + W_t^s H_{jt}^s + P_t I_{jt} + P_t \Phi(D_{jt}^E) + R_{t-1}^b B_{j,t-1} = P_{jt}^m Y_{jt} + B_{jt}$$

$$B_{jt} \leq \xi_t (P_t K_{jt} + \nu P_t Z_{jt})$$

$$Y_{jt}^m = Y_{jt}^m \left( \frac{P_{jt}^m}{P_t^m} \right)^{-\theta_m}$$

$$K_{j,t+1} = [1 - \delta(u_{jt})] K_{jt} + [1 - \frac{\phi_k}{2} \left( \frac{I_{jt}}{(1+g^y)I_{j,t-1}} - 1 \right)^2] I_{jt}$$

$$Z_{j,t+1} = (1 - \delta_z) Z_{jt} + X_{jt}$$

$$Y_{jt}^m = A_t Z_{jt}^\zeta (u_{jt} K_{jt}^y)^\alpha (H_{jt}^u)^{1-\alpha-\zeta}$$

$$X_{jt} = \chi Z_{jt}^\zeta (u_{jt} K_{jt}^z)^\alpha (H_{jt}^s)^{1-\alpha-\zeta}$$

Denoting  $\mu_{jt}$ ,  $\mu_{jt}^y$ ,  $\mu_{jt}^k$ ,  $\mu_{jt}^z$ ,  $\mu_{jt}^b$  as Lagrange multipliers associated with the budget constraint, demand scheme, physical capital accumulation, intangible capital accumulation, and borrowing constraint respectively, the maximization of dividends yields the following first order conditions:

$$1 = \mu_{jt} P_t [1 + \kappa (D_t^E / (1+g^y)^t - d^E)]$$

$$P_{jt}^m = \frac{\theta_m}{\theta_m - 1} \frac{\mu_{jt}^y P_t}{\mu_{jt} P_t}$$

$$W_t^u \mu_{jt} = (1 - \alpha - \zeta) \mu_{jt}^y \frac{Y_{jt}^m}{H_{jt}^u}$$

$$W_t^s \mu_{jt} = (1 - \alpha - \zeta) \mu_{jt}^z \frac{X_{jt}}{H_{jt}^s}$$

$$\begin{aligned}
\mu_{jt}P_t &= \mu_{jt}^k \left[ 1 - \frac{\phi_k}{2} \left( \frac{I_{jt}}{(1+g^y)I_{j,t-1}} - 1 \right)^2 + \phi_k \left( \frac{I_{jt}}{(1+g^y)I_{j,t-1}} - 1 \right) \left( \frac{I_{jt}}{(1+g^y)I_{j,t-1}} \right) \right. \\
&\quad \left. - \mathbb{E}_t M_{t,t+1} \mu_{j,t+1}^k \left[ \phi_k \left( \frac{I_{j,t+1}}{(1+g^y)I_{j,t}} - 1 \right) \left( \frac{I_{j,t+1}}{(1+g^y)I_{j,t}} \right) \right]^2 \right] \\
\mu_{jt}^k [\delta_1 + \delta_2(u_{jt} - 1)] K_{jt} &= \alpha (\mu_{jt}^y \frac{Y_{jt}^m}{u_{jt}} + \mu_{jt}^z \frac{X_{jt}}{u_{jt}}) \\
\mu_{jt}P_t - \mathbb{E}_t M_{t,t+1} \mu_{j,t+1} P_{t+1} \frac{R_t^b}{\pi_{t+1}} &= \mu_{jt}^b \\
\mu_{jt}^k &= \mathbb{E}_t M_{t,t+1} \left[ \alpha \mu_{jt}^y \frac{Y_{jt}^m}{K_{jt}^y} + \mu_{j,t+1}^k (1 - \delta_{t+1}^k) \right] + \mu_{jt}^b \xi_t \\
\mu_{jt}^k &= \mathbb{E}_t M_{t,t+1} \left[ \alpha \mu_{jt}^z \frac{X_{jt}}{K_{jt}^z} + \mu_{j,t+1}^k (1 - \delta_{t+1}^k) \right] + \mu_{jt}^b \xi_t \\
\mu_{jt}^z &= \mathbb{E}_t M_{t,t+1} \left[ \zeta \mu_{jt}^z \frac{X_{jt}}{Z_{jt}} + \zeta \mu_{jt}^y \frac{Y_{jt}^m}{Z_{jt}} + \mu_{j,t+1}^z (1 - \delta_z) \right] + \nu \mu_{jt}^b \xi_t
\end{aligned}$$

Since all intermediate firms make the same choice, we can drop the  $j$  index.

