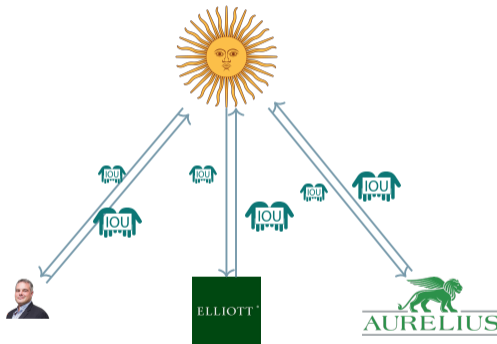
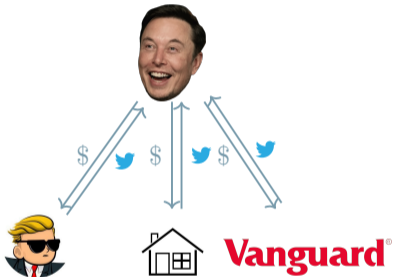

A Theory of Holdouts

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Exchange Offers and Holdout Problems



The Puzzle

The holdout problem is surprising as it has an "easy" solution:

Contingent proposal requiring unanimity makes all agents pivotal (Segal 99)

Almost never used in practice

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Why? Limited commitment!

Provides a unified framework for holdout problems

Two types of players:

Agents endowed with outstanding securities

Principal, the residual claimant, offers new securities for old

Two frictions:

Collective action problem among agents

Limited commitment (L.C.) of the principal

Full Commitment Benchmarks:

B1: Same new securities used in equilibrium independent of existing securities

B2: No role for policy intervention: Efficient outcome attained

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Limited Commitment (L.C.) Results:

R1: Different new securities, depending on initial securities's payoff sensitivity

Key: Payoff sensitivity determines credibility of punishment

R2: Role of policy intervention: Increasing commitment partially can backfire

Key: Commitment also helps in renegotiation

Holdout	Full Commitment	Limited Commitment
Specific Security	Classic Papers e.g., Grossman–Hart 80 (Cash)	No Optimal Contracting Pitchford–Wright 12 (Cash)
General Securities	No Holdout Problems e.g., Segal 99	My Paper

Model Setup

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Players: N agents (A_i) and a principal (P)

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3. Given $\mathbf{h} = (h_1, \dots, h_N)$, P chooses to honor at cost c or renegotiate
 - If honored, asset value $v(\mathbf{h})$ realized; Everyone paid according to securities
 - Else, repeat if P not committed

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NB: Static when $\mathbf{R} = (R_1, \dots, R_N)$ renego.-proof

What do we mean by “Contracts”

Payoffs: Specific Securities

Suppose no new securities and all holdouts get $w \leq v$ collectively

Equity $\alpha = (\alpha_1, \dots, \alpha_N)$: A_i gets paid $\alpha_i w$

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w/o seniority : A_i gets paid $\min \left\{ D_i, \frac{(1-h_i) D_i}{(1-h) \cdot D} w \right\}$

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But how to model general contracts that can be arbitrary?

Payoffs: General Securities

Securities are *vector functions* mapping asset value & agents' securities to payoffs

$$\mathbf{R}(v, \mathbf{h}) \mapsto \mathbb{R}^N \quad \text{New securities}$$

$$\mathbf{R}^O(v, \mathbf{h}|\mathbf{R}) \mapsto \mathbb{R}^N \quad \text{Original securities}$$

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A_i 's payoff:

$$u_i := h_i R_i^O + (1 - h_i) R_i$$

P's gross payoff:

$$J(\mathbf{h}|\mathbf{R}) := v(\mathbf{h}) - \left[\mathbf{h} \cdot \mathbf{R}^O + (\mathbf{1} - \mathbf{h}) \cdot \mathbf{R} \right]$$

Model: Weak Consistency

Weak consistency (cf. Aumann–Maschler 85, Moulin 00)

$$R_i^O(v, \mathbf{h} | \mathbf{R}) = R_i^O\left(v - \underbrace{(1-h) \cdot \mathbf{R}}_{=: x \text{ ("dilution")}}, \mathbf{h}\right)$$

↓
Eqm. asset value $v(h)$

↑
Holdout profile

P cannot selectively dilute \implies cannot punish holdouts without punishing herself

Model: Payoff Sensitivity

How payoff $R_i^O(w, \mathbf{h})$ varies with $w := v - (\mathbf{1} - \mathbf{h}) \cdot \mathbf{R}$ measured by left derivative

