A Collusion-Proof Efficient Dynamic Mechanism

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Motivation

Efficiency and **Dominant Strategy** Incentive Compatibility in **dynamic settings**:

> *In general, it is impossible to implement the efficient allocation rule in dominant strategies in dynamic setting*
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(Perfect) Bayesian Nash equilibria:

- **Multiple equilibria**
- **Dynamic coordination/collusion undermines efficiency**
- We show that in most celebrated dynamic mechanisms **efficient eq might not survive Iterated Elimination of Weakly Dominated Strategies (IEWDS)...**

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We also construct
a modified mechanism that approximately achieves the same property in environments without transfers.

Related Literature

Collusion-proof **static** mechanisms: **Che and Kim (2006)**, Laffont and Martimort (1997, 2000), Cremer and Riordan (1985), **Safronov (2017)**

Efficient dynamic mechanisms: Skrzypacz and Toikka (2015), **Bergemann and Välimäki (2010)**, **Athey and Segal (2013)**

Optimal dynamic mechanisms: Pavan, Segal and Toikka (2014), Bergemann and Välimäki (2019)

Collusion with persistent private info: Athey and Bagwell (2001, 2008), Miller (2012)

Repeated implementation: **Jackson and Sonnenschein (2007)**, **Ball et al. (2022)**, Lee and Sabourian (2009, 2013), Renou and Mezzetti (2017), Renou and Tomala (2015)

Dynamic Pivot (BV10), BTM (AS13), and Guaranteed Utility Mechanism (GUM)

	BV10 DP	AS13 BTM	GUM
Incentive Compatibility			
efficient PBE	YES	YES	
efficient PBE survive IEWDS	NO	NO	
all PBEs/BNEs are efficient	NO	NO	

- Dynamic Pivot extends AGV
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Collusion			
collusion-proof	NO	NO	YES
Properties, Robustness, Extensions			
balanced budget	NO	YES	YES
IR (exiting and re-entering)	YES	NO	YES

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A (pure) strategy $s^i = \{s_t^i\}_{t=1}^T$:

$$s_t^i : (\boldsymbol{\Theta^i})^t \times \mathcal{H}_{t-1} \to \boldsymbol{\Theta^i}.$$

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2. **transfer rule**

$$y_t^i = y_t^i(h_t)$$

   as a function of public history.

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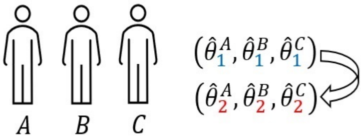
[Transfers]
Externality payments are **bilateral**:

- every agent j receives the transfer from i,
  which compensates j by the externality imposed on j by i.

GUM: Example



A B C
$(\hat{\theta}_1^A, \hat{\theta}_1^B, \hat{\theta}_1^C)$
$(\hat{\theta}_2^A, \hat{\theta}_2^B, \hat{\theta}_2^C)$

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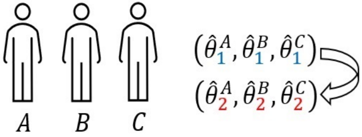

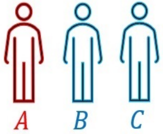

"effect" of $\hat{\theta}_2^A$:

$(\hat{\theta}_1^A, \hat{\theta}_1^B, \hat{\theta}_1^C)$

versus

$(\hat{\theta}_2^A, \hat{\theta}_1^B, \hat{\theta}_1^C)$

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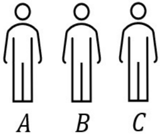

$(\hat\theta_1^A, \hat\theta_1^B, \hat\theta_1^C)$

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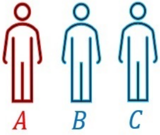

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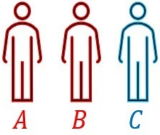

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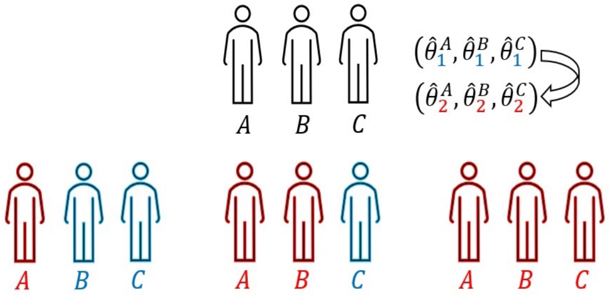

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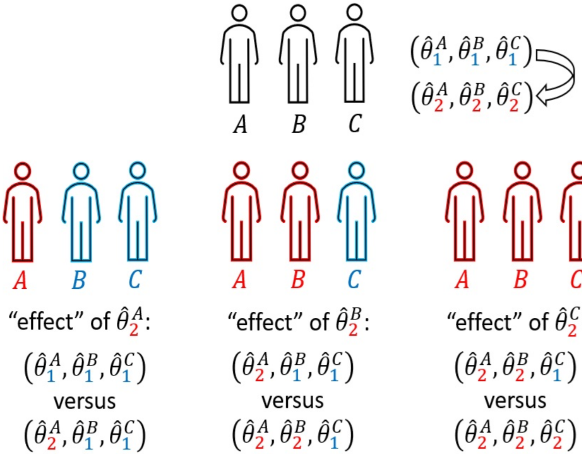
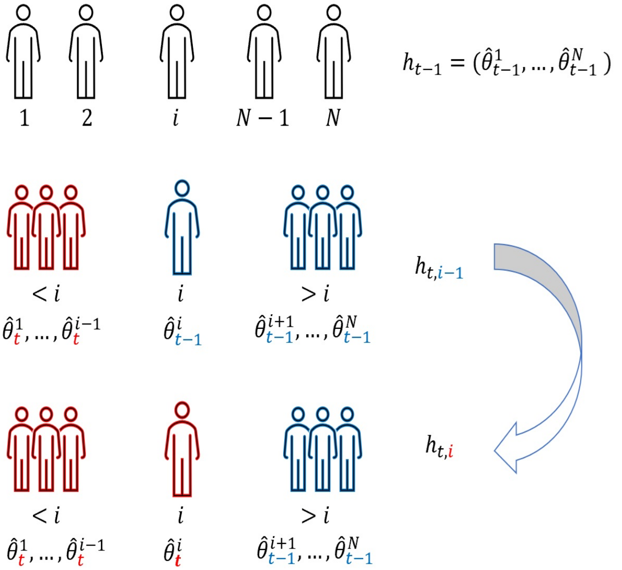