### Innovation Model I

In the first innovation model, intangible (or technology) is created using tangible goods as the input. The intangible production function is given by

$$X_{jt} = \chi \Phi_t N_{jt}$$

$\Phi_t$  is an aggregate efficiency coefficient of technology creation (Comin & Gertler 2006).

$$\Phi_t = \chi \left( \frac{N_t}{Z_{t-1}} \right)^{\gamma_z - 1}, \quad 0 < \gamma_z < 1$$

where  $\gamma_z$  is the elasticity of technology with respect to the input  $N_t$ .

The firm budget constraint becomes

$$W_t^u H_{jt}^u + P_t N_{jt} + P_t I_{jt} + P_t \Phi (D_{jt}^E) + R_{t-1}^b B_{j,t-1} = P_{jt}^m Y_{jt} + B_{jt}$$

In equilibrium, we have the following equilibrium condition for  $N_t$ .

$$\mu_t P_t = \chi \mu_t^z \Phi_t$$

The input-sided measured of intangible investment is given by

$$I_t^z = P_t N_t$$

### Innovation Model II

In the second innovation model, intangible (or technology) is created using both tangible goods and skilled labor (Queralto 2020). The intangible production function is given by

$$X_{jt} = \chi N_{jt}^{1-\gamma_x} (Z_{t-1} H_{jt}^s)^{\gamma_x}, \quad 0 < \gamma_x < 1$$

The firm budget constraint becomes

$$W_t^u H_{jt}^u + W_t^s H_{jt}^s + P_t N_{jt} + P_t I_{jt} + P_t \Phi(D_{jt}^E) + R_{t-1}^b B_{j,t-1} = P_{jt}^m Y_{jt} + B_{jt}$$

In equilibrium, we have the following equilibrium condition for  $N_t$  and  $H_t^s$ .

$$\mu_t P_t = (1 - \gamma_x) \mu_t^z \frac{X_t}{N_t}$$

$$W_t^s \mu_t = \gamma_x \mu_t^z \frac{X_t}{H_t^s}$$

The input-sided measured of intangible investment is given by

$$I_t^z = P_t N_t + W_t^s H_t^s$$

In both Innovation Models, the resource constraint becomes

$$Y_t = C_t + I_t + N_t + G_t + \frac{\phi_p}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 Y_t + \frac{\kappa}{2} \left[ \frac{D_t^E}{(1+g^y)^t} - d^E \right]^2$$

## Appendix DIII Stationary Equilibrium Conditions

The model can be detrended with deterministic growth trend  $(1+g^y)^t$ . Lower case expressions are used to represent detrended real variables. Let  $y_t = \frac{Y_t}{(1+g^y)^t}$ ,  $x_t = \frac{X_t}{(1+g^y)^t}$ ,  $c_t = \frac{C_t}{(1+g^y)^t}$ ,  $i_t = \frac{I_t}{(1+g^y)^t}$ ,  $g_t = \frac{G_t}{(1+g^y)^t}$ ,  $z_t = \frac{Z_t}{(1+g^y)^t}$ ,  $k_t = \frac{K_t}{(1+g^y)^t}$ ,  $k_t^y = \frac{K_t^y}{(1+g^y)^t}$ ,  $k_t^z = \frac{K_t^z}{(1+g^y)^t}$ ,  $b_t = \frac{B_t}{P_t(1+g^y)^t}$ ,  $w_t^u = \frac{W_t^u}{P_t(1+g^y)^t}$ ,  $w_t^s = \frac{W_t^s}{P_t(1+g^y)^t}$ ,  $p_t^m = \frac{P_t^m}{P_t}$ ,  $mc_t^f = \frac{MC_t^f}{P_t}$ ,  $d_t^E = \frac{D_t^E}{P_t(1+g^y)^t}$ ,  $p_t^E = \frac{P_t^E}{P_t(1+g^y)^t}$ ,  $v_t = \frac{V_t}{(1+g^y)^t}$ ,  $util_t = \frac{U_t}{(1+g^y)^t}$ ,  $gdp_t^a = \frac{GDP_t^a}{(1+g^y)^t}$ ,  $gdp_t = \frac{GDP_t}{(1+g^y)^t}$ ,  $i_t^z = \frac{I_t^z}{(1+g^y)^t}$ ,  $z_t^m = \frac{Z_t^m}{(1+g^y)^t}$ ,  $\mu_t' = \mu_t P_t$ ,