Equity: A_i has an equity stake $\alpha_i \in (0, 1)$, then

$$R_i^O(w, \mathbf{h}) = \alpha_i w \quad \implies \quad \frac{\partial R_i^O(w, \mathbf{h})}{\partial w} = \alpha_i < 1$$

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Principal: The residual claimant

$$J(\mathbf{h}|\mathbf{R}) = w - \mathbf{h} \cdot \mathbf{R}^O \quad \implies \quad \frac{\partial J(\mathbf{h}|\mathbf{R})}{\partial w} = 1 - \sum_{i=1}^N \frac{\partial R_i^O(w, \mathbf{h})}{\partial w} h_i$$

Assumptions

A1 (Inefficient Holdouts): Weakly lower value when more agents hold out

$v(h)$ is weakly decreasing in h

A2 (Payoff Regularity): Existing securities have “reasonable” payoffs

$w \mapsto h \cdot R^O(w, h)$ is increasing and 1-Lipschitz $\forall h$

A3 (Moderate Cost): Cost neither too large nor too small

$v(\mathbf{0}) > c > v(\mathbf{0}) - \sum_{i=1}^N R_i^O(v(e_i), e_i)$ where $h = e_i$ is profile when only A_i holds out

Solution Concepts

Principal's Problem

P chooses R to maximize value $J(\mathbf{0})$ at $h = \mathbf{0}$

$$\max_R \underbrace{v(\mathbf{0}) - \sum_{i=1}^N R_i(v(\mathbf{0}), \mathbf{0})}_{J(\mathbf{0}|R)}$$

such that

A_i incentive compatible to accept at $\mathbf{0}$ (IC)

P unwilling to renegotiate upon deviation (only with L.C.) (RP)

Incentive Compatibility for Agents at $\mathbf{0}$

R is incentive compatible at $\mathbf{0}$ ($R \in \mathcal{I}(\mathbf{0})$) if

$$R_i(v(\mathbf{0}), \mathbf{0}) \geq R_i^O \left(v(e_i) - \sum_{j \neq i} R_j(v(e_i), e_i), e_i \right) \quad (\text{IC})$$

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NB: Only if no renegotiation on path (similar for off-path h)

What are feasible actions in renegotiation if agents deviate?

Credibility for Principal w. Limited Commitment

Exchange offer R is **credible** at h if (cf. Pearce 87, Farrel–Maskin 89, Ray 94)

R is IC at h for all agents

At deviation profile \hat{h} , P unwilling to renegotiate to any offer \tilde{R} **credible** at \hat{h}

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$$\mathcal{C}(\mathbf{h}) = \left\{ \mathbf{R} \in \mathcal{I}(\mathbf{h}) : J(\hat{\mathbf{h}}|\mathbf{R}) \geq \delta J(\hat{\mathbf{h}}|\tilde{\mathbf{R}}) \quad \forall \tilde{\mathbf{R}} \in \mathcal{C}(\hat{\mathbf{h}}) \quad \forall \hat{\mathbf{h}} : \|\hat{\mathbf{h}} - \mathbf{h}\| = 1 \right\}$$

Thm1: $\mathcal{C}(\cdot)$ exists and is unique for any $\delta \in [0, 1]$ Existence

Setup Summary

A_i 's payoff depends on credible punishment when he holds out

Credibility of punishment depends on credible offers in renegotiation

Weak consistency disciplines feasible punishment on P vis-à-vis A_i

P's payoff sensitivity to punishment characterizes credible punishment

Analysis Framework

Efficiency (First Best)

Efficiency achieved if everyone tenders $h = 0$

Follows from A1 : $v(h)$ decreasing in h

How Different Elements Add Up

Coordinated Agents: FB achieved by Coase Thm. (No holdout problems)

↓ + collective action problem

Dispersed Agents: FB not achieved with cash (Classic holdout problems)

↓ + flexible contractual space

Benchmarks

↓ + limited commitment

Main Results

Benchmarks: Full Commitment

Full Commitment: Holdout Problems w. Cash

Result: There is no R non-contingent that implements $h = \mathbf{0}$ (only result requiring A3)

Full Commitment: Holdout Problems w. Cash

Result: There is no R non-contingent that implements $h = \mathbf{0}$ (only result requiring A3)

Intuition: A_i benefits from the deal when others participate

Impact on deal not fully internalized and costly for P to compensate

Incentive to free-ride impedes value enhancement

Essential force underlines Grossman–Hart, Bulow–Rogoff, etc

Full Commitment: One Solution to All

B1: No heterogeneity in the exchange offers

Proof with $v(\mathbf{1})$ normalized to 0:

P implements $\mathbf{h} = \mathbf{0}$ by offering small $R_i > 0$ only if all agents agree

$$u_i = \begin{cases} 0 & \text{if } h_i = 1 \\ R_i > 0 & \text{if } h_j = 0 \forall j \end{cases} \implies h_i = 0 \text{ weakly dominates } h_i = 1$$

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B2: Efficiency achieved: No role for policy intervention

Limited Commitment Results

**R0: Lack of Commitment
Undermines Restructuring**

R0: Unanimity Fails with Limited Commitment

Result: Unanimity doesn't implement $h = \mathbf{0}$ when P has L.C.

