How do Platforms Appeal to Buyers?*

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Abstract

In this paper, we consider a model of platform competition to examine the mechanisms through which asymmetric platforms attract different agents. Specifically, we analyze how platforms strategically choose different attributes to appeal to the buyers. We consider a two-stage game where heterogeneous platforms simultaneously choose features on the buyers' side in the first stage and membership fees in the second stage. Our results show that the equilibrium values of attributes depend significantly on the relative strengths of cross-network effects along with the degree of heterogeneity between platforms. Buyers' decisions to join a platform therefore are influenced not only by the membership fees and cross-network effects but also by the range of functionalities offered by the platform. Furthermore, even though such attributes are offered solely on the buyers' side in our model, sellers' participation is also significantly affected by them via their interactions with the membership fees and cross-network effects.

1 Introduction

In recent times, there has been a significant surge in the volume of electronic commerce attributable to the widespread availability of the Internet. According to Euromonitor International's 2018 report, the proportion of retail sales conducted online accounted for 13.7% and 17% in the United States and the United Kingdom, respectively, while globally, it represented 11.5% of all retail sales. These figures translate into substantial revenue, with online retail sales reaching over \$400 billion, \$86 billion, and \$1.7 trillion for the USA, UK, and worldwide, respectively.[1]

E-commerce typically involves buying and selling goods or services through online platforms, which is a business model connecting buyers and sellers, enabling them to engage in value-

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[1]Data is taken from Exhibit 7 of Amazon.com, 2019 Case 716-402.

creating exchanges. It is common that a dominant platform is present in this type of market, such as *Amazon* in the online retailing sector[2], *Airbnb* in lodging services[3], and *Uber* in the ride-hailing industry[4], among others. The underlying factors that contribute to these platforms' successful attraction and retention of agents have generated significant scholarly and practical interest. One potential explanation for their success is the platform's ability to serve as an intermediary between agents and actively shape the business model. It is this active involvement that may give rise to heterogeneity among platforms, and may, in turn, affect agents' incentives and valuations regarding which platform to join.

In this study, we present a framework for analysing how platforms appeal to agents, specifically on buyers' side. Our argument is that buyers' decisions to join a platform are not only based on membership fees and cross-network effects but also on other attributes platforms offer.[5] The combination of these three elements determines which platform buyers find most appealing. Buyers are more inclined to join a platform that has built a favourable reputation and brand image over time by offering a diverse range of features. As the quality of platform's features increases, buyers' perception of the platform's benefits improves, resulting in a stronger reputation and brand image, thereby increasing the likelihood of buyers joining the platform.

A specific example is *Amazon*, which not only works as an intermediary between buyers and sellers but also has an active function adopting a customer-centric approach to generate attributes that create value. For buyers, the platform's benefits proposition transcends beyond product pricing. It extends to the ability to appeal to and initiate a loyal customer base, enhancing their browsing experience through the provision of flexible delivery options, an extensive product assortment, swift checkout processes, and a lenient refund and return policy. On the seller side, having their products affiliated with Amazon's brand name enhances their credibility with customers and leverages the platform's Prime audience. Wells et al. (2019) observed that the majority of attributes developed by *Amazon* are primarily buyer-oriented. *Amazon* strives to attract buyers to its site by developing various attributes to meet their needs.

This paper makes a two-fold contribution to the existing literature on two-sided markets. Firstly, we introduce the platform's features as a form of vertical product differentiation on the buyers' side, shedding light on the importance of quality attributes in shaping market structure. Secondly, we analyse the intricate interactions between these quality attributes and cross-group network effects to gain insights into the resulting market configurations. By exploring these dimensions, our study expands the understanding of two-sided markets and offers valuable insights for market participants and policymakers alike.

Vertical differentiation refers to the differentiation of products or services offered by platforms based on their perceived quality, features, or attributes that cater to the distinct needs of both sides of the market. Platforms offer different levels of quality to enhance their features, functionality, user experience, or service level to attract and retain users on both sides

[2] Amazon.com, 2019 Case 716-402.

[3] World's Leading Online Travel Accommodation Marketplace 2020, accessed August 2021

[4] Global Top 100 Brands 2019

[5] For simplification purposes attributes, features and characteristics are used interchangeably throughout the chapter.

of the market. Rather than attempting to capture all possible features a platform may have, we integrate them into a single variable representing buyers' perception of the quality of the platform.

Our model builds on the framework of Armstrong (2006), where equilibrium membership fees depend on cross-group network effects, and the literature on vertical differentiation, including Mussa and Rosen (1978); Gabszewicz and Thisse (1979); Shaked and Sutton (1982, 1983), which identify consumer income as a source of differentiation. We extend Armstrong (2006) model by introducing the level of features offered on the buyers' side as a strategic variable on the vertical dimension. This allows for the existence of asymmetric platforms in equilibrium, as shown by Gabszewicz and Wauthy (2014).

The provision of attributes by platforms creates a competitive advantage in attracting agents in a two-sided market. This competitive advantage can be understood as heterogeneity within a vertically differentiated product space, where agents prefer platforms offering more attributes compared to those offering fewer attributes. The concept of vertical product differentiation space was first explored by scholars such as Mussa and Rosen (1978); Gabszewicz and Thisse (1979); Shaked and Sutton (1982, 1983). Mussa and Rosen (1978) investigated a monopoly pricing model for quality differentiated goods, and found that a monopolist cannot price discriminate in the usual way, but rather assigns a price-quality pair to customers to partially discriminate against them, thereby reducing the quality sold to customers compared to a competitive market. Gabszewicz and Thisse (1979) analysed a non-cooperative price equilibrium between firms, where consumers have different willingness to pay for quality improvements, and found that with less income disparity, the firm selling the lowest quality product will exit the market. Moreover, when consumers' tastes are less differentiated, Cournot's equilibrium price is near zero. Shaked and Sutton (1982, 1983) studied vertical differentiation in a competitive market and found that firms differentiate themselves by choosing distinct qualities to lower price competition and earn positive profits.

Subsequently, the seminal works of Economides (1989); Neven and Thisse (1990) were the first to jointly examine both horizontal and vertical product differentiation spaces. Horizontal differentiation pertains to the range of products offered, while vertical differentiation refers to the quality of the products sold in the market. Both studies yield comparable results, showing that firms maximise one dimension (variety) while minimising the other characteristic (quality) to gain a larger market share and increase profits. Building on these findings, Irmen and Thisse (1998) extended the previous models to include multiple characteristics and report similar results, indicating that firms choose to maximise differentiation in the dominant characteristic and minimise the remaining attributes to reduce price competition.

These models have undergone extensions to encompass a diverse range of sectors. Degryse (1996) explored banking services, Baake and Boom (2001) examined markets with network externalities. Inderst and Irmen (2005) focused on space and time as strategic variables in horizontal product differentiation, specifically in the retail markets, and Hansen and Nielsen (2011) investigated price as a proxy for quality in the trade between two countries. Garella and Lambertini (2014) identifies situations in which firms select maximum differentiation in both characteristics by studying economies of scope. Finally, Barigozzi and Ma (2018) developed a general specification model that allows for general consumer preference distributions, general production cost functions (increasing and convex), and firms selecting any arbitrary number of quality characteristics.

Recent studies have explored the intersection of two-sided markets and vertical differentiation. For instance, Gabszewicz and Wauthy (2014) introduced heterogeneity among participants and found that platform competition with cross-group externalities and vertical differentiation can result in the equilibrium coexistence of asymmetric platforms. Zennyo (2016) investigated vertically differentiated two-sided markets and found that in a sequential game, both platforms charged the same per-transaction fee in equilibrium, even with quality asymmetries. Under certain conditions, a low-quality platform was found to have higher profits than a high-quality platform. Roger (2017) studied two-sided markets where platforms compete for agents on both sides of the market, and concluded that when cross-group externalities are too strong, pure-strategy equilibrium may not exist. Lastly, Etro (2021) considered the differences between device-funded and ad-funded platforms. His results showed that device-funded platforms are more aligned with consumers because they provide high-quality products and services, while ad-funded platforms offer products at competitive prices and free services.

The seminal models of Caillaud and Jullien (2003); Armstrong (2006); Rochet and Tirole (2003, 2006) analysing two-sided markets have been extended in various directions by subsequent research. Belleflamme and Toulemonde (2009); Hagiu (2009); Belleflamme and Toulemonde (2016); Belleflamme and Peitz (2019a) introduced competition among sellers and investigate how pricing equilibrium, product variety, and the optimal number of platforms are affected in the presence of a monopolistic or duopolistic platform. Their findings indicate that while consumers and producers prefer product variety, platforms prefer to minimise differentiation among them. Weyl (2010) proposed a nonlinear tariff that is conditional on the participation of agents on both sides in order to address the problem of equilibrium multiplicity. Choi (2010); Choi et al. (2017) investigated the impact of tying in a two-sided market where agents can use multiple platforms. They find that allowing multi-homing can improve welfare through tying. Gao (2018) analysed the effects of overlapping agents on both sides of a platform. Finally, Karle et al. (2020); Jeitschko and Tremblay (2020) examined how agents endogenously determine whether to singlehome or multihome.

Our model consists of two stages, where agents can join one platform (singlehome) only and platforms simultaneously determine the level of attributes they offer on the buyers' side in the first stage, and then determine membership fees in the second stage. We find equilibrium membership fees follow Armstrong (2006) result, but are adjusted by the differences in attributes offered by platforms on the buyers' side, and weighted by the cross-group network effect one side exercises on the other side.

Our first key finding is that the difference in attributes on the buyers' side between two competing platforms not only affects their behaviour but also has an impact on the sellers' side as a result of the presence of cross-group network effects on both sides of the market. We analyse two different scenarios based on the strength of these cross-group network effects. The first scenario establishes identical indirect network effects on both sides of the market. The second scenario analyses when the network effects are distinct. We find that when both cross-network effects are equal, the sellers' equilibrium membership fee remains as Armstrong (2006) stated, indicating that the difference in attributes on the buyers' side only impacts buyers' decisions.

We establish conditions for a max-min strategy to enhance profits, as demonstrated in the early works of Economides (1989) and Neven and Thisse (1990) and the generalized model of Irmen and Thisse (1998). Specifically, we identified two scenarios where such a strategy is effective: when the cross-group network effects on both sides of the market are equal, and when the cross-group network effect buyers have on sellers is greater than the impact sellers exert on buyers. In the former situation, platforms differentiate themselves as much as possible on attributes on buyers' side (vertical dimension) and as little as possible on the product differentiation cost (horizontal dimension). In the latter setting, platforms differentiate themselves as little as possible on attributes on buyers' side and as much as possible on the horizontal dimension to maximise profits. Furthermore, we find conditions for a max-max strategy to maximise profits, as seen in recent studies by Garella and Lambertini (2014); Barigozzi and Ma (2018). In particular, we find platforms differentiate as much as possible on both dimensions when the cross-group network effect exerted by sellers on buyers outweighs those exercised by buyers on sellers.

The paper is structured as follows. Section 2 outlines the model primitives, while Section 3 presents the solution to stage 2 of the model to obtain equilibrium membership fee configurations. Section 4 provides the solution to stage 1 of the model, deriving equilibrium attribute configurations on buyers' side. In addition, Section 5 analyses and compares market structure where the cross-group network effects on both sides of the market are identical and opposite. In both cases, we express the strategic variables as a function of the model parameters and provide intuitive explanations for the results. The paper concludes in Section 6.

2 Model

This chapter considers a model of platform competition with cross-group external effects and attributes on the buyers' side. There are three different players: platforms, buyers and sellers. The model follows Armstrong (2006) considering two platforms that are horizontally differentiated and charge access fees to both sides of the market. Buyers and sellers whom we refer to as agents, make a decision to join a single platform, a scenario known as singlehoming. In this model we introduce the level of attributes q_b as a strategic variable capturing various platform features on buyers' side: the higher the value of q_b, the more attractive the platform is for buyers, given membership fees.

Two platforms engage in competition through membership fees and attributes offered on the buyers' side. This setup is designed to facilitate interactions between a unit mass of sellers and buyers, generating positive cross-group network effects. Positioned at the extremes of a unit interval, the platforms exhibit horizontal differentiation à la Hotelling and bear a constant cost of f_b and f_s for serving buyers and sellers, respectively. Buyers and sellers, uniformly distributed across this interval, face a cost of visiting a platform that increases linearly in distance, τ_b and τ_s, respectively. This cost can be interpreted as a potential mismatch with buyers' and sellers' preferences. Considering our focal point is the relationship between the cross-group network effects and the attributes a platform offers on the buyers' side, we assume, that the cost associated with visiting a platform is homogeneous across both platforms and both sides. This means that both buyers and sellers face the same disutility cost when their preferences are mismatched, and we defined it as $\tau_b = \tau_s = \tau$.

Buyers, upon joining the platform, purchase one unit of product from each active seller on the same platform. For each trade, buyers and sellers obtain a cross-group network effect of υ and π, respectively; which can also be seen as gains from trade. Additionally, there exists a stand-alone benefit of R_b for buyers and R_s for sellers when they visit the platform, a benefit uniform across both platforms. We define η_b^i and η_s^i as the mass of buyers and sellers joining platform i, $i = 1, 2$. The membership fees charged to buyers and sellers on platform i are denoted as p_b^i and p_s^i, respectively.

In addition, buyers receive q_b^i for the attributes platform i offers. Platform i's for $i = 1, 2$ production cost of providing these attributes on the buyers' side is set as $C^i\left(q_b^i\right) = \frac{1}{2}\alpha^i\left(q_b^i\right)^2$. The parameter α^i captures the efficiency of platform i developing characteristics on the buyers' side. We assume $0 < \alpha^1 < \alpha^2$, meaning platform 1 is more efficient in developing these attributes compared to platform 2. This is possible, either because it can produce more features with the same inputs or deliver the same level of features at a lower cost. As a result, platforms are heterogeneous in terms of both product differentiation and the characteristics they offer on the buyers' side.

Therefore, buyers and sellers, respectively, obtain a surplus of visiting platform i, $i = 1, 2$, of:[6]

$$\nu_b^i = R_b + q_b^i + \upsilon\eta_s^i - p_b^i \tag{1a}$$

$$\nu_s^i = R_s + \pi\eta_b^i - p_s^i \tag{1b}$$

The model consists of two stages. In the first stage, platforms simultaneously choose characteristics on the buyers' side, and in the second stage, they simultaneously choose membership fees. Then, buyers and sellers choose which platform to join. In the next sections, we analyse different cases using the previous framework.

The model parameters must meet the following assumptions.[7]

Assumption 1. $\tau > \frac{\pi+\upsilon}{2}$ if $\pi > \upsilon$ or $\frac{\pi+\upsilon}{2} < \tau < \frac{\pi+2\upsilon}{3}$ if $\upsilon > \pi$

Assumption 2. $\alpha^i > \frac{2\tau}{\Sigma}$, $i = 1, 2$ where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$

Assumption 1 is developed on the second-order conditions of the platform maximisation problem at stage 2 of the model and from the conditions to guarantee equilibrium market

[6] We assume that the market is fully covered, implying that buyers and sellers do not have an outside option to interact. This assumption is standard in the literature, as evidenced by Choi (2010); Hagiu (2009).

[7] For further details on both assumptions see Appendix A.1.

shares are restricted to a unit interval. This condition stipulates that the degree of product differentiation must fall within a range defined by the cross-group network effects. This condition also is needed to have positive equilibrium attributes[8].

Assumption 2 guarantees the existence of positive equilibrium attributes. This condition means that the parameter measuring platform i efficiency in developing attributes on the buyers' side is not negligible. This condition guarantees that the second-order conditions of the platform maximisation problem at stage 1 of the model will be satisfied.

3 Equilibrium membership fees

We develop a two-stage model of two-sided markets with vertical differentiation where agents singlehome. We solve our model using backward induction. In this section, we solve the second stage of the game where platforms choose simultaneously membership fees and then agents choose simultaneously which platform to join, assuming the level of attributes on the buyers' side as given. We then obtain market shares and platform profits at equilibrium, offering some insights into the results.

We identify a buyer (b) and a seller (s) positioned at locations x_b and x_s within a unit interval, respectively, who are indifferent between joining platform 1 and 2, such that $\nu_k^1 - \tau x_k = \nu_k^2 - \tau(1 - x_k)$ where $k = b, s$. Buyers and sellers located between 0 and x_b or x_s visit platform 1, while those positioned between x_b or x_s and 1 visit platform 2. Consequently, we have $\eta_b^1 = x_b$, $\eta_b^2 = (1 - x_b)$, $\eta_s^1 = x_s$ and $\eta_s^2 = (1 - x_s)$, with the total number of buyers and sellers on both platforms being $\eta_b^1 + \eta_b^2 = \eta_s^1 + \eta_s^2 = 1$. We then determine the proportion of buyers and sellers for platform i, $i = 1, 2$ using the expressions for the indifferent buyer and seller along with expressions for x_b and x_s and the surpluses given by Equations (1a) and (1b):[9]

$$\eta_b^i = \frac{1}{2} + \frac{\tau\left(q_b^i - q_b^j\right) + \tau\left(p_b^j - p_b^i\right) + \upsilon\left(p_s^j - p_s^i\right)}{2\left(\tau^2 - \pi\upsilon\right)} \tag{2a}$$

$$\eta_s^i = \frac{1}{2} + \frac{\pi\left(q_b^i - q_b^j\right) + \tau\left(p_s^j - p_s^i\right) + \pi\left(p_b^j - p_b^i\right)}{2\left(\tau^2 - \pi\upsilon\right)} \tag{2b}$$

We are interested in a solution where both platforms remain active. This implies that not only do buyers' and sellers' market shares decrease when their own side's membership fee increases, but also when the membership fee of the other side increases.[10] In other words, the market shares of both sides are influenced by changes in fees on either side of the market.[11]

[8] Equilibrium attributes are defined in Definition 2 and equilibrium market shares are defined in Equations (5a) and (5b)

[9] See Appendix A.2 for further details on how market shares are determined.

[10] Alternatively, if the cross-group network effects outweigh the opportunity cost associated with mismatched preferences on both sides of the market, i.e., $\tau^2 < \pi\upsilon$, both sides' market shares would become an increasing function of their membership fee. Consequently, both buyers and sellers would opt for the same platform, leading to a tipping point in the market.

[11] The partial derivative of Equations (2a) and (2b) concerning both membership fees are negative, as long as $\tau > \sqrt{\pi\upsilon}$. Considering Assumption 1, expressed as $\tau - \frac{\pi+\upsilon}{2} > 0$, we can show that $\tau - \sqrt{\pi\upsilon} > 0$. This can be derived from the inequality $(\pi + \upsilon)^2 > 4\pi\upsilon$ which simplifies to $(\pi - \upsilon)^2 > 0$ if $\pi \neq \upsilon$.

Definition 1. *An equilibrium at stage two of the model is a pair* p_b^i, p_s^i *such that* p_b^i *and* p_s^i *solves the platform maximisation problem* $\max_{\{p_b^i, p_s^i\}} \Pi^i = (p_b^i - f_b)\eta_b^i(p_b^i, p_b^j, p_s^i, p_s^j, q_b^i, q_b^j) + (p_s^i - f_s)\,\eta_s^i\left(p_b^i, p_b^j, p_s^i, p_s^j, q_b^i, q_b^j\right) - \frac{\alpha^i (q_b^i)^2}{2}$ *for each* $i, j = 1, 2,\ i \neq j$

From the first-order conditions for platform i's maximisation problem, the following best response functions are obtained:[12]

$$p_b^i = \frac{f_b + \tau + p_b^j}{2} + \frac{\left(q_b^i - q_b^j\right)}{2} - \frac{\upsilon\left(p_s^i - p_s^j\right)}{2\tau} - \frac{\pi\left(\upsilon + p_s^i - f_s\right)}{2\tau} \tag{3a}$$

$$p_s^i = \frac{f_s + \tau + p_s^j}{2} + \frac{\pi\left(q_b^i - q_b^j\right)}{2\tau} - \frac{\pi\left(p_b^i - p_b^j\right)}{2\tau} - \frac{\upsilon\left(\pi + p_b^i - f_b\right)}{2\tau} \tag{3b}$$

The best strategy for platform i when the difference in characteristics $q_b^i - q_b^j$ on the buyers' side is positive[13] is to increase the membership fee on both sides of the market. At the same time, platform i's best response is to increase the membership fee on both sides when the other platform increases its fees on either side ($\partial p_b^i / \partial p_b^j \equiv \partial p_b^i / \partial p_s^j > 0$). However, platform i decreases the membership fee on one side when the membership fee on the other side increases, ($\partial p_b^i / \partial p_s^i \equiv \partial p_s^i / \partial p_b^i < 0$). Following Bulow et al. (1985), membership fees' best responses are, *for a certain side*, strategic complements amongst platforms; whilst they are strategic substitutes between sides for a *certain* platform.

Although platform i's best response is to increase both sides' membership fee when the difference in attributes is positive, on the sellers' side the best response is boosted when the cross-group network effect buyers exert on sellers π increases. This behaviour is common in two-sided markets where sellers benefit as more buyers join the platform. Platform i developed attributes on buyers' side appealing to more buyers because they can enjoy more features, but also appealing to more sellers given the cross-group network effect.

Next, we solve the best response functions given in Equations (3a) and (3b) to obtain the equilibrium membership fees as a function of the model parameters and the difference in attributes on the buyers' side:[14]

$$p_b^i = f_b + \tau - \pi + \left[\frac{3\tau^2 - \pi\left(\pi + 2\upsilon\right)}{9\tau^2 - \left(2\pi + \upsilon\right)\left(\pi + 2\upsilon\right)}\right]\left(q_b^i - q_b^j\right) \tag{4a}$$

$$p_s^i = f_s + \tau - \upsilon + \left[\frac{\tau\left(\pi - \upsilon\right)}{9\tau^2 - \left(2\pi + \upsilon\right)\left(\pi + 2\upsilon\right)}\right]\left(q_b^i - q_b^j\right) \tag{4b}$$

First, notice the difference in attributes $q_b^i - q_b^j$ affects equilibrium membership fees on both sides of the market, even though they were developed only on the buyers' side. The sellers'

[12]See Appendix A.3 for details.

[13]When we refer to the difference of a strategic variable: *membership fees*, *market-shares*, *attributes* and *platforms' profits*, it is always between both platforms.

[14]We are interested in obtaining an equilibrium where both platforms are active. Therefore, Assumption 1 guarantees, platform i's profit function is concave and the second-order conditions of the maximisation problem are satisfied. See Appendix A.4 for more details.

side is affected by the difference in characteristics on the other side because of the cross-group network effects one side exerts on the other side. Therefore platforms adjust sellers' membership fees taking into account the difference in features on the buyers' side.

Both agents' equilibrium membership fees on platform i are a function of two terms. The first term is Armstrong (2006) result, the cost of serving buyers and sellers f_b and f_s, the disutility for mismatch preference τ, and the cross-group network effect this side exerts on the other side, π for buyers and υ for sellers. The second term captures the difference in attributes developed on the buyers' side $q_b^i - q_b^j$. This extra markup could be positive or negative depending on which side exerts a stronger cross-network effect on the other side.

In a one-sided market, a firm typically increases its prices as it offers more attributes to customers. However, in a two-sided market, pricing dynamics are influenced by the interplay of cross-group network effects on both sides of the market. As a result, membership fees on one side may actually decrease despite platforms offering additional features, as they can offset this decrease by charging a higher fee on the other side, using the indirect network effects present in the market.

We summarise our discussion in the next proposition:

Proposition 1. *For* $\left(q_b^i - q_b^j\right) > 0$, *whenever this difference in attributes increases, platform* i,

(i) Increases buyers' and decreases sellers' equilibrium membership fees, whenever the cross-group network effect experienced by buyers is higher than the one experienced by sellers $(i.e., \upsilon > \pi)$;

(ii) Increases sellers' and decreases buyers' equilibrium membership fees, whenever the influence exerted on sellers by buyers outweighs the impact on buyers by sellers $(i.e., \pi > \upsilon)$.

Proof: See Appendix A.5

Platform i appeals to more agents by increasing the features on buyers' side, attracting more buyers directly and more sellers indirectly since the cross-group network effect. This creates a positive loop considering more agents are attracted on both sides, i.e buyers join platform i given there are more features developed for them, sellers join as well because more buyers joined, then more buyers,..., and this behaviour continues.

Platform i, decides to charge a lower fee on the side that exerts a stronger cross-group network effect on the other side. On the one hand, platform i decreases buyers' fee if the influence buyers exert on sellers is higher than sellers on buyers, $(\pi > \upsilon)$. On the contrary, platform i decreases sellers' fee if the cross-group network effect sellers exert on buyers is higher than the impact buyers exert on sellers $(\upsilon > \pi)$.

Equilibrium market shares and profits (stage 2)

At equilibrium, buyers' and sellers' market shares for platform i where $i, j = 1, 2,\ i \neq j$, are:[15]

$$\eta_b^i = \frac{1}{2} + \left[\frac{3\tau}{2\left[9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right]}\right]\left(q_b^i - q_b^j\right) \tag{5a}$$

$$\eta_s^i = \frac{1}{2} + \left[\frac{(\pi + 2\upsilon)}{2\left[9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right]}\right]\left(q_b^i - q_b^j\right) \tag{5b}$$

As with equilibrium membership fees, we find that even when platform features are exclusively developed on buyers' side, the difference in attributes impacts both sides' market shares. Sellers join platform i even in the absence of tailored attributes for them, through the influence of cross-group network effects. Furthermore, buyers' and sellers' market shares experience an increase when there is a positive difference in attributes developed on the buyers' side $(q_b^i - q_b^j)$, regardless of which side places a higher value on interaction with the other side.[16]

Platform i can increase its position in the market by developing more attributes on buyers' side. Buyers and sellers will be drawn to join platform i, buyers will join to enjoy more features developed for them and sellers will join because they can interact with more buyers (cross-group network effects).

As we already have the equilibrium membership fees and market shares on both sides of the market, we can compute equilibrium profits for platform i as:

$$\Pi^i = \tau - \frac{(\pi + \upsilon)}{2} + \frac{\tau\left(q_b^i - q_b^j\right)^2 + \Omega\left(q_b^i - q_b^j\right)}{2\left[9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right]} - \frac{\alpha^i\left(q_b^i\right)^2}{2} \tag{6}$$

where $\Omega \equiv 6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)$.

