The share of housing wealth and the decline in real interest rates[*]

Markus Knell[**]
Oesterreichische Nationalbank

August 2024

Abstract

The recent decades have been characterized by three macroeconomic trends: a decline in real interest rates, an increase in the wealth-to-income ratio and an increase in the share of housing wealth. This paper examines whether the observed decline in real interest rates can be reconciled with the other developments within a standard economic model that includes physical capital and housing stock as assets. Theoretical modeling and numerical calibrations reveal that while a reduction in interest rates boosts asset valuations and aggregate wealth ratios, the quantitative relationships observed empirically are not straightforwardly replicated in a standard setup. In particular, a comparison between 1980 and 2017 that assumes a decrease in the return to capital from 10% to 7%, alongside declines in productivity and population growth and an increase in public debt, shows an only modest effect on the wealth-to-income ratio and the share of housing wealth. To address these discrepancies, the paper explores additional explanatory factors, particularly the role of outright owners. Numerical calibrations suggest that an increase in the fraction of outrightly owned houses and a higher markup produce results closely aligned with empirical data. The model extends to regional differences, successfully explaining divergent trends in wealth aggregates between the US and Europe by accounting for higher markups in the US and a greater prevalence of outright owners in Europe.

Keywords: Housing, Wealth, Interest Rate, Saving
JEL-Classification: D14, D31, G51, R21

[*] I thank Volker Grossmann, Ricardo Reis, Martin Summer, two anonymous referees and participants at various conferences for valuable comments and suggestions. The views expressed in this paper do not necessarily reflect those of the Oesterreichische Nationalbank.
[**] OeNB, Economic Studies Division, Otto-Wagner-Platz 3, POB-61, A-1011 Vienna; Phone: (++43-1) 40420 7218, email: markus.knell@oenb.at.

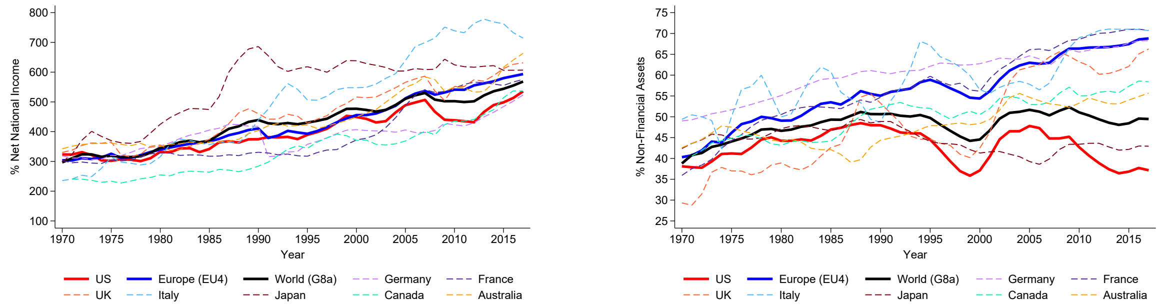
1 Introduction

Real interest rates have exhibited a constant decline over the recent decades. A seminal study by Laubach & Williams (2003) documented a decrease of approximately 3 percentage points (pp) in the long-run real interest rate in the United States since 1980. This decline was subsequently corroborated by various studies, including Summers & Rachel (2019), who estimated a similarly sized decrease in the *global* real interest rate since 1980. This empirical trend has given rise to a substantial body of literature aiming to elucidate its underlying causes. The primary explanatory factors highlighted in this literature include shifts in savings patterns due to demographic aging (Eggertsson et al. 2019, Auclert et al. 2021) or increasing income inequality (Mian et al. 2021*b*), advancements in technological progress (Gordon 2014) and a global savings glut (Bernanke 2005, Caballero et al. 2017). A comprehensive discussion of various explanatory channels can be found in Rachel & Smith (2015) and Mian et al. (2021*a*). Summers & Rachel (2019) and Platzer & Peruffo (2022) employ large-scale quantitative models to assess the relative importance of these different channels in explaining the decline in interest rates.

The level of real interest rates, however, was not the only macroeconomic variable that underwent a considerable shift over recent decades. At least two other important macroeconomic magnitudes have shown pronounced changes during the same period. On the one hand, the private wealth-to-income ratio has increased from around 300% in 1970 and 340% in 1980 to approximately 570% in the year 2017 as illustrated in Figure 1a and summarized in Table 1. This trend is similar for the world average and for the developments in the US and Europe taken separately.[1] On the other hand, the data also indicate that the general rise in the wealth-to-income ratio was especially driven by an increase in housing wealth. As shown in Figure 1b the world average for the share of housing in total wealth increased from around 40% in 1970 to 47% in 1980 and to approximately 50% in 2017. In this case, however, more pronounced cross-country differences can be observed, particularly between the US and the European countries. While the housing wealth share in the US remained basically constant between 1970 and today, the share increased considerably for the EU4, from 42% in 1970 to almost 70% in 2017.

In this paper I investigate whether and under which assumptions the long-run trends

[1] The “world” corresponds to the GDP-weighted average of 8 large economies which I term the “G8a” (i.e., the G7 plus Australia). “Europe”, on the other hand, is equated with the EU4 (Germany, France, the United Kingdom and Italy).



(a) Wealth-to-Income Ratio

(b) Share of Housing Wealth

Figure 1: Panel (a) shows the private wealth-to-income ratio from 1970 to 2017 for a number of countries and regions. Panel (b) does the same for the share of housing wealth. Data source: *World Inequality Database* (WID.world).

of these three macroeconomic magnitudes are compatible with each other. In particular, I do not take a stand on the sources behind the development of real interest rates and treat their decline by between 3 to 4 pp since 1980 as a fact. The central question of the paper is whether this observed decline can be aligned with the observed increases in the average wealth-to-income ratio and the average share of housing wealth, as well as with the cross-country patterns. The first — theoretical — part of the paper discusses how and through which channels an exogenous decline in real interest rates affects the various wealth aggregates in a standard economic model. In the second — numerical — part I investigate whether a realistic calibration of the theoretical model leads to results that are also quantitatively in line with the observed patterns.

The theoretical framework assumes that there exist two types of assets: physical capital and the housing stock (partitioned into rented and owner-occupied houses). Both of these assets need financing and their level will thus depend (negatively) on the interest rate. For physical capital this arises from the assumptions of a Cobb-Douglas production function and competitive factor markets, implying an inverse relationship between capital demand and the return on capital r_k. As far as housing is concerned, I assume that the housing stock is fixed (or that it grows at least more slowly than the general economy) and that housing expenditures are a fixed fraction of households' income (an assumption that is in line with the empirical facts). Since the purchasing price of the housing stock is determined by the present value of the discounted stream of (actual and imputed) rental incomes (cf. Poterba 1984, Himmelberg et al. 2005) this implies that the housing sector's financing demand also depends negatively on the interest rate. Taken together, the

Table 1: The wealth-to-income ratio and the share of housing wealth for various regions (in %)

	World (weighted)		**World** (unweighted)		**US**		**Europe** (EU4)	
	β	$\frac{\beta_H}{\beta}$	β	$\frac{\beta_H}{\beta}$	β	$\frac{\beta_H}{\beta}$	β	$\frac{\beta_H}{\beta}$
1970	297	39	269	36	323	38	281	42
1980	346	47	339	47	332	44	333	49
2017	568	50	598	59	536	37	595	69

Note: The "world" refers to a group of core countries termed the G8a (i.e., the G7 countries plus Australia), "Europe" to the EU4 countries (Germany, France, United Kingdom, Italy). The values in the first two columns are weighted with GDP based on PPP. Data source: Wealth Inequality Database (WID).

standard model gives rise to simple steady state expressions for the aggregate wealth-to-income ratio β, the capital-wealth-to-income ratio β_K and the housing-wealth-to-income ratio β_H. All of these ratios encompass a negative relation between aggregate wealth and the level of interest rates. The relative sizes of the subaggregates β_K and β_H depend on a number of parameters like the depreciation rates, the rate of economic growth, the capital share and the housing expenditure share. The fact that the latter two magnitudes are often in a similar range explains why empirical data show a share of housing wealth β_H/β that typically hovers around 50%.

The stylized model can then be used to investigate how an exogenous decrease in the return to capital r_k will affect the wealth-to-income ratio β and the share of housing wealth β_H/β. In the benchmark exercise I compare the steady states in 1980 and in 2017 by using standard values for the structural parameters and by assuming a decrease in r_k from 10% to 7% together with a decrease in the productivity growth rate g and in the population growth rate n that is in line with the empirical data. It turns out that this decline has only a rather modest effect on the wealth-to-income ratio and almost no effect on the share of housing wealth. There are several reasons behind this finding. First, the rate of return r_k appears in the denominators of the expressions for both housing wealth and capital wealth. Therefore, a decrease in the interest rate increases both types of wealth, potentially leaving their ratio nearly unchanged. Second, the effect of a decline by 3 pp also depends on the initial level of the return on capital. For instance, a decrease in r_k from 10% to 7% (as I assume) has a smaller effect on the wealth aggregates than a decrease from 6% to 3%. The use of the latter assumption would, however, be misguided since it does not align with empirical data and furthermore leads to implausibly high levels of the wealth-to-income ratio. Third, the assumption of a simultaneous decrease in g and n further cushions the impact of the decline in r_k. An exclusive decrease in r_k by 3 pp would, e.g., imply a slightly higher (but still insufficient) increase in the share of housing wealth. It would, however, be wrong to treat the decline in the interest rate as an isolated event, in particular since the reductions in the growth rates of productivity and the population have been identified as important drivers of the decline in the first place (Summers & Rachel 2019, Platzer & Peruffo 2022).[2]

Since the basic model is not capable to replicate the observed data one has to look for additional explanatory factors. The theoretical analysis highlights the significance of owner-occupiers in understanding the relationship between interest rates and the share of housing wealth. It is particularly crucial to consider outright owners, i.e., households that own their homes without a mortgage. In the model, I account for this by assuming that a certain fraction of households κ_N^{oo} is in the possession of a certain fraction of the housing stock κ_H^{oo} . This constellation is meant to capture the empirically relevant fact that some owners of houses and land simply hold on to their property, either because they are not able or because they are not willing to trade (e.g., due to legal restrictions of due to their preference to "age in place"). Outright ownership might play a crucial role for the changes in wealth aggregates for the following reason: for renters and owner-occupiers with mortgages, a reduction in the aggregate housing supply has no first-order effect on their housing wealth. The reduction in housing supply leads to an increase in house prices while leaving their total housing expenditures, and thus their housing wealth, essentially unchanged. However, the increase in house prices raises the value of the housing stock owned by outright owners, resulting in an increase in their housing wealth and pari passu in aggregate housing wealth. Therefore, changes in the relative size of the housing stocks controlled by renters, owner-occupiers with mortgages, and outright owners can significantly affect how the share of housing wealth changes over time and how it reacts to declines in the interest rate.

The magnitude of the effects sketched in the theoretical part of the paper are assessed in the numerical section. I use various data sources to find realistic values for the main

[2] In fact, these models typically also identify other important drivers behind the decline in interest rates, such as increases in income inequality or in life expectancy. In the stylized framework of the present paper these factors are not explicitly modelled and are subsumed in the exogenous decline in interest rates. For the purpose of this paper this is an inconsequential omission since the drivers do not appear directly in the formulas for capital and housing demand.

model parameters. Besides the productivity growth rate g and the population growth rate n these are the depreciation rates (for capital and housing), the labor share, the degree of market power μ (which is due to the assumption of monopolistic competition in the goods market), the relative shares of dwellers and dwellings and the risk wedges between the return to capital r_k and other interest rates. Furthermore, I now also introduce government bonds as a third asset category. Using the benchmark calibration for the world economy in 1980 and assuming a return to capital of $r_k = 10\%$ implies steady state values for the wealth-to-income ratio of $\beta = 340\%$ and for the share of housing wealth of $\beta_H/\beta = 47\%$. Both of these values are in line with the observed data (see Table 1). A decrease in r_k by 3 pp together with the observed changes in g (from 3.11% to 1.93%), in n (from 0.79% to 0.54%) and in public debt (from 20% to 70%) implies an increase in the wealth ratios to $\beta = 480\%$ and $\beta_H/\beta = 48\%$, respectively. These numbers are lower than empirically observed. One could get closer to the real-world pattern if one would assume a decline in the interest rate to 6% which, however, seems to exceed the values found in the related literature (Summers & Rachel 2019, Holston et al. 2023). Alternatively, however, one can also consider the empirically more plausible case that there have been changes in additional parameter values. In particular, I focus on two crucial variables: the (gross) markup μ and the relative abundance of outrightly owned houses, i.e. the fraction of houses in the possession of outright owners to their share in the population ($\kappa_H^{oo}/\kappa_N^{oo}$). An increase in this fraction can, e.g., be due to a preference shift or to the fact that more owners have paid down their mortgage and decide to hold on to their home until high age. I show that an extension of the baseline case where $\kappa_H^{oo}/\kappa_N^{oo}$ is assumed to increase from 100% to 120% and the mark-up from 10% to 20% leads to an increase in the share of housing wealth to 52% even if r_k declines only from 10% to 7%. Allowing for changes in additional parameters (like the depreciation rates, the share of renters and the housing expenditure share) leads to results that are even closer aligned with the empirical trends. In a final specification I also consider the case where part of the monopoly profits are included in the valuation of firms. This specification leads to particularly promising results. A decrease in the rate of return from 10% to 7% now implies an increase in the wealth-to-income ratio to 561% and in the share of housing wealth to 52%. This outcome is very close to the empirical pattern for the world economy (i.e. for the G8a) as reported in Table 1.

In the final part of the paper I show that the model is also capable to explain the different developments of wealth aggregates in the US and in Europe as long as one takes the different trends in the two regions for some crucial parameters into account. In particular, I take the model where part of monopoly profits are included in the valuation of firms but now assume that the change in average values was driven by heterogeneous regional developments. In particular, I follow the literature (De Loecker & Eeckhout 2021) and assume that the level of the markup in 1980 and its increase until 2017 was higher in the US (moving from 15% to 30%) than in Europe (with an assumed move from 5% to 10%). The opposite is true for the size and the weight of outright owners where the data suggest that their importance was always larger in Europe than in the US. I assume that for the US the term $\kappa_H^{oo}/\kappa_N^{oo}$ stayed constant at 80% while it increased from 120% to 160% in Europe. Using these values (together with some additional minor changes) the model implies for the US an increase in the wealth-to-income ratio from 364% to 574% and a slight decrease in the share of housing wealth from 44% to 43%. For the EU4, on the other hand, the results indicate an increase in β from 332% to 646% and a steep increase in β_H/β from 50% to 68%. These numbers are fairly close to the empirically observed values in Figure 1 and in Table 1.

The results of the paper are relevant for a number of current debates. First, it has been frequently argued that housing is a crucial ingredient to understand the changes in wealth both over time and across countries (Bonnet et al. 2014, Rognlie 2016). The paper highlights that the decline in interest rates is sufficient to explain the intertemporal and interregional pattern once additional economic changes are taken into account. Second, the paper emphasizes the important role of outright owners in order to understand different developments, e.g. between the US and Europe. Given that housing markets are organized in highly different ways across countries and that one can observe different shares of renters, owners with mortgage and outright owners the model can also be used to assess possible or likely future developments for housing wealth and the share of housing wealth. Inter alia this will depend, e.g., on the impact of regulatory or behavioural changes on the size of outright ownership. Third, the model could be further extended to endogenize the share of dwellers and to study the question how to deal with sharply increasing house prices and the challenges to affordability.

Related literature: The paper takes the decrease in the equilibrium real interest rate that happened since (at least) the 1980s as an exogenously given starting point. Important contributions to this literature have been mentioned at the beginning of the introduction, notably the quantitative models developed by Eggertsson et al. (2019), Summers & Rachel (2019) and Platzer & Peruffo (2022). A second area of related literature focuses on the components of wealth and their evolution over time. Notable contributions in this domain include Piketty (2011), Piketty & Zucman (2014) and Bauluz et al. (2022). Within this context, Bonnet et al. (2014), Bonnet et al. (2021) and Rognlie (2016) have discussed the role of housing in explaining observed trends in aggregate wealth. The long-term trajectory of house prices and the parallel increase in household mortgage lending are documented in Knoll et al. (2017*a*) and Jordà et al. (2016).

Furthermore, the paper is also related to the literature on housing and macroeconomics. The majority of research in this strand of research focuses on the issue of short-run fluctuations, with a particular emphasis on developments following the onset of the Great Financial Crisis (Favilukis et al. 2017, Justiniano et al. 2019, Kaplan et al. 2020). Papers that deal with long-run developments are Borri & Reichlin (2018), Grossmann, Larin, Löfflad & Steger (2021), Grossmann, Larin & Steger (2021) and Lisack et al. (2021). In particular the latter two articles are closely related to the topic of this paper. Grossmann, Larin & Steger (2021) present a complementary explanation for the increase in the share of housing wealth, centered on a model that distinguishes between land and structures and in which productivity increases are weaker in the construction sector than in the non-housing sector. The model leads to a steady state in which house prices grow at a different rate than the rest of the economy. The same is also true in my model, although it follows from the direct assumption about the growth rate of the housing stock and not from an explicit model of housing production. Grossmann, Larin & Steger (2021) also study the development of aggregate wealth and they show that their calibrated model is able to explain most of the observed increase in the housing wealth-to-income ratio since 1950 for US, UK, France, and Germany. Different to my model, however, they refer to transitional dynamics (where a transitional decline in the interest rate is associated with an increase in housing wealth) while I focus on a comparison between steady states and emphasize the importance of additional changes like the increase in market power and the influence of outright owners.[3] Lisack et al. (2021), on the other hand, use a calibrated OLG model similar to Summers & Rachel (2019) and Platzer & Peruffo (2022) but with the inclusion of housing. Their focus is on the reaction of the house price to the decline in interest rates (that comes out of their model). They show that the model is able to explain about 85 percent of the observed increase in real house prices and most of the increase in housing wealth, but they do not focus on the *share* of housing wealth which is the core of this paper. Finally, a number of papers have studied the relation between the level of interest rates and house prices (and thus also housing wealth). Miles & Monro

[3]Details on the comparison between the two approaches can be found in Appendix A.2.

(2021), e.g., argue that the entire increase in the house-price-to-income ratio in the UK between 1985 and 2018 can be attributed to the decline in the real risk-free interest rate.

The paper is structured as follows. The asset supply side of the model is presented in the subsequent section. In section 3 I derive the expressions for the wealth-to-income ratio and the share if housing wealth for the case where all households are renters while in section 4 I introduce owner-occupiers. The numerical results are contained in section 5 and section 6 concludes.

2 The model

In the following I describe the (asset) supply side of the model. Since the paper focuses on the relation between the levels and changes of interest rates and the size and changes of wealth aggregates I leave the asset demand side unspecified and rather treat the interest rates as exogenously given. In Appendix D I sketch one approach of endogenizing the interest rate (using a Solow-type model).

2.1 Non-housing production

The final output of non-housing (or "normal" or "numeraire") goods and services Y_{Nt} is produced by assembling a continuum $j \in [0,1]$ of intermediate goods $Y_{It}(j)$ according to a CES aggregator:

$$Y_{Nt} = \left(\int_0^1 Y_{It}(j)^{\frac{1}{\mu}} \, \mathrm{d}a \right)^{\mu}, \tag{1}$$

where $\mu \geq 1$ captures the elasticity of substitution between the inputs. Each of the j intermediate goods is produced according to a standard Cobb-Douglas production function:

$$Y_{It}(j) \quad = \quad F\left[K_t(j), L_t(j)\right] = K_t(j)^{\alpha} \left[\mathcal{A}_t L_t(j)\right]^{1-\alpha}, \tag{2}$$

where $K_t(j)$ stands for physical capital, $L_t(j)$ for the amount of employed labor and where average labor productivity $\mathcal{A}_t$ is assumed to grow at a constant rate g (i.e. $\mathcal{A}_t = \mathcal{A}_0 e^{gt}$ with $\mathcal{A}_0$ predetermined).

Given the aggregator (1) each intermediate good firm faces a downward sloping demand curve for its product and it chooses its profit-optimizing price $P_{It}(j)$ as a markup over marginal costs. Assuming perfect competition in the input markets the equilibrium factor prices come out as:

$$\mathcal{R}_{kt} = P_{It}(j)\frac{\alpha}{\mu}\frac{Y_{It}(j)}{K_t(j)}, \mathcal{W}_t = P_{It}(j)\frac{1-\alpha}{\mu}\frac{Y_{It}(j)}{L_t(j)}, \tag{3}$$

where $\mathcal{R}_{kt}$ and $\mathcal{W}_t$ stand for the rental rate of capital and the wage, respectively.

In a symmetric equilibrium all intermediate firms will set the same price and choose the same factor inputs, i.e. $P_{It}(j) = P_{It}$, $K_t(j) = K_t$, $L_t(j) = L_t$. Total population (and thus the labor supply) is assumed to grow at rate n (i.e. $L_t = L_0 e^{nt}$ with L_0 given). Normalizing the price of intermediate goods to $P_{It} = 1$ and noting that in the symmetric equilibrium it holds that $Y_{Nt} = Y_{It}$ one can write the net return on capital r_{kt} as:

$$r_{kt} = \frac{\alpha}{\mu}\frac{Y_{Nt}}{K_t} - \delta_k, \tag{4}$$

where δ_k stands for the rate of capital depreciation.

2.2 Factor shares and the distribution of profits

The parameter μ captures the (gross) markup of the monopolistically competitive firm over marginal costs. It is also the source of "pure profits" in the economy. In particular, the proceeds of the "normal" production are divided between aggregate capital income Y_{Kt}, aggregate labor income Y_{Lt} and pure profits Π_t , i.e. $Y_{Nt} = Y_{Lt} + Y_{Kt} + \Pi_t$ where $Y_{Lt} = \mathcal{W}_t L_t$ and $Y_{Kt} = \mathcal{R}_{kt} K_t$. The shares of capital income, labor income and profits are defined as:

$$\varphi_K = \frac{\mathcal{R}_{kt}K_t}{Y_{Nt}}, \varphi_L = \frac{\mathcal{W}_t L_t}{Y_{Nt}}, \varphi_\Pi = \frac{\Pi_t}{Y_{Nt}}, \tag{5}$$

with $\varphi_K + \varphi_L + \varphi_\Pi = 1$.[4] One can use $\mathcal{R}_{kt} = \frac{\alpha}{\mu}\frac{Y_{Nt}}{K_t}$ and $\mathcal{W}_t = \frac{1-\alpha}{\mu}\frac{Y_{Nt}}{L_t}$ (from the factor price equations (3)) to write:

$$\varphi_L = \frac{1-\alpha}{\mu}, \tag{6a}$$

$$\varphi_\Pi = \frac{\mu - 1}{\mu}. \tag{6b}$$

As is apparent from equation (6b), the production function (2) with constant returns to scale might lead to excessive pure profits for higher values of μ. In Appendix A.1.1 I show

[4]Note that the factor shares are defined in relation to the output of "non-housing" production Y_{Nt} and not in relation to GDP_t (see section 2.4). This choice is in line with the definitions of the empirical literature that typically excludes housing in the measurement of factor shares (Gutiérrez & Piton 2020).

how a specification with increasing returns to scale based on Ball & Mankiw (2022) can be used to remedy this implication.

It is also important to specify who will be the recipient of the profits. In the literature it is often assumed that the profits are distributed to the workers (mostly in proportion to their labor income if there are differences in productivity). This assumption, however, seems difficult to align with real-world practices (in particular since it implies that measured firm profits should be zero). It appears more reasonable to assume that profits accrue to the firms (or to the firm owners). This has, however, implications for the value of firms and the stock of available assets and I come back to this issue in section 2.5.