$$1 = \mathbb{E}_t M_{t,t+1} \frac{R_t}{\pi_{t+1}} \quad (D1)$$

$$\psi^u(h_t^u)^\eta = \frac{w_t^u}{c_t - h/(1+g^y)c_{t-1}} \quad (D2)$$

$$\psi^s(h_t^s)^\eta = \frac{w_t^s}{c_t - h/(1+g^y)c_{t-1}} \quad (D3)$$

$$R_t^E = (1 + g^y) \frac{p_t^E + d_t^E}{p_{t-1}^E} \quad (D4)$$

$$util_t = \log(c_t - h/(1 + g^y)c_{t-1}) e^{-\frac{\psi^u (H_t^u)^{1+\eta} + \psi^s (H_t^s)^{1+\eta}}{1+\eta}} \quad (D5)$$

$$v_t = [\varepsilon_t^d u_t^{(1-\sigma)/\theta_v} + \beta(1 + g^y)^{(1-1/\psi)} (E_t v_{t+1}^{1-\sigma})^{1/\theta_v}]^{\theta_v/(1-\sigma)} \quad (D6)$$

$$M_{t,t+1} = \frac{\beta}{(1 + g^y)^\psi} \frac{\varepsilon_{t+1}^d}{\varepsilon_t^d} \left( \frac{util_{t+1}}{util_t} \right)^{(1-\sigma)/\theta_v} \left( \frac{c_t - h/(1 + g^y)c_{t-1}}{c_{t+1} - h/(1 + g^y)c_t} \right) \left( \frac{v_{t+1}^{1-\sigma}}{\mathbb{E}_t v_{t+1}^{1-\sigma}} \right)^{\theta_v/(1-\sigma)} \quad (D7)$$

$$y_t = A_t z_t^\zeta (u_t k_t^y)^\alpha (H_t^u)^{1-\alpha-\zeta} \quad (D8)$$

$$x_t = \chi z_t^\zeta (u_t k_t^z)^\alpha (H_t^s)^{1-\alpha-\zeta} \quad (D9)$$

$$(1 + g^y)k_{t+1} = [1 - \delta(u_t)]k_t + [1 - \frac{\phi_k}{2}(\frac{i_t}{i_{t-1}} - 1)^2]i_t \quad (D10)$$

$$(1 + g^y)z_{t+1} = (1 - \delta_z)z_t + x_t \quad (D11)$$

$$\delta(u_t) = \delta_k + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2 \quad (D12)$$

$$b_t \leq \xi_t(k_t + \nu z_t) \quad (D13)$$

$$1 = \mu_t' [1 + \kappa(d_t^E - d^E)] \quad (D14)$$

$$p_t^m = \frac{\theta_m}{\theta_m - 1} \frac{\mu_t^y}{\mu_t'} \quad (D15)$$

$$w_t^u \mu_t' = (1 - \alpha - \zeta) \mu_t^y \frac{y_t}{H_t^u} \quad (D16)$$

$$w_t^s \mu_t' = (1 - \alpha - \zeta) \mu_t^z \frac{x_t}{H_t^s} \quad (D17)$$

$$\begin{aligned} \mu_t' &= \mu_{jt}^k [1 - \frac{\phi_k}{2}(\frac{i_t}{i_{t-1}} - 1)^2 + \phi_k(\frac{i_t}{i_{t-1}} - 1)(\frac{i_t}{i_{t-1}}) \\ &\quad - \mathbb{E}_t M_{t,t+1} \mu_{t+1}^k [\phi_k(\frac{i_{t+1}}{i_t} - 1)(\frac{i_{t+1}}{i_t})^2] \end{aligned} \quad (D18)$$

$$\mu_t^k [\delta_1 + \delta_2(u_t - 1)] k_t = \alpha (\mu_t^y \frac{y_t}{u_t} + \mu_t^z \frac{x_t}{u_t}) \quad (D19)$$

$$\mu_t' - \mathbb{E}_t M_{t,t+1} \mu_{t+1}' \frac{R_t^b}{\pi_{t+1}} = \mu_t^b \quad (D20)$$