Equilibrium profits are equal to the degree of product differentiation on both sides of the market (τ), adjusted downwards by the cross-group network effects, π and υ as in Armstrong (2006) main result. Furthermore, profits are adjusted by two additional elements. The first term, $\frac{1}{2\Sigma}\left[\tau\left(q_b^i - q_b^j\right)^2 + \Omega\left(q_b^i - q_b^j\right)\right]$, where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ is an extra markup associated with the difference in attributes on buyers' side and the second component $\frac{\alpha^i\left(q_b^i\right)^2}{2}$ is the cost of developing these attributes.

We can see from Equation (6) that platform i's equilibrium profits increase when additional attributes on buyers' side are developed.[17] When platforms offer new and innovative features,

[15] For more details on how to derive market shares see Appendix A.6.

[16] Partially differentiate equilibrium market-shares in Equations (5a) and (5b) respect the difference in attributes on buyers' side. The numerator is always positive and to show the denominator $9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ is positive we use Assumption 1 by making the left side of both inequalities equal to compare the right side, showing that Assumption 1 right side is greater and therefore the condition is positive. That is, Assumption 1 can be transform to be $\tau^2 > \frac{(\pi+\upsilon)^2}{4}$, then we have $\frac{(\pi+\upsilon)^2}{4} > \frac{(2\pi+\upsilon)(\pi+2\upsilon)}{9}$ which simplifies to $9\left(\pi^2 + 2\pi\upsilon + \upsilon^2\right) - 4\left(2\pi^2 + 5\pi\upsilon + 2\upsilon^2\right) > 0$ and then simplifies to $(\pi - \upsilon)^2 > 0$ if $\pi \neq \upsilon$.

[17] This can be seen by partially differentiate Equation (6) with respect to q_b^i.That is $\frac{\partial \Pi^i}{\partial q_b^i} = \frac{2q_b^i\left(\tau - \alpha^i\Sigma\right) - 2\tau q_b^j + \Omega}{2\Sigma} >$

they can appeal to more agents (buyers and sellers) and increase customer satisfaction leading to higher profits.

4 Equilibrium attributes

In this section, we find the equilibrium values of attributes on buyers' side at stage 1 of the model. Platform i differentiates by the features offered on buyers' side, measured by q_b^i. There is a cost of providing q_b^i of $C^i\left(q_b^i\right) = \frac{1}{2}\alpha^i\left(q_b^i\right)^2$, where $i = 1, 2$ and $\alpha^2 > \alpha^1 > 0$. The parameter α^i measures the efficiency platform i has in developing attributes on buyers' side. The fact that platform 1 is more efficient in developing attributes can be related to specialisation in certain technology, experience in having a better understanding of buyers' needs, or innovation by investing more in research and development.

In stage 1 platforms simultaneously choose the characteristics' levels on buyers' side q_b^i, $i = 1, 2$. We can state the next definition:[18]

Definition 2. *An equilibrium at stage one of the model is q_b^i such that q_b^i solves the platform maximisation problem* $\max_{\{q_b^i\}} \Pi^i \equiv \left(p_b^i - f_b\right)\eta_b^i\left(q_b^i\right) + \left(p_s^i - f_s\right)\eta_s^i\left(q_b^i\right) - \frac{\alpha^i\left(q_b^i\right)^2}{2}$ *for each* $i = 1, 2$.

From the first-order conditions for platform i's maximisation problem, we obtained the following best response function:

$$q_b^i = \frac{-\tau q_b^j}{\left(\alpha^i\Sigma - \tau\right)} + \frac{6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)}{2\left(\alpha^i\Sigma - \tau\right)} \text{ for each } i, j = 1, 2,\ i \neq j \tag{7}$$

where $\Sigma \equiv 9\tau^2 - (\pi + 2\upsilon)(2\pi + \upsilon)$

Note that attributes are strategic substitutes considering the best response function in Equation (7). Platform i's employs a strategy of increasing attributes on buyers' side whenever its competitor takes the opposite approach.[19]

Solving the best response function in Equation (7) for $i = 1, 2$ we obtain the equilibrium attributes on buyers' side as a function of the model parameters, that is:[20]

$$q_b^i = \frac{\left(\alpha^j\Sigma - 2\tau\right)\left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{2\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \text{ for each } i, j = 1, 2,\ i \neq j \tag{8}$$

where $\Sigma = 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$.

We observe from Equation (8) that the rivals efficiency parameter in developing attributes is what differentiates equilibrium attributes on buyers' side between both platforms. Platform 1 increases attributes when platform 2 becomes less efficient in developing characteristics on

0 if $q_b^i > \frac{2\tau q_b^j - \Omega}{2\left(\tau - \alpha^i\Sigma\right)}$, where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$.

[18] See Appendix A.7 for more details.

[19] The partial derivative of Equation (7) respect to q_b^j is negative. $\partial q_b^i / \partial q_b^j = -\frac{\tau}{\left(\alpha^i\Sigma - \tau\right)}$, where $\alpha^i\Sigma - \tau$ is positive as long as Assumption 2 holds.

[20] Assumption 2 guarantees, platform i profit function is concave and the second-order conditions of the maximisation problem at stage 1 of the model are satisfied. See Appendices A.1 and A.8 for more details.

buyers' side (higher α^{2})[21], as long as Assumption 1 and Assumption 2 hold.[22] Platform 1 enhances attributes offered on buyers' side to appeal buyers and sellers, establishing itself as a leading intermediary in the industry.

Finally, we define the difference in attributes on buyers' side, using Equation (8) as:

$$\Delta q_b^i \equiv q_b^i - q_b^j = \frac{\left(\alpha^i - \alpha^j\right)\left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{2\left[\alpha^i \alpha^j \Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{9}$$

for each $i, j = 1, 2$, $i \neq 2$ and $\Sigma = 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$.

The equilibrium difference in attributes on buyers' side in Equation (9) is positive for platform 1 and negative for platform 2 considering platform 1 is more efficient in developing features compared to platform 2 ($\alpha^{2} > \alpha^{1}$). [23]

Therefore, recognising the significance of the impacts that the difference in cross-group network effects has on platform attributes and overall market equilibrium (fees, market shares and profits), our focus now shifts towards a comprehensive analysis of these effects in the subsequent section.

5 Analysis of cross-group network effects on market configurations

In this section, we study how cross-group network effects shape the structure and dynamics of the market. We explore two distinct scenarios to gain insights into the interactions between platform's attributes and cross-group network effects. Firstly, we consider a benchmark case where cross-group network effects are identical on both sides of the market. Secondly, we explore a scenario where the cross-side network impacts are allowed to differ.

5.1 Benchmark scenario: Identical cross-group network effects, $\pi = \upsilon$

In this section, we develop a benchmark scenario where the cross-group network effects are identical on both sides of the market, ($\pi = \upsilon$). We use the game's solution at stage 1 to obtain the strategic variables as a function of the model's parameters. We use the superscript "bs" to denote the equilibrium market structures. Furthermore, we provide some intuition for the results that are going to help us to examine asymmetric network effects in the next section.

Using equilibrium attributes at Equation (8) and the fact that $\pi = \upsilon$, benchmark equilibrium attributes on buyers' side are:

$$\left(q_b^i\right)^{bs} = \frac{9\alpha^j\left(\tau^2 - \pi^2\right) - 2\tau}{3\left[9\alpha^i\alpha^j\left(\tau^2 - \pi^2\right) - \left(\alpha^i + \alpha^j\right)\tau\right]} \text{ for each } i, j = 1, 2,\ i \neq j \tag{10}$$

[21] To avoid confusion between squared parameters and parameters of platform *2*, italic numbers will be used instead of normal numerals 1 and 2 when referring to platforms *1* and *2* in the mathematical expressions.

[22] Partially differentiate q_b^{1} in Equation (8) respect to α^{2}. $\partial q_b^{1}/\partial\alpha^{2} = \frac{\Omega\tau}{2\Sigma A^2}\left(\alpha^{1}\Sigma - 2\tau\right) > 0$, where $\Omega \equiv 6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)$, $\Sigma = 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ and $A \equiv \left(\alpha^{1}\alpha^{2}\Sigma - \left(\alpha^{1} + \alpha^{2}\right)\tau\right)$

[23] Appendix A.9 shows conditions for positive equilibrium attributes, which applies to the difference in attributes in Equation (9).

Equilibrium attributes on buyers' side on Equation (10) are positive as long as Assumption 1 and Assumption 2 holds[24], and considering identical cross-group network effects on both sides of the market we can state the next proposition:

Proposition 2. *Equilibrium attributes on the buyers' side decrease in the product differentiation parameter τ and increase in the cross-group network effect ($\pi = \upsilon$). Moreover, an increase in the cross-group network effect is stronger in the platform that is more efficient in developing attributes.*

Proof: See Appendix B.1

Proposition 2 says that an increase in the product differentiation parameter τ (in the horizontal dimension) prompts the platforms to reduce attributes on the buyers' side thereby differentiating less on the vertical dimension. As the product differentiation parameter τ increases across both sides of the market, the platform no longer has any incentives to further enhance attributes on the buyers' side. This is due to the costs associated with simultaneous differentiation on both the horizontal and vertical dimensions.

Instead, to gain a competitive advantage, platform 1 opts for a broader degree of product differentiation, catering to a wide range of preferences from both buyers and sellers. Rather than focusing on increasing the level of features on buyers' side for a specific set of preferences, platform 1 engages in less intense competition for the same pool of agents as the degree of product differentiation expands. Consequently, agents become more captive and there is reduced pressure to develop additional attributes on the buyers' side.

We notice also from Proposition 2 that platform 1 increases the attributes on buyers' side whenever the cross-group network effects increase because this attracts directly more buyers and more sellers, given the cross-side network effects. This creates a positive loop where the more agents use platform 1, the more valuable it becomes to buyers and sellers, which in turn attracts even more agents. Considering platform 1 is more efficient in developing attributes than platform 2, $\alpha^2 > \alpha^1$, this outcome is more pronounced on platform 1.

Corollary 1. *The difference in attributes on buyers' side decreases when there is a higher product differentiation on both sides of the market and increases when the cross-group network effects become stronger.*

Proof: See Appendix B.1

Corollary 1 extends the proven arguments on Proposition 2 to the difference in attributes on buyers' side. For this reason, the intuition is the same as in Proposition 2.

Equilibrium membership fees

We now obtain equilibrium membership fees, market shares and platform profits as a function of the model parameters.

[24]When $\upsilon = \pi$ Assumption 1 turns to $\tau > \pi$ and Assumption 2 turns to $\alpha^i > \frac{2\tau}{9\sigma}$, where $\sigma \equiv \tau^2 - \pi^2$.

For the equilibrium membership fees we have:

$$\left(p_b^i\right)^{bs} = f_b + \tau - \pi + \frac{\left(\alpha^j - \alpha^i\right)\sigma}{9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau} \;=\; f_b + \tau - \pi + \frac{1}{3}\left(\Delta q_b^i\right)^{bs} \tag{11a}$$

$$\left(p_s^i\right)^{bs} = f_s + \tau - \upsilon; \quad \upsilon = \pi \tag{11b}$$

for each $i, j = 1, 2,\ i \neq 2$ and where $\sigma \equiv \tau^2 - \pi^2$.

When the cross-group network effects are identical on both sides of the market $\pi = \upsilon$, platforms charge symmetric fees on sellers' side. This is a consequence that the difference in attributes on buyers' side does not influence sellers' fees when the cross-network effects are the same. Both platforms charge sellers the same fee as in Armstrong (2006) seminal model.

However, buyers' equilibrium membership fee is higher on platform 1 than it would have been without the development of specific features for them. This is due to the extra markup denoted by $\frac{1}{3}\left(\Delta q_b^i\right)^{bs}$, which is positive for platform 1 considering ($\alpha^2 > \alpha^1$). Consequently, platform 1 lacks the option to discern which side values interaction more, and thus, cannot adjust the fee accordingly when the cross-group network effects are identical on both sides of the market.

We examine the effects of the model parameters on the difference in equilibrium fees between the two platforms, under the assumption that platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$). Hence, we set the following:

Proposition 3. *The difference in equilibrium fees buyers pay decreases when there is a greater heterogeneity between platforms (higher τ) and increases when platforms become more valuable for both groups (stronger $\pi = \upsilon$). In addition, buyers' fees are more expensive in the platform which is more efficient in developing attributes whenever the cross-group network effect is stronger.*

Proof: See Appendix B.2

Proposition 3 reveals that as the product differentiation parameter increases ($\tau \uparrow$), platform 1 reduces buyers' fees because the difference in attributes between platforms decreases. This fee reduction serves as an incentive to attract more buyers. Then, it raises sellers' fees, as indicated in Equation (11b), to compensate for the decrease in buyers' fees. Conversely, when the cross-group network effect ($\pi = \upsilon$) increases, it raises buyers' fees as it has developed more attributes to enhance their experience. Simultaneously, it lowers sellers' fees to encourage greater participation from sellers, as observed in Equation (11b).[25]

An increase in the cross-group network effect has a greater impact on buyers' equilibrium membership fee in platform 1. This is because platform 1 is more proficient in developing features, which attracts a larger number of buyers. Consequently, it exploits this by charging buyers a higher fee, allowing it to extract a greater portion of buyers' surplus.

[25]The proof for Proposition 3 is straightforward, partially differentiate Equation (11a) with respect to the model parameters. For details see Appendix B.2.

Equilibrium market shares and profits

Using equations Equations (5a), (5b) and (10) we obtain the following equilibrium market shares:[26]

$$\left(\eta_b^i\right)^{bs} = \frac{1}{2} + \frac{\left(\alpha^j - \alpha^i\right)\tau}{2\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{12a}$$

$$\left(\eta_s^i\right)^{bs} = \frac{1}{2} + \frac{\left(\alpha^j - \alpha^i\right)\pi}{2\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{12b}$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\sigma \equiv \tau^2 - \pi^2$.

Platform 1 gains a larger market share among both buyers and sellers considering it is more efficient in developing attributes on buyers' side, $(\alpha^2 > \alpha^1)$. Equilibrium market shares on both sides increase when the cross-group network effect is stronger $(\pi = \upsilon)$. Platform 1 becomes more valuable to both buyers and sellers as the cross-group network effects strengthen, resulting in the development of more attributes for buyers. This positive feedback loop contributes to a rapid expansion of its market share, potentially leading to its dominance in the market.[27]

Using equilibrium membership fees in Equations (11a) and (11b) and equilibrium market shares Equations (12a) and (12b) we obtain equilibrium profits as a function of the equilibrium features configurations:

$$\left(\Pi^i\right)^{bs} = \tau - \pi + \frac{9\sigma\left(\alpha^j - \alpha^i\right)\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right] - \alpha^i\left(9\alpha^j\sigma - 2\tau\right)\left(9\alpha^i\sigma - 2\tau\right)}{18\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2} \tag{13}$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\sigma = \tau^2 - \pi^2$.

Platform i's equilibrium profits are a function of two terms. The first term $(\tau - \pi)$ is similar to Armstrong (2006) having product differentiation on both sides of the market $(\tau_b = \tau_s = \tau)$ and cross-group network effects $(\pi = \upsilon)$. The second term is an extra markup related to the difference in attributes on buyers' side between both platforms, which is positive for platform 1 because is more efficient in developing attributes and as long as Assumption 2 holds.[28]

We obtain some insights into platforms' strategy to maximise profits, under the assumption that platform 1 is more efficient in developing attributes compared to platform 2, $(\alpha^2 > \alpha^1)$, in the following proposition:

Proposition 4. *The difference in equilibrium profits decreases as the degree of product differentiation intensifies (higher τ) and increases the more valuable it becomes for both buyers and sellers since the cross-group network effect $(\pi = \upsilon)$ turns stronger.*

Proof: See Appendix B.6

Proposition 4 contrasts with Armstrong (2006) where equilibrium platforms' profits are

[26] Condition for buyers' and sellers' market shares distributed in the unit interval is $\alpha^i > \frac{2\tau}{9\sigma}$. For details see Appendix B.3.

[27] Partially differentiate Equations (12a) and (12b) respect the model parameters. For details see Appendix B.4.

[28] See Appendix B.5 for details.

increasing on the degree of product differentiation (τ) and decreasing on cross-group network effects ($\pi = \upsilon$). In our benchmark scenario, the effects in equilibrium profits are the opposite.

As platform 1 becomes more horizontally differentiated ($\tau \uparrow$), there is a decrease in the development of attributes on buyers' side. Consequently, the number of buyers joining the platform decreases, along with the number of sellers, considering the cross-group network effect. As a result, platform 1 has a smaller pool of agents to charge additional fees to, leading to a decline in the difference in equilibrium profits.

Conversely, an increase in cross-group network effects leads to an increase in attributes on buyers' side. This attracts a larger number of buyers and sellers, taking into account the cross-effect of the networks. In response, platform 1 charges a higher fee on buyers' side and a lower fee on the sellers' side, as indicated in Proposition 3. Accordingly, it charges an additional fee per additional agent, resulting in higher profits.

These findings align with the early work conducted by Economides (1989) and Neven and Thisse (1990) and the generalised model by Irmen and Thisse (1998). These studies suggest that platforms' profit-maximising strategy involves maximising differentiation on one dimension while minimising differentiation on the other dimension. In the current scenario, platform i increases the vertical dimension by developing attributes on buyers' side when the horizontal dimension, representing the product differentiation parameter on both sides of the market, decreases.

5.2 Non-Identical cross-group network effects, $\pi \neq \upsilon$

In this section, our objective is to analyse the presence of asymmetric cross-group network effects. To ensure that the analysis remains tractable without sacrificing its essence, we simplify the model by setting the side that exerts a weaker network effect on the other side to zero.[29]

The first case we consider is when buyers value interactions more than sellers or when the cross-group network effect sellers exert on buyers is greater than vice versa ($\upsilon > \pi$). To keep our analysis tractable, we normalise the value of π to zero. The second case we examine is when sellers value interaction more than buyers or when the cross-group network effect buyers exert on sellers is greater than vice versa ($\pi > \upsilon$). Again, for simplicity, we normalise the value of υ to zero. By using the game's solution at stage 1, we obtain the strategic variables as functions of the model's parameters and gain insights into the results.

Equilibrium attributes

Using equilibrium attributes in Equation (8) we obtain platforms equilibrium attributes on buyers' side for two different scenarios:

$$\text{When } \upsilon > \pi \ \ (\pi = 0)\,, \quad \left(q_b^i\right)\Big|_{\upsilon>\pi} = \frac{\left(\alpha^j\sigma_\upsilon - 2\tau\right)\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)}{2\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{14a}$$

[29] Ideally, what we mean is that the network effect exerted by this side is negligible compared to the magnitude of the network effect originating from the other side.

$$\text{When } \pi > \upsilon \ \ (\upsilon = 0)\,, \quad \left(q_b^i\right)\Big|_{\pi>\upsilon} = \frac{\left(\alpha^j\sigma_\pi - 2\tau\right)(3\tau+\pi)(2\tau-\pi)}{2\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right]} \tag{14b}$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$ and $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$.

We can observe in Equations (14a) and (14b) that equilibrium attributes on buyers' side $\left(q_b^i\right)\Big|_{\upsilon>\pi}$ and $\left(q_b^i\right)\Big|_{\pi>\upsilon}$ are positive if Assumption 1 and Assumption 2 holds.[30] Then we can state the next proposition:

Proposition 5. *The difference in equilibrium attributes on buyers' side decreases as the degree of product differentiation increases and rises with a stronger cross-group network effect, as long as $\tau > 4\upsilon$ when the cross-group network effect sellers exert on buyers is greater than the effect exerted by buyers on sellers, $\upsilon > \pi$, $\pi = 0$.*

Proof: See Appendix C.1

Propositions Proposition 2 and Proposition 5 provide similar insights regarding equilibrium attributes on buyers' side. Regardless of whether the cross-group network effects are identical or if one side exerts a stronger network effect on the other, these propositions establish that equilibrium attributes on buyers' side unambiguously decrease with a higher degree of product differentiation ($\tau \uparrow$) and increase with stronger cross-group network effects ($\pi, \upsilon \uparrow$).

Proposition 5 is based on the observation that as the degree of product differentiation (τ) increases, platform 1 engages in less aggressive competition for both agents. This is because the unique and distinct nature of its services reduces the need to develop additional attributes on buyers' side to attract them. Conversely, when there is a stronger relationship between the two groups, characterised by increased features on buyers' side, given higher cross-group network effects, the platform becomes more valuable to both agents. The growth of one group enhances the value of the other group, resulting in mutual growth. When the effect sellers exert on buyers is stronger than vice versa, $\upsilon > \pi$, $\pi = 0$, the degree of product differentiation has to exceed a certain threshold ($\tau > 4\upsilon$), for an increase in attributes on buyers' side to attract more participants, as it becomes more costly (τ was $\frac{\upsilon}{2}$ and now is 4υ) for them to join and can feel discouraged. Therefore, platform 1 starts developing more attributes to appeal to more buyers and eventually more sellers given the cross-group network effect.

Equilibrium market shares and profits

We proceed to obtain the equilibrium market shares on both sides of the market using Equations (5a) and (5b) and equilibrium attributes in Equation (8)[31]

$$\eta_b^i = \frac{1}{2} + \frac{\left(\alpha^j - \alpha^i\right)3\tau\left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)\right]}{4\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i+\alpha^j\right)\tau\right]} \tag{15a}$$

[30] Assumption 1 and Assumption 2 turn to $\tau > \frac{\pi}{2}$ and $\alpha^i > \frac{2\tau}{\sigma_\pi}$ respectively, when $\pi > \upsilon$ $(\upsilon = 0)$. Conversely, they turn to $\frac{\upsilon}{2} < \tau < \frac{2\upsilon}{3}$ and $\alpha^i > \frac{2\tau}{\sigma_\upsilon}$ respectively, when $\upsilon > \pi$ $(\pi = 0)$. Where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$ and $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$.

[31] Buyers' and sellers' market shares are distributed in the unit interval as long as Assumption 1 and Assumption 2 hold. For more details see Appendix C.2.

$$\eta_s^i = \frac{1}{2} + \frac{\left(\alpha^j - \alpha^i\right)(\pi + 2\upsilon)\left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon - \pi)\right]}{4\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{15b}$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$.

Based on the equilibrium market shares in Equations (15a) and (15b), we can conclude that platform 1 gains a competitive advantage over its rival by being more efficient in developing attributes on buyers' side ($\alpha^2 > \alpha^1$). This advantage remains regardless of whether the cross-group network effects are identical ($\pi = \upsilon$), as mentioned in Section 5.1, or if the indirect network effect exerted by sellers on buyers is larger ($\upsilon > \pi, \pi = 0$), or if the cross-group network effect exerted by buyers on sellers is stronger ($\pi > \upsilon, \upsilon = 0$) in Section 5.2. Platform 1 outperforms platform 2 because it is capable of producing more features on buyers' side with fewer resources and/or in less time.

The following claim captures the impact of model parameters τ and π, υ, on buyers' and sellers' equilibrium market shares under the assumption that platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$).[32]

Claim 1. *Buyers' and sellers' equilibrium market shares decrease when platform 1 is more heterogeneous in the horizontal dimension (higher τ) and increase when the cross-group network effects become stronger (higher υ, π).*

Proof: See Appendix C.3

The claim states that as platform 1 becomes more heterogeneous in terms of the degree of product differentiation ($\tau \uparrow$), the number of attributes on buyers' side decreases. This reduction diminishes the incentives for buyers and sellers to join the platform. Conversely, as the cross-group network effects increase, platform 1 becomes more valuable, attracting more participants on both sides of the market.[33]

The next step is to obtain platform i equilibrium profits as a function of the equilibrium features in Equation (8):

$$\Pi^i = \tau - \frac{\pi + \upsilon}{2} + \left[\frac{\left(\alpha^j - \alpha^i\right)\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right] - \alpha^i\left(\alpha^j\Sigma - 2\tau\right)\left(\alpha^i\Sigma - 2\tau\right)}{8\Sigma^2\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\right]\Omega^2 \tag{16}$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ and $\Omega \equiv 6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)$.

Platform i's equilibrium profits are a function of two terms. The first term is similar to Armstrong (2006), product differentiation cost and cross-side network effects on both sides of the market $\tau - \frac{\pi + \upsilon}{2}$. The second term is a markup related to the difference in attributes on buyers' side, which is positive for platform 1 because $\alpha^2 > \alpha^1$ and as long as Assumption 2 holds.[34]

[32] As we observe equilibrium market shares on buyers' side is $\eta_b^i = \frac{1}{2} + \frac{3\tau}{2\Sigma}\Delta q_b i$ and on sellers' side is $\eta_s^i = \frac{1}{2} + \frac{(\pi+2\upsilon)}{2\Sigma}\Delta q_b i$

[33] The detailed derivation of these results can be found in Appendix C.3, where Equations (15a) and (15b) are partially differentiated with respect the model parameters.

[34] See Appendix C.4 for details.

Case 1: When sellers exert a stronger influence on buyers: $\upsilon > \pi \, (\pi = 0)$**.**

Equilibrium membership fees and Platform Profits

In this case, we have:

$$\left(p_b^i\right)\Big|_{\upsilon>\pi,\ \pi=0} = f_b + \tau + \frac{3\left(\alpha^j - \alpha^i\right)\tau^2\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)}{2\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{17a}$$

$$\left(p_s^i\right)\Big|_{\upsilon>\pi,\ \pi=0} = f_s + \tau - \upsilon - \frac{\left(\alpha^j - \alpha^i\right)\tau\upsilon\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)}{2\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{17b}$$

for each $i, j = 1, 2,\ i \neq 2$ and where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$.

Note that the extra markup on Equations (17a) and (17b) is positive in platform 1 considering $\alpha^2 > \alpha^1$ and as long as Assumption 1 and Assumption 2 hold. Therefore, when the cross-group network effect sellers exert on buyers outweighs the effect buyers exert on sellers ($\upsilon > \pi$), platform 1 implements a pricing strategy that deviates from the seminal results by Armstrong (2006). Specifically, platform 1 charges on buyers' side an additional markup while reducing sellers' subscription fees. That is $\left(p_b^1\right)\Big|_{\upsilon>\pi,\ \pi=0} > \left(p_b^1\right)^{Armstrong}$ and $\left(p_s^1\right)\Big|_{\upsilon>\pi,\ \pi=0} < \left(p_s^1\right)^{Armstrong}$.