2.3 Housing

The total housing stock is denoted by $\overline{H}_t$. It is plausible to assume that the housing supply increases with the size of the population (an assumption that is often maintained in the related literature). In particular, I assume that $\overline{H}_t = \overline{H}_0 e^{n\chi t}$, where $0 \leq \chi \leq 1$ is a parameter that captures the fact that the housing supply might not fully keep pace with population growth.[5]

In the model there are renters and owners. The housing stocks available for renters and owners are denoted by $\overline{H}_t^r$ and $\overline{H}_t^o$, respectively. For the latter I furthermore assume that only a part $\overline{H}_t^{om}$ of the owner-occupied houses are actually on the market while a part $\overline{H}_t^{oo}$ is held by direct/dynastic/outright owners that stick to their dwelling, maybe because of sluggishness or out of a sense of family obligations. It thus holds that:

$$\overline{H}_t = \overline{H}_t^r + \overline{H}_t^o = \overline{H}_t^r + \overline{H}_t^{om} + \overline{H}_t^{oo} = \left(\kappa_H^r + \kappa_H^{om} + \kappa_H^{oo}\right) \overline{H}_t, \tag{7}$$

where κ_H^r, κ_H^{om} and κ_H^{oo} denote the shares of the total housing stock that are allocated to the three types of dwellings with $\kappa_H^r + \kappa_H^{om} + \kappa_H^{oo} = 1$. In the following I describe the housing market for rented and owned property in more detail.

[5]This formulation can also be interpreted as the reduced form of a more elaborated set-up that explicitly models the production of housing units with land and structures as inputs. For a model along these lines see Grossmann, Larin & Steger (2021) as is further discussed in Appendix A.2. The fact that the growth rate of the housing stock $n\chi$ lags behind the general growth rate $g + n$ captures the fact that residential land is the crucial factor for the development of housing (Knoll et al. 2017*b*).

2.3.1 Rented houses

For each type of dwelling there are two important prices. For the rental properties $\overline{H}_t^r$ the price for housing services P_{st}^r (the "rent") indicates how much a tenant has to pay per unit of housing in order to *use* the housing services for one period. On the other hand, the house price P_{ht}^r states how much an investor has to pay in order to *purchase* one unit of the rental housing stock. Furthermore, it is assumed that the value of the housing stock depreciates at a constant rate δ_h.[6]

The rent and the purchase price are closely related to each other (see Himmelberg et al. 2005, Svensson 2023). In particular, the advantage of holding a rental unit is twofold. On the one hand, an investor gets the rent P_{st}^r that is paid for using the unit diminished by the amount $\delta_h P_{ht}^r$ that is needed to hold its service value intact. On the other hand, the investor also benefits from any appreciation in the value of the housing unit, i.e. $\dot{P}_{ht}^r = \frac{dP_{ht}^r}{dt}$. The rate of return r_{ht} on investments into rental housing is thus given by:

$$r_{ht} = \frac{P_{st}^r - \delta_h P_{ht}^r + \dot{P}_{ht}^r}{P_{ht}^r} = \frac{P_{st}^r}{P_{ht}^r} - \delta_h + \frac{\dot{P}_{ht}^r}{P_{ht}^r}.$$

This expression can be solved for the purchasing price P_{ht}^r:

$$P_{ht}^r = \frac{P_{st}^r}{r_{ht} + \delta_h - \frac{\dot{P}_{ht}^r}{P_{ht}^r}}, \tag{8}$$

with r_{ht} now interpreted as the appropriate rate of return used to calculate the present value.

2.3.2 Owner-occupied houses on the market

For owner-occupiers the situation is somewhat different. The "buying owners", i.e. the ones that have to actually purchase their home, face a per unit price of P_{ht}^o. In order to highlight the parallel to the renters it is instructive to assume that the owner-occupiers are completely flexible in their behavior and that they are constantly buying and re-selling their homes (abstracting from any transaction costs, credit constraints or other financial frictions). Furthermore, it is assumed that these purchases are entirely financed

[6] Alternatively, one could also assume that the depreciation were proportional to the rent P_{st}^r or to aggregate labor income Y_{Lt}. The formulation where depreciation is proportional to the house value is, however, most commonly employed in the related literature, in particular since it is assumed to include also other factors like property taxes. See, e.g., Poterba (1984).

by mortgages with a mortgage interest rate r_{mt}. While occupying their dwelling, households have to pay the maintenance costs δ_m while at the same time benefiting from the valuation gains. The user cost of owning (i.e. "the imputed rent") is thus given by $P^o_{st} = P^o_{ht}\left(r_{mt} + \delta_m - \frac{\dot{P}^o_{ht}}{P^o_{ht}}\right)$. For the owner segment there thus holds a condition parallel to the rental market expressions (8):

$$P^o_{ht} = \frac{P^o_{st}}{r_{mt} + \delta_m - \frac{\dot{P}^o_{ht}}{P^o_{ht}}}. \tag{9}$$

The housing stock of the outright owners is not traded on the market but it is valued at the price P^o_{ht}. I come back to this issue later as it turns out to be an important element in the reaction of housing wealth to interest rate changes.

2.3.3 Steady state for the rented and owner-occupied markets

In the steady state house prices grow at the rate g_h where:[7]

$$g_h \equiv g + n(1-\chi). \tag{10}$$

The rates of return will also be constant in the steady state even though they do not have to be equal (e.g. due to different risk-return profiles). In particular, I assume that:

$$r_{ht} = r_{kt} - \xi_h, r_{mt} = r_{kt} - \xi_m, \tag{11}$$

where ξ_h and ξ_m are risk wedges with $\xi_h \geq 0$ and $\xi_m \geq \xi_h$ (such that $r_{kt} \geq r_{ht} \geq r_{mt}$).[8] The steady-state price-to-rent ratios are thus given by:

$$\frac{P^r_{ht}}{P^r_{st}} = \frac{1}{r_h + \delta_h - g_h}, \frac{P^o_{ht}}{P^o_{st}} = \frac{1}{r_m + \delta_m - g_h}. \tag{12}$$

[7] This follows from the fact that in equilibrium the total value of houses has to grow at the same rate as the output of normal goods $n + g$. For the rented segment it thus has to hold that $\frac{d\left(P^r_{ht}\overline{H}^r_t\right)}{dt}\frac{1}{P^r_{ht}\overline{H}^r_t} = \frac{\dot{P}^r_{ht}}{P^r_{ht}} + \frac{\dot{\overline{H}}^r_t}{\overline{H}^r_t} = n + g$. From $\overline{H}_t = \overline{H}_0 e^{n\chi t}$ it follows that $\frac{\dot{\overline{H}}^r_t}{\overline{H}^r_t} = \chi n$ and thus $\frac{\dot{P}^r_{ht}}{P^r_{ht}} = n + g - \chi n = g + n(1-\chi)$. Parallel reasoning also holds for the owner-occupied market with P^o_{ht} and $\overline{H}^{om}_t$ (noting that only this part of the owned stock is actually on the market).

[8] I assume here that these wedges are real resource costs (maybe capturing insurance activities or costs of financial intermediation). For models where the risk premia are derived from explicit assumptions involving stochastic returns, risk aversion and portfolio choices see, e.g., Piazzesi & Schneider (2016).

These relations are well-known from the asset-market approach of the housing market (cf. Poterba 1984, Himmelberg et al. 2005).

2.4 National accounting

In order to calculate easily comparable wealth-to-income ratios it is necessary to first define an income concept that is in line with the conventions concerning net domestic and net national product (Piketty & Zucman 2014, Grossmann, Larin & Steger 2021). Following the current practice, gross domestic product is defined as:[9]

$$GDP_t = Y_t = Y_{Nt} + P^r_{st}\overline{H}^r_t + P^o_{st}\overline{H}^o_t \tag{13}$$

where Y_{Nt} stands for the gross domestic production of final normal goods (see (2)) while $P^r_{st}\overline{H}^r_t$ and $P^o_{st}\overline{H}^o_t$ capture the production of housing services in the rented and owner-occupied segments, respectively.[10]

Physical capital K_t and the value of the rented and owner-occupied housing stock depreciate at the rates δ_k, δ_h and δ_m, respectively. The net domestic product NDP_t is thus given by:

$$NDP_t = GDP_t - \delta_k K_t - \delta_h P^r_{ht}\overline{H}^r_t - \delta_m P^o_{ht}\overline{H}^o_t. \tag{14}$$

2.5 Aggregate asset supply and wealth-to-income ratios

2.5.1 Aggregate asset supply

The aggregate supply of assets consists of business assets B_t, the housing stock $\overline{H}_t$ and a possible stock of government bonds $\mathcal{D}_t$. The business assets correspond to the total value of firms which subsumes not only the value of the capital stock K_t but potentially also the present value of future profits. In particular, in Appendix A.1.2 I show that the total

[9] In this paper I focus on a closed economy and thus abstract from net foreign assets and thus also from the distinction between domestic and national products.

[10] Note that this formulation of national income excludes capital gains. As shown, e.g., by Robbins (2018) and Fagereng et al. (2019) the neglect of capital gains can lead to inconsistencies in the context of theoretical models. In order to deal with this issue they propose the use of the Haig-Simmons definition of national income $GDP^{HS}_t = Y_{Nt} + P^r_{st}\overline{H}^r_t + P^o_{st}\overline{H}^o_t + \dot{P}^r_{ht}\overline{H}^r_t + \dot{P}^o_{ht}\overline{H}^o_t$ where $\dot{P}^j_{ht} = \frac{dP^j_{ht}}{dt}$ for $j \in \{r, o\}$. I come back to this issue. For the main part of the paper, however, I stick to the traditional definition for the sake of comparison.

value of business assets can be written as:

$$B_t = K_t + \zeta_\Pi \frac{\varphi_\Pi Y_{Nt}}{r_{kt} - (g+n)}, \tag{15}$$

where ζ_Π stands for the fraction of profits that are included in the valuation of firms. In a situation where firms make zero profits ($\varphi_\Pi = 0$) or where (for whatever reason) the market valuation of these profits is zero ($\zeta_\Pi = 0$) one arrives at the benchmark case where the value of business assets just corresponds to the value of the capital stock, i.e. $B_t = K_t$.

The aggregate asset supply (or equivalently the total demand for wealth) can be written as:

$$W_t^d = W_{Bt} + W_{Ht} + W_{Dt}, \tag{16}$$

where $W_{Bt} = B_t$, $W_{Ht} = W_{Hrt} + W_{Hot}$ with $W_{Hrt} = P_{ht}^r \overline{H}_t^r$, $W_{Hot} = P_{ht}^o \overline{H}_t^o$ and $W_{Dt} = \mathcal{D}_t$. Alternatively, one can also split the aggregate asset supply in the supply of financial (or liquid) assets and the value of owner-occupied assets: $W_t^d = W_{Ft}^d + W_{Ot}^d$ where $W_{Ft}^d = W_{Bt} + W_{Hrt} + W_{Mt} + W_{Dt}$ with $W_{Mt} = M_t$ denoting the value of outstanding mortgages and where $W_{Ot}^d = W_{Hot} - W_{Mt} = P_{ht}^o \overline{H}_t^{oo} + \left(P_{ht}^o \overline{H}_t^{om} - M_t\right)$ stands for the net worth of the stock of owner-occupied housing (i.e. its market value minus the value of outstanding mortgage debt). Note that for the assumption of continuous mortgage-financing the total value of mortgages equals the value of the self-acquired stock, i.e. $M_t = P_{ht}^o \overline{H}_t^{om}$.

In Appendix B.1 I discuss the balance sheets of households in more detail and I relate it to the definitions and the notation of the *Wealth Inequality Database* (Alvaredo et al. 2016) which is my primary data source for the wealth aggregates.

2.5.2 Interest rates

It is assumed that the entire supply of financial assets W_{Ft}^d is held by financial funds that operate under the condition of perfect competition. The funds collect all financial savings in the economy, undertake all investments on behalf of the customers (the households) and hand out the returns which constitute the households' asset income. Using the definition of financial wealth from above ($W_{Ft}^d = W_{Bt} + W_{Hrt} + W_{Mt} + W_{Dt}$) the average interest rate is given by:

$$r_t = \frac{B_t}{W_{Ft}^d} r_{kt} + \frac{P_{ht}^r \overline{H}_t^r}{W_{Ft}^d} r_{ht} + \frac{M_t}{W_{Ft}^d} r_{mt} + \frac{D_t}{W_{Ft}^d} r_{dt}, \tag{17}$$

where the interest rates $r_{ht} = r_{kt} - \xi_h$ and $r_{mt} = r_{kt} - \xi_m$ have been defined in equation (11) and where similarly $r_{dt} = r_{kt} - \xi_d$. Note that in the definition of the interest (17) it is assumed that the return on equity and the return on capital is identical ($r_{bt} = r_{kt}$). This follows from a standard no-arbitrage argument as discussed in Appendix A.1.2.[11] There I also derive an alternative expression $\widetilde{r}_{kt}$ for the return on capital that is based on national account data and that is sometimes used in the related literature (Gomme et al. 2015, Reis 2022).

2.5.3 Wealth-to-income ratios

Total wealth and the various subaggregates can be related to any of the concepts of national income that have been discussed above. The wealth-to-normal-goods ratio, e.g., is defined as:

$$\beta_t^N = \frac{W_t^d}{Y_{Nt}} \tag{18}$$

which is a useful concept for the following calculations since it allows for compact expressions.

In a similar fashion one can define $\beta_{xt}^N = \frac{W_{xt}}{Y_{Nt}}$ for $x \in \{K, H, H_r, H_o, D, M\}$. For later reference one can use equations (4), (8) and (9) to write:[12]

$$\beta_{Kt}^N = \frac{\alpha/\mu}{r_{kt} + \delta_k}, \tag{19}$$

$$\beta_{Hrt}^N = \frac{P_{st}^r \overline{H}_t^r}{Y_{Nt}} \frac{1}{r_{ht} + \delta_h - g_h}, \tag{20}$$

$$\beta_{Hot}^N = \frac{P_{st}^o \overline{H}_t^o}{Y_{Nt}} \frac{1}{r_{mt} + \delta_m - g_h}. \tag{21}$$

The ratio β_t^N (and all other ratios) can be easily transformed into alternative wealth-to-income ratios as discussed in Appendix A.3. The related empirical literature, e.g., mostly divides aggregate wealth by the net domestic product. In this case (which I will

[11] In a general equilibrium setup one would also specify the savings behavior of households to derive a schedule for the supply of financial wealth (or the demand for assets) W_{Ft}^s. As stated repeatedly, this paper disregards this demand side and instead treats the equilibrium interest rate r_{kt} as given.

[12] For the two subgroups of owner-occupied houses the ratios are $\beta_{Homt}^N = \frac{P_{st}^o \overline{H}_t^{om}}{Y_{Nt}} \frac{1}{r_{mt}+\delta_m - g_h} = \frac{\kappa_H^{om}}{\kappa_H^{om}+\kappa_H^{oo}} \beta_{Hot}^N$ and $\beta_{Hodt}^N = \frac{P_{st}^o \overline{H}_t^{oo}}{Y_{Nt}} \frac{1}{r_{mt}+\delta_m - g_h} = \frac{\kappa_H^{oo}}{\kappa_H^{om}+\kappa_H^{oo}} \beta_{Hot}^N$.

define as β_t) one can write:

$$\begin{aligned}\beta_t \equiv \beta_t^{NDP} &= \frac{W_t^d}{NDP_t} = \beta_t^N \frac{Y_{Nt}}{NDP_t} \\ &= \beta_t^N \frac{1}{1 + \frac{P_{st}^r \overline{H}_t^r}{Y_{Nt}} + \frac{P_{st}^o \overline{H}_t^o}{Y_{Nt}} - \frac{\delta_h P_{ht}^r \overline{H}_t^r}{Y_{Nt}} - \frac{\delta_m P_{ht}^o \overline{H}_t^o}{Y_{Nt}} - \frac{\delta_k K_t}{Y_{Nt}}}.\end{aligned}$$

For realistic calibrations it typically holds that $\frac{Y_{Nt}}{NDP_t} \approx 1$ and thus $\beta_t \approx \beta_{Nt}$ (see Appendix A.3).

3 The case without homeowners

3.1 Set-up

I start the theoretical part with a simple specification. The main goal of this exercise is to investigate whether and under which assumptions the implications of the model are likely to be in line with the empirical observations as summarized in Tabke 1. In particular, the central question is whether a decrease in interest rates between 3 pp and 4 pp (depending on whether one takes 1980 or 1970 as the respective starting point) is compatible with an increase in β between 250 pp and 300 pp and with an increase in $\frac{\beta_H}{\beta}$ between 3 pp and 11 pp. In section 5.2 I will furthermore investigate whether and how one could also make sense of the different cross-country experiences where, e.g., the housing wealth share increased by almost 20 pp in Europe while it *decreased* in the US.

In order to be able to highlight the main mechanisms I start in this section with the case where all households are renters ($\kappa_H^r = 1$). Furthermore, I assume that the renters spend a constant share γ of their total expenditures $\mathcal{E}_t$ on rents, i.e. $P_{st}^r \overline{H}_t^r = \gamma \mathcal{E}_t$. The empirical data suggest an average value of $\gamma = 0.17$ (see Appendix B). Furthermore, empirically household consumption expenditure are typically around 60% of GDP which suggests that $\mathcal{E}_t = \varepsilon Y_t$ with $\varepsilon = 0.6$.[13] One can thus write:

$$P_{st}^r \overline{H}_t^r = \gamma \mathcal{E}_t = \gamma \varepsilon Y_t = \gamma \varepsilon \left(Y_{Nt} + P_{st}^r \overline{H}_t^r \right),$$

[13]This implies that rents are a share $\gamma\varepsilon = 0.17 \times 0.6 = 10.2\%$ of GDP. The empirical data typically report that households spend between 12% and 13% of GDP on housing services (for the EU in 2019, e.g., 12.3%). These services, however, also include expenditures on utilities (water, electricity, gas etc.) while I focus for theoretical reasons only on the rental part. As discussed in Appendix B the expenditures on utilities amount to almost a quarter of total housing expenditures which explains the difference between the percentages.

where the last lines uses the definition for GDP in equation (13) for the case with $\overline{H}_t^o = 0$. This can be solved to give:

$$P_{st}^r \overline{H}_t^r = \frac{\gamma\varepsilon}{1-\gamma\varepsilon} Y_{Nt} = \phi Y_{Nt}, \tag{22}$$

where $\phi \equiv \frac{\gamma\varepsilon}{1-\gamma\varepsilon}$. Using the numerical values from above one gets that $\phi = \frac{0.17\times 0.6}{1-0.17\times 0.6} = 0.114$. In other words, households spend 11.4% of the production value of "non-housing goods" Y_{Nt} on their rents. If one abstracts for the moment from government bonds ($\beta_D^N = 0$) and from the valuation of profits ($\zeta_\Pi = 0$) one can then use equations (19) and (20) to derive the steady state expressions:

$$\beta^N = \frac{\alpha}{\mu}\frac{1}{r_k+\delta_k} + \phi\frac{1}{r_h+\delta_h-g_h}, \tag{23}$$

$$\frac{\beta_H}{\beta_K} = \frac{\phi\mu}{\alpha}\frac{r_k+\delta_k}{r_h+\delta_h-g_h} = \frac{\phi\mu}{\alpha}\frac{r_k+\delta_k}{r_k-\xi_h+\delta_h-(g+(1-\chi)n)}. \tag{24}$$

Note that the share of housing wealth $\frac{\beta_H}{\beta}$ is directly (and positively) related to the latter expression since $\frac{\beta_H}{\beta} = \frac{\beta_H}{\beta_H+\beta_K} = \frac{1}{\left(\frac{\beta_H}{\beta_K}\right)^{-1}+1}$.

3.2 The equilibrium share of housing wealth

The data show that for most countries the share of housing wealth lies between 40% and 60% and is often close to 50%. The model suggests that this "round number" is more or less a coincidence that follows from the size of the expenditure share γ, the share of total households expenditures in GDP ε (and thus crucially on the savings rate), the capital coefficient α and the gross markup μ (or put differently on the labor share). For the extreme case with $\delta_k = \delta_h - \xi_h - g_h$ the fraction is independent of the interest rate r_k and given by $\frac{\beta_H}{\beta_K} = \frac{\phi\mu}{\alpha}$. In this case capital and housing wealth are of equal size ($\beta_H = \beta_K$ or $\frac{\beta_H}{\beta} = 50\%$) if $\phi = \frac{\alpha}{\mu}$ or $\gamma\varepsilon \approx \frac{\alpha}{\mu}$, i.e. the share of housing expenditures is approximately equal to the capital share. For typical values like $\mu = 1$ and $\alpha = 1/3$ (or $\alpha = 1/4$) this would imply somewhat excessive values for the expenditure shares ($\gamma = 42\%$ or $\gamma = 33\%$, respectively, continuing to assume that $\varepsilon = 0.6$).

Going beyond this extreme case one can calibrate the model with values that roughly correspond to the situation around 1980 for the group of high income countries. The values for the productivity growth rate $g = 3.11\%$ and the population growth rate and $n = 0.79\%$ are shown in Table 2. The calibration choices for the other crucial parameter values are discussed in Appendix B and are summarized in Table 3. In particular, for the situation around 1980 I use $\gamma = 0.17$, $\varepsilon = 0.6$ (as has been stated above) together with $\mu = 1.1$, $\varphi_L = 66\%$, $\alpha = 0.274$, $\delta_k = 0.05$, $\delta_h = 0.025$, $\xi_h = 0$ and $\chi = 0.5$. For the moment I stick to the renter-only case ($\kappa^r_N = 100\%$) and to the assumption that future profits are not included in the business valuations ($\zeta_\Pi = 0$ and thus $\beta_B = \beta_K$). Assuming a capital return of $r_k = 10\%$ one gets a housing wealth share of $\frac{\beta_H}{\beta} = 43.2\%$ and a corresponding wealth-to-income ratio of $\beta = 293\%$. These are in fact plausible values that are broadly in line with the target values for 1980 from Table 1. In the numerical calibrations in section 5 I will show that a more encompassing approach leads to an even better fit.

Table 2: Productivity growth and population growth for various regions (in %)

	High Income		US		Europe (EU4)	
	g	**n**	**g**	**n**	**g**	**n**
$\overline{1970}$	4.58	1.04	3.36	1.07	4.34	0.58
$\overline{1980}$	3.11	0.79	3.45	0.98	2.67	0.18
$\overline{2017}$	1.93	0.54	2.08	0.71	1.42	0.37

Note: The values for the productivity growth rate g, the population growth rate n and the definition of "high income countries" come from the World Bank. $\overline{1970}$, $\overline{1980}$ and $\overline{2017}$ correspond to the average values for the time spans 1966-1974, 1976-1984 and 2011-2017, respectively.

3.3 The reaction to changes in the interest rate

In the next step one can analyze the reaction of an exogenous fall in interest rates from the starting value $r_k = 10\%$ with $\frac{\beta_H}{\beta} = 43.2\%$. Under the assumption that all other parameter values stay constant, a decline in the interest rate by 3 pp is associated with an increase in the share of housing wealth by 4.5 pp ($\frac{\beta_H}{\beta} = 47.7\%$ for $r_k = 7\%$). This implied increase is roughly in line (or even above) the observed increase of 3 pp (from 47% to 50%) for the "weighted world average" between 1980 and 2017 (see Table 1). The change is, however, too low when compared to the increase in the housing share from 1970 to 2017 (by 11 pp) and it also does not conform with the much larger increases for the group of European countries (or the unweighted world average). Even a decline in the interest rate by 4 pp would not change this conclusion since this implies only an increase by 7 pp ($\frac{\beta_H}{\beta} = 50.1\%$ for $r_k = 6\%$).