If $\hat\theta_2^B \uparrow U_A$ by 20 and $\downarrow U_C$ by 5,
transfers are $(-20, 15, 5)$:
A pays 20 to **B**
B pays 5 to **C**.

GUM



1
2
i
$N-1$
N
$h_{t-1} = (\hat{\theta}^1_{t-1}, \ldots, \hat{\theta}^N_{t-1})$
$< i$
i
$> i$
$\hat{\theta}^1_t, \ldots, \hat{\theta}^{i-1}_t$
$\hat{\theta}^i_{t-1}$
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$h_{t,i}$

GUM: Transfers

anticipated payoff of agent j is:

$$\Upsilon^{j}_{t,i} = \mathbb{E}^{\mu(h_{t,i},\chi^*)}\Big[\sum_{t'=0}^{T} u^j\big(\tilde{\theta}^j_{t'}, x(\tilde{\theta}_{t'})\big)\Big].$$

recall $h_{t,i} = \big(\, \hat{\theta}_0, \hat{\theta}_1, ..., \hat{\theta}_{t-1}, (\hat{\theta}^0_t, \hat{\theta}^1_t, ..., \hat{\theta}^i_t)\,\big)$

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budget-balanced as every payment is from one agent to another.

Why "**Guaranteed Utility** Mechanism"?

Mechanism satisfies the *Guaranteed Utility Property* (GUP) if there exists a strategy profile $s_* \in \mathcal{S}^{\mathcal{I}}$ and a vector $C \in \mathbb{R}^N$ such that:

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$$\sup_{s \in \mathcal{S}^{\mathcal{I}}} \sum_{i \in \mathcal{I}} \mathbb{E}\big[U^i(s)\big] = \sum_{i \in \mathcal{I}} C^i.$$

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GUM satisfies GUP: each agent **guarantees**

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Proposition

All BNEs (PBEs) in GUM are efficient and utility-equivalent.

Collusion-Proofness

For $L \subseteq \mathcal{I}$, a side contract is $\bar{s}^L = \{\bar{s}_t^L\}_{t=1}^T$, where

$$\bar{s}_t^L : \left(\boldsymbol{\Theta}^{\boldsymbol{L}}\right)^t \times \mathcal{H}_t \to \boldsymbol{\Theta}^{\boldsymbol{L}} \times \mathbb{R}^{|L|}.$$

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$\bar{s}^{*\mathcal{I}} \in \bar{\mathcal{S}}^{\mathcal{I}}$ is a **weak equilibrium** if:

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Ex: Bayesian Nash Equilibrium is a weak equilibrium.

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Ex: Bayesian Nash Equilibrium is a weak equilibrium.

A mechanism is **collusion-proof** if
all weak equilibria are utility-equivalent.

The Main Result

Theorem

Guaranteed Utility Mechanism is collusion-proof.

Further Properties and Extensions

- Participation constraint: allowing exiting and re-entering
- Results hold verbatim if agents can observe true past types
- Can extend to allow agents to take private actions
- Easy to achieve symmetry by averaging over orderings
- (in progress, separate paper)
  agents' initial types are **private**: tendering model extension

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Our transfer-free GUM **achieves efficiency with an error that does not depend on the type space size** unlike in JS07.

Related Literature: A Closer Look

	BV10 DP	AS13 BTM	GUM
Incentive Compatibility			
efficient PBE	YES	YES	YES
efficient PBE survive IEWDS	NO	NO	YES
all PBEs/BNEs are efficient	NO	NO	YES
Collusion			
collusion-proof	NO	NO	YES
Properties, robustness, extensions			
balanced budget	NO	YES	YES
exiting and re-entering	YES	NO	YES
observing past types	YES	NO	YES
observing same-period types	YES	NO	NO
private actions	NO	YES	YES

Additive Externality (if time permits...)

(recall) **externality** i's report $\hat{\theta}_t^i$ imposes on j is:

$$\gamma_t^{i \to j} = \Upsilon_{t,i}^j - \Upsilon_{t,i-1}^j.$$

total externality *simultaneous* updating of all the agent's reports at period t imposes on the agent's j payoff is

$$\gamma_t^{\mathcal{I} \to j} = \Upsilon_{t,N}^j - \Upsilon_{t-1,N}^j.$$

externality is **additive** if the sum of the externalities across all the agents is equal to the total externality:

$$\sum_{i \in \mathcal{I}} \gamma_t^{i \to j} = \gamma_t^{\mathcal{I} \to j} \quad \forall j, \ \forall t.$$