Next, we characterise the impacts on the difference in equilibrium fees considering platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$)

Proposition 6a. *For* $\upsilon > \pi \, (\pi = 0)$*, the difference in equilibrium membership fees*

(i) On buyers' side decreases and on sellers' side increases when τ *increases.*

(ii) On buyers' side increases and sellers' side decreases as the cross-group network effect becomes stronger (i.e., when υ *increases).*

Proof: See Appendix C.5.

According to Proposition 6a, as the degree of product differentiation increases ($\tau \uparrow$), there is no need for platform 1 to develop additional attributes on buyers' side. Platform 1 is perceived as offering unique and distinct services compared to the other platform. As a result, the features on buyers' side decrease, discouraging buyers from joining it.

To counteract this potential decrease in buyer participation, the platform adjusts its pricing strategy by charging a lower fee on buyers' side. This lower fee is aimed at attracting and retaining buyers. To compensate for the revenue loss from lower buyer fees, the platform charges a higher fee on sellers' side. The higher fee is justified by the increased participation of sellers due to the positive cross-group network effect.

This finding contrasts with the results of Armstrong (2006), where membership fees on both sides of the market increase as the degree of product differentiation increases. The difference arises from the fact that in our model, platforms adjust their pricing strategies indirectly by manipulating the features developed on buyers' side, rather than directly adjusting the membership fees.

Furthermore, when the cross-group network effect exerted by sellers on buyers is stronger

($\upsilon > \pi$), platform 1 increases the attributes on buyers' side. This strategy aims to appeal to more buyers and incentivise their participation in the platform. Consequently, it charges a higher fee to buyers, reflecting the additional value provided through the developed attributes. Additionally, the stronger cross-group network effect encourages more sellers to join the platform, as they benefit from the increased buyer participation. To attract and retain sellers, platform 1 charges them a lower fee.

This result aligns with existing findings in the literature on two-sided markets as in Armstrong (2006); Jullien et al. (2021), where platforms often adjust their pricing strategies by charging a lower subscription fee on the side that exerts a more substantial influence on the other side. In this particular scenario, sellers have a more prominent effect on buyers. By charging a lower fee to sellers, platform i promotes their participation, which, in turn, attracts more agents on both sides of the market.

The next step is to obtain the difference in platforms' equilibrium profits using Equation (16):

$$\Delta\Pi^i_{\upsilon>\pi} = \left(\alpha^j - \alpha^i\right)\left[\frac{2\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right] + \left(\alpha^j\sigma_\upsilon - 2\tau\right)\left(\alpha^i\sigma_\upsilon - 2\tau\right)}{8\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\right]\Omega_\upsilon^2 \tag{18}$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$ and $\Omega_\upsilon \equiv (3\tau + 2\upsilon)(2\tau - \upsilon)$.

Next, we characterise the impacts on the difference in equilibrium profits considering platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$)

Proposition 7a. *For $\upsilon > \pi$ (i.e., sellers exert a stronger cross-group network effect on buyers' side) the difference in equilibrium profits increases as the degree of product differentiation and the indirect network effect grow. The impact of the cross-group network effect holds as long as $\tau > 4\upsilon$.*

Proof: See Appendix C.6

Proposition 7a shows that when the cross-group network effect exerted by sellers on buyers is stronger, $\upsilon > \pi, (\pi = 0)$, the difference in equilibrium profits increases. This is because as platforms become more valuable to buyers (indicated by higher υ), the profit-increasing strategy involves developing additional attributes if the degree of product differentiation τ is big enough as 4υ. The intuition on why platform 1 develop more attributes is the same as in Proposition 5. This prompts participants from both sides of the market to join, resulting in an additional fee per buyer and seller and ultimately leading to an increase in the platform's profits.

On the contrary, when the degree of product differentiation is below 4υ the difference in equilibrium profits decreases as the cross-group network effect exerted by sellers on buyers increases. This occurs because fewer attributes are developed, discouraging both buyers and sellers (given the cross-group network effect) from joining the platform. Consequently, this behaviour impacts platform revenue by reducing the number of participants available to charge fees, ultimately decreasing its profits.

The result on Proposition 7a aligns with more recent research by Garella and Lambertini

(2014) and Barigozzi and Ma (2018), which suggests that platforms strive to differentiate themselves on both dimensions to maximise profits. Specifically, platforms aim to increase the degree of product differentiation in the horizontal dimension by becoming more heterogeneous, and in the vertical dimension by enhancing features on buyers' side, as buyers are highly valued by platforms. By pursuing these strategies, platforms can effectively increase their profits in the market.

Case 2: When buyers exert a stronger influence on sellers, $\pi > \upsilon\,(\upsilon = 0)$.

Equilibrium membership fees and Platform Profits

In this case, we have:

$$\left(p_b^i\right)\Big|_{\pi>\upsilon,\ \upsilon=0} = f_b + \tau - \pi + \frac{\left(\alpha^j - \alpha^i\right)\left(3\tau^2 - \pi^2\right)\left(3\tau + \pi\right)\left(2\tau - \pi\right)}{2\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{19a}$$

$$\left(p_s^i\right)\Big|_{\pi>\upsilon,\ \upsilon=0} = f_s + \tau + \frac{\left(\alpha^j - \alpha^i\right)\tau\pi\left(3\tau + \pi\right)\left(2\tau - \pi\right)}{2\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{19b}$$

for each $i, j = 1, 2,\ i \neq 2$ and where $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$.

Note that the additional markup on Equation (19b) is positive in platform 1 considering $\alpha^2 > \alpha^1$ and as long as Assumption 1 and Assumption 2 hold. However, on Equation (19a), it turns negative when $3\tau^2 - \pi^2 < 0$ holds true, provided that $\tau < \frac{\pi}{\sqrt{3}}$.[35] When the cross-group network effect exerted by buyers on sellers is stronger than the effect sellers have on buyers ($\pi > \upsilon$), platform 1 also adopts a pricing strategy that deviates from the seminal results presented in Armstrong (2006) as in case 1. Specifically, platform 1 charges a lower subscription fee for buyers, $\left(p_b^1\right)\Big|_{\pi>\upsilon} < \left(p_b^1\right)^{Armstrong}$. Additionally, platform 1 applies an extra markup on sellers' side, $\left(p_s^1\right)\Big|_{\pi>\upsilon} > \left(p_s^1\right)^{Armstrong}$. This sets the stage to develop the following proposition:

Proposition 6b. *For $\pi > \upsilon\,(\upsilon = 0)$, the difference in equilibrium membership fees*

(i) On buyers' side increases and sellers' side decreases when τ increases.

(ii) On buyers' side decreases and on sellers' side increases as the cross-group network effect becomes stronger (i.e., when π increases).

Proof: See Appendix C.5.

It is noteworthy that platform 1's pricing strategy in Proposition 6b is the opposite of Proposition 6a. The reason is as a consequence of the reversal in the strength of the cross-group network effects, form $\upsilon > \pi$, $\pi = 0$ to $\pi > \upsilon$, $\upsilon = 0$.

According to Proposition 6b, platform 1 adjusts its pricing strategy by lowering the equilibrium fee for sellers, acknowledging their higher valuation of interaction with the other side of the market, ($\pi > \upsilon$). This adjustment is in response to a reduction in features on buyers' side, given an increase in the degree of product differentiation (τ).

[35] This condition is compatible with Assumption 1 since $\frac{\pi}{2} < \tau < \frac{\pi}{\sqrt{3}}$.

On the one hand, this strategy discourages buyers from joining the platform, and as a result, it also affects the sellers' participation due to the cross-group network effect. On the other hand, sellers fee reduction attracts more of them and, in turn, encourages buyers to join the platform due to the positive cross-group network effect. However, to compensate for the fee decrease on sellers' side, platform 1 charges a higher fee to buyers.

Furthermore, when the cross-group network effect exerted by buyers on sellers is stronger ($\pi > \upsilon$), platform 1 develops more attributes on buyers' side to appeal to a larger number of participants. This increased attractiveness of the platform to sellers, who value interaction more, leads to a higher equilibrium fee charged to them. At the same time, the platform adopts a pricing policy of lowering buyers' subscription fees. This strategy creates a positive feedback loop, as the lower fees attract more buyers, which in turn further enhances the benefits of platform 1.

The next step is to obtain the difference in platforms' equilibrium profits using Equation (16):

$$\Delta\Pi^i_{\pi>\upsilon} = \left(\alpha^j - \alpha^i\right)\left[\frac{2\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right] + \left(\alpha^j\sigma_\pi - 2\tau\right)\left(\alpha^i\sigma_\pi - 2\tau\right)}{8\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right]^2}\right]\Omega_\pi^2 \tag{20}$$

for each $i, j = 1, 2$, $i \neq 2$ and where $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$ and $\Omega_\pi \equiv (3\tau + \pi)(2\tau - \pi)$.

Next, we characterise the impacts on the difference in equilibrium profits considering platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$)

Proposition 7b. *For $\pi > \upsilon$ (i.e., buyers exert a stronger cross-group network effect on sellers' side) the difference in equilibrium profits increases as the degree of product differentiation grows and decreases as the cross-group network effect rises.*

Proof: See Appendix C.6

It is important to notice that contrary to the previous scenario where the cross-group network effects on both sides are identical when the indirect network effects on both sides of the market are different, the difference in equilibrium profits increase in the degree of product differentiation τ as in the seminal model of Armstrong (2006).

Proposition 7a and Proposition 7b specify that when platforms are more heterogeneous (higher τ) the difference in equilibrium profits increases whether one side influences the other more or vice versa. The mechanism by which this occurs is as follows:

- Platform 1 offers unique and differentiated services compared to the other platform, there is no obligation to develop additional attributes on buyers' side. Consequently, the features available to buyers decrease, which can lead to a decrease in their motivation to continue using or joining platform 1 on both sides of the market.
- If buyers value interaction more than sellers ($\upsilon > \pi$), the platform charges them a lower fee. To balance this, charges a higher fee on sellers' side, as more sellers are expected to join due to the cross-group network effect. This combination of pricing strategies leads to

an increase in the difference in equilibrium profits.

- Conversely, when the cross-group network effect exerted by buyers on sellers is stronger ($\pi > \upsilon$), platform 1 adjusts its pricing strategy by lowering sellers' equilibrium fees. This strategy encourages more buyers to join, driven by the cross-group network effect. To offset the fee decrease on the sellers' side, it charges buyers a higher fee.

As seen in Proposition 7b the result driven from the cross-group network effect may seem counterintuitive. As platform 1 becomes more valuable for both agents (higher π), it develops more features on buyers' side, attracting more participants and generating additional fees per agent. However, the increase in sellers' cross-group network effect enhances their value, leading platforms to compete more intensely to attract sellers. This intensified competition prompts platforms to develop even more attributes on buyers' side (an increase in π increases the difference in equilibrium attributes), escalating competition further. Finally, this results in a decrease in the difference in equilibrium profits.

As in the scenario where the cross-group network effects on both sides of the market are identical, Proposition 7b aligns with the earlier work of Economides (1989) and Neven and Thisse (1990), as well as the generalised model of Irmen and Thisse (1998). This result suggests that platforms strive to maximise their differentiation on one dimension while minimising it on the other to increase profits. Specifically, platforms focus on increasing differentiation in the horizontal dimension by becoming more heterogeneous, while reducing differentiation in the vertical dimension by developing fewer features on buyers' side when the cross-group network effect exerted by sellers decreases.

6 Conclusion

We have introduced a two-stage model for a two-sided market that incorporates the concept of vertical differentiation. By analysing the intricate interplay between quality attributes and cross-group network effects, our research provides valuable insights into various market configurations. This study enables us to explore the relation of price competition, cross-group network effects and platform's quality between two-sided platforms that are differentiated both horizontally and vertically, thus extending the seminal findings of Armstrong (2006); Rochet and Tirole (2002, 2006).

We introduced platform attributes on the buyers' side to account for the vertical dimension. In the first stage of the model, platforms selected the level of attributes they offer to buyers simultaneously. In the second stage, platforms simultaneously chose membership fees. The equilibrium membership fees, market shares, and profits were determined by the difference in attributes on the buyers' side. Although the features were developed only on the buyers' side, they also influenced decisions on the sellers' side. As a result, we demonstrate that vertical differentiation allows for the existence of asymmetric platforms in equilibrium. Overall, our contribution is to provide a comprehensive model that captures the dynamics of competition in two-sided markets with vertical differentiation.

Our study examines two scenarios depending on the strength of the cross-group network effects. Specifically, we consider the following scenarios: Firstly, we explore a case where the indirect network effects on both sides of the market are identical. Secondly, we centre our attention where sellers' cross-group network effect on buyers is stronger than buyers' impact on sellers, normalising sellers' network effect to zero. Then, we analyse where buyers' cross-group network effect on sellers is stronger than sellers' impact on buyers, normalising buyers' network effect to zero. By examining these scenarios, we contribute to the existing literature on two-sided markets by offering insights into the influence of cross-group network effects and attributes as a vertical differentiation variable on platform competition. This knowledge can be leveraged to devise effective strategies that enhance platform performance and support overall market welfare.

Our analysis shows platforms use attributes on the buyers' side as the main trigger to adjust their strategies to appeal to agents and boost profits. We find that the more heterogeneous platforms are (measured by the degree of product differentiation), the fewer attributes they develop on the buyers' side. Whereas the more valuable platforms become given a stronger cross-group network effect, the more attributes are offered on the buyers' side. This mechanism drives platforms to adjust equilibrium membership fees and profits. Our analysis also uncovers interesting insights into the impact of model parameters on equilibrium membership fees, which are contingent on the relative strength of cross-group network effects between the two sides of the market. By providing such granular insights, platforms design optimal pricing strategies in two-sided markets with attributes on the buyers' side.

We also identify the optimal conditions for platforms to maximise their profits by strategically balancing the degree of product differentiation on the horizontal dimension and attributes on the buyers' side on the vertical dimension. This finding aligns with previous research conducted by Garella and Lambertini (2014) and Barigozzi and Ma (2018). Specifically, we observe that this optimal strategy occurs when the cross-group network effect exerted by sellers on buyers is stronger than the impact buyers have on sellers. Moreover, we establish the conditions under which it is optimal to maximise one dimension while minimising the other dimension to enhance profitability. This pattern is consistent with earlier studies, including Economides (1989) and Neven and Thisse (1990), as well as the generalised model proposed by Irmen and Thisse (1998). Particularly, we observe that this optimal strategy occurs when the cross-group network effect exerted by buyers on sellers is stronger than the effect that sellers have on buyers.

Our findings shed light on the strategic trade-offs platforms face in two-sided markets with vertical differentiation seen as attributes on the buyers' side and provide important insights for platform managers and policymakers seeking to optimise their pricing strategies. By understanding the optimal conditions for maximising profits, platforms can enhance their performance and contribute to the overall welfare of the market.

Furthermore, our findings can provide valuable insights for regulators seeking to establish minimum quality standards to identify opportunities to enhance social welfare. However, it is crucial to consider the influence of cross-group network effects on price competition and, consequently, on the welfare of participants. This entails understanding how interactions between buyers and sellers across horizontal and vertical differentiation affect two-sided market dynamics and overall welfare.

One potential extension of the study involves incorporating features on the sellers' side, which would contribute to a more comprehensive model that better reflects real-world dynamics. Additionally, enabling both buyers and sellers to engage in multihoming would provide valuable insights into how platforms define their pricing strategies. In addition, a welfare analysis can be included by comparing the aggregate surpluses of buyers and sellers across the different scenarios. By including these additional features, a more thorough understanding of the platform's decision-making processes can be attained.

A Model

A.1 Model Assumptions

In this section, we show how the model assumptions are defined.

Second-order conditions

First, to guarantee a unique equilibrium where both platforms remain active, the second-order conditions of the platform maximisation problem must be satisfied in both stages of the game. Specifically, the sufficient conditions required for the second-order conditions at stage two are detailed in Appendix A.4 and are (i) $\tau > \sqrt{\pi\upsilon}$ and (ii) $\tau > \frac{(\pi+\upsilon)}{2}$.

Now, we determine which of the two conditions is more stringent, ensuring the other is also met. Initially, since the left side of both inequalities is equal, we compare the right sides to identify the greater one. This yields $\frac{\pi+\upsilon}{2} > \sqrt{\pi\upsilon}$, which can be rewritten as $(\pi+\upsilon)^2 > 4\pi\upsilon$. Further simplification leads to $\pi^2 + 2\pi\upsilon + \upsilon^2 > 4\pi\upsilon$, which simplifies to $(\pi-\upsilon)^2 > 0$ if $\pi \neq \upsilon$. Therefore, if condition (ii) holds, condition (i) is satisfied. Thereby Assumption 1 is established.

Second, the sufficient condition that needs to be set for the second order conditions of the platform maximisation problem at stage one to be satisfied is obtained in Appendix A.8 and is (i) $\alpha^i > \frac{\tau}{\Sigma}$. This condition is satisfied as condition $\alpha^i > \frac{2\tau}{\Sigma}$ is more stringent (this condition guarantees positive equilibrium attributes and is going to be shown next).

Positive Equilibrium Attributes

Third, the conditions to have positive attributes in equilibrium obtained in Appendix A.9 are (i) $\alpha^j > \frac{2\tau}{\Sigma}$, (ii) $\tau > \frac{(\pi+\upsilon)}{2}$ and (iii) $\alpha^i > \frac{\alpha^j\tau}{\Sigma-\tau}$. Next, we show that these conditions are satisfied. For the first condition, we use the fact that platform 1 is more efficient in developing attributes than platform 2, that is $\alpha^2 > \alpha^1$, as was defined in Section 2. Therefore if $\alpha^2 > \alpha^1$ and $\alpha^2 > \frac{2\tau}{\Sigma}$ we derive $\alpha^1 > \frac{2\tau}{\Sigma}$. Then $\alpha^i > \frac{2\tau}{\Sigma}$ for $i = 1, 2$. Thereby Assumption 2 is established. The second condition (ii) $\tau > \frac{(\pi+\upsilon)}{2}$ is the same as Assumption 1. The third condition (iii) $\alpha^i > \frac{\alpha^j\tau}{\Sigma-\tau}$ is satisfied if Assumption 2 is more stringent. We show this by comparing the right side of both inequalities, then if the right side of Assumption 2 is greater, the condition is satisfied. Next, comparing the right side we have $\frac{2\tau}{\Sigma} > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ which simplifies to $2\left(\alpha^j\Sigma - \tau\right) - \alpha^j\Sigma > 0$ and simplifies to $\alpha^j\Sigma - 2\tau > 0$ if $\alpha^j > \frac{2\tau}{\Sigma}$, which is the same Assumption 2. Therefore if Assumption 2 holds, condition $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ is satisfied.

Equilibrium market shares

Fourth, the conditions to have equilibrium market shares on both sides within the unit interval, $0 < \eta_b^i < 1$ and $0 < \eta_s^i < 1$, obtained in Appendix C.2 are (i) $\tau < \frac{\pi+2\upsilon}{3}$ if $\upsilon > \pi$ or $\tau > \frac{\pi+2\upsilon}{3}$ if $\pi > \upsilon$. Furthermore, (ii) $\tau > \sqrt{\frac{(\pi+\upsilon)(\pi+2\upsilon)}{6}}$ and (iii) $\tau > \frac{\pi+2\upsilon}{3}$. Now, we show these conditions are satisfied using Assumption 1. For (i) $\tau > \frac{\pi+2\upsilon}{3}$, we compare the right sides of the inequalities to show that the right side of Assumption 1 is more stringent and therefore condition (i) is met. That is $\frac{\pi+\upsilon}{2} > \frac{\pi+2\upsilon}{3}$ which simplifies to $3(\pi+\upsilon) > 2(\pi+2\upsilon)$, which further simplifies to $\pi - \upsilon > 0$ if $\pi > \upsilon$. For (ii), we use the same method comparing the right side of both inequalities and showing the right side of Assumption 1 is more stringent and therefore condition (ii) is satisfied. That is $\frac{\pi+\upsilon}{2} > \sqrt{\frac{(\pi+\upsilon)(\pi+2\upsilon)}{6}}$ which simplifies to $3(\pi+\upsilon) - 2(\pi+2\upsilon) > 0$ and further simplifies to $\pi - \upsilon > 0$ if $\pi > \upsilon$. For (iii) we have $\frac{\pi+\upsilon}{2} > \frac{(\pi+2\upsilon)}{3}$ which turns to $3(\pi+\upsilon) > 2(\pi+2\upsilon)$ which simplifies to $\pi - \upsilon > 0$ if $\pi > \upsilon$.

To summarise, the assumptions we are establishing are (i) $\tau > \frac{\pi+\upsilon}{2}$ if $\pi > \upsilon$, and $\frac{\pi+\upsilon}{2} < \tau < \frac{\pi+2\upsilon}{3}$ if $\upsilon > \pi$, (ii) $\alpha^i > \frac{2\tau}{\Sigma}$.

A.2 Market's Shares

To get the proportion of buyers and sellers at Equations (2a) and (2b) we use Equations (1a) and (1b). For buyers $\eta_b^i = \frac{1}{2} + \frac{\nu_b^i - \nu_b^j}{2\tau}$ turns to $\eta_b^i = \frac{1}{2} + \frac{1}{2\tau}\left[R_b + q_b^i + \upsilon\eta_s^i - p_b^i - \left(R_b + q_b^j + \upsilon\eta_s^j - p_b^j\right)\right]$ turns to $2\tau\eta_b^i = \tau + \upsilon\left(\eta_s^i - \eta_s^j\right) + q_b^i - q_b^j + \left(p_b^j - p_b^i\right)$. Then, since $\eta_b^i + \eta_b^j = 1$ and $\eta_s^i + \eta_s^j = 1$ we have $2\tau\eta_b^i = \tau + \upsilon\left(2\eta_s^i - 1\right) + \left(q_b^i - q_b^j\right) + \left(p_b^j - p_b^i\right)$ and turns to $\eta_b^i = \frac{\tau + \left(2\eta_s^i - 1\right)\upsilon + \left(q_b^i - q_b^j\right) + \left(p_b^j - p_b^i\right)}{2\tau}$.

For sellers $\eta_s^i = \frac{1}{2} + \frac{\nu_s^i - \nu_s^j}{2\tau}$ turns to $\eta_s^i = \frac{1}{2} + \frac{1}{2\tau}\left[R_s + \pi\eta_b^i - p_s^i - \left(R_s + \pi\eta_b^j - p_s^j\right)\right]$ turns to $2\tau\eta_s^i = \tau + \pi\left(\eta_b^i - \eta_b^j\right) + \left(p_s^j - p_s^i\right)$. Then, since $\eta_b^i + \eta_b^j = 1$ and $\eta_s^i + \eta_s^j = 1$ we have $2\tau\eta_s^i = \tau + \pi\left(2\eta_b^i - 1\right) + \left(p_s^j - p_s^i\right)$ and turns to $\eta_s^i = \frac{\tau + \left(2\eta_b^i - 1\right)\pi + \left(p_s^j - p_s^i\right)}{2\tau}$. Then we have:

$$\eta_b^i = \frac{\tau + \left(2\eta_s^i - 1\right)\upsilon + \left(q_b^i - q_b^j\right) + \left(p_b^j - p_b^i\right)}{2\tau} \tag{1}$$

$$\eta_s^i = \frac{\tau + \left(2\eta_b^i - 1\right)\pi + \left(p_s^j - p_s^i\right)}{2\tau} \tag{2}$$

We solve the previous system of equations to obtain η_b^i and η_s^i as a function of membership fees. First, we find the value of $\left(2\eta_s^i - 1\right)$ from equation (2) and substitute this value into equation 1 and then solve for η_b^i. That is, from equation (2) we have $2\eta_s^i - 1 = \frac{1}{\tau}\left[\left(2\eta_b^i - 1\right)\pi + \left(p_s^j - p_s^i\right)\right]$, then we substitute it in equation (1) $2\tau\eta_b^i = \tau + \left(q_b^i - q_b^j\right) + \left(p_b^j - p_b^i\right) + \frac{\upsilon}{\tau}\left[\left(2\eta_b^i - 1\right)\pi + \left(p_s^j - p_s^i\right)\right]$ turns to $2\left(\tau^2 - \pi\upsilon\right)\eta_b^i = \left(\tau^2 - \pi\upsilon\right) - \pi\upsilon + \tau\left(q_b^i - q_b^j\right) + \tau\left(p_b^j - p_b^i\right) + \upsilon\left(p_s^j - p_s^i\right)$. Then it turns to $\eta_b^i = \frac{1}{2} + \frac{\tau\left(q_b^i - q_b^j\right) + \upsilon\left(p_s^j - p_s^i\right) + \tau\left(p_b^j - p_b^i\right)}{2\left(\tau^2 - \pi\upsilon\right)}$. Then we substitute the previous result into equation (2) to get $\eta_s^i = \frac{1}{2} + \frac{\pi\left(q_b^i - q_b^j\right) + \pi\left(p_b^j - p_b^i\right) + \tau\left(p_s^j - p_s^i\right)}{2\left(\tau^2 - \pi\upsilon\right)}$. The solution for the system of equations (1) and (2) are:

$$\eta_b^i = \frac{1}{2} + \frac{\tau\left(q_b^i - q_b^j\right) + \upsilon\left(p_s^j - p_s^i\right) + \tau\left(p_b^j - p_b^i\right)}{2\left(\tau^2 - \pi\upsilon\right)}$$

$$\eta_s^i = \frac{1}{2} + \frac{\pi\left(q_b^i - q_b^j\right) + \pi\left(p_b^j - p_b^i\right) + \tau\left(p_s^j - p_s^i\right)}{2\left(\tau^2 - \pi\upsilon\right)}$$

A.3 Maximisation Problem - stage 2

Platforms maximise the next expression concerning both sides' membership fees to have:

$$\max_{\{p_b^i, p_s^i\}} \Pi^i \equiv \left(p_b^i - f_b\right) \eta_b^i \left(p_b^i, p_s^i, p_b^j, p_s^j\right) + \left(p_s^i - f_s\right) \eta_s^i \left(p_b^i, p_s^i, p_b^j, p_s^j\right) - \frac{\alpha^i \left(q_b^i\right)^2}{2}$$

The first-order conditions for platform $i = 1, 2$:

$$\frac{\partial \Pi^i}{\partial p_b^i} = \eta_b^i + \frac{\partial \eta_b^i}{\partial p_b^i} \left(p_b^i - f_b\right) + \frac{\partial \eta_s^i}{\partial p_b^i} \left(p_s^i - f_s\right) = 0$$

$$\frac{\partial \Pi^i}{\partial p_s^i} = \frac{\partial \eta_b^i}{\partial p_s^i} \left(p_b^i - f_b\right) + \eta_s^i + \frac{\partial \eta_s^i}{\partial p_s^i} \left(p_s^i - f_s\right) = 0$$