Table 3: Parameter values from the related literature

Description	**Symbol**	**Value** (1980)	**Value** (2017)	**Source**
Housing expenditure share	γ	17%	17.5%	OECD (2023)
HH total consumption share	ε	60%	60%	National Accounts
Depreciation rate of capital	δ_k	5%	7%	Dalgaard & Olsen (2021)
Depreciation rate of housing	δ_h, δ_m	2.5%	2%	Kaplan et al. (2020)
(Gross) Markup	μ	1.1	1.2	De Loecker & Eeckhout (2021)
Labor Share	φ_L	66%	60%	Gutiérrez & Piton (2020)
Risk wedge (commercial real estate)	ξ_h	0%	0%	Jordà et al. (2019)
Risk wedge (mortgage interest rate)	ξ_m	2%	2%	Jordà et al. (2019)
Risk wedge (government bonds)	ξ_d	5%	5%	Jordà et al. (2019)
Share of renters	κ_N^r	45%	38%	Jordà et al. (2016)
Share of outright owners	κ_N^{oo}	29%	30%	OECD (2023)
Share of outrightly owned houses	κ_H^{oo}	29%	36%	OECD (2023), ECB (2021)
Elasticity of housing supply	χ	0.5	0	Benchmark values

Note: The table reports the values for the initial period (around 1980) and the current period (around 2017) and the respective sources (databases or related literature). The data are discussed in Appendix B.2.

As a side remark I want to mention that one could have thought that the model would imply a much stronger increase in the housing share due to the impact of r_k in the denominator of β_H. This conjecture has to be qualified, however, by taking two factors into consideration. On the one hand, it has to be noted that the interest rate r_k appears in the expressions of both β_H *and* β_K. A reduction in r_k will thus increase both types of wealth thereby weakening the effect on the ratio $\frac{\beta_H}{\beta_K}$. In fact, the related literature (Grossmann, Larin & Steger 2021, Lisack et al. 2021) often emphasizes the ability of the models to explain the increase in housing wealth β_H while less focus is directed to the implications for the housing wealth share $\frac{\beta_H}{\beta}$. On the other hand, the size of the reaction depends on the starting value of r_k. A reduction in the interest from 10% to 7% has a smaller effect than a reduction from 6% to 3% (where the model would imply an increase from 49.8% to 64.5%). Assuming such low values of the interest rate is, however, contradicted by at least two empirical facts. First, the existing evidence on the returns to capital point to values that are considerably higher than 6%. For 1980, e.g., Jordà et al. (2019) report values for the return on equity that are between 8% and 11%. Second, the levels of the total wealth-to-income-ratios that are associated with lower interest rates are much higher than the empirically observed values (e.g., a value of $\beta = 1080\%$ for $r_k = 3\%$).

3.4 Comparison to the standard model without housing

In this context it is also instructive to have a brief look at the model without housing that is commonly used in the related literature on the decline in interest rates. In this case the wealth-to-income ratio is simply given by $\beta = \beta_K = \frac{\alpha}{\mu}\frac{1}{r_k+\delta_k}$. Setting $\frac{\alpha}{\mu} = 0.25$ (e.g. as above $\alpha = 0.274$ and $\mu = 1.1$) a wealth-to-income ratio of 340% implies that $r_k + \delta_k = \frac{0.25}{3.4} = 0.073$. For commonly used choices of the depreciation rate δ_k this will thus imply rather low values for the return on capital (e.g., for $\delta_k = 5\%$ one gets $r_k = 2.3\%$). As an alternative it would of course be more reasonable not to target the wealth-to-income ratio but rather the *capital*-to-income ratio (thus excluding housing wealth) which amounted to 215% in 1980 for the G8a countries (and to 210% for the US). In this case one gets $r_k = 0.118 - \delta_k$ which implies (for $\delta_k = 5\%$) an equilibrium capital return of $r_k = 6.8\%$ which is closer to the observed values (although still somewhat too low). In Appendix A.4, I briefly discuss the magnitudes of the equilibrium interest rates that arise in the models from the related literature (Summers & Rachel 2019, Platzer & Peruffo 2022).

3.5 The reaction to changes in the interest rate and other parameters

So far I have simply looked at the effect of an exogenous shift in the interest rate on the wealth ratios without considering the possibility that other parameters might change at the same time. This is of course not an innocuous omission, in particular since the interest rate movement itself has to be understood as a consequence of the general economic development.

This issue can be illustrated by looking at changes in the growh for productivity and population. As reported in Table 2 these growth rates declined for the high income countries from $g = 3.11\%$ to $g = 1.93\%$ and from $n = 0.79\%$ to $n = 0.54\%$ between 1980 and 2017. By taking these changes into account it becomes even more difficult to explain a large increase in the share of housing wealth. In particular, the housing wealth share now moves from $\frac{\beta_H}{\beta} = 43.2\%$ (for $r_k = 10\%$) to $\frac{\beta_H}{\beta} = 42.8\%$ (for $r_k = 7\%$) and $\frac{\beta_H}{\beta} = 44.3\%$ (for $r_k = 6\%$), thus implying a stagnation or even a reduction in the share of housing wealth.

The reason for this result is that the decrease in g and n lowers the equilibrium growth rate of house prices g_h which works into the opposite direction than the reduction in r_k

in the denominator of β_H. This can be illustrated by focusing again on the extreme case with $\delta_k = \delta_h = 0$ and $\chi = 1$. In this case one has $\frac{\beta_H}{\beta_K} = \frac{\phi\mu}{\alpha}\frac{r_k}{r_k - g}$. If r_k and g change by the same factor then there is no effect on $\frac{\beta_H}{\beta_K}$ and thus on the housing wealth share. In fact, this is to a certain extent the case in the numerical example used above where g was assumed to decrease by 38% (from $g = 3.11\%$ to $g = 1.93\%$) while r_k decreased by 30% (from $r_k = 10\%$ to $r_k = 7\%$).[14] This is reflected in the muted response where the housing wealth share stays basically constant around 43% even though the interest rate has decreased by 3 pp.

4 The case with homeowners

The discussion in the last section has indicated that the simple model is not capable to explain a sizeable increase in the share of housing wealth as could be observed for a longer time period and especially for some groups of countries. In order to deal with this shortcoming it is thus necessary to allow for changes in a wider set of parameters and to expand the analysis. In particular, it will turn out to be important to have a closer look at the housing structure. For this reason I now move beyond the situation of a renter-only society ($\kappa_H^r = 1$) and instead assume that there also exist owner-occupiers either with or without mortgages (outright owners). Furthermore, I distinguish between the share of households in each group (κ_N^j) and the fraction of the housing stock that is under the control of these groups (κ_H^j) where $j \in \{r, om, oo\}$ and where $\sum_j \kappa_N^j = \sum_j \kappa_H^j = 1$.

4.1 Two types of owner-occupiers

As described in section 2.3 the owner-occupiers with mortgages are assumed to be completely flexible, constantly buying and reselling their homes without transaction costs or financial frictions. These purchases are entirely financed by mortgages at the mortgage interest rate r_m. As mentioned above this means that the imputed rent of these owners is given by $P_{st}^o = P_{ht}^o (r_m + \delta_m - g_h)$.

For the direct owners the situation is different. Their houses are assumed to not being traded on the market but to rather be passed on from generation to generation. This assumption is of course highly stylized, but it allows to consider the empirically

[14]Note that the positive correlation between r_k and g is not a hypothetical scenario. On the contrary, this relation comes out of many theoretical models. For the assumption of an infinitely lived agent one gets, for example, that in the steady state $r_k = \sigma g + n + \rho$ (where ρ stands for the rate of time preference and σ for the intertemporal elasticity of substitution).

important group of outright owners in a straightforward and tractable manner. The stylized assumption of completely passive outright owners who simply use and pass on their inherited home is thereby meant to capture three observable phenomena. First, a certain proportion of real estate is under the control of real estate trusts (fee tails, entails, fideicommis etc.) that prohibit (or at least considerably restricts) heirs to sell the inherited land. Second, a certain percentage of the population does not seem to be *willing* to trade their home even if they were *able* to do so, for example because they feel an obligation to a generation-old "family house" etc. Third, and probably most relevant, in the real world the two groups of owners with mortgages and outright owners are not separate entities but many households rather assume these roles in sequence. They start out as mortgage-holders and turn into outright owners after having paid of their debts from when on they simply stick to their home until death ("aging in place") without reacting to house price fluctuations (Cocco & Lopes 2020, French et al. 2023). The assumed structure is also meant to capture this constellation in a tractable manner.

4.2 The equilibrium share of housing wealth with homeowners

I now assume that both renters and buying owners spend an identical share of their total expenditures $\mathcal{E}_t^r$ and $\mathcal{E}_t^{om}$, respectively, on housing services, i.e. $P_{st}^r \overline{H}_t^r = \gamma \mathcal{E}_t^r$ and $P_{st}^o \overline{H}_t^{om} = \gamma \mathcal{E}_t^{om}$. Assuming that the share of total expenditures to GDP for renters and buying owners is proportional to their population share ($\mathcal{E}_t^r = \varepsilon \kappa_N^r Y_t$, $\mathcal{E}_t^{om} = \varepsilon \kappa_N^{om} Y_t$) one can write: $P_{st}^r \overline{H}_t^r = \kappa_N^r \gamma \varepsilon Y_t$ and $P_{st}^o \overline{H}_t^{om} = \kappa_N^{om} \gamma \varepsilon Y_t$. Furthermore, I assume that the housing stocks of rental and self-bought houses adjust such that their service price is equal ($P_{st}^r = P_{st}^o$).[15] In Appendix A.5 I show that under these assumptions one can calculate the ratio of housing wealth to capital wealth as:

$$\frac{\beta_H}{\beta_K} = \frac{\widetilde{\phi}\mu}{\alpha} \left[\kappa_N^r \frac{r_k + \delta_k}{r_h + \delta_h - g_h} + \left(\frac{1 - \kappa_N^{oo}}{1 - \kappa_H^{oo}} - \kappa_N^r \right) \frac{r_k + \delta_k}{r_m + \delta_m - g_h} \right]. \tag{25}$$

where $\widetilde{\phi} = \frac{\gamma\varepsilon}{1 - \gamma\varepsilon \frac{1-\kappa_N^{oo}}{1-\kappa_H^{oo}}}$. The share of housing wealth depends on the relative size of the interest rates r_k, r_h and r_m, on the population share of renters κ_N^r and on the term $\frac{1-\kappa_N^{oo}}{1-\kappa_H^{oo}}$ which captures the "relative housing abundance" of direct owners. For $r_m = r_h$, $\delta_m = \delta_h$ and $\kappa_H^{oo} = \kappa_N^{oo}$ equation (25) reduces to expression (24): $\frac{\beta_H}{\beta_K} = \frac{\phi\mu}{\alpha} \frac{r_k + \delta_k}{r_h + \delta_h - g_h}$. The distinction

[15] In fact, the equality between actual rents and imputed rents (as it is assumed in the asset pricing approach to house pricing) is typically refuted in empirical data (Duca et al. 2021). It is nevertheless a reasonable benchmark for an equilibrium analysis.

between renters and owners does not play a role in this situation since the rent and imputed rent are identical per assumption ($P^r_{st} = P^o_{st}$) and the same is true for the house prices ($P^r_{ht} = P^o_{ht}$) since $r_m = r_h$ and $\delta_m = \delta_h$. If these equalities no longer hold then the housing structure will become important. A larger share of owners with mortgages might, e.g., increase the share $\frac{\beta_H}{\beta_K}$ if $r_m < r_h$ (or $\delta_m < \delta_h$) since their houses have a higher price. Of crucial importance is the situation of outright owners which I study in detail in the next section.

4.3 The role of outright owners

In order to focus clearly on the role of the outright owners I abstract from all other influences and assume again that $r_m = r_h$ and $\delta_m = \delta_h$. In this case equation (25) simplifies to:

$$\begin{aligned}\frac{\beta_H}{\beta_K} &= \frac{\widetilde{\phi}\mu}{\alpha}\frac{r_k+\delta_k}{r_h+\delta_h-g_h}\frac{1-\kappa_N^{oo}}{1-\kappa_H^{oo}} \\ &= \frac{\gamma\varepsilon}{1-\gamma\varepsilon\frac{1-\kappa_N^{oo}}{1-\kappa_H^{oo}}}\frac{\mu}{\alpha}\frac{r_k+\delta_k}{r_h+\delta_h-g_h}\frac{1-\kappa_N^{oo}}{1-\kappa_H^{oo}}. \qquad (26)\end{aligned}$$

When the share of the directly owned housing stock is identical to the population share of the direct owners then $\frac{1-\kappa_N^{oo}}{1-\kappa_H^{oo}} = 1$. In this case the existence of outright owners has no effect on the equilibrium prices and the equilibrium housing portfolio share since the same number of houses and house owners is removed from the market. If, however, the direct owners possess houses that are on average larger than the average houses in the rest of the market (i.e. if $\kappa_H^{oo} > \kappa_N^{oo}$) then this has an effect on prices and the housing share. If, in the extreme case, the direct owners command over almost the entire housing stock ($\kappa_H^{oo} \to 1$) the share $\frac{\beta_H}{\beta_K}$ goes to infinity. The reason for this mechanism is the following. The eager buyers (i.e. the owners with mortgages) are competing for the scarce stock of available houses thereby driving up the price P^o_{ht}. This by itself, however, has no effect on their own housing wealth since their total expenditures on housing services is fixed by $P^o_{st}\overline{H}^{om}_t = \kappa_N^{om}\gamma\varepsilon Y_t$. The smaller stock of available houses $\overline{H}^{om}_t$ causes an increase in the price P^o_{st} but the *total* expenditures $P^o_{st}\overline{H}^{om}_t$ and pari passu also the total housing wealth of buying owners $P^o_{ht}\overline{H}^{om}_t$ is unaffected.[16] The increase in P^o_{ht}, however, also increases

[16] In fact, this is basically also the reason why the share of housing wealth do not seem to be considerably larger in countries with higher population densities or with smaller volumes of the per capita housing stock.

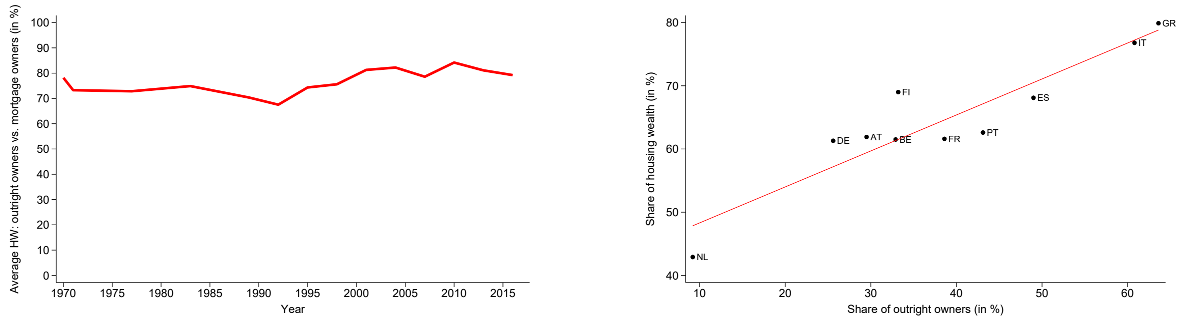
the value of the non-traded housing stock of the outright owners. They suddenly face a "windfall gain" in the valuation of their housing stock. Per assumption, however, they do not react to this price change either because they are not able or because they are not willing to do so. The former might be the case because the housing asset is part of an entail (an inalienable estate), the latter because the owners prefer to stay in their family home and age in place.

As an example, assume that $\xi_h = \xi_m = 2\%$ and $\kappa_N^r = 45\%$. In the absence of direct owners one has that $\frac{\beta_H}{\beta} = 49.4\%$. If the share of direct owners is increased to $\kappa_N^{oo} = 30\%$ then the effect depends on the size of the associated housing stock. If $\kappa_H^{oo} = 30\%$ then there is no effect and still $\frac{\beta_H}{\beta} = 49.4\%$. If, however, $\kappa_H^{oo} = 36\%$ then the housing share increases to 51.9% and for $\kappa_H^{oo} = 48\%$ it is considerably higher at $\frac{\beta_H}{\beta} = 57.8\%$.

4.4 Data on outright owners

The available data on the structure of outright ownership is rather scarce as I discuss in Appendix B. For the US one can use the *SCF+* database that has been constructed by Kuhn et al. (2020). These data document that the share of renters stayed constant from 1980 through the 2010s while one could observe a slight increase (decrease) in the share of owners with (without) mortgages. What is more, the data also suggest that in the US the average value of the outrightly owned houses was always below the value of the houses with mortgages and that this relation also stayed constant over time around a value of 80%. This is illustrated Figure 2a. For this reason I will later use $\frac{\kappa_H^{oo}}{\kappa_N^{oo}} = 0.8$ in the calibration for the US.

For Europe it is difficult to find comparable data that span a longer time period. As discussed in the appendix, it is, however, possible to come up with indirect evidence for the relative importance of outright owners. In particular, assume that the share of houses in the possession of outright owners is a multiple z of the proportion of outright owners in society, i.e. $\kappa_H^{oo} = z\kappa_N^{oo}$. One can now linearize the share of $\frac{\beta_H}{\beta_K}$ from equations (25) or (26) around a baseline value of κ_N^{oo} and similarly for the share $\frac{\beta_H}{\beta}$. If one uses — for the sake of illustration — a linearization of (26) around $\kappa_N^{oo} = 0$ one gets an approximation of the form $\frac{\beta_H}{\beta_K} \approx G_1 + G_2(z-1)\kappa_N^{oo}$ and similarly $\frac{\beta_H}{\beta} \approx G_3 + G_4(z-1)\kappa_N^{oo}$ for some coefficients G_x. The share of housing wealth increases with the share of outright owners κ_N^{oo} where the degree of dependence depends on the multiplier z. For $z = 1$ one is back to the benchmark results where the share of housing wealth is independent of the share of outright owners. One can use data from the HFCS and the OECD (from the late 2010s)



(a) Housing wealth of outright owners vs. mortgage owners (US)

(b) Share of outright owners vs. share of housing wealth (Europe)

Figure 2: The data come from the $SCF+$ database (Kuhn et al. 2020) and from the third wave of the HFCS (ECB, 2021). Panel (a) shows the ratio of the average value of outrightly owned houses to the average value of houses that are owned with a mortgage for the US. Panel (b) contrasts the share of outright owners to the share of housing wealth for a number of European countries.

to regress the share of housing wealth on the share of outright owners for a group of European countries (including three of the EU4 countries). This is illustrated in Figure 2b. One observes a clear relation between the share of housing wealth and the share of outright owners as predicted by the model with $z > 1$. If one assumes identical values for all parameters except κ_N^{oo} and uses the benchmark calibration then a value of $z = 1.6$ gives the best fit for the slope of the regression line in Figure 2b if one linearizes around $\kappa_N^{oo} = 0.35$ (see Appendix B.2.8). This is the value for $\kappa_H^{oo}/\kappa_N^{oo}$ that I use later for Europe in the year 2017. For the starting year 1980, on the other hand, I use $\kappa_H^{oo}/\kappa_N^{oo} = 1.2$ such that the global value is the benchmark $\kappa_H^{oo}/\kappa_N^{oo} = 1$.

5 Numerical examples

After having discussed the crucial mechanisms that influence the share of housing wealth in the theoretical model I turn to numerical examples based on realistic calibrations in order to study their relative importance in an even more detailed setting. I start with the results for the world average (i.e. the G8a countries) before turning to a comparisons between the US and the EU4.

5.1 World average

5.1.1 Calibration

The calibration uses again the parameter values discussed before and in Table 3. For housing I assume that in 1980 45% of households are renters, 26% owners with a mortgage and 29% outright owners. For the benchmark case I also assume that the share of the outrightly owned stock is identical to the share of outright owners ($\kappa_H^{oo} = \kappa_N^{oo} = 0.29$). In addition, I now also allow for government debt and for risk wedges. In particular, for the year 1980 I set $\beta_D^N = 20\%$ together with $\xi_d = 5\%$, $\xi_h = 0\%$ and $\xi_m = 2\%$.

5.1.2 Results

Table 4: Reaction of aggregate wealth and housing wealth to parameter changes

	Model	$\boldsymbol{\beta}$ (in %) 1980 ($r_k \approx 10\%$)	2017 ($r_k = 7\%$)	2017 ($r_k = 6\%$)	$\frac{\boldsymbol{\beta_H}}{\boldsymbol{\beta}}$ (in %) 1980 ($r_k \approx 10\%$)	2017 ($r_k = 7\%$)	2017 ($r_k = 6\%$)
1	**Benchmark**	**340**	**483**	**554**	**47**	**48**	**50**
	Changes in μ and $\kappa_H^{oo}/\kappa_N^{oo}$						
2	$\zeta_\Pi = 0$	340	480	553	47	52	55
3	$\zeta_\Pi = 1/4$	340	574	676	47	43	44
	Change in many parameters						
4	$\zeta_\Pi = 0$	340	489	572	47	59	63
5	$\zeta_\Pi = 1/4$	340	561	671	47	52	54

Note: The table reports the wealth-to-income ratio β and the housing wealth share $\frac{\beta_H}{\beta}$ for various specifications of the model. The values for the baseline parameters in 1980 and the assumed changes for the year 2017 are described in the text and in Appendix C. In particular, for the gross markup I assume an increase from $\mu = 1.1$ to $\mu = 1.2$ and for the relative abundance of outrightly owned houses an increase from $\kappa_H^{oo}/\kappa_N^{oo} = 1$ to $\kappa_H^{oo}/\kappa_N^{oo} = 1.2$. In rows 3 and 5 it is assumed that part of the present value of pure profits are also included in the valuation of businesses ($\zeta_\Pi = 1/4$). In order to have the same initial situation this requires to set $\delta_k = 9.4\%$ and $r_k = 10.2\%$ (in 1980).

In row 1 of Table 4 I report the results of the baseline specification. For the year 1980 the values of the wealth-to-income ratio and the share of housing wealth come out as $\beta = 340\%$ and $\frac{\beta_H}{\beta} = 47\%$, respectively, when the exogenous interest rate is again set (close) to 10%.[17] For the current period, on the other hand, I consider two variants

[17]In fact, I set $r_k = 9.84\%$ in order to get target the values $\beta = 340\%$ and $\frac{\beta_H}{\beta} = 47\%$ that correspond to the empirically observed values reported in Table 1.