$$\mu_t^k = \mathbb{E}_t M_{t,t+1} [\alpha \mu_{jt}^y \frac{y_t}{k_t^y} + \mu_{t+1}^k (1 - \delta_{t+1}^k)] + \mu_t^b \xi_t \quad (D21)$$

$$\mu_t^k = \mathbb{E}_t M_{t,t+1} [\alpha \mu_t^z \frac{x_t}{k_t^z} + \mu_{t+1}^k (1 - \delta_{t+1}^k)] + \mu_t^b \xi_t \quad (D22)$$

$$\mu_t^z = \mathbb{E}_t M_{t,t+1} [\zeta \mu_t^z \frac{x_t}{z_t} + \zeta \mu_t^y \frac{y_t}{z_t} + \mu_{t+1}^z (1 - \delta_z)] + \nu \mu_t^b \xi_t \quad (D23)$$

$$p_t^m = \frac{\theta_f - 1}{\theta_f} + \frac{\phi_p}{\theta_f} \left[ \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} - \mathbb{E}_t M_{t,t+1} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} \frac{(1 + g^y) y_{t+1}}{y_t} \right] \quad (D24)$$



$$y_t = c_t + i_t + g_t + \frac{\phi_p}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 y_t + \frac{\kappa}{2} (d_t^E - d^E)^2 \quad (\text{D25})$$

$$k_t = k_t^y + k_t^z \quad (\text{D26})$$

$$H_t = H_t^u + H_t^s \quad (\text{D27})$$

$$R_t = R_{t-1}^{\rho_r} \left[ R \left( \frac{\pi_t}{\pi} \right)^{\rho_\pi} \left( \frac{y_t}{y_{t-1}} \right)^{\rho_y} \right]^{1-\rho_r} \quad (\text{D28})$$

$$V_t^M = 100 \sqrt{4 \text{VAR}(R_{t+1}^E)} \quad (\text{D29})$$

$$gdp_t = c_t + i_t + g_t \quad (\text{D30})$$

$$gdp_t^a = c_t + i_t + x_t + g_t \quad (\text{D31})$$

$$i_t^z = r_t^{k,z} k_t^z + w_t^s H_t^s \quad (\text{D32})$$

$$(1 + g^y) z_{t+1}^m = (1 - \delta_z) z_t^m + i_t^z \quad (\text{D33})$$

where  $\mu_{jt}$ ,  $\mu_{jt}^y$ ,  $\mu_{jt}^k$ ,  $\mu_{jt}^z$ ,  $\mu_{jt}^b$  as Lagrange multipliers associated with the budget constraint, demand scheme, physical capital accumulation, intangible capital accumulation, and borrowing constraint respectively.

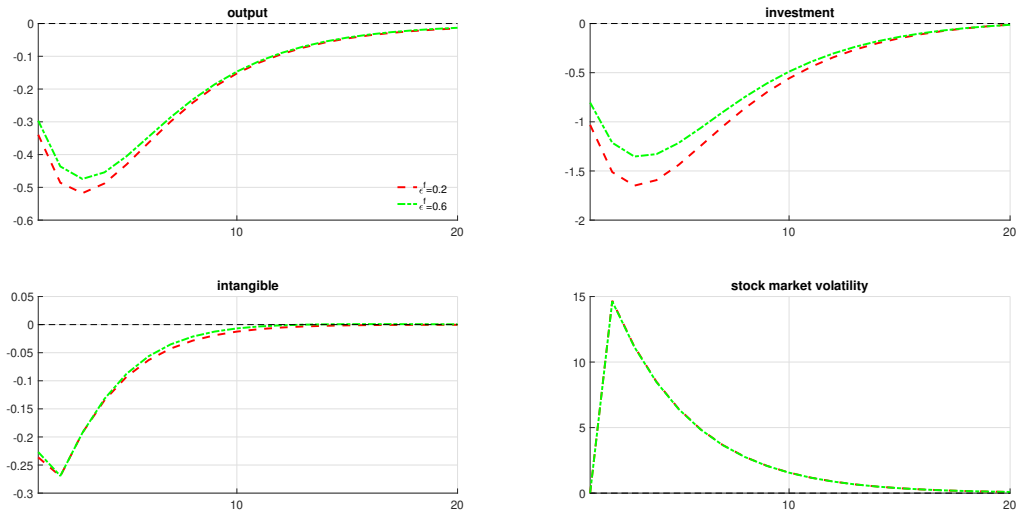
## Appendix E Additional Quantitative Results