Using Equations (2a) and (2b) the first-order conditions for platform i turn to Platform 1 first-order conditions:

$$\frac{\partial \Pi^1}{\partial p_b^1} = \frac{1}{2} + \frac{\tau \left(q_b^1 - q_b^2\right) + \tau \left(p_b^2 - p_b^1\right) + \upsilon \left(p_s^2 - p_s^1\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\tau \left(p_b^1 - f_b\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\pi \left(p_s^1 - f_s\right)}{2\left(\tau^2 - \pi\upsilon\right)} = 0$$
$$\frac{\partial \Pi^1}{\partial p_s^1} = \frac{1}{2} + \frac{\pi \left(q_b^1 - q_b^2\right) + \tau \left(p_s^2 - p_s^1\right) + \pi \left(p_b^2 - p_b^1\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\tau \left(p_s^1 - f_s\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\upsilon \left(p_b^1 - f_b\right)}{2\left(\tau^2 - \pi\upsilon\right)} = 0$$

Platform 2 first-order conditions:

$$\frac{\partial \Pi^2}{\partial p_b^2} = \frac{1}{2} + \frac{\tau \left(q_b^2 - q_b^1\right) + \tau \left(p_b^1 - p_b^2\right) + \upsilon \left(p_s^1 - p_s^2\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\tau \left(p_b^2 - f_b\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\pi \left(p_s^2 - f_s\right)}{2\left(\tau^2 - \pi\upsilon\right)} = 0$$
$$\frac{\partial \Pi^2}{\partial p_s^2} = \frac{1}{2} + \frac{\pi \left(q_b^2 - q_b^1\right) + \tau \left(p_s^1 - p_s^2\right) + \pi \left(p_b^1 - p_b^2\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\tau \left(p_s^2 - f_s\right)}{2\left(\tau^2 - \pi\upsilon\right)} - \frac{\upsilon \left(p_b^2 - f_b\right)}{2\left(\tau^2 - \pi\upsilon\right)} = 0$$

From the first-order conditions on both platforms, we obtain:

$$2\tau p_b^i + (\pi + \upsilon)\, p_s^i - \tau p_b^j - \upsilon p_s^j = \tau f_b + \pi f_s + \left(\tau^2 - \pi\upsilon\right) + \tau \left(q_b^i - q_b^j\right) \tag{b1}$$
$$(\pi + \upsilon)\, p_b^i + 2\tau p_s^i - \pi p_b^j - \tau p_s^j = \tau f_s + \upsilon f_b + \left(\tau^2 - \pi\upsilon\right) + \pi \left(q_b^i - q_b^j\right) \tag{b2}$$
$$-\tau p_b^i - \upsilon p_s^i + 2\tau p_b^j + (\pi + \upsilon)\, p_s^j = \tau f_b + \pi f_s + \left(\tau^2 - \pi\upsilon\right) + \tau \left(q_b^j - q_b^i\right) \tag{b3}$$
$$-\pi p_b^i - \tau p_s^i + (\pi + \upsilon)\, p_b^j + 2\tau p_s^j = \tau f_s + \upsilon f_b + \left(\tau^2 - \pi\upsilon\right) + \pi \left(q_b^j - q_b^i\right) \tag{b4}$$

Then, we solve for p_s^j in equation (b3) and then substitute it into equations (b1), (b2) and (b4) to obtain:

$$\tau\left(2\pi + \upsilon\right) p_b^i + \pi\left(\pi + 2\upsilon\right) p_s^i + \tau\left(\upsilon - \pi\right) p_b^j = \tau\left(\pi + 2\upsilon\right) f_b + \pi\left(\pi + 2\upsilon\right) f_s + \left(\tau^2 - \pi\upsilon\right)\left(\pi + 2\upsilon\right) + \tau\pi\left(q_b^i - q_b^j\right) \tag{b5}$$

$$-\left[\tau^2-(\pi+\upsilon)^2\right]p_b^i+\tau\,(2\pi+\upsilon)\,p_s^i+\left[2\tau^2-\pi\,(\pi+\upsilon)\right]p_b^j=\left[\tau^2+\upsilon\,(\pi+\upsilon)\right]f_b$$
$$+\tau\,(2\pi+\upsilon)\,f_s+(\tau+(\pi+\upsilon))\left(\tau^2-\pi\upsilon\right)-\left(\tau^2-\pi\,(\pi+\upsilon)\right)\left(q_b^i-q_b^j\right) \quad \text{(b6)}$$

$$\left[2\tau^2-\pi\,(\pi+\upsilon)\right]p_b^i+\tau\,(\upsilon-\pi)\,p_s^i-\left[4\tau^2-(\pi+\upsilon)^2\right]p_b^j=-\left[2\tau^2-\upsilon\,(\pi+\upsilon)\right]f_b$$
$$+\tau\,(\upsilon-\pi)\,f_s-(2\tau-(\pi+\upsilon))\left(\tau^2-\pi\upsilon\right)+\left[2\tau^2-\pi\,(\pi+\upsilon)\right]\left(q_b^i-q_b^j\right) \quad \text{(b7)}$$

Then, we solve for p_b^j in equation (b7) and substitute it into equation (b5) and (b6) to obtain:

$$\tau\left[6\tau^2-(\pi+\upsilon)^2-2\pi\upsilon\right]p_b^i+\left[\tau^2\,(5\pi+\upsilon)-\pi\,(\pi+\upsilon)\,(\pi+2\upsilon)\right]p_s^i=\tau\big[6\tau^2-(\pi+\upsilon)^2$$
$$-2\pi\upsilon\big]f_b+\left[\tau^2\,(5\pi+\upsilon)-\pi\,(\pi+\upsilon)\,(\pi+2\upsilon)\right]f_s+\left[6\tau^2-(\pi+\upsilon)\,(\pi+2\upsilon)\right]\left(\tau^2-\pi\upsilon\right)$$
$$+\tau\,(\upsilon-\pi)\left(\tau^2-\pi\upsilon\right)+2\tau\left(\tau^2-\pi\upsilon\right)\left(q_b^i-q_b^j\right) \quad \text{(b8)}$$

$$\left[\tau^2\,(\pi+5\upsilon)-\upsilon\,(\pi+\upsilon)\,(2\pi+\upsilon)\right]p_b^i+\tau\left[6\tau^2-(\pi+\upsilon)^2-2\pi\upsilon\right]p_s^i=\big[\tau^2\,(\pi+5\upsilon)$$
$$-\upsilon\,(\pi+\upsilon)\,(2\pi+\upsilon)\big]f_b+\tau\left[6\tau^2-(\pi+\upsilon)^2-2\pi\upsilon\right]f_s+\left[6\tau^2-(\pi+\upsilon)\,(2\pi+\upsilon)\right]\left(\tau^2-\pi\upsilon\right)$$
$$+\tau\,(\pi-\upsilon)\left(\tau^2-\pi\upsilon\right)+(\pi+\upsilon)\left(\tau^2-\pi\upsilon\right)\left(q_b^i-q_b^j\right) \quad \text{(b9)}$$

Next, we solve for p_s^i in equation (b9) and then substitute it into equation (b8) to express p_b^i as a function of the model parameter and the attributes developed on buyers' side. Subsequently, we substitute this outcome into equation (b9) to obtain:

$$p_b^i=f_b+\tau-\pi+\left[\frac{3\tau^2-\pi\,(\pi+2\upsilon)}{9\tau^2-(2\pi+\upsilon)\,(\pi+2\upsilon)}\right]\left(q_b^i-q_b^j\right)$$
$$p_s^i=f_s+\tau-\upsilon-\left[\frac{\tau\,(\upsilon-\pi)}{9\tau^2-(2\pi+\upsilon)\,(\pi+2\upsilon)}\right]\left(q_b^i-q_b^j\right)$$

for $i,j=1,2,\quad i\neq j$.

A.4 Second-order conditions at stage 2

We obtain the following second-order conditions from the profit maximisation problem at stage 2 of the game in Appendix A.3, which define the Hessian matrix as:

$$\mathrm{H}=\begin{pmatrix}\Pi^i_{p_b^ip_b^i}\equiv\frac{\partial^2\Pi^i}{\partial(p_b^i)^2}=-\frac{\tau}{(\tau^2-\pi\upsilon)} & \Pi^i_{p_b^ip_s^i}\equiv\frac{\partial^2\Pi^i}{\partial p_b^i\,\partial p_s^i}=-\frac{(\pi+\upsilon)}{2(\tau^2-\pi\upsilon)}\\ \Pi^i_{p_s^ip_b^i}\equiv\frac{\partial^2\Pi^i}{\partial p_s^i\,\partial p_b^i}=-\frac{(\pi+\upsilon)}{2(\tau^2-\pi\upsilon)} & \Pi^i_{p_s^ip_s^i}\equiv\frac{\partial^2\Pi^i}{\partial(p_s^i)^2}=-\frac{\tau}{(\tau^2-\pi\upsilon)}\end{pmatrix}$$

In order to guarantee that platforms' profits reach a maximum with equilibrium fees in Equations (4a) and (4b) a sufficient condition is having H negative definite, indicating that $|H|>0$, and either $\Pi^i_{p_bp_b}<0$ or $\Pi^i_{p_sp_s}<0$. To show $\Pi^i_{p_bp_b}$ and $\Pi^i_{p_sp_s}$ are negative, the denominator $\tau^2 - \pi\upsilon$ must be positive because the numerator is always positive, then we get $\tau^2 > \pi\upsilon$ that turns to $\tau > \sqrt{\pi\upsilon}$. To show $|H| > 0$ we have $\frac{\tau^2}{(\tau^2-\pi\upsilon)^2} - \frac{(\pi+\upsilon)^2}{4(\tau^2-\pi\upsilon)^2} > 0$ that turns to $4\tau^2 - (\pi+\upsilon)^2 > 0$, that turns to $\tau > \frac{\pi+\upsilon}{2}$.

In summary, for the second-order conditions defined by the Hessian matrix to be negative definite, the following conditions must hold (i) $\tau > \sqrt{\pi\upsilon}$ and (ii) $\tau > \frac{\pi+\upsilon}{2}$.

Now, we determine which of the two conditions is more stringent, ensuring that the other condition is also met. Initially, since the left side of both inequalities is equal, we compare the right sides to identify the greater one. That is $\frac{\pi+\upsilon}{2} > \sqrt{\pi\upsilon}$, which turns to $(\pi+\upsilon)^2 > 4\pi\upsilon$. Further simplification leads to $\pi^2 + 2\pi\upsilon + \upsilon^2 > 4\pi\upsilon$, which simplifies to $(\pi-\upsilon)^2 > 0$ if $\pi \neq \upsilon$. Therefore, if condition (ii) holds, condition (i) is satisfied.

A.5 Proof of Proposition 1

Proof. Partially differentiate equilibrium membership fees at stage one of the game in Equation 4a and Equation 4b regarding the difference in attributes on buyers' side. First, we define $\Delta q_b^i \equiv q_b^i - q_b^j$. Now, on buyers' side we have $\frac{\partial p_b^i}{\partial \Delta q_b^i} = \frac{3\tau^2 - \pi(\pi+2\upsilon)}{9\tau^2-(2\pi+\upsilon)(\pi+2\upsilon)}$. To demonstrate that the previous expression is positive is sufficient to show both the numerator and denominator are positive. The denominator is positive if this condition $9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$ is positive. We use Assumption 1 to show $9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$ is positive. First, we make the left side of both inequalities equivalent to compare the right side, showing that Assumption 1 right side is greater and therefore the condition is positive. Assumption 1 can be transform to be $\tau^2 > \frac{(\pi+\upsilon)^2}{4}$, then we have $\frac{(\pi+\upsilon)^2}{4} > \frac{(2\pi+\upsilon)(\pi+2\upsilon)}{9}$ which simplifies to $9\left(\pi^2+2\pi\upsilon+\upsilon^2\right) - 4\left(2\pi^2+5\pi\upsilon+2\upsilon^2\right) > 0$ and then simplifies to $(\pi-\upsilon)^2 > 0$ if $\pi \neq \upsilon$. Therefore, $9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$ is positive.

Following, we use the same method to show $3\tau^2 - \pi(\pi+2\upsilon)$ is positive by comparing the right side of both inequalities and showing Assumption 1 right side is greater, so the condition is positive. That is $\frac{(\pi+\upsilon)^2}{4} > \frac{\pi(\pi+2\upsilon)}{3}$ which turns to $3\upsilon^2 - 2\pi\upsilon - \pi^2 > 0$ which simplifies to $(3\upsilon+\pi)(\upsilon-\pi) > 0$ if $\upsilon > \pi$. Therefore $\partial p_b^i / \partial \Delta q_b^i > 0$ if $\upsilon > \pi$.

On sellers' side we have $\frac{\partial p_s^i}{\partial \Delta q_b^i} = \frac{\tau_s(\pi-\upsilon)}{9\tau^2-(2\pi+\upsilon)(\pi+2\upsilon)}$ which is positive if $\pi > \upsilon$, considering we showed the denominator $9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$ is always positive as long as Assumption 1 holds. Therefore $\partial p_s^i / \partial \Delta q_b^i > 0$ if $\pi > \upsilon$. □

A.6 Buyers and sellers Market-shares

We obtain equilibrium market shares at stage two of the model in Equations (5a) and (5b) using membership fees in Equations (4a) and (4b). First, we compute the difference in membership fees on both sides of the market, $p_b^j - p_b^i = -\frac{2}{\Sigma}\left[3\tau^2 - \pi(\pi+2\upsilon)\right]\left(q_b^i - q_b^j\right)$ and $p_s^j - p_s^i = -\frac{2\tau}{\Sigma}(\pi-\upsilon)\left(q_b^i - q_b^j\right)$. Where $\Sigma \equiv 9\tau^2 - (2\pi+\upsilon)(2\upsilon+\pi)$. Then we substitute these expressions into Equations (2a) and (2b) to get:

$$\eta_b^i = \frac{1}{2} + \frac{\tau\left[9\tau^2 - (2\pi+\upsilon)(2\upsilon+\pi) - 2\left(3\tau^2 - \pi(\pi+2\upsilon)\right) - 2\upsilon(\pi-\upsilon)\right]\left(q_b^i - q_b^j\right)}{2\left(9\tau^2 - (2\pi+\upsilon)(2\upsilon+\pi)\right)\left(\tau^2 - \pi\upsilon\right)}$$
$$= \frac{1}{2} + \frac{3\tau^2 - \upsilon(\pi+2\upsilon) - 2\upsilon(\pi-\upsilon)}{2\left(9\tau^2 - (2\pi+\upsilon)(2\upsilon+\pi)\right)\left(\tau^2 - \pi\upsilon\right)}$$

$$\eta_b^i = \frac{1}{2} + \left[\frac{3\tau}{2\left[9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)\right]}\right]\left(q_b^i - q_b^j\right)$$
$$\eta_s^i = \frac{1}{2} + \frac{\left[\pi\left(9\tau^2 - (2\pi+\upsilon)(2\upsilon+\pi)\right) - 2\tau^2(\pi-\upsilon) - 2\pi\left(3\tau^2 - \pi(\pi+2\upsilon)\right)\right]\left(q_b^i - q_b^j\right)}{2\left(9\tau^2 - (2\pi+\upsilon)(2\upsilon+\pi)\right)\left(\tau^2 - \pi\upsilon\right)}$$
$$= \frac{3\tau^2\pi - \pi\upsilon(\pi+2\upsilon) - 2\tau^2(\pi-\upsilon)}{2\left(9\tau^2 - (2\pi+\upsilon)(2\upsilon+\pi)\right)\left(\tau^2 - \pi\upsilon\right)}$$
$$\eta_s^i = \frac{1}{2} + \left[\frac{(\pi+2\upsilon)}{2\left[9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)\right]}\right]\left(q_b^i - q_b^j\right)$$

A.7 Attributes Maximisation Problem - Stage 1

The first-order conditions of the platform i, $i = 1, 2$ maximisation problem at stage 1 come from maximising Equation 6, that is:

$$\frac{\partial \Pi^1}{\partial q_b^1} = \frac{2\tau\left(q_b^1 - q_b^2\right) + \left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)\right]}{2\left[9\tau^2 - (2\pi-\upsilon)(\pi+2\upsilon)\right]} - \alpha^1 q_b^1 = 0$$
$$\frac{\partial \Pi^2}{\partial q_b^2} = \frac{2\tau\left(q_b^2 - q_b^1\right) + \left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)\right]}{2\left[9\tau^2 - (2\pi-\upsilon)(\pi+2\upsilon)\right]} - \alpha^2 q_b^2 = 0$$

From the first-order conditions on both platforms, we obtain:

$$2\left[\alpha^i\left[9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)\right] - \tau\right] q_b^i = -2\tau q_b^j + 6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi) \quad \text{(b10)}$$
$$2\left[\alpha^j\left[9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)\right] - \tau\right] q_b^j = -2\tau q_b^i + 6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi) \quad \text{(b11)}$$

Then, we solve for q_b^j on both equations (b10) and (b11), then we compare them to get q_b^i, that is:

$$\frac{1}{2\tau}\left[-2\left(\alpha^i\Sigma - \tau\right) q_b^i + \left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)\right]\right] =$$
$$\frac{1}{2\left(\alpha^j\Sigma - \tau\right)}\left[-2\tau q_b^i + \left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)\right]\right]$$

Where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$

$$\begin{aligned}
2\left(\alpha^i\Sigma - \tau\right)\left(\alpha^j\Sigma - \tau\right)q_b^i + \tau\left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon - \pi)\right] &= 2\tau^2 q_b^i \\
&+ \left(\alpha^j\Sigma - \tau\right)\left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)\right] \\
2\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]q_b^i &= \left(\alpha^j\Sigma - 2\tau\right)\left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)\right] \\
q_b^i = \frac{\left(\alpha^j\Sigma - 2\tau\right)\left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)\right]}{2\Sigma\left[\alpha^i\alpha^j\Sigma - (\alpha^i + \alpha^j)\tau\right]} &\quad \text{for } i,j = 1,2, \quad i \neq j
\end{aligned}$$

where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$

A.8 Second-order conditions at stage 1

We obtain the following second-order condition from the profit maximisation at stage 1 of the game in Appendix A.4 as:

$$\Pi^i_{q_b^i q_b^i} \equiv \frac{\partial^2 \Pi^i}{\partial (q_b^i)^2} = \frac{\tau}{9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)} - \alpha^i = 0$$

To guarantee that platforms' profits reach a maximum at stage 2 of the game with equilibrium attributes in Equation (8), a sufficient condition is to have the previous second partial derivative negative. To show $\Pi^i_{q_b^i q_b^i} < 0$ is sufficient to have $\alpha^i > \frac{\tau}{\Sigma}$ where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$

A.9 Positive Equilibrium Attributes

In order to achieve positive equilibrium attributes in Equation (8), we require the following: $q_b^i = \frac{\left(\alpha^j\Sigma - 2\tau\right)\left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)\right]}{2\Sigma\left[\alpha^i\alpha^j\Sigma - (\alpha^i+\alpha^j)\tau\right]} > 0$. Where $\Sigma \equiv 9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$. Initially, we see that q_b^i is made up of three different elements. Let's call $\left(\alpha^j\Sigma - 2\tau\right)$ part one, $\left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)\right]$ part two and $2\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]$ part three. Then it is sufficient to show that all three parts are positive to confirm positive equilibrium attributes.

Firstly, element number one $\left(\alpha^j\Sigma - 2\tau\right)$ is positive if $\alpha^j > \frac{2\tau}{\Sigma}$. Now, we show it is satisfied using the fact that platform 1 is more efficient in developing attributes than platform 2, that is $\alpha^2 > \alpha^1$ as was defined in Section 2. Therefore if $\alpha^2 > \alpha^1$ and $\alpha^2 > \frac{2\tau}{\Sigma}$ we derive $\alpha^1 > \frac{2\tau}{\Sigma}$. Then we obtain $\alpha^i > \frac{2\tau}{\Sigma}$ for $i = 1,2$ which is Assumption 2.

Secondly, for element number two $6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)$ to be positive, we determine a value for τ that ensures the entire expression is positive. We rearrange the expression as a quadratic polynomial in τ, that is $6\tau^2 - (\pi-\upsilon)\tau - (\pi+\upsilon)(\pi+2\upsilon)$. Then, employing the quadratic formula to find the roots, we obtain $\tau = \frac{-[-(\pi-\upsilon)] \pm \sqrt{[-(\pi-\upsilon)]^2 - 4(\pi+\upsilon)(\pi+2\upsilon)(-6)}}{12}$ which simplifies to $\tau = \frac{(\pi-\upsilon) \pm (5\pi+7\upsilon)}{12}$. The first root is $\tau_{r_1} = \frac{\pi+\upsilon}{2}$ and the second root is $\tau_{r_2} = \frac{-(\pi+2\upsilon)}{3}$. Since the square term of the polynomial in τ is positive, the expression $6\tau^2 - (\pi-\upsilon)\tau - (\pi+\upsilon)(\pi+2\upsilon)$ is positive for values outside both roots, that is for $\tau > \frac{\pi+\upsilon}{2}$ and for $\tau < \frac{-(\pi+2\upsilon)}{3}$. Since transportation cost τ is positive by definition, values for $\tau < \frac{-(\pi+2\upsilon)}{3}$ are dismissed. Consequently, $6\tau^2 - (\pi-\upsilon)\tau - (\pi+\upsilon)(\pi+2\upsilon) > 0$ if $\tau > \frac{\pi+\upsilon}{2}$, as stated in

Assumption 1.

Finally, we show the third component is positive. First, we show $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ is positive by using Assumption 1. We make the left side of both inequalities equal to compare the right side, showing that Assumption 1 right side is greater and therefore proving $\Sigma > 0$. That is, Assumption 1 can be transform to be $\tau^2 > \frac{(\pi+\upsilon)^2}{4}$, then we have $\frac{(\pi+\upsilon)^2}{4} > \frac{(2\pi+\upsilon)(\pi+2\upsilon)}{9}$ which simplifies to $9\left(\pi^2 + 2\pi\upsilon + \upsilon^2\right) - 4\left(2\pi^2 + 5\pi\upsilon + 2\upsilon^2\right) > 0$ and simplifies to $(\pi - \upsilon)^2 > 0$ if $\pi \neq \upsilon$. So we have demonstrated Σ is positive. Now, for $\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]$ to be positive it is sufficient to have $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma - \tau}$. Then, we show it is satisfied if Assumption 2 is more stringent than the previous condition. We compare the right side of both inequalities, that is $\frac{2\tau}{\Sigma} > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ which simplifies to $2\left(\alpha^j\Sigma - \tau\right) - \alpha^j\Sigma > 0$ and simplifies to $\alpha^j\Sigma - 2\tau > 0$ if $\alpha^j > \frac{2\tau}{\Sigma}$, which is Assumption 2. Then if Assumption 2 holds, condition $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ is satisfied, which leads us to have proven the third element of q_b^i to be positive.

Summarising, $q_b^i > 0$ if $\alpha^j > \frac{2\tau}{\Sigma}$, $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ and $\tau > \frac{(\pi+\upsilon)}{2}$ which are satisfied as long as Assumption 1 and Assumption 2 hold.

B Benchmark Scenario: $\pi = \upsilon$

When $\upsilon = \pi$ Assumption 1 turns to $\tau > \pi$ and Assumption 2 turns to $\alpha^i > \frac{2\tau}{9\sigma}$, where $\sigma \equiv \tau^2 - \pi^2$.

B.1 Proof of Proposition 2 and Corollary 1

Proof. We prove Proposition 2 by partially differentiate Equation (10) with respect to τ and π under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$.

$$\frac{\partial\left(q_b^i\right)^{bs}}{\partial\tau} = \frac{3\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]\left(18\alpha^j\tau - 2\right) - 3\left(9\alpha^j\sigma - 2\tau\right)\left[18\alpha^i\alpha^j\tau - \left(\alpha^i + \alpha^j\right)\right]}{9\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2}$$
$$= \frac{18\alpha^i\alpha^j\left(9\alpha^j\tau - 1\right) - 2\tau\left(\alpha^i + \alpha^j\right)\left(9\alpha^j\tau - 1\right) - 18\alpha^i\alpha^j\tau\left(9\alpha^j\sigma - \tau\right) + \left(\alpha^i + \alpha^j\right)\left(9\alpha^j\sigma - \tau\right)}{3\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2}$$
$$\frac{\partial\left(q_b^i\right)^{bs}}{\partial\tau} = \frac{18\alpha^i\alpha^j\left(2\tau^2 - \sigma\right) - 9\alpha^j\left(\alpha^i + \alpha^j\right)\left(2\tau^2 - \sigma\right)}{3\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2} = \frac{-3\alpha^j\left(\alpha^j - \alpha^i\right)\left(\tau + \pi^2\right)}{\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2}$$

$$\frac{\partial\left(q_b^i\right)^{bs}}{\partial\pi} = \frac{-54\alpha^j\pi\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right] + 54\alpha^i\alpha^j\pi\left(9\alpha^j\sigma - 2\tau\right)}{9\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2}$$
$$\frac{\partial\left(q_b^i\right)^{bs}}{\partial\pi} = \frac{6\alpha^j\pi\left[-2\alpha^i\tau + \left(\alpha^i + \alpha^j\right)\tau\right]}{\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2} = \frac{6\alpha^j\tau\pi\left(\alpha^j - \alpha^i\right)}{\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2}$$

Now, to know the signs of both partial derivatives for platform 1, we need to find out the signs of their elements. The denominators are positive given they are squared. The elements on the numerators are positive considering platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$). Therefore, $\frac{\partial\left(q_b^i\right)^{bs}}{\partial\tau} = \frac{-3\alpha^j\left(\alpha^j-\alpha^i\right)\left(\tau+\pi^2\right)}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} < 0$ and $\frac{\partial\left(q_b^i\right)^{bs}}{\partial\pi} =$

$\frac{6\alpha^j(\alpha^j-\alpha^i)\tau\pi}{[9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau]^2} > 0.$

$$\text{Furthermore,} \quad \frac{\partial\left(q_b^i\right)^{bs}}{\partial \pi} - \frac{\partial\left(q_b^j\right)^{bs}}{\partial \pi} = \frac{6\alpha^j\left(\alpha^j-\alpha^i\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} - \frac{6\alpha^j\left(\alpha^i-\alpha^j\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} = \frac{6\left(\alpha^i+\alpha^j\right)\left(\alpha^j-\alpha^i\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} > 0$$

The difference between the partial derivatives of the equilibrium attributes with respect to the cross-group network effect on both platforms, $\frac{\partial(q_b^1)^{bs}}{\partial\pi}$ and $\frac{\partial(q_b^2)^{bs}}{\partial\pi}$ is positive given the same argument shown previously.