(one with $r_k = 7\%$ and one with $r_k = 6\%$). In addition to the exogenous change in the interest rate the benchmark specification in row 1 only involves changes in the basic parameters g, n and β_D^N. It turns out that the values for the year 2017 ($\beta = 554\%$, $\frac{\beta_H}{\beta} = 50\%$) are roughly in line with the observed weighted world average (see Table 1) if one uses the assumption of $r_k = 6\%$. This necessary decrease in the interest rate is, however, rather large when compared to the empirical observations. Furthermore, the results implies that the benchmark model (with changes in only g, n and public debt) will have difficulties to explain the observed developments for some (especially European) countries that have shown much larger increases in the share of housing wealth (up to 20 pp). As a logical next step I therefore investigate whether the assumption of changes in additional structural parameters might offer the potential to better match these empirical patterns. In Appendix C I report the results for a number of additional specifications where I distinguish between calibrations that only change one additional parameter at a time, two parameters or many parameters. The changes are based on findings from the empirical literature as summarized in Table 3 and in Appendix B. In rows 2 and 3 of Table 4 I look at a central case where the average markup increases from 10% to 20% (with an accompanying reduction in the labor share) and where the relative abundance of outrightly owned houses increases from $\kappa_H^{oo}/\kappa_N^{oo} = 100\%$ to $\kappa_H^{oo}/\kappa_N^{oo} = 120\%$. As I will discuss later, this is an combination that has the potential to explain the cross-country variation since one might argue that the increase in the markup was a dominant phenomenon for the US while the change in the ownership structure was concentrated in Europe. Returning to the world average, this constellation leads to a larger reaction than in the benchmark case. The housing wealth for 2017 now comes out as 52% with an almost identical associated wealth-to-income ratio of 480% (for $r_k = 7\%$). Another channel that might be important to understand the pattern of intertemporal and interregional differences has to do with the role of profits. In particular, so far I have looked at the case where future profits are not included in the valuation of firms (i.e. $\zeta_\Pi = 0$). In fact, this assumption is typically maintained in models that include pure profits caused by the existence of markups. In this case it holds that $\beta_B = \beta_K$. It seems, however, reasonable to assume that at least some part of future profits are already included in the market valuation of businesses. In row 3 of Table 4 I show the result for $\zeta_\Pi = 0.25$.[18] As one can see one now gets a much bigger effect on the wealth-to-income ratio (which increases to $\beta = 574\%$ for $r_k = 7\%$) while for the housing wealth share one now observes a *decrease* to 43%. The reason for

[18] In order to get the same starting values for 1980 as before I had to adapt the value for the capital depreciation rate to $\delta_k = 9.4\%$ and the initial interest rate to $r_k = 10.2\%$.

this reaction is that an increase in the markup leads to a larger increase in the valuation of future profits which, on the one hand, increases aggregate wealth while, on the other hand, diminishing the role of housing wealth.

The implied decrease in the housing wealth share can be prevented by allowing for parallel changes in additional parameters. In particular, in rows 4 and 5 of Table 4 I show the results if one also considers changes in γ, δ_k, δ_m and κ_N^j in addition to the changes in g, n, β_D^n, μ and $\kappa_H^{oo}/\kappa_N^{oo}$. As can be seen in row 4 this has a considerable impact on the housing wealth share while the increase in the wealth-to-income ratio is still somewhat too low. If, however, one assumes that part of the profits are included in business valuations ($\zeta_\Pi = 1/4$) then one gets a pattern that is very close to the empirical observations for the world (G8a) average. A decrease in the interest rate by 3 pp is associated with an increase in the wealth-to-income ratio from 340% to 561% and an increase in the housing wealth share from 47% to 52%.

5.2 US and Europe

So far I have treated the world (the G8a countries) as a uniform entity and I have used parameter values that were meant to capture world-wide averages (see Appendix B). This followed the arguments by Summers & Rachel (2019) and others who have stressed that the decline in the equilibrium interest rate is a global phenomenon and that one needs a global perspective in order to explain it. At the same time, however, one has to acknowledge that product markets and even more so housing markets show considerable regional peculiarities that have to be taken into account in order to explain cross-country differences.

In this section I thus take the uniform world specification in row 5 of Table 4 as the starting point and now allow for cross-country differences in the levels and changes of crucial parameters. In particular, I will focus on two blocks — the US and the EU4 — and I will use the empirical literature to choose a realistic, region-specific calibration.

5.2.1 Calibration

The main discrepancy between regions is assumed to be found in the degrees of market power and in the influence of outright owners. In particular, I follow the literature and assume that the markup in 1980 was higher in the US than in Europa ($\mu^{US} = 1.15$, $\mu^{EU4} = 1.05$) and that it also increased more rapidly in the US (up to $\mu^{US} = 1.3$ in 2017 while $\mu^{EU4} = 1.1$). On the other hand, the size and the weight of outright owners was always larger in Europe than in the US. As discussed in section 4.4 I assume that in 1980 the outright owners in the US controlled an underproportional fraction of the housing stock ($\kappa_H^{oo,US}/\kappa_N^{oo,US} = 0.8$) while the opposite was true for Europe ($\kappa_H^{oo,EU4}/\kappa_N^{oo,EU4} = 1.2$). For 2017 I assume that this rate stayed constant for the US while it increased to 1.6 within the EU4. This differential levels and developments are likely due to a number of factors. First, in the US the housing market is organized in a special manner and many households are actively re-financing their mortgages in order to benefit from interest rate developments. In most European countries, on the other hand, the mortgage markets are organized differently and certain instruments (like reverse mortgages) are basically non-existent. Second, there seem to be behavioral differences between the two continents where in some European countries housing is a family affair with stronger intergenerational linkages thereby putting more weight on the importance of outright ownership. Finally, it has to be recognized that in Europe there was a considerable increase in homeownership over recent decades. Many of these new homewoners initially financed their purchases with mortgages most of which have been paid of in the meanwhile. This means that a sizeable fraction of outright owners are now between the ages 50 to 70 and they might stick to their home for the case of emergency. In other words, the increasing share of outrightly owned houses might be a transitory phenomenon that will adjust once the transition to a higher share of owners is completed.

Besides these main differences in the markups and the housing structure I also follow the available data and include some (minor) differences in the expenditure share for housing ($\gamma^{US} = 0.16$, $\gamma^{EU4} = 0.18$ in 1980 and $\gamma^{EU4} = 0.19$ in 2017) and the depreciation rate δ_m (which is assumed to have decreased in Europe from 3% to 2% capturing various measures to make ownership more attractive). Note that all of these region-specific values are chosen in a way that the averages closely correspond to the parameter values that have been used for the uniform examples before.[19]

5.2.2 Results

The results for the regional differentiation is shown in Table 5. The first line containing the average for the "world economy" (the G8a) corresponds to row 5 in Table 4 while

[19]In fact, all of the uniform parameters are the average of the US and EU4 values. For the reported averages β and $\frac{\beta_H}{\beta}$ in Table 5 I use, however, the relative GDP-weights from the data which come out as 53/47 (in 1980) and 60/40 (in 2017). For the comparisons it should be noted that the world (G8a) includes 3 additional countries (Australia, Canada and Japan) that are not included in the region-specific comparisons of this section.

the following lines show the results for the US, for Europe and for the weighted average of the US and Europe, respectively. The results of Table 5 indicate that for the chosen calibration the implied values for the world average *and* the region-specific patterns are qualitatively (and often quantitatively) close to the empirical observations summarized in Table 1. In particular, the empirical data for the US suggest an increase in the wealth-to-income ratio from 332% to 536% and a decrease in the housing wealth share from 44% to 37%. The numerical example shows a similar increase in the former magnitude (from 364% to 574%) and a stagnation (or even slight decrease) in the latter (from 44% to 43%). For the EU4, on the other hand, the data report an increase in the wealth-to-income ratio from 333% to 595% and a steep increase in the housing wealth share from 49% to 69%. The results of the model are again close to this development with an increase in β from 332% to 646% and an increase in $\frac{\beta_H}{\beta}$ from 50% to 68%. The stagnation in the housing wealth share for the US is driven by the larger increase in business wealth β_B (due to the increase in the markup) while the marked increase of housing wealth in Europe is a consequence of the increased influence of outrightly owned houses.

Table 5: Aggregate wealth and housing wealth for various country groups

Region	$\boldsymbol{\beta}$ (in %) 1980 ($r_k \approx 10\%$)	$\boldsymbol{\beta}$ (in %) 2017 ($r_k = 7\%$)	$\frac{\beta_H}{\beta}$ (in %) 1980 ($r_k \approx 10\%$)	$\frac{\beta_H}{\beta}$ (in %) 2017 ($r_k = 7\%$)
World (G8a)	340	561	47	52
US	364	574	44	43
Europe (EU4)	332	646	50	68
Average (US, EU4)	349	603	47	53

Note: The table reports the wealth-to-income ratio β and the housing wealth share $\frac{\beta_H}{\beta}$ for various regions. The benchmark in the first row corresponds to row 5 in Table 4. The values for the baseline parameters in 1980 and the assumed changes for the year 2017 are described in the text and in Appendix C. For the central parameters it is assumed that for the US: $\mu = 1.15$ (1980), $\mu = 1.30$ (2017), while $\kappa_H^{oo} = 0.8\kappa_N^{oo}$ in both period. For Europe: $\mu = 1.05$ (1980), $\mu = 1.1$ (2017), while $\kappa_H^{oo} = 1.2\kappa_N^{oo}$ (1980) and $\kappa_H^{oo} = 1.6\kappa_N^{oo}$ (2017).

6 Conclusion

In this paper I study the compatibility of long-run trends in three macroeconomic magnitudes: the decline in real interest rates, the increase in the average wealth-to-income ratio, and the average share of housing wealth across different countries. The central question is whether the observed decline in real interest rates can be aligned with these trends within a standard economic model that includes physical capital and housing stock as asset. The theoretical model and numerical calibrations indicate that while an exogenous reduction in interest rates boosts asset valuations and aggregate wealth ratios, the quantitative relationships observed empirically are not straightforwardly reproduced in a standard set-up. In particular, for the benchmark specification I compare the steady states of the model for the years 1980 and 2017. Assuming a decrease in the return to capital from 10% to 7%, alongside decreases in productivity and population growth rates and an increase in public debt has only a modest effect on the wealth-to-income ratio and almost no effect on the share of housing wealth.

Given that the basic model cannot fully replicate the observed data, the paper explores additional explanatory factors. It highlights the significance of owner-occupiers, particularly outright owners, in understanding the relationship between interest rates and the share of housing wealth. Outright owners, who own their homes without a mortgage, experience an increase in their housing wealth as house prices rise without reacting to it, thereby impacting aggregate housing wealth. The numerical calibration shows that assuming an increase in the fraction of outrightly owned houses and a higher markup (together with the assumption that part of the monopoly profits are included in the valuation of firms) lead to results that are well aligned with the empirical data. In particular, an assumed decline in the return on capital from 10% to 7% between 1980 and 2017 is associated with an increase in the wealth-to-income ratio for the world average from 340% to 561% and an increase in the housing wealth share from 47% to 52%. These numbers are close to the real-world trends.

Finally, the paper extends the analysis to regional differences, particularly between the US and Europe. By adjusting the model to account for higher markups in the US and a greater prevalence of outright owners in Europe, the model also successfully explains the divergent trends in wealth aggregates across these regions. In order to make prediction about the future path of the wealth-to-income ratio and the share of housing wealth it is important to also take likely developments on the housing market into account, in particular the involving the role of outright owners.

References

Alvaredo, F., Atkinson, A., Chancel, L., Piketty, T., Saez, E. & Zucman, G. (2016), Distributional National Accounts (DINA) Guidelines: Concepts and Methods used in WID.world, Working Papers 201602, World Inequality Lab.

Auclert, A., Malmberg, H., Martenet, F. & Rognlie, M. (2021), Demographics, Wealth, and Global Imbalances in the Twenty-First Century, Technical report, National Bureau of Economic Research.

Ball, L. & Mankiw, N. G. (2022), 'Market Power in Neoclassical Growth Models', *The Review of Economic Studies* **90**(2), 572–596.

Bauluz, L., Novokmet, F. & Schularick, M. (2022), The Anatomy of the Global Saving Glut, Technical report.

Bernanke, B. S. (2005), The Global Saving Glut and the U.S. Current Account Deficit, Technical report.

Bonnet, O., Bono, P.-H., Chapelle, G. & Wasmer, E. (2014), Does Housing Capital Contribute to Inequality? A Comment on Thomas Piketty's Capital in the 21st Century, Discussion Paper 2014-07, Sciences Po Departement of Economics.

Bonnet, O., Chapelle, G., Trannoy, A. & Wasmer, E. (2021), 'Land Is Back, It Should Be Taxed, It Can Be Taxed', *European Economic Review* **134**, 103696.

Borri, N. & Reichlin, P. (2018), 'The Housing Cost Disease', *Journal of Economic Dynamics and Control* **87**(C), 106–123.

Caballero, R. J., Farhi, E. & Gourinchas, P.-O. (2017), 'The Safe Assets Shortage Conundrum', *Journal of Economic Perspectives* **31**(3), 29–46.

Cesa-Bianchi, A., Harrison, R. & Sajedi, R. (2023), Global R*, Cepr discussion paper no. 18518.

Chambers, D., Spaenjers, C. & Steiner, E. (2021), 'The Rate of Return on Real Estate: Long-Run Micro-Level Evidence', *The Review of Financial Studies* **34**(8), 3572–3607.

Cocco, J. F. & Lopes, P. (2020), 'Aging in Place, Housing Maintenance, and Reverse Mortgages', *The Review of Economic Studies* **87**(4), 1799–1836.

Dalgaard, C.-J. & Olsen, M. (2021), Has the Real Rate of Return "depreciated"?, Technical report, Draft.

De Loecker, J. & Eeckhout, J. (2021), Global Market Power, Technical report, Draft.

De Loecker, J., Eeckhout, J. & Unger, G. (2020), 'The Rise of Market Power and the Macroeconomic Implications', *The Quarterly Journal of Economics* **135**(2), 561–644.

Del Negro, M., Giannone, D., Giannoni, M. P. & Tambalotti, A. (2019), 'Global Trends in Interest Rates', *Journal of International Economics* **118**, 248–262.

Duca, J. V., Muellbauer, J. & Murphy, A. (2021), 'What Drives House Price Cycles? International Experience and Policy Issues', *Journal of Economic Literature* **59**(3), 773–864.

ECB (2021), The Household Finance and Consumption Survey, Wave 2017, European Central Bank (ECB), Statistical Tables.

Eggertsson, G. B., Mehrotra, N. R. & Robbins, J. A. (2019), 'A Model of Secular Stagnation: Theory and Quantitative Evaluation', *American Economic Journal: Macroeconomics* **11**(1), 1–48.

Eichholtz, P., Korevaar, M., Lindenthal, T. & Tallec, R. (2021), 'The Total Return and Risk to Residential Real Estate', *The Review of Financial Studies* **34**(8), 3608–3646.

Fagereng, A., Holm, M. B., Moll, B. & Natvik, G. (2019), Saving Behavior Across the Wealth Distribution: The Importance of Capital Gains, Working Paper 26588, National Bureau of Economic Research.

Favilukis, J., Ludvigson, S. C. & van Nieuwerburgh, S. (2017), 'The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk Sharing in General Equilibrium', *Journal of Political Economy* **125**(1), 140–223.

French, E., Jones, J. B. & McGee, R. (2023), 'Why Do Retired Households Draw Down Their Wealth So Slowly?', *Journal of Economic Perspectives* **37**(4), 91–113.

Gomme, P., Ravikumar, B. & Rupert, P. (2015), 'Secular Stagnation and Returns on Capital', *Economic Synopses* (19).

Gordon, R. J. (2014), The Turtle's Progress: Secular Stagnation Meets the Headwinds, *in* R. Baldwin & C. Teulings, eds, 'Secular Stagnation', CEPR, United Kingdom, pp. 47–60.

Grossmann, V., Larin, B., Löfflad, H. T. & Steger, T. (2021), 'Distributional Consequences of Surging Housing Rents', *Journal of Economic Theory* **196**, 105275.

Grossmann, V., Larin, B. & Steger, T. M. (2021), Das House-Kapital, FSES Working Papers 523, Faculty of Economics and Social Sciences, University of Freiburg/Fribourg Switzerland.

Gutiérrez, G. & Piton, S. (2020), 'Revisiting the Global Decline of the (Non-Housing) Labor Share', *American Economic Review: Insights* **2**(3), 321–338.

Harding, J. P., Rosenthal, S. S. & Sirmans, C. (2007), 'Depreciation of Housing Capital, Maintenance, and House Price Inflation: Estimates from a Repeat Sales Model', *Journal of Urban Economics* **61**(2), 193–217.

Himmelberg, C., Mayer, C. & Sinai, T. (2005), 'Assessing High House Prices: Bubbles, Fundamentals and Misperceptions', *Journal of Economic Perspectives* **19**(4), 67–92.

Holston, K., Laubach, T. & Williams, J. C. (2023), Measuring the Natural Rate of Interest after COVID-19, Staff Reports 1063, Federal Reserve Bank of New York.

Jordà, Ò., Knoll, K., Kuvshinov, D., Schularick, M. & Taylor, A. M. (2019), 'The Rate of Return on Everything, 1870–2015', *The Quarterly Journal of Economics* **134**(3), 1225–1298.

Jordà, Ò., Schularick, M. & Taylor, A. M. (2016), 'The Great Mortgaging: Housing Finance, Crises and Business Cycles', *Economic Policy* **31**(85), 107–152.

Justiniano, A., Primiceri, G. E. & Tambalotti, A. (2019), 'Credit Supply and the Housing Boom', *Journal of Political Economy* **127**(3), 1317–1350.

Kaplan, G., Mitman, K. & Violante, G. L. (2020), 'The Housing Boom and Bust: Model Meets Evidence', *Journal of Political Economy* **128**(9), 3285–3345.

Karabarbounis, L. & Neiman, B. (2014), 'The Global Decline of the Labor Share', *The Quarterly journal of economics* **129**(1), 61–103.

Knoll, K., Schularick, M. & Steger, T. (2017*a*), 'No Price Like Home: Global House Prices, 1870-2012', *American Economic Review* **107**(2), 331–53.

Knoll, K., Schularick, M. & Steger, T. (2017*b*), 'No Price Like Home: Global House Prices, 1870-2012', *American Economic Review* **107**(2), 331–53.

Kuhn, M., Schularick, M. & Steins, U. I. (2020), 'Income and Wealth Inequality in America, 1949–2016', *Journal of Political Economy* **128**(9), 3469–3519.

Laubach, T. & Williams, J. C. (2003), 'Measuring the Natural Rate of Interest', *The Review of Economics and Statistics* **85**(4), 1063–1070.

Lisack, N., Sajedi, R. & Thwaites, G. (2021), 'Population Aging and the Macroeconomy', *International Journal of Central Banking* **17**(2), 43–80.

Mankiw, N. G. (2022), 'Government Debt and Capital Accumulation in an Era of Low Interest Rates', *Brookings Papers on Economic Activity* **2022**, 219–231.

McKay, A. & Wieland, J. F. (2021), 'Lumpy Durable Consumption Demand and the Limited Ammunition of Monetary Policy', *Econometrica* **89**(6), 2717–2749.

Mian, A. R., Straub, L. & Sufi, A. (2021*a*), 'What explains the decline in r*? Rising income inequality versus demographic shifts', (2021-12).

Mian, A., Straub, L. & Sufi, A. (2021*b*), 'Indebted Demand', *The Quarterly Journal of Economics* **136**(4), 2243–2307.

Miles, D. & Monro, V. (2021), 'UK House Prices and Three Decades of Decline in the Risk-free Real Interest Rate', *Economic Policy* **36**(108), 627–684.

OECD (2023), Affordable Housing Database, , OECD, Paris.

Piazzesi, M. & Schneider, M. (2016), Housing and Macroeconomics, Vol. 2 of *Handbook of Macroeconomics*, Elsevier, pp. 1547–1640.

Piketty, T. (2011), 'On the Long-Run Evolution of Inheritance: France 1820–2050', *The Quarterly Journal of Economics* **126**(3), 1071–1131.

Piketty, T. & Zucman, G. (2014), 'Capital is Back: Wealth-Income Ratios in Rich Countries 1700-2010', *The Quarterly Journal of Economics* **129**(3), 1255–1310.

Platzer, J. & Peruffo, M. (2022), 'Secular Drivers of the Natural Rate of Interest in the United States: A Quantitative Evaluation', *IMF Working Papers* **2022**(030).

Poterba, J. M. (1984), 'Tax Subsidies to Owner-Occupied Housing: An Asset-Market Approach', *The Quarterly Journal of Economics* **99**(4), 729–752.

Rachel, L. & Smith, T. (2015), Secular Drivers of the Global Real Interest Rate, Bank of England Working Papers 571, Bank of England.

Reis, R. (2022), Which r-star, public bonds or private investment? Measurement and policy implications, mimeo, London School of Economics.

Robbins, J. (2018), Capital Gains and the Distribution of Income in the United States, mimeo, Brown University.

Rognlie, M. (2016), 'Deciphering the Fall and Rise in the Net Capital Share: Accumulation or Scarcity?', *Brookings Papers on Economic Activity* **2015**(1), 1–69.

Sommer, K. & Sullivan, P. (2018), 'Implications of US Tax Policy for House Prices, Rents, and Homeownership', *American Economic Review* **108**(2), 241–274.

Summers, L. H. & Rachel, L. (2019), On Falling Neutral Real Rates, Fiscal Policy and the Risk of Secular Stagnation, *in* 'Brookings Papers on Economic Activity BPEA Conference Drafts, March', Vol. 7, pp. 1–54.

Svensson, L. E. (2023), Are Swedish House Prices Too High? Why the Price-to-Income Ratio Is a Misleading Indicator, mimeo.

Appendices

A Details of the model

A.1 Supply of normal goods

In this section of the appendix I collect a number of proofs and extensions of the model that are related to the production of normal goods.

A.1.1 The model with increasing returns to scale

In the paper I have used the standard assumption of a production function with constant returns to scale. In the case with a sizeable gross markup μ this might, however, lead to an implausibly high share of pure profit $\varphi_\Pi = \frac{\mu-1}{\mu}$ (e.g. for $\mu = 1.3$ one has $\varphi_\Pi = 23\%$). In order to circumvent this implication one can follow Ball & Mankiw (2022) and use a set-up with increasing returns based on overhead labor. In particular, let's assume that the j intermediate good j is now produced according to the production function:

$$Y_{It}(j) \quad = \quad F\left[K_t(j), L_t(j), X_t(j)\right] = K_t(j)^\alpha \left[\mathcal{A}_t\left(L_t(j) - X_t(j)\right)\right]^{1-\alpha}, \tag{27}$$

where the variable $X_t(j)$ represents fixed overhead labor. For $X_t(j) = 0$ production is characterized by constant returns to scale while for $X_t(j) > 0$ firms exhibit increasing returns to scale due to the fixed costs. This set-up is useful in order to find reasonable calibrations but it should be noted that the assumption of increasing returns to scale is not crucial for the analytical results. The factor prices now come out as:

$$\mathcal{R}_{kt} = P_{It}(j)\frac{\alpha}{\mu}\frac{Y_{It}(j)}{K_t(j)}, \mathcal{W}_t = P_{It}(j)\frac{1-\alpha}{\mu}\frac{Y_{It}(j)}{L_t(j) - X_t(j)}. \tag{28}$$

The factor shares, on the other hand, are now given by:

$$\varphi_\Pi = 1 - \frac{\alpha}{\mu} - \varphi_L, \tag{29a}$$

$$\varphi_L = \frac{1-\alpha}{\mu}\frac{1}{1-\frac{X}{L}}, \tag{29b}$$

where X/L is the steady state ratio of the fixed overhead labor X to flexible labor L. This is a system of two equations in five free parameters (α, μ, φ_L, φ_Π and $\frac{X}{L}$) which is useful for the determination of reasonable calibrations. In particular, one can choose three parameters while the remaining two parameters are determined endogenously by equations (29a) and (29b). For example, in the most straightforward approach one can choose the gross markup μ, the labor share φ_L and the profit share φ_Π from the literature and let α and X/L be determined by the system.[20] The standard case in the literature (and the case used in the main text) abstracts from increasing returns to scale. In this case one thus has $X = 0$ implying that the free choice of μ and φ_L leaves α and φ_Π to be determined by equations (6). This might cause an excessive profit share $\varphi_\Pi = \frac{\mu-1}{\mu}$ for high values of the gross markup μ. For the sake of brevity, I do not discuss alternative calibrations in the paper.