Figure 13 displays impulse responses of the uncertainty shock based on alternative values of the SS credit constraint  $\epsilon_f$ , including a case with tighter credit constraint and the other with looser credit constraint. Comparing the two cases, tangible goods and physical investment tend to be more responsive if there is a lower value in  $\epsilon_f$ , which also indicates a relatively tight credit constraint. This result implies that credit conditions could be important in affecting the transmission of the uncertainty shock particularly for the tangible sector. Conversely, the IRFs of intangible goods do not show pronounced differences between the two cases, implying that the intangible sector is relatively insensitive to the tightness of the credit constraint.

Figure 14 plots impulse responses of the uncertainty shock based on alternative values of the equity adjustment cost  $\kappa$ , including a case with higher adjustment cost and the other with lower adjustment cost. Comparing the two cases, both the two types of investment show significant differences in the response to the uncertainty shock. Results based Figures refirf-u-epsilon and 14 together deliver implications consistent with literature (Brown et al. 2009, Bianchi et al. 2019)—equity is relatively more important source of finance for intangibles while credits are relatively more important for financing physical investment. Hence, the two types of investment exhibit different sensitivity two the frictions in credit and equity markets.

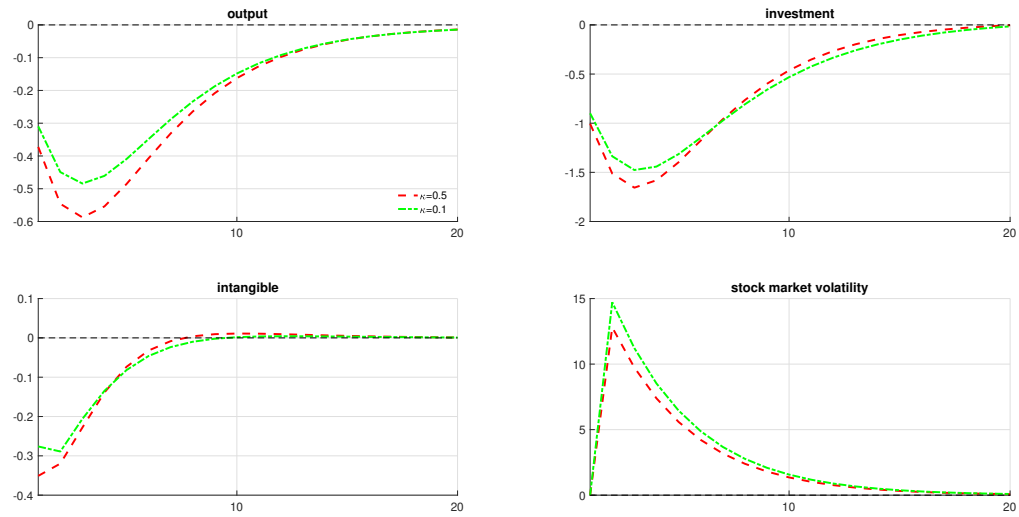
To investigate the sensitivity of impulse responses to the intangible-specific uncertainty shock, I consider different values for persistence of the intangible productivity shock and that of the intangible-specific uncertainty shock. Overall, Figures 15 and 16 confirm the key finding in the main analysis—the intangible-specific

Figure 13: Uncertainty Shock: Alternative Credit Constraints



*Note:* variables are expressed as percentage deviations from the trend.