Next, we prove Corollary 1 by partially differentiate $\left(\Delta q_b^i\right)^{bs} = q_b^i - q_b^j$ with respect to τ and π, considering platform 1 is more efficient in developing attributes compared to platform 2, ($\alpha^2 > \alpha^1$). Firstly, We use Equation (10) to compute $\left(\Delta q_b^i\right)^{bs}$, which is $\left(\Delta q_b^i\right)^{bs} = \frac{(9\alpha^j\sigma-2\tau)-(9\alpha^i\sigma-2\tau)}{3[9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau]}$, which simplifies to $\left(\Delta q_b^i\right)^{bs} = \frac{3\sigma(\alpha^j-\alpha^i)}{9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau}$. Then, we have:

$$\frac{\partial\left(\Delta q_b^i\right)^{bs}}{\partial\tau} = \frac{6\tau\left(\alpha^j-\alpha^i\right)\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]-3\sigma\left(\alpha^j-\alpha^i\right)\left[18\alpha^i\alpha^j\tau-\left(\alpha^i+\alpha^j\right)\right]}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$$
$$= \frac{3\left(\alpha^j-\alpha^i\right)\left[-2\left(\alpha^i+\alpha^j\right)\tau^2+\sigma\left(\alpha^i+\alpha^j\right)\right]}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} = \frac{-3\left(\alpha^i+\alpha^j\right)\left(\alpha^j-\alpha^i\right)\left(\tau+\pi^2\right)}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$$

$$\frac{\partial\left(\Delta q_b^i\right)^{bs}}{\partial\pi} = \frac{6\pi\left(\alpha^j-\alpha^i\right)\left[-9\alpha^i\alpha^j\sigma+\left(\alpha^i+\alpha^j\right)\tau+9\alpha^i\alpha^j\sigma\right]}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2} = \frac{6\left(\alpha^i+\alpha^j\right)\left(\alpha^j-\alpha^i\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$$

The partial derivatives $\frac{\partial(\Delta q_b^i)^{bs}}{\partial\tau} < 0$ is negative and $\frac{\partial(\Delta q_b^i)^{bs}}{\partial\pi} > 0$ is positive as established using the same reasoning presented in the proof of Proposition 2 □

B.2 Proof of Proposition 3

Proof. We prove Proposition 3 by partially differentiating the difference in buyers' equilibrium membership fees with respect to τ and π using Equation (11a).

Firstly, we manipulate the expression for the difference in buyers' equilibrium membership fees in the following way: $\left(\Delta p_b^i\right)^{bs} = \left(p_b^i\right)^{bs} - \left(p_b^j\right)^{bs} = f_b+\tau-\pi+\frac{1}{3}\left(\Delta q_b^i\right)^{bs} - \left[f_b+\tau-\pi+\frac{1}{3}\left(\Delta q_b^i\right)^{bs}\right]$ $= \frac{2\sigma(\alpha^j-\alpha^i)}{9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau}$. Then we see that $\left(\Delta p_b^i\right)^{bs} = \frac{2}{3}\left(\Delta q_b^i\right)^{bs}$.

Now, we obtain $\frac{\partial(\Delta p_b^i)^{bs}}{\partial\tau} = \frac{2}{3}\frac{\partial(\Delta q_b^i)^{bs}}{\partial\tau}$. We have shown that $\frac{\partial(\Delta q_b^i)^{bs}}{\partial\tau} < 0$ in the proof of Proposition 2, therefore $\frac{\partial(\Delta p_b^i)^{bs}}{\partial\tau} < 0$. Next, we compute $\frac{\partial(\Delta p_b^i)^{bs}}{\partial\pi} = \frac{2}{3}\frac{\partial(\Delta q_b^i)^{bs}}{\partial\pi}$. We have shown that $\frac{\partial(\Delta q_b^i)^{bs}}{\partial\pi} > 0$ in the proof of Proposition 2, therefore $\frac{\partial(\Delta p_b^i)^{bs}}{\partial\pi} > 0$.

Finally, we compute $\frac{\partial(p_b^i)^{bs}}{\partial\pi} - \frac{\partial(p_b^j)^{bs}}{\partial\pi} = -1 + \frac{6(\alpha^i+\alpha^j)(\alpha^j-\alpha^i)\tau\pi}{[9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau]^2} - \left[-1+\frac{6(\alpha^i+\alpha^j)(\alpha^i-\alpha^j)\tau\pi}{[9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau]^2}\right]$ which simplifies to $\frac{6(\alpha^i+\alpha^j)(\alpha^i-\alpha^j)\tau\pi}{[9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau]^2}$. Then, considering platform 1 is more efficient in developing attributes compared to platform 2, $(\alpha^2 > \alpha^1)$ we get $\frac{\partial (p_b^i)^{bs}}{\partial \pi} - \frac{\partial (p_b^j)^{bs}}{\partial \pi} > 0$. □

B.3 Market-shares conditions

We obtain conditions for buyers' and sellers' market shares to be distributed in the unit interval using Equations (12a) and (12b)

$0 < (\eta_b^i)^{bs} < 1$. For $(\eta_b^i)^{bs} > 0$ we have $\frac{1}{2} + \frac{(\alpha^j - \alpha^i)\tau}{2[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau]} > 0$. This inequality turns into $(\alpha^j - \alpha^i)\tau > -9\alpha^i\alpha^j\sigma + (\alpha^i + \alpha^j)\tau$, which simplifies to $\alpha^i(9\alpha^j\sigma - 2\tau) > 0$ if $\alpha^j > \frac{2\tau}{9\sigma}$. This condition holds under Assumption 2 when $\pi = \upsilon$. For $(\eta_b^i)^{bs} < 1$ we get $9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau - (\alpha^j - \alpha^i)\tau > 0$. This inequality simplifies to $\alpha^j(9\alpha^i\sigma - 2\tau) > 0$ if $\alpha^i > \frac{2\tau}{9\sigma}$. This condition holds under Assumption 2 when $\pi = \upsilon$.

$0 < (\eta_s^i)^{bs} < 1$. For $(\eta_s^i)^{bs} > 0$ we have $\frac{1}{2} + \frac{(\alpha^j - \alpha^i)\pi}{2[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau]} > 0$. This inequality turns into $(\alpha^j - \alpha^i)\pi > -9\alpha^i\alpha^j\sigma + (\alpha^i + \alpha^j)\tau$, which simplifies to $9\alpha^i\alpha^j\sigma - \alpha^i(\tau + \pi) - \alpha^j(\tau - \pi) > 0$ if $\alpha^i > \frac{\alpha^j(\tau - \pi)}{9\alpha^j\sigma - (\tau + \pi)}$. Now, if the right side of Assumption 2 is greater than the right side of $\alpha^i > \frac{\alpha^j(\tau - \pi)}{9\alpha^j\sigma - (\tau + \pi)}$ the condition is satisfied. That is $\frac{2\tau}{9\sigma} > \frac{\alpha^j(\tau - \pi)}{9\alpha^j\sigma - (\tau + \pi)}$ turns to $18\alpha^j\sigma\tau - 2\tau(\tau + \pi) - 9\alpha^j\sigma(\tau - \pi) > 0$. This inequality simplifies to $(\tau + \pi)(9\alpha^j\sigma - 2\tau) > 0$. This inequality is positive under Assumption 2 when $\pi = \upsilon$. For $(\eta_s^i)^{bs} < 1$ we get $9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau - (\alpha^j + \alpha^i)\pi > 0$ if $\alpha^j > \frac{\alpha^i(\tau - \pi)}{9\alpha^i\sigma - (\tau + \pi)}$. We follow the same method comparing the right side of this condition and Assumption 2. That is $\frac{2\tau}{9\sigma} > \frac{\alpha^i(\tau - \pi)}{9\alpha^i\sigma - (\tau + \pi)}$ turns to $18\alpha^i\sigma\tau - 2\tau(\tau + \pi) - 9\alpha^i\sigma(\tau - \pi) > 0$. This inequality simplifies to $(\tau + \pi)(9\alpha^i\sigma - 2\tau) > 0$. This inequality is positive under Assumption 2 when $\pi = \upsilon$.

In summary, as long as $\alpha^i > \frac{2\tau}{9\sigma}$, for $i = 1, 2$, which is guaranteed by Assumption 2 when $\pi = \upsilon$, then the conditions $0 < (\eta_b^i)^{bs} < 1$ and $0 < (\eta_s^i)^{bs} < 1$ are satisfied.

B.4 Impacts on Equilibrium Market-shares

We compute the impacts on buyers' and sellers' equilibrium market shares respect parameters τ and π using Equations (12a) and (12b).

Firstly, we manipulate the expression for buyers' and sellers' equilibrium market shares in Equations (12a) and (12b) as follows: We know that $(\Delta q_b^i)^{bs} = \frac{3\sigma(\alpha^j - \alpha^i)}{9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau}$ then $(\eta_b^i)^{bs} = \frac{1}{2} + \frac{(\alpha^j - \alpha^i)\tau}{2[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau]}$ can be rewritten as $(\eta_b^i)^{bs} = \frac{1}{2} + \frac{\tau}{6\sigma}(\Delta q_b^i)^{bs}$, and $(\eta_s^i)^{bs} = \frac{1}{2} + \frac{(\alpha^j - \alpha^i)\pi}{2[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau]}$ can be rewritten as $(\eta_s^i)^{bs} = \frac{1}{2} + \frac{\pi}{6\sigma}(\Delta q_b^i)^{bs}$.

Next, we compute the partial derivatives on buyers' side, $\frac{\partial (\eta_b^i)^{bs}}{\partial \tau} = \frac{1}{6^2\sigma^2}\left[6(\Delta q_b^i)^{bs}(\sigma - 2\tau^2) + 6\sigma\partial(\Delta q_b^i)^{bs}/\partial\tau\right]$, which simplifies to $\frac{\partial (\eta_b^i)^{bs}}{\partial \tau} = \frac{1}{6\sigma^2}\left[\sigma\left(\frac{\partial(\Delta q_b^i)^{bs}}{\partial \tau}\right) - (\Delta q_b^i)^{bs}(\tau^2 + \pi^2) + \right]$. All the elements of the partial derivative are positive except $\frac{\partial(\Delta q_b^i)^{bs}}{\partial \tau}$, which we demonstrated in the proof of Proposition 2 that $\frac{\partial(\Delta q_b^i)^{bs}}{\partial \tau} < 0$, consequently, it follows that $\frac{\partial (\eta_b^i)^{bs}}{\partial \tau} < 0$. $\frac{\partial (\eta_b^i)^{bs}}{\partial \pi} = \frac{\tau}{18\sigma^2}\left[3\sigma\left(\frac{\partial(\Delta q_b^i)^{bs}}{\partial \pi}\right) + \pi(\Delta q_b^i)^{bs}\right]$. We demonstrated in the proof of Proposition 2 that $\frac{\partial(\Delta q_b^i)^{bs}}{\partial \pi} > 0$, and all elements of the partial derivative are positive, consequently, it follows that

$\frac{\partial \left(\eta_b^i\right)^{bs}}{\partial \pi} > 0$.

Next, we compute the partial derivatives on sellers' side, $\frac{\partial \left(\eta_s^i\right)^{bs}}{\partial \tau} = \frac{\pi}{6\sigma^2}\Big[\sigma \left(\frac{\partial \left(\Delta q_b^i\right)^{bs}}{\partial \tau}\right) - 2\tau \left(\Delta q_b^i\right)^{bs}\Big]$. All the elements of the partial derivative are positive except $\frac{\partial \left(\Delta q_b^i\right)^{bs}}{\partial \tau}$, which we demonstrated in the proof of Proposition 2 that $\frac{\partial \left(\Delta q_b^i\right)^{bs}}{\partial \tau} < 0$, consequently, it follows that $\frac{\partial \left(\eta_s^i\right)^{bs}}{\partial \tau} < 0$. $\frac{\partial \left(\eta_s^i\right)^{bs}}{\partial \pi} = \frac{1}{6\sigma^2}\Big[\left(\Delta q_b^i\right)^{bs}\left(\sigma + 2\pi^2\right) + \pi\sigma \left(\frac{\partial \left(\Delta q_b^i\right)^{bs}}{\partial \pi}\right)\Big]$ which simplifies to $\frac{\partial \left(\eta_s^i\right)^{bs}}{\partial \pi} = \frac{1}{6\sigma^2}\left[\left(\Delta q_b^i\right)^{bs}\left(\tau^2 + \pi^2\right) + \pi\sigma \left(\frac{\partial \left(\Delta q_b^i\right)^{bs}}{\partial \pi}\right)\right]$. We demonstrated in the proof of Proposition 2 that $\frac{\partial \left(\Delta q_b^i\right)^{bs}}{\partial \pi} > 0$, and all elements of the partial derivative are positive, consequently, it follows that $\frac{\partial \left(\eta_s^i\right)^{bs}}{\partial \pi} > 0$.

B.5 Positive Equilibrium Profits

We show the conditions for Equilibrium profits in Equation (13) for platform 1 to be positive. We notice Equation (13) is compose of two elements, the first is $\tau - \pi$ and the second is $\frac{9\sigma\left(\alpha^j - \alpha^i\right)\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i+\alpha^j\right)\tau\right] - \alpha^i\left(9\alpha^j\sigma - 2\tau\right)\left(9\alpha^i\sigma - 2\tau\right)}{18\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i+\alpha^j\right)\tau\right]^2}$. The first component is positive under Assumption 1 when $\pi = \upsilon$. To determine if the second element is positive, we can partially differentiate it with respect to α^2 and evaluate the result when $\alpha^2 = \frac{2\tau}{9\sigma}$.

$$\begin{aligned}
\frac{\partial part2}{\partial \alpha^2} &= 18\Big[\left[9\alpha^1\alpha^2\sigma - \left(\alpha^1+\alpha^2\right)\tau\right]^2\left[9\sigma\left[9\alpha^1\alpha^2\sigma - \left(\alpha^1+\alpha^2\right)\tau\right] + 9\sigma\left(\alpha^2 - \alpha^1\right)\right.\\
&\left(9\alpha^1\sigma - \tau\right) - 9\alpha^1\sigma\left(9\alpha^1\sigma - 2\tau\right)\Big] - 2\left(9\alpha^1\sigma - \tau\right)\left[9\alpha^1\alpha^2\sigma - \left(\alpha^1+\alpha^2\right)\tau\right]\left[9\sigma\left(\alpha^2 - \alpha^1\right)\right.\\
&\left[9\alpha^1\alpha^2\sigma - \left(\alpha^1+\alpha^2\right)\tau\right] - \alpha^1\left(9\alpha^2\sigma - 2\tau\right)\left(9\alpha^1\sigma - 2\tau\right)\Big]\Big] \Big/ 18^2\left[9\alpha^1\alpha^2\sigma - \left(\alpha^1+\alpha^2\right)\tau\right]^4\\
&= \Big[18\sigma\left[9\alpha^1\alpha^2\sigma - \left(\alpha^1+\alpha^2\right)\tau\right]\left[9\alpha^1\alpha^2\sigma - 9\left(\alpha^1\right)^2\sigma - 2\tau\left(\alpha^2 - \alpha^1\right)\right] - 18\sigma\left(\alpha^2 - \alpha^1\right)\\
&\quad \left(9\alpha^1\sigma - \tau\right)\left[9\alpha^1\alpha^2\sigma - \left(\alpha^1+\alpha^2\right)\tau\right] + 2\alpha^1\left(9\alpha^1\sigma - \tau\right)\left(9\alpha^1\sigma - 2\tau\right)\\
&\quad \left(9\alpha^2\sigma - 2\tau\right)\Big] \Big/ 18\left[9\alpha^1\alpha^2\sigma - \left(\alpha^1+\alpha^2\right)\tau\right]^3
\end{aligned}$$

$$\begin{aligned}
&= \Big[18\sigma\left[9\alpha^1\alpha^2\sigma - \left(\alpha^1+\alpha^2\right)\tau\right]\left[\alpha^2\left(9\alpha^1\sigma - \tau\right) - \alpha^1\left(9\alpha^1\sigma - \tau\right)\right] - 18\sigma\left(\alpha^2 - \alpha^1\right)\\
&\quad \left(9\alpha^1\sigma - \tau\right)\left[9\alpha^1\alpha^2\sigma - \left(\alpha^1+\alpha^2\right)\tau\right] + 2\alpha^1\left(9\alpha^1\sigma - \tau\right)\left(9\alpha^1\sigma - 2\tau\right)\\
&\left(9\alpha^2\sigma - 2\tau\right)\Big] \Big/ 18\left[9\alpha^1\alpha^2\sigma - \left(\alpha^1+\alpha^2\right)\tau\right]^3 = \frac{\alpha^1\left(9\alpha^1\sigma - \tau\right)\left(9\alpha^1\sigma - 2\tau\right)\left(9\alpha^2\sigma - 2\tau\right)}{9\left[9\alpha^1\alpha^2\sigma - \left(\alpha^1+\alpha^2\right)\tau\right]^3}\\
\frac{\partial part2}{\partial \alpha^2} &= \frac{\alpha^1\left(9\alpha^1\sigma - \tau\right)\left(9\alpha^1\sigma - 2\tau\right)\left(9\alpha^2\sigma - 2\tau\right)}{9\left[\alpha^2\left(9\alpha^1\sigma - 2\tau\right) + \tau\left(\alpha^2 - \alpha^1\right)\right]^3}
\end{aligned}$$

As it can be observed, for values of α^1 and α^2 greater than $\frac{2\tau}{9\sigma}$, the equilibrium profits on platform 1 in Equation (13) are always positive. This condition $\alpha^i > \frac{2\tau}{9\sigma}$, for $i = 1, 2$ is Assumption 2 when $\pi = \upsilon$.

B.6 Proof of Proposition 4

Proof. We prove Proposition 4 by partially differentiating the difference in equilibrium profits with respect to τ and π using Equation (13) under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$.

$$\frac{\partial\left(\Delta\Pi^i\right)^{bs}}{\partial\tau} = 18\left(\alpha^j-\alpha^i\right)\Big[18\tau\left[27\alpha^i\alpha^j\sigma-4\left(\alpha^i+\alpha^j\right)\tau\right]+9\sigma\left[54\alpha^i\alpha^j\sigma-4\left(\alpha^i+\alpha^j\right)\right]$$
$$+8\tau\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2-2\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]\left[18\alpha^i\alpha^j\tau-\left(\alpha^i+\alpha^j\right)\right]$$
$$\left[9\sigma\left[27\alpha^i\alpha^j\sigma-4\left(\alpha^i+\alpha^j\right)\tau\right]+4\tau^2\right]\Big]\Big/18^2\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^4$$
$$=\left(\alpha^j-\alpha^i\right)\Big[9\big[54\alpha^i\alpha^j\sigma-2\left(\alpha^i+\alpha^j\right)\left(\sigma+2\tau^2\right)\big]\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]+4\tau\big[9\alpha^i\alpha^j\sigma$$
$$-\left(\alpha^i+\alpha^j\right)\tau\big]-9\sigma\left[18\alpha^i\alpha^j\tau-\left(\alpha^i+\alpha^j\right)\right]\left[27\alpha^i\alpha^j\sigma-4\left(\alpha^i+\alpha^j\right)\tau\right]$$
$$-4\tau^2\left[18\alpha^i\alpha^j\tau-\left(\alpha^i+\alpha^j\right)\right]\Big]\Big/9\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^3$$
$$=\frac{\left(\alpha^j-\alpha^i\right)\left(\sigma-2\tau^2\right)\left[9\alpha^i\alpha^j\left(\alpha^i+\alpha^j\right)\sigma-2\tau\left(\alpha^i+\alpha^j\right)^2+4\alpha^i\alpha^j\tau\right]}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^3}$$
$$\frac{\partial\left(\Delta\Pi^i\right)^{bs}}{\partial\tau}=\frac{-\left(\alpha^j-\alpha^i\right)\left(\tau^2+\pi^2\right)\left[\left(\alpha^i\right)^2\left(9\alpha^j\sigma-2\tau\right)+\left(\alpha^j\right)^2\left(9\alpha^i\sigma-2\tau\right)\right]}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^3}$$

To guarantee that $\frac{\partial(\Delta\Pi^i)^{bs}}{\partial\tau}$ is negative for platform 1, it is sufficient for all of its components to be positive. The denominator can be expressed as $\left[\alpha^j\left(9\alpha^j\sigma-2\tau\right)+\tau\left(\alpha^j-\alpha^i\right)\right]^3$, which is positive if Assumption 2 holds when $\pi=\upsilon$. The elements in the numerator are all positive, based on the same reasoning as for the denominator, provided that platform 1 is more efficient in developing attributes, $\alpha^2>\alpha^1$ and Assumption 2 holds when $\pi=\upsilon$.

$$\frac{\partial\left(\Delta\Pi^i\right)^{bs}}{\partial\pi}=\left(\alpha^j-\alpha^i\right)\Big[-18\pi\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2\Big[\left[27\alpha^i\alpha^j\sigma-4\left(\alpha^i+\alpha^j\right)\tau\right]$$
$$+27\alpha^i\alpha^j\sigma\Big]+36\alpha^i\alpha^j\pi\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]\left[9\sigma\left[27\alpha^i\alpha^j\sigma-4\left(\alpha^i+\alpha^j\right)\tau\right]+4\tau^2\right]\Big]$$
$$\Big/18\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^4$$
$$=2\pi\left(\alpha^j-\alpha^i\right)\Big[-\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]\left[27\alpha^i\alpha^j\sigma-2\left(\alpha^i+\alpha^j\right)\tau\right]$$
$$+9\alpha^i\alpha^j\sigma\left[27\alpha^i\alpha^j\sigma-4\left(\alpha^i+\alpha^j\right)\tau\right]+4\alpha^i\alpha^j\tau^2\Big]\Big/\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^3$$
$$=\frac{2\tau\pi\left(\alpha^j-\alpha^i\right)\left[9\alpha^i\alpha^j\left(\alpha^i+\alpha^j\right)\sigma-2\left(\alpha^i\right)^2\tau-2\left(\alpha^j\right)^2\tau\right]}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^3}$$
$$\frac{\partial\left(\Delta\Pi^i\right)^{bs}}{\partial\pi}=\frac{2\left(\alpha^j-\alpha^i\right)\tau\pi\left[\left(\alpha^i\right)^2\left(9\alpha^j\sigma-2\tau\right)+\left(\alpha^j\right)^2\left(9\alpha^i\sigma-2\tau\right)\right]}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^3}$$

To guarantee that $\frac{\partial(\Delta\Pi^i)^{bs}}{\partial\pi}$ is positive for platform 1, it is sufficient for all of its components to be positive. The denominator can be expressed as $\left[\alpha^j\left(9\alpha^j\sigma-2\tau\right)+\tau\left(\alpha^j-\alpha^i\right)\right]^3$, which is positive if Assumption 2 holds when $\pi = \upsilon$. The elements in the numerator are all positive, based on the same reasoning as for the denominator, provided that platform 1 is more efficient in developing attributes, $\alpha^2 > \alpha^1$ and Assumption 2 holds when $\pi = \upsilon$. $\square$

C Scenario: $\upsilon \neq \pi$

When $\upsilon > \pi$ $(\pi = 0)$ Assumption 1 turns to $\frac{\upsilon}{2} < \tau < \frac{2\upsilon}{3}$ and Assumption 2 turns to $\alpha^i > \frac{2\tau}{\sigma_\upsilon}$. Conversely, when $\pi > \upsilon$ $(\upsilon = 0)$ Assumption 1 turns to $\tau > \frac{\pi}{2}$ and Assumption 2 turns to $\alpha^i > \frac{2\tau}{\sigma_\pi}$, where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$ and $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$.

C.1 Proof of Proposition 5

Proof. We prove Proposition 5 by partially differentiating the difference in equilibrium attributes with respect to τ, υ and π using Equations (14a) and (14b) under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$.