A.1.2 The value of businesses including future profits

In section 2.5 I stated an expression for the value of business assets B_t that includes the present value of future profits (see equation (15)). This can be derived as follows. First, assume that there exists a fixed number of shares $Q_t^b = Q^b$ that certify the ownership rights in the firms and that trade at price P_t^b. The rate of return r_{bt} of investing in these shares is thus given by:

$$r_{bt} = \frac{\mathcal{R}_{kt}K_t - (\delta_k + g + n)K_t + \frac{dP_t^b}{dt}Q_t^b + \zeta_\Pi \Pi_t}{P_t^b Q_t^b}.$$

Firms earn the total return to capital $\mathcal{R}_{kt}K_t$ but they have to invest $(\delta_k + g + n)K_t$ in order to replace the worn-out capital stock and also to increase the capital stock at rate $(g+n)$ in order to remain on the balanced growth path. On the other hand, they benefit from increases in the shares prices where in equilibrium it holds that $\frac{dP_t^b}{dt} = (g+n)P_t^b$. Finally, it seems reasonable to assume that the share valuation also includes some part ζ_Π of the profits. Using from section 2 that $\mathcal{R}_{kt} = \frac{\alpha}{\mu}\frac{Y_{Nt}}{K_t}$ and $\varphi_\Pi = \frac{\Pi_t}{Y_{Nt}}$ and assuming that the return on equity is the same as the return on capital (i.e. $r_{bt} = r_{kt}$) one can derive

[20] In particular, the remaining parameters are then given by: $\alpha = \mu(1 - \varphi_L - \varphi_\Pi)$ and $\frac{X}{L} = \frac{1-\frac{1}{\mu}-\varphi_\Pi}{\varphi_L}$. Alternatively, one could also choose μ, φ_L and α and let φ_Π and X/L be endogenously determined. In this case $\varphi_\Pi = 1 - \frac{\alpha}{\mu} - \varphi_L$ and $\frac{X}{L} = 1 - \frac{1-\alpha}{\mu\varphi_L}$. Note that one can also write $\varphi_\Pi = 1 - \frac{1}{\mu} - \varphi_L\frac{X}{L}$ which highlights that $\varphi_\Pi = \frac{\mu-1}{\mu}$ for $X = 0$.

the equilibrium value of business assets as:

$$\begin{aligned} B_t &= P_t^b Q_t^b = \frac{\frac{\alpha}{\mu} Y_{Nt} - (\delta_k + g + n) K_t + \zeta_\Pi \varphi_\Pi Y_{Nt}}{r_{kt} - (g+n)} \\ &= K_t + \zeta_\Pi \frac{\varphi_\Pi Y_{Nt}}{r_{kt} - (g+n)}, \end{aligned} \tag{30}$$

where for the last line I use the definition from (4) that $r_{kt} = \frac{\alpha}{\mu}\frac{Y_{Nt}}{K_t} - \delta_k$. This is shown as equation (15) in the text.

A.1.3 The national-account based measure for the return on capital

The return on investing in shares r_{bt} is per assumption the same as the return on capital r_{kt}. This follows from a simple no-arbitrage argument. A financial investor should be indifferent between buying a unit of capital and renting it to the firms or investing into the firm directly (assuming identical risk profiles). This can also be seen differently by dividing the proceeds of the investment in the shares by the purchase value. In particular, one can insert the value for $P_t^b Q_t^b$ from (30) into the expressions for r_{bt}. This can be transformed as follows:

$$\begin{aligned} r_{bt} &= \frac{\mathcal{R}_{kt} K_t - (\delta_k + g + n) K_t + \frac{dP_t^b}{dt} Q_t^b + \zeta_\Pi \varphi_\Pi Y_{Nt}}{P_t^b Q_t^b} \\ &= \frac{\mathcal{R}_{kt} K_t - (\delta_k + g + n) K_t + P_t^b Q_t^b (g+n) + \zeta_\Pi \varphi_\Pi Y_{Nt}}{P_t^b Q_t^b} \\ &= \frac{\mathcal{R}_{kt} K_t - (\delta_k + g + n) K_t + \zeta_\Pi \varphi_\Pi Y_{Nt}}{P_t^b Q_t^b} + (g+n) \\ &= \frac{\mathcal{R}_{kt} K_t - (\delta_k + g + n) K_t + \zeta_\Pi \varphi_\Pi Y_{Nt}}{\frac{\frac{\alpha}{\mu} Y_{Nt} - (\delta_k + g + n) K_t + \zeta_\Pi \varphi_\Pi Y_{Nt}}{r_{kt} - (g+n)}} + (g+n) \\ &= r_{kt} - (g+n) + (g+n) = r_{kt}. \end{aligned} \tag{31}$$

This is of course not a result but rather a number of transformations that follow from the assumption that $r_{bt} = r_{kt}$. The step-by-step presentation is, however, useful when comparing the rate of return on equity with another measure that is often used in the empirical literature. In particular, as briefly mentioned in the text, in this literature the marginal product of capital is measured as capital income per unit of capital based on data from the national accounts (Gomme et al. 2015, Reis 2022). In these data sources, however, the net payments to capital $\mathcal{R}_{kt} K_t - \delta_k K_t$ are typically combined with the profits

$\Pi_t = \varphi_\Pi Y_{Nt}$. This (national-accounts-based) measure for the return on capital $\widetilde{r}_{kt}$ is thus given by:

$$\begin{aligned} \widetilde{r}_{kt} &= \frac{\mathcal{R}_{kt}K_t - \delta_k K_t + \Pi_t}{K_t} = \frac{\alpha}{\mu}\frac{Y_{Nt}}{K_t} - \delta_k + \varphi_\Pi \frac{Y_{Nt}}{K_t} \\ &= r_{kt} + \varphi_\Pi \frac{Y_{Nt}}{K_t}. \end{aligned} \tag{32}$$

A comparison between the transformations of r_{bt} and the definition of $\widetilde{r}_{kt}$ reveals that the difference is that the return on equity uses the total firm valuation $P_t^b Q_t^b$ in the denominator and also takes into account the increases in share prices. The measured rate of return $\widetilde{r}_{kt}$, on the other hand, only use K_t as the reference point in the denominator (and abstracts from valuation gains and from the necessary expansion of the capital stock). The two measures can also be related by using the well-known concept of Tobin's q defined as:

$$q = \frac{P_t^b Q_t^b}{K_t}. \tag{33}$$

Note that one can write $r_{bt} = r_{kt}$ (assuming $\zeta_\Pi = 1$) as:[21]

$$\begin{aligned} r_{kt} &= \frac{\mathcal{R}_{kt}K_t - \delta_k K_t + \varphi_\Pi Y_{Nt}}{P_t^b Q_t^b} + (g+n) - \frac{(g+n)K_t}{P_t^b Q_t^b} \\ &= \frac{K_t}{P_t^b Q_t^b}\left(\frac{\mathcal{R}_{kt}K_t - \delta_k K_t + \varphi_\Pi Y_{Nt}}{K_t} + (g+n)\left(\frac{P_t^b Q_t^b}{K_t} - 1\right)\right) \\ &= \frac{1}{q}\left(\widetilde{r}_{kt} + (g+n)(q-1)\right). \end{aligned}$$

From this it follows that:

$$\widetilde{r}_{kt} = q r_{kt} + (1-q)(g+n). \tag{34}$$

A similar expression has also been derived in Ball & Mankiw (2022). Equations (32) and (34) indicate that this national-account-based measure $\widetilde{r}_{kt}$ will be larger than r_{kt}. What is more, the two measures might develop differently over time. An increase in μ will, e.g., decrease $r_{kt} = \frac{\alpha}{\mu} - \delta_k$ but increase φ_Π and q (where in the simplest case $\varphi_\Pi = \frac{\mu-1}{\mu}$). This pattern is in fact broadly in line with existing empirical evidence. The data on returns to equity in Jordà et al. (2019) are based on publicly traded equities and are thus a good proxy for r_{kt}. As discussed above, the data on the returns on equity in Jordà et al. (2019) indicate in fact a (slight) decrease in r_{kt} from 1950 up to today. On the

[21] This follows from the fourth line of the transformation in the equation block (31).

other hand, the papers that use the national-accounts-based measure (see e.g. Gomme et al. 2015, Reis 2022) report a constancy of measured $\widetilde{r}_{kt}$. This divergent development might be a reflection of a situation where the markup and profits have increased and the decline in r_{kt} has been compensated by an increase in the profit term $\varphi_\Pi \frac{Y_{Nt}}{K_t}$. An explanation along these lines can, e.g., also be found in Eggertsson et al. (2019).[22]

A.2 Housing supply

In the model I assume that the housing supply increases with the size of the population, although probably not to a full extent. In particular, I specify that $\overline{H}_t = \overline{H}_0 e^{n\chi t}$, where $0 \leq \chi \leq 1$ accounts for the reaction of the housing supply to population growth. As discussed in the text, this leads to a steady state growth rate of the house price given by (10), i.e. $g_h = g + n(1 - \chi)$.

In contrast to the reduced form used in this paper, Grossmann, Larin & Steger (2021) present a framework that explicitly models the production of housing by the use of structures and land. In particular, the numeraire good Y is produced with capital K, labor L^Y and land $\mathcal{L}^Y$:[23]

$$Y = K^{\alpha_K} \left(B^Y L^Y\right)^{\alpha_L} \left(\mathcal{L}^Y\right)^{1-\alpha_K-\alpha_L},$$

where B^Y (which captures productivity growth) is assumed to grow at the constant rate g_Y.

Houses are produced by special firms that combine residential structure X and residential land $\mathcal{L}^X$:

$$H = X^\nu \left(\mathcal{L}^X\right)^{1-\nu}.$$

Finally, construction firms produce new structures I^X by employing materials $\mathcal{M}$ and labor L^X according to:

$$I^X = \mathcal{M}^\eta \left(B^X L^X\right)^{1-\eta},$$

where B^X is the productivity in the construction sector that is assumed to grow at rate

[22] "[One] argument against the presence of secular stagnation conditions in the United States is that the measured return on capital is stable (see, for example Gomme et al. 2015). The argument is that the most relevant measure of return is not the return on government debt but the return on productive capital. One problem with that argument is that business income does not just measure capital income but also pure profits. And over the past decades, there is some evidence that competition has decreased and monopoly rents have risen. If an increase in monopoly rents cancels out the decrease in competitive returns to capital, this would lead to a stable measured average return on capital" (Eggertsson et al. 2019, p.44).

[23] Here and in the following I sometimes change the notation from Grossmann, Larin & Steger (2021) such as to adapt it to the notation used in the present paper.

g_X.

Grossmann, Larin & Steger (2021) solve the model and they derive the steady state equations. It has to be noted, however, that they assume that the population and labor supply are allowed to grow over time but that they converge to a finite limit as time approaches infinity. Therefore their steady state equations do not include population growth. In order to facilitate a comparison to my expression for g_h I assume for the sake of the argument that L^Y and L^X grow at rate n (even though strictly spoken this does not seem to be possible in the framework of Grossmann, Larin & Steger (2021)). In this case, the steady state growth rate for house prices comes out as (see Table 2 in Grossmann, Larin & Steger 2021):

$$\mathfrak{g}_{P^H} = (1 - \nu\eta)\frac{\alpha_L}{1 - \alpha_K}(g_Y + n) - \nu(1 - \eta)(g_X + n).$$

Using the same calibration as Grossmann, Larin & Steger (2021) (for the US: $g_X = 0$, $g_Y = 0.019$, $\nu = 0.709$, $\eta = 0.485$, $\alpha_K = 0.274$, $\alpha_L = 0.613$ and — as stated above — $n = 0$) this leads to $\mathfrak{g}_{P^H} = 0.01$.

In order to relate this formulation to my assumption I first abstract from land in the goods production function: $1 - \alpha_K = \alpha_L$. Together with $g_X = 0$ (but now allowing for $n > 0$) and denoting $g_Y = g$ one thus has $\mathfrak{g}_{P^H} = (1 - \nu\eta)(g + n) - \nu(1 - \eta)n = g(1 - \nu\eta) + n(1 - \nu)$. This expression is very similar to my own steady state term $g_h \equiv g + n(1 - \chi)$. In fact, for $\eta = 0$ one gets that $\mathfrak{g}_{P^H} = g + n(1 - \nu)$ which is identical to my formulation. Note that in the calibration by Grossmann, Larin & Steger (2021) it holds that $\nu\eta$ is rather small ($\nu\eta = 0.71 \times 0.485 = 0.35$) and thus $\mathfrak{g}_{P^H} \approx g + n(1 - \nu)$ is an acceptable approximation. Furthermore, note that their choice of $\nu = 0.709$ is in the neighbourhood of my benchmark calibration where $\chi = 0.5$. In fact, for some of the other countries the calibration in Grossmann, Larin & Steger (2021) is even closer to this value: $\nu = 0.52$ (UK), $\nu = 0.11$ (FR), $\nu = 0.59$ (GER).

Different to the present paper, Grossmann, Larin & Steger (2021) to not attempt to explain the shift in the share of housing wealth as a steady state event (associated with a change in the economic structure) but they rather see it as a transitional phenomenon. In fact, their model has the property (see Corollary 2.1) that "in the economy's steady-state equilibrium the wealth-to-income ratio, the housing wealth-to-income ratio, and the non-housing wealth-to-income ratio are constant". The same is not true for the prices related to housing that all increase in the steady state if two conditions are fulfilled: "i) technological progress in the construction sector lags behind the technological progress of the rest of the economy and ii) housing production is more land-intensive than non-housing production". In fact, due to the richness of their model Grossmann, Larin & Steger (2021) can distinguish between the different (steady-state) developments of the price of residential land, the price of residential structures and the house price. It should be noted, however, that a positive steady-state growth rate of the house price also occurs in my model. In fact, as discussed above I implicitly use a model without construction (and thus without technological progress in the construction sector) and thus $g_X = 0$. The first condition is thus fulfilled since $g_X < g_Y = g$. I furthermore assume implicitly that housing production is more land-intensive than the non-housing production (where I abstract from land and $1 - \alpha_K = \alpha_L$, see equation (2)). For $g = g_X = 0$ and $\chi = 1$ it also holds that $g_h = 0$ in my framework, similar to the results in Grossmann, Larin & Steger (2021).

The increase in the housing wealth-to-income ratio is explained as a transitional phenomenon in Grossmann, Larin & Steger (2021): "Our analysis suggests that neoclassical convergence forces explain the evolution of the housing wealth-to-income ratio: initial values of physical capital and residential structures—exogenous model variables in the initial period—were considerably below their steady-state values in all considered economies. Intuitively, a low stock of residential structures implies a high marginal productivity of structures in producing housing services, leading to high residential investment at the start of the transition. An initially low and subsequently growing capital stock results in rising housing demand and a declining interest rate, pushing the house price up" (p.5). The interest rates are endogenous in the model of Grossmann, Larin & Steger (2021) and the authors are therefore not able to study the impact of a change in interest rates for house prices or the housing wealth. They state, however, that "the endogenous dynamics of the interest rate are in line with the empirical observation of declining interest rates. The real interest rates declines because capital starts below its steady state and accumulates over time, reducing the marginal product of capital and hence the interest rate. Our explanation for rising house prices is therefore in line with the empirical trend in the real interest rate" (p 29ff.). In fact, their steady state interest rate is given by (see Grossmann, Larin & Steger 2021, p.48):

$$r = \rho + ((1-\gamma)\sigma + \gamma)g_Y + \gamma(\sigma - 1)g_X,$$

where ρ stands for the rate of time preference, σ for the intertemporal elasticity of substitution and γ for the housing expenditure share. Using their calibration ($\gamma = 0.19$, $\sigma = 10/3$,

$\rho = 0.051$, $g_Y = 0.019$, $g_X = 0$) implies a steady state interest rate of $r = 10.6\%$. But this refers to the final steady state, i.e. also to the current situation and is thus considerable larger then my (assumed) values of $r_k = 7\%$ or $r_k = 6\%$.

Summing uo, the analysis in Grossmann, Larin & Steger (2021) can be regarded as complementary to the results of this paper. While they use a rich setup that models the construction of housing and look at the transitional dynamics I focus on the ownership structure (where Grossmann, Larin & Steger (2021) abstract from owner-occupiers) and on steady-state comparisons.

A.3 National accounting

As noted in section 2.4 total wealth and the various subaggregates of wealth demand and wealth supply can be related to each of the income concepts Y_{Nt}, GDP_t and NDP_t (and also to GNP_t and NNP_t if one would include an open economy structure). The use of one or the other income concept depends on its usefulness and/or tractability. In the text I have shown the formulas for β_t^N and β_t^{NDP}, i.e. using the domestic production (excluding housing services) Y_{Nt} and the net domestic product NDP_t, respectively. The transformation between the various concepts is straightforward using appropriate multiplicative factors. These factors can also be applied to all different "partitions" of wealth. For example:

$$\beta_{Bt}^N = \frac{B_t}{Y_{Nt}}, \beta_{Kt}^N = \frac{K_t}{Y_{Nt}}, \beta_{Ht}^N = \frac{P_{ht}^r \overline{H}_t^r + P_{ht}^o \overline{H}_t^o}{Y_{Nt}}, \beta_{Dt}^N = \frac{D_t}{Y_{Nt}}, \beta_{Mt}^N = \frac{M_t}{Y_{Nt}}. \tag{35}$$

If one wants to calculate the wealth-to-GDP-ratio then one has to take the definition (13) into account $GDP_t = Y_{Nt} + P_{st}^r \overline{H}_t^r + P_{st}^o \overline{H}_t^o$. It thus holds for any wealth concept $x \in \{B, K, H, H_r, H_o, D, M\}$ that:

$$\beta_{xt}^{GDP} = \beta_{xt}^N \frac{Y_{Nt}}{GDP_t} = \beta_{xt}^N \frac{1}{1 + \frac{P_{st}^r \overline{H}_t^r}{Y_{Nt}} + \frac{P_{st}^o \overline{H}_t^o}{Y_{Nt}}}. \tag{36}$$

Focusing on the net domestic product the parallel relation holds:

$$\beta_{xt}^{NDP} \equiv \beta_{xt} = \beta_{xt}^N \frac{Y_{Nt}}{NDP_t} \tag{37}$$

which has already been shown for $\beta_t^{NDP} = \beta_t$ in section 2.4.

As far as the size of $\frac{Y_{Nt}}{NDP_t}$ is concerned it is instructive to look at the steady state with

$r_{kt} = r_{ht} = r_{mt} = r$, $\beta_D^N = 0$ and $\zeta_\Pi = 0$. In this case the ratio of net domestic product to non-housing output can be written as:

$$\frac{NDP_t}{Y_{Nt}} = 1 + r\beta^N - \frac{\alpha}{\mu} = 1 + \frac{\alpha}{\mu}\frac{\beta_H}{\beta_K} - \delta_k \beta^N, \tag{38}$$

where $\frac{\beta_H}{\beta_K} = \frac{P_{ht}^r \overline{H}_t^r + P_{ht}^o \overline{H}_t^o}{K_t}$ is the ratio of housing wealth to physical capital wealth. The proof of equation (38) follows below. The equation emphasizes that in the absence of housing ($\frac{\beta_H}{\beta_K} = 0$) the net domestic product is always smaller than the non-housing output (with $NDP_t = Y_{Nt}$ for $\delta_k = 0$). This, however, is no longer true for the general situation where it might be the case that the inclusion of housing services exactly counterbalances the subtraction of depreciation such that again $NDP_t = Y_{Nt}$. This will happen if $r\beta^N = \frac{\alpha}{\mu}$ which is not an implausible condition (e.g. $\alpha = 0.3$, $\mu = 1.1$ with $\varphi_L = \frac{1-\alpha}{\mu} = 64\%$, $r = 8\%$, $\beta^N = 341\%$). For most of the following calibrations it will hold that $\frac{NDP_t}{Y_{Nt}}$ is between 95% and 100%.

The proof of equation (38) is based on the Haig-Simmons definition of national output (see footnote 10, in particular: $GDP_t^{HS} = Y_{Nt} + P_{st}^r \overline{H}_t^r + P_{st}^o \overline{H}_t^o + \dot{P}_{ht}^r \overline{H}_t^r + \dot{P}_{ht}^o \overline{H}_t^o$). One can then write:

$$\begin{aligned}
\frac{NDP_t}{Y_{Nt}} &= 1 + \frac{P_{st}^r \overline{H}_t^r}{Y_{Nt}} + \frac{P_{st}^o \overline{H}_t^o}{Y_{Nt}} + \frac{\dot{P}_{ht}^r \overline{H}_t^r}{Y_{Nt}} + \frac{\dot{P}_{ht}^o \overline{H}_t^o}{Y_{Nt}} - \frac{\delta_h P_{ht}^r \overline{H}_t^r}{Y_{Nt}} - \frac{\delta_m P_{ht}^o \overline{H}_t^o}{Y_{Nt}} - \frac{\delta_k K_t}{Y_{Nt}} \\
&= 1 + r_{ht}\frac{P_{ht}^r \overline{H}_t^r}{Y_{Nt}} + r_{mt}\frac{P_{ht}^o \overline{H}_t^o}{Y_{Nt}} - \frac{\delta_k K_t}{Y_{Nt}}
\end{aligned} \tag{39}$$

where the transformation follows from equations (8) $\left(P_{st}^r = P_{ht}^r\left(r_{ht} + \delta_h - \frac{\dot{P}_{ht}^r}{P_{ht}^r}\right)\right)$ and (9) $(P_{st}^o = P_{ht}^o\left(r_{mt} + \delta_m - \frac{\dot{P}_{ht}^o}{P_{ht}^o}\right))$ which implies that $\overline{H}_t^r\left(P_{st}^r + \dot{P}_{ht}^r - \delta_h P_{ht}^r\right) = r_{ht} P_{ht}^r \overline{H}_t^r$ and $\overline{H}_t^o\left(P_{st}^o + \dot{P}_{ht}^o - \delta_m P_{ht}^o\right) = r_{mt} P_{ht}^o \overline{H}_t^o$. In the next step one can use $\beta^N = \beta_H^N + \beta_K^N$, where $\beta_H^N = \frac{P_{ht}^r \overline{H}_t^r + P_{ht}^o \overline{H}_t^o}{Y_{Nt}}$ and $\beta_K^N = \frac{K_t}{Y_{Nt}}$. One can thus write:

$$\begin{aligned}
\frac{NDP_t}{Y_{Nt}} &= 1 + r\frac{P_{ht}^r \overline{H}_t^r}{Y_{Nt}} + r\frac{P_{ht}^o \overline{H}_t^o}{Y_{Nt}} - \frac{\delta_k K_t}{Y_{Nt}} \\
&= 1 + r\left(\beta^N - \beta_K^N\right) - \delta_k \beta_K^N = 1 + r\beta^N - (r + \delta_k)\beta_K^N \\
&= 1 + r\beta^N - \frac{\alpha}{\mu},
\end{aligned}$$

where I use the steady state condition $r_{kt} = r_{ht} = r_{mt} = r$ together with $r_{kt} = \frac{\alpha}{\mu}\frac{Y_{Nt}}{K_t} - \delta_k = \frac{\alpha}{\mu}\frac{1}{\beta_K^N} - \delta_k$ from equation (4) which implies that $(r + \delta_k)\beta_K^N = \frac{\alpha}{\mu}$. Using $r = \frac{\alpha}{\mu\beta_K^N} - \delta_k$ the expression above can also be written as:

$$\begin{aligned}\frac{NDP_t}{Y_{Nt}} &= 1 + r\beta_H^N - \delta_k \beta_K^N = 1 + \beta_H^N \left(\frac{\alpha}{\mu \beta_K^N} - \delta_k \right) - \delta_k \beta_K^N \\ &= 1 + \frac{\alpha}{\mu} \frac{\beta_H^N}{\beta_K^N} - \delta_k \left(\beta_K^N + \beta_H^N \right) = 1 + \frac{\alpha}{\mu} \frac{\beta_H}{\beta_K} - \delta_k \beta^N,\end{aligned}$$

where in the last line I use $\frac{\beta_H^N}{\beta_K^N} = \frac{\beta_H}{\beta_K}$ since the income concept used in the denominator drops out (and where $\beta_K = \frac{K_t}{NDP_t}$, $\beta_H = \frac{P_{ht}^r \overline{H}_t^r + P_{ht}^o \overline{H}_t^o}{NDP_t}$).