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Unanimity gives P nothing when agents deviate

P not willing to execute threat ex post, carrying out the deal

Anticipating this, everyone holds out

No value enhancement to start with

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NB: Seeing off-eqm non-credible offers, per subgame perfection,

A_i correctly "believes" P will offer credible ones when he deviates

Takeaways

T0: Holdout problems appear to be coordination failures (Sturzenegger–Zettelmeyer 07)

...but are essentially commitment problems

**R1: Optimal Contracts Depends on
Holdout's Payoff Sensitivity**

R1: Optimal Contracts \Leftarrow Holdout's Payoff Sensitivity

No contracts do better than cash when punishment hurts P and renegotiation costless

Payoff sensitivity serves as sufficient stat for arbitrary initial securities

R1: Optimal Contracts \iff Holdout's Payoff Sensitivity

No contracts do better than cash when punishment hurts P and renegotiation costless

Payoff sensitivity serves as sufficient stat for arbitrary initial securities

Dilution credible for debt holdout \implies Senior debt effective

Dilution not credible for equity holdout \implies Cash optimal

Debt restructuring: Senior debt offering credible

Senior debt dilutes the claim of the holdout in default by

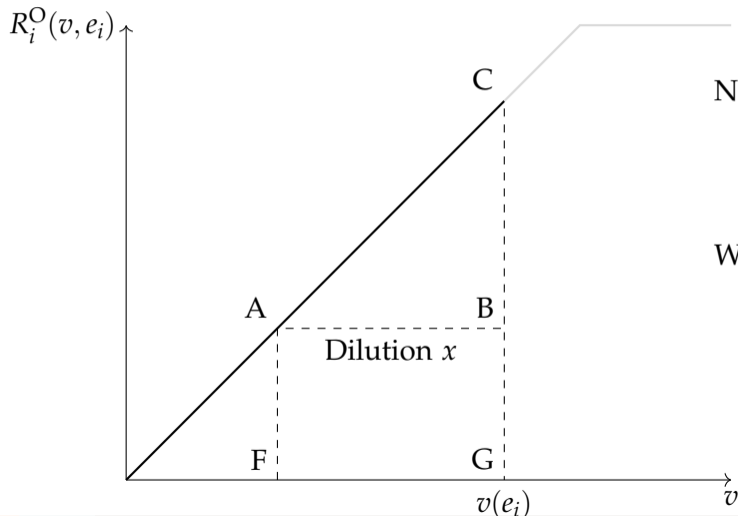
$$\frac{\partial R_i^O(\omega, \mathbf{h})}{\partial \omega} = 1$$

And that of the principal by

$$\frac{\partial J(\mathbf{h}|\mathbf{R})}{\partial \omega} = 1 - \frac{\partial R_i^O(\omega, \mathbf{h})}{\partial \omega} = 0$$

Diluting the holdout does not change the P's payoff \Rightarrow (RP) met

Graphic Representation: Credible dilution w. Debt



No Dilution:

P gets nothing

A_i gets CG

With Dilution x :

P gets nothing

A_i gets $AF = CG - x$

R1 Proof: Offering Priority Not Credible in Takeovers

Takeovers: Offering priority not credible

Priority dilutes the equity stake of the holdout by

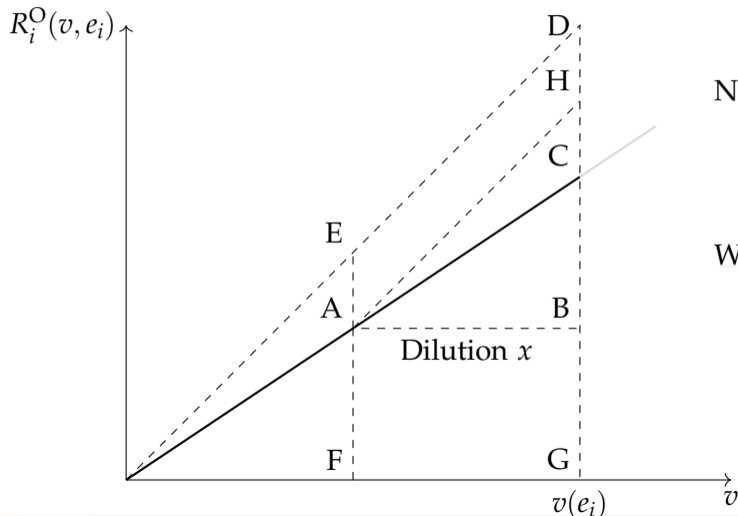
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And that of the principal by

$$\frac{\partial J(\mathbf{h}|\mathbf{R})}{\partial w} = 1 - \frac{\partial R_i^O(w, \mathbf{h})}{\partial w} = 1 - \alpha_i > 0$$