Case 1. $\upsilon > \pi$**,** $\pi = 0$

$$\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau} = \frac{\left(\alpha^j - \alpha^i\right)\left[\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]\left(12\tau + \upsilon\right) - \left[18\alpha^i\alpha^j\tau - \left(\alpha^i + \alpha^j\right)\right]\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right]}{2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}$$

$$= \frac{\left(\alpha^j - \alpha^i\right)}{2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\left[\left[\alpha^i\left(\alpha^j\sigma_\upsilon - 2\tau\right) - \tau\left(\alpha^j - \alpha^i\right)\right]\left(12\tau + \upsilon\right)\right.$$

$$\left. - \left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\left[2\alpha^i\left(9\alpha^j\tau - 1\right) - \left(\alpha^j - \alpha^i\right)\right]\right]$$

$$= \frac{-\left(\alpha^j - \alpha^i\right)}{2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\left[\alpha^i\left[2\left(9\alpha^j\tau - 1\right)\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right) - \left(\alpha^j\sigma_\upsilon - 2\tau\right)\left(12\tau + \upsilon\right)\right]\right.$$

$$\left. + \left(\alpha^j - \alpha^i\right)\left[\tau\left(12\tau + \upsilon\right) - \left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right]\right]$$

Next, $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}$ is negative if $\alpha^i\left[2\left(9\alpha^j\tau - 1\right)\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right) - \left(\alpha^j\sigma_\upsilon - 2\tau\right)\left(12\tau + \upsilon\right)\right] + \left(\alpha^j - \alpha^i\right)\left[\tau\left(12\tau + \upsilon\right) - \left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right]$ is positive. The first part $\alpha^i\left[2\left(9\alpha^j\tau - 1\right)\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right) - \left(\alpha^j\sigma_\upsilon - 2\tau\right)\left(12\tau + \upsilon\right)\right]$ can be rearranged as $\alpha^i\left[\alpha^j\left[18\tau\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right) - \sigma_\upsilon\left(12\tau + \upsilon\right)\right] - 2\left[\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right) - \tau\left(12\tau + \upsilon\right)\right]\right]$, and is positive if $\alpha^j > \frac{2[(3\tau+2\upsilon)(2\tau-\upsilon)-\tau(12\tau+\upsilon)]}{18\tau(3\tau+2\upsilon)(2\tau-\upsilon)-\sigma_\upsilon(12\tau+\upsilon)}$. We use Assumption 2 to show the condition is satisfied by making the left side on both inequalities equal and comparing the right side. Then showing that Assumption 2 right side is greater we guarantee the condition is positive. That is $\frac{2\tau}{\sigma_\upsilon} > \frac{2[(3\tau+2\upsilon)(2\tau-\upsilon)-\tau(12\tau+\upsilon)]}{18\tau(3\tau+2\upsilon)(2\tau-\upsilon)-\sigma_\upsilon(12\tau+\upsilon)}$, which simplifies to $\left(18\tau^2 - \sigma_\upsilon\right)\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right) > 0$ and finally turns to $\left(9\tau^2 + 2\upsilon^2\right)\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right) > 0$ if $\tau > \frac{\upsilon}{2}$. Then, the first part is negative. The second part $\left(\alpha^j - \alpha^i\right)\left[\tau\left(12\tau + \upsilon\right) - \left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right]$ simplifies to $2\left(\alpha^j - \alpha^i\right)\left(3\tau^2 + \upsilon^2\right)$, which is positive for platform 1 given $\alpha^2 > \alpha^1$

Therefore, $\frac{\partial(\Delta q_b^i)_{\upsilon>\pi}}{\partial\tau} < 0$ under Assumption 1 and Assumption 2 when $\upsilon > \pi$, $\pi = 0$.

$$\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\upsilon} = \frac{\left(\alpha^j-\alpha^i\right)\Big[\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right](\tau-4\upsilon)+4\alpha^i\alpha^j\upsilon\,(3\tau+2\upsilon)\,(2\tau-\upsilon)\Big]}{2\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$$
$$= \frac{\left(\alpha^j-\alpha^i\right)\Big[\left[\alpha^j\left(\alpha^i\sigma_\upsilon-2\tau\right)+\left(\alpha^j-\alpha^i\right)\tau\right](\tau-4\upsilon)+4\alpha^i\alpha^j\upsilon\,(3\tau+2\upsilon)\,(2\tau-\upsilon)\Big]}{2\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$$

Therefore, $\frac{\partial(\Delta q_b^i)_{\upsilon>\pi}}{\partial\upsilon}$ is positive under Assumption 2 when $\upsilon > \pi$, $\pi = 0$ and as long as $\tau > 4\upsilon$.

Case 2. $\pi > \upsilon$**,** $\upsilon = 0$

$$\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau} = \frac{\left(\alpha^j-\alpha^i\right)\Big[\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right](12\tau-\pi)-\left[18\alpha^i\alpha^j\tau-\left(\alpha^i+\alpha^j\right)\right](3\tau+\pi)\,(2\tau-\pi)\Big]}{2\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$$
$$= \frac{\left(\alpha^j-\alpha^i\right)}{2\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\left[\alpha^i\left(\alpha^j\sigma_\pi-2\tau\right)-\tau\left(\alpha^j-\alpha^i\right)\right](12\tau-\pi)$$
$$-(3\tau+\pi)\,(2\tau-\pi)\left[2\alpha^i\left(9\alpha^j\tau-1\right)-\left(\alpha^j-\alpha^i\right)\right]\Bigg]$$

$$= \frac{-\left(\alpha^j-\alpha^i\right)}{2\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\alpha^i\left[2\left(9\alpha^j\tau-1\right)(3\tau+2\upsilon)\,(2\tau-\upsilon)-\left(\alpha^j\sigma_\pi-2\tau\right)(12\tau-\pi)\right]$$
$$+\left(\alpha^j-\alpha^i\right)\left[\tau\,(12\tau-\pi)-\left(3\tau+\pi\right)(2\tau-\pi)\right]\Bigg]$$

Next, $\frac{\partial(\Delta q_b^i)_{\pi>\upsilon}}{\partial\tau}$ is negative if $\alpha^i\left[2\left(9\alpha^j\tau-1\right)\left(3\tau+\pi\right)(2\tau-\pi)-\left(\alpha^j\sigma_\pi-2\tau\right)(12\tau-\pi)\right]+\left(\alpha^j-\alpha^i\right)\left[\tau\,(12\tau-\pi)-\left(3\tau+\pi\right)(2\tau-\pi)\right]$ is positive. The first part $\alpha^i\left[2\left(9\alpha^j\tau-1\right)\left(3\tau+\pi\right)(2\tau-\pi)-\left(\alpha^j\sigma_\pi-2\tau\right)(12\tau-\pi)\right]$ can be rearranged as $\alpha^i\left[\alpha^j\left[18\tau\left(3\tau+\pi\right)\left(2\tau-\pi\right)-\sigma_\pi\,(12\tau-\pi)\right]-2\left[(3\tau+\pi)\,(2\tau-\pi)-\tau\left(12\tau-\pi\right)\right]\right]$, and is positive if $\alpha^j > \frac{2[(3\tau+\pi)(2\tau-\pi)-\tau(12\tau-\pi)]}{18\tau(3\tau+\pi)(2\tau-\pi)-\sigma_\pi(12\tau-\pi)}$. We use Assumption 2 to show the condition is satisfied by making the left side on both inequalities equal and comparing the right side. Then showing that Assumption 2 right side is greater we guarantee the condition is positive. That is $\frac{2\tau}{\sigma_\pi} > \frac{2[(3\tau+\pi)(2\tau-\pi)-\tau(12\tau-\pi)]}{18\tau(3\tau+\pi)(2\tau-\pi)-\sigma_\pi(12\tau-\pi)}$, which simplifies to $\left(18\tau^2-\sigma_\pi\right)(3\tau+\pi)\,(2\tau-\pi) > 0$ and finally turns to $\left(9\tau^2+2\pi^2\right)(3\tau+\pi)\,(2\tau-\pi) > 0$ if $\tau > \frac{\pi}{2}$. Then, the first part is negative. The second part $\left(\alpha^j-\alpha^i\right)\left[\tau\,(12\tau-\pi)-\left(3\tau+\pi\right)(2\tau-\pi)\right]$ simplifies to $\left(\alpha^j-\alpha^i\right)\left(6\tau^2+\pi^2\right)$, which is positive for platform 1 given $\alpha^2 > \alpha^1$

Therefore, $\frac{\partial(\Delta q_b^i)_{\pi>\upsilon}}{\partial\tau} < 0$ under Assumption 1 and Assumption 2 when $\pi > \upsilon$, $\upsilon = 0$.

$$\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi} = \frac{\left(\alpha^j-\alpha^i\right)\Big[-\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right](\tau+2\pi)+4\alpha^i\alpha^j\pi\,(3\tau+\pi)\,(2\tau-\pi)\Big]}{2\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$$
$$= \frac{\left(\alpha^j-\alpha^i\right)\left[\alpha^i\left[\alpha^j\left[4\pi\,(3\tau+\pi)\,(2\tau-\pi)-\sigma_\pi\,(\tau+2\pi)\right]+\tau\,(\tau+2\pi)\right]+\alpha^j\tau\,(\tau+2\pi)\right]}{2\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$$

To determine the sign of $\frac{\partial(\Delta q_b^i)_{\pi>\upsilon}}{\partial\pi}$ it is sufficient to find the sign of the following expression $\alpha^i\big[\alpha^j\big[4\pi\,(3\tau+\pi)\,(2\tau-\pi)-\sigma_\pi\,(\tau+2\pi)\big]+\tau\,(\tau+2\pi)\big]+\alpha^j\tau\,(\tau+2\pi)$. We use Assumption 2 to show the condition is positive. We make the left side of both expressions equal and compare the right side. Then showing that Assumption 2 right side is greater we guarantee the condition is positive. That is $\frac{2\tau}{\sigma_\upsilon}>\frac{-\alpha^j\tau(\tau+2\pi)}{B}$, where $B\equiv\alpha^j\big[4\pi\,(3\tau+\pi)\,(2\tau-\pi)-\sigma_\pi\,(\tau+2\pi)\big]+\tau\,(\tau+2\pi)$. Then we have $\alpha^j\big[8\pi\,(3\tau+\pi)\,\big(2\tau-\pi\big)-\sigma_\pi\,(\tau+2\pi)\big]+2\tau\,(\tau+2\pi)>0$. Then we use the same method of comparing the right side of Assumption 2 and the previous inequality and show the condition is satisfied. That is $\frac{2\tau}{\sigma_\pi}>\frac{-2\tau(\tau+2\pi)}{8\pi(3\tau+\pi)(2\tau-\pi)-\sigma_\pi(\tau+2\pi)}$, which simplifies to $8\pi\,(3\tau+\pi)\,(2\tau-\pi)>0$, this condition is satisfied under Assumption 1 when $\pi>\upsilon$, $\upsilon=0$.

Therefore, $\frac{\partial(\Delta q_b^i)_{\pi>\upsilon}}{\partial\pi}>0$ under Assumption 1 and Assumption 2 when $\pi>\upsilon$, $\upsilon=0$. $\square$

C.2 Market-shares conditions

Buyers and sellers equilibrium market shares in Equations (15a) and (15b) must satisfy conditions $0<\eta_b^i<1$ and $0<\eta_s^i<1$, respectively.

Firstly, for platform 1, both equilibrium market shares are positive because all of their elements are positive. Given platform 1 is more efficient in developing attributes, $\alpha^2-\alpha^1>0$ and given Assumption 1 and Assumption 2 hold. That is $6\tau^2-(\pi-\upsilon)\tau-(\pi+\upsilon)(\pi+2\upsilon)>0$ when $\tau>\frac{\pi+\upsilon}{2}$, as was demonstrated previously in Appendix A.9. Furthermore, $\big[\alpha^i\alpha^j\Sigma-\big(\alpha^i+\alpha^j\big)\,\tau\big]$ is positive, which can be observed when rewritten as $\alpha^j\,\big(\alpha^i\Sigma-2\tau\big)+\big(\alpha^j-\alpha^i\big)\,\tau$, where $\Sigma\equiv 9\tau^2-(2\pi+\upsilon)\,(\pi+2\upsilon)$.

For $\eta_b^i<1$ we have $\eta_b^i=\frac{1}{2}+\frac{3\tau\left(\alpha^j-\alpha^i\right)\left[6\tau^2-(\pi-\upsilon)\tau-(\pi+\upsilon)(\pi+2\upsilon)\right]}{4\Sigma[\alpha^i\alpha^j\Sigma-(\alpha^i+\alpha^j)\tau]}<1$. This inequality can be rewritten as $2\Sigma\big[\alpha^i\alpha^j\Sigma-\big(\alpha^i+\alpha^j\big)\,\tau\big]>3\tau\,\big[6\tau^2-(\pi-\upsilon)\,\tau-(\pi+\upsilon)\,(\pi+2\upsilon)\big]\,\big(\alpha^j-\alpha^i\big)$, which simplifies to $2\alpha^j\Sigma\,\big(\alpha^i\Sigma-2\tau\big)+\big(\alpha^j-\alpha^i\big)\,\tau\big[2\Sigma-3\big[6\tau^2-(\pi+\upsilon)\,(\pi+2\upsilon)+\tau\,(\upsilon-\pi)\big]\big]>0$. To show that the previous condition is positive, it is sufficient to demonstrate that $2\Sigma-3\big[6\tau^2-(\pi+\upsilon)\,(\pi+2\upsilon)+\tau\,(\upsilon-\pi)\big]$ is positive, considering that the other elements are positive. This expression turns to $18\tau^2-2\,(2\pi+\upsilon)\,(\pi+2\upsilon)-18\tau^2+3\,(\pi+\upsilon)\,(\pi+2\upsilon)-3\tau\,(\upsilon-\pi)>0$ which simplifies to $(\upsilon-\pi)\,[(\pi+2\upsilon)-3\tau]>0$ if $\tau<\frac{\pi+2\upsilon}{3}$ and $\upsilon>\pi$ or $\tau>\frac{\pi+2\upsilon}{3}$ and $\pi>\upsilon$. We use Assumption 1 to show the previous condition is satisfied by comparing the right side of both inequalities. Therefore showing that if Assumption 1 right side is greater the condition is satisfied. That is $\frac{(\pi+\upsilon)}{2}>\frac{(\pi+2\upsilon)}{3}$ which simplifies to $\pi-\upsilon>0$ if $\pi>\upsilon$.

For $\eta_s^i<1$ we have $\eta_s^i=\frac{1}{2}+\frac{(\pi+2\upsilon)\left(\alpha^j-\alpha^i\right)\left[6\tau^2-(\pi-\upsilon)\tau-(\pi+\upsilon)(\pi+2\upsilon)\right]}{4\Sigma[\alpha^i\alpha^j\Sigma-(\alpha^i+\alpha^j)\tau]}<1$. This inequality can be rewritten as which turns to $2\Sigma\big[\alpha^i\alpha^j\Sigma-\big(\alpha^i+\alpha^j\big)\tau\big]-(\pi+2\upsilon)\,\big(\alpha^j-\alpha^i\big)\,\big[6\tau^2-(\pi-\upsilon)\,\tau-(\pi+\upsilon)\,(\pi+2\upsilon)\big]>0$, which simplifies to $2\alpha^j\Sigma\,\big(\alpha^i\Sigma-2\tau\big)+\big(\alpha^j-\alpha^i\big)\,\big[2\tau\Sigma-(\pi+2\upsilon)\,\big[6\tau^2-(\pi-\upsilon)\,\tau-(\pi+\upsilon)\,(\pi+2\upsilon)\big]\big]>0$. To show that the previous condition is positive, it is sufficient to demonstrate that $2\tau\Sigma-(\pi+2\upsilon)\,\big[6\tau^2-(\pi-\upsilon)\,\tau-(\pi+\upsilon)\,(\pi+2\upsilon)\big]$ is positive, considering that the other elements are positive. This expression turns to $18\tau^3-2\tau\,(2\pi+\upsilon)\,(\pi+2\upsilon)-6\tau^2\,(\pi+2\upsilon)+(\pi+2\upsilon)^2\,(\pi+\upsilon)-\tau\big(\upsilon-\pi\big)\big(\pi+2\upsilon\big)>0$ which simplifies to $6\tau^2\big[3\tau-(\pi+2\upsilon)\big]-3\tau\,(\pi+2\upsilon)\,(\pi+\upsilon)+(\pi+2\upsilon)^2\,\big(\pi+\upsilon\big)>0$ and then simplifies to $\big[6\tau^2-(\pi+\upsilon)\,(\pi+2\upsilon)\big]\,[3\tau-(\pi+2\upsilon)]>0$. For the previous condition to be positive, it is sufficient to have expressions $6\tau^2-(\pi+\upsilon)\,(\pi+2\upsilon)>0$ and $3\tau-(\pi+2\upsilon)>0$. We use Assumption

1 to show the first condition is satisfied by making the left side on both inequalities equal and comparing the right side. Then showing that Assumption 1 right side is greater we guarantee the condition is positive. First, Assumption 1 can be expressed as $\tau^2 > \frac{(\pi+\upsilon)^2}{4}$, then comparing the right side we have $\frac{(\pi+\upsilon)^2}{4} > \frac{(\pi+\upsilon)(\pi+2\upsilon)}{6}$ which simplifies to $3(\pi+\upsilon) - 2(\pi+2\upsilon) > 0$ an finally simplifies to $\pi - \upsilon > 0$ if $\pi > \upsilon$. Moreover, $3\tau - (\pi + 2\upsilon) > 0$ if $\tau > \frac{(\pi+2\upsilon)}{3}$. This condition is met if Assumption 1 holds as was previously shown for $\eta_b^i < 1$.

In summary conditions $0 < \eta_b^i < 1$ and $0 < \eta_s^i < 1$ are satisfied if $\tau < \frac{\pi+2\upsilon}{3}$ and $\upsilon > \pi$ or $\tau > \frac{\pi+\upsilon}{2}$ and $\pi > \upsilon$ which is stated in Assumption 1 and $\alpha^i > \frac{2\tau}{\Sigma}$ which is stated in Assumption 2.

C.3 Proof of Claim 1

Proof. We prove Claim 1 by partially differentiating the equilibrium market shares on both sides with respect to τ, υ and π using Equations (15a) and (15b) under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$.

Case 1. $\upsilon > \pi$**,** $\pi = 0$

We use Equations (15a) and (15b) to compute the equilibrium market shares on buyers' and sellers' sides, and then we use Equation (14a) to express the market shares as a function of the difference in attributes in equilibrium.

$$
\begin{aligned}
\left(\eta_b^i\right)_{\upsilon>\pi} &= \frac{1}{2} + \frac{3\tau\left(\alpha^j - \alpha^i\right)(3\tau+2\upsilon)(2\tau-\upsilon)}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i+\alpha^j\right)\tau\right]} = \frac{1}{2} + \frac{3\tau}{2\sigma_\upsilon}\left(\Delta q_b^i\right)_{\upsilon>\pi} \\
\frac{\partial\left(\eta_b^i\right)_{\upsilon>\pi}}{\partial\tau} &= \frac{3}{2\sigma_\upsilon^2}\left[\sigma_\upsilon\left[\left(\Delta q_b^i\right)_{\upsilon>\pi} + \tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right] - 18\tau^2\left(\Delta q_b^i\right)_{\upsilon>\pi}\right] \\
&= -\frac{3}{2\sigma_\upsilon^2}\left[\left(9\tau^2+2\upsilon^2\right)\left(\Delta q_b^i\right)_{\upsilon>\pi} - \sigma_\upsilon\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right]
\end{aligned}
$$

According to Proposition 5 and its proof in Appendix C.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}$ is negative.

Therefore $\partial \left(\eta_b^i\right)_{\upsilon>\pi} / \partial\tau < 0$.

$$\frac{\partial \left(\eta_b^i\right)_{\upsilon>\pi}}{\partial \upsilon} = \frac{3\left(\alpha^j - \alpha^i\right)\tau}{4\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i+\alpha^j\right)\tau\right](\tau - 4\upsilon)$$
$$+\left[8\upsilon\alpha^i\alpha^j\sigma_\upsilon - 4\tau\upsilon\left(\alpha^i+\alpha^j\right)\right](3\tau+2\upsilon)(2\tau-\upsilon)\Bigg]$$
$$= \frac{3\left(\alpha^j - \alpha^i\right)\tau}{4\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\tau\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i+\alpha^j\right)\tau\right] - 4\upsilon\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i+\alpha^j\right)\tau\right]$$
$$+8\upsilon\alpha^i\alpha^j\sigma_\upsilon(3\tau+2\upsilon)(2\tau-\upsilon) - 4\tau\upsilon\left(\alpha^i+\alpha^j\right)(3\tau+2\upsilon)(2\tau-\upsilon)\Bigg]$$
$$= \frac{3\left(\alpha^j - \alpha^i\right)\tau}{4\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\tau\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j-\alpha^i\right)\tau\right] + 4\alpha^i\alpha^j\sigma_\upsilon\upsilon\big[2(3\tau+2\upsilon)$$
$$(2\tau-\upsilon) - \sigma_\upsilon\big] + 4\left(\alpha^i+\alpha^j\right)\tau\upsilon\left[\sigma_\upsilon - (3\tau+2\upsilon)(2\tau-\upsilon)\right]\Bigg]$$

$$= \frac{3\left(\alpha^j - \alpha^i\right)\tau}{4\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\tau\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j-\alpha^i\right)\tau\right]$$
$$+4\alpha^i\alpha^j\sigma_\upsilon\upsilon\left[3\tau^2 + 2\tau\upsilon - 2\upsilon^2\right] + 4\left(\alpha^i+\alpha^j\right)\tau^2\upsilon(3\tau-\upsilon)\Bigg]$$

To determine the sign of $\frac{\partial\left(\eta_b^i\right)_{\upsilon>\pi}}{\partial\upsilon}$ it is sufficient to examine the sign of $3\tau^2 + 2\tau\upsilon - 2\upsilon^2$ and $3\tau - \upsilon$ as the remaining elements of the partial derivative are positive. Specifically, $\tau\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j - \alpha^i\right)\tau\right]$ is positive under Assumption 2 when $\upsilon > \pi$, $\pi = 0$, and given platform 1 is more efficient in developing attributes $\alpha^2 > \alpha^1$.

Next, $3\tau^2 + 2\upsilon\tau - 2\upsilon^2$ is a quadratic polynomial in τ. We use the quadratic formula to find the values of τ that make the expression positive. The roots are $\tau = \frac{1}{3}\left(-\upsilon \pm \upsilon\sqrt{6}\right)$. Thus, the first root is $\tau_{r_1} = \frac{\sqrt{6}-1}{3}\upsilon$ and the second root is $\tau_{r_2} = -\frac{\sqrt{6}+1}{3}\upsilon$. Since the square term of the polynomial in τ is positive, $3\tau^2 + 2\upsilon\tau - 2\upsilon^2$ is positive for values outside both roots, meaning $\tau > \frac{\sqrt{6}-1}{3}\upsilon$ and $\tau < -\frac{\sqrt{6}+1}{3}\upsilon$. Given that transportation cost τ is positive by definition, we dismiss the negative root. Therefore, $3\tau^2 + 2\upsilon\tau - 2\upsilon^2$ is positive if $\tau > \frac{\sqrt{6}-1}{3}\upsilon$. This condition is satisfied under Assumption 1 when $\upsilon > \pi$, $\pi = 0$. Namely, if the right side of $\tau > \frac{\upsilon}{2}$ is greater than the right side of the previous condition, we guarantee it holds true. That is $\frac{\upsilon}{2} > \frac{\sqrt{6}-1}{3}\upsilon$, which results in $3 > 2.89$. Moreover, $3\tau - \upsilon$ is positive if $\tau > \frac{\upsilon}{3}$, which is also satisfied under Assumption 1 when $\upsilon > \pi$, $\pi = 0$ similar to the the previous condition. That is $\frac{\upsilon}{2} > \frac{\upsilon}{3}$, which results in $3 > 2$.

Therefore, $\frac{\partial (\eta_b^i)_{\upsilon>\pi}}{\partial \upsilon}$ is positive under Assumption 1 and Assumption 2 when $\upsilon > \pi$, $\pi = 0$.

$$\left(\eta_s^i\right)_{\upsilon>\pi} = \frac{1}{2} + \frac{\upsilon\left(\alpha^j - \alpha^i\right)(3\tau + 2\upsilon)(2\tau - \upsilon)}{2\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} = \frac{1}{2} + \frac{\upsilon}{\sigma_\upsilon}\left(\Delta q_b^i\right)_{\upsilon>\pi}$$
$$\frac{\partial\left(\eta_s^i\right)_{\upsilon>\pi}}{\partial\tau} = -\frac{\upsilon}{\sigma_\upsilon^2}\left[18\tau\left(\Delta q_b^i\right)_{\upsilon>\pi} - \sigma_\upsilon\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right]$$

According to Proposition 5 and its proof in Appendix C.1, we know that $\frac{\partial(\Delta q_b^i)_{\upsilon>\pi}}{\partial\tau}$ is negative. Therefore $\partial\left(\eta_s^i\right)_{\upsilon>\pi}/\partial\tau < 0$.

$$\frac{\partial\left(\eta_s^i\right)_{\upsilon>\pi}}{\partial\upsilon} = \frac{\left(\alpha^j - \alpha^i\right)}{2\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]\left[\upsilon(\tau - 4\upsilon)\right.$$
$$\left. + (3\tau + 2\upsilon)(2\tau - \upsilon)\right] + \upsilon(3\tau + 2\upsilon)(2\tau - \upsilon)\left[8\upsilon\alpha^i\alpha^j\sigma_\upsilon - 4\tau\upsilon\left(\alpha^i + \alpha^j\right)\right]\Bigg]$$
$$= \frac{\left(\alpha^j - \alpha^i\right)}{2\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]\left[\tau\upsilon + (3\tau + 2\upsilon)(2\tau - \upsilon)\right]$$

$$+4\alpha^i\alpha^j\sigma_\upsilon\upsilon^2\left[2(3\tau + 2\upsilon)(2\tau - \upsilon) - \sigma_\upsilon\right] + 4\tau\upsilon^2\left(\alpha^i + \alpha^j\right)\left[\sigma_\upsilon - (3\tau + 2\upsilon)(2\tau - \upsilon)\right]\Bigg]$$
$$= \frac{\left(\alpha^j - \alpha^i\right)}{2\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j - \alpha^i\right)\tau\right]\left[\tau\upsilon\right.$$
$$\left. + (3\tau + 2\upsilon)(2\tau - \upsilon)\right] + 4\alpha^i\alpha^j\sigma_\upsilon\upsilon^2\left[3\tau^2 + 2\tau\upsilon - 2\upsilon^2\right] + 4\tau^2\upsilon^2\left(\alpha^i + \alpha^j\right)(3\tau - \upsilon)\Bigg]$$

To determine the sign of $\frac{\partial(\eta_s^i)_{\upsilon>\pi}}{\partial\upsilon}$ it is sufficient to examine the sign of $3\tau^2 + 2\tau\upsilon - 2\upsilon^2$ and $3\tau - \upsilon$ as the remaining elements of the partial derivative are positive. Specifically, $\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j - \alpha^i\right)\tau\right]$ is positive under Assumption 2 when $\upsilon > \pi$, $\pi = 0$, and $(3\tau + 2\upsilon)(2\tau - \upsilon)$ is positive under Assumption 1 when $\upsilon > \pi$, $\pi = 0$ and given platform 1 is more efficient in developing attributes $\alpha^2 > \alpha^1$.

As it has been shown in $\frac{\partial(\Delta q_b^i)_{\upsilon>\pi}}{\partial\upsilon} > 0$ that both conditions $3\tau^2 + 2\tau\upsilon - 2\upsilon^2$ and $3\tau - \upsilon$ are positive, we conclude that $\frac{\partial(\eta_s^i)_{\upsilon>\pi}}{\partial\upsilon}$ is also positive under Assumption 1 and Assumption 2 when $\upsilon > \pi$, $\pi = 0$.