A.4 The interest rate in the related literature without housing

In section 3.4 I have shown that in models without housing the equilibrium interest is simply given by $r_k = \frac{\alpha}{\mu\beta} - \delta_k$ where $\beta = \beta_K$. One can compare the results of this back-of-the-envelope calculation to the main papers of the related literature. It should be noted at the outset that the models of these papers are constructed in a way such as to explain the decline in the natural interest rate which is equated with a safe interest rate. Therefore they mostly try to target a value of $r^* = 4\%$ around 1980 and a value of $r^* = 1\%$ around 2017. Summers & Rachel (2019), e.g, base their calibration on a value of $K/Y = 225\%$ for the initial situation in 1970 (which is in line with the data from the WID). Furthermore, they choose $\alpha = 0.33$, $\mu = 1$ and $\delta_k = 10\%$. Using the expression from above this implies that $r_k = \frac{0.33}{2.25} - 0.1 = 4.66\%$. In fact, this is very close to the interest rate of 4.5% reported in their paper. Platzer & Peruffo (2022), on the other hand, refer in their basic calibration to the period 2010-2018 for which they target a value of $K/Y = 370\%$. This is somewhat lager than the capital-to-income ratio reported in the WID for the US in 2017 (346%) and much larger than the value for the G8a countries (269%). They choose $\alpha = 0.302$, $\mu = 1.175$ and $\delta_k = 5\%$. Furthermore they also explicitly introduce an "intermediation wedge" of 2% between the return on capital and the return on government bonds. Thus $r = r_k - 0.02 = \frac{0.302/1.175}{3.7} - 0.05 - 0.02 = -0.1\%$. This is again close to their target value for the safe interest rate of $r = 0.53\%$.

A.5 Derivations of the case with homeowners

In this appendix I show how to derive equation (25) of the paper. In particular, starting with $P_{st}^r \overline{H}_t^r = \kappa_N^r \gamma \varepsilon Y_t$ and $P_{st}^o \overline{H}_t^{om} = \kappa_N^{om} \gamma \varepsilon Y_t$ and using the definition for GDP in equation

(13) ($Y_t = Y_{Nt} + P^r_{st}\overline{H}^r_t + P^o_{st}\overline{H}^o_t$) one can derive that:

$$\begin{aligned} P^r_{st}\overline{H}^r_t &= \kappa^r_N \frac{\gamma\varepsilon}{1-\gamma\varepsilon\left(1-\kappa^{oo}_N+\kappa^{om}_N\frac{\kappa^{oo}_H}{\kappa^{om}_H}\right)} Y_{Nt} = \kappa^r_N \widetilde{\phi} Y_{Nt}, \\ P^o_{st}\overline{H}^{om}_t &= \kappa^{om}_N \frac{\gamma\varepsilon}{1-\gamma\varepsilon\left(1-\kappa^{oo}_N+\kappa^{om}_N\frac{\kappa^{oo}_H}{\kappa^{om}_H}\right)} Y_{Nt} = \kappa^{om}_N \widetilde{\phi} Y_{Nt}, \end{aligned} \tag{40}$$

where $\widetilde{\phi} \equiv \frac{\gamma\varepsilon}{1-\gamma\varepsilon\left(1-\kappa^{oo}_N+\kappa^{om}_N\frac{\kappa^{oo}_H}{\kappa^{om}_H}\right)}$. Note that in the absence of outright owners ($\kappa^{oo}_N = \kappa^{oo}_H = 0$) it holds that $\widetilde{\phi} = \phi = \frac{\gamma\varepsilon}{1-\gamma\varepsilon}$.

Note that the house price formed in the segment of the self-buying owners is used to assess the value of the stock of the directly owned houses. Since $\overline{H}^o_t = \overline{H}^{om}_t + \overline{H}^{oo}_t = \overline{H}^{om}_t\left(1+\frac{\kappa^{oo}_H}{\kappa^{om}_H}\right)$ one can use (20) and (21) to write the ratio of the steady state housing-wealth-to-income ratios (of owners) as $\beta^N_{Ho} = \beta^N_{Hom} + \beta^N_{Hoo} = \frac{\kappa^{om}_N\widetilde{\phi}}{r_m+\delta_m-g_h}\left(1+\frac{\kappa^{oo}_H}{\kappa^{om}_H}\right)$ where $\beta^N_{Hom} = \frac{P^o_{ht}\overline{H}^{om}_t}{Y_{Nt}}$ is the value of the housing stock of the buying owners with mortgages. Or step-by-step:

$$\begin{aligned} \beta^N_{Ho} &= \frac{P^o_{ht}\overline{H}^o_t}{Y_{Nt}} = \frac{P^o_{ht}\left(\overline{H}^{om}_t+\overline{H}^{oo}_t\right)}{Y_{Nt}} = \frac{P^o_{st}}{(r_m+\delta_m-g_h)Y_{Nt}}\left(\overline{H}^{om}_t+\overline{H}^{oo}_t\right) \\ &= \frac{\kappa^{om}_N\widetilde{\phi}}{(r_m+\delta_m-g_h)\overline{H}^{om}_t}\left(\overline{H}^{om}_t+\overline{H}^{oo}_t\right) = \frac{\kappa^{om}_N\widetilde{\phi}}{r_m+\delta_m-g_h}\left(1+\frac{\kappa^{oo}_H}{\kappa^{om}_H}\right) \end{aligned}$$

where the last line uses $P^o_{st}\overline{H}^{om}_t = \kappa^{om}_N\widetilde{\phi}Y_{Nt}$.

As mentioned in the text I assume that the actual rent and the imputed rent are equal ($P^r_{st} = P^o_{st}$). In a fully-fledged model this would, e.g., be the case if one assumes that in equilibrium renters and buying owners are indifferent between the possible choice (and if their are no intrinsic utility differences between owning and renting). This requires that the housing stocks adjust such that this condition is fulfilled. In particular, one can use the expressions $P^r_{st}\overline{H}^r_t = \kappa^r_N\widetilde{\phi}Y_{Nt}$ and $P^o_{st}\overline{H}^{om}_t = \kappa^{om}_N\widetilde{\phi}Y_{Nt}$ together with $P^r_{st} = P^o_{st}$ to derive that $\frac{\kappa^r_N}{\kappa^r_H} = \frac{\kappa^{om}_N}{\kappa^{om}_H}$. For given population shares of renters (κ^r_N), owners with mortgage (κ^{om}_N) and outright owners (κ^{oo}_N) and a given share of outrightly owned houses κ^{oo}_H one can derive the equilibrium shares of rented and self-acquired houses as:

$$\kappa^r_H = \kappa^r_N\frac{1-\kappa^{oo}_H}{1-\kappa^{oo}_N}, \kappa^{om}_H = \kappa^{om}_N\frac{1-\kappa^{oo}_H}{1-\kappa^{oo}_N}. \tag{41}$$

If there are no direct owners ($\kappa^{oo}_H = \kappa^{oo}_N = 0$) or if the share of directly owned houses also corresponds to their population share ($\kappa_H^{oo} = \kappa_N^{oo}$) then the service price for rented and self-acquired houses is the same if $\kappa_H^r = \kappa_N^r$ and $\kappa_H^{om} = \kappa_N^{om}$. Using (41) one can calculate that:

$$\begin{aligned}
\beta_{Ho}^N &= \frac{\kappa_N^{om}\widetilde{\phi}}{r_m + \delta_m - g_h}\left(1 + \frac{\kappa_H^{oo}}{\kappa_H^{om}}\right) = \frac{\kappa_N^{om}\widetilde{\phi}}{r_m + \delta_m - g_h}\left(1 + \frac{\kappa_H^{oo}}{\kappa_N^{om}}\frac{1-\kappa_N^{oo}}{1-\kappa_H^{oo}}\right) \\
&= \frac{\widetilde{\phi}}{r_m + \delta_m - g_h}\left(\kappa_N^{om} + \kappa_H^{oo}\frac{1-\kappa_N^{oo}}{1-\kappa_H^{oo}}\right) = \frac{\widetilde{\phi}}{r_m + \delta_m - g_h}\left(1 - \kappa_N^r + \frac{\kappa_H^{oo} - \kappa_N^{oo}}{1-\kappa_H^{oo}}\right)
\end{aligned}$$

where the last line uses $\kappa_N^{om} = 1 - \kappa_N^r - \kappa_N^{oo}$. The total housing wealth ratio $\beta_H^N = \beta_{Hr}^N + \beta_{Ho}^N$ can thus be calculated as:

$$\beta_H^N = \frac{\kappa_N^r\widetilde{\phi}}{r_h + \delta_h - g_h} + \frac{\widetilde{\phi}}{r_m + \delta_m - g_h}\left(\frac{1-\kappa_N^{oo}}{1-\kappa_H^{oo}} - \kappa_N^r\right)$$

and the ratio of housing wealth to physical wealth as equation (25) which is here repeated:

$$\frac{\beta_H}{\beta_K} = \frac{\widetilde{\phi}\mu}{\alpha}\left[\kappa_N^r\frac{r_k + \delta_k}{r_h + \delta_h - g_h} + \left(\frac{1-\kappa_N^{oo}}{1-\kappa_H^{oo}} - \kappa_N^r\right)\frac{r_k + \delta_k}{r_m + \delta_m - g_h}\right]. \tag{25}$$

As noted above, the housing share depends on a number of parameters and in general it is not clear how it will react to changes in the economic structure.

B Calibration

B.1 Data on wealth

The data for wealth come from the *World Inequality Database* (WID, see WID.world) and were retrieved at the end of 2023. The database builds on the work and the conceptual framework developed by Piketty & Zucman (2014). It is extensively explained on the homepage, in particular in the code dictionary (https://wid.world/codes-dictionary). In the following, I summarize the main concepts and relate them to the framework used in the model of the paper. For this purpose I closely follow the notation (with some minor adaptations) introduced in an early version of the WID (Alvaredo et al. 2016, p.44ff.). My main focus is on private wealth W_{pt} which is defined as:

$$W_{pt} = B_{pt} + H_{pt} + F_{pt} - L_{pt}, \tag{42}$$

where B_{pt} stand for national business and other non-financial assets owned by the private sector, H_{pt} for national housing assets owned by the private sector, F_{pt} for financial assets owned by the private sector and L_{pt} for financial liabilities (debt, bonds, loans etc.) of the private sector. The financial assets can be further split into:

$$F_{pt} = F^C_{pt} + F^E_{pt} + F^I_{pt},$$

where F^C_{pt} stand for currency/deposits/bonds/loans, F^E_{pt} for equities/shares/offshore and F^I_{pt} for pension-funds/life-insurances. In the notation of the WID code dictionary equation (42) corresponds to:

`pweal=pwbus+pwhou+pwfin-pwdeb`.

Public wealth W_{gt}, on the other hand, is defined in a completely parallel fashion:

$$W_{gt} = B_{gt} + H_{gt} + F_{gt} - L_{gt}, \tag{43}$$

where all variables are also defined in an analogous manner.

For corporate wealth this is somewhat more complicated since one has to distinguish between the book value of corporations BV_{ct} and the market equity value of corporations EV_{ct}. The book-value is defined as:

$$BV_{ct} = B_{ct} + H_{ct} + F_{ct} - DL_{ct}, \tag{44}$$

where B_{ct}, H_{ct} and F_{ct} are again defined in an analogous manner to before and where DL_{ct} stands for the debt liabilities of the corporate sector (i.e. the non-equity corporate liabilities: debt, bonds, loans, etc.). The market equity value of the corporate sector EV_{ct}, on the other hand, corresponds to the equity corporate liabilities (i.e. to total market equity value of domestic quoted and unquoted corporations). $L_{ct} = DL_{ct} + EV_{ct}$ denotes the total financial liabilities of the corporate sector (debt and equity). Finally, $W_{ct} \equiv BV_{ct} - EV_{ct}$ stands for residual corporate wealth. In the WID code dictionary this is denoted as: `cwres=cwboo-cwdeq`. One can thus rewrite (44) as

$$W_{ct} = BV_{ct} - EV_{ct} = B_{ct} + H_{ct} + F_{ct} - L_{ct}. \tag{45}$$

If the book value of corporations corresponds to the market equity value then there is no "extra" wealth of the corporate sector since the value of the assets of the corporations is already included in the financial assets of the other sectors that own the corporate sector (i.e. the private sector, the government sector and the foreign sector).

Market-value national wealth W_{nt} is defined as the sum of private wealth and government wealth (i.e. residual corporate wealth W_{ct} is set to zero):

$$W_{nt} = W_{pt} + W_{gt} = B_{nt} + H_{nt} + NFA_t, \tag{46}$$

where $B_{nt} = B_{pt}+B_{gt}+B_{ct}$, $H_{nt} = H_{pt}+H_{gt}+H_{ct}$ and $NFA_t = F_{nt}–L_{nt}$ stands for the net foreign assets owned by domestic sectors with $F_{nt} = F_{pt}+F_{gt}+F_{ct}$ and $L_{nt} = L_{pt}+L_{gt}+L_{ct}$. In the WID code dictionary (46) is written as `nweal=pweal+gweal=nwbus+nwhou+nwnxa`.

Viewed from a different angle there are five types of *financial* assets that can be held by the private sector: government bonds L_{gt}, private bonds L_{pt}, corporate bonds DL_{ct}, equity EV_{ct} and foreign assets FA_t. Denoting the holding of the private sector by an expression "p" in parentheses one can thus write:

$$F_{pt} = L_{gt}(p) + L_{pt}(p) + DL_{ct}(p) + EV_{ct}(p) + FA_t(p).$$

Parallel equations also hold for F_{gt}, F_{ct} and NFA_t.[24] The expression for F_{pt} can be inserted into equation (42) to give:

$$W_{pt} = B_{pt} + H_{pt} + L_{gt}(p) + L_{pt}(p) + DL_{ct}(p) + EV_{ct}(p) + FA_t(p) - L_{pt}.$$

In other words, private wealth is the sum of all private sector financial assets F_{pt} (which are themselves holdings of different types of bonds, equity shares and foreign financial assets), of directly owned private businesses B_{pt}, privately owned housing assets H_{pt} (in particular owner-occupied houses) minus financial liabilities of the private sector (in particular mortgage debt).

In the model of the paper I make the following simplifying assumptions. First, I assume that the book and equity values of corporations are equal (i.e. $W_{ct} = 0$) and that market-value national wealth W_{nt} is therefore given by (46). Second, I abstract from a foreign sector and set $NFA_t = 0$. Third, I assume that the public sector does not hold any assets and that it only has the government bonds $\mathcal{D}_t$ as its liabilities L_{gt} on the balance sheet, i.e. $W_{gt} = -L_{pt} = -\mathcal{D}_t$. Fourth, households do not hold direct business assets (B_{pt}), but

[24]In particular, one has that $F_{jt} = L_{gt}(j) + L_{pt}(j) + DL_{ct}(j) + EV_{ct}(j) + FA_t(j)$ for $j \in \{p, g, c\}$. At the same time $L_{gt} = \sum_k L_{gt}(k)$, $L_{pt} = \sum_k L_{pt}(k)$, $DL_{ct} = \sum_k DL_{ct}(k)$ and $EV_{ct} = \sum_k EV_{ct}(k)$, for $j \in \{p, g, c, f\}$ and $NFA_t = FA_t(p) + FA_t(g) + FA_t(c) - (L_{gt}(f) + L_{pt}(f) + DL_{ct}(f) + EV_{ct}(f))$. It can be shown that these equations lead again to equation (46), i.e. $W_{nt} = W_{pt} + W_{gt} = B_{nt} + H_{nt} + NFA_t$.

they rather own the entire value of business B_t (which includes the value of the capital stock K_t and perhaps also the present value of profits) via equities. Fifth, households hold the residential housing stock $\overline{H}_t^r$ as commercial real estate (i.e. as financial assets) and the owner-occupied stock $\overline{H}_t^o$ as direct private asset. Sixth, the only liabilities of the private sector are the mortgage loans, i.e. $L_{pt} = M_t$ where these loans are on the other hand also a financial assets that is part of F_{pt}. Summarizing the last points one can thus write:

$$F_{pt} = B_t + \overline{H}_t^r + \mathcal{D}_t + M_t \tag{47}$$

while total private wealth can be written as:

$$\begin{aligned} W_{pt} &= B_{pt} + H_{pt} + F_{pt} - L_{pt} = 0 + \overline{H}_t^o + F_{pt} - M_t \\ &= \overline{H}_t^o + B_t + \overline{H}_t^r + \mathcal{D}_t + M_t - M_t = K_t + \overline{H}_t + \mathcal{D}_t. \end{aligned} \tag{48}$$

This corresponds to equation (16) in section 2.5, i.e. $W_t^d = W_{Bt} + W_{Hrt} + W_{Hot} + W_{Dt}$. On the other hand, the definition of financial (or liquid) assets W_{Ft}^d introduced there ($W_{Ft}^d = W_{Bt} + W_{Hrt} + W_{Mt} + W_{Dt}$) exactly corresponds to the magnitude F_{pt}. The net worth of the stock of owner-occupiers $W_{Ot}^d = W_{Hot} - W_{Mt}$, on the other hand, corresponds to $H_{pt} - L_{pt}$.

The equilibrium interest rate r_t is a weighted average of the various available financial assets as specified in 2.5 and repeated here for convenience:[25]

$$r_t = \frac{B_t}{W_{Ft}^d} r_{kt} + \frac{P_{ht}^r \overline{H}_t^r}{W_{Ft}^d} r_{ht} + \frac{M_t}{W_{Ft}^d} r_{mt} + \frac{D_t}{W_{Ft}^d} r_{dt}. \tag{17}$$

For the empirical data I follow the related literature (in particular Piketty & Zucman 2014) and I use the (private-)wealth-to-(net-)income ratio:

$$\beta_t = \frac{W_{pt}}{NDP_t} = \frac{\texttt{pweal}}{\texttt{nninc}}.$$

For the share of housing wealth I also follow Piketty & Zucman (2014) and define it as the share of national housing assets in national non-financial assets, i.e.:

$$\frac{\beta_{Ht}}{\beta_t} = \frac{H_{nt}}{B_{nt} + H_{nt}} = \frac{\texttt{nwhou}}{\texttt{nwnfa}}.$$

[25] In a fully fledged model the interest rate would be an endogenous variable that is determined by equating the supply of available *financial* assets $W_{Ft}^d = F_{pt}$ with the demand for financial assets W_{Ft}^s by the household sector that follows, e.g., from some intertemporal maximization problem.

The reason for this definition is the following. Housing assets are not only held (directly or indirectly) by the private sector but also by the government and (in particular) by the corporate sector. A focus on H_{pt} would ignore these additional housing assets and might give a distorted picture. I do not use the entire national wealth $W_{nt} = B_{nt} + H_{nt} + NFA_t$ in the denominator since net foreign assets are no distinguished between financial and non-financial assets. In particular, "in case foreign residents — either private individuals, governments or corporations — own domestic non-financial assets, this is accounted for as if they own financial assets in a domestic fictitious corporation, which then owns the domestic non-financial assets" (Alvaredo et al. 2016, p.50). Again this could distort the measured share of housing wealth. The use of different definition does, however, lead to similar qualitative results.

B.2 Other variables and parameters

In the following I discuss the choice of the main parameter values that have been summarized in Table 3 of the paper and that have been used to calibrate the model for the years around 1980 and around 2017.

B.2.1 Rates of return and risk wedges ξ_h, ξ_m and ξ_d

The literature on the development on interest rates is large and constantly increasing. I will only focus on a number of central papers that are often quoted in the related literature.

A standard reference for the empirical estimates of the development of the natural interest rate are the estimates by Laubach & Williams (2003) which are continuously updated (see Holston et al. 2023). The estimates refer to short-term interest rates (for the US they are based on the annualized federal funds rate, for the Euro Area the three-month rate). The estimates indicate a decrease of the natural rate for the US from 4.2% (1970) to 3.6% (1980) to 0.9% (2017), for Canada from 5.2% (1970) to 4.5% (1980) to 1.6% (2017) and for the Euro Area from 2.8% (1980) to 0.4% (2017). Overall a decrease between 3 pp and 3.5 pp.

Del Negro et al. (2019) study the development of interest rates for a sample of seven countries where they also account for the co-movements in interest rates across countries. They use either government securities or close substitutes to measure the level of (safe) interest rates and find that the world real interest decreased from around 3% im 1980 to almost zero in 2016.

Cesa-Bianchi et al. (2023) use a method similar to Del Negro et al. (2019) to calculate estimates for a "global r^* for a panel of 31 countries again referring to a short-term real interest rate. They find a decline from 2.5% (1980) to basically 0% in recent years.

Beside the empirical literature there exist a number of papers that take the empirical estimates as a starting point in order to explain the decrease with the help of theoretical models. Eggertsson et al. (2019), e.g., use their model of secular stagnation to generate a fall in the natural interest rate from 2.55% (in 1970) to -1.47% (a decrease of 4.02 pp) which they contrast to the similar decrease in the real Federal Funds rate that was observed over this period.

Summers & Rachel (2019), on the other hand, report a decline in the natural interest rate (measured by long-term interest rates) of about 3 pp between 1971 and 2016 (from 3.5% to 0.5% where the decrease from 1980 onwards was around 2.5 pp).

The model in Platzer & Peruffo (2022) is associated with a decline in the (natural) real interest rate of 4.26 pp between the 1965 steady state to the 2015 steady (with a decline between 1975 and 2015 of 3.12 pp) down to a rate of 0.53%.

So far I have only discussed papers that focus on the development of safe assets (either government papers or short-term interest rates). Jordà et al. (2019), on the other hand, present more encompassing data on the rates of return for a group of 16 countries. In particular, they include data for short and long-term bonds but also for the return on stocks and housing. Some of their main findings are collected in Table B1. The authors themselves summarize their findings as follows: "In terms of total returns, residential real estate and equities have shown very similar and high real total gains, on average about 7% a year. [...] The observation that housing returns are similar to equity returns, but much less volatile, is puzzling" (p.1229). "Safe returns have been low on average in the full sample, falling in the 1%–3% range for most countries and peacetime periods" (ibd.). The last two observations give rise to a stable risk premia between risky and safe assets of 4%–5%. This has been the rationale for choosing $\xi_d = 5\%$. On the other hand, the similar (long-term) similarity between the rates of return on equity and housing (and the fact that the latter even risk-dominates the former) has motivated the choice of $\xi_h = 0$.[26] Note, however, that these data only refer to investments in commercial real estate. For mortgage rates it is more reasonable and in line with the empirical evidence to assume

[26] Later studies based on more detailed data (Chambers et al. 2021, Eichholtz et al. 2021) have challenged some of the findings related to the returns on housing included in Jordà et al. (2019) (especially the one about the superiority of risk-adjusted returns of housing). Even these studies, however, do not seem to support risk wedges on residential housing investments that are larger than 2-3%.

the existence of a risk wedge. In particular, the available evidence suggests that mortgage interest rates are typically between 2 pp to 3 pp above government bonds rates which itself are assumed to have a risk discount of around 5 pp ($\xi_d = 5\%$). I thus assume in the initial calibration that $\xi_m = 2\%$ and assume for the calibration of the later period that the mortgage discount rate increases to $\xi_m = 2.5\%$.