Diluting the holdout means diluting the principal \Rightarrow (RP) violated

Graphic Representation: Non-credible dilution w. Equity



No Dilution:

P gets CD

A_i gets CG

With Dilution x :

P gets $EA = DH < CD$

A_i gets $AF > CG - x$

T1: Securities with higher priority are attractive to dilute
... and thus more vulnerable to dilution

Debt contracts are

most sensitive in distress so that credible dilution facilitates restructuring

least sensitive in normal times so that no excessive dilution

**R2: Higher Commitment Could
Backfire**

A contract R is a $(2^N + 1)$ dimensional object! Hard to characterize!

P 's continuation payoff at h only depends eqm. punishment $x(h)$

A contract \mathbf{R} is a $(2^N + 1)$ dimensional object! Hard to characterize!

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Fully characterized by dynamics of

min punishment $\underline{x}(\mathbf{h})$ so that (IC) met

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Commitment δ only affect P through credibility constraint (i.e., through $x(\mathbf{h})$)

Limited Commitment: Equity Example

With equity, $\bar{x}(\mathbf{h}) = \underline{x}(\mathbf{h})$ (Recall R1)

Max punishment \bar{x} satisfies recursion with initial condition $\bar{x}(\mathbf{1}) = 0$

$$\bar{x}(\mathbf{h}) = (1 - \delta)v(\mathbf{h}) + \delta \sum_{i \in \xi(\mathbf{h})} \alpha_i (v(\mathbf{h} + e_i) - \bar{x}(\mathbf{h} + e_i))$$

Punishment = Loss due to discounting + Discounted payoff to tendering shares

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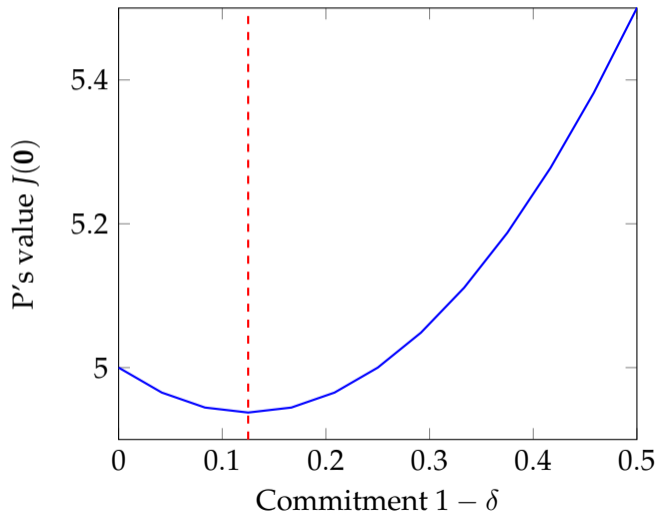
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Note: \bar{x} has an oscillating structure

At \mathbf{h} if P can impose higher punishment upon deviation $\mathbf{h} + e_i$

\implies P more willing to renegotiate at $\mathbf{h} \implies$ Lower credible punishment at \mathbf{h}

R2: Higher Commitment Might Backfire: 3-agent case



Consider path A_i, A_j deviate sequentially

(+) Higher commitment makes punishment to A_i at e_i more credible

Lower on-path payment to $A_i \implies$ Higher value to P

(-) Higher commitment also makes punishment to A_j at $e_i + e_j$ more credible

Lower payment to A_j at $e_i \implies$ Less credible punishment to A_i

\implies Higher on path payment to $A_i \implies$ Lower value to P

Second (-) effect dominates when commitment low as renegotiation more likely

T2: Ability to punish holdouts tomorrow

...limits ability to punish holdouts today

Conclusion

Holdout problems are essentially commitment problems

Holdout problems are essentially commitment problems

Credible punishment depends on holdout's payoff sensitivity

Holdout problems are essentially commitment problems

Credible punishment depends on holdout's payoff sensitivity

Commitment to punishing holdouts could backfire via renegotiation