Case 2. $\pi > \upsilon$**,** $\upsilon = 0$

We use Equations (15a) and (15b) to compute the equilibrium market shares on buyers' and sellers' sides, and then we use Equation (14b) to express the market shares as a function of the difference in attributes in equilibrium.

$$\left(\eta_b^i\right)_{\pi>\upsilon} = \frac{1}{2} + \frac{3\tau\left(\alpha^j-\alpha^i\right)(3\tau+\pi)(2\tau-\pi)}{4\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]} = \frac{1}{2} + \frac{3\tau}{2\sigma_\pi}\left(\Delta q_b^i\right)_{\pi>\upsilon}$$

$$\begin{aligned}\frac{\partial\left(\eta_b^i\right)_{\pi>\upsilon}}{\partial\tau} &= \frac{3}{2\sigma_\pi^2}\left[\sigma_\pi\left[\left(\Delta q_b^i\right)_{\pi>\upsilon}+\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}\right)\right]-18\tau^2\left(\Delta q_b^i\right)_{\pi>\upsilon}\right] \\ &= -\frac{3}{2\sigma_\pi^2}\left[\left(9\tau^2+2\pi^2\right)\left(\Delta q_b^i\right)_{\pi>\upsilon}-\sigma_\pi\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}\right)\right]\end{aligned}$$

According to Proposition 5 and its proof in Appendix C.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}$ is negative. Therefore $\partial\left(\eta_b^i\right)_{\pi>\upsilon}/\partial\tau<0$.

$$\frac{\partial\left(\eta_b^i\right)_{\pi>\upsilon}}{\partial\pi} = \frac{3\tau}{2\sigma_\pi^2}\left[\sigma_\pi\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi}\right)+4\pi\left(\Delta q_b^i\right)_{\pi>\upsilon}\right]$$

According to Proposition 5 and its proof in Appendix C.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi}$ is positive. Therefore $\partial\left(\eta_b^i\right)_{\pi>\upsilon}/\partial\pi>0$.

$$\left(\eta_s^i\right)_{\pi>\upsilon} = \frac{1}{2} + \frac{\pi\left(\alpha^j-\alpha^i\right)(3\tau+\pi)(2\tau-\pi)}{4\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]} = \frac{1}{2} + \frac{\pi}{2\sigma_\pi}\left(\Delta q_b^i\right)_{\pi>\upsilon}$$

$$\frac{\partial\left(\eta_s^i\right)_{\pi>\upsilon}}{\partial\tau} = -\frac{\pi}{2\sigma_\pi^2}\left[18\tau\left(\Delta q_b^i\right)_{\pi>\upsilon}-\sigma_\pi\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}\right)\right]$$

According to Proposition 5 and its proof in Appendix C.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\tau}$ is negative. Therefore $\partial\left(\eta_s^i\right)_{\pi>\upsilon}/\partial\tau<0$.

$$\begin{aligned}\frac{\partial\left(\eta_s^i\right)_{\pi>\upsilon}}{\partial\pi} &= \frac{1}{2\sigma_\pi^2}\left[\sigma_\pi\left[\left(\Delta q_b^i\right)_{\pi>\upsilon}+\pi\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi}\right)\right]+4\pi^2\left(\Delta q_b^i\right)_{\pi>\upsilon}\right] \\ &= \frac{1}{2\sigma_\pi^2}\left[\left(9\tau^2+2\pi^2\right)\left(\Delta q_b^i\right)_{\pi>\upsilon}+\pi\sigma_\pi\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi}\right)\right]\end{aligned}$$

According to Proposition 5 and its proof in Appendix C.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial\pi}$ is positive. Therefore $\partial\left(\eta_s^i\right)_{\pi>\upsilon}/\partial\pi>0$. □

C.4 Positive Equilibrium Profits

We show the conditions for Equilibrium profits in Equation (16) for platform 1 to be positive. We notice Equation (16) is compose of two elements, the first is $\tau-\frac{\pi+\upsilon}{2}$ and the second is $\frac{\left[\Sigma\left(\alpha^j-\alpha^i\right)\left[\alpha^i\alpha^j\Sigma-\left(\alpha^i+\alpha^j\right)\tau\right]-\alpha^i\left(\alpha^j\Sigma-2\tau\right)\left(\alpha^i\Sigma-2\tau\right)\right]\Omega^2}{8\Sigma^2\left[\alpha^i\alpha^j\Sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$. The first component is positive under Assumption 1. To determine if the second element is positive, we can partially differentiate it with respect to α^2 and evaluate the result when $\alpha^2 = \frac{2\tau}{9\sigma}$.

$$\frac{\partial part2}{\partial \alpha^2} = 8\Sigma^2 \Big[\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right]^2 \left[\Sigma\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right] + \Sigma\left(\alpha^2-\alpha^1\right)\right.$$
$$\left.\left(\alpha^1\Sigma-\tau\right) - \alpha^1\Sigma\left(\alpha^1\Sigma-2\tau\right)\right] - 2\left(\alpha^1\Sigma-\tau\right)\left[\alpha^1\alpha^2\Sigma - \left(\alpha^1+\alpha^2\right)\tau\right]\left[\Sigma\left(\alpha^2-\alpha^1\right)\right.$$
$$\left.\left[\alpha^1\alpha^2\Sigma-\left(\alpha^1+\alpha^2\right)\tau\right] - \alpha^1\left(\alpha^2\Sigma-2\tau\right)\left(\alpha^1\Sigma-2\tau\right)\right]\Big] \Big/ 18^2\left[\alpha^1\alpha^2\Sigma-\left(\alpha^1+\alpha^2\right)\tau\right]^4$$
$$= \Big[2\Sigma\left[\alpha^1\alpha^2\Sigma-\left(\alpha^1+\alpha^2\right)\tau\right]\left[\alpha^1\alpha^2\Sigma-\left(\alpha^1\right)^2\Sigma-\tau\left(\alpha^2-\alpha^1\right)\right] - 2\Sigma\left(\alpha^2-\alpha^1\right)$$
$$\left(\alpha^1\Sigma-\tau\right)\left[\alpha^1\alpha^2\Sigma-\left(\alpha^1+\alpha^2\right)\tau\right] + 2\alpha^1\left(\alpha^1\Sigma-\tau\right)\left(\alpha^1\Sigma-2\tau\right)$$
$$\left(\alpha^2\Sigma-2\tau\right)\Big] \Big/ 8\Sigma^2\left[\alpha^1\alpha^2\Sigma-\left(\alpha^1+\alpha^2\right)\tau\right]^3$$
$$= \Big[\Sigma\left[\alpha^1\alpha^2\Sigma-\left(\alpha^1+\alpha^2\right)\tau\right]\left[\alpha^2\left(\alpha^1\Sigma-\tau\right)-\alpha^1\left(\alpha^1\Sigma-\tau\right)\right] - \Sigma\left(\alpha^2-\alpha^1\right)$$
$$\left(\alpha^1\Sigma-\tau\right)\left[\alpha^1\alpha^2\Sigma-\left(\alpha^1+\alpha^2\right)\tau\right] + \alpha^1\left(\alpha^1\Sigma-\tau\right)\left(\alpha^1\Sigma-2\tau\right)$$
$$\left(\alpha^2\Sigma-2\tau\right)\Big] \Big/ 4\Sigma^2\left[\alpha^1\alpha^2\Sigma-\left(\alpha^1+\alpha^2\right)\tau\right]^3 = \frac{\alpha^1\left(\alpha^1\Sigma-\tau\right)\left(\alpha^1\Sigma-2\tau\right)\left(\alpha^2\Sigma-2\tau\right)}{4\Sigma^2\left[\alpha^1\alpha^2\Sigma-\left(\alpha^1+\alpha^2\right)\tau\right]^3}$$
$$\frac{\partial part2}{\partial \alpha^2} = \frac{\alpha^1\left(\alpha^1\Sigma-\tau\right)\left(\alpha^1\Sigma-2\tau\right)\left(\alpha^2\Sigma-2\tau\right)}{4\Sigma^2\left[\alpha^2\left(\alpha^1\Sigma-2\tau\right)+\tau\left(\alpha^2-\alpha^1\right)\right]^3}$$

As it can be observed, for values of α^2 greater than $\frac{2\tau}{\Sigma}$, the equilibrium profits on platform 1 in Equation (16) are always positive.

C.5 Proof of Propositions 6a and 6b

Proof. We prove Proposition 6a and Proposition 6b by partially differentiating the difference in equilibrium market shares with respect to the parameters of the model τ, υ and π using Equations (17a) and (17b) when the cross-group network effect sellers exert on buyers are stronger than vice versa $\upsilon > \pi$, $\pi = 0$; and Equations (19a) and (19b) when the cross-group network effect buyers exert on sellers are stronger than vice versa $\pi > \upsilon$, $\upsilon = 0$, under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$.

Case 1. $\upsilon > \pi$, $\pi = 0$

We use Equation (17a) to compute the difference in equilibrium membership fees on buyers' side. Then, we use Equation (14a) to express this difference as a function of the difference in equilibrium attributes as

$$\Delta\left(p_b^i\right)_{\upsilon>\pi} = \frac{3\left(\alpha^j-\alpha^i\right)\tau^2\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)}{\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]} = \frac{6\tau^2}{\sigma_\upsilon}\left(\Delta q_b^i\right)_{\upsilon>\pi}$$

$$\frac{\partial \Delta\left(p_b^i\right)_{\upsilon>\pi}}{\partial \tau} = \frac{6}{\sigma_\upsilon^2}\left[\sigma_\upsilon\left[2\tau\left(\Delta q_b^i\right)_{\upsilon>\pi} + \tau^2\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial \tau}\right)\right] - 18\tau^3\left(\Delta q_b^i\right)_{\upsilon>\pi}\right]$$
$$= \frac{6\tau}{\sigma_\upsilon^2}\left[-2\left(\Delta q_b^i\right)_{\upsilon>\pi}\left(18\tau^2 - \sigma_\upsilon\right) + \sigma_\upsilon\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial \tau}\right)\right]$$
$$= \frac{6\tau}{\sigma_\upsilon^2}\left[-2\left(\Delta q_b^i\right)_{\upsilon>\pi}\left(9\tau^2 + 2\upsilon^2\right) + \sigma_\upsilon\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial \tau}\right)\right]$$

According to Proposition 5 and its proof in Appendix C.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial \tau}$ is negative. Therefore $\frac{\partial \Delta\left(p_b^i\right)_{\upsilon>\pi}}{\partial \tau} < 0$.

$$\frac{\partial \Delta\left(p_b^i\right)_{\upsilon>\pi}}{\partial \upsilon} = \frac{3\tau^2\left(\alpha^j - \alpha^i\right)}{\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\left[\left[\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]\left(\tau - 4\upsilon\right)\right.\right.$$
$$\left. + \left[8\upsilon\alpha^i\alpha^j\sigma_\upsilon - 4\tau\upsilon\left(\alpha^i + \alpha^j\right)\right]\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right]$$
$$= \frac{3\tau^2\left(\alpha^j - \alpha^i\right)}{\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\left[\tau\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j - \alpha^i\right)\tau\right] + 4\alpha^i\alpha^j\sigma_\upsilon\upsilon\left[2\left(3\tau + 2\upsilon\right)\right.\right.$$
$$\left.\left.\left(2\tau - \upsilon\right) - \sigma_\upsilon\right] + 4\left(\alpha^i + \alpha^j\right)\tau\upsilon\left[\sigma_\upsilon - \left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right]\right]$$
$$= \frac{3\tau^2\left(\alpha^j - \alpha^i\right)}{\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\left[\tau\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j - \alpha^i\right)\tau\right]\right.$$
$$\left. + 4\alpha^i\alpha^j\sigma_\upsilon\upsilon\left[3\tau^2 + 2\tau\upsilon - 2\upsilon^2\right] + 4\left(\alpha^i + \alpha^j\right)\tau^2\upsilon\left(3\tau - \upsilon\right)\right]$$

To determine the sign of $\frac{\partial \Delta\left(p_b^i\right)_{\upsilon>\pi}}{\partial \upsilon}$ it is sufficient to examine the sign of $3\tau^2 + 2\tau\upsilon - 2\upsilon^2$ and $3\tau - \upsilon$ as the remaining elements of the partial derivative are positive. Specifically, $\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j - \alpha^i\right)\tau\right]$ is positive under Assumption 2 when $\upsilon > \pi$, $\pi = 0$, and $\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)$ is positive under Assumption 1 when $\upsilon > \pi$, $\pi = 0$ and given platform 1 is more efficient in developing attributes $\alpha^2 > \alpha^1$.

As it has been shown in the proof of Claim 1 in Appendix C.3, specifically in $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial \upsilon} > 0$ that both conditions $3\tau^2 + 2\tau\upsilon - 2\upsilon^2$ and $3\tau - \upsilon$ are positive, we conclude that $\frac{\partial \Delta\left(p_b^i\right)_{\upsilon>\pi}}{\partial \upsilon}$ is also positive under Assumption 1 and Assumption 2 when $\upsilon > \pi$, $\pi = 0$.

Next, we use Equation (17b) to compute the difference in equilibrium membership fees on sellers' side. Then, we use Equation (14a) to express this difference as a function of the difference in equilibrium attributes as

$$\Delta\left(p_s^i\right)_{\upsilon>\pi} = -\frac{\left(\alpha^j - \alpha^i\right)\tau\upsilon\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)}{\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} = -\frac{2\tau\upsilon}{\sigma_\upsilon}\left(\Delta q_b^i\right)_{\upsilon>\pi}$$

$$\frac{\partial \Delta\left(p_s^i\right)_{\upsilon>\pi}}{\partial \tau} = \frac{-2\upsilon}{\sigma_\upsilon^2}\left[\sigma_\upsilon\left[\left(\Delta q_b^i\right)_{\upsilon>\pi} + \tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right] - 18\tau^2\left(\Delta q_b^i\right)_{\upsilon>\pi}\right]$$
$$= \frac{-2\upsilon}{\sigma_\upsilon^2}\left[-\left(\Delta q_b^i\right)_{\upsilon>\pi}\left(18\tau^2 - \sigma_\upsilon\right) + \sigma_\upsilon\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right]$$
$$= \frac{2\upsilon}{\sigma_\upsilon^2}\left[\left(\Delta q_b^i\right)_{\upsilon>\pi}\left(9\tau^2 + 2\upsilon^2\right) - \sigma_\upsilon\tau\left(\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}\right)\right]$$

According to Proposition 5 and its proof in Appendix C.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\tau}$ is negative. Therefore $\frac{\partial\Delta\left(p_s^i\right)_{\upsilon>\pi}}{\partial\tau} > 0$.

$$\frac{\partial \Delta\left(p_s^i\right)_{\upsilon>\pi}}{\partial \upsilon} = -\frac{\left(\alpha^j - \alpha^i\right)\tau}{\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]\left[\upsilon\left(\tau - 4\upsilon\right)\right.$$
$$\left.+\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right] + \upsilon\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\left[8\upsilon\alpha^i\alpha^j\sigma_\upsilon - 4\tau\upsilon\left(\alpha^i + \alpha^j\right)\right]\Bigg]$$
$$= -\frac{\left(\alpha^j - \alpha^i\right)\tau}{\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]\left[\tau\upsilon + \left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right]$$
$$+4\alpha^i\alpha^j\sigma_\upsilon\upsilon^2\left[2\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right) - \sigma_\upsilon\right] + 4\tau\upsilon^2\left(\alpha^i + \alpha^j\right)\left[\sigma_\upsilon - \left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right]\Bigg]$$

$$= -\frac{\left(\alpha^j - \alpha^i\right)\tau}{\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j - \alpha^i\right)\tau\right]\left[\tau\upsilon\right.$$
$$\left.+\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)\right] + 4\alpha^i\alpha^j\sigma_\upsilon\upsilon^2\left[3\tau^2 + 2\tau\upsilon - 2\upsilon^2\right] + 4\tau^2\upsilon^2\left(\alpha^i + \alpha^j\right)\left(3\tau - \upsilon\right)\Bigg]$$

To determine the sign of $\frac{\partial\Delta\left(p_s^i\right)_{\upsilon>\pi}}{\partial\upsilon}$ it is sufficient to examine the sign of $3\tau^2+2\tau\upsilon-2\upsilon^2$ and $3\tau-\upsilon$ as the remaining elements of the partial derivative are positive. Specifically, $\sigma_\upsilon\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right) + \left(\alpha^j - \alpha^i\right)\tau\right]$ is positive under Assumption 2 when $\upsilon > \pi$, $\pi = 0$, and $\left(3\tau + 2\upsilon\right)\left(2\tau - \upsilon\right)$ is positive under Assumption 1 when $\upsilon > \pi$, $\pi = 0$ and given platform 1 is more efficient in developing attributes $\alpha^2 > \alpha^1$.

As it has been shown in the proof of Claim 1 in Appendix C.3, specifically in $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial\upsilon} > 0$ that both conditions $3\tau^2 + 2\tau\upsilon - 2\upsilon^2$ and $3\tau - \upsilon$ are positive, we conclude that $\frac{\partial\Delta\left(p_s^i\right)_{\upsilon>\pi}}{\partial\upsilon}$ is also negative under Assumption 1 and Assumption 2 when $\upsilon > \pi$, $\pi = 0$.

Case 2. $\pi > \upsilon$ **,** $\upsilon = 0$

We use Equation (19a) to compute the difference in equilibrium membership fees on buyers' side. Then, we use Equation (14b) to express this difference as a function of the difference in equilibrium attributes as

$$\Delta\left(p_b^i\right)_{\pi>\upsilon} = \frac{\left(\alpha^j - \alpha^i\right)\left(3\tau^2 - \pi^2\right)\left(3\tau + \pi\right)\left(2\tau - \pi\right)}{\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]} = \frac{2\left(3\tau^2 - \pi^2\right)}{\sigma_\pi}\left(\Delta q_b^i\right)_{\pi>\upsilon}$$

$$\frac{\partial \Delta \left(p_b^i\right)_{\pi>\upsilon}}{\partial \tau} = \frac{2}{\sigma_\pi^2}\left[\sigma_\pi \left[6\tau \left(\Delta q_b^i\right)_{\pi>\upsilon} + \left(3\tau^2 - \pi^2\right)\left(\frac{\partial \left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \tau}\right)\right] - 18\tau \left(3\tau^2 - \pi^2\right)\left(\Delta q_b^i\right)_{\pi>\upsilon}\right]$$

$$= \frac{2}{\sigma_\pi}\left[6\tau \left(\Delta q_b^i\right)_{\pi>\upsilon}\left(\sigma_\pi - 9\tau^2 + 3\pi^2\right) + \sigma_\pi \left(3\tau^2 - \pi^2\right)\left(\frac{\partial \left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \tau}\right)\right]$$

$$= \frac{2}{\sigma_\pi}\left[6\tau\pi^2 \left(\Delta q_b^i\right)_{\pi>\upsilon} + \sigma_\pi \left(3\tau^2 - \pi^2\right)\left(\frac{\partial \left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \tau}\right)\right]$$

According to Proposition 5 and its proof in Appendix C.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \tau}$ is negative. Furthermore, as was mentioned in the intuition of the equilibrium membership fees in Equations (19a) and (19b) $3\tau^2 - \pi^2$ is negative as long as $\tau < \frac{\pi}{\sqrt{3}}$, which is compatible with Assumption 1 when $\pi > \upsilon$, $\upsilon = 0$ since $\frac{\pi}{2} < \tau < \frac{\pi}{\sqrt{3}}$. Therefore $\frac{\partial \Delta\left(p_b^i\right)_{\pi>\upsilon}}{\partial \tau} > 0$.

$$\frac{\partial \Delta \left(p_b^i\right)_{\pi>\upsilon}}{\partial \pi} = \frac{2}{\sigma_\pi^2}\left[\sigma_\pi \left[-2\pi \left(\Delta q_b^i\right)_{\pi>\upsilon} + \left(3\tau^2 - \pi^2\right)\left(\frac{\partial \left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \tau}\right)\right] - 4\pi \left(3\tau^2 - \pi^2\right)\left(\Delta q_b^i\right)_{\pi>\upsilon}\right]$$

$$= \frac{2}{\sigma_\pi}\left[2\pi \left(\Delta q_b^i\right)_{\pi>\upsilon}\left(-\sigma_\pi + 6\tau^2 - 2\pi^2\right) + \sigma_\pi \left(3\tau^2 - \pi^2\right)\left(\frac{\partial \left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \pi}\right)\right]$$

$$= -\frac{2}{\sigma_\pi}\left[6\tau^2\pi \left(\Delta q_b^i\right)_{\pi>\upsilon} - \sigma_\pi \left(3\tau^2 - \pi^2\right)\left(\frac{\partial \left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \pi}\right)\right]$$

According to Proposition 5 and its proof in Appendix C.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \pi}$ is negative. Furthermore, as was mentioned previously $3\tau^2 - \pi^2$ is negative as long as $\tau < \frac{\pi}{\sqrt{3}}$, which is compatible with Assumption 1 when $\pi > \upsilon$, $\upsilon = 0$ since $\frac{\pi}{2} < \tau < \frac{\pi}{\sqrt{3}}$. Therefore $\frac{\partial \Delta\left(p_b^i\right)_{\pi>\upsilon}}{\partial \pi} < 0$.

We use Equation (19b) to compute the difference in equilibrium membership fees on sellers' side. Then, we use Equation (14b) to express this difference as a function of the difference in equilibrium attributes as

$$\Delta \left(p_s^i\right)_{\pi>\upsilon} = \frac{\left(\alpha^j - \alpha^i\right)\tau\pi\left(3\tau + \pi\right)\left(2\tau - \pi\right)}{\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]} = \frac{2\tau\pi}{\sigma_\pi}\left(\Delta q_b^i\right)_{\pi>\upsilon}$$

$$\frac{\partial \Delta \left(p_s^i\right)_{\pi>\upsilon}}{\partial \tau} = \frac{2\pi}{\sigma_\pi^2}\left[\sigma_\pi \left[\left(\Delta q_b^i\right)_{\pi>\upsilon} + \tau\left(\frac{\partial \left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \tau}\right)\right] - 18\tau^2 \left(\Delta q_b^i\right)_{\pi>\upsilon}\right]$$

$$= -\frac{2\pi}{\sigma_\pi^2}\left[\left(\Delta q_b^i\right)_{\pi>\upsilon}\left(9\tau^2 + 2\pi^2\right) - \sigma_\pi\tau\left(\frac{\partial \left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \tau}\right)\right]$$

According to Proposition 5 and its proof in Appendix C.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial \tau}$ is negative.

Therefore $\frac{\partial \Delta\left(p_s^i\right)_{\pi>\upsilon}}{\partial \tau} < 0$.

$$\begin{aligned}\frac{\partial \Delta\left(p_s^i\right)_{\pi>\upsilon}}{\partial \pi} &= \frac{2\tau}{\sigma_\pi^2}\left[\sigma_\pi\left[\left(\Delta q_b^i\right)_{\pi>\upsilon} + \pi\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \pi}\right)\right] + 4\pi^2\left(\Delta q_b^i\right)_{\pi>\upsilon}\right] \\ &= \frac{2\tau}{\sigma_\pi^2}\left[\left(\Delta q_b^i\right)_{\pi>\upsilon}\left(9\tau^2 + 2\pi^2\right) + \pi\sigma_\pi\left(\frac{\partial\left(\Delta q_b^i\right)_{\pi>\upsilon}}{\partial \pi}\right)\right]\end{aligned}$$

According to Proposition 5 and its proof in Appendix C.1, we know that $\frac{\partial\left(\Delta q_b^i\right)_{\upsilon>\pi}}{\partial \pi}$ is positive. Therefore $\frac{\partial \Delta\left(p_s^i\right)_{\pi>\upsilon}}{\partial \pi} > 0$. □

C.6 Proof of Propositions 7a and 7b

Proof. We prove Proposition 7a and Proposition 7b by partially differentiating the difference in equilibrium profits with respect to the parameters of the model τ, υ and π when the cross-group network effect sellers exert on buyers are stronger than vice versa $\upsilon > \pi$, $\pi = 0$; and when the cross-group network effect buyers exert on sellers are stronger than vice versa $\pi > \upsilon$, $\upsilon = 0$, under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$.