As far as the development over time is concerned it has to be noted that there are considerable (and often long-lasting) swings in the rates of return. It is thus difficult to detect long-run trends by the use of simple averaging. Nevertheless, in Table B1 I look at the rates of return on equity, housing, bills and bonds for three time periods (1950-1970, 1970-1990, 1990-2016) and different groups of countries. The data seems to indicate that the risky rate of return might have fallen between 3 pp and 4 pp over the last decades. On the other hand, the data show now clear time trend for the safe rate. This is not entirely consistent with other evidence that often show the opposite (a constant return in capital and a sharply decreasing safe interest rate) (Reis 2022).

B.2.2 Productivity growth rate g, population growth rate n and gross savings rate s

The values for g (real GDP growth), n (population growth) and s (savings rate) correspond to the data for the group of high income countries in the World Development Indicators database. In particular, for g I use the variable `NY.GDP.MKTP.KD.ZG` (annual percentage growth rate of GDP at market prices), for n the variable `SP.POP.GROW` (annual population growth rate defined as the exponential rate of growth of midyear population from the previous to the current year) and for s the variable `NY.GDS.TOTL.ZS` (gross domestic savings calculated as GDP less final consumption expenditure). In each case I transform the annual values to "period averages" $\overline{1970}$, $\overline{1980}$ and $\overline{2017}$. Therefore I use the arithmetic averages for the years 1966-1974, 1976-1984 and 2011-2017, respectively. These are shown in Tables 2 and D1 for the world (G8a), the US and the EU4 countries for three points in time.

B.2.3 Depreciation rate for physical capital δ_k

In the literature one can find many values for the depreciation rate for physical capital. The online appendix of Eggertsson et al. (2019), e.g., refers to three papers that use values between 5% and 15%. McKay & Wieland (2021), one the other hand, use a value of 6.8% (an average from 1970 to 2019) for the depreciation of durable assets which includes in

Table B1: Rates of return based on Jorda et al. (2019) (in %)

Country	**Period**	**Bills**	**Bonds**	**Equity**	**Housing**
World	Full Sample	1.3	2.5	7.1	6.7
World	Post 1950	—	—	8.2	6.4
World	Post 1980	—	—	9.0	5.5
7 countries	Post 1950	1.3	3.9	7.6	5.8
7 countries	1950-1970	1.2	-0.3	10.6	7.8
7 countries	1970-1990	2.2	2.8	8.7	6.5
7 countries	1990-2016	1.0	5.2	6.6	5.2
US	Post 1950	1.3	4.0	8.1	5.9
US	1950-1970	1.3	-1.3	10.6	5.0
US	1970-1990	2.7	3.8	7.4	6.2
US	1990-2016	0.7	5.2	7.9	6.0
EU4	Post 1950	1.5	4.0	7.7	6.2
EU4	1950-1970	0.8	0.4	9.8	10.1
EU4	1970-1990	1.7	2.0	8.1	6.5
EU4	1990-2016	1.5	5.8	7.0	5.3

Note: The values report real returns. The first three lines are from the paper and the online appendix of Jordà et al. (2019). The first line from Table A.5 (including war periods), lines two and three from Table VII in the paper. The other lines are based on own calculations using the dataset provided by Jordà et al. (2019). The category "7 countries" refers to the group G8a except Canada. All country averages are (GDP)-weighted.

their definition also residential housing. This would suggest a value around $\delta_k = 10\%$ which would give an overall rate of depreciation of around 7% if the share of housing wealth is between 40% and 50%.

The standard choice of $\delta_k = 5\%$ is, however, in line with the benchmark assumption in many textbooks and also, e.g., with the back-of-the-envelope calibration in Mankiw (2022). It is based on the recent study by Dalgaard & Olsen (2021) who use data from the Penn World Tables to come up with a GDP-weighted average depreciation rate for the group of advanced economies. They calculate a value of $\delta = 3.5\%$ (for 1980) that has increased to around $\delta = 4.5\%$ in 2017. The use of more detailed data from the Bureau of Economic Analysis (BEA) more or less confirms this development for the US where the rate of depreciation is reported to have increased from 4.7% to 5.7%. The use of $\delta_k = 5\%$ and $\delta_h = \delta_m = 2.5\%$ leads to an average rate of depreciation of $\delta = 3.8\%$ if one assume a share of housing wealth of 47% (as is observed in the present data) ($3.8\% = 0.47 \times 2.5\% + 0.53 \times 5\%$). In line with the results in Dalgaard & Olsen (2021) I sometimes assume that the value for δ_k increased to 7% in 2017 which implies a shift in the average δ by about 1 pp. The reason for this recent increase is related to an increasing importance of information technology and software hat depreciates at a faster speed. For the cases with $\varphi_\Pi = 1/4$ I use a higher depreciation rate of $\delta_k = 9.4\%$ in order to get the same initial situation. This value implies an average depreciation rate of around 5% which is above the results reported in Dalgaard & Olsen (2021) but still in the ballpark of common assumptions. For δ_k I do not use regional differences.

B.2.4 Depreciation rate for housing capital δ_h

The depreciation rate of housing structures is often assumed to be 1.5% (Kaplan et al. 2020, Sommer & Sullivan 2018, Grossmann, Larin, Löfflad & Steger 2021). A number of papers, however, also include housing-related taxes which are assumed to be around 1% (Kaplan et al. 2020, Sommer & Sullivan 2018) and I thus use $\delta_h = \delta_m = 2.5\%$. This is also in line with the results in Harding et al. (2007). In this paper the authors use data from the American Housing Survey to show that housing has depreciated at roughly 2.5 percent per year over the period 1983-2001 (gross of maintenance).

For the development over time it has been argued (Svensson 2023) that the increase in the share of land in the total valuation of houses had the consequence that the depreciation-related share of structures has decreased. Since the depreciation rate is related to the house price this change in the relative weight of the components of the house price can be captured by a decrease in δ_h or δ_m. In the paper I use the assumption that the depreciation rates stayed constant for the US but decreased for Europe (this is meant to capture measures that made home-ownership more attractive).

B.2.5 Labor share and markup (φ_L and μ)

The reference value for the labor share in calibrations is 66%. Many studies have confirmed this value (at least until the 1980ies). Karabarbounis & Neiman (2014), e.g., report a value of 64% for the value of the (weighted) global labor share in 1980 while Gutiérrez & Piton (2020) report values of around 69% for Europe (EU28) in 1980 and around 63% for the US (based on the adjusted business sector; see Figure 1). I take 66% as the (average) target value for 1980. Karabarbounis & Neiman (2014) report that the global labor share decreased by about 5 pp until 2015. This finding, however, is challenged by Gutiérrez & Piton (2020) who argue that countries differ in their definitions of corporate sectors. In particular, the US exclude all self-employed and most dwellings from the corporate sectors while they are included by most other countries. Gutiérrez & Piton (2020) show that the "harmonized" series show constant (or even slightly increasing) labor shares for all major advanced countries except the US and Canada. For the US they report a decrease to 60% (in 2015) while for Europe only a slight decrease to about 68%. For the country-specific calibration I assume a constant labor share for the EU and a decrease for the US to below 60%.

There exists some controversy about the mechanism behind the decline in the labor share. Karabarbounis & Neiman (2014), e.g., argue that it is due to a within-firm substitution of labor for capital (e.g. due to a decrease in the price of investment goods). The problem with this line of explanation, however, is that estimates for the elasticity of substitution between capital and labor are also ambiguous and mostly not larger than 1 (which would be needed to be consistent with the substitution story). De Loecker et al. (2020), on the other hand, argue that the decline in labor share is the consequence of the increase in the rise in (global) market power. In particular, De Loecker et al. (2020) report that for the US the markup has risen from 21% (1980) to 61% (2016) while for the global markup they calculated in De Loecker & Eeckhout (2021) an increase from 17% (1980) to 60% (2016).

In the simple model used in the paper such a large increase in the markup would imply an implausible decline in the labor share. In particular, the labor share is given by $ls = \frac{1-\alpha}{\mu}$. I choose $\alpha = 0.274$ and $\mu = 1.1$ for the initial situation (in 1980) such that the (average or "global") labor share amounts to 66% ($\frac{1-0.274}{1.1} = 0.66$). Note that for the capital coefficient α Eggertsson et al. (2019) again report a wide variety of assumptions used in the macroeconomic literature ranging from 0.25 to 0.36 (which includes my assumption of $\alpha = 0.274$).

For the later time period in 2017 I assume that the capital coefficient stays constant but that there is an increase in the gross markup to $\mu = 1.2$ thus implying a decline in the labor share to 60.5%. This is roughly in line with the observed decrease by 6 pp. In fact, it should be noted that the empirical specification in De Loecker et al. (2020) is based on a framework that allows for other variable factors besides labor. As a consequence this framework is not characterized by a 1:1 relation between the increase in the markup and the decline in the labor share. In fact, the authors provide estimates that indicate that an increase in the firm markup by 10% will decrease the labor share by only between 2% and 2.4%. Furthermore, a different assumption about the production function (with overheads as a specific production factor) also leads to lower estimates of the markup (from 1 in 1980 to 1.3 in 2016, see Figure XIII). This is similar to the overhead model presented in Appendix A.1.1.

Altogether I use $\alpha = 0.274$ and (for the average values) $\mu = 1.1$ (in 1980) and $\mu = 1.2$ (in 2017) such that the labor share decreases from 66% to 60.5%. When differentiating between regions I use $\mu = 1.05$ (Europe, 1980) and $\mu = 1.15$ (US, 1980) with corresponding labor shares of 69% and 63%, respectively. It is assumed that the value for Europe has slightly increased to $\mu = 1.1$ while the one for the US has increased to $\mu = 1.3$ in 2017 (thus implying a rather low labor share of 56% which, however, is close to the average value for various estimates reported in Figure B.1 of the online appendix to Gutiérrez & Piton (2020)).

B.2.6 Housing expenditure share γ

As far as housing is concerned, one can start with the share of housing-related expenditure as a percentage of total household consumption expenditures in the OECD Affordable Housing Database (OECD 2023). In particular, Figure HC 1.1.3 reports an average share of housing expenditures of 22.8% across all OECD member states. This number, however, also includes expenditures on electricity, gas, water etc. If one only considers the numbers for actual and imputed rents the expenditure share comes out as 16.7% or (if one also adds the expenditures for maintenance and repair of the dwelling) as 17.5%. These data thus suggest the choice of $\gamma = 0.17$. The data also suggest some (smaller) regional difference and I assume for the US a share of $\gamma^{US} = 0.16$ while for the EU4 I assume that the share increased from $\gamma = 0.18$ (1980) to $\gamma = 0.19$ (2017).

B.2.7 Homeownership rates

The share of renters and owners κ_N^r and κ_N^o (=κ_N^{om}+κ_N^{oo}) come from Jordà et al. (2016). In Table 3 they report homeownership rate for various years (1950, 1960, ..., 2010) and various countries (Canada, Germany, France, Italy, Switzerland, United Kingdom, United States). These contain six of the eight countries of the core country sample in Piketty & Zucman (2014) (only Australia and Japan are missing). Calculating weighted averages gives a value of $\kappa_N^r = 0.45$ and $\kappa_N^o = 0.55$ (for 1980) and $\kappa_N^r = 0.38$ and $\kappa_N^o = 0.62$ (for 2010). I use the latter values for the year 2017.

Table 3 in Jordà et al. (2016) indicates values of $\kappa_N^r = 0.36$ for the US and $\kappa_N^o = 0.50$ for the EU4 in 1980. For the later year they give $\kappa_N^r = 0.35$ for the US and $\kappa_N^r = 0.40$ for the EU4. Alternatively one can also use the data from Table HM1.3.A1 of the Affordable Housing Database in OECD (2023). The values for renters around 2017 are very similar both for the EU4 (41.2%) and for the US (36.1%). The advantage of the OECD data is that they also distinguish between owners with mortgage and outright owners. For the OECD the (unweighted) average among member states around the year 2020 are 28.5% (renters), 23.3% (owners with mortgage) and 48.2% (outright owners). Using the same data the (weighted) average for the EU4 in 2017 comes out as 41.2% (renters), 20.8% (owners with mortgage) and 38.0% (outright owners) and 36.1% (renters), 40.2% (owners with mortgage) and 23.8% (outright owners). For the US one could also use data from the $SCF+$ (Kuhn et al. 2020) which are discussed in the following section and which are also broadly in line with the other data sources.

B.2.8 Importance of outright owners

For the calibration of the model one also needs data for the change in the share of outright owners over time and also for the relation between the share of outright owners and the value of the housing stock that is in their possession. Unfortunately, these data do not seem to be available for all countries and it is therefore necessary to piece together a number of data sources in order to get some rough estimates.

As discussed above, one can use the data from the OECD (2023) to get estimates for the share of outright owners around 2017. In order to come up with comparable data for earlier periods one has to look at specific countries.

For the US I use the $SCF+$ database that has been constructed by Kuhn et al. (2020) by compiling various waves of the Survey of Consumer Finances (SCF) in order to create a consistent time series of household balance sheets from 1950 to 2016. The data contain two variables that are highly useful for the question at hand. First, the variable `house` measures the asset value of the house that is inhabited by the household. Second, the variable `hdebt` reports the housing debt on owner-occupied real estate. From this information one can identify renters (`house` $= 0$), owners with mortgage (`house` > 0 and `hdebt` > 0) and outright owners (`house` > 0 and `hdebt` $= 0$) and their respective shares. These data indicate that the share of renting households stayed more or less constant at 36% between 1983 and 2016 while the share of owners with mortgages increased from 37% to 42% and the share of outright owners decreased from 26% to 22%. For the UK, on the other hand, data from the English Housing Survey document that the share of households who rent increased from 16% in 1981 to around 23% in 2018. At the same time the share of owners with a mortgage decreased from 47% to 35% while the share of outright owners increased from 37% to 41%. For the rest of the countries, OECD data are only available for the time span from 2010 to 2020 and they show only little movements with the (unweighted) average share of renters staying at 18%-19% and the average shares of owners with and without a mortgage at 23% and 48%, respectively.

All of these data refer, however, only to the population share of dwellers (to κ_N^j in my notation) and not to the relative value of the housing stock that they control (to κ_H^j in my notation). In order to get information about the latter data I employ different evidence as briefly discussed in section 4.4. Using again the $SCF+$ database from Kuhn et al. (2020) one can also calculate for the US the value of the dwelling (i.e. of the variable `house`) for the two groups of owners. The ratio of these two values (i.e. the average value of the house of outright owners to the average value of the house of owners with a mortgage) and its development since 1970 has already been shown in Figure 2a in section 4.4. This ratio corresponds to the expression $\kappa_H^{oo}/\kappa_N^{oo}$. In order to see this note that using the notation of the paper the average value of outrightly owned houses in a certain year is given by $\overline{W}_H^{oo} = \frac{P_h^o \kappa_H^{oo} \overline{H}}{\kappa_N^{oo} N}$, where N is the number of households in this period. Similarly, the average value of houses owned with a mortgage is given by $\overline{W}_H^{om} = \frac{P_h^o \kappa_H^{om} \overline{H}}{\kappa_N^{om} N}$ and thus the ratio of these two comes out as $\frac{\overline{W}_H^{oo}}{\overline{W}_H^{om}} = \frac{\kappa_H^{oo}}{\kappa_H^{om}} \frac{\kappa_N^{om}}{\kappa_N^{oo}}$. Assuming for simplicity that $\kappa_H^{om} = \kappa_N^{om}$ it follows that $\frac{\overline{W}_H^{oo}}{\overline{W}_H^{om}} = \frac{\kappa_H^{oo}}{\kappa_N^{oo}}$. [27]

In the paper I assume a stylized structure in which it is assumed that all not outrightly

[27]Using instead the formulas (41) it follows that $\frac{\overline{W}_H^{oo}}{\overline{W}_H^{om}} = \frac{\kappa_H^{oo}}{\kappa_N^{oo}} \frac{1-\kappa_N^{oo}}{1-\kappa_H^{oo}} \approx \frac{\kappa_H^{oo}}{\kappa_N^{oo}} \left(1 - \kappa_N^{oo}(1 - \frac{\kappa_H^{oo}}{\kappa_N^{oo}})\right)$. If the

owned houses are fully financed by mortgages. In the real world this is of course not true and the mortgage share is lower, either due to the role of equity (i.e. a LTV ratio that is below 100%) or because the mortgage has already been partly repaid. For the US, e.g., the share of debt on the owner-occupied real estate $\frac{\texttt{hdebt}}{\texttt{house}}$ has increased from 44% in 1970 to around 52% around 2015 (average of 2013 and 2016). In order to make the data comparable with the model one could thus also add the paid-off part of the mortgage to the outrightly owned housing stock. In this case it holds that $\overline{W}_H^{oo} = \frac{P_h^o \overline{H}\left(\kappa_H^{oo}+(1-d)\kappa_H^{om}\right)}{N\left(\kappa_N^{oo}+(1-d)\kappa_N^{om}\right)}$, where d is the fraction of mortgage debt to the house value of mortgage-financed houses. This gives a picture that is qualitatively identical to Figure 2a, only that now the share hovers around 90%.

For Europe I was not able to find a dataset that is comparable to the $SCF+$ and that allows for a direct calculation of $\kappa_H^{oo}/\kappa_N^{oo}$. There exists, however, indirect evidence that the housing structure is important for the wealth-to-income ratios and the share of housing wealth. In particular, in a standard model the tenure choice would not have an impact on the accumulation or composition of wealth (since renters and owners would, e.g., adjust their financial wealth such as to meet an identical target of total wealth). This, however, is not reflected in the available data. In Figure B1 I plot the share of outright owners vs. the share of housing wealth in panel (a) (which has already been shown in Figure 2b) and the wealth-to-income ratio (panel b) for a group of European countries. The housing wealth variable comes from the third wave of the HFCS (see ECB 2021) and is defined as the sum of the value of the household's main residence plus the value of other real estate minus the value of outstanding mortgage debt divided by the net wealth. Using gross values in both the numerator and the denominator give very similar values for the housing share. The share of outright owners comes from Table HM1.3.3 in the OECD's Affordable Housing Database (OECD 2023). Interestingly, for the UK the data from the WID show a share of housing wealth of 66% in 2017 while the survey evidence quoted above report a share of outright owners of 41%. This datapoint thus also fits almost perfectly on the regression line. For the US, on the other hand, the WID and the $SCF+$ suggest a share of outright owners of 22% with a share of housing wealth of 37% which is somewhat below the line. Focusing on panel (a), an increase in the share of outright owners by 10 pp increases the share of housing wealth by about 5 pp (in other words, the slope of the regression line is around 0.5). A similar picture emerges if one contrasts the share of outright owners with the total wealth-to-income ratio in panel (b). Also here one

share of outright owners is not too large the ratio $\frac{\overline{W}_H^{oo}}{\overline{W}_H^{om}} = \frac{\kappa_H^{oo}}{\kappa_N^{oo}}$ is thus an acceptable approximation.

gets a positive correlation although now the relation is less clear-cut and the regression coefficients are no longer statistically significant.

The correlation between the share of outright owners and the share of housing wealth in Figure B1a can be used to get an idea about reasonable values for the calibration for Europe. In particular, one can use the formulas (25) or (26) (or the corresponding expression for $\frac{\beta_h}{\beta}$) to investigate which value of $\frac{\kappa_H^{oo}}{\kappa_N^{oo}}$ gives a relation that is in line with the empirical evidence. In particular, I start with the expression in (26) that $\frac{\beta_H}{\beta_K} = \frac{\gamma\varepsilon}{1-\gamma\varepsilon\frac{1-\kappa_N^{oo}}{1-\kappa_H^{oo}}}\frac{\mu}{\alpha}\frac{r_k+\delta_k}{r_h+\delta_h-g_h}\frac{1-\kappa_N^{oo}}{1-\kappa_H^{oo}}$. I now assume that the share of possessed houses of outright owners is z times their population share, i.e. $\kappa_H^{oo} = z\kappa_N^{oo}$. Assuming $r_h = r_k$ and defining $\breve{\alpha} = \frac{\varepsilon\gamma\mu(\delta_k+r_k)}{\alpha(\delta_h-g_h+r_k)}$ the linearization of $\frac{\beta_H}{\beta_K}$ and $\frac{\beta_H}{\beta}$ around $\kappa_N^{oo} = 0$ gives:

$$
\begin{aligned}
\frac{\beta_H}{\beta_K} &\approx \frac{\breve{\alpha}}{(1-\varepsilon\gamma)^2}\left(1-\varepsilon\gamma+\kappa_N^{oo}(z-1)\right), \\
\frac{\beta_H}{\beta} &\approx \frac{\breve{\alpha}}{(1+\breve{\alpha}-\varepsilon\gamma)^2}\left(1+\breve{\alpha}-\varepsilon\gamma+\kappa_N^{oo}(z-1)\right).
\end{aligned}
$$

These equations have been referred to as $\frac{\beta_H}{\beta_K} \approx G_1+G_2(z-1)\kappa_N^{oo}$ and $\frac{\beta_H}{\beta} \approx G_3+G_4(z-1)\kappa_N^{oo}$ in section 4.4 of the paper. The slope of the regression line in a scatter-plot of $\frac{\beta_H}{\beta}$ vs. κ_N^{oo} (as in Figure B1a) thus corresponds to $\frac{\breve{\alpha}(z-1)}{(1+\breve{\alpha}-\varepsilon\gamma)^2}$. Using the benchmark calibration for 2017 (including $r_k = 7\%$) one can ask which value for z leads to a slope that is equal to 0.57 (as is the case in Figure B1a). This implies a large value of $z = 3.1$. The linearization around $\kappa_N^{oo} = 0$ is, however, an extreme assumption. It is more reasonable to use a linearization around the average value of κ_N^{oo} in the sample which is approximately 35%. For this linearization it turns out that a value of $z = 1.57$ leads to a slope of 0.57. This is the rationale behind the chosen calibration of $\frac{\kappa_H^{oo}}{\kappa_N^{oo}} = 1.6$ for Europe in 2017 which has been used in section 5 of the paper. A thorough investigation of these empirical issues is a topic for future research.