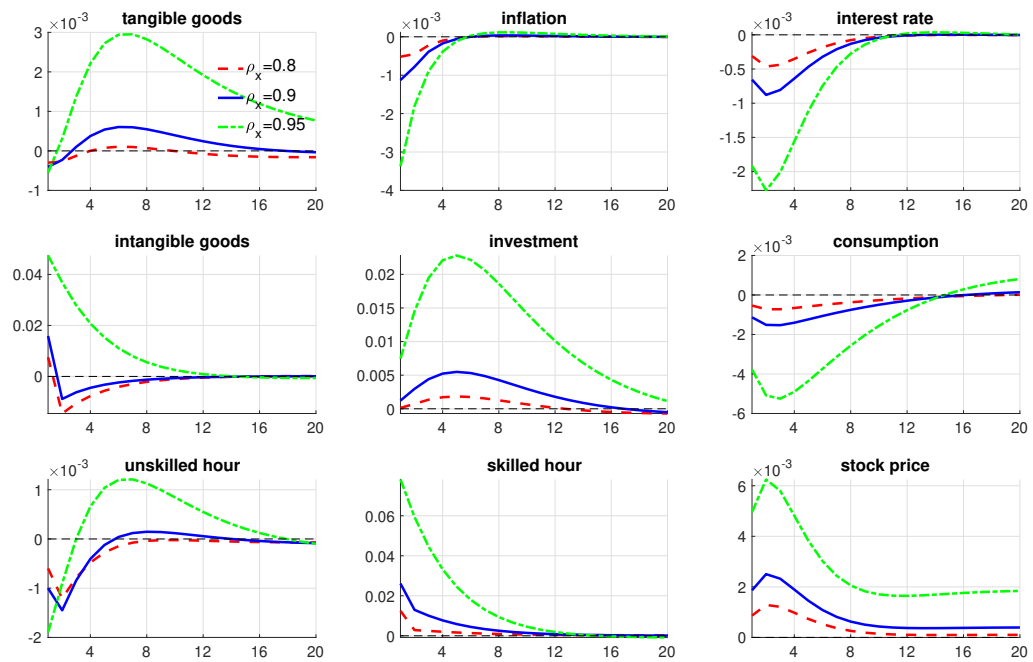
Figure 14: Uncertainty Shock: Alternative Equity Adjustment Costs



*Note:* variables are expressed as percentage deviations from the trend.

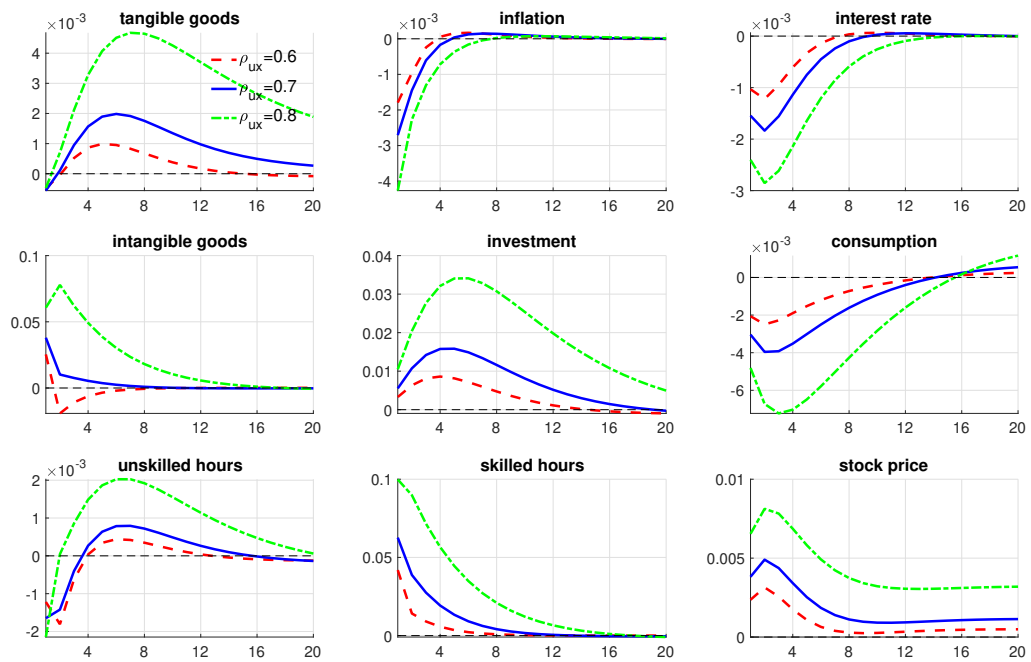
uncertainty shock leads to an expansion in the mid-to-long run, and such effects are more pronounced on the intangible sector than on the tangible sector. When the two persistence parameters are relatively high, the expansionary effects tend to be more significant, though the magnitude is still incomparable with the recessionary effects from the demand-sided uncertainty shock.

Figure 15: Intangible Uncertainty Shock: Alternative Persistence for Intangible Productivity Shock



*Note:* variables are expressed as percentage deviations from the trend.

Figure 16: Intangible Uncertainty Shock: Alternative Persistence for Intangible uncertainty Shock



Note: variables are expressed as percentage deviations from the trend.