Case 1. $\upsilon > \pi$**,** $\pi = 0$

We use Equations (17a) and (17b) and Equations (15a) and (15b) when $\upsilon > \pi$, $\pi = 0$ to compute the difference in equilibrium profits. Then we use Equation (14a) to express this difference as a function of the difference in equilibrium attributes as

$$\Delta\Pi^i_{\upsilon>\pi} = \left[(p_b)^i_{\upsilon>\pi} - f_b\right](\eta_b)^i_{\upsilon>\pi} + \left[(p_s)^i_{\upsilon>\pi} - f_s\right](\eta_s)^i_{\upsilon>\pi} - \frac{\alpha^i}{2}\left(q^i_b\right)^2_{\upsilon>\pi}$$

$$- \left[(p_b)^j_{\upsilon>\pi} - f_b\right](\eta_b)^j_{\upsilon>\pi} - \left[(p_s)^j_{\upsilon>\pi} - f_s\right](\eta_s)^j_{\upsilon>\pi} + \frac{\alpha^j}{2}\left(q^j_b\right)^2_{\upsilon>\pi}$$

$$= \left[\tau + \frac{3\tau^2}{\sigma_\upsilon}\Delta\left(q^i_b\right)_{\upsilon>\pi}\right]\left[\frac{1}{2} + \frac{3\tau}{2\sigma_\upsilon}\Delta\left(q^i_b\right)_{\upsilon>\pi}\right] - \left[\tau + \frac{3\tau^2}{\sigma_\upsilon}\Delta\left(q^j_b\right)_{\upsilon>\pi}\right]\left[\frac{1}{2} + \frac{3\tau}{2\sigma_\upsilon}\Delta\left(q^j_b\right)_{\upsilon>\pi}\right]$$

$$+ \left[\tau - \upsilon - \frac{\tau\upsilon}{\sigma_\upsilon}\Delta\left(q^i_b\right)_{\upsilon>\pi}\right]\left[\frac{1}{2} + \frac{\upsilon}{\sigma_\upsilon}\Delta\left(q^i_b\right)_{\upsilon>\pi}\right] - \left[\tau - \upsilon - \frac{\tau\upsilon}{\sigma_\upsilon}\Delta\left(q^j_b\right)_{\upsilon>\pi}\right]\left[\frac{1}{2} + \frac{\upsilon}{\sigma_\upsilon}\Delta\left(q^j_b\right)_{\upsilon>\pi}\right]$$

$$+ \frac{\alpha^j}{2}\left[\frac{\left(\alpha^i\sigma_\upsilon - 2\tau\right)^2(3\tau + 2\upsilon)^2(2\tau - \upsilon)^2}{4\sigma^2_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\right] - \frac{\alpha^i}{2}\left[\frac{\left(\alpha^j\sigma_\upsilon - 2\tau\right)^2(3\tau + 2\upsilon)^2(2\tau - \upsilon)^2}{4\sigma^2_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\right]$$

$$= \frac{3\tau^2}{\sigma_\upsilon}\left[\Delta\left(q^i_b\right)_{\upsilon>\pi} - \Delta\left(q^j_b\right)_{\upsilon>\pi}\right] + \frac{9\tau^3}{2\sigma^2_\upsilon}\left[\Delta\left(q^i_b\right)^2_{\upsilon>\pi} - \Delta\left(q^j_b\right)^2_{\upsilon>\pi}\right]$$

$$+ \frac{\upsilon(\tau - 2\upsilon)}{2\sigma_\upsilon}\left[\Delta\left(q^i_b\right)_{\upsilon>\pi} - \Delta\left(q^j_b\right)_{\upsilon>\pi}\right] - \frac{\tau\upsilon^2}{\sigma^2_\upsilon}\left[\Delta\left(q^i_b\right)^2_{\upsilon>\pi} - \Delta\left(q^j_b\right)^2_{\upsilon>\pi}\right]$$

$$+ \frac{(3\tau + 2\upsilon)^2(2\tau - \upsilon)^2}{8\sigma^2_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\left[\alpha^j\left(\alpha^i\sigma_\upsilon - 2\tau\right)^2 - \alpha^i\left(\alpha^j\sigma_\upsilon - 2\tau\right)^2\right]$$

$$= \frac{6\tau^2\left(\alpha^j - \alpha^i\right)(3\tau + 2\upsilon)(2\tau - \upsilon)}{2\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} + \frac{9\tau^3}{2\sigma^2_\upsilon}\left[\Delta\left(q^i_b\right)_{\upsilon>\pi} + \Delta\left(q^j_b\right)_{\upsilon>\pi}\right]$$

$$\left[\Delta\left(q^i_b\right)_{\upsilon>\pi} - \Delta\left(q^j_b\right)_{\upsilon>\pi}\right] + \frac{2\upsilon(\tau - 2\upsilon)\left(\alpha^j - \alpha^i\right)(3\tau + 2\upsilon)(2\tau - \upsilon)}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]}$$

$$- \frac{\tau\upsilon^2}{\sigma^2_\upsilon}\left[\Delta\left(q^i_b\right)_{\upsilon>\pi} + \Delta\left(q^j_b\right)_{\upsilon>\pi}\right]\left[\Delta\left(q^i_b\right)_{\upsilon>\pi} - \Delta\left(q^j_b\right)_{\upsilon>\pi}\right]$$

$$+ \frac{(3\tau + 2\upsilon)^2(2\tau - \upsilon)^2}{8\sigma^2_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\left[\alpha^j\left(\alpha^i\right)^2\sigma^2_\upsilon + 4\alpha^j\tau^2 - \alpha^i\left(\alpha^j\right)^2\sigma^2_\upsilon - 4\alpha^i\tau^2\right]$$

Since $\Delta\left(q^i_b\right)_{\upsilon>\pi} = \frac{\left(\alpha^j - \alpha^i\right)(3\tau+2\upsilon)(2\tau-\upsilon)}{2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i+\alpha^j\right)\tau\right]}$, we have $\Delta\left(q^i_b\right)_{\upsilon>\pi} + \Delta\left(q^j_b\right)_{\upsilon>\pi} = 0$, then

$$= \frac{6\tau^2}{\sigma_\upsilon}\Delta\left(q^i_b\right)_{\upsilon>\pi} + \frac{\upsilon(\tau - 2\upsilon)}{\sigma_\upsilon}\Delta\left(q^i_b\right)_{\upsilon>\pi} - \frac{\left[\alpha^i\alpha^j\sigma^2_\upsilon - 4\tau^2\right](3\tau + 2\upsilon)(2\tau - \upsilon)}{4\sigma^2_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]}\Delta\left(q^i_b\right)_{\upsilon>\pi}$$

$$\Delta\Pi^i_{\upsilon>\pi} = \frac{\Delta\left(q^i_b\right)_{\upsilon>\pi}}{\sigma_\upsilon}(3\tau + 2\upsilon)(2\tau - \upsilon) - \frac{\left[\alpha^i\alpha^j\sigma^2_\upsilon - 4\tau^2\right](3\tau + 2\upsilon)(2\tau - \upsilon)}{4\sigma^2_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]}\Delta\left(q^i_b\right)_{\upsilon>\pi}$$

Next, we partially differentiate $\Delta\Pi^i_{\upsilon>\pi}$ respect τ and υ, obtaining:

$$\frac{\partial\Delta\Pi^i_{\upsilon>\pi}}{\partial\tau} = \frac{1}{\sigma^2_\upsilon}\left[\sigma_\upsilon\left[\frac{\partial\Delta\left(q^i_b\right)_{\upsilon>\pi}}{\partial\tau}(3\tau + 2\upsilon)(2\tau - \upsilon) + (12\tau + \upsilon)\Delta\left(q^i_b\right)_{\upsilon>\pi}\right]\right.$$

$$-18\tau\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}\Bigg]-\frac{1}{4\sigma_\upsilon^4\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon^2\Big[\frac{\partial\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\partial\tau}$$

$$\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]+\Delta\left(q_b^i\right)_{\upsilon>\pi}\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]\left(12\tau+\upsilon\right)+4\tau\Delta\left(q_b^i\right)_{\upsilon>\pi}$$

$$\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\left(9\alpha^i\alpha^j\sigma_\upsilon-2\right)\Big]\Big[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\Big]-\Big[\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]\left(3\tau+2\upsilon\right)$$

$$\left(2\tau-\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}\Big]\Big[54\alpha^i\alpha^j\sigma_\upsilon^2\tau-\left(\alpha^i+\alpha^j\right)\sigma_\upsilon\left(36\tau^2+\sigma_\upsilon\right)\Big]\Bigg]$$

Next, we evaluate the partial derivative when $\tau=\frac{\upsilon}{2}$ getting:

$$\begin{aligned}
\left.\frac{\partial\Delta\Pi_{\upsilon>\pi}^i}{\partial\tau}\right|_{\tau=\frac{\upsilon}{2}} &= \frac{1}{\sigma_\upsilon}\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}-\frac{\Delta\left(q_b^i\right)_{\upsilon>\pi}\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]\left(12\tau+\upsilon\right)}{4\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}\\
&= \frac{\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[1-\frac{\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}\right]\\
&= \frac{\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[\frac{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]-\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}\right]\\
&= \frac{\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[\frac{3\alpha^i\alpha^j\sigma_\upsilon^2-4\sigma_\upsilon\left(\alpha^i+\alpha^j\right)\tau+4\tau^2}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}\right]\\
&= \frac{\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[\frac{3\alpha^i\alpha^j\sigma_\upsilon^2-6\alpha^j\sigma_\upsilon\tau-2\alpha^i\sigma_\upsilon\tau+4\tau^2+2\alpha^j\sigma_\upsilon\tau-2\alpha^i\sigma_\upsilon\tau}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}\right]\\
&= \frac{\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[\frac{3\alpha^j\sigma_\upsilon\left(\alpha^i\sigma_\upsilon-2\tau\right)-2\tau\left(\alpha^i\sigma_\upsilon-2\tau\right)+2\sigma_\upsilon\tau\left(\alpha^j-\alpha^i\right)}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}\right]\\
&= \frac{\left(12\tau+\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[\frac{\left(3\alpha^j\sigma_\upsilon-2\tau\right)\left(\alpha^i\sigma_\upsilon-2\tau\right)+2\sigma_\upsilon\tau\left(\alpha^j-\alpha^i\right)}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]}\right]\\
\left.\frac{\partial\Delta\Pi_{\upsilon>\pi}^i}{\partial\tau}\right|_{\tau=\frac{\upsilon}{2}} &= \frac{7\upsilon^3\Delta\left(q_b^i\right)_{\upsilon>\pi}\left[\left(\frac{3}{4}\alpha^j\upsilon-1\right)\left(\frac{1}{4}\alpha^j\upsilon-1\right)+\frac{\upsilon}{4}\left(\alpha^j-\alpha^i\right)\right]}{\frac{\upsilon^5}{4}\left[\alpha^i\alpha^j\frac{u}{4}-\frac{1}{2}\left(\alpha^i+\alpha^j\right)\right]}\\
&= \left[\frac{7\left[\left(3\alpha^j\upsilon-4\right)\left(\alpha^j\upsilon-4\right)+4\upsilon\left(\alpha^j-\alpha^i\right)\right]}{\upsilon^2\left[\alpha^j\left(\alpha^i\upsilon-4\right)+2\left(\alpha^j-\alpha^i\right)\right]}\right]\Delta\left(q_b^i\right)_{\upsilon>\pi}
\end{aligned}$$

The previous expression is positive under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2>\alpha^1$; and for τ values greater than $\frac{\upsilon}{2}$ (Assumption 1); and under Assumption 2 which turns to $\alpha^i>\frac{4}{\upsilon}$ when $\tau=\frac{\upsilon}{2}$. Therefore, $\frac{\partial\Delta\Pi_{\upsilon>\pi}^i}{\partial\tau}>0$.

$$\frac{\partial\Delta\Pi_{\upsilon>\pi}^i}{\partial\upsilon}=\frac{1}{\sigma_\upsilon^2}\Bigg[\sigma_\upsilon\left[\frac{\partial\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\partial\upsilon}\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)+\left(\tau-4\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}\right.$$

$$\left.+4\upsilon\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\Delta\left(q_b^i\right)_{\upsilon>\pi}\right]-\frac{1}{4\sigma_\upsilon^4\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}\Bigg[\sigma_\upsilon^2\Big[\frac{\partial\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\partial\upsilon}$$

$$\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]+\Delta\left(q_b^i\right)_{\upsilon>\pi}\left[\alpha^i\alpha^j\sigma_\upsilon^2-4\tau^2\right]\left(\tau-4\upsilon\right)$$

$$-8\alpha^i\alpha^j\upsilon\sigma_\upsilon\tau\Delta\left(q_b^i\right)_{\upsilon>\pi}\left(3\tau+2\upsilon\right)\left(2\tau-\upsilon\right)\Big]\Big[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\Big]$$

$$+ 4\upsilon\sigma_\upsilon \Big[\left[\alpha^i\alpha^j\sigma_\upsilon^2 - 4\tau^2\right](3\tau + 2\upsilon)(2\tau - \upsilon)\Delta\left(q_b^i\right)_{\upsilon>\pi}\Big]\Big[3\alpha^i\alpha^j\sigma_\upsilon - 2\left(\alpha^i + \alpha^j\right)\tau\Big]\Big]$$

Next, we evaluate the partial derivative when $\tau = \frac{\upsilon}{2}$ getting:

$$\begin{aligned}
\frac{\partial \Delta \Pi^i_{\upsilon>\pi}}{\partial \upsilon}\Big|_{\tau=\frac{\upsilon}{2}} &= \frac{1}{\sigma_\upsilon}(\tau - 4\upsilon)\Delta\left(q_b^i\right)_{\upsilon>\pi} - \frac{\Delta\left(q_b^i\right)_{\upsilon>\pi}\left[\alpha^i\alpha^j\sigma_\upsilon^2 - 4\tau^2\right](\tau - 4\upsilon)}{4\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} \\
&= \frac{(\tau - 4\upsilon)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[1 - \frac{\left[\alpha^i\alpha^j\sigma_\upsilon^2 - 4\tau^2\right]}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]}\right]
\end{aligned}$$

The right side of the previous expression is the same as in $\frac{\partial \Delta \Pi^i_{\upsilon>\pi}}{\partial \tau}$, consequently we obtain:

$$\begin{aligned}
&= \frac{(\tau - 4\upsilon)\Delta\left(q_b^i\right)_{\upsilon>\pi}}{\sigma_\upsilon}\left[\frac{\left(3\alpha^j\sigma_\upsilon - 2\tau\right)\left(\alpha^i\sigma_\upsilon - 2\tau\right) + 2\sigma_\upsilon\tau\left(\alpha^j - \alpha^i\right)}{4\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]}\right] \\
\frac{\partial \Delta \Pi^i_{\upsilon>\pi}}{\partial \upsilon}\Big|_{\tau=\frac{\upsilon}{2}} &= \left[\frac{(\tau - 4\upsilon)\left[\left(3\alpha^j\upsilon - 4\right)\left(\alpha^j\upsilon - 4\right) + 4\upsilon\left(\alpha^j - \alpha^i\right)\right]}{\upsilon^2\left[\alpha^j\left(\alpha^i\upsilon - 4\right) + 2\left(\alpha^j - \alpha^i\right)\right]}\right]\Delta\left(q_b^i\right)_{\upsilon>\pi}
\end{aligned}$$

The previous expression is positive under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$; and for τ values greater than 4υ; and under Assumption 2 which turns to $\alpha^i > \frac{4}{\upsilon}$ when $\tau = \frac{\upsilon}{2}$. Therefore, $\frac{\partial \Delta \Pi^i_{\upsilon>\pi}}{\partial \upsilon} > 0$.

Case 2. $\pi > \upsilon$, $\upsilon = 0$

We use Equations (19a) and (19b) and Equations (15a) and (15b) when $\pi > \upsilon$, $\upsilon = 0$ to compute the difference in equilibrium profits. Then we use Equation (14b) to express this difference as a function of the difference in equilibrium attributes as:

$$\begin{aligned}
\Delta \Pi^i_{\pi>\upsilon} &= \left[(p_b)^i_{\pi>\upsilon} - f_b\right](\eta_b)^i_{\pi>\upsilon} + \left[(p_s)^i_{\pi>\upsilon} - f_s\right](\eta_s)^i_{\pi>\upsilon} - \frac{\alpha^i}{2}\left(q_b^i\right)^2_{\pi>\upsilon} \\
&\quad - \left[(p_b)^j_{\pi>\upsilon} - f_b\right](\eta_b)^j_{\pi>\upsilon} - \left[(p_s)^j_{\pi>\upsilon} - f_s\right](\eta_s)^j_{\pi>\upsilon} + \frac{\alpha^j}{2}\left(q_b^j\right)^2_{\pi>\upsilon} \\
&= \left[\tau - \pi + \frac{3\tau^2 - \pi^2}{\sigma_\pi}\Delta\left(q_b^i\right)_{\pi>\upsilon}\right]\left[\frac{1}{2} + \frac{3\tau}{2\sigma_\pi}\Delta\left(q_b^i\right)_{\pi>\upsilon}\right] \\
&\quad - \left[\tau - \pi + \frac{3\tau^2 - \pi^2}{\sigma_\pi}\Delta\left(q_b^j\right)_{\pi>\upsilon}\right]\left[\frac{1}{2} + \frac{3\tau}{2\sigma_\pi}\Delta\left(q_b^j\right)_{\pi>\upsilon}\right] \\
&\quad + \left[\tau + \frac{\tau\pi}{\sigma_\pi}\Delta\left(q_b^i\right)_{\pi>\upsilon}\right]\left[\frac{1}{2} + \frac{\pi}{2\sigma_\pi}\Delta\left(q_b^i\right)_{\pi>\upsilon}\right] - \left[\tau + \frac{\tau\pi}{\sigma_\pi}\Delta\left(q_b^j\right)_{\pi>\upsilon}\right]\left[\frac{1}{2} + \frac{\pi}{2\sigma_\pi}\Delta\left(q_b^j\right)_{\pi>\upsilon}\right] \\
&\quad + \frac{\alpha^j}{2}\left[\frac{\left(\alpha^i\sigma_\pi - 2\tau\right)^2(3\tau + \pi)^2(2\tau - \pi)^2}{4\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\right] - \frac{\alpha^i}{2}\left[\frac{\left(\alpha^j\sigma_\pi - 2\tau\right)^2(3\tau + \pi)^2(2\tau - \pi)^2}{4\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\right] \\
&= \frac{6\tau^2 - 3\tau\pi - \pi^2}{2\sigma_\pi}\left[\Delta\left(q_b^i\right)_{\pi>\upsilon} - \Delta\left(q_b^j\right)_{\pi>\upsilon}\right] + \frac{3\tau\left(3\tau^2 - \pi^2\right)}{2\sigma_\pi^2}\left[\Delta\left(q_b^i\right)^2_{\pi>\upsilon} - \Delta\left(q_b^j\right)^2_{\pi>\upsilon}\right] \\
&\quad + \frac{\tau\pi}{\sigma_\pi}\left[\Delta\left(q_b^i\right)_{\pi>\upsilon} - \Delta\left(q_b^j\right)_{\pi>\upsilon}\right] - \frac{\tau\pi^2}{2\sigma_\pi^2}\left[\Delta\left(q_b^i\right)^2_{\pi>\upsilon} - \Delta\left(q_b^j\right)^2_{\pi>\upsilon}\right]
\end{aligned}$$

$$+\frac{(3\tau+\pi)^2(2\tau-\pi)^2}{8\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau\right]^2}\left[\alpha^j\left(\alpha^i\sigma_\pi-2\tau\right)^2-\alpha^i\left(\alpha^j\sigma_\pi-2\tau\right)^2\right]$$

As was mentioned in $\frac{\partial\Delta\Pi^i_{\pi>\upsilon}}{\partial\tau}$, $\Delta\left(q_b^i\right)_{\pi>\upsilon}+\Delta\left(q_b^j\right)_{\pi>\upsilon}=0$, then $\Delta\left(q_b^i\right)^2_{\pi>\upsilon}-\Delta\left(q_b^j\right)^2_{\pi>\upsilon}=\left[\Delta\left(q_b^i\right)_{\pi>\upsilon}+\Delta\left(q_b^j\right)_{\pi>\upsilon}\right]\left[\Delta\left(q_b^i\right)_{\pi>\upsilon}-\Delta\left(q_b^j\right)_{\pi>\upsilon}\right]=0$, Furthermore, $\Delta\left(q_b^i\right)_{\pi>\upsilon}-\Delta\left(q_b^j\right)_{\pi>\upsilon}=\frac{(3\tau+\pi)(2\tau-\pi)}{2[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau]}\left[2\left(\alpha^j-\alpha^i\right)\right]$, then it turns to $\Delta\left(q_b^i\right)_{\pi>\upsilon}-\Delta\left(q_b^j\right)_{\pi>\upsilon}=2\Delta\left(q_b^i\right)_{\pi>\upsilon}$, therefore we obtain:

$$\Delta\Pi^i_{\pi>\upsilon}=\frac{(3\tau+\pi)(2\tau-\pi)}{\sigma_\pi}\Delta\left(q_b^i\right)_{\pi>\upsilon}-\frac{\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right](3\tau+\pi)(2\tau-\pi)}{4\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau\right]}\Delta\left(q_b^i\right)_{\pi>\upsilon}$$

Next, we partially differentiate $\Delta\Pi^i_{\pi>\upsilon}$ respect τ and π, obtaining:

$$\begin{aligned}\frac{\partial\Delta\Pi^i_{\pi>\upsilon}}{\partial\tau}&=\frac{1}{\sigma_\pi^2}\left[\sigma_\pi\left[\frac{\partial\Delta\left(q_b^i\right)_{\pi>\upsilon}}{\partial\tau}(3\tau+\pi)(2\tau-\pi)+(12\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}\right]\right.\\ &\left.-18\tau(3\tau+\pi)(2\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}\right]-\frac{1}{4\sigma_\pi^4\left[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau\right]^2}\left[\sigma_\pi^2\left[\frac{\partial\Delta\left(q_b^i\right)_{\pi>\upsilon}}{\partial\tau}\right.\right.\\ &(3\tau+\pi)(2\tau-\pi)\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right]+\Delta\left(q_b^i\right)_{\pi>\upsilon}\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right](12\tau-\pi)+4\tau\Delta\left(q_b^i\right)_{\pi>\upsilon}\\ &\left.(3\tau+\pi)(2\tau-\pi)\left(9\alpha^i\alpha^j\sigma_\pi-2\right)\right]\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]-\left[\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right](3\tau+\pi)\right.\\ &\left.\left.(2\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}\right]\left[54\alpha^i\alpha^j\sigma_\pi^2\tau-\left(\alpha^i+\alpha^j\right)\sigma_\pi\left(36\tau^2+\sigma_\pi\right)\right]\right]\end{aligned}$$

Next, we evaluate the partial derivative when $\tau=\frac{\pi}{2}$ and obtain:

$$\begin{aligned}\left.\frac{\partial\Delta\Pi^i_{\pi>\upsilon}}{\partial\tau}\right|_{\tau=\frac{\pi}{2}}&=\frac{1}{\sigma_\pi}(12\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}-\frac{\Delta\left(q_b^i\right)_{\pi>\upsilon}\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right](12\tau-\pi)}{4\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau\right]}\\ &=\frac{(12\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}}{\sigma_\pi}\left[1-\frac{\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right]}{4\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau\right]}\right]\end{aligned}$$

The right side of the previous expression is the same as in $\frac{\partial\Delta\Pi^i_{\upsilon>\pi}}{\partial\tau}$, consequently we obtain:

$$\begin{aligned}&=\frac{(12\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}}{\sigma_\pi}\left[\frac{\left(3\alpha^j\sigma_\pi-2\tau\right)\left(\alpha^i\sigma_\pi-2\tau\right)+2\sigma_\pi\tau\left(\alpha^j-\alpha^i\right)}{4\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau\right]}\right]\\ \left.\frac{\partial\Delta\Pi^i_{\pi>\upsilon}}{\partial\tau}\right|_{\tau=\frac{\pi}{2}}&=\frac{5\pi^3\Delta\left(q_b^i\right)_{\pi>\upsilon}\left[\left(\frac{3}{4}\alpha^j\pi-1\right)\left(\frac{1}{4}\alpha^j\pi-1\right)+\frac{\pi}{4}\left(\alpha^j-\alpha^i\right)\right]}{\frac{\pi^5}{4}\left[\alpha^i\alpha^j\frac{\pi}{4}-\frac{1}{2}\left(\alpha^i+\alpha^j\right)\right]}\\ &=\left[\frac{5\left[\left(3\alpha^j\pi-4\right)\left(\alpha^j\pi-4\right)+4\upsilon\left(\alpha^j-\alpha^i\right)\right]}{\pi^2\left[\alpha^j\left(\alpha^i\pi-4\right)+2\left(\alpha^j-\alpha^i\right)\right]}\right]\Delta\left(q_b^i\right)_{\pi>\upsilon}\end{aligned}$$

The previous expression is positive under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2>\alpha^1$; and for τ values greater than $\frac{\pi}{2}$ (Assumption 1); and under Assumption 2 which turns to $\alpha^i>\frac{4}{\pi}$ when $\tau=\frac{\pi}{2}$. Therefore,

$\frac{\partial \Delta \Pi^i_{\pi > \upsilon}}{\partial \tau} > 0$.

$$\frac{\partial \Delta \Pi^i_{\pi>\upsilon}}{\partial \pi} = \frac{1}{\sigma_\pi^2}\left[\sigma_\pi \left[\frac{\partial \Delta\left(q_b^i\right)_{\pi>\upsilon}}{\partial \pi}(3\tau+\pi)(2\tau-\pi) - (\tau+2\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}\right]\right.$$

$$\left. +4\pi(3\tau+\pi)(2\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}\right] - \frac{1}{4\sigma_\pi^4\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right]^2}\left[\sigma_\pi^2\left[\frac{\partial \Delta\left(q_b^i\right)_{\pi>\upsilon}}{\partial \pi}\right.\right.$$

$$(3\tau+\pi)(2\tau-\pi)\left[\alpha^i\alpha^j\sigma_\pi^2 - 4\tau^2\right] - \Delta\left(q_b^i\right)_{\pi>\upsilon}\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right](\tau+2\pi)$$

$$\left. -8\alpha^i\alpha^j\pi\sigma_\pi\tau\Delta\left(q_b^i\right)_{\pi>\upsilon}(3\tau+\pi)(2\tau-\pi)\right]\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right]$$

$$\left. +4\pi\sigma_\pi\left[\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right](3\tau+\pi)(2\tau-\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}\right]\left[3\alpha^i\alpha^j\sigma_\pi - 2\left(\alpha^i+\alpha^j\right)\tau\right]\right]$$

Next, we evaluate the partial derivative when $\tau = \frac{\pi}{2}$ getting:

$$\left.\frac{\partial \Delta \Pi^i_{\pi>\upsilon}}{\partial \pi}\right|_{\tau=\frac{\pi}{2}} = -\frac{1}{\sigma_\pi}(\tau+2\pi)\Delta\left(q_b^i\right)_{\upsilon>\pi} + \frac{\Delta\left(q_b^i\right)_{\pi>\upsilon}\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right](\tau+2\pi)}{4\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right]}$$

$$= -\frac{(\tau+2\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}}{\sigma_\pi}\left[1 - \frac{\left[\alpha^i\alpha^j\sigma_\pi^2-4\tau^2\right]}{4\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right]}\right]$$

The right side of the previous expression is the same as in $\frac{\partial \Delta \Pi^i_{\pi>\upsilon}}{\partial \tau}$, consequently we get

$$= -\frac{(\tau+2\pi)\Delta\left(q_b^i\right)_{\pi>\upsilon}}{\sigma_\pi}\left[\frac{\left(3\alpha^j\sigma_\pi - 2\tau\right)\left(\alpha^i\sigma_\pi - 2\tau\right) + 2\sigma_\pi\tau\left(\alpha^j-\alpha^i\right)}{4\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right]}\right]$$

$$\left.\frac{\partial \Delta \Pi^i_{\pi>\upsilon}}{\partial \upsilon}\right|_{\tau=\frac{\pi}{2}} = -\left[\frac{5\left[\left(3\alpha^j\pi-4\right)\left(\alpha^j\pi-4\right)+4\pi\left(\alpha^j-\alpha^i\right)\right]}{2\pi^2\left[\alpha^j\left(\alpha^i\pi-4\right)+2\left(\alpha^j-\alpha^i\right)\right]}\right]\Delta\left(q_b^i\right)_{\pi>\upsilon}$$

The previous expression is positive under the assumption that platform 1 is more efficient in developing attributes on buyers' side than platform 2, $\alpha^2 > \alpha^1$; and for τ values greater than $\frac{\pi}{2}$; and under Assumption 2 which turns to $\alpha^i > \frac{4}{\pi}$ when $\tau = \frac{\pi}{2}$. Therefore, $\frac{\partial \Delta \Pi^i_{\pi>\pi}}{\partial \pi} < 0$.

□

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