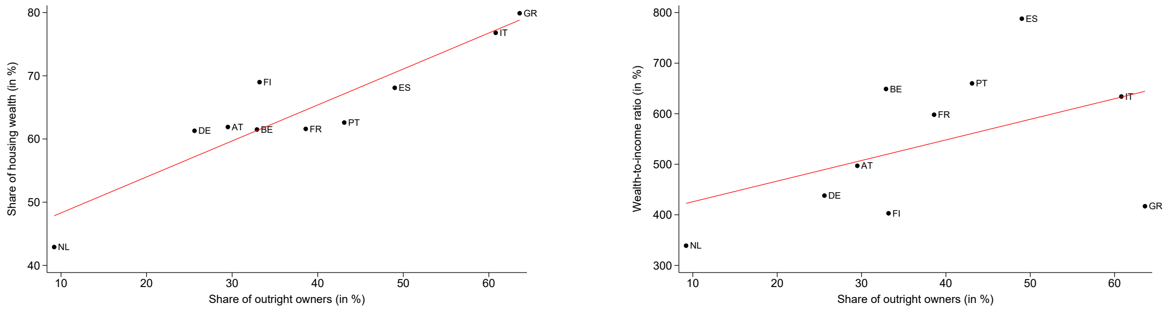
C Additional specifications

Table C1 is an extension of Table 4 in the paper and it shows the results of many specifications with one parameter change or various combinations of parameter changes. The results can be compared to the benchmark case where it is assumed that only g, n, and β_D^N change and to the empirical pattern as presented in Table 1. Rows 2 to 8 of Table C1 consider an increase in the markup from 10% to 20% (with an accompanying reduction in

Table C1: Reaction of aggregate wealth and housing wealth to parameter changes

	Model	$\boldsymbol{\beta}$ (in %) 1980 ($r_k \approx 10\%$)	2017 ($r_k = 7\%$)	2017 ($r_k = 6\%$)	$\frac{\beta_H}{\beta}$ (in %) 1980 ($r_k \approx 10\%$)	2017 ($r_k = 7\%$)	2017 ($r_k = 6\%$)
1	**Benchmark**	**340**	**483**	**554**	**47**	**48**	**50**
	Change in one parameter						
2	μ	340	461	529	47	50	52
3	δ_k	340	463	529	47	51	54
4	δ_m	340	498	578	47	50	53
5	κ_N^j	340	489	562	47	48	51
6	κ_H^{oo}	340	502	579	47	50	53
7	χ	340	494	570	47	49	51
8	ξ_m	340	498	577	47	49	52
	Change in two parameters						
9	δ_m, κ_H^{oo}	340	515	600	47	52	55
10	μ, κ_H^{oo}	340	480	553	47	52	55
11	μ, κ_H^{oo} ($\zeta_\Pi = 1/4$)	340	574	676	47	43	44
	Change in many parameters						
12	$\zeta_\Pi = 0$	340	489	572	47	59	63
13	$\zeta_\Pi = 1/4$	340	561	671	47	52	54

Note: The table reports the results of the wealth-to-income ratio β and the housing wealth share $\frac{\beta_H}{\beta}$ for various specifications of the model. The benchmark parameter values for 1980 are $\varphi_L = 66\%$ $\mu = 1.1$, $\alpha = 0.274$, $\gamma = 0.17$, $\delta_k = 0.05$, $\delta_h = \delta_m = 0.025$, $\xi_h = 0$, $\xi_m = 0.02$, $\xi_d = 0.05$ and $\chi = 0.5$. The housing market is characterized by $\kappa_N^r = 0.45$, $\kappa_N^{om} = 0.26$ and $\kappa_H^{oo} = \kappa_N^{oo} = 0.29$. In addition $g = 3.11\%$, $n = 0.79\%$, $\beta_D^N = 20\%$ and I assume an interest rate that is approximately 10% (in fact $r_k = 9.84\%$). For 2017 I assume $g = 1.93\%$, $n = 0.54\%$ and $\beta_D^N = 70\%$ and I consider two alternative values for the interest rate ($r_k = 7\%$ and $r_k = 6\%$). In rows 2 to 6 there are changes in specific parameters values for the year 2017. In particular: $\mu = 1.2$ (row 2) , $\delta_k = 0.07$ (row 3), $\delta_m = 0.02$ (row 4), $\kappa_N^r = 0.38$ and $\kappa_N^{om} = 0.32$ and $\kappa_H^{oo} = \kappa_N^{oo} = 0.30$ (row 5), $\kappa_H^{oo} = 0.35$ ($\kappa_H^{oo}/\kappa_N^{oo} = 1.2$) (row 6), $\xi = 0$ (row 7) and $\xi_m = 2.5\%$ (row 8). In rows 9 to 11 it is assumed that two parameters change: $\kappa_H^{oo} = 0.35$ and $\delta_m = 0.02$ (row 9), $\mu = 1.2$ and $\kappa_H^{oo}/\kappa_N^{oo} = 1.2$ (row 10) and in row 11 it is assumed in addition that part of the present value of pure profits are also included in the valuation of businesses ($\zeta_\Pi = 0.25$). In order to have the same initial situation this requires to set $\delta_k = 9.4\%$ and $r_k = 10.2\%$ (in 1980). In rows 12 and 13 finally, it is assumed that many parameters change at the same time.



(a) Wealth-to-income ratio vs. share of outright owners

(b) Share of housing wealth vs. share of outright owners

Figure B1: The data come from the third wave of the HFCS (ECB, 2021) and the OECD's Affordable Housing Database (2020 or latest year available) (OECD, 2023).

the labor share), an increase in the rate of depreciation δ_k (from 5% to 7%), a reduction in the depreciation of housing δ_m (from 2.5% to 2%), a change in the mix of dwellers (less renters, more owners with mortgages) an increase in the stock of outrightly owned houses (from $\kappa_H^{oo} = \kappa_N^{oo}$ to $\kappa_H^{oo} = 1.2\kappa_N^{oo}$), an increase in the risk wedges ξ_m (from 2% to 2.5%) and a reduction in the growth rate of the housing stock (χ moves from 0.5 to 0).

The magnitude of the parameter changes are broadly in line with empirical regularities as explained in Appendix B. The rationale behind these changes can be briefly summarized as follows. The increase in the markup has been related to technological changes and in particular to the changes in market power (De Loecker et al. 2020). The increase in the depreciation rate of capital has been documented, e.g., by Dalgaard & Olsen (2021) and is a consequence of the increasing importance of information technologies, software products etc. that show a faster degree of obsolescence. The decrease in δ_m, on the other hand, is primarily meant to capture a higher attractiveness of housing via the tax system. The increase in the risk wedge on mortgages ξ_m can be related to changes in the financial structure and can also be found in the empirical data. In fact, one could also be inclined to assume increases in the risk wedges ξ_h and/or larger increases in the risk wedge ξ_m. This would lead to higher values of β and $\frac{\beta_H}{\beta}$. The problem with the assumption of a considerable increase in the wedge ξ_h is, however, that they are not in line with the empirical evidence. Jordà et al. (2019), e.g., have shown that the rates of returns on equity and on housing are very similar. In fact, taken the lower volatility of house prices into account Jordà et al. (2019) have argued that the risk-adjusted returns of housing are even larger than the ones of equity investment. The decline in χ could be the result of sluggish housing construction, arguably due to overly strict zoning laws or to NIMBY attitudes. The changes in the ownership structure of dweller is based on empirical data from Jordà et al. (2016) and OECD (2023) and is explained in the text.

As can be seen, many of these parameter changes have similar effects on $\frac{\beta_H}{\beta}$ where there seem to be largest for changes in δ_k, δ_m and in the relative size of outright owners ($\kappa_H^{oo}/\kappa_N^{oo}$). Assuming $\kappa_H^{oo}/\kappa_N^{oo} = 1.2$, e.g., justifies an increase in the housing wealth share to 50% already for the decrease in the interest rate by 3 pp. In this case, however, the increase in β is somewhat too low (only up to 502%). If one allows for the simultaneous change in two parameters at the same time then it is possible to get even larger effects as is illustrated in rows 9 to 11. A parallel decrease in δ_m together with the change in $\kappa_H^{oo}/\kappa_N^{oo}$ leads to a housing wealth share of 52% and a wealth-to-income ratio of 515% (for $r_k = 7\%$). In row 10, on the other hand, I look at the case that combines an increase in the average gross markup to $\mu = 1.2$ with an increase in the ratio of outrightly owned houses to $\kappa_H^{oo}/\kappa_N^{oo} = 120\%$. This case has already been studied in the text and is here only repeated for the sake of comparison. The same is true for the cases 11 to 13.

D Endogenizing the decline in the interest rate

In the paper I have studied the effect of an exogenous decline in real interest rates on important wealth aggregates. I maintained this assumption in order to be better able to focus on the main mechanisms involved in the relation. In this appendix, however, I want to briefly sketch how one can amend the model with a wealth supply schedule based on individual savings behavior. In the related literature, most paper use a framework where households have an intertemporal utility function and where their savings decisions depend on preference parameters as well as on demographic variables and the design of the pension system (if the model is set in an OLG framework as is the case in Summers & Rachel 2019, Platzer & Peruffo 2022). In the following, I will use a simpler Solow-model-type framework with constant saving rates which is sufficient to highlight the main mechanisms and connections that become apparent in a general equilibrium framework.

D.1 Setup

As a starting point I assume that households save a constant fraction s of their income Y_t (i.e. of the gross domestic product). Total savings S_t are thus given by:

$$S_t = sY_t = s\left(Y_{Nt} + P^r_{st}\overline{H}^r_t + P^o_{st}\overline{H}^o_t\right), \tag{49}$$

where I use the definition for GDP from equation (13).

Total wealth demand, on the other hand, was written in equation (16) as:

$$W^d_t = W_{Bt} + W_{Ht} + W_{Dt} = B_t + P^r_{ht}\overline{H}^r_t + P^o_{ht}\overline{H}^o_t + \mathcal{D}_t,$$

with B_t defined in equation (15). In the following, I set $\zeta_\Pi = 0$ such that $B_t = K_t$. Wealth demand thus changes over time according to:

$$\begin{aligned}\dot{W}^d_t = \frac{dW^d_t}{dt} &= \frac{dK_t}{dt} + \frac{d\left(P^r_{st}\overline{H}^r_t\right)}{dt} + \frac{d\left(P^o_{st}\overline{H}^o_t\right)}{dt} + \frac{d\mathcal{D}_t}{dt} \\ &= (g+n)\left(K_t + P^r_{st}\overline{H}^r_t + P^o_{st}\overline{H}^o_t + \mathcal{D}_t\right),\end{aligned} \tag{50}$$

where I use the fact that in the steady state all asset values grow at the rate $(g+n)$, i.e. $\frac{\dot{K}_t}{K_t} = g+n$ etc. Total savings has to be equal to these requirements for new financing plus expenditures that are necessary to replace the ongoing depreciations. The accumulation equation thus reads as:

$$\begin{aligned}\dot{W}^d_t &= S_t - \delta_k K_t - \delta_h P^r_{ht}\overline{H}^r_t - \delta_m P^o_{ht}\overline{H}^o_t \\ &= sY_t - \delta_k K_t - \delta_h P^r_{ht}\overline{H}^r_t - \delta_m P^o_{ht}\overline{H}^o_t \\ &= s\left(Y_{Nt} + P^r_{st}\overline{H}^r_t + P^o_{st}\overline{H}^o_t\right) - \delta_k K_t - \delta_h P^r_{ht}\overline{H}^r_t - \delta_m P^o_{ht}\overline{H}^o_t.\end{aligned} \tag{51}$$

D.2 Results

One can substitute $\dot{W}^d_t$ from (50) into equation (51) and note that (from (12)) $P^r_{st} = P^r_{ht}(r_h + \delta_h - g_h)$ and $P^o_{st} = P^o_{ht}(r_m + \delta_m - g_h)$ (where I already assume steady state interest rates) to arrive at:

$$\begin{aligned}(g+n)\left(K_t + P^r_{st}\overline{H}^r_t + P^o_{st}\overline{H}^o_t + \mathcal{D}_t\right) = \\ s\left(Y_{Nt} + P^r_{ht}\overline{H}^r_t(r_h+\delta_h-g_h) + P^o_{ht}\overline{H}^o_t(r_m+\delta_m-g_h)\right) \\ -\delta_k K_t - \delta_h P^r_{ht}\overline{H}^r_t - \delta_m P^o_{ht}\overline{H}^o_t.\end{aligned}$$

Or:

$$\begin{aligned}K_t(g+n+\delta_k) + P^r_{st}\overline{H}^r_t(g+n+\delta_h) + P^o_{st}\overline{H}^o_t(g+n+\delta_m) + \mathcal{D}_t(g+n) = \\ s\left(Y_{Nt} + P^r_{ht}\overline{H}^r_t(r_h+\delta_h-g_h) + P^o_{ht}\overline{H}^o_t(r_m+\delta_m-g_h)\right).\end{aligned}$$

Note that $\frac{\beta_H}{\beta_K} = \frac{P^r_{ht}\overline{H}^r_t + P^o_{ht}\overline{H}^o_t}{K_t}$ and $\frac{\beta_{Hr}}{\beta_{Ho}} = \frac{P^r_{ht}\overline{H}^r_t}{P^o_{ht}\overline{H}^o_t}$ which implies $\frac{P^o_{ht}\overline{H}^o_t}{K_t} = \frac{\beta_H}{\beta_K}\frac{1}{1+\frac{\beta_{Hr}}{\beta_{Ho}}}$. Dividing both sides of the above equation by K_t (and noting that $\beta^N_K = \frac{K_t}{Y_{Nt}}$ and $\beta^N_D = \frac{\mathcal{D}_t}{Y_{Nt}}$) leads to an expressions for s:

$$s = \frac{(g+n+\delta_k) + \frac{\beta_H}{\beta_K}\left(g+n+\delta_h + \frac{1}{1+\frac{\beta_{Hr}}{\beta_{Ho}}}(\delta_m-\delta_h)\right) + \frac{\beta^N_D}{\beta^N_K}(g+n)}{\frac{1}{\beta^N_K} + \frac{\beta_H}{\beta_K}\left(r_h+\delta_h-g_h + \frac{1}{1+\frac{\beta_{Hr}}{\beta_{Ho}}}(r_m+\delta_m-r_h-\delta_h)\right)}. \tag{52}$$

Equation (52) gives the savings rate s that is associated with the choice of an interest rate r_k. This is the case since all parameters and expressions on the right-hand side of equation (52) are functions of the structural (or chosen) parameters. In particular, the following three expressions hold where the first follows from equation (4), the second from equation (25) and the third from $\beta^N_{Ho} = \frac{\kappa^{om}_N\widetilde{\phi}}{r_m+\delta_m-g_h}\left(1+\frac{\kappa^{oo}_H}{\kappa^{om}_H}\right)$ (see section A.5) and $\beta^N_{Hr} = \frac{\kappa^r_N\widetilde{\phi}}{r_h+\delta_h-g_h}$:

$$\begin{aligned}\beta^N_K &= \frac{\alpha/\mu}{r_k+\delta_k}, \\ \frac{\beta_H}{\beta_K} &= \frac{\widetilde{\phi}\mu}{\alpha}\left[\kappa^r_N\frac{r_k+\delta_k}{r_h+\delta_h-g_h} + \left(\frac{1-\kappa^{oo}_N}{1-\kappa^{oo}_H} - \kappa^r_N\right)\frac{r_k+\delta_k}{r_m+\delta_m-g_h}\right], \\ \frac{\beta_{Hr}}{\beta_{Ho}} &= \frac{\kappa^r_N}{\kappa^{om}_N}\frac{r_m+\delta_m-g_h}{r_h+\delta_h-g_h}\frac{1}{1+\frac{\kappa^{oo}_H}{\kappa^{om}_H}}.\end{aligned}$$

Finally, one can use $r_h = r_k - \xi_h$, $r_m = r_k - \xi_m$, $g_h = g + n(1-\chi)$ and $\widetilde{\phi} \equiv \frac{\gamma\varepsilon}{1-\gamma\varepsilon\left(1-\kappa^{oo}_N+\kappa^{om}_N\frac{\kappa^{oo}_H}{\kappa^{om}_H}\right)}$ to confirm the claim that equation (52) is a relation between the savings rate s and the interest rate r_k while all other parameters (g, n, β^N_D, α, μ, δ_k, δ_h, δ_m, ξ_h, ξ_m, κ^j_N, κ^j_H)

are calibrated. There are two different ways to look at equation (52). On the one hand, one can assume an exogenous savings rate s and derive the equilibrium interest rate r_k as the endogenous outcome. This is the way the Solow model is normally derived in the textbook expositions. Alternatively, however, one can also fix a target interest rate r_k and investigate which savings rate s would be necessary to "implement" this interest rate or which change in s would be implied by a certain interest rate path (e.g. a decrease from 10% to 7%).

D.3 Special cases

In the following, I show the results for two special case. First, the standard cases without housing and second the case with homogeneous housing (i.e. $r_m = r_h$, $\delta_m = \delta_h$, $\kappa_H^j = \kappa_N^j$ for $j \in \{r, om, oo\}$).

D.3.1 No housing

Without housing it holds that $\beta^H = 0$ and thus equation (52) reduces to:

$$s = \beta_K^N (g + n + \delta_k)$$

where for simplicity I also set $\beta_D^N = 0$. This equation can be solved for the equilibrium wealth-to-income ratio for an exogenously given savings rate s, i.e. $\beta_K^N = \beta_K = \frac{s}{g+n+\delta_k}$. This ratio is thus independent of the interest rate and is just a function of g, n, δ_k and s. The equilibrium interest can then be derived from $\beta_K^N = \frac{\alpha/\mu}{r_k+\delta_k}$ which comes out as $r_k = \frac{\alpha}{\mu}\frac{g+n+\delta_k}{s} - \delta_k$. This is the expression that can be found in standard textbooks and it is, e.g., also used by Mankiw (2022) in order to explain the shift in the equilibrium interest rate since the 1980ies.[28]

As mentioned above, the alternative perspective treats the interest rate r_k as given and calculates the savings rate that is compatible with this chosen interest rate. Using $s = \beta_K^N(g + n + \delta_k)$ and $\beta_K^N = \frac{\alpha/\mu}{r_k+\delta_k}$ this comes out as $s = \frac{\alpha}{\mu}\frac{g+n+\delta_k}{r_k+\delta_k}$. Using (as before) $\alpha = 0.274$, $\mu = 1.1$, $\delta_k = 5\%$, $g = 3.11\%$, $n = 0.79\%$ and $r_k = 10\%$ leads to a savings rate

[28] In fact, Mankiw (2022) attempts to explain the decline in the global interest rate from 1975 to 2020 by using a basic calibration with $\alpha = 1/3$ and $\delta_k = 5\%$ together with $n + g = 4.1\%$ and $s = 22.2\%$ (for the initial half of the period) and $n + g = 2.8\%$ and $s = 25.1\%$ (for the second half). The equilibrium interest rate comes out as $r_k = 8.7\%$ (for the first half) and $r_k = 5.4\%$ (for the second half), a decrease by 3.3 pp. Assuming in addition (as is done in a later part in Mankiw 2022) $\mu = 1.1$ in the first and $\mu = 1.3$ in the second half implies interest rates of $r_k = 7.4\%$ and $r_k = 3\%$, respectively.

of $s = 14.8\%$ and a capital-to-income ratio of $\beta_K = 166\%$. Both of these values seem too low.

As a third perspective one could thus choose a target capital-to-income ratio and let r_k and s be determined endogenously. In particular, choosing the same target rate as in section 3.4, i.e. $\beta_K^N = 215\%$ (which corresponds to the 1980 average for the G8a countries) leads to an implied savings rate of 19.1% and an interest rate (as stated above) of $r_k = 6.6\%$. For $\beta_D^N = 20\%$ the savings rate is slightly higher at $s = 19.9\%$. The slower growth rates for 2017 ($g = 1.93\%$, $n = 0.54\%$) imply a slight increase in savings to $s = 19.8\%$ (for $\beta_D^N = 0\%$) if $\beta_K^N = 215\%$ (as in the data) or $s = 21.5\%$ (for $\beta_D^N = 70\%$). This increase in the savings rate is not completely in line with the data as reported in Table D1. For the group of high income countries the savings rate is reported to have decreased from 24.6% (1980) to 23% (2017) and even more markable for the US (from 22.7% to 17.4%). Only for the EU4 countries one could observe a tiny increase from 21.0% to 21.9%.

Table D1:
Savings rate for various regions (in %)

	High Income	**US**	**Europe (EU4)**
$\overline{1980}$	24.6	22.7	21.0
$\overline{2017}$	23.0	17.4	21.9

Note: The data for the gross savings rate s come from the World Bank. $\overline{1980}$ and $\overline{2017}$ correspond to the average values for the time spans 1976-1984 and 2011-2017, respectively.

D.3.2 Homogeneous housing

As a second case it is interesting to look at a situation with homogenous housing, i.e. $r_m = r_h = r_k$, $\delta_m = \delta_h$, $\kappa_H^j = \kappa_N^j$ for $j \in \{r, om, oo\}$). Now (52) reduces to:

$$s = \frac{\alpha}{\mu}(1 - \gamma\varepsilon)\frac{g + n + \delta_k}{r_k + \delta_k} + \gamma\varepsilon\frac{g + n + \delta_h}{r_k + \delta_h - g_h}.$$

This defines implicitly the equilibrium interest rate for a given exogenous savings rate s. Alternatively one can again view it as the savings rate that is compatible with a chosen interest rate r_k.

For the benchmark calibration, e.g., $g = 3.11\%$, $n = 0.79\%$, $\delta_k = 5\%$, $\delta_h = 2.5\%$, $\alpha = 0.274$, $\mu = 1.1$, $\chi = 0.5$, $\varepsilon = 0.6$, $\gamma = 0.17$ and $r_k = 10\%$ one gets that $s = 23.6\%$. A decrease in growth to $g = 1.93\%$ and $n = 0.54\%$ leads to less wealth demand. So if one were to assume that r_k stays constant this would imply a considerable decrease in the savings rate to $s = 18.2\%$ such as to lower wealth supply. If, on the other hand, one would assume that the growth rates stay the same but the interest rate decreases to $r_k = 7\%$ then this requires a massive increase wealth supply and thus and increase in the savings rate to $s = 30.2\%$. If both events (the decrease in the interest rate and the slowdown in growth) happen, however, at the same time, then the two effects can cancel each other. For the chosen parameter values, e.g., one gets that the equilibrium savings rates shows only a tiny move from 23.6% to 23.1%.

D.4 Numerical results

In a final step I calculate the savings rate that follow from equation (52) for three cases that have been shown in Table 4. In particular, I look at the basic case where only g, n and β_D^N are assumed to change, at the case where in addition also μ and $\kappa_H^{oo}/\kappa_N^{oo}$ are assumed to change and finally at the case very many parameters are assumed to change. As shown in Table D2 the different cases are associated with increases in the savings rate from 22.7% to values between 23.4% to 24.9%. Comparing these values to the observed values reported in Table D1 one can conclude that that average magnitude seem to be quite accurate. The models fail, however, to capture the observed decrease in the savings rate for the average of high income countries. In fact this is a property that seems to be valid for the majority of papers in the related literature. Although the papers typically do not report the associated pattern of savings rate it seems to be the case that the decline is mainly driven by an increase in savings—either due to population ageing or due to increasing inequality (with the assumption of higher propensities to save for higher income groups). The model with owner occupiers (and with outright owners in particular) has the potential to resolve this discrepancy. One would, e.g., naturally assume that the propensity to save from imputed rents might be different from the propensity to save from other sources of income. This is also related to the recent discussion about "capital gains saving" (Fagereng et al. 2019). I leave a thorough investigation of this issue for future research.

Table D2:
Savings rates (in %) associated with the outcomes in Table 4

Model	World 1980	World 2017
Benchmark	22.7	23.9
Changes in μ and $\kappa_H^{oo}/\kappa_N^{oo}$	22.7	23.4
Changes in many parameters	22.7	24.9

Note: The table shows the savings rate that are implied by the assumptions of rows 1, 2 and 4 of Table 4. The columns in the year 1980 are associated with an interest rate of $r_k = 10\%$, the ones in the year 2018 with a rate of $r_k = 7